THE

CIVIL ENGINEER'S POCKET-BOOK.

THE AUTHOR

Dedicates this Book

TO HIS FRIEND

BENJAMIN H. LATROBE, Esq.,

CIVIL ENGINEER.

THE

CIVIL ENGINEER'S POCKET-BOOK,

OF

Mensuration, Trigonometry, Surveying, Hydraulics, Hydrostatics, Instruments and their Adjustments, Strength of Materials, Masonry, Principles of Wooden and Iron Roof and Bridge Trusses, Stone Bridges and Culverts, Trestles, Pillars, Suspension Bridges, Dams, Railroads, Turnouts, Turning-Platforms, Water Stations, Cost of Earthwork, Foundations, Retaining Walls.

Etc., Etc., Etc.

IN ADDITION TO WHICH THE ELUCIDATION OF CERTAIN IMPORTANT PRINCIPLES OF CONSTRUCTION IS MADE IN A MORE SIMPLE MANNER THAN HERETOFORE.

By JOHN C. TRAUTWINE,
CIVIL ENGINEER PHILADELPHIA.

ILLUSTRATED WITH 650 ENGRAVINGS FROM ORIGINAL DESIGNS.

Eighth Thousand.

REVISED AND CORRECTED.

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PREFACE.

SHOULD experts in engineering complain that they do not find anything of interest in this volume, the writer would merely remind them that it was not his intention that they should. The book has been prepared for *young* members of the profession; and one of the leading objects has been to elucidate in plain English, a few important elementary principles which the savants have enveloped in such a haze of mystery as to render pursuit hopeless to any but a confirmed mathematician.

Comparatively few engineers are good mathematicians; and in the writer's opinion, it is fortunate that such is the case; for nature rarely combines high mathematical talent, with that practical tact, and observation of outward things, so essential to a successful engineer.

There have been, it is true, brilliant exceptions; but they are very rare. But few even of those who have been tolerable mathematicians when young, can, as they advance in years, and become engaged in business, spare the time necessary for retaining such accomplishments.

Nearly all the scientific principles which constitute the foundation of civil engineering are susceptible of complete and satisfactory explanation to any person who *really* possesses only so much elementary knowledge of arithmetic and natural philosophy as is *supposed* to be taught to boys of twelve or fourteen in our public schools.*

* Let two little boys weigh each other on a platform scale. Then when they balance each other on their board see-saw, let them see (and measure for themselves) that the lighter one is farther from the fence-rail on which their board is placed, in the same proportion as the heavier boy outweighs the lighter one. They will then have learned the grand principle of the *lever*. Then let them measure and see that the light one *see-saws farther* than the heavy one, in the same proportion; and they *will have acquired the principle of virtual velocities.* Explain to them that equal *of moments means nothing* more than that when they seat themselves at their

The little that is beyond this, might safely be intrusted to the savans. Let them work out the results, and give them to the engineer *in intelligible language.* We could afford to take their words for it, because such things are their specialty; and because we know that they are the best qualified to investigate them. On the same principle we intrust our lives to our physician, or to the captain of the vessel at sea. Medicine and seamanship are their respective specialties.

If there is any point in which the writer may hope to meet the approbation of proficients, it is in the accuracy of the tables. The pains taken in this respect have been very great. Most of the tables have been entirely recalculated expressly for this book; and one of the results has been the detection of a great many errors in those in common use. He trusts that none will be found exceeding one, or sometimes two, in the last figure of any table in which great accuracy is required. There are many errors to that amount, especially where

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ured distances on their see-saw, *they balance each other.* Let them see that the weight of the heavy boy, when multiplied by his distance in feet from the fence-rail amounts to just as much as the weight of the light one when multiplied by his distance. Explain to them that each of the amounts is in *foot-pounds.* Tell them that the lightest one, because he see-saws so much faster than the other, will bump against the ground just as hard as the heavy one; and that this means that *their momentums are equal.* The boys may then go in to dinner, and probably puzzle their big lout of a brother who has just passed through college with high honors. They will not forget what they have learned, for they learned it *as play,* without any ear-pulling, spanking, or keeping in. Let their bats and balls, their marbles, their swings, &c, once become their philosophical apparatus, and children may be taught (*really* taught) many of the most important principles of engineering before they can read or write.

It is the ignorance of these principles, so easily taught even to children, that constitutes what is popularly called "THE PRACTICAL ENGINEER;" which, in the great majority of cases, means simply an ignoramus, who blunders along without knowing any other reason for what he does, than that he has seen it done so before. And it is this same ignorance that causes employers to prefer this practical man to one who is conversant with principles. They, themselves, were spanked, kept in, &c, when boys, because they could not master leverage, equality of moments, and virtual velo-ities, enveloped in x's, p's, Greek letters, square-roots, cube-roots, &c, and they urally set down any man as a fool who could. They turn up their noses at science, reaming that the word means simply, *knowing why.* And it must be confessed ey are not altogether without reason; for the savans appear to prepare their ith the express object of preventing purchasers, (they have but few *readers,*) ning why.

the recalculation was very tedious, and where, consequently, interpolation was resorted to. They are too small to be of practical importance. He knows, however, the almost impossibility of avoiding larger errors *entirely;* and will be glad to be informed of any that may be detected, except the final ones alluded to, that they may be corrected in case another edition should be called for. Tables which are absolutely reliable, possess an intrinsic value that is not to be measured by money alone. With this consideration the volume has been made a trifle larger than would otherwise have been necessary, in order to admit the stereotyped sines and tangents from his book on railroad curves. These have been so thoroughly compared with standards prepared independently of each other, that the writer believes them to be absolutely correct.

In order to reduce the volume to pocket-size, smaller type has been used than would otherwise have been desirable.

Many abbreviations of common words in frequent use have been introduced, such as abut, cen, diag, hor, vert, pres, &c, instead of abutment, center, diagonal, horizontal, vertical, pressure, &c. They can in no case lead to doubt; while they appreciably reduce the thickness of the volume.

Where prices have been added, they are placed in footnotes. They are intended merely to give an approximate or comparative idea of value; for constant fluctuations prevent anything farther.

The addresses of a few manufacturing establishments have also been inserted in notes, in the belief that they might at times be found convenient. They have been given without the knowledge of the proprietors.

The writer is frequently asked to name good elementary books on civil engineering; but regrets to say that there are very few such in our language. "Civil Engineering," by Prof. Mahan of West Point; "Roads and Railroads," by the late Prof. Gillespie; and the "Manual for Railroad Engineers," by George L. Vose, Civ. Eng, and Professor of Civil Engineering in Bowdoin College, Brunswick, Maine, are th best. The last, published by Lee & Shepard, Boston, 1873, is most complete work of its class with which the writer is acqua

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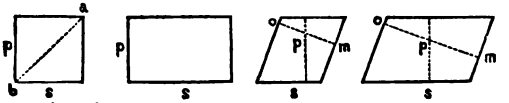
MENSURATION.

PARALLELOGRAMS.

Square. *Rectangle.* *Rhombus.* *Rhomboid.*



A PARALLELOGRAM is any figure of four straight sides, the opposite ones of which are parallel. Thes are but four, as in the above figs. In the square and rhombus all the four sides are equal; in th rectangle and rhomboid only the opposite ones are equal. In any parallelogram the four angl amount to four right angles, or 360°; and any two diagonally opposite angles are equal to each othe hence, having one angle given, the other three can readily be found. In a square, or a rhombus, diag divides each of two angles into two equal parts; but in the two other parallelograms it does not

To find the area of any parallelogram.

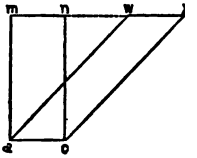
Multiply any side, as S, by the perp height, or dist *p* to the opposite side. If the side *m* be take then the dotted perp dist *m o* becomes the height.

The diag a b *of any square* is equal to one side mult by 1.41421; and a side is equal to $\frac{\text{diagonal}}{1.41421}$

The side of a square equal in area to a given circle, is equal to diam × .88623.

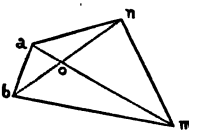
The side of the greatest square that can be inscribed in a given circle, is equal to diam × .707107.

The side of a square mult by 1.520 gives the side of an equilateral triangle of the same area. All parallelograms which have equal bases *a c*, and equal perp heights *n c*, have also equal areas; thus the area of *a c m n* is equal to that of *a c w x*; and the area of each is twice that of a triangle having the same base, and perp height. The area of a square inscribed in a circle is equal to twice the square of the rad.



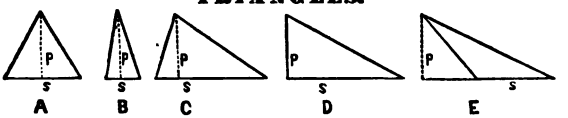
In every parallelogram, the 4 squares drawn on its sides have a united area equal to that of the two squares drawn on its 2 diags. If a larger square be drawn on the diag *a b* of a smaller square, its area will be twice that of said smaller square. Either diag of any parallelogram divides it into two equal triangles; and the 2 diags divide it into 4 triangles of equal areas. The two diags of any parallelogram divide each other into two equal parts. Any line drawn through the centre of a diag divides the parallelogram into 2 equal parts.

REMARK 1. — *The area of any fig whatever that is enclosed by four straight lines*, may be found thus: Mult together the two diags *a m*, *n b*; and the nat sine of the least angle *a o b*, or *n o m*, formed by their intersection. Div the product by 2. This is useful in land surveying, when obstacles, as is often the case, make it difficult to measure the sides of the fig or field; while it may be easy to measure the diags; and after finding their point of intersection *o*, to measure the required angle.



REM. 2. — The sides of a parallelogram, triangle, and many other figs. may be found, *when only the* **area** *and* **angles** are given, thus: Assume some particular one of its sides to be of the length 1; and calculate what its area would be if that were the case. Then as the sq rt of the area thus found is to this side 1, so is the sq rt of the actual given area, to the correspondin actual side of the fig.

TRIANGLES.



We speak here of *plane* triangles only; that is, of those having straight sides. Since the area of any triangle is equal to half that of a parallelogram which has the same base and perp height, therefore,

To find the area of any triangle, having its base and perp height.

Mult its base S by its height, or perp dist *p* to the opposite angle; and div the prod by 2 Although this applies to all plane triangles, the following is convenient for an equilateral one, when its perp height is not known: Square one side; and mult the square by the decimal .4330. Any side may be assumed as the base of a triangle; but the perp height must always be measd from the side so assumed; to do which, the side must sometimes be prolonged, as in Fig E; but the prolongation is not to be considered as a part of the base.

The perp height of an equilateral triangle is equal to one side × .8660. Hence one of its sides is equal to the perp height div by .8660; or to perp height × 1.1547. Or, to find a side, mult the sq rt of its area by 1.520. The side of an equilateral triangle, mult by .658 = side of a square of the same area; or div by 1.3468 it gives the diam of a circle of same area.

Having two sides, and the included angle.

Mult together the two sides, and the nat sine of the included angle; div by 2.

EXAMPLE. — *Sides* 650 ft and 960 ft; included angle 69° 20'. By the table we find the n .9356; therefore, $\frac{650 \times 960 \times .9356}{2} = 297966.6$ square ft area.

B

Having the three sides.

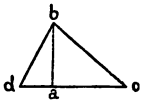
Add them together; div the sum by 2; from the half sum, subtract each side separately; mult the half sum and the three remainders continuously together; take the sq rt of the prod.

Ex.—The 3 sides = 20, 30, 40 ft. Here $20 + 30 + 40 = 90$; and $\frac{90}{2} = 45$. And $45 - 20 = 25$; $45 - 30 = 15$; and $45 - 40 = 5$. And $45 \times 25 \times 15 \times 5 = 84375$; and the sq rt of 84375 is 290.47 sq ft, area reqd.

Having one side and the two adjacent angles.

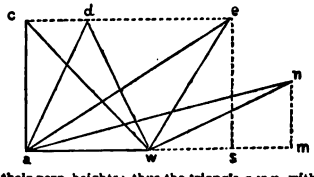
Add the 2 angles together; take the sum from 180°; the rem will be the angle opp the given side. Find the nat sine of this angle; also find the nat sines of the other angles, and mult them together. Then as the nat sine of the single angle, is to the prod of the nat sines of the other 2 angles, so is the *square* of the given side to *double* the reqd area.

Having the three angles, and the perp height, *a b*.



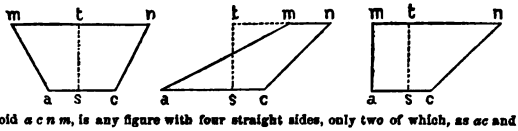
Find the nat sines of the three angles; mult together the sines of the angles *d* and *o*; div the sine of the angle *b* by the prod; mult the quot by the square of the perp height *a b*; div by 2. See Remark 7. after Parallelograms.

A triangle is *equilateral* when all its sides are equal, as A; *isosceles* when only two sides are equal, as B; *scalene* when all the sides are unequal, as C, D, and E; *acute-angled* when all its angles are acute, or each less than 90°, as A, B, and C; *right-angled* when it contains a right angle, as D; *obtuse-angled* when it contains an obtuse angle, or one greater than 90°, as E.



All the three angles of any triangle are equal to two right angles, or 180°; therefore, if we know two of them, we can find the third by subtracting their sum from 180°. All triangles which have equal bases, and equal perp heights, have also equal areas; thus the areas of *a w c*, *a w d*, and *a w e*, are equal to each other. The area of any triangle is equal to half that of any parallelogram which has an equal base, and an equal perp height. The areas of triangles which have equal bases, but diff perp heights, are to each other as, or in proportion to, their perp heights; thus the triangle *a w n*, with a perp height *m n*, equal to but one-half that (*s e*) of the three other triangles, has also but half the area of either of those others.

TRAPEZOIDS.

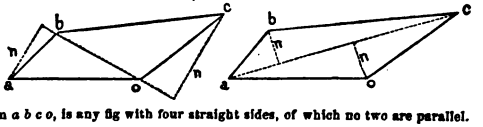


A trapezoid *a c n m*, is any figure with four straight sides, only two of which, as *ac* and *n m*, are parallel.

To find the area of any trapezoid.

Add together the two parallel sides, *a c* and *m n*; mult the sum by the perp dist *s t* between them; div the prod by 2. See the following rules for trapeziums, which are all equally applicable to trapezoids; also see Remarks after Parallelograms.

TRAPEZIUMS.



A trapezium *a b c o*, is any fig with four straight sides, of which no two are parallel.

To find the area of any trapezium, having given the diag *b o*, or *a c*, between either pair of opposite angles; and also the two perps, *n*, *n*, from the other two angles.

Add together these two perps; mult the sum by the diag; div the prod by 2.

Having the four sides; and either pair of opposite angles, as *a b c*, *a o c*; or *b a o*, and *b c o*.

Consider the trapezium as divided into two triangles, in each of which are given two sides and the *included angle*. Find the area of each of these triangles as directed under the preceding head "Triangles," and add them together.

Having the four angles, and either pair of opposite sides.

Begin with one of the sides, and the two angles at its ends. If the sum of these two angles exceeds bisect each of them from 180°, and make use of the rems instead of the angles themselves. Consider this side and its two adjacent angles (or the two rems, as the case may be) as those of a triangle; and find its area as directed for that case under the preceding head "Triangle." Do

the same with the other given side, and its two adjacent angles, (or their rems, as the case may be.) Subtract the least of the areas thus found, from the greatest; the rem will be the reqd area.

Having three sides; and the two included angles.

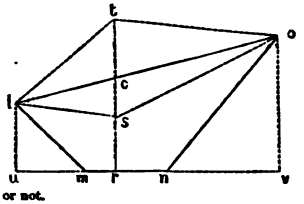
Mult together the middle side, and one of the adjacent sides; mult the prod by the nat sine of their included angle; call the result *a*.* Do the same with the middle side and its other adjacent side, and the nat sine of the other included angle; call the result *b*. Add the two angles together; find the diff between their sum and 180°, whether greater or less; find the nat sine of this diff; mult together the two given sides which are *opposite* one another; mult the prod by the nat sine just found; call the result *c*. Add together the results *a* and *b*; then, if the sum of the two given angles is *less* than 180°, subtract *c* from the sum of *a* and *b*; *half* the rem will be the area of the trapezium. But if the sum of the two given angles be *greater* than 180°, add together the three results *a*, *b*, and *c*; *half* their sum will be the area.

Having the two diagonals, and either angle formed by their intersection.

See Remarks after Parallelograms.

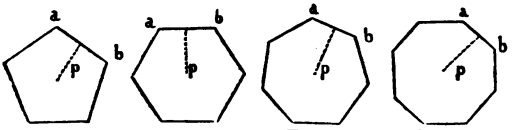
In railroad measurements

Of excavation and embankment, the trapezium *l m n o* frequently occurs; as well as the two 5-sided figures *l m n o t* and *l m n o s*; in all of which *m n* represents the roadway; *r s*, *r c*, and *r t* the center-depths or heights; *l u* and *o v* the side-depths or heights, as given by the level; *l m* and *n o* the side-slopes.



The same general rule for area applies to all three of these figs; namely, mult the extreme hor width *u v* by *half* the center depth *r s*, *r c*, or *r t*, as the case may be. Also mult one *fourth* of the width of roadway *m n*, by the *sum* of the two side-depths *l u* and *o v*. Add the two prods together; the sum is the reqd area. This rule applies whether the two side-slopes *m l* and *n o* have the same angle of inclination or not.

POLYGONS.



Pentagon. 5 sides. *Hexagon.* 6 sides. *Heptagon.* 7 sides. *Octagon.* 8 sides.

All straight-sided figs of more than 4 sides, are called polygons; if all the sides are equal, it is a *regular* polygon; if not, *irregular*. Of course their number is infinite; but those in most common use are the four shown above.

TABLE OF POLYGONS.

Number of Sides.	Name of Polygon.	Areas.	Radii.	Sides.	Angle contained between two sides.	Angle at center of circle.
3	Equilateral triangle.	.4330	.5774	1.7320	60°	120°
4	Square.	1.	.7071	1.4142	90°	90°
5	Pentagon.	1.7205	.8507	1.1756	108°	72°
6	Hexagon.	2.5981	1.	1.	120°	60°
7	Heptagon.	3.6339	1.1524	.8678	128° 34′.29	51° 25′.71
8	Octagon.	4.8284	1.3066	.7654	135°	45°
9	Nonagon.	6.1818	1.4619	.6840	140°	40°
10	Decagon.	7.6942	1.6180	.6180	144°	36°
11	Undecagon.	9.3656	1.7747	.5635	147° 16′.36	32° 43′.64
12	Dodecagon.	11.1962	1.9319	.5176	150°	30°

To find the area of any regular polygon.

Mult together one of its sides, *a b*; the perp *p* drawn from the center of the fig to the center of its side; and the number of its sides. Div the prod by 2. Or, square one of its sides; and mult said square by the number in the foregoing table, in the column of areas.

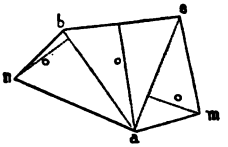
Having a side of a regular polygon, to find the rad of a circumscribing circle.

Mult the side by the corresponding number in the column of radii.

Having the rad of a circumscribing circle, to find the sid of the inscribed regular polygon.

Mult the rad by the corresponding number in the column of sides.

* To find the sine of an angle greater than 90°, subtract the angle from 180°, and find the nat for the sine of an angle in equal to the sine of its supplement.

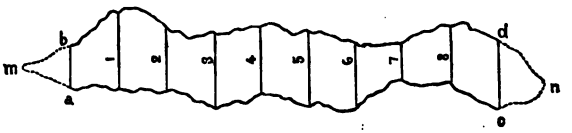


To find the area of any irregular polygon, *a n b c m.*

Div it into triangles, as *a n b*, *a m c*, and *a b c*; in each of which find the perp dist *o*, between its base *a b*, *a c*, or *b c*; and the opposite angle *n*, *m*, or *a*; mult each base by its perp dist; add all the prods together; div by 2.

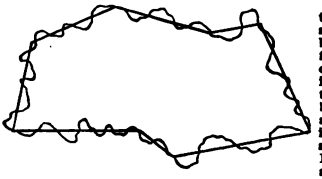
To find the area of a long irregular fig, as *a b c d.*

Between its two ends *a b*, and *c d*,



space off *equal* dists, (the shorter they are the more accurate will be the result,) through which draw the intermediate parallel lines 1, 2, 3, &c, across the breadth of the fig. Measure the lengths of these intermediate lines, and add them together; to the sum add one *half* of the two end breadths *a b* and *c d*. Mult the entire sum by one of the equal spaces between the parallel lines. The prod will be the area. This rule answers as well if either one or both the ends terminate in points, as at *m* and *n*. In the last of these cases, both *a b* and *c d* will be included in the intermediate lines; and half the two end breadths will be 0, or nothing.

To find the area of a fig whose outline is extremely irregular.



Draw lines around it which shall enclose within them (as nearly as can be judged by eye) as much space not belonging to the fig, as they exclude space belonging to it. The area of the simplified fig thus formed, being in this manner rendered equal to that of the complicated one, may be calculated by dividing it into triangles, &c. By using a piece of fine thread, the proper position for the new boundary lines may be found, before drawing them in. Small irregular areas may be found from a drawing, by laying upon it a piece of transparent paper carefully ruled into small squares, each of a given area, say 10, 20, or 100 sq ft each; and by first counting the whole squares, and then adding the fractions of squares.

CIRCLES.

A circle is the area included within a curved line of such a character that every point in it is equally distant from a certain point within it, called its center. The curved line itself is called the circumference, or periphery of the circle; or very commonly it is called the circle.

To find the circumference.

Mult the diam by 3.1416; more correctly, by 3.141593. Or as 113 is to 355, so is diam to circumf, as approximately as the last; or mult the diam by $3\frac{1}{7}$; which gives the circumf, only about 1 part in 2500 too great; or mult the area by 12.566, and take the sq rt of the quot; or use the following table of circles.

To find the diam.

Div the circumf by 3.1416, or, as 355 is to 113, so is circumf to diam; or, mult the circumf by 7; and div the prod by 22, which gives the diam too small by only about one part in 2500; or mult the area by 1.2732; and take the sq rt of the prod; or use the following table of circles.

The diam is to the circumf more exactly as 1 to 3.14159265.

To find the diam of a circle equal in area to a given square.

Mult one side of the square by 1.12838.

To find the rad of a circle to circumscribe a given square.

Mult one side by .7071; or take ½ the diag.

To find the side of a square equal in area to a given circle.

Mult the diam by .88623.

To find the side of the greatest square that can be inscribed in a given circle.

Mult diam by .7071. The area of the greatest square that can be inscribed in a circle is equal to twice the square of the rad. The diam × by 1.3468 gives the side of an equilateral triangle of equal area.

Having the chord or span: and the rise or height (improperly *called the versed sine*) of a circular arc, to find the rad.

Square one half the chord; also square the whole rise; add these squares together; div the sum by twice the rise. This gives a ready method of ascertaining the rad of any railroad curve already constructed.

Having the chord, and rad, to find the rise.

Square the rad; also square half the chord: take the last square from the first; take sq rt of the rem and subtract it from the rad. Useful in bending rails, &c.

Having the rad, and rise, to find the chord.

From rad subtract rise; square the rem; also square the rad; from this last square take the first; take sq rt of rem; and mult it by 2.

Having rise of arc, and diam of circle, to find the chord.

From the diam take the rise; mult the rem by the rise; take sq rt of prod and mult it by 2.

The rad of a circle is equal to half the diam; or to circumf mult by .159155. It is also equal to one side of an inscribed hexagon; consequently, as a chord of the circle, it subtends 60°.

Half the chord of an arc, if div by the rad of the circle, will give the nat sine of *half* the angle subtended by the whole chord, or arc.

Table of Minutes converted into decimals of a Degree.

A second is the same decimal of a min, as a min is of a deg. To reduce seconds to a dec of a deg, mult them by .0002778. Fifteen sec, or ¼ min = .004167 deg; and ½ min = .008333 deg; and ¾ min = .012500 deg. 360° = 21600 minutes = 1296000 seconds.

Min.	Deg.	Min.	Deg.	Min.	Deg.	Min.	Deg.
1	.016666	16	.266666	31	.516666	46	.766666
2	.033333	17	.283333	32	.533333	47	.783333
3	.050000	18	.300000	33	.550000	48	.800000
4	.066666	19	.316666	34	.566666	49	.816666
5	.083333	20	.333333	35	.583333	50	.833333
6	.100000	21	.350000	36	.600000	51	.850000
7	.116666	22	.366666	37	.616666	52	.866666
8	.133333	23	.383333	38	.633333	53	.883333
9	.150000	24	.400000	39	.650000	54	.900000
10	.166666	25	.416666	40	.666666	55	.916666
11	.183333	26	.433333	41	.683333	56	.933333
12	.200000	27	.450000	42	.700000	57	.950000
13	.216666	28	.466666	43	.716666	58	.966666
14	.233333	29	.483333	44	.733333	59	.983333
15	.250000	30	.500000	45	.750000	60	1.000000

To find the area of a circle.

Square the diam; mult this square by .7854; or more accurately by .78539816; or square the circumf; mult this square by .07958; or more accurately by .07957747; or mult half the diam by half the circumf; or refer to the following table of areas of circles.

The area of a circle is to the area of any circumscribed straight-sided fig, as the circumf of the circle is to the circumf or periphery of the fig. The area of a square inscribed in a circle, is equal to twice the square of the rad. Of a circle in a square, = square × .7854.

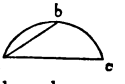
It is convenient to remember, in rounding off a square corner *a b c*, by a quarter of a circle, that the shaded area *a b c* is equal to about $\frac{1}{5}$ part (correctly .2146) of the whole square *a b c d*.

To find the breadth of a circular ring.

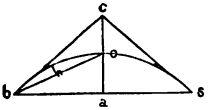
Having its area, and the diam of its outer circle. Find the area of the whole circle. From it take the area of the ring. Mult the rem by 1.2732. Take the sq rt of the prod. This sq rt will be the diam of the inner circle. Take it from the diam of the outer one; and div the rem by 2, for the reqd breadth.

To find the length of a circular arc.

Find the chord *a b* of half the arc;* and mult it by 8. From the prod take the chord *a c* of the whole arc; div the rem by 3. This is a close approximation for *flat* arcs. It always gives them a little too short. When greater accuracy is required, add as follows, for semicircles, $\frac{1}{64}$th part; rise $\frac{1}{10}$, $\frac{1}{152}$; rise ⅓ $\frac{1}{285}$; rise $\frac{3}{10}$, $\frac{1}{364}$; rise ¼, $\frac{1}{670}$; rise $\frac{1}{5}$, $\frac{1}{1430}$; rise ⅙, $\frac{1}{3300}$; rise $\frac{1}{10}$, $\frac{1}{20000}$. Or refer to the two tables of arcs on pages 21 and 23.



Remark.—It may frequently be of use to remember, that in any arc *b o s*, not exceeding 29°, or in other words, *whose chord b s is at least sixteen times its rise*, the middle ordinate *a o*, will be one half of *a c*, quite near enough for many purposes; *b c* and *s c* being tangents to the arc.† And vice versa, if in such an arc we make *o c* equal *a o*, then will c be, very nearly, the point at which tangents from the ends of the arc will meet. Also the middle ordinate *n*, of the half arc *o b*, or *o s*, will be approximately ¼ of *a o*, the mid ord of the whole arc. Indeed, this last observation will apply near enough for many approximate uses even if the arc be as great as 45°; for if in that case we take ¼ of *o a* for the ord *n*, it will then be but 1 part in 103 too small; and therefore the principle may often be used in drawings for finding points in a curve of too great rad to be drawn by the dividers; for in the same manner, ¼ of *n* will be the mid ord for the arc *n b* or *n o*; and so on to any extent.



In an arc of 90°, The chord = rad × 1.4142; or = rise × 4.8283.

The rise = rad × .2929; or = $\frac{\text{chord}}{4.8283}$.

Length of arc = rad × 1.5708; or = chord × 1.11072.

* *To find the chord of half the arc*, add together the square of the rise; and the square of span; and take the sq rt of the sum.

† *At 29°, o c thus found will be but about* $\frac{1}{33}$ *part too short.*

TABLE OF CIRCLES.

Circumferences or areas intermediate of those in the table, may be found by simple arithmetical proportion. The diameters, &c, are in inches; but it is plain that if the diams are taken as feet, yards, &c, the other parts will also be in those same measures. No errors.

Diam. Ins.	Circumf. Ins.	Area. Sq. Ins.	Diam. Ins.	Circumf. Ins.	Area. Sq. Ins.	Diam. Ins.	Circumf. Ins.	Area. Sq. Ins.	Diam. Ins.	Circumf. Ins.	Area. Sq Ins.
1-64	.049087	.00019	3. 1/2	10.9956	9.6211	10 1/2	31.8086	80.516	19 1/4	60.4757	291.04
1-32	.098175	.00077	9-16	11.1949	9.9678	1/2	32.2013	82.516	3/8	60.8684	294.83
3-64	.147262	.00173	5/8	11.3883	10.321	5/8	32.5940	84.541	1/2	61.2611	298.65
1-16	.196350	.00307	11-16	11.5846	10.680	3/4	32.9867	86.590	5/8	61.6538	302.49
3-32	.294524	.00690	3/4	11.7810	11.045	7/8	33.3794	88.664	3/4	62.0465	306.35
1/8	.392699	.01227	13-16	11.9773	11.416	3/4	33.7721	90.763	7/8	62.4392	310.24
5-32	.490874	.01917	7/8	12.1737	11.793	7/8	34.1648	92.886	20.	62.8319	314.16
3-16	.589049	.02761	15-16	12.3700	12.177	11.	34.5575	95.033	1/8	63.2246	318.10
7-32	.687223	.03758	4.	12.5664	12.566	1/8	34.9502	97.205	1/4	63.6173	322.06
1/4	.785398	.04909	1-16	12.7627	12.962	1/4	35.3429	99.402	3/8	64.0100	326.05
9-32	.883573	.06213	1/8	12.9591	13.364	3/8	35.7356	101.62	1/2	64.4026	330.06
5-16	.981748	.07670	3-16	13.1554	13.772	1/2	36.1283	103.87	5/8	64.7953	334.10
11-32	1.07992	.09281	1/4	13.3518	14.186	5/8	36.5210	106.14	3/4	65.1880	338.16
3/8	1.17810	.11045	5-16	13.5481	14.607	3/4	36.9137	108.43	7/8	65.5807	342.25
13-32	1.27627	.12962	3/8	13.7445	15.033	7/8	37.3064	110.75	21.	65.9734	346.36
7-16	1.37445	.15033	7-16	13.9408	15.466	12.	37.6991	113.10	1/8	66.3661	350.50
15-32	1.47262	.17257	1/2	14.1372	15.904	1/8	38.0918	115.47	1/4	66.7588	354.66
1/2	1.57080	.19635	9-16	14.3335	16.349	1/4	38.4845	117.86	3/8	67.1515	358.84
17-32	1.66897	.22166	5/8	14.5299	16.800	3/8	38.8772	120.28	1/2	67.5442	363.05
9-16	1.76715	.24850	11-16	14.7262	17.257	1/2	39.2699	122.72	5/8	67.9369	367.28
19-32	1.86532	.27688	3/4	14.9226	17.721	5/8	39.6626	125.19	3/4	68.3296	371.54
5/8	1.96350	.30680	13-16	15.1189	18.190	3/4	40.0553	127.68	7/8	68.7223	375.83
21-32	2.06167	.33824	7/8	15.3153	18.665	7/8	40.4480	130.19	22.	69.1150	380.13
11-16	2.15984	.37122	15-16	15.5116	19.147	13.	40.8407	132.73	1/8	69.5077	384.46
23-32	2.25802	.40574	5.	15.7080	19.635	1/8	41.2334	135.30	1/4	69.9004	388.82
3/4	2.35619	.44179	1-16	15.9043	20.129	1/4	41.6261	137.89	3/8	70.2931	393.20
25-32	2.45437	.47937	1/8	16.1007	20.629	3/8	42.0188	140.50	1/2	70.6858	397.61
13-16	2.55254	.51849	3-16	16.2970	21.135	1/2	42.4115	143.14	5/8	71.0785	402.04
27-32	2.65072	.55914	1/4	16.4934	21.648	5/8	42.8042	145.80	3/4	71.4712	406.49
7/8	2.74889	.60132	5-16	16.6897	22.166	3/4	43.1969	148.49	7/8	71.8639	410.97
29-32	2.84707	.64504	3/8	16.8861	22.691	7/8	43.5896	151.20	23.	72.2566	415.48
15-16	2.94524	.69029	7-16	17.0824	23.221	14.	43.9823	153.94	1/8	72.6493	420.00
31-32	3.04342	.73708	1/2	17.2788	23.758	1/8	44.3750	156.70	1/4	73.0420	424.56
1.	3.14159	.78540	9-16	17.4751	24.301	1/4	44.7677	159.48	3/8	73.4347	429.13
1-16	3.33794	.88664	5/8	17.6715	24.850	3/8	45.1604	162.30	1/2	73.8274	433.74
1/8	3.53429	.99402	11-16	17.8678	25.406	1/2	45.5531	165.13	5/8	74.2201	438.36
3-16	3.73064	1.1075	3/4	18.0642	25.967	5/8	45.9458	167.99	3/4	74.6128	443.01
1/4	3.92699	1.2272	13-16	18.2605	26.535	3/4	46.3385	170.87	7/8	75.0055	447.69
5-16	4.12334	1.3530	7/8	18.4569	27.109	7/8	46.7312	173.78	24.	75.3982	452.39
3/8	4 31969	1.4849	15-16	18.6532	27.688	15.	47.1239	176.71	1/8	75.7909	457.11
7-16	4.51604	1.6230	6.	18.8496	28.274	1/8	47.5166	179.67	1/4	76.1836	461.86
1/2	4.71239	1.7671	1/8	19.2423	29.465	1/4	47.9093	182.65	3/8	76.5763	466.64
9-16	4.90874	1.9175	1/4	19.6350	30.680	3/8	48.3020	185.66	1/2	76.9690	471.44
5/8	5.10509	2.0739	3/8	20.0277	31.919	1/2	48.6947	188.69	5/8	77.3617	476.26
11-16	5.30144	2.2365	1/2	20.4204	33.183	5/8	49.0874	191.75	3/4	77.7544	481.11
3/4	5.49779	2.4053	5/8	20.8131	34.472	3/4	49.4801	194.83	7/8	78.1471	485.98
13-16	5 69414	2.5802	3/4	21.2058	35.785	7/8	49.8728	197.93	25.	78.5398	490.87
7/8	5.89049	2.7612	7/8	21.5984	37.122	16.	50.2655	201.06	1/8	78.9325	495.79
15-16	6.08684	2.9483	7.	21.9911	38.485	1/8	50.6582	204.22	1/4	79.3252	500.74
2.	6.28319	3.1416	1/8	22.3838	39.871	1/4	51.0509	207.39	3/8	79.7179	505.71
1-16	6.47953	3.3410	1/4	22.7765	41.282	3/8	51.4436	210.60	1/2	80.1106	510.71
1/8	6.67588	3.5466	3/8	23.1692	42.718	1/2	51.8363	213.82	5/8	80.5033	515.72
3-16	6 87223	3.7583	1/2	23.5619	44.179	5/8	52.2290	217 08	3/4	80.8960	520.77
1/4	7.06858	3.9761	5/8	23.9546	45 664	3/4	52.6217	220.35	7/8	81.2887	525.84
5-16	7.26493	4.2000	3/4	24.3473	47.173	7/8	53.0144	223.65	26.	81.6814	530.93
3/8	7.46128	4.4301	7/8	24.7400	48.707	17.	53.4071	226.98	1/8	82.0741	536.05
7-16	7.65763	4.6664	8.	25.1327	50.265	1/8	53.7998	230.33	1/4	82.4668	541.19
1/2	7.85398	4.9087	1/8	25.5254	51.849	1/4	54.1925	233.71	3/8	82.8595	546.35
9-16	8.05033	5.1572	1/4	25 9181	53.456	3/8	54.5852	237.10	1/2	83.2522	551.55
5/8	8.24668	5.4119	3/8	26.3108	55.088	1/2	54.9779	240.53	5/8	83.6449	556.76
11-16	8.44303	5.6727	1/2	26.7035	56.745	5/8	55.3706	243.98	3/4	84.0376	562.00
3/4	8.63938	5.9396	5/8	27.0962	58.426	3/4	55.7633	247.45	7/8	84.4303	567.27
13-16	8.83573	6.2126	3/4	27.4889	60.132	7/8	56.1560	250.95	27.	84.8230	572.56
7/8	9.03208	6.4918	7/8	27.8816	61.862	18.	56.5487	254.47	1/8	85.2157	577.87
15-16	9.22843	6.7771	9.	28.2743	63.617	1/8	56.9414	258.02	1/4	85.6084	583.21
3.	9.42478	7.0686	1/8	28.6670	65.397	1/4	57.3341	261.59	3/8	86.0011	588.57
1-16	9.62113	7.3662	1/4	29.0597	67.201	3/8	57.7268	265.18	1/2	86.3938	593.96
1/8	9.81748	7.6699	3/8	29.4524	69.029	1/2	58.1195	268.80	5/8	86.7865	599.37
3-16	10.0138	7.9798	1/2	29.8451	70.882	5/8	58.5122	272.45	3/4	87.1792	604.81
1/4	10.2102	8.2958	5/8	30.2378	72.760	3/4	58.9049	276.12	7/8	87.5719	610.27
5-16	10.4065	8.6179	3/4	30.6305	74 662	7/8	59.2976	279.81	28.	87.9646	615.75
3/8	10.6029	8 9462	7/8	31.0232	76.589	19.	59.6903	283.53	1/8	88.3573	621.26
7-16	10.7992	9.2806	10.	31.4159	78.540	1/8	60.0830	287.27	1/4	88.7500	626.80

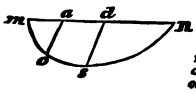
TABLE OF CIRCLES—(Continued.)

Diam. Ins.	Circumf. Ins.	Area. Sq. Ins.	Diam. Ins.	Circumf. Ins.	Area. Sq. Ins.	Diam. Ins.	Circumf. Ins.	Area. Sq. Ins.	Diam. Ins.	Circumf. Ins.	Area. Sq. Ins.
28⅝	89.1427	632.36	38.	119.381	1134.1	47⅝	149.618	1781.4	57¼	179.856	2574.3
½	89.5354	637.94	⅛	119.773	1141.6	¾	150.011	1790.8	⅜	180.249	2585.4
⅝	89.9281	643.55	¼	120.166	1149.1	⅞	150.404	1800.1	½	180.642	2596.7
¾	90.3208	649.18	⅜	120.559	1156.6	48.	150.796	1809.6	⅝	181.034	2608.0
⅞	90.7135	654.84	½	120.951	1164.2	⅛	151.189	1819.0	¾	181.427	2619.4
29.	91.1062	660.52	⅝	121.344	1171.7	¼	151.582	1828.5	⅞	181.820	2630.7
⅛	91.4969	666.23	¾	121.737	1179.3	⅜	151.975	1837.9	58.	182.212	2642.1
¼	91.8916	671.96	⅞	122.129	1186.9	½	152.367	1847.5	⅛	182.605	2653.5
⅜	92.2843	677.71	39.	122.522	1194.6	⅝	152.760	1857.0	¼	182.998	2664.9
½	92.6770	683.49	⅛	122.915	1202.3	¾	153.153	1866.5	⅜	183.390	2676.4
⅝	93.0697	689.30	¼	123.308	1210.0	⅞	153.545	1876.1	½	183.783	2687.8
¾	93.4624	695.13	⅜	123.700	1217.7	49.	153.938	1885.7	⅝	184.176	2699.3
⅞	93.8551	700.98	½	124.093	1225.4	⅛	154.331	1895.4	¾	184.569	2710.9
30.	94.2478	706.86	⅝	124.486	1233.2	¼	154.723	1905.0	⅞	184.961	2722.4
⅛	94.6405	712.76	¾	124.878	1241.0	⅜	155.116	1914.7	59.	185.354	2734.0
¼	95.0332	718.69	⅞	125.271	1248.8	½	155.509	1924.4	⅛	185.747	2745.6
⅜	95.4259	724.64	40.	125.664	1256.6	⅝	155.902	1934.2	¼	186.139	2757.2
½	95.8186	730.62	⅛	126.056	1264.5	¾	156.294	1943.9	⅜	186.532	2768.8
⅝	96.2113	736.62	¼	126.449	1272.4	⅞	156.687	1953.7	½	186.925	2780.5
¾	96.6040	742.64	⅜	126.842	1280.3	50.	157.080	1963.5	⅝	187.317	2792.2
⅞	96.9967	748.69	½	127.235	1288.2	⅛	157.472	1973.3	¾	187.710	2803.9
31.	97.3894	754.77	⅝	127.627	1296.2	¼	157.865	1983.2	⅞	188.103	2815.7
⅛	97.7821	760.87	¾	128.020	1304.2	⅜	158.258	1993.1	60.	188.496	2827.4
¼	98.1748	766.99	⅞	128.413	1312.2	½	158.650	2003.0	⅛	188.888	2839.2
⅜	98.5675	773.14	41.	128.805	1320.3	⅝	159.043	2012.9	¼	189.281	2851.0
½	98.9602	779.31	⅛	129.198	1328.3	¾	159.436	2022.8	⅜	189.674	2862.9
⅝	99.3529	785.51	¼	129.591	1336.4	⅞	159.829	2032.8	½	190.066	2874.8
¾	99.7456	791.73	⅜	129.983	1344.5	51.	160.221	2042.8	⅝	190.459	2886.6
⅞	100.138	797.98	½	130.376	1352.7	⅛	160.614	2052.8	¾	190.852	2898.6
32.	100.531	804.25	⅝	130.769	1360.8	¼	161.007	2062.9	⅞	191.244	2910.5
⅛	100.924	810.54	¾	131.161	1369.0	⅜	161.399	2073.0	61.	191.637	2922.5
¼	101.316	816.86	⅞	131.554	1377.2	½	161.792	2083.1	⅛	192.030	2934.5
⅜	101.709	823.21	42.	131.947	1385.4	⅝	162.185	2093.2	¼	192.423	2946.5
½	102.102	829.58	⅛	132.340	1393.7	¾	162.577	2103.3	⅜	192.815	2958.5
⅝	102.494	835.97	¼	132.732	1402.0	⅞	162.970	2113.5	½	193.208	2970.6
¾	102.887	842.39	⅜	133.125	1410.3	52.	163.363	2123.7	⅝	193.601	2982.7
⅞	103.280	848.83	½	133.518	1418.6	⅛	163.756	2133.9	¾	193.993	2994.8
33.	103.673	855.30	⅝	133.910	1427.0	¼	164.148	2144.2	⅞	194.386	3006.9
⅛	104.065	861.79	¾	134.303	1435.4	⅜	164.541	2154.5	62.	194.779	3019.1
¼	104.458	868.31	⅞	134.696	1443.8	½	164.934	2164.8	⅛	195.171	3031.3
⅜	104.851	874.85	43.	135.088	1452.2	⅝	165.326	2175.1	¼	195.564	3043.5
½	105.243	881.41	⅛	135.481	1460.7	¾	165.719	2185.4	⅜	195.957	3055.7
⅝	105.636	888.00	¼	135.874	1469.1	⅞	166.112	2195.8	½	196.350	3068.0
¾	106.029	894.62	⅜	136.267	1477.6	53.	166.504	2206.2	⅝	196.742	3080.3
⅞	106.421	901.26	½	136.659	1486.2	⅛	166.897	2216.6	¾	197.135	3092.6
34.	106.814	907.92	⅝	137.052	1494.7	¼	167.290	2227.0	⅞	197.528	3104.9
⅛	107.207	914.61	¾	137.445	1503.3	⅜	167.683	2237.5	63.	197.920	3117.2
¼	107.600	921.32	⅞	137.837	1511.9	½	168.075	2248.0	⅛	198.313	3129.6
⅜	107.992	928.06	44.	138.230	1520.5	⅝	168.468	2258.5	¼	198.706	3142.0
½	108.385	934.82	⅛	138.623	1529.2	¾	168.861	2269.1	⅜	199.098	3154.5
⅝	108.778	941.61	¼	139.015	1537.9	⅞	169.253	2279.6	½	199.491	3166.9
¾	109.170	948.42	⅜	139.408	1546.6	54.	169.646	2290.2	⅝	199.884	3179.4
⅞	109.563	955.25	½	139.801	1555.3	⅛	170.039	2300.8	¾	200.277	3191.9
35.	109.956	962.11	⅝	140.194	1564.0	¼	170.431	2311.5	⅞	200.669	3204.4
⅛	110.348	969.00	¾	140.586	1572.8	⅜	170.824	2322.1	64.	201.062	3217.0
¼	110.741	975.91	⅞	140.979	1581.6	½	171.217	2332.8	⅛	201.455	3229.6
⅜	111.134	982.84	45.	141.372	1590.4	⅝	171.609	2343.5	¼	201.847	3242.2
½	111.527	989.80	⅛	141.764	1599.3	¾	172.002	2354.3	⅜	202.240	3254.8
⅝	111.919	996.78	¼	142.157	1608.2	⅞	172.395	2365.0	½	202.633	3267.5
¾	112.312	1003.8	⅜	142.550	1617.0	55.	172.788	2375.8	⅝	203.025	3280.1
⅞	112.705	1010.8	½	142.942	1626.0	⅛	173.180	2386.6	¾	203.418	3292.8
36.	113.097	1017.9	⅝	143.335	1634.9	¼	173.573	2397.5	⅞	203.811	3305.6
⅛	113.490	1025.0	¾	143.728	1643.9	⅜	173.966	2408.3	65.	204.204	3318.3
¼	113.883	1032 1	⅞	144.121	1652.9	½	174.358	2419.2	⅛	204.596	3331.1
⅜	114.275	1039.2	46.	144.513	1661.9	⅝	174.751	2430.1	¼	204.989	3343.9
½	114.668	1046.3	⅛	144.906	1670.9	¾	175.144	2441.1	⅜	205.382	3356.7
⅝	115.061	1053.5	¼	145.299	1680.0	⅞	175.536	2452.0	½	205.774	3369.6
¾	115.454	1060.7	⅜	145.691	1689.1	56.	175.929	2463.0	⅝	206.167	3382.4
⅞	115.846	1068 0	½	146.084	1698.2	⅛	176.322	2474.0	¾	206 560	3395.3
37.	116.239	1075.2	⅝	146.477	1707.4	¼	176.715	2485.0	⅞	206.952	3408.2
⅛	116.632	1082.5	¾	146.869	1716.5	⅜	177.107	2496.1	66.	207.345	3421.2
¼	117.024	1089.8	⅞	147.262	1725.7	½	177.500	2507.2	⅛	207.738	3434.2
⅜	117.417	1097.1	47.	147.655	1734.9	⅝	177.893	2518.3	¼	208.131	3447.3
½	117.810	1104.5	⅛	148.048	1744.2	¾	178.285	2529.4	⅜	208.523	3460
⅝	118.202	1111.8	¼	148.440	1753.5	⅞	178.678	2540.6	½	208.916	347
¾	118.595	1119.2	⅜	148.833	1762.7	57.	179.071	2551.8	⅝	209.309	34
⅞	118.988	1126.7	½	149.226	1772.1	⅛	179.463	2563.0	¾	209.701	3

TABLE OF CIRCLES—(Continued.)

Diam. Ins.	Circumf. Ins.	Area. Sq. Ins.	Diam. Ins.	Circumf. Ins.	Area. Sq. Ins.	Diam. Ins.	Circumf. Ins.	Area. Sq. Ins.	Diam. Ins.	Circumf. Ins.
⅛	210.094	3512.5	75¼	236.405	4447.4	83⅝	262.716	5492.4	92.	289.027
67.	210.487	3525.7	⅜	236.798	4462.2	¾	263.108	5508.8	⅛	289.419
⅛	210.879	3538.8	½	237.190	4477.0	⅞	263.501	5525.3	¼	289.812
¼	211.272	3552.0	⅝	237.583	4491.8	84.	263.894	5541.8	⅜	290.205
⅜	211.665	3565.2	¾	237.976	4506.7	⅛	264.286	5558.3	½	290.597
½	212.058	3578.5	⅞	238.368	4521.5	¼	264.679	5574.8	⅝	290.990
⅝	212.450	3591.7	76.	238.761	4536.5	⅜	265.072	5591.4	¾	291.383
¾	212.843	3605.0	⅛	239.154	4551.4	½	265.465	5607.9	⅞	291.775
⅞	213.236	3618.3	¼	239.546	4566.4	⅝	265.857	5624.5	93.	292.168
68.	213.628	3631.7	⅜	239.939	4581.3	¾	266.250	5641.2	⅛	292.561
⅛	214.021	3645.0	½	240.332	4596.3	⅞	266.643	5657.8	¼	292.954
¼	214.414	3658.4	⅝	240.725	4611.4	85.	267.035	5674.5	⅜	293.346
⅜	214.806	3671.8	¾	241.117	4626.4	⅛	267.428	5691.2	½	293.739
½	215.199	3685.3	⅞	241.510	4641.5	¼	267.821	5707.9	⅝	294.132
⅝	215.592	3698.7	77.	241.903	4656.6	⅜	268.213	5724.7	¾	294.524
¾	215.984	3712.2	⅛	242.295	4671.8	½	268.606	5741.5	⅞	294.917
⅞	216.377	3725.7	¼	242.688	4686.9	⅝	268.999	5758.3	94.	295.310
69.	216.770	3739.3	⅜	243.081	4702.1	¾	269.392	5775.1	⅛	295.702
⅛	217.163	3752.8	½	243.473	4717.3	⅞	269.784	5791.9	¼	296.095
¼	217.555	3766.4	⅝	243.866	4732.5	86.	270.177	5808.8	⅜	296.488
⅜	217.948	3780.0	¾	244.259	4747.8	⅛	270.570	5825.7	½	296.881
½	218.341	3793.7	⅞	244.652	4763.1	¼	270.962	5842.6	⅝	297.273
⅝	218.733	3807.3	78.	245.044	4778.4	⅜	271.355	5859.6	¾	297.666
¾	219.126	3821.0	⅛	245.437	4793.7	½	271.748	5876.5	⅞	298.059
⅞	219.519	3834.7	¼	245.830	4809.0	⅝	272.140	5893.5	95.	298.451
70.	219.911	3848.5	⅜	246.222	4824.4	¾	272.533	5910.6	⅛	298.844
⅛	220.304	3862.2	½	246.615	4839.8	⅞	272.926	5927.6	¼	299.237
¼	220.697	3876.0	⅝	247.008	4855.2	87.	273.319	5944.7	⅜	299.629
⅜	221.090	3889.8	¾	247.400	4870.7	⅛	273.711	5961.8	½	300.022
½	221.482	3903.6	⅞	247.793	4886.2	¼	274.104	5978.9	⅝	300.415
⅝	221.875	3917.5	79.	248.186	4901.7	⅜	274.497	5996.0	¾	300.807
¾	222.268	3931.4	⅛	248.579	4917.2	½	274.889	6013.2	⅞	301.200
⅞	222.660	3945.3	¼	248.971	4932.7	⅝	275.282	6030.4	96.	301.593
71.	223.053	3959.2	⅜	249.364	4948.3	¾	275.675	6047.6	⅛	301.986
⅛	223.446	3973.1	½	249.757	4963.9	⅞	276.067	6064.9	¼	302.378
¼	223.838	3987.1	⅝	250.149	4979.5	88.	276.460	6082.1	⅜	302.771
⅜	224.231	4001.1	¾	250.542	4995.2	⅛	276.853	6099.4	½	303.164
½	224.624	4015.2	⅞	250.935	5010.9	¼	277.246	6116.7	⅝	303.556
⅝	225.017	4029.2	80.	251.327	5026.5	⅜	277.638	6134.1	¾	303.949
¾	225.409	4043.3	⅛	251.720	5042.3	½	278.031	6151.4	⅞	304.342
⅞	225.802	4057.4	¼	252.113	5058.0	⅝	278.424	6168.8	97.	304.734
72.	226.195	4071.5	⅜	252.506	5073.8	¾	278.816	6186.2	⅛	305.127
⅛	226.587	4085.7	½	252.898	5089.6	⅞	279.209	6203.7	¼	305.520
¼	226.980	4099.8	⅝	253.291	5105.4	89.	279.602	6221.1	⅜	305.913
⅜	227.373	4114.0	¾	253.684	5121.2	⅛	279.994	6238.6	½	306.305
½	227.765	4128.2	⅞	254.076	5137.1	¼	280.387	6256.1	⅝	306.698
⅝	228.158	4142.5	81.	254.469	5153.0	⅜	280.780	6273.7	¾	307.091
¾	228.551	4156.8	⅛	254.862	5168.9	½	281.173	6291.2	⅞	307.483
⅞	228.944	4171.1	¼	255.254	5184.9	⅝	281.565	6308.8	98.	307.876
73.	229.336	4185.4	⅜	255.647	5200.8	¾	281.958	6326.4	⅛	308.269
⅛	229.729	4199.7	½	256.040	5216.8	⅞	282.351	6344.1	¼	308.661
¼	230.122	4214.1	⅝	256.433	5232.8	90.	282.743	6361.7	⅜	309.054
⅜	230.514	4228.5	¾	256.825	5248.9	⅛	283.136	6379.4	½	309.447
½	230.907	4242.9	⅞	257.218	5264.9	¼	283.529	6397.1	⅝	309.840
⅝	231.300	4257.4	82.	257.611	5281.0	⅜	283.921	6414.9	¾	310.232
¾	231.692	4271.8	⅛	258.003	5297.1	½	284.314	6432.6	⅞	310.625
⅞	232.085	4286.3	¼	258.396	5313.3	⅝	284.707	6450.4	99.	311.018
74.	232.478	4300.8	⅜	258.789	5329.4	¾	285.100	6468.2	⅛	311.410
⅛	232.871	4315.4	½	259.181	5345.6	⅞	285.492	6486.0	¼	311.803
¼	233.263	4329.9	⅝	259.574	5361.8	91.	285.885	6503.9	⅜	312.196
⅜	233.656	4344.5	¾	259.967	5378.1	⅛	286.278	6521.8	½	312.588
½	234.049	4359.2	⅞	260.359	5394.3	¼	286.670	6539.7	⅝	312.981
⅝	234.441	4373.8	83.	260.752	5410.6	⅜	287.063	6557.6	¾	313.374
¾	234.834	4388.5	⅛	261.145	5426.9	½	287.456	6575.5	⅞	313.767
⅞	235.227	4403.1	¼	261.538	5443.3	⅝	287.848	6593.5	100.	314.159
75.	235.619	4417.9	⅜	261.930	5459.6	¾	288.241	6611.5		
⅛	236.012	4432.6	½	262.323	5476.0	⅞	288.634	6629.6		

To find ordinates, *ds*, *ao*, &c., of a circular arc *mnn*



First find the one *ds* at the center, thus. Square the rad. Squa *or half* the chord *m m*. Subtract this last square from the fi *Take the sq rt* of the rem. Subtract this sq rt from the rad for *For any other ord a o*, subtract *d s* from rad. Call the rem *t*. *rad*. Square *d a*. Take this last square from the first one. T *of the rem*. From this sq rt take *t*. The rem will be *a o*. *ords*, *see pp* 416, 632.

To find the length of a circular arc by the following table.

Knowing its chord and height, divide the height by the chord. Find in the column of heights the number equal to this quotient. Take out the corresponding number from the column of lengths. Multiply this last number by the length of the given chord.

TABLE OF CIRCULAR ARCS.

No errors.

H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.
.001	1.00001	.076	1.01533	.151	1.05973	.226	1.13108	.301	1.22636
.002	1.00001	.077	1.01573	.152	1.06051	.227	1.13219	.302	1.22778
.003	1.00002	.078	1.01614	.153	1.06130	.228	1.13331	.303	1.22920
.004	1.00004	.079	1.01656	.154	1.06209	.229	1.13444	.304	1.23063
.005	1.00007	.080	1.01698	.155	1.06288	.230	1.13557	.305	1.23206
.006	1.00010	.081	1.01741	.156	1.06368	.231	1.13671	.306	1.23349
.007	1.00013	.082	1.01784	.157	1.06449	.232	1.13785	.307	1.23492
.008	1.00017	.083	1.01828	.158	1.06530	.233	1.13900	.308	1.23636
.009	1.00022	.084	1.01872	.159	1.06611	.234	1.14015	.309	1.23781
.010	1.00027	.085	1.01916	.160	1.06693	.235	1.14131	.310	1.23926
.011	1.00032	.086	1.01961	.161	1.06775	.236	1.14247	.311	1.24070
.012	1.00038	.087	1.02006	.162	1.06858	.237	1.14363	.312	1.24216
.013	1.00045	.088	1.02052	.163	1.06941	.238	1.14480	.313	1.24361
.014	1.00053	.089	1.02098	.164	1.07025	.239	1.14597	.314	1.24507
.015	1.00061	.090	1.02145	.165	1.07109	.240	1.14714	.315	1.24654
.016	1.00069	.091	1.02192	.166	1.07194	.241	1.14832	.316	1.24801
.017	1.00078	.092	1.02240	.167	1.07279	.242	1.14951	.317	1.24948
.018	1.00087	.093	1.02289	.168	1.07365	.243	1.15070	.318	1.25095
.019	1.00097	.094	1.02339	.169	1.07451	.244	1.15189	.319	1.25243
.020	1.00107	.095	1.02389	.170	1.07537	.245	1.15308	.320	1.25391
.021	1.00117	.096	1.02440	.171	1.07624	.246	1.15428	.321	1.25540
.022	1.00128	.097	1.02491	.172	1.07711	.247	1.15549	.322	1.25689
.023	1.00140	.098	1.02542	.173	1.07799	.248	1.15670	.323	1.25838
.024	1.00153	.099	1.02593	.174	1.07888	.249	1.15791	.324	1.25988
.025	1.00167	.100	1.02645	.175	1.07977	.250	1.15912	.325	1.26138
.026	1.00182	.101	1.02698	.176	1.08066	.251	1.16034	.326	1.26288
.027	1.00196	.102	1.02752	.177	1.08156	.252	1.16156	.327	1.26437
.028	1.00210	.103	1.02806	.178	1.08246	.253	1.16279	.328	1.26588
.029	1.00225	.104	1.02860	.179	1.08337	.254	1.16402	.329	1.26740
.030	1.00240	.105	1.02914	.180	1.08428	.255	1.16526	.330	1.26892
.031	1.00256	.106	1.02970	.181	1.08519	.256	1.16650	.331	1.27044
.032	1.00272	.107	1.03025	.182	1.08611	.257	1.16774	.332	1.27196
.033	1.00289	.108	1.03082	.183	1.08704	.258	1.16899	.333	1.27349
.034	1.00307	.109	1.03139	.184	1.08797	.259	1.17024	.334	1.27502
.035	1.00327	.110	1.03196	.185	1.08890	.260	1.17150	.335	1.27656
.036	1.00345	.111	1.03254	.186	1.08984	.261	1.17276	.336	1.27810
.037	1.00364	.112	1.03312	.187	1.09079	.262	1.17403	.337	1.27964
.038	1.00384	.113	1.03371	.188	1.09174	.263	1.17530	.338	1.28118
.039	1.00405	.114	1.03430	.189	1.09269	.264	1.17657	.339	1.28273
.040	1.00426	.115	1.03490	.190	1.09365	.265	1.17784	.340	1.28428
.041	1.00447	.116	1.03551	.191	1.09461	.266	1.17912	.341	1.28583
.042	1.00469	.117	1.03611	.192	1.09557	.267	1.18040	.342	1.28739
.043	1.00492	.118	1.03672	.193	1.09654	.268	1.18169	.343	1.28895
.044	1.00515	.119	1.03734	.194	1.09752	.269	1.18299	.344	1.29052
.045	1.00539	.120	1.03797	.195	1.09850	.270	1.18429	.345	1.29209
.046	1.00563	.121	1.03860	.196	1.09949	.271	1.18559	.346	1.29366
.047	1.00587	.122	1.03923	.197	1.10048	.272	1.18689	.347	1.29523
.048	1.00612	.123	1.03987	.198	1.10147	.273	1.18820	.348	1.29681
.049	1.00638	.124	1.04051	.199	1.10247	.274	1.18951	.349	1.29839
.050	1.00665	.125	1.04116	.200	1.10347	.275	1.19082	.350	1.29997
.051	1.00692	.126	1.04181	.201	1.10447	.276	1.19214	.351	1.30156
.052	1.00720	.127	1.04247	.202	1.10548	.277	1.19346	.352	1.30315
.053	1.00748	.128	1.04313	.203	1.10650	.278	1.19479	.353	1.30474
.054	1.00776	.129	1.04380	.204	1.10752	.279	1.19612	.354	1.30634
.055	1.00805	.130	1.04447	.205	1.10855	.280	1.19746	.355	1.30794
.056	1.00834	.131	1.04515	.206	1.10958	.281	1.19880	.356	1.30954
.057	1.00864	.132	1.04584	.207	1.11062	.282	1.20014	.357	1.31115
.058	1.00895	.133	1.04652	.208	1.11165	.283	1.20149	.358	1.31276
.059	1.00926	.134	1.04722	.209	1.11269	.284	1.20284	.359	1.31437
.060	1.00957	.135	1.04792	.210	1.11374	.285	1.20419	.360	1.31599
.061	1.00989	.136	1.04862	.211	1.11479	.286	1.20555	.361	1.31761
.062	1.01021	.137	1.04932	.212	1.11584	.287	1.20691	.362	1.31923
.063	1.01054	.138	1.05003	.213	1.11690	.288	1.20827	.363	1.32086
.064	1.01088	.139	1.05075	.214	1.11796	.289	1.20964	.364	1.32249
.065	1.01123	.140	1.05147	.215	1.11904	.290	1.21102	.365	1.32413
.066	1.01158	.141	1.05220	.216	1.12011	.291	1.21239	.366	1.32577
.067	1.01193	.142	1.05293	.217	1.12118	.292	1.21377	.367	1.32741
.068	1.01228	.143	1.05367	.218	1.12225	.293	1.21515	.368	1.32905
.069	1.01264	.144	1.05441	.219	1.12334	.294	1.21654	.369	1.33069
.070	1.01301	.145	1.05516	.220	1.12444	.295	1.21794	.370	1.33234
.071	1.01338	.146	1.05591	.221	1.12554	.296	1.21933	.371	1.33398
.072	1.01376	.147	1.05667	.222	1.12664	.297	1.22073	.372	1.33563
.073	1.01414	.148	1.05743	.223	1.12774	.298	1.22213	.373	1.33728
.074	1.01453	.149	1.05819	.224	1.12885	.299	1.22354	.374	1.33893
.075	1.01493	.150	1.05896	.225	1.12997	.300	1.22495	.375	1.34

TABLE OF CIRCULAR ARCS—(Continued.)

H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.	Lengths.	H'ghts.
.376	1.34229	.401	1.38496	.426	1.42845	.451	1.47265	.476
.377	1.34393	.402	1.38671	.427	1.43127	.452	1.47553	.477
.378	1.34563	.403	1.38846	.428	1.43309	.453	1.47842	.478
.379	1.34731	.404	1.39021	.429	1.43491	.454	1.48131	.479
.380	1.34899	.405	1.39196	.430	1.43673	.455	1.48420	.480
.381	1.35068	.406	1.39372	.431	1.43856	.456	1.48509	.481
.382	1.35237	.407	1.39548	.432	1.44039	.457	1.48899	.482
.383	1.35406	.408	1.39724	.433	1.44222	.458	1.48889	.483
.384	1.35575	.409	1.39900	.434	1.44405	.459	1.49079	.484
.385	1.35744	.410	1.40077	.435	1.44589	.460	1.49269	.485
.386	1.35914	.411	1.40254	.436	1.44773	.461	1.49460	.486
.387	1.36084	.412	1.40432	.437	1.44957	.462	1.49651	.487
.388	1.36254	.413	1.40610	.438	1.45142	.463	1.49842	.488
.389	1.36425	.414	1.40788	.439	1.45327	.464	1.50033	.489
.390	1.36596	.415	1.40966	.440	1.45512	.465	1.50224	.490
.391	1.36767	.416	1.41145	.441	1.45697	.466	1.50416	.491
.392	1.36939	.417	1.41324	.442	1.45883	.467	1.50608	.492
.393	1.37111	.418	1.41503	.443	1.46069	.468	1.50800	.493
.394	1.37283	.419	1.41682	.444	1.46255	.469	1.50992	.494
.395	1.37455	.420	1.41861	.445	1.46441	.470	1.51185	.495
.396	1.37628	.421	1.42041	.446	1.46628	.471	1.51378	.496
.397	1.37801	.422	1.42221	.447	1.46815	.472	1.51571	.497
.398	1.37974	.423	1.42402	.448	1.47002	.473	1.51764	.498
.399	1.38148	.424	1.42583	.449	1.47189	.474	1.51958	.499
.400	1.38322	.425	1.42764	.450	1.47377	.475	1.52152	.500

To find the length of a circular arc by the following on next page.

Knowing the rad of the circle, and the measure of the arc in deg, min, &c.

Rule. Add together the lengths in the table found respectively opposite to the deg, r the arc. Mult the sum by the rad of the circle.

Ex. In a circle of 12.43 feet rad, is an arc of 13 deg, 27 min, 8 sec. How long is the arc

Here, opposite 13 deg in the table, we find, .2268928
" 27 min " " " " .0078540
" 8 sec " " " " .0000388

Sum = .2347856

And .2347856 × 12.43 or rad = 2.918385 feet, the reqd length of arc.

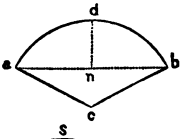
The length of 1 degree of a circular arc is equal to .017453 292 520 of its radius.
" " " 1 minute " " " " " .000290 888 209 " " "
" " " 1 second " " " " " .000004 848 137 " " "

An arc of 1 degree of the earth's equatorial great circle is but about $6\frac{1}{10}$ feet longer tha
Its length is 69.16 land or statute miles. Earth's equatorial rad = 3962.5705 miles.

An arc of 1 degree, to a rad of 1 mile, is 92.1534 feet; a minute is 1.5359 feet; a second is foot; or very nearly 5-sixteenths of an inch. An arc of 1 deg. to a rad of 100 feet = 1.7453

To find the length of a revolution of the thread of a s

Square the outer circumf of the screw; also square the dist apart of the threads fro center. Take the sq rt of the sum.

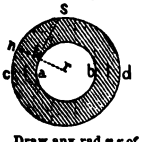


To find the area of a circular sector

Rule 1. Mult the length of the arc *a d b*, by the rad *c a* divide prod by 2.

Rem. Knowing the chord *a b*, and the height *n d*; or th the number of degs in the arc, the length of the arc itself c from either the preceding or the following table.

Rule 2. As 360° is to the number of degs in the angle the area of the entire circle to the area of the sector.



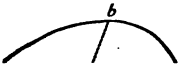
To find the area of a circular ring.

Find the areas of the two circles, and take the least from the gr mult the sum of the two diams, *a b*, *c d*, by their diff; mult the pro Or mult together the thickness *c a*; the mean diam *l l*; and 3.1416.

To find the rad of a circle which shall ha same area of section as a given circul *c s d a b*.

Draw any rad *s r* of the outer circle; and from where said rad cuts the inner circle at *t*, *right* angles to it. Then will *t s* be the reqd rad.

To find the area of a circular segment *a b c d*.



By the table on pages 24 and 25 having the chord *height d b*; and the diam of the circle. Div the height diam of the circle; find in the first column, the number n quot; take out from the col of areas, the corresponding are area by the square of the diam.

Rem. 2. When the height $d\,b$ is not more than ¼th of the chord $a\,c$, the area may be found approximately thus. Mult the chord by the height, and take ⅔ds of the prod.

The following formula is quite close for flat segs;

and does not err more than 1 part in 800, in any high one not exceeding a semicircle.

$$\text{Area of segment} = 1\tfrac{1}{3}\,\text{Height} \times \sqrt{\left(\frac{\text{Chord}^2}{4}\right) + \left((\text{Height}^2 \times .388)\right)}$$

Having the area of a segment reqd to be cut off from a given circle, to find its chord, and height.

Div the area by the square of the diam of the circle; look for the quot in the col of areas in the table which follows the next one; take out from the table the corresponding height; and mult it by the diam. The prod will be the reqd height. Then from the diam, take the actual height thus found; mult the rem by the actual height; take the sq rt of the prod; mult it by 2, for the reqd chord.

TABLE OF LENGTHS OF CIRCULAR ARCS, WHOSE RADIUS IS 1.

Referred to on the preceding page. No errors.

Deg.	Length.	Deg.	Length.	Deg.	Length.	Min.	Length.	Sec.	Length.
1	.0174533	61	1.0646508	121	2.1118484	1	.0002909	1	.0000048
2	.0349066	62	1.0821041	122	2.1293017	2	.0005818	2	.0000097
3	.0523599	63	1.0995574	123	2.1467550	3	.0008727	3	.0000145
4	.0698132	64	1.1170107	124	2.1642083	4	.0011636	4	.0000194
5	.0872665	65	1.1344640	125	2.1816616	5	.0014544	5	.0000242
6	.1047198	66	1.1519173	126	2.1991149	6	.0017453	6	.0000291
7	.1221730	67	1.1693706	127	2.2165682	7	.0020362	7	.0000339
8	.1396263	68	1.1868239	128	2.2340214	8	.0023271	8	.0000388
9	.1570796	69	1.2042772	129	2.2514747	9	.0026180	9	.0000436
10	.1745329	70	1.2217305	130	2.2689280	10	.0029089	10	.0000485
11	.1919862	71	1.2391838	131	2.2863813	11	.0031998	11	.0000533
12	.2094395	72	1.2566371	132	2.3038346	12	.0034907	12	.0000582
13	.2268928	73	1.2740904	133	2.3212879	13	.0037815	13	.0000630
14	.2443461	74	1.2915436	134	2.3387412	14	.0040724	14	.0000679
15	.2617994	75	1.3089969	135	2.3561945	15	.0043633	15	.0000727
16	.2792527	76	1.3264502	136	2.3736478	16	.0046542	16	.0000776
17	.2967060	77	1.3439035	137	2.3911011	17	.0049451	17	.0000824
18	.3141593	78	1.3613568	138	2.4085544	18	.0052360	18	.0000873
19	.3316126	79	1.3788101	139	2.4260077	19	.0055269	19	.0000921
20	.3490659	80	1.3962634	140	2.4434610	20	.0058178	20	.0000970
21	.3665191	81	1.4137167	141	2.4609142	21	.0061087	21	.0001018
22	.3839724	82	1.4311700	142	2.4783675	22	.0063995	22	.0001067
23	.4014257	83	1.4486233	143	2.4958208	23	.0066904	23	.0001115
24	.4188790	84	1.4660766	144	2.5132741	24	.0069813	24	.0001164
25	.4363323	85	1.4835299	145	2.5307274	25	.0072722	25	.0001212
26	.4537856	86	1.5009832	146	2.5481807	26	.0075631	26	.0001261
27	.4712389	87	1.5184364	147	2.5656340	27	.0078540	27	.0001309
28	.4886922	88	1.5358897	148	2.5830873	28	.0081449	28	.0001357
29	.5061455	89	1.5533430	149	2.6005406	29	.0084358	29	.0001406
30	.5235988	90	1.5707963	150	2.6179939	30	.0087266	30	.0001454
31	.5410521	91	1.5882496	151	2.6354472	31	.0090175	31	.0001503
32	.5585054	92	1.6057029	152	2.6529005	32	.0093084	32	.0001551
33	.5759587	93	1.6231562	153	2.6703538	33	.0095993	33	.0001600
34	.5934119	94	1.6406095	154	2.6878070	34	.0098902	34	.0001648
35	.6108652	95	1.6580628	155	2.7052603	35	.0101811	35	.0001697
36	.6283185	96	1.6755161	156	2.7227136	36	.0104720	36	.0001745
37	.6457718	97	1.6929694	157	2.7401669	37	.0107629	37	.0001794
38	.6632251	98	1.7104227	158	2.7576202	38	.0110538	38	.0001842
39	.6806784	99	1.7278760	159	2.7750735	39	.0113446	39	.0001891
40	.6981317	100	1.7453293	160	2.7925268	40	.0116355	40	.0001939
41	.7155850	101	1.7627825	161	2.8099801	41	.0119264	41	.0001988
42	.7330383	102	1.7802358	162	2.8274334	42	.0122173	42	.0002036
43	.7504916	103	1.7976891	163	2.8448867	43	.0125082	43	.0002085
44	.7679449	104	1.8151424	164	2.8623400	44	.0127991	44	.0002133
45	.7853982	105	1.8325957	165	2.8797933	45	.0130900	45	.0002182
46	.8028515	106	1.8500490	166	2.8972466	46	.0133809	46	.0002230
47	.8203047	107	1.8675023	167	2.9146999	47	.0136717	47	.0002279
48	.8377580	108	1.8849556	168	2.9321531	48	.0139626	48	.0002327
49	.8552113	109	1.9024089	169	2.9496064	49	.0142535	49	.0002376
50	.8726646	110	1.9198622	170	2.9670597	50	.0145444	50	.0002424
51	.8901179	111	1.9373155	171	2.9845130	51	.0148353	51	.0002473
52	.9075712	112	1.9547688	172	3.0019663	52	.0151262	52	.0002521
53	.9250245	113	1.9722221	173	3.0194196	53	.0154171	53	.0002570
54	.9424778	114	1.9896753	174	3.0368729	54	.0157080	54	.0002618
55	.9599311	115	2.0071286	175	3.0543262	55	.0159989	55	.0002666
56	.9773844	116	2.0245819	176	3.0717795	56	.0162897	56	.0002715
57	.9948377	117	2.0420352	177	3.0892328	57	.0165806	57	.0002763
58	1.0122910	118	2.0594885	178	3.1066861	58	.0168715	58	.0002812
59	1.0297443	119	2.0769418	179	3.1241394	59	.0171624	59	.0002860
60	1.0471976	120	2.0943951	180	3.1415927	60	.0174533	60	.0002909

TABLE OF AREAS OF CIRCULAR SEGMENTS.

The heights are in parts of the diam of the circle.
For the use of this table see near bottom of page 22.
No errors equal to 1 in the last figure.

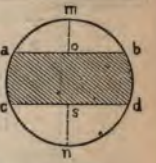
Height.	Areas.	Height.	Areas.	Height.	Areas.	Height.	Areas.	Height.	Area
.001	.000042	.064	.021168	.127	.057991	.190	.103900	.253	.156
.002	.000119	.065	.021660	.128	.058658	.191	.104686	.254	.157
.003	.000219	.066	.022155	.129	.059328	.192	.105472	.255	.157
.004	.000337	.067	.022653	.130	.059999	.193	.106261	.256	.158
.005	.000471	.068	.023155	.131	.060673	.194	.107051	.257	.159
.006	.000619	.069	.023660	.132	.061349	.195	.107843	.258	.160
.007	.000779	.070	.024168	.133	.062027	.196	.108636	.259	.161
.008	.000952	.071	.024680	.134	.062707	.197	.109431	.260	.162
.009	.001135	.072	.025196	.135	.063389	.198	.110227	.261	.163
.010	.001329	.073	.025714	.136	.064074	.199	.111025	.262	.164
.011	.001533	.074	.026236	.137	.064761	.200	.111824	.263	.164
.012	.001746	.075	.026761	.138	.065449	.201	.112625	.264	.165
.013	.001969	.076	.027290	.139	.066140	.202	.113427	.265	.166
.014	.002199	.077	.027821	.140	.066833	.203	.114231	.266	.167
.015	.002438	.078	.028356	.141	.067528	.204	.115036	.267	.168
.016	.002685	.079	.028894	.142	.068225	.205	.115842	.268	.169
.017	.002940	.080	.029435	.143	.068924	.206	.116651	.269	.170
.018	.003202	.081	.029979	.144	.069626	.207	.117460	.270	.171
.019	.003472	.082	.030526	.145	.070329	.208	.118271	.271	.171
.020	.003749	.083	.031077	.146	.071034	.209	.119084	.272	.172
.021	.004032	.084	.031630	.147	.071741	.210	.119898	.273	.173
.022	.004322	.085	.032186	.148	.072450	.211	.120713	.274	.174
.023	.004619	.086	.032746	.149	.073162	.212	.121530	.275	.175
.024	.004922	.087	.033308	.150	.073875	.213	.122348	.276	.176
.025	.005231	.088	.033873	.151	.074590	.214	.123167	.277	.177
.026	.005546	.089	.034441	.152	.075307	.215	.123988	.278	.178
.027	.005867	.090	.035012	.153	.076026	.216	.124811	.279	.179
.028	.006194	.091	.035586	.154	.076747	.217	.125634	.280	.180
.029	.006527	.092	.036162	.155	.077470	.218	.126459	.281	.180
.030	.006866	.093	.036742	.156	.078194	.219	.127286	.282	.181
.031	.007209	.094	.037324	.157	.078921	.220	.128114	.283	.182
.032	.007559	.095	.037909	.158	.079650	.221	.128943	.284	.183
.033	.007913	.096	.038497	.159	.080380	.222	.129773	.285	.184
.034	.008273	.097	.039087	.160	.081112	.223	.130605	.286	.185
.035	.008638	.098	.039681	.161	.081847	.224	.131438	.287	.186
.036	.009008	.099	.040277	.162	.082582	.225	.132273	.288	.187
.037	.009383	.100	.040875	.163	.083320	.226	.133109	.289	.188
.038	.009763	.101	.041477	.164	.084060	.227	.133946	.290	.189
.039	.010148	.102	.042081	.165	.084801	.228	.134784	.291	.189
.040	.010538	.103	.042687	.166	.085545	.229	.135624	.292	.190
.041	.010932	.104	.043296	.167	.086290	.230	.136465	.293	.191
.042	.011331	.105	.043908	.168	.087037	.231	.137307	.294	.192
.043	.011734	.106	.044523	.169	.087785	.232	.138151	.295	.193
.044	.012142	.107	.045140	.170	.088536	.233	.138996	.296	.194
.045	.012555	.108	.045759	.171	.089288	.234	.139842	.297	.195
.046	.012971	.109	.046381	.172	.090042	.235	.140689	.298	.196
.047	.013393	.110	.047006	.173	.090797	.236	.141538	.299	.197
.048	.013818	.111	.047633	.174	.091555	.237	.142388	.300	.198
.049	.014248	.112	.048262	.175	.092314	.238	.143239	.301	.199
.050	.014681	.113	.048894	.176	.093074	.239	.144091	.302	.200
.051	.015119	.114	.049529	.177	.093837	.240	.144945	.303	.200
.052	.015561	.115	.050165	.178	.094601	.241	.145800	.304	.201
.053	.016008	.116	.050805	.179	.095367	.242	.146656	.305	.202
.054	.016458	.117	.051446	.180	.096135	.243	[illegible]	.306	.203
.055	.016912	.118	.052090	.181	.096904	.244	.148371	.307	.204
.056	.017369	.119	.052737	.182	.097675	.245	.149231	.308	.205
.057	.017831	.120	.053385	.183	.098447	.246	.150091	.309	.206
.058	.018297	.121	.054037	.184	.099221	.247	.150953	.310	.207
.059	.018766	.122	.054690	.185	.099997	.248	.151816	.311	.208
.060	.019239	.123	.055346	.186	.100774	.249	.152681	.312	.209
.061	.019716	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

ABLE OF AREAS OF CIRCULAR SEGMENTS—(CONTINUED.)

Height.	Areas.	Height.	Areas.	Height.	Areas.	Height.	Areas.	Height.	Areas.
.316	.212941	.353	.247845	.390	.283593	.427	.319959	.464	.356730
.317	.213871	.354	.248801	.391	.284569	.428	.320949	.465	.357728
.318	.214802	.355	.249758	.392	.285545	.429	.321938	.466	.358725
.319	.215734	.356	.250715	.393	.286521	.430	.322928	.467	.359723
.320	.216666	.357	.251673	.394	.287499	.431	.323919	.468	.360721
.321	.217600	.358	.252632	.395	.288476	.432	.324909	.469	.361719
.322	.218534	.359	.253591	.396	.289454	.433	.325900	.470	.362717
.323	.219469	.360	.254551	.397	.290432	.434	.326891	.471	.363715
.324	.220404	.361	.255511	.398	.291411	.435	.327883	.472	.364714
.325	.221341	.362	.256472	.399	.292390	.436	.328874	.473	.365712
.326	.222278	.363	.257433	.400	.293370	.437	.329866	.474	.366711
.327	.223216	.364	.258395	.401	.294350	.438	.330858	.475	.367710
.328	.224154	.365	.259358	.402	.295330	.439	.331851	.476	.368708
.329	.225094	.366	.260321	.403	.296311	.440	.332843	.477	.369707
.330	.226034	.367	.261285	.404	.297292	.441	.333836	.478	.370706
.331	.226974	.368	.262249	.405	.298274	.442	.334829	.479	.371705
.332	.227916	.369	.263214	.406	.299256	.443	.335823	.480	.372704
.333	.228858	.370	.264179	.407	.300238	.444	.336816	.481	.373704
.334	.229801	.371	.265145	.408	.301221	.445	.337810	.482	.374703
.335	.230745	.372	.266111	.409	.302204	.446	.338804	.483	.375702
.336	.231689	.373	.267078	.410	.303187	.447	.339799	.484	.376702
.337	.232634	.374	.268046	.411	.304171	.448	.340793	.485	.377701
.338	.233580	.375	.269014	.412	.305156	.449	.341788	.486	.378701
.339	.234526	.376	.269982	.413	.306140	.450	.342783	.487	.379701
.340	.235473	.377	.270951	.414	.307125	.451	.343778	.488	.380700
.341	.236421	.378	.271921	.415	.308110	.452	.344773	.489	.381700
.342	.237369	.379	.272891	.416	.309096	.453	.345768	.490	.382700
.343	.238319	.380	.273861	.417	.310082	.454	.346764	.491	.383700
.344	.239268	.381	.274832	.418	.311068	.455	.347760	.492	.384699
.345	.240219	.382	.275804	.419	.312055	.456	.348756	.493	.385699
.346	.241170	.383	.276776	.420	.313042	.457	.349752	.494	.386699
.347	.242122	.384	.277748	.421	.314029	.458	.350749	.495	.387699
.348	.243074	.385	.278721	.422	.315017	.459	.351745	.496	.388699
.349	.244027	.386	.279695	.423	.316005	.460	.352742	.497	.389699
.350	.244980	.387	.280669	.424	.316993	.461	.353739	.498	.390699
.351	.245935	.388	.281643	.425	.317981	.462	.354736	.499	.391699
.352	.246890	.389	.282618	.426	.318970	.463	.355733	.500	.392699

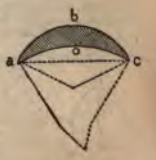
To find the area of a circular zone *a b c d*.

Knowing the diam of the circle; the chords *a b*, and *c d*; and the heights *m*, and *s n*, of the segments *a m b*, and *c n d*. First find the area of the entire circle; then by means of the preceding table of circular segments, find the areas of the two segments *a m b*, and *c n d*; and subtract them from the area of the circle.



To find the area of a circular lune *a b c o*.

A lune is a crescent-shaped fig, comprised between two arcs of circles of different diams. Having the chord *a c*, and the heights of the two segments *a o c*, and *a b c*, find the areas of those segments; take the least of these areas from the greatest; the rem is evidently the area of the lune.



THE ELLIPSE.

Fig 1

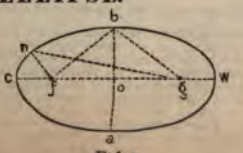


Fig 2

C

An ellipse is a curve, *s q e s*, Fig 1, formed by an oblique section of either a cone or a cylinder, passing through its curved surface, without touching the base. Its nature is such that if two lines, as *n f* and *n q*, Fig 2, be drawn from any point *n* in its periphery or circumf, to two certain points *f* and *g*, situated in its long diam *c w*, (and called the foci of the ellipse.) they will be equal to any other two lines, as *b f*, and *b g*, drawn from any other point, as *b*, in the circumf, to the foci *f* and *g*; also any two such lines will together be equal to the long diam *c w*. The line *c w* dividing the ellipse into two equal parts lengthwise, is called its transverse, or major axis, or long diam; and *a b*, which divides it equally at right-angles to *c w*, is called the conjugate, or minor axis, or short diam. To find the position of the foci of an ellipse, from either end, as *b*, of the short diam, measure off the dists *b f* and *b g*, Fig 2, each equal to *o c*, or one-half the long diam.

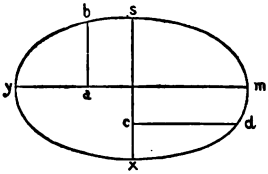


Fig 3

The parameter of an ellipse is a certain length obtained thus; as the long diam : short diam : : short diam : parameter; or the

$$\text{parameter is equal to } \frac{\text{short diam}^2}{\text{long diam}}.$$

Any line *b a*, or *c d*, Fig 3, drawn from the circumf, to, and at right angles to, either diam, is called an *ordinate;* and the part *a y*, or *c x* of that diam, between the ord and the circumf, is called an *abscissa*, or *abscise*.

To find the length of any ordinate *a b*, drawn to the long diam *y m*.

Knowing the abscise *y a*, and the two diams.

$$\text{Ord. } a\,b = \left(\frac{\text{short diam}}{\text{long diam}}\right) \times \sqrt{(\text{long diam} \times y\,a) - y\,a^2}$$

Or in words, div the short diam by the long one; call the quot *q*. Mult the long diam by the abscise *y a*; from the prod subtract the square of the abscise *y a*; take the sq rt of the rem; mult this sq rt by *q*.

Or, mult together the two parts *y a* and *a m*, Fig 3, into which the ord divides the long diam; take the sq rt of the prod; mult this sq rt by the short diam; div the prod by the long diam.

Rem. Neither of these rules (nor any other) applies also to ordinates like *c d*, drawn to the short diam *x s*.

To find the circumf of an ellipse.

Mathematicians have furnished practical men with no simple working rule for this purpose. The so-called approximate rules do not deserve the name. They are as follows, D being the long diam: and *d* the short one.*

Rule 1. Circumf $= 3.1416\ \frac{D+d}{2}$; Rule 2. $3.1416 \sqrt{\left(\frac{D^2+d^2}{2}\right)}$; Rule 3. $2.2215 \sqrt{D^2+d^2}$; this is the same as Rule 2, but in a diff shape. Rule 4. $2\sqrt{D^2 + 1.4674\,d^2}$. Now, in an ellipse whose long and short diams are 10 and 2, the circumf is actually 21, very approximately; but rule 1 gives it $= 18.85$; rule 2, or 3, $= 22.65$; and rule 4, $= 20.51$. Again, if the diams be 10 and 6, the circumf actually $= 25.59$; but rule 4 gives 24.72. These examples show that none of the rules usually given are reliable. The following one by the writer, is sufficiently exact for ordinary purposes; not being in error probably more than 1 part in 1000. When D is not more than 5 times as long as *d*, then, calling the diff between them, Diff, the

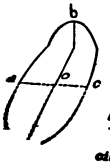
$$\text{Circumf} = 3.1416 \sqrt{\left(\frac{D^2+d^2}{2}\right) - \frac{\text{Diff}^2}{8.8}}$$

If D is more than 5 times *d*, then instead of dividing Diff² by 8.8, div it by the number in the following table:

D = 6 *d*	9	D = 14 *d*	9.6	D = 40 *d*	9.98
7	9.2	16	9.68	50	10.04
8	9.3	18	9.75	60	10.10
9	9.35	20	9.8	70	10.17
10	9.4	25	9.87	80	10.23
12	9.5	30	9.92	100	10.35

In words, this rule is as follows: Square both D and *d*; add these squares together; div the sum by 2; call the quot A. Next subtract *d* from D; square the Diff: div this square by 8.8 (or by the proper number from the table); subtract the quot from A; take the sq rt of the rem; mult this sq rt by 3.1416.

The following table of semi-elliptic arcs, has been prepared by this rule.



To use this table, div the height or rise of the arc, by its span or chord. The quot will be the height of an arc whose span is 1. Find this quot in the column of heights; and take out the corresponding number from the col. of lengths. Mult this *number* by the actual span. The prod will be the reqd length.

When the height becomes .500 of the chord (as at the end of the table) the ellipse *becomes a circle.* When the height exceeds .500 of the chord, as in *a b c*, then take *a o, or half the* chord, as the rise; and div this rise by the long diam *b d*, for the *quot to be looked for* in the col of heights; and to be mult by long diam. We thus *get the* arc *b a d*, which is evidently equal to *a b c*.

* *The full table in Mr. Haswell's book, Adcock's, and others, is incorrect, especially on the first page.*

TABLE OF LENGTHS OF SEMI-ELLIPTIC ARCS.

(Original.)

Heights.	Lengths.	Heights.	Lengths.	Heights.	Lengths.	Heights.	Lengths.
.005	1.000	.130	1.079	.255	1.219	.380	1.390
.01	1.001	.135	1.084	.260	1.226	.385	1.397
.015	1.002	.140	1.089	.265	1.233	.390	1.404
.02	1.003	.145	1.094	.270	1.239	.395	1.412
.025	1.004	.150	1.099	.275	1.245	.400	1.419
.03	1.006	.155	1.104	.280	1.252	.405	1.426
.035	1.008	.160	1.109	.285	1.259	.410	1.434
.04	1.011	.165	1.115	.290	1.265	.415	1.441
.045	1.014	.170	1.120	.295	1.272	.420	1.449
.05	1.017	.175	1.125	.300	1.279	.425	1.456
.055	1.020	.180	1.131	.305	1.285	.430	1.464
.06	1.023	.185	1.137	.310	1.292	.435	1.471
.065	1.026	.190	1.142	.315	1.298	.440	1.479
.07	1.029	.195	1.147	.320	1.305	.445	1.486
.075	1.032	.200	1.153	.325	1.312	.450	1.494
.08	1.036	.205	1.159	.330	1.319	.455	1.501
.085	1.039	.210	1.165	.335	1.325	.460	1.509
.09	1.043	.215	1.171	.340	1.332	.465	1.517
.095	1.046	.220	1.177	.345	1.339	.470	1.524
.100	1.051	.225	1.183	.350	1.346	.475	1.532
.105	1.055	.230	1.189	.355	1.353	.480	1.540
.110	1.059	.235	1.196	.360	1.361	.485	1.547
.115	1.064	.240	1.202	.365	1.368	.490	1.555
.120	1.069	.245	1.207	.370	1.375	.495	1.563
.125	1.074	.250	1.213	.375	1.382	.500	1.571

To find the area of an ellipse.

Mult the two diams together; mult the prod by .7854. Ex. $D = 10$; $d = 6$. Then $10 \times 6 \times .7854 = 47.124$ area. The area of an ellipse is a mean proportional between the areas of two circles, described on its two diams; therefore it may be found by mult together the areas of those two circles; and taking the sq rt of the prod. The area of an ellipse is therefore always greater than that of the circular section of the cylinder from which it may be supposed to be derived.

To find the diam of a circle of the same area as a given ellipse.

Mult together the 2 diams; take sq rt of prod.

To find the area of an elliptic segment whose base is parallel to either diam.

Div the height of the segment, by that diam of which said height is a part. From the table of circular segments take out the tabular area opposite the quot. Mult together this area, the long diam, and the short diam.

To draw an ellipse.

Having its long and short diams *a b* and *c d*, Fig 4.

Rule 1. From either end of the short diam, as *c*, lay off the dists *c f*, *c f'*, each equal to *a e*, or to one-half of the long diam. The points *f*, *f'* are the foci of the ellipse. Prepare a string, *f' n f*, or *f' g f*, with a loop at each end; the total length of string from end to end of loop, being equal to the long diam. Place pins at *f* and *f'*; and placing the loops over them, trace the curve by a pencil, which in every position, as at *n*, or *g*, keeps the string *f' n f*, or *f' g f*, equally stretched all the time.

Note. Owing to the difficulty of keeping the string equally stretched, this method is not as satisfactory as the following.

Rule 2. On the edge of a strip of paper *w s*, mark *w l* equal to half the short diam; and *w s* equal half the long diam. Then in whatever position this strip be placed, keeping *l* on the long diam, and *s* on the short diam, *w* will mark a point in the circumf of the ellipse. We may thus obtain as many such points as we please; and then draw the curve through them by hand.

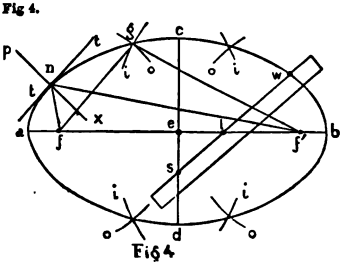


Fig 4

Rule 3. From the two foci f and f', Fig 4, with a rad equal to any part whatever of the long di *describe 4 short arcs, o o o o; also* with a rad equal to the remaining part of the long diam, des *4 other arcs, i i i i. The intersections of these four pairs of arcs,* will give four points in the cir *In this manner any number of such points may be found,* and the curve to be drawn by hand.

To draw a tangent *t t*, at any point *n* of an ellipse.

Draw *n f* and *n f'*, to the foci; bisect the angle *f n f'* by the line *x p*; draw *t n t* at right angles to *x p*.

To draw a joint *n p*, of an elliptic arch, from any point *n*, in the arch.

Proceed as in the foregoing rule for a tangent, only omitting *t t*; *n p* will be the reqd joint.

THE PARABOLA.

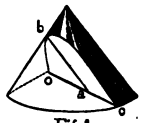


Fig 1

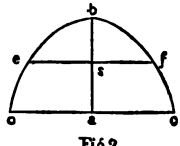


Fig 2

The common or conic parabola,

o b c, Fig 1, is a curve formed by cutting a cone in a direction *b a*, parallel to its side. The curved line *o b c* itself is called the *perimeter* of the parabola; the line *o c* is called its *base*; *b a* its height or *axis*; *b* its *apex* or *vertex*; any line *e s*, or *o a*, Fig 2, drawn from the curve, to, and at right angles to, the axis, is an *ordinate*; and the part *s b*, or *a b*, of the axis, between the ordinate and the apex *b*, is an *abscissa*. The *focus* of a parabola is that point in the axis, where the abscissa *b s*, is equal to one-half of the ord *e s*. The dist from the apex to the focus, is called the *focal dist*. The focus may be entirely beyond or outside of the curve itself. Its dist from the apex is found thus: square any ord, as *o a*; div this square by the abscissa *b a* of that ord; div the quot by 4. The nature of the parabola is such that its abscissas, as *b s*, *b a*, &c, are to each other as, or in proportion to, the squares of their respective ords *e s*, *o a*, &c; that is, as $b\,s : b\,a :: e\,s^2 : o\,a^2$; or $b\,s : e\,s^2 :: b\,a : o\,a^2$. If the square of any ord be divided by its abscissa, the quot will be a constant quantity; that is, it will be equal to the square of any other ord divided by its abscissa. This quot or constant quantity is also equal to a certain quantity called the *parameter* of the ellipse. Therefore the parameter may be found by squaring *e s*, or *o a*, (one *half* of the base,) and dividing said square by the height *b s*, or *b a*, as the case may be. If the square of any ord be divided by the parameter, the quot will be the abscissa of that ord.

To find the length of a parabolic curve.

The so-called approximate rule given by various pocket-books, is, like those for the ellipse, entirely unreliable. It is

$$\sqrt{\text{Half base}^2 + 1\tfrac{1}{4}\text{times the (Height}^2) \times 2}.$$

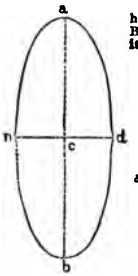


Fig 4

This rule is, however, close (about 1 per ct in excess) for parabolas, whose height is not more than 1-10th of the base; and still more so for flatter ones. But in a parabola whose height is 2, and base 1, it gives a curve of 4.73; whereas it is actually but about 4.2; being an error of nearly 13 per cent, or 1 in 7½. The following by the writer is correct within perhaps 1 part in 800, in all cases; and will therefore answer for many ordinary purposes.

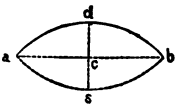


Fig 3

Let *a d b*, Fig 3, or *n a d*, Fig 4, be the parabola, in which are given the base *a b* or *n d*; and the height *c d* or *c a*. Imagine the complete fig *a d b s*, or *n a d b*, to be drawn; and in *either* case, assume its *long* diam *a b* to be the chord or base; and one-half the short diam, or *c d*, to be the height, of a circular arc. Find the length of this circular arc, by means of the rule and table given for that purpose. Then div the chord or base *a b*, or *n d* of the parabola, by its height *c d* or *c a*. Look for the quot in the column of bases in the following table, and take from the table, the corresponding multiplier. Mult the length of the circular segment by this; the prod will be the length of arc *a d b*, or *n a d*, as the case may be. For bases of parabolas less than .05 of the height, or greater than 10 times the height, the multiplier is 1, and is very approximate; or in other words, the parabola will be of almost exactly the same length as the circular arc.

To find the area of a parabola *m a n b*.

Mult its base *m n*, Fig 5, by its height *a b*; and take ⅔ds of the prod. The area of any segment, as *u b v*, whose base *u v* is parallel to *m n*, is found in the same way, using *u v* and *s b*, instead of *m n* and *a b*.

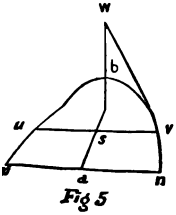


Fig 5

To find the area of a parabolic zone, or frustum, as *m n u v*.

Rule 1. First find by the preceding rule the area of the whole parabola *m b n*; then that of the segment *u b v*; and subtract the last from the first.

Rule 2. From the cube of *m n*, take the cube of *u v*; call the diff *a*. From the square of *m n*, take the square of *u v*; call the diff *a*. Div *a* by *a*. Mult the quot by ⅔ds of the height *a s*.

Original.

Base.	Mult.	Base.	Mult.	Base.	Mult.	Base.	Mult.
.05	1.000	1.10	.999	2.15	.949	3.20	.983
.10	1.001	1.15	.997	2.20	.951	3.30	.984
.15	1.002	1.20	.995	2.25	.954	3.40	.985
.20	1.004	1.25	.993	2.30	.956	3.50	.986
.25	1.006	1.30	.990	2.35	.958	3.60	.987
.30	1.007	1.35	.987	2.40	.960	3.70	.988
.35	1.007	1.40	.984	2.45	.962	3.80	.989
.40	1.008	1.45	.980	2.50	.963	3.90	.990
.45	1.009	1.50	.977	2.55	.965	4.00	.991
.50	1.010	1.55	.974	2.60	.967	4.25	.992
.55	1.010	1.60	.970	2.65	.969	4.50	.993
.60	1.010	1.65	.966	2.70	.970	4.75	.994
.65	1.011	1.70	.963	2.75	.972	5.00	.995
.70	1.011	1.75	.960	2.80	.973	5.25	.996
.75	1.010	1.80	.957	2.85	.975	5.50	.997
.80	1.009	1.85	.953	2.90	.976	5.75	.998
.85	1.008	1.90	.950	2.95	.978	6.00	.998
.90	1.006	1.95	.946	3.00	.979	7.00	.999
.95	1.004	2.00	.942	3.05	.980	8.00	1.000
1.00	1.002	2.05	.944	3.10	.981	10.00	1.000
1.05	1.001	2.10	.946	3.15	.982		

To draw a parabola,

c o s, Fig 6. Make *o t* equal to the height *e o*. Draw *c t* and *s t*; and divide each of them into any number of equal parts; numbering them as in the Fig. Join 1, 1; 2, 2; 3, 3, &c; then draw the curve by hand. It will be observed that the intersections of the lines 1, 1; 2, 2, &c, do not give points in the curve; but a portion of each of those lines forms a tangent to the curve. By increasing the number of divisions on *c t* and *s t*, an almost perfect curve is formed, scarcely requiring to be touched up by hand. In practice it is best first to draw only the center portions of the two lines which cross each other just above *o*; and from them to work downward; actually drawing only that small portion of each successive lower line, which is necessary to indicate the curve.

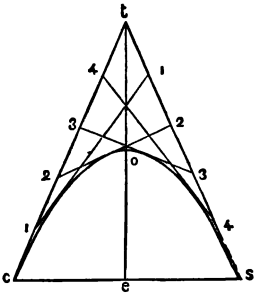


Fig 6

Or the parabola may be drawn thus:

Let *b a*, Fig 7, be the base; and *a d* the height. Draw the rectangle *b n m c*; div each half of the base into any number of equal parts, and number them from the center each way. Div *n b*, and *m c* into the same number of equal parts; and number them from the top, downward. From the points on *b a* draw vert lines; and from those at the sides draw lines to *d*. Then the intersections of lines 1, 1; 2, 2, &c, will form points in the parabola. As in the preceding case, it is not necessary to draw the entire lines; but merely portions of them, as shown between *d* and *c*.

Or a parabola may be drawn by first div the height *a b*, Fig 5, into any number of parts, either equal or unequal; and then calculating the ordinates *u s*, &c; thus, as the height *a b* : square of half base *a m* : : any abscisa *b s* : square of its ord *u s*. Take the sq rt for *u s*.

Rem.—When the height of a parabola is not greater than 1-10th part its base, the curve coincides so *very* closely with that of a circular segment, that in the preparation of drawings for suspension bridges, &c., the circular arc may be employed; or if no great accuracy is reqd, the circle may be used even when the height is as great as 1/4th base.

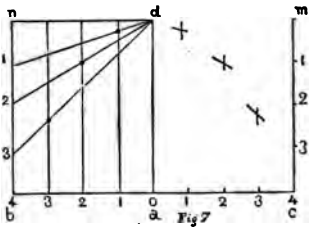


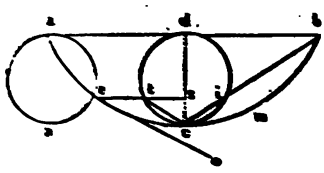
Fig 7

To draw a tangent *w v*, Fig. 5, to a parabola, from any point *v*.

Draw *v s* perp to axis *a b*; prolong *a b* until *b w* equals *s b*. Join *w v*.

The Cycloid,

a o b is the curve described by a point *a* in the circumf of a circle, *a n*, during one complete r[illegible] of the circle, rolled along a straight line *a b*; which is called the base of the cycloid.



base a b is evidently equal to the circumf of t
generating circle a n; and the axis, or bei
d e c, is equal to its diam. The length a c b =
times diam of a n. The area a c b d = 3 (ti
area of circle a n; and its cen of grav is at ⅓t
of d d, measured from the vertex c. To draw
tang e n, from any point e; draw e a at rig
angles to the axis d c; on d e describe the gene
ating circle d f e; join f c: from e draw e o parall
to f c. The cycloid is the *curve of quickest descen*
so that a body would fall from b to c along the cur
b n c, in less time than along the inclined pla
b f c, or any other line, except a vertical one.

PARALLELOPIPEDS.

Fig 1 Fig 2 Fig 3 Fig 4

A parallelopiped is any solid contained within six sides, all of which are parallelograms; and those of each opposite pair, parallel to each other. There are but four of these; corresponding to the four parallelograms, namely, the cube, Fig 1, which has all its sides equal squares; and all its angles right angles: the rectangular prism, Fig 2, which has all its angles right angles; each pair of opposite faces equal: but not all of its faces equal; the *Rhomb*, Fig 3, which has all its sides equal rhombuses: the *Rhomboic* prism, Fig 4; its faces rhombuses, or rhomboids; each pair of opposite faces equal, but not all its faces equal.

To find the solidity of any parallelopiped.

Mult the area of any face, as a, by the *perp dist*, p, to the opposite face.

The diag of a cube, or the diam of its circumscribing sphere, is equal to one of its edges mult by [illegible]. The diag of a rhomb, or of a rhomboic prism, cannot be calculated by means of its sides, and their angles.

PRISMS.

5 6 7 8 9 10

A prism is any solid whose two ends are parallel, similar, and equal; and whose sides are parallelograms, as Figs 5 to 10. Consequently the foregoing parallelopipeds are prisms. A right prism [illegible] sides are perp to its ends, as 5, 6, 7: when not so, the prism is oblique, as 8, 9, 10. When all the sides, or the figs which form the ends are equal, the angles included between those sides are also equal, the prism is then said to be regular; otherwise, irregular.

To find the solidity of any prism,

[illegible] right or oblique, regular or irregular: mult the area of one end by the *perp dist*, p, to the other end; or mult the area measured perp to the sides by the actual length a b; Figs. 5 to 10.

To find the solidity of any frustum* of any prism,

Whose cross section, perp to its sides, is either any triangle; any parallelogram; a square, (as Fig 10n) is supposed to be; or a regular polygon of any number of sides; no matter how the two ends of the frustum may be inclined with regard to each other; or whether one, or neither of them, is parallel to the base of the original prism. Measure and add together the lengths of all the parallel edges 11, 22, 33, 44. Div the sum by the number of them, for a mean length. Mult the area of cross section, perp to the sides, by this mean leng for the solidity.

This rule may be used for constructing individuals, [illegible] quantity of earth to be removed from a "turnpike." The irregular surface of the ground is then divided out in squares, the cut-line being circumscribed horizontally, where intersected by their sides. These squares should be of such a size that without material error each of them may be considered to [illegible]

* Generally adopted "frustum."

a plane surface, either hor, or inclined. The depth of the hor bottom of the pit being determined on; and the levels being taken at every corner of the squares, we are thereby furnished with the lengths of the four parallel vertical edges of each of the resulting frustums of earth. In Figs 10¼ y may be supposed to represent one of these frustums.*

If the frustum is that of an *irregular* 4-sided, or polygonal prism, first div its cross section perp to its *sides*, into triangles, by lines drawn from any one of its angles, as *a*, Fig 10½. Calculate the area of each of these triangles separately; then consider the entire frustum to be made up of so many triangular ones; calculate the solidity of each of these by the preceding rule for triangular frustums; and add them together, for the solidity of the entire frustum.

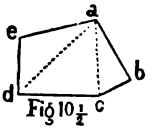


Fig 10½

The solidity of any frustum whatever, of any prism whatever,

Or of a cylinder, may be found thus. Consider either end to be the base; and find its area. Also find the cen of grav c of the other end, and the *perp* dist *n c*, from the base to said cen of grav. Mult this dist by the area of the base. Fig. 10¾.

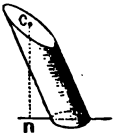


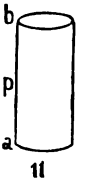
Fig 10¾

To find the surface of any prism; Figs. 5 to 10.

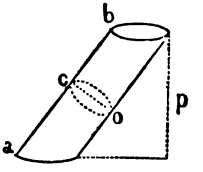
Whether right or oblique; regular, or irregular; mult the circumf of one end, by the *perp* dist *p* to the other end; this gives the surf of the sides; to which add the surf or area of the two ends, when reqd. Or mult the circumf measd perp to the sides by the actual length *a b*; then add the ends.

CYLINDERS.

A cylinder is any solid whose ends are parallel, similar, and equal *curved* figs; and whose sections parallel to the ends are every where the same as the ends. Hence there are circular, elliptic, (or cylindroids), and other cylinders; but when not otherwise expressed, the circular one is understood. A *right* cylinder is one whose ends are perp to its sides, as Fig 11; when otherwise, it is *oblique*, as Fig 12. If the ends of a right circular cylinder be cut so as to make it oblique, it becomes an elliptic one; because then both its ends, and all sections parallel to them, are ellipses. An oblique circular cylinder seldom occurs; it may be conceived of by imagining the two ends of Fig 12 to be circles, united by straight lines forming its curved sides.



11



12

To find the solidity of any cylinder.

Whether circular, elliptic, &c; right or oblique; as in prisms, mult the area of one end by the perp dist, *p*, to the other end. Or mult the area measd perp to the sides, by the actual length *a b*, Fig 11, 12. The solidity of a cylinder is 3 times that of a cone of the same base and height.

To find the surface of any cylinder,

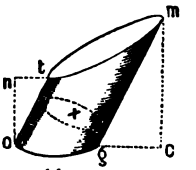
As in prisms, mult the circumf of one end by the *perp* dist *p* to the other end; this gives the surf of the sides; to which add that of the ends when reqd. Or mult the circumf measd perp to the sides as at *c c*, Fig 12, by the actual length *a b*, Figs 11 and 12, and add the ends.

Rem.—The solidity of a right cylinder whose height equals its diam, is to the solidity of its inscribed sphere, as 3 is to 2; the *curved* surf of the cylinder (that is, its sides,) is equal to the surf of the sphere: and the entire surf of the cylinder, including its ends, is to the surf of the sphere as 3 to 2; consequently the surf of the two ends of such a cylinder, is equal to half the surf of its sides. Any *slant* end, *c*, fig 10¾, is an ellipse, of greater area than a circle.

To find the solidity of a cylindric ungula, when the cutting plane does not pass through the base.

Figs 13 and 14; mult the area of its base, by half the sum of its greatest and least perp lengths *o n*, and *c m*. Or mult the area measd perp to the sides, as at *x*, Fig 14, by the half sum of the greatest and least actual lengths *o t* and *g m*.

13

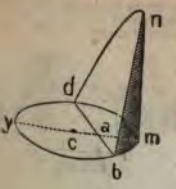


14

To find the surface of a cylindric ungula, when the cutting plane does not pass through the base.

Mult the circumf of its base, by the half sum of its greatest and least perp lengths *o n*, and *c m*. The prod will be the curved surf of the sides; to which add the ends if reqd. Or mult its circumf *measured perp to its sides*, as at *x*, Fig 14, by the half sum of the greatest and least actual *lengths o t, g m*. *Add the ends if reqd.*

* *Our text-books on mensuration strangely do not give rules for frustums of prisms, although such are of frequent occurrence.*



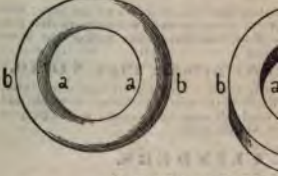
Cylindric ungula w passes thr

making $m\,a$, less than $m\,c$, or solidity, cube $a\,b$, and take ⅔ d the base $a\,d\,m\,b$, by $a\,c$. Take height $m\,n$; div the prod by a

For the co

Mult the diam $m\,y$ of the cir length of the arc $d\,m\,b$, by $a\,c$ by the height $m\,n$; div the prod

CIRCULAR RIN



Mult the
ner and o
Mult togeth

For the solidity

area of transverse section of the bar of which th
ter diams, $a\,a$, and $b\,b$, of the ring; mult the pro

For the surface

her the *girt* of the bar; the half sum of the two d

PYRAMIDS AND

A pyramid,

3, is any solid which has for its base, a plane fig of
gles, all terminating at one point d, called its *ape*
Bed is *regular*; otherwise *irregular*. For regular fi

A cone,

15, is a solid, of which the base is a curved fig; a
a line, of which one end is stationary at a certa
ing carried around the circumf of the base, whic
a pyramid, or cone, is a straight line $d\,o$ in Figs
entre of the base. When the axis is perp to th
ne; when otherwise, as Figs 3, 5, an *oblique* c
lar cone, Fig 4, is understood. If such a cone be
ll be an ellipse; and the cone $d\,t\,t$ becomes an
que circular cone, or an oblique elliptic one, acco

To find the solidity of any py

regular or irregular; right or oblique; mult the a
Figs 1 to 5. Every pyramid, or cone, has one-thi
ying the same area of base, and the same perp hei
base and height; in other words, a cone, hemisph
solidities as 1, 2, 3.

find the surface of any regular r cone.

the circumf or outline of its base, by the *slant* height;
the sides; to which add that of the base if reqd. In
from d to the middle of one of the equal sides, and not
unable to devise any measurement

the surface of an irre

, each side must be calculated as
rea of base if reqd.

nd the solid- of any frus- m of any pyr- id, or cone, hen the base d top are par- lel.

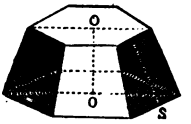


Fig 6

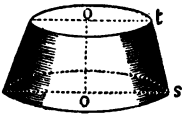


Fig 7

x 1. Whether regular gular, right or oblique, gether the areas of the nd top, and the area of ean proportional; mult m by ½d of the perp o o, Figs 6 and 7, between the base and top.

. For the area of the mean proportional, (which is *not* the area halfway between, and parallel he ends,) mult together the areas of base and top; and take the sq rt of the prod.

x 2. This applies only to the right circular conical frust. Add together the squares of the two ; and the prod of the two diams; mult the sum by ½d the perp height o o; mult the prod by .7854.

nd the surface of any frustum of a regular right pyramid, r cone, Figs 6 and 7, when the base and top are parallel.

together the circumfs of the two ends; mult the sum by the *slant* height s t; take half the prod. ives the surf of the sides: to which add that of the ends when reqd. In the frustum of the ld, the slant height must be mead between the middles s and t of two corresponding sides of se, and top. Fig. 6.

he pyramid is not regular; or if it is oblique, then the surf of the sides must be obtained for ide separately as a trapezoid.

WEDGES.

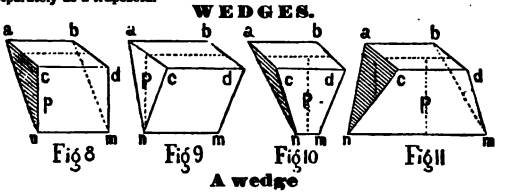


Fig 8 Fig 9 Fig 10 Fig 11

A wedge

ally defined to be a solid, Figs 8 and 9, generated by a plane triangle, *a n a*, moving parallel to in a straight line. This definition requires that the two triangular ends of the wedge should be el; but a wedge may be shaped as in Fig 10 or 11. We would therefore propose the following tion, which embraces all the figs; besides various modifications of them. A solid of five plane one of which is a parallelogram *a b c d*, two opposite sides of which, as *a c* and *b d*, are united ans of two triangular faces *a c n*, and *b d m*, to an edge or line *n m*, parallel to the other opposite s *b* and *c d*. The parallelogram *a b c d* may be either rectangular, or not; the two triangular may be similar, or not; and the same with regard to the other two faces. The following rule s equally to all.

To find the solidity of any wedge.

together the length of the edge *m n*, and twice the length *a b* or *c d* of the back; mult the sum perp height *p*, from the edge to the back; mult this prod by the breadth of the back, mead e its two sides *a b* and *c d*; div this last prod by 6.

PRISMOIDS.

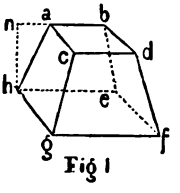


Fig 1

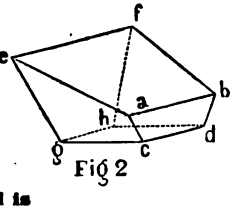


Fig 2

A prismoid is

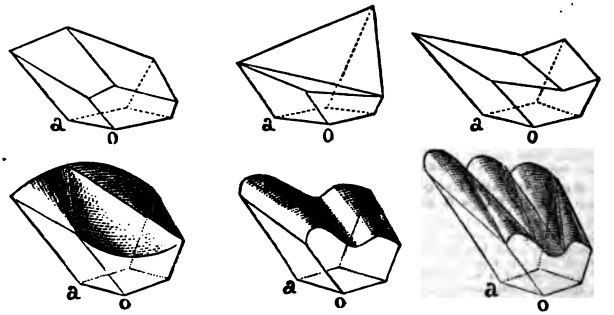
lid bounded by six plane surfaces, of which but two, as *a b c d* and *e f g h*, Figs 1 and 2, are l.

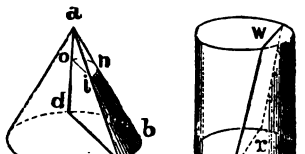
To find the solidity of any prismoid.

together the areas of the two parallel surfaces; and four times the area of the section take *r between them, and parallel* to them; mult the sum by the perp dist between the two parall *nd div the prod by 6. In Fig 1, h n* is the perp dist; and in Fig 2, which represents a railr *ss, h is g c.*

ending rule is the well-known "prismoidal formula;" the very extended applicati *other solids than those* which fall strictly within the definition of the prismoid, w

discovered and made known by Ellwood Morris, civ eng of Philadelphia, in 1840. It embraces all parallelopipeds, prisms, pyramids, cones, wedges, &c. whether regular or irregular, right or oblique; together with their frustums when cut parallel to their bases; indeed all solids whatever having two parallel faces, or sides, provided these two faces are united by surfaces, whether plane or curved, upon which, and through every point of which, a straight line may be drawn from one of the parallel faces to the other. The following six Figs represent a few such solids; they may be regarded as one-chain lengths of railroad cuttings; *a o* being the perp dist between the two parallel ends.





The prismoidal formula applies also to the sphere, hemisphere, and other spherical segments; also to any sections such as *a b c d*, and *o n i d b c*, of the cone, in which the sides *a d*, *a c*, or *o d*, *i c*, are straight; as they are only when the cutting plane *a d c* passes through the apex, or top *a*. Also to sections of the cylinder, when the cutting plane *w x* passes through both parallel ends. But it does not apply to circular spindles; nor to cylindric, nor conical ungulas; that is, to those portions of cylinders or cones in which the cutting plane passes through a curved side. In these last three, a straight line cannot be drawn through each and every point, from one parallel face to the other; and which shall at the same time coincide with the surface on which it is drawn.

SPHERES OR GLOBES.

A sphere

Is a solid generated by the revolution of a semicircle around its diam. Any line passing entirely through a sphere, and through its center, is called its *axis*, or diam. Any circle described on the surface of a sphere, from the center of the sphere as the center of the circle, is called a *great circle* of that sphere; in other words, any entire circumf of a sphere is a great circle. A sphere has a greater content or solidity than any other solid with the same amount of surface; so that if the shape of a sphere be any way changed, its content will be reduced.

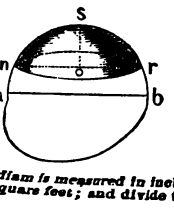
To find the solidity of a sphere.*

Cube its diam; mult said cube by .5236. Or cube its circumf; and mult by .01689. Or mult its surface by its diam; and div by 6. Or refer to the following table of spheres. The solidity of a sphere is ⅔ that of its circumscribing cylinder; or .5236 that of its circumscribing cube; or to 4 great circles × ⅙ diam.

To find the surface of a sphere.*

Square its diam; mult said square by 3.1416. Or mult the diam by the circumf. Or square its circumf; and mult by .3183. The surf of a sphere is equal to 4 times the area of its great circle; or to the area of a circle whose diam is twice as great as that of the sphere; or to the curved surf of its circumscribing cylinder.

To find the solidity of a spherical segment.



Rule 1. Square the rad *o n*, of its base; mult this square by 3; to the prod add the square of its height *o s*; mult the sum by the height *o s*; and mult this last prod by .5236.

Rule 2. Mult the diam *a b of the sphere* by 3; from the prod take twice the height *o s* of the segment; mult the rem by the square of the height *o s*; and mult this prod by .5236.

*In the following table the surfaces and solidities will be inches, feet, yards, &c, according as the diams are considered to be in inches, feet, yards, &c.

The solidity of a sphere being ⅔ds that of its circumscribing cylinder, if we add to any solidity in the table, its half, we obtain that of a cylinder of the same diam as the sphere, and whose height equals its diam.

If the diam is measured in inches, divide the surfaces in the table by 144, if it is required to reduce them to square feet; and divide the solidities by 1728, if required in cubic feet.

SPHERES. (ORIGINAL.)

Some errors of 1 in the last figure only.

Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.
1-64	.00077		13-32	18.190	7.2949	⅝	170.87	210.03	¼	921.33	2629.6
1-32	.00307	.00002	7-16	18.666	7.5829	⅝	176.71	220.89	¼	934.83	2687.6
3-64	.00690	.00005	15-32	19.147	7.8783	⅝	182.66	232.13	⅜	948.43	2746.5
1-16	.01227	.00013	½	19.635	8.1813	⅝	188.69	243.73	½	962.12	2806.2
5-32	.02761	.00043	17-32	20.129	8.4919	⅞	194.83	255.72	⅝	975.91	2866.8
⅛	.04909	.00102	9-16	20.629	8.8103	8.	201.06	268.08	¾	989.80	2928.2
7-32	.07670	.00200	19-32	21.135	9.1366	⅛	207.39	280.85	⅞	1003.8	2990.5
3-16	.11045	.00345	⅝	21.648	9.4708	¼	213.82	294.01	18.	1017.9	3053.6
7-32	.15033	.00548	21-32	22.166	9.8131	⅜	220.36	307.58	⅛	1032.1	3117.7
¼	.19635	.00818	11-16	22.691	10.164	½	226.98	321.56	¼	1046.4	3182.6
9-32	.24851	.01165	23-32	23.222	10.522	⅝	233.71	335.95	⅜	1060.8	3248.5
5-16	.30680	.01598	¾	23.758	10.889	¾	240.53	350.77	½	1075.2	3315.3
11-32	.37123	.02127	25-32	24.302	11.265	⅞	247.45	366.02	⅝	1089.8	3382.9
⅜	.44179	.02761	13-16	24.850	11.649	9.	254.47	381.70	¾	1104.5	3451.5
13-32	.51848	.03511	27-32	25.405	12.041	⅛	261.59	397.83	⅞	1119.3	3521.0
7-16	.60132	.04385	⅞	25.967	12.443	¼	268.81	414.41	19.	1134.1	3591.4
15-32	.69028	.05393	29-32	26.535	12.853	⅜	276.12	431.44	⅛	1149.1	3662.8
½	.78540	.06545	15-16	27.109	13.272	½	283.53	448.92	¼	1164.2	3735.0
17-32	.88664	.07850	31-32	27.688	13.700	⅝	291.04	466.87	⅜	1179.3	3808.2
9-16	.99403	.09319	3.	28.274	14.137	¾	298.65	485.31	½	1194.6	3882.5
19-32	1.1075	.10960	1-16	29.465	15.039	⅞	306.36	504.21	⅝	1210.0	3957.6
⅝	1.2272	.12783	⅛	30.680	15.979	10.	314.16	523.60	¾	1225.4	4033.7
21-32	1.3530	.14798	3-16	31.919	16.957	⅛	322.06	543.48	⅞	1241.0	4110.8
11-16	1.4849	.17014	¼	33.183	17.974	¼	330.06	563.86	20.	1256.7	4188.8
23-32	1.6230	.19442	5-16	34.472	19.031	⅜	338.16	584.74	⅛	1272.4	4267.8
¾	1.7671	.22089	⅜	35.784	20.129	½	346.36	606.13	¼	1288.3	4347.8
25-32	1.9175	.24967	7-16	37.122	21.268	⅝	354.66	628.04	⅜	1304.2	4428.8
13-16	2.0739	.28084	½	38.484	22.449	¾	363.05	650.46	½	1320.3	4510.9
27-32	2.2365	.31451	9-16	39.872	23.674	⅞	371.54	673.42	⅝	1336.4	4593.9
⅞	2.4053	.35077	⅝	41.283	24.942	11.	380.13	696.91	¾	1352.7	4677.9
29-32	2.5802	.38971	11-16	42.719	26.254	⅛	388.83	720.95	⅞	1369.0	4763.0
15-16	2.7611	.43143	¾	44.179	27.611	¼	397.61	745.51	21.	1385.5	4849.1
31-32	2.9483	.47603	13-16	45.664	29.016	⅜	406.49	770.64	⅛	1402.0	4936.3
1.	3.1416	.52360	⅞	47.173	30.466	½	415.48	796.33	¼	1418.6	5024.3
1-32	3.3410	.57424	15-16	48.708	31.965	⅝	424.56	822.58	⅜	1435.4	5113.5
1-16	3.5466	.62804	4.	50.265	33.510	¾	433.73	849.40	½	1452.2	5203.7
3-32	3.7583	.68511	1-16	51.848	35.106	⅞	443.01	876.79	⅝	1469.2	5295.1
⅛	3.9761	.74551	⅛	53.456	36.751	12.	452.39	904.78	¾	1486.2	5387.4
5-32	4.2000	.80939	3-16	55.089	38.448	⅛	461.87	933.34	⅞	1503.3	5480.8
3-16	4.4301	.87681	¼	56.745	40.195	¼	471.44	962.52	22.	1520.5	5575.3
7-32	4.6664	.94786	5-16	58.427	41.994	⅜	481.11	992.28	⅛	1537.9	5670.8
¼	4.9088	1.0277	⅜	60.133	43.847	½	490.87	1022.7	¼	1555.3	5767.6
9-32	5.1573	1.1013	7-16	61.863	45.752	⅝	500.73	1053.6	⅜	1572.8	5865.2
5-16	5.4119	1.1839	½	63.617	47.713	¾	510.71	1085.3	½	1590.4	5964.1
11-32	5.6728	1.2704	9-16	65.397	49.729	⅞	520.77	1117.5	⅝	1608.2	6064.1
⅜	5.9396	1.3611	⅝	67.201	51.801	13.	530.93	1150.3	¾	1626.0	6165.2
13-32	6.2126	1.4561	11-16	69.030	53.929	⅛	541.19	1183.8	⅞	1643.9	6267.3
7-16	6.4919	1.5553	¾	70.883	56.116	¼	551.55	1218.0	23.	1661.9	6370.6
15-32	6.7771	1.6590	13-16	72.759	58.359	⅜	562.00	1252.7	⅛	1680.0	6475.0
½	7.0686	1.7671	⅞	74.663	60.663	½	572.55	1288.3	¼	1698.2	6580.6
17-32	7.3663	1.8799	15-16	76.589	63.026	⅝	583.20	1324.4	⅜	1716.5	6687.3
9-16	7.6699	1.9974	5.	78.540	65.450	¾	593.95	1361.2	½	1735.0	6795.2
19-32	7.9798	2.1196	1-16	80.516	67.935	⅞	604.80	1398.6	⅝	1753.5	6904.2
⅝	8.2957	2.2468	⅛	82.516	70.482	14.	615.75	1436.8	¾	1772.1	7014.3
21-32	8.6180	2.3789	3-16	84.541	73.092	⅛	626.80	1475.6	⅞	1790.8	7125.6
11-16	8.9461	2.5161	¼	86.591	75.767	¼	637.95	1515.1	24.	1809.6	7238.2
23-32	9.2805	2.6586	5-16	88.664	78.505	⅜	649.17	1555.3	⅛	1828.5	7351.9
¾	9.6211	2.8062	⅜	90.763	81.308	½	660.52	1596.3	¼	1847.5	7466.7
25-32	9.9678	2.9592	7-16	92.887	84.178	⅝	671.95	1637.9	⅜	1866.6	7583.0
13-16	10.321	3.1177	½	95.033	87.113	¾	683.49	1680.3	½	1885.8	7700.1
27-32	10.680	3.2818	9-16	97.205	90.118	⅞	695.13	1723.3	⅝	1905.1	7818.6
⅞	11.044	3.4514	⅝	99.401	93.189	15.	706.85	1767.2	¾	1924.4	7938.3
29-32	11.416	3.6270	11-16	101.62	96.331	⅛	718.69	1811.7	⅞	1943.9	8059.2
15-16	11.793	3.8083	¾	103.87	99.541	¼	730.63	1857.0	25.	1963.5	8181.3
31-32	12.177	3.9956	13-16	106.14	102.82	⅜	742.65	1903.0	⅛	1983.2	8304.7
2.	12.566	4.1888	⅞	108.44	106.18	½	754.77	1949.8	¼	2002.9	8429.2
1-32	12.962	4.3882	15-16	110.75	109.60	⅝	767.00	1997.4	⅜	2022.9	8554.9
1-16	13.364	4.5939	6.	113.10	113.10	¾	779.32	2045.7	½	2042.8	8682.0
3-32	13.772	4.8059	⅛	117.87	120.31	⅞	791.73	2094.8	⅝	2062.9	8810.3
⅛	14.186	5.0243	¼	122.72	127.83	16.	804.25	2144.7	¾	2083.0	8939.9
5-32	14.607	5.2493	⅜	127.68	135.66	⅛	816.85	2195.3	⅞	2103.4	9070.8
3-16	15.033	5.4809	½	132.73	143.79	¼	829.57	2246.8	26.	2123.7	9202.8
7-32	15.466	5.7190	⅝	137.89	152.25	⅜	842.40	2299.1	⅛	2144.2	9336.
¼	15.904	5.9641	¾	143.14	161.03	½	855.29	2352.1	¼	2164.7	9470
9-32	16.349	6.2161	⅞	148.49	170.14	⅝	868.31	2406.0	⅜	2185.5	960
5-16	16.800	6.4751	7.	153.94	179.59	¾	881.42	2460.6	½	2206.2	97
11-32	17.258	6.7412	⅛	159.49	189.39	⅞	894.62	2516.1	⅝	2227.1	
⅜	17.721	7.0144	¼	165.13	199.53	17.	907.93	2572.4	¾	2248.0	

SPHERES—(Continued.)

Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.
⅛	2269.1	10164	⅛	4214.1	25724	⅛	6756.5	52222	⅛	9896.0
27.	2290.2	10306	¼	4243.0	26088	¼	6792.9	52645	¼	9940.2
⅛	2311.5	10450	⅜	4271.8	26254	⅜	6829.5	53071	⅜	9984.4
¼	2332.8	10595	37.	4300.9	26522	½	6866.1	53499	½	10029
⅜	2354.3	10741	⅛	4330.0	26792	⅝	6902.9	53929	⅝	10073
½	2375.8	10889	¼	4359.2	27063	47.	6939.9	54362	¾	10118
⅝	2397.5	11038	⅜	4388.5	27337	⅛	6976.8	54797	⅞	10163
¾	2419.2	11189	½	4417.9	27612	¼	7013.9	55234	57.	10207
⅞	2441.1	11341	⅝	4447.5	27889	⅜	7050.9	55674	⅛	10252
28.	2463.0	11494	¾	4477.1	28168	½	7088.3	56115	¼	10297
⅛	2485.1	11649	⅞	4506.8	28449	⅝	7125.6	56559	⅜	10342
¼	2507.2	11805	38.	4536.5	28731	¾	7163.1	57006	½	10387
⅜	2529.5	11962	⅛	4566.5	29016	⅞	7200.7	57455	⅝	10432
½	2551.8	12121	¼	4596.4	29302	48.	7238.3	57906	¾	10478
⅝	2574.3	12281	⅜	4626.5	29590	⅛	7276.0	58360	⅞	10523
¾	2596.7	12443	½	4656.7	29880	¼	7313.9	58815	58.	10568
⅞	2619.4	12606	⅝	4686.9	30173	⅜	7351.9	59274	⅛	10614
29.	2642.1	12770	¾	4717.3	30466	½	7389.9	59734	¼	10660
⅛	2665.0	12936	⅞	4747.9	30762	⅝	7428.0	60197	⅜	10706
¼	2687.8	13103	39.	4778.4	31059	¾	7466.3	60663	½	10751
⅜	2710.9	13272	⅛	4809.0	31359	⅞	7504.5	61131	⅝	10798
½	2734.0	13442	¼	4839.9	31661	49.	7543.1	61601	¾	10844
⅝	2757.3	13614	⅜	4870.8	31964	⅛	7581.6	62074	⅞	10890
¾	2780.5	13787	½	4901.7	32270	¼	7620.1	62549	59.	10936
⅞	2804.0	13961	⅝	4932.7	32577	⅜	7658.9	63026	⅛	10983
30.	2827.4	14137	¾	4964.0	32886	½	7697.7	63506	¼	11029
⅛	2851.1	14315	⅞	4995.3	33197	⅝	7736.7	63989	⅜	11076
¼	2874.8	14494	40.	5026.8	33510	¾	7775.7	64474	½	11122
⅜	2898.7	14674	⅛	5058.1	33825	⅞	7814.8	64961	⅝	11169
½	2922.5	14856	¼	5089.6	34143	50.	7854.0	65450	¾	11216
⅝	2946.6	15039	⅜	5121.3	34462	⅛	7893.3	65941	⅞	11263
¾	2970.6	15224	½	5153.1	34783	¼	7932.8	66436	60.	11310
⅞	2994.9	15411	⅝	5184.9	35106	⅜	7972.2	66934	⅛	11357
31.	3019.1	15599	¾	5216.8	35431	½	8011.8	67433	¼	11404
⅛	3043.6	15788	⅞	5248.9	35758	⅝	8051.6	67935	⅜	11452
¼	3068.0	15979	41.	5281.1	36087	¾	8091.4	68439	½	11499
⅜	3092.7	16172	⅛	5313.3	36418	⅞	8131.3	68946	⅝	11547
½	3117.3	16366	¼	5345.6	36751	51.	8171.2	69456	¾	11595
⅝	3142.1	16561	⅜	5378.1	37086	⅛	8211.4	69967	⅞	11642
¾	3166.9	16758	½	5410.7	37423	¼	8251.6	70482	61.	11690
⅞	3192.0	16957	⅝	5443.3	37763	⅜	8292.0	70999	⅛	11738
32.	3217.0	17157	¾	5476.0	38104	½	8332.3	71519	¼	11786
⅛	3242.2	17359	⅞	5508.9	38448	⅝	8372.8	72040	⅜	11834
¼	3267.4	17563	42.	5541.9	38792	¾	8413.4	72565	½	11882
⅜	3292.9	17768	⅛	5574.9	39140	⅞	8454.1	73092	⅝	11931
½	3318.3	17974	¼	5608.0	39490	52.	8494.8	73622	¾	11980
⅝	3343.9	18182	⅜	5641.3	39841	¼	8535.8	74154	⅞	12028
¾	3369.6	18392	½	5674.5	40194	¼	8576.8	74689	62.	12076
⅞	3395.4	18604	⅝	5708.0	40551	⅜	8617.8	75226	⅛	12126
33.	3421.2	18817	¾	5741.5	40908	½	8658.9	75767	¼	12174
¼	3447.3	19032	⅞	5775.2	41268	⅝	8700.4	76309	⅜	12223
¼	3473.3	19248	43.	5808.8	41630	¾	8741.7	76854	½	12272
⅜	3499.5	19466	⅛	5842.7	41994	⅞	8783.2	77401	⅝	12322
½	3525.7	19685	¼	5876.5	42360	53.	8824.8	77952	¾	12371
⅝	3552.1	19907	⅜	5910.7	42729	⅛	8866.4	78505	⅞	12420
¾	3578.5	20129	½	5944.7	43099	¼	8908.2	79060	63.	12469
⅞	3605.1	20354	⅝	5978.9	43472	⅜	8950.1	79617	¼	12519
34.	3631.7	20580	¾	6013.2	43846	½	8992.0	80178	¼	12568
⅛	3658.5	20808	⅞	6047.7	44224	⅝	9034.1	80741	⅜	12618
¼	3685.3	21037	44.	6082.1	44602	¾	9076.4	81308	½	1:668
⅜	3712.3	21268	¼	6116.8	44984	⅞	9118.5	81876	⅝	12718
½	3739.3	21501	⅜	6151.5	45367	54.	9160.8	82448	¾	12768
⅝	3766.5	21736	½	6186.3	45753	⅛	9203.2	83021	⅞	12819
¾	3793.7	21972	⅝	6221.2	46141	¼	9246.0	83598	64.	12868
⅞	3821.1	22210	¾	6256.1	46530	⅜	9288.5	84177	¼	12918
35.	3848.5	22449	⅞	6291.2	46922	½	9331.2	84760	¼	12969
⅛	3876.1	22691	45.	6326.5	47317	⅝	9374.1	85344	⅜	13019
¼	3903.7	22934	⅛	6361.7	47713	¾	9417.2	85931	½	13070
⅜	3931.5	23179	¼	6397.2	48112	⅞	9460.2	86521	⅝	13121
½	3959.2	23425	⅜	6432.7	48513	55.	9503.2	87114	¾	13172
⅝	3987.2	23674	½	6468.3	48916	⅛	9546.5	87709	⅞	13222
¾	4015.2	23924	⅝	6503.9	49321	¼	9590.0	88307	65.	13273
⅞	4043.3	24176	¾	6539.7	49729	⅜	9633.5	88908	⅛	13324
36.	4071.5	24429	⅞	6575.5	50139	½	9676.8	89511	¼	13376
⅛	4099.9	24685	46.	6611.6	50551	⅝	9720.6	90117	⅜	13427
¼	4128.3	24942	⅛	6647.6	50965	¾	9764.4	90726	½	13478
⅜	4156.9	25201	¼	6683.7	51382	⅞	9808.1	91338	⅝	13530
½	4185.5	25461	⅜	6720.0	51801	56.	9852.0	91953	¾	13582

SPHERES—(Continued.)

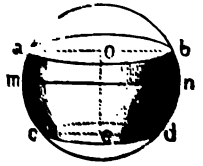
Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.	Diam.	Surface.	Solidity.
⅛	13633	149680	⅛	17437	216505	¼	21708	300743	¾	26446	404406
66.	13685	150533	⅜	17496	217597	⅜	21773	302100	⅞	26518	406060
¼	13737	151390	½	17554	218698	½	21839	303463	92.	26590	407721
⅜	13789	152251	⅝	17613	219792	⅝	21904	304831	⅛	26663	409384
½	13841	153114	75.	17672	220894	¾	21970	306201	¼	26735	411054
⅝	13893	153980	⅛	17731	222001	⅞	22036	307576	⅜	26808	412726
¾	13946	154850	¼	17790	223111	⅛	22102	308957	½	26880	414405
⅞	13998	155724	⅜	17849	224224	84.	22167	310340	⅝	26953	416086
⅛	14050	156600	½	17908	225341	⅛	22234	311728	¾	27026	417774
67.	14103	157480	⅝	17968	226463	¼	22300	313118	⅞	27099	419464
⅛	14156	158363	¾	18027	227588	⅜	22366	314514	93.	27172	421161
¼	14208	159250	⅞	18087	228716	½	22432	315915	⅛	27245	422862
⅜	14261	160139	76.	18146	229848	⅝	22499	317318	¼	27318	424567
½	14314	161032	⅛	18206	230984	¾	22565	318726	⅜	27391	426277
⅝	14367	161927	¼	18266	232124	⅞	22632	320140	½	27464	427991
¾	14420	162827	⅜	18326	233267	85.	22698	321556	⅝	27538	429710
⅞	14474	163731	½	18385	234414	⅛	22765	322977	¾	27612	431433
68.	14527	164637	⅝	18446	235566	¼	22832	324402	⅞	27686	433160
⅛	14580	165547	¾	18506	236719	⅜	22899	325831	94.	27759	434894
¼	14634	166460	⅞	18566	237879	½	22966	327264	⅛	27833	436630
⅜	14688	167376	77.	18626	239041	⅝	23034	328702	¼	27907	438373
½	14741	168295	⅛	18687	240206	¾	23101	330142	⅜	27981	440118
⅝	14795	169218	¼	18748	241376	⅞	23168	331588	½	28055	441871
¾	14849	170145	⅜	18809	242551	86.	23235	333039	⅝	28130	443625
⅞	14903	171074	½	18869	243728	⅛	23303	334492	¾	28204	445387
69.	14957	172007	⅝	18930	244908	¼	23371	335951	⅞	28278	447151
⅛	15012	172944	¾	18992	246093	⅜	23439	337414	95.	28353	448920
¼	15066	173883	⅞	19053	247283	½	23506	338882	⅛	28428	450695
⅜	15120	174828	78.	19114	248475	⅝	23575	340352	¼	28503	452475
½	15175	175774	⅛	19175	249672	¾	23643	341829	⅜	28577	454259
⅝	15230	176723	¼	19237	250873	⅞	23711	343307	½	28652	456047
¾	15284	177677	⅜	19298	252077	87.	23779	344792	⅝	28727	457839
⅞	15339	178635	½	19360	253284	⅛	23847	346281	¾	28802	459638
70.	15394	179595	⅝	19422	254496	¼	23916	347772	⅞	28878	461439
⅛	15449	180559	¾	19483	255713	⅜	23984	349269	96.	28953	463248
¼	15504	181525	⅞	19545	256932	½	24053	350771	⅛	29028	465059
⅜	15560	182497	79.	19607	258155	⅝	24122	352277	¼	29104	466875
½	15615	183471	⅛	19669	259383	¾	24191	353785	⅜	29180	468697
⅝	15670	184449	¼	19732	260613	⅞	24260	355301	½	29255	470524
¾	15726	185430	⅜	19794	261848	88.	24328	356819	⅝	29331	472354
⅞	15782	186414	½	19856	263088	⅛	24398	358342	¾	29407	474189
71.	15837	187402	⅝	19919	264330	¼	24467	359869	⅞	29483	476029
⅛	15892	188394	¾	19981	265577	⅜	24536	361400	97.	29559	477874
¼	15949	189389	⅞	20044	266829	½	24606	362935	⅛	29636	479725
⅜	16005	190387	80.	20106	268083	⅝	24676	364476	¼	29712	481579
½	16061	191389	⅛	20170	269342	¾	24745	366019	⅜	29788	483438
⅝	16117	192395	¼	20232	270604	⅞	24815	367568	½	29865	485302
¾	16174	193404	⅜	20296	271871	89.	24885	369122	⅝	29942	487171
⅞	16230	194417	½	20358	273141	⅛	24955	370678	¾	30018	489045
72.	16286	195433	⅝	20422	274416	¼	25025	372240	⅞	30095	490924
⅛	16343	196453	¾	20485	275694	⅜	25095	373806	98.	30172	492808
¼	16400	197476	⅞	20549	276977	½	25165	375378	⅛	30249	494695
⅜	16456	198502	81.	20612	278263	⅝	25236	376954	¼	30326	496588
½	16513	199532	⅛	20676	279553	¾	25306	378531	⅜	30404	498486
⅝	16570	200566	¼	20740	280847	⅞	25376	380115	½	30481	500388
¾	16628	201604	⅜	20804	282145	90.	25447	381704	⅝	30558	502296
⅞	16685	202645	½	20867	283447	⅛	25518	383297	¾	30636	504208
73.	16742	203689	⅝	20932	284754	¼	25589	384894	⅞	30713	506125
⅛	16799	204737	¾	20996	286064	⅜	25660	386496	99.	30791	508047
¼	16857	205789	⅞	21060	287378	½	25730	388102	⅛	30869	509975
⅜	16914	206844	82.	21124	288696	⅝	25802	389711	¼	30947	511906
½	16972	207903	⅛	21189	290019	¾	25873	391327	⅜	31025	513843
⅝	17030	208966	¼	21253	291345	⅞	25944	392945	½	31103	515785
¾	17088	210032	⅜	21318	292674	91.	26016	394570	⅝	31181	517730
⅞	17146	211102	½	21382	294010	⅛	26087	396197	¾	31259	519682
74.	17204	212175	⅝	21448	295347	¼	26159	397831	⅞	31338	521638
⅛	17262	213252	¾	21512	296691	⅜	26230	399468	100.	31416	523598
¼	17320	214333	⅞	21578	298036	½	26302	401109			
⅜	17379	215417	83.	21642	299388	⅝	26374	402756			

To find the curved surface of a spherical segment.

See last Fig. *Rule 1. Mult the diam b of the sphere from which the segment is cut, by 3.14*
mult the prod by the height o s of the seg. Add the area of base reqd. Rem. Having the diam
of the seg. and its height o s, the diam a b of the sphere may be found thus: Div the square of
the diam s r, by its height o s; to the quot add the height o s. Rule 2. The curved surf of
a segment, (last Fig.) or of a zone, (next Fig.) bears the same proportion to the surf of the

D

sphere, that the height of the seg or zone bears to the diam of the sphere. Therefore, surf of the whole sphere, either by rule or from the preceding table; mult it by the hei or zone; div the prod by diam of sphere. RULE 3. Mult the circumf of the sphere by of the seg.

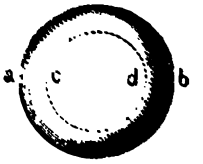


To find the solidity of a spheric

Add together the square of the rad *e d*, the squa and ⅓d of the square of the perp height *e o*; mult 1.5708; and mult this prod by the height *e o*.

To find the curved surface of a leal zone.

RULE 1. Mult together the diam *m n* of the sphere *e e* of the zone, and the number 3.1416. Or see prec for surf of segments. Rule 2. Mult the circumf of t the height of the zone.



To find the solidity of a hollow leal shell.

Take from the foregoing table the solidities of two sp the diams *a b*, and *c d*. Subtract the least from the gr *a c* or *b d* is the thickness of the shell.

THE REGULAR BODI

A regular body

is one which has all its sides, and its solid angles, similar and equal to each other. There are but five namely the *Tetraedron*, bounded by four equilateral triangles; the *Hexaedron, or cube*, the *Octaedron*, bounded by eight equilateral triangles; the *Dodecaedron*, twelve equilateral pentagons, and the *Icosaedron*, bounded by twenty equilateral triang

To find the solidity, or the surface of any of the regular bodies.

For the solidity, cube the length of one of its edges; mult said cube by the correspond in the following column of solidities. For the surface, square the length of one of its said square by the corresponding number in the following column of surfs.

Names.	Surfaces.	Solidities.
Tetraedron	1.7320	.1178
Hexaedron	6.	1.
Octaedron	3.4641	.4714
Dodecaedron	20.5458	7.6631
Icosaedron	5.0802	2.1817

THE ELLIPSOID, OR SPHEROID,

is a solid generated by the revolution of an ellipse around either its long or its short di [illegible] Fig 1. it is an oblate spheroid; when around the short a [illegible]

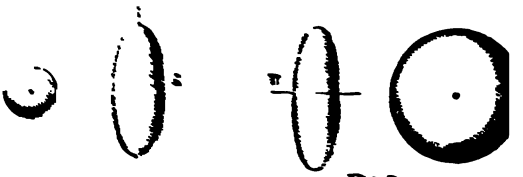


Fig 1 Fig 2

For the solidity.

[illegible] the square of the revolving one: and m [illegible]

THE PARABOLOID, OR PARABOLIC CONOI

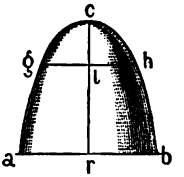
[illegible] a parabola e c b, around its axis e r.

For its solidity.

[illegible] mult together the square of the

For the solidity of a frustum,

h, the ends of which are perp to the axis $r\,c$; add together the
es of the two diams $a\,b$ and $g\,h$; mult the sum by the height $r\,l$;
the prod by the decimal .3927.



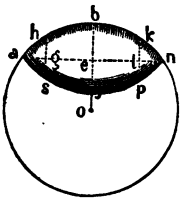
To find the surface of a paraboloid,

the rad $a\,r$ of its base, by 6.2832; div the prod by 12 times the
of the height $r\,c$; call the quot p. Then add together the square
rad $a\,r$, and 4 times the square of the height $r\,c$. Cube the
take the sq rt of this cube; from the sq rt subtract the cube of
d $a\,r$. Mult the rem by p.
er the solidity, or the surface of a frustum, $a\,b\,g\,h$, when $g\,h$ is
el to $a\,b$, may be found by calculating for the whole paraboloid,
e the upper portion $c\,g\,h$, as two separate paraboloids, and taking their diff.

THE CIRCULAR SPINDLE,

olid $a\,b\,n\,y$ generated by the revolution of a circular segment
s a, around its chord $a\,n$ as an axis.

To find its solidity.



e 1. First find the area of $a\,b\,e$, or half the generating circular
st. Then to the square of $a\,e$, add the square of $b\,e$; div the sum
; from the quot take $b\,e$; mult the rem by the area of $a\,e\,b$; call
d p. Cube $a\,e$; div the cube by 3; from the quot take p. Mult the
12.5664.
e 2. When the dist $o\,e$ is known, from the center of the circle to
of the spindle, then mult that dist $o\,e$, by the area of $a\,b\,e$; call
d p; cube $a\,e$; div the cube by 3; from the quot take p; mult the
12.5664.

To find its surface.

1. First find the length of the circular arc $a\,b\,n$; and mult it by
$o\,e$ from the center of the circle to the center of the spindle. Call
d p. Next mult the length $a\,n$ of the spindle, by the rad $o\,b$ of the circle. From the prod
mult the rem by 6.2832.
2. First find the length of the arc $a\,b\,n$. Square $a\,e$; also square $b\,e$; add these squares
r: div their sum by $b\,y$; call the quot s; and mult it by $a\,n$; call the prod p. Next from s take
ult the rem by the length of the arc $a\,b\,n$. Subtract the prod from p; mult the rem by 6.2832.

ind the solidity of a middle zone of a circular spindle,

$s\,k\,p$

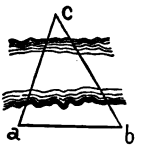
$$\left(\left(a\,e^2 - \frac{g\,e^2}{3}\right) \times g\,e\right) - \left(o\,e \times \text{area of } g\,h\,l\,k\right)\Bigg) \times 6.2832.$$

PLANE TRIGONOMETRY.

e trigonometry teaches how to find certain unknown parts of plane, or straight-sided tri-
by means of other parts which are known; and thus enables us to measure inaccessible dis-
&c. A triangle consists of six parts, namely, three sides, and three angles; and if we know
ree of these, (except the three angles, and in the ambiguous case under "Case 2,") we can find
er three. The following four cases include the whole subject; the student should commit them
ory.

e 1. Having any two angles, and one side, to find the other sides and angle.

the two angles together; and subtract their sum from 180°; the rem
the third angle. And for the sides, as



e of the angle : Sine of the angle : : given side : reqd side.
the given side : opp the reqd side : :

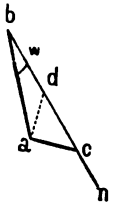
the side thus found, as the given one; and in the same manner find
rd side. As a practical example of the use of Case 1, if we measure
e or side $a\,b$ along one shore of a river; and also the two angles
$a\,c$, to an object c on the opposite shore, we can calculate the dist $a\,c$,
to that object; or by drawing the triangle on paper, to a scale, we can
ard measure $a\,c$, or $b\,c$ by the scale.

2. Having two sides, and the angle opposite to one of them,

the other side, and angles. This is of use only when that given side which is opp the given
is as long, or longer than the other given side.

Side opp the given angle : Side opp the reqd angle : : Sine of angle opp the former : Sine of angle opp the latter.

ng found the sine, take out the corresponding angle from the table of nat sines.
When the given side a c. next Fig. opposite the given angle a b c, is *shorter* than the other given
then the above proportion alone does not enable us to determine the other parts; for it
after having drawn the side a b, and laid down the given angle a b c, and extended it inde
ard a, we cannot tell whether the length of a b is to be drawn from a to c, or from a t



If $a\,c$ is as long or longer than $a\,b$, there can be no doubt; for in that case it cannot be drawn toward b, but only toward n, as in the Fig. Therefore, in taking field-notes, the two sides and the angle opp one of them, must not be depended upon as sufficient data.

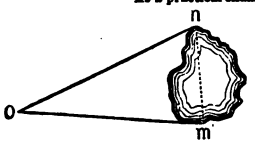
Case 3. Having two sides, and the angle included between them.

Take the angle from 180°; the rem will be the sum of the two unknown angles. Div this sum by 2; and find the nat tang of the quot. Then as

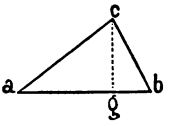
The sum of the two given sides : Their diff : : Tang of half the sum of the two unknown angles : Tang of half their diff.

Take from the table of nat tang, the angle opposite this last tang. Add this angle to the half sum of the two unknown angles, and it will give the angle opp the longest given side; and subtract it from the same half sum, for the angle opp the shortest given side. Having thus found the angles, find the third side by Case 1.

As a practical example of the use of Case 3, we can ascertain the dist $n\,m$ across a deep pond, by measuring two lines $n\,o$ and $m\,o$; and the angle $n\,o\,m$. From this data we may calculate $n\,m$; or by drawing the two sides, and the angle on paper, by a scale, we can afterward measure $n\,m$ on the drawing.



Case 4. Having the three sides,



To find the three angles; upon one side $a\,b$ as a base, draw (or suppose to be drawn) a perp $c\,g$ from the opposite angle c. Find the diff between the other two sides, $a\,c$ and $c\,b$: also their sum. Then, as

The base : Sum of the other two sides : : Diff of other two sides : Diff of the two parts $a\,g$ and $b\,g$, of the base.

Add *half* this diff of the parts, to *half* the base $a\,b$; the sum will be the longest part $a\,g$; which taken from the whole base, gives the shortest part $g\,b$. By this means we get in each of the small triangles $a\,c\,g$ and $c\,g\,b$, two sides, (namely, $a\,c$ and $a\,g$; and $c\,b$ and $g\,b$;) and an angle (namely, the right angle $c\,g\,a$, or $c\,g\,b$) opposite to one of the given sides. Therefore, use Case 2 for finding the angles a and b. When that is done, take their sum from 180°, for the angle $a\,c\,b$.

The following are useful examples of trigonometry.

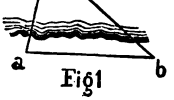


Fig 1

Ex. 1. To find the dist from a to an inaccessible object c.

Measure a line $a\,b$; and from its ends measure the angles $c\,a\,b$ and $c\,b\,a$. Thus having found one side and two angles of the triangle $a\,b\,c$, calculate $a\,c$ by means of Case 1. Or if extreme accuracy is not reqd, draw the line $a\,b$ on paper to any convenient scale; then by means of a protractor lay off the angles $c\,a\,b$, $c\,b\,a$; and draw $a\,c$ and $c\,b$; then measure $a\,c$ by the same scale. When possible, it is well to make $a\,b$ about as long as $a\,c$ is supposed to be; but when $a\,c$ is very long, this often cannot be done; and we must be content with a much shorter base $a\,b$; in which case care should be taken to measure the angles with great accuracy, as a slight error in them will then cause a greater error in $a\,c$.

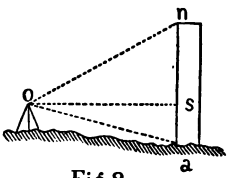
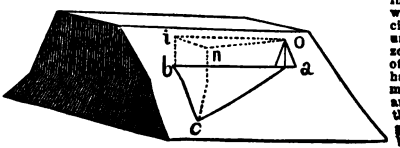


Fig 2

Ex. 2. To find the height of a *vertical* object, $n\,a$.

Place the instrument for measuring angles, at any convenient spot o; also meas the dist $o\,a$: or if $o\,a$ cannot be actually measd in consequence of some obstacle, calculate it by the same process as $a\,c$ in Fig 1. Then, first directing the instrument horizontally,* as $o\,s$, measure the angle of depression, $s\,o\,a$, say 12°; also the angle $s\,o\,n$, say 30°. These two angles added together, give the angle $a\,o\,n$, 42°. Now, in the small triangle $o\,s\,a$ we have the angle $o\,s\,a$ equal to 90°, because $a\,n$ is vert, and $o\,s$ hor; and since the three angles of any triangle are equal to 180°, if we subtract the angles $o\,s\,a$ (90°), and $s\,o\,a$ (12°) from 180°, the rem (78°) will be the angle $o\,a\,s$ or $o\,a\,n$. Therefore, in the triangle $o\,n\,a$, we have one side $o\,a$: and two angles $a\,o\,n$, and $o\,a\,n$, to calculate the side $a\,n$ by Case 1.

* The instrument in our examples, is supposed to measure both vert and hor angles, on the same principle as a theodolite, or the common engineer's transit with a graduated vert arc. The sides of such triangles as are upon the ground (in distinction to the imaginary sides of vert triangles, such as in Figs 2, 3, 4, &c.) must be measd hor, as well as their angles. The graduated hor circle of the instrument evidently measures the angle between two objects horizontally, no matter how much higher one of them may be than the other: one perhaps requiring the telescope of the instrument to be directed upward toward it; and the other downward. If, therefore, the sides of triangles lying upon sloping ground, are not also measd hor, there can be no accordance between the two. Thus



Rmk. If, as in Fig 3, it should be necessary to ascertain the *vert* height $a\,n$ from a point o, entirely above it, then both the angles measd at o, namely, $s\,o\,n$, and $s\,o\,a$, will be angles of depression, or below the hor line $o\,s$ assumed to measure them from. In this case we have the side $o\,a$ as before; the angle $n\,o\,a = s\,o\,a - s\,o\,n$; and the angle $o\,a\,n = 180° - (o\,s\,a\ (90°)$, and $s\,o\,a;)$ to calculate $a\,n$ by Case 1.

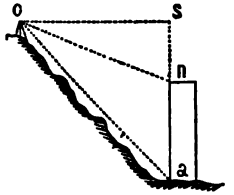


Fig. 3

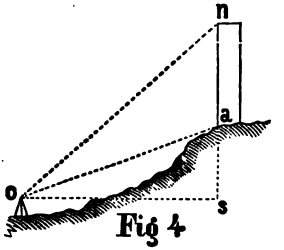


Fig 4

Or if, as in Fig 4, the observations are to be taken from a point o, entirely below the object $a\,n$, then both the angles $s\,o\,a$, $s\,o\,n$, will be angles of elevation, or *above* the assumed hor line $o\,s$. Here we have in the triangle $o\,n\,a$, the given side $o\,a$ as before; the angle $a\,o\,n = s\,o\,n - s\,o\,a$; and the angle $o\,n\,a = 180° - (o\,s\,n\ (90°)$, and $n\,o\,s,)$ to calculate $a\,n$ by Case 1.

If the object $a\,n$, as in Fig 5, instead of being vert, is *inclined*; and instead of its vert height, we wish to find its length $a\,n$, we must first ascertain its angle $y\,t\,i$ of inclination to the horizon; to which angle each of the angles $o\,s\,n$ will be equal. To find this angle $y\,t\,i$, suspend a plumb-line $i\,y$, of any convenient known length, from the object $a\,n$; and measure also $y\,t$ horizontally. Then say as

$y\,i : i\,y :: 1 :$ nat tang of angle $y\,t\,i$.

From the table of nat tangs take out the angle $y\,t\,i$ found opposite this nat tang; and use it for the angles $o\,s\,n$ or $o\,s\,a$; instead of the 90° of Figs 3 and 4. Also when the object inclines, the side $s\,o$ of the triangle must be measd in line, or in range with the inclination. If the object, as the rock $a\,n$, Fig 6, is curved or irregular, a pole $a\,s$ may be planted sloping in the direction $a\,n$; and

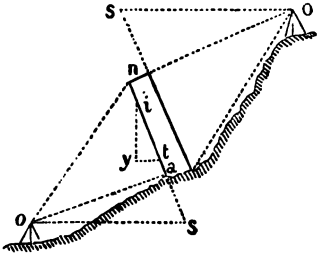


Fig 5

in the triangle $a\,b\,c$, upon sloping ground, the instrument at o, measures the hor angle $i\,o\,n$; and not the angle $b\,a\,c$. Therefore, the side which corresponds with this hor angle $i\,o\,n$, is the hor dist $i\,n$; and not the sloping dist $b\,c$. In other words, when sides and angles are on sloping ground, we do not seek their *actual* measures; but their *hor* ones. This remark applies to all surveying for farms, railroads, triangulations of countries, &c, &c; and the want of a strict attention to it, is one cause of the small errors, almost unavoidable, (and fortunately, of but trifling consequence in practice), which occur in all ordinary field operations.

When a sextant is used, the angles between objects at diff altitudes, as p and q, in this figure, may be measd hor, by first planting two vert rods o and s, in range with the objects; and then taking the hor angle $o\,n\,s$, subtended by the rods.

Angles may be measd with considerable accuracy without any instrument, thus: Measure 100 feet toward each object, and drive stakes; measure the dist across from one stake to the other. Half this dist will be the sine of *half* the angle to a rad of 100; and if we move the decimal point two places to the left, we get the *nat sine of* this one half of the angle to a rad of 1, as in the tables. Thus, suppose the dist to be 80.64 feet; then 40.32 is the sine of half the angle; and .4032 will be the nat sine, opposite to which in the table of nat sines we find the angle 23° 47'; which mult by 2, gives 47° 34', the reqd angle. If obstacles prevent measuring *toward* the objects, we may measure directly *from* them; because, when two lines intersect, the opposite angles are equal. A rough measurement may be made by sticking three pins vert, and a few ins apart, into a small piece of board, nailed hor to the top of a post. *The pins would occupy* the positions n o s, of the last figure. Pencil-lines may then *be drawn, connecting the pin-holes*; and the angle be measd with a protractor. By nailing *plane of board vert to a tree, and then* drawing upon it a short hor line, by means of a pocket c *masters' spirit-level, vert angles of* elevation and depression may be taken roughly in the same *In this way the writer has at times* availed himself of the outer door of a house, by opening it u *pointed toward some mountain-peak*, the dist of which he knew approximately; but of the hei

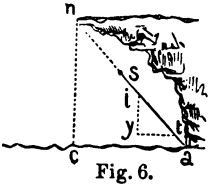


Fig. 6.

its angle $y\,t\,i$ of inclination with the horizon found as before in which case the dist $a\,n$ is calculated. Or if the *vert* height $c\,n$ is sought, the point c may first be found by sighting upward along a plumb-line held above the head.

Ex. 3. To find the approximate height $s\,x$, of a mountain,

Of which, perhaps, only the very summit, x, is visible above interposing forests, or other obstacles; but the dist, $m\,i$, of which is known. In this case, first direct the instrument hor, as $m\,h$; and then measure the angle $i\,m\,x$. Then in the triangle $i\,m\,x$ we have one side $m\,i$; the measd angle $i\,m\,x$, and the angle $m\,i\,x$ (90°), to find $i\,x$ by Case 1. But to this $i\,x$ we must add $i\,o$, equal to the height $y\,m$ of the instrument above the ground: and also $o\,s$. Now, $o\,s$ is *apparently* due entirely to the curvature of the earth, which is equal to very nearly 8 ins, or .667 ft in one mile; and increases as the squares of the dists; being 4 times 8 ins in 2 miles; 9 times 8 ins in 3 miles, &c. But this is somewhat diminished by the refraction of the atmosphere; which varies with temperature, moisture, &c; but always tends to make the object x appear higher than it actually is.* At an average, this deceptive elevation amounts to about $\frac{1}{7}$th part of the curvature of the earth; and like the latter, it varies with the squares of the dists. Consequently if we subtract $\frac{1}{7}$ part from 8 ins, or .667 ft, we have at once the combined effect of curvature and refraction for one mile, equal to 6.857 ins, or .5717 ft; and for other dists, as shown in the following table, by the use of which we avoid the necessity of making *separate* allowances for curvature and refraction.

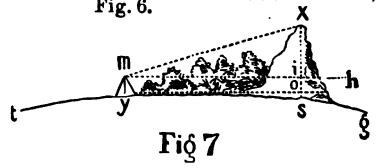


Fig 7

Table of allowances to be added for curvature of the earth; and for refraction; combined.†

Dist. in yards.	Allow. feet.	Dist. in miles.	Allow. feet.	Dist. in miles.	Allow. feet.	Dist. in miles.	Allow. feet.
100	.002	1/4	.036	6	20.6	20	229
150	.004	1/2	.143	7	28.0	22	277
200	.007	3/4	.321	8	36.6	25	357
300	.017	1	.572	9	46.3	30	514
400	.030	1 1/4	.893	10	57.2	35	700
500	.046	1 1/2	1.29	11	69.2	40	915
600	.066	1 3/4	1.75	12	82.3	45	1158
700	.090	2	2.29	13	96.6	50	1429
800	.118	2 1/2	3.57	14	112	55	1729
900	.149	3	5.14	15	129	60	2058
1000	.185	3 1/2	7.00	16	146	70	2801
1200	.266	4	9.15	17	165	80	3659
1500	.415	4 1/2	11.6	18	185	90	4631
2000	.738	5	14.3	19	206	100	5717

The young student must distinguish between a *hor* line, and a *level* one. A straight line cannot be a level one. The curve of the earth, as exemplified in an expanse of quiet water, is level. In Fig 7, if we suppose the curved line $t\,y\,s\,g$ to represent the surface of the sea, then the points $t\,y\,s$ and g are on a level with each other; that is, they are equally dist from the center of the earth; but the hor line $y\,o$ is out of level by the dist $o\,s$. The height of x above y, is not $o\,x$, but $s\,x$. In short dists, the diff between a hor line and level one, is not appreciable; and in such cases it becomes a matter of great convenience in practice to regard them as the same thing.

Ex. 4. If the inaccessible vert height $c\,d$, Fig 8,

Is so situated that it can not reach it at all, then place the instrument for measuring angles, at any convenient station, and in range between n and d, plant two staffs, whose tops o and i shall range precisely with n though they need not be on the same level or hor line with it. Measure $n\,o$: also from n measure the angles $o\,n\,d$ and $o\,n\,c$. Then move the instrument to the precise spot previously

which he had no idea. The necessary allowances for curvature and refraction, may be taken from the table given above on this page.

A triangle whose sides are in the proportion of 3, 4, and 5, is right-angled; and one whose sides are as 7, 7, and 9.9, contains 1 right angle; and 2 angles of 45° each. As it is frequently necessary to *lay down angles of 45° and 90° on the ground*, these proportions may be used for the purpose, by *staking a portion of a tape line* or chain into such a triangle, and driving a stake at each angle.

* *Doubts have been expressed by good authority*, as to the last part of this common assumption.

† *In common levelling*, the allowances are subtracted. It is a different case.

the top *o* of the staff; and from *o* measure the angles *i o d* and *d o c*. This being done, sub-

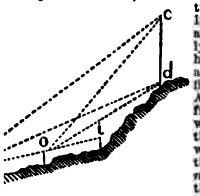


Fig 8

tract the angle *i o c* from 180°; the rem will be the angle *c o n*.* Consequently in the triangle *n o c*, we have one side *n o*, and two angles, *c n o* and *c o n*, to find by Case I the side *o c*. Again, take the angle *i o d* from 180°; the remainder will be the angle *n o d*, so that in the triangle *d n o* we have one side *n o*, and the two angles *d n o* and *n o d*, to find by Case I the side *o d*. Finally, in the triangle *c o d*, we have two sides *c o* and *o d*, and their included angle *c o d*, to find *c d*, the reqd vert height.

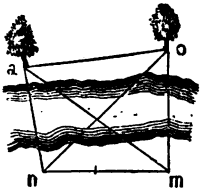


Fig 9

l were in a valley, or on a hill, and the observations reqd to be made from either higher and, the operation would be precisely the same.

5. See Ex 10.

the dist *a o*, Fig 9, between two entirely inaccessible objects,

le *n m*; at *n* measure the angles *a n m* and *o n m*; also at *m* measure the angles *o m n*, and being done, we have in the triangle *a n m*, one side *n m*, Fig 9, and the angles *a n m*, and s, by Case 1, we can calculate the side *a n*. e triangle *o m n* we have one side *n m*, and s *o m n*, and *m n o*; hence, by Case 1, we can side *n o*. This being done, we have in the *o*, two sides *a n*, and *n o*; and their included hence, by Case 3, we can calculate the side the reqd dist. It is plain that in this manner n also the position or direction of the inaccess- for we can calculate the angle *n a o*; and can duce that of *a o*; and thus be enabled to run l to it, if required. By drawing *n m* on pa- le, and laying down the four measd angles, may be measd upon the drawing by the same scale.

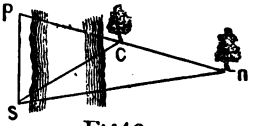


Fig 10

tion of the inaccessible dist *c n*, Fig 10, be such that a stake *p* in line with it, we may proceed thus: Place nt at any suitable point *s*, and take the angles *p s c* o find the angle *c p s*, and measure the dist *p s*. Then *e p s* c find *s c* by Case 1; again, the exterior angle equal to the two interior and opposite angles *c p s*, have in the triangle *c s n*, one side and two angles Case 1.

'o find a dist *a b*, Fig 11, of which ends only are accessible.

l *b*, measure any two lines *a c*, *b c*, meeting at *c*; also angle *a c b*. Then in the triangle *a b c* we have two included angle, to find the third side *a b* by Case 3.

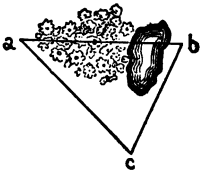


Fig 11

'o find the vert height *o m*, of a ill, above a given point *i*.

nstrument at *i*; measure *a m*. Directing nt hor, as *a n*, take the angle *n a m*. Then, is 90° Fig 12, we have one side *a m*, and *n a m* and *a n m*, to find *n m* by Case 1. al to *a i*, the height of the instrument. ill is a long one, add for curvature of the r refraction, as explained in Example 3, e instrument may be placed at the top of an angle of depression measured; instead f elevation *n a m*.

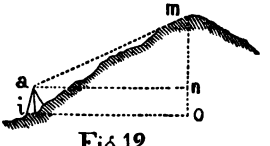


Fig 12

plain, that if the height *o m* be previously re wish to ascertain the dist from its sum- point *i*, the same measurement as before, *n a m*, will enable us to calculate *a m* by Ex. 2, if the height *n a* be known, the angles measd in that example, will enable us e dist *a o*; so also in Figs 3, 4, 5, and 7; in all of which the process is so plain as to ther explanation.

height of a vert object **by means of its shadow.** Plant one end of k vert *in the ground*; and measure its shadow; also measure the length of the shadow *. Then, as the length of the shadow* of the stick is to the length of the stick abov

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hen *two straight lines, as o c*, and *n i*, intersect each other at any inclination whatev angles *c o n* and *i o c* amount to 180°. Therefore, if one of them is given, we can *tracting the given* one from 180°.

ground, so is the length of the shadow of the object, to its height. If the object is inclined must be equally inclined.

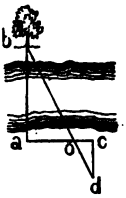


Fig 13

The following examples may be regarded as substitutes for strict t try: and will at times be useful, in case a table of sines, &c, is not at making trigonometrical calculations.

Ex. 8. To find the dist $a\,b$, of which one en is accessible.

Drive a stake at any convenient point a; from a lay off any angle the line $a\,c$, at any convenient point c, drive a stake; and from c lay of $a\,c\,d$, equal to the angle $b\,a\,c$. In the line $c\,d$, at any convenient p drive a stake. Then, standing at d, and looking at b, place a stake with $d\,b$; and at the same time in the line $a\,c$. Measure $a\,o$, $o\,c$, and from the principle of similar triangles, as

$$o\,c : c\,d :: a\,o : a\,b.$$

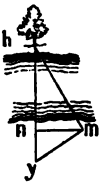


Fig 14

Or thus:

Fig 13, $m\,h$ being the dist, place a stake at n; and lay off the angle h At any convenient dist $n\,m$, place a stake m. Make the angle $h\,m\,y =$ place a stake at y, in range with $h\,n$. Measure $n\,y$ and $n\,m$; then, principle of similar triangles, as

$$n\,y : n\,m :: n\,m : n\,h.$$

Or thus, Fig 13. Lay off the angle $h\,n\,m = 90°$, placing a stake convenient dist $n\,m$. Measure $n\,m$. Also measure the angle $n\,m\,h$. tang of $n\,m\,h$ by Table p. 108. Mult this nat tang by $n\,m$. The prod

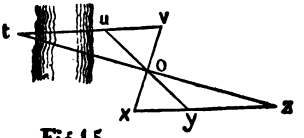


Fig 15

Or thus, without meas any angle;

$t\,u$ being the dist. Make $u\,v$ of any e length, in range with $t\,u$. Measure any $o\,x$ equal to it, in range. Measure $v\,o$ equal to it in range. Place a stake x in r both $x\,y$, and $t\,s$. Then will $y\,z$ be bot $t\,u$, and parallel to it.

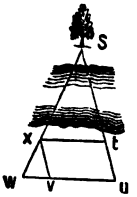


Fig 16

Or thus, without measuring any ang

Drive two stakes t and u, in range with the object s. From t l convenient dist $t\,x$, in any direction. From u lay off $u\,w$ paral placing w in range with $x\,s$. Make $u\,v$ equal to $t\,x$. Measure $w\,v$, $x\,t$. Then, as

$$w\,v : v\,x :: x\,t : t\,s.$$

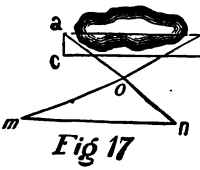


Fig 17

Ex. 9. To find the dist $a\,b$, of whic ends only are accessible.

From a lay off the angle $b\,a\,c$; and from b, the angle a $90°$. Make $a\,c$ and $b\,d$ equal to each other. Then, $c\,d =$ $a\,b$ may be considered as the dist across the river in Figs 16; and be ascertained in the same way. Or measure any 17, $a\,o$; and make $o\,n$ in line and equal to it. Also me and make $o\,m$ in line and equal to it. Then will $m\,n$ be lel to $a\,b$, and equal to it.

See Ex. 4. **To find the entirely ccessible dist $y\,z$, and also its direction.**

convenient points a and b, from each of which be seen, drive stakes. Then we have the four four-sided figure, in which are given the directions sides, and of its two diags. This data enables us the ground, the small four-sided fig $a\,c\,o\,i$, exactly e large one. Thus, in the line $a\,b$ place a stake $c\,o$ parallel to $b\,z$; o being at the same time in diag $a\,z$. Also, from c make $c\,i$ parallel to $b\,y$; e same time in range of $a\,y$. Then will $i\,o$ be in ection as $y\,z$, or parallel to it. Measure $a\,c$, $a\,b$, evidently, from the principle of similar figures, as

$$a\,c : a\,b :: i\,o : y\,z.$$

e a visible line, such as a fence or road, we could de it into any required portions. Thus, if we wish take halfway between y and z, first place one half- i and o; then standing at a, by means of signals, on in range on $y\,z$. Or, to find along $a\,b$, a point t $t\,y$, first make $o\,t\,s = 90°$; and measure $a\,s$. Then,

$$o\,t : a\,s :: y\,z : a\,t.$$

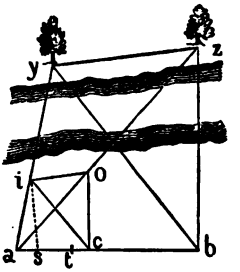


Fig 18

x. 11. To find the position of a point, n, Fig 19,

f two angles a n b *and* b n c, *taken from it to the three objects* a b c, *whose positions and dists apart are known.*

this problem is more frequent in marine than in land surveying. It is chiefly employed ing the position n of a hich soundings are being a coast. As the boat point to point to take ngs, it becomes necessary fresh observation at each der to define its position . An observation consists urement by a sextant of gles $a\,n\,b$, $b\,n\,c$, to the sig- revious ly arranged on the en practicable, this method ejected; and the observa- to the boat at the same two observers on shore, at tations. The boat to show the proper moment. The itious mode of fixing the the map, is to draw three ng the two angles, and ex- initely, on a piece of trans- r. Place the paper upon the map, and move it about until the three lines pass through ations; then prick through the point n wherever it happens to come. f the transparent paper, an instrument called a *station pointer* may be used when there oints to be fixed.

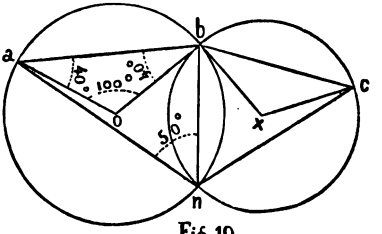


Fig 19

osition of the point n can be found more correctly by describing two circles, as in Fig 19, ch shall pass through n and two of the station points. The question is to find the centers o such circles. This is very simple. We know that the angle $a\,o\,b$ at the center of a circle is eat as any angle $a\,n\,b$ at the circumf of the same circle, when both are subtended by the $a\,b$. Consequently, if the angle $a\,n\,b$, observed from the boat, is say, 50°, the angle $a\,o\,b$ °. And, since the three angles of every plane triangle are equal to 180°, the two angles $b\,a$ are together equal to $180° - 100° = 80°$. And, since the two sides $a\,o$ and $b\,o$ are radii of the same circle), therefore, the angles $o\,a\,b$ and $o\,b\,a$ are equal; and each equal to

Consequently, on the map we have only to lay down at a and b, two angles of 40°; the

ntersection will be the center of the circle $a\,b\,n$. Proceed in the same way with the angle d the center x. Then the intersection of the two circles at n will be the point sought.

TABLE of Contents in Cub Ft, and in U. S. Gallons

Of 231 cub ins (or 7.4805 gals to a cub ft); and for one foot of length of the cylinder. For the contents for a greater diam than any in the table, take the quantity opposite one-*half* said diam; and mult it by 4. Thus, the number of cub ft in one ft length of a pipe 80 inches in diam is equal to 8.727 × 4 = 34.908 cub ft. So also with gallons, and areas. No errors.

Diam. in Ins.	Diam. in decimals of a foot.	For 1 ft. in length. Cub. Feet. Also area in sq. ft.	For 1 ft. in length. Gallons of 231 Cub. Ins.	Diam. in Ins.	Diam. in decimals of a foot.	For 1 ft in length. Cub. Feet. Also area in sq. ft.	For 1 ft in length. Gallons of 231 Cub. Ins.	Diam. in Ins.	Diam. in decimals of a foot.	For 1 ft. in length. Cub. Feet. Also area in sq. ft.	For 1 ft. in length. Gallons of 231 Cub. Ins.
¼	.0208	.0003	.0025	¾	.5625	.2485	1.859	19.	1.583	1.969	14.73
5-16	.0260	.0005	.0040	7.	.5833	.2673	1.999	½	1.625	2.074	15.51
⅜	.0313	.0008	.0057	¼	.6042	.2867	2.145	20.	1.667	2.182	16.32
7-16	.0365	.0010	.0078	½	.6250	.3068	2.295	½	1.708	2.292	17.15
½	0417	.0014	.0102	¾	.6458	.3276	2.450	21.	1.750	2.405	17.99
9-16	.0469	.0017	.0129	8.	.6667	.3491	2.611	½	1.792	2.521	18.86
⅝	.0521	.0021	.0159	¼	.6875	.3712	2.777	22.	1.833	2.640	19.75
11-16	.0573	.0026	.0193	½	.7083	.3941	2.948	½	1.875	2.761	20.66
¾	.0625	.0031	.0230	¾	.7292	.4176	3.125	23.	1.917	2.885	21.58
13-16	.0677	.0036	.0269	9.	.7500	.4418	3.305	½	1.958	3.012	22.53
⅞	.0729	.0042	.0312	¼	.7708	.4667	3.491	24.	2.000	3.142	23.50
15-16	.0781	.0048	.0359	½	.7917	.4922	3.682	25.	2.083	3.409	25.50
1.	.0833	.0055	.0408	¾	.8125	.5185	3.879	26.	2.167	3.687	27.58
¼	.1042	.0085	.0638	10.	.8333	.5454	4.080	27.	2.250	3.976	29.74
½	.1250	.0123	.0918	¼	.8542	.5730	4.286	28.	2.333	4.276	31.99
¾	.1458	.0167	.1249	½	.8750	.6013	4.498	29.	2.417	4.587	34.31
2.	.1667	.0218	.1632	¾	.8958	.6303	4.715	30.	2.500	4.909	36.72
¼	.1875	.0276	.2066	11.	.9167	.6600	4.937	31.	2.583	5.241	39.21
½	.2083	.0341	.2550	¼	.9375	.6903	5.164	32.	2.667	5.585	41.78
¾	.2292	.0412	.3085	½	.9583	.7213	5.396	33.	2.750	5.940	44.43
3.	.2500	.0491	.3672	¾	.9792	.7530	5.633	34.	2.833	6.305	47.16
¼	.2708	.0576	.4309	12.	1 Foot.	.7854	5.875	35.	2.917	6.681	49.98
½	.2917	.0668	.4998	½	1.042	.8522	6.375	36.	3.000	7.069	52.88
¾	.3125	.0767	.5738	13.	1.083	.9218	6.895	37.	3.083	7.467	55.86
4.	.3333	.0873	.6528	½	1.125	.9940	7.436	38.	3.167	7.876	58.92
¼	.3542	.0985	.7369	14.	1.167	1.069	7.997	39.	3.250	8.296	62.06
½	.3750	.1104	.8263	½	1.208	1.147	8.578	40.	3.333	8.727	65.28
¾	.3958	.1231	.9206	15.	1.250	1.227	9.180	41.	3.417	9.168	68.58
5.	.4167	.1364	1.020	½	1.292	1.310	9.801	42.	3.500	9.621	71.97
¼	.4375	.1503	1.125	16.	1.333	1.396	10.44	43.	3.583	10.085	75.44
½	.4583	.1650	1.234	½	1.375	1.485	11.11	44.	3.667	10.559	78.99
¾	.4792	.1803	1.349	17.	1.417	1.576	11.79	45.	3.750	11.045	82.62
6	.5000	.1963	1.469	½	1.458	1.670	12.49	46.	3.833	11.541	86.33
¼	.5208	.2131	1.594	18.	1.500	1.767	13.22	47.	3.917	12.048	90.13
½	.5417	.2304	1.724	½	1.542	1.867	13.96	48.	4.000	12.566	94.00

For the weight of water in 1 foot length of pipe. See Table 2½, page 540.

DIGGING, &c, OF WELLS.

For diams twice as great as those in the table, for the cub yds of digging, take out those opposite *one half* of the greater diam; and mult them by 4. Thus, for the cub yds in each foot of depth of a well 31 feet in diam, first take out from the table those opposite the diam of 15½ feet; namely, 6.989. Then 6.989 × 4 = 27.956 cub yds reqd for the 31 ft diam. But for the stone lining or walling, bricks or plastering, mult the tabular quantity opposite *half* the greater diam, by 2. Thus, the perches of stone walling for each foot of depth of a well of 31 ft diam, will be 2.073 × 2 = 4.146. If the wall is more or less than one foot thick, within usual moderate limits, it will generally be near enough for practice to assume that the number of perches, or of bricks, will increase or decrease in the same proportion.

The size of the bricks is taken at 8¼ × 4 × 2 inches; and to be laid dry, or without mortar. In practice an addition of about 5 per cent should be made for waste. The brick lining is supposed to be 1 brick thick, or 8¼ ins.

Be careful to observe that the diams to be used for the digging, are greater than those for the walling, bricks, or plastering. No errors.

Diam. in Feet.	For each foot of depth. For this col use the Diameter of the Digging. Cub Yds. of Digging.	For these three cols use the diam in clear of the lining. Stone Lining 1 ft thick. Perches of 25 Cub Ft.	No. of Bricks in a Lining 1 Brick thick.	Square Yards of Plastering.	Diam. in Feet.	For each foot of depth. For this col use the Diameter of the Digging. Cub Yds. of Digging.	For these three cols use the diam in clear of the lining. Stone Lining 1 ft thick. Perches of 25 Cub Ft.	No. of Bricks in a Lining 1 Brick thick.	Square Yards of Plastering.
1.	.0291	.2513	57	.3491	¼	5.107	1.791	750	4.625
¼	.0455	.2827	71	.4364	½	5.301	1.822	764	4.713
½	.0654	.3142	85	.5236	¾	5.500	1.854	778	4.800
¾	.0891	.3456	99	.6109	14.	5.701	1.885	792	4.887
2.	.1164	.3770	114	.6982	¼	5.907	1.916	806	4.974
¼	.1473	.4084	128	.7855	½	6.116	1.948	820	5.062
½	.1818	.4398	142	.8727	¾	6.329	1.979	834	5.149
¾	.2200	.4712	156	.9600	15.	6.545	2.011	849	5.236
3.	.2618	.5027	170	1.047	¼	6.765	2.042	863	5.323
¼	.3073	.5341	184	1.135	½	6.989	2.073	877	5.411
½	.3563	.5655	198	1.222	¾	7.216	2.105	891	5.498
¾	.4091	.5969	212	1.309	16.	7.447	2.136	905	5.585
4.	.4654	.6283	227	1.396	¼	7.681	2.168	919	5.673
¼	.5254	.6597	241	1.484	½	7.919	2.199	933	5.760
½	.5890	.6912	255	1.571	¾	8.161	2.231	948	5.847
¾	.6563	.7226	269	1.658	17.	8.407	2.262	962	5.934
5.	.7272	.7540	283	1.745	¼	8.656	2.293	976	6.022
¼	.8018	.7854	297	1.833	½	8.908	2.325	990	6.109
½	.8799	.8168	311	1.920	¾	9.165	2.356	1004	6.196
¾	.9617	.8482	326	2.007	18.	9.425	2.388	1018	6.283
6.	1.047	.8796	340	2.095	¼	9.688	2.419	1032	6.371
¼	1.136	.9111	354	2.182	½	9.956	2.450	1046	6.458
½	1.229	.9425	368	2.269	¾	10.23	2.482	1061	6.545
¾	1.325	.9739	382	2.356	19.	10.50	2.513	1075	6.633
7.	1.425	1.005	396	2.444	¼	10.78	2.545	1089	6.720
¼	1.529	1.037	410	2.531	½	11.06	2.576	1103	6.807
½	1.636	1.068	425	2.618	¾	11.35	2.608	1117	6.894
¾	1.747	1.100	439	2.705	20.	11.64	2.639	1131	6.982
8.	1.862	1.131	453	2.793	¼	11.93	2.670	1145	7.069
¼	1.980	1.162	467	2.880	½	12.22	2.702	1160	7.156
½	2.102	1.194	481	2.967	¾	12.52	2.733	1174	7.243
¾	2.227	1.225	495	3.054	21.	12.83	2.765	1188	7.331
9.	2.356	1.257	509	3.142	¼	13.14	2.796	1202	7.418
¼	2.489	1.288	523	3.229	½	13.45	2.827	1216	7.505
½	2.625	1.319	538	3.316	¾	13.76	2.859	1230	7.593
¾	2.765	1.351	552	3.404	22.	14.08	2.890	1244	7.680
10.	2.909	1.382	566	3.491	¼	14.40	2.922	1259	7.767
¼	3.056	1.414	580	3.578	½	14.73	2.953	1273	7.854
½	3.207	1.445	594	3.665	¾	15.06	2.985	1287	7.942
¾	3.362	1.477	608	3.753	23.	15.39	3.016	1301	8.029
11.	3.520	1.508	622	3.840	¼	15.72	3.047	1315	8.116
¼	3.682	1.539	637	3.927	½	16.06	3.079	1329	8.203
½	3.847	1.571	651	4.014	¾	16.41	3.110	1343	8.291
¾	4.016	1.602	665	4.102	24.	16.76	3.142	1357	8.378
12.	4.189	1.634	679	4.189	¼	17.11	3.173	1372	8.465
¼	4.365	1.665	693	4.276	½	17.46	3.204	1386	8.552
½	4.545	1.696	707	4.364	¾	17.82	3.236	1400	8.640
¾	4.729	1.728	721	4.451	25.	18.18	3.267	1414	8.727
13.	4.916	1.759	736	4.538					

When *perches* are *mentioned in any contract*, it is necessary, in order to prevent fraud, to specify the number of *cub feet contained in the perch*; for stone-quarriers have one perch, stone-masons a rder, &c. *Engineers, on this account, contract by* the *cubic yard*. The perch should be done a with entirely; whether as a measure of length, area, or solidity. Perches × .926 = cub yds.

Square Roots and Cube Roots of Numbers from .1 to

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Sq. Rt.	C. Rt.	No.	Sq. Rt.
.1	.01	.001	.316	.464	.7	2.387	1.786	.4	3.661
.15	.0225	.0034	.387	.531	.8	2.408	1.797	.6	3.688
.2	.04	.008	.447	.585	.9	2.429	1.807	.8	3.715
.25	.0625	.0156	.500	.630	6.	2.449	1.817	14.	3.742
.3	.09	.027	.548	.669	.1	2.470	1.827	.2	3.768
.35	.1225	.0429	.592	.705	.2	2.490	1.837	.4	3.795
.4	.16	.064	.633	.737	.3	2.510	1.847	.6	3.821
.45	.2025	.0911	.671	.766	.4	2.530	1.857	.8	3.847
.5	.25	.125	.707	.794	.5	2.550	1.866	15.	3.873
.55	.3025	.1664	.742	.819	.6	2.569	1.876	.2	3.899
.6	.36	.216	.775	.843	.7	2.588	1.885	.4	3.924
.65	.4225	.2746	.806	.866	.8	2.608	1.895	.6	3.950
.7	.49	.343	837	.888	.9	2.627	1.904	.8	3.975
.75	.5625	.4219	.866	.909	7.	2.646	1.913	16.	4.
.8	.64	.512	.894	.928	.1	2.665	1.922	.2	4.025
.85	.7225	.6141	.922	.947	.2	2.683	1.931	.4	4.050
.9	.81	.729	.949	.965	.3	2.702	1.940	.6	4.074
.95	.9025	.8574	.975	.983	.4	2.720	1.949	.8	4.099
1.	1.000	1.000	1.000	1.000	.5	2.739	1.957	17.	4.123
.05	1.103	1.158	1.025	1.016	.6	2.757	1.966	.2	4.147
1.1	1.210	1.331	1.049	1.032	.7	2.775	1 975	.4	4.171
.15	1.323	1.521	1.072	1.048	.8	2.793	1.983	.6	4.195
1.2	1.440	1.728	1.095	1.063	.9	2.811	1.992	.8	4.219
.25	1.563	1.953	1.118	1.077	8.	2.828	2.000	18.	4.243
1.3	1.690	2.197	1.140	1.091	.1	2.846	2.008	.2	4.266
.35	1.823	2.460	1.162	1.105	.2	2.864	2.017	.4	4.290
1.4	1.960	2.744	1.183	1.119	.3	2.881	2.025	.6	4.313
.45	2.103	3.049	1.204	1.132	.4	2.898	2.033	.8	4.336
1.5	2.250	3.375	1.225	1.145	.5	2.915	2.041	19.	4.359
.55	2.403	3.724	1.245	1.157	.6	2.933	2.049	.2	4.382
1.6	2.560	4.096	1 265	1.170	.7	2.950	2.057	.4	4.405
.65	2.723	4.492	1.285	1.182	.8	2.966	2.065	.6	4.427
1.7	2.890	4.913	1.304	1.193	.9	2.983	2.072	.8	4.450
.75	3.063	5.359	1.323	1.205	9.	3.	2.080	20.	4.472
1.8	3.240	5.832	1.342	1.216	.1	3.017	2.088	.2	4.494
.85	3.423	6.332	1.360	1.228	.2	3.033	2.095	.4	4.517
1.9	3.610	6.859	1.378	1.239	.3	3.050	2.103	.6	4.539
.95	3.803	7.415	1.396	1.249	.4	3.066	2.110	.8	4.561
2.	4.000	8.000	1.414	1.260	.5	3.082	2.118	21.	4.583
.1	4.410	9.261	1.449	1.281	.6	3.098	2.125	.2	4.604
.2	4.840	10.65	1.483	1.301	.7	3.114	2 133	.4	4.626
.3	5.290	12.17	1.517	1.320	.8	3.130	2.140	.6	4.648
.4	5.760	13.82	1.549	1.339	.9	3.146	2.147	.8	4.669
.5	6.250	15.63	1.581	1.357	10.	3.162	2.154	22.	4.690
.6	6.760	17.58	1.612	1.375	.1	3.178	2.162	.2	4.712
.7	7.290	19.68	1.643	1.392	.2	3.194	2.169	.4	4.733
.8	7.840	21.95	1.673	1.409	.3	3.209	2.176	.6	4.754
.9	8.410	24.39	1.703	1.426	.4	3.225	2.183	.8	4.775
3.	9.	27.	1.732	1.442	.5	3.240	2.190	23.	4.796
.1	9.61	29.79	1.761	1.458	.6	3.256	2.197	.2	4.817
.2	10.24	32.77	1.789	1.474	.7	3.271	2.204	.4	4.837
.3	10.89	35.94	1.817	1.489	.8	3.286	2.210	.6	4.858
.4	11.56	39.30	1.844	1.504	.9	3.302	2.217	.8	4.879
.5	12.25	42.88	1.871	1.518	11.	3.317	2.224	24.	4.899
.6	12.96	46.66	1.897	1.533	.1	3.332	2.231	.2	4.919
.7	13.69	50.65	1.924	1.547	.2	3.347	2.237	.4	4.940
.8	14 44	54.87	1.949	1.560	.3	3.362	2.244	.6	4.960
.9	15.21	59.32	1.975	1.574	.4	3 376	2.251	.8	4.980
4.	16.	64.	2.	1.587	.5	3.391	2.257	25.	5.
.1	16.81	68.92	2.025	1.601	.6	3.406	2.264	.2	5.020
.2	17.64	74.09	2.049	1 613	.7	3.421	2.270	.4	5.040
.3	18.49	79.51	2.074	1.626	.8	3.435	2.277	.6	5.060
.4	19.36	85.18	2.098	1.639	.9	3.450	2.283	.8	5.079
.5	20.25	91.13	2.121	1.651	12.	3.464	2 289	26.	5.099
.6	21.16	97.34	2.145	1.663	.1	3.479	2.296	.2	5.119
.7	22.09	103.8	2.168	1.675	.2	3.493	2.302	.4	5.138
.8	23.04	110.6	2.191	1.687	.3	3.507	2.308	.6	5.158
.9	24.01	117.6	2.214	1.698	.4	3.521	2.315	.8	5.177
5.	25.	125.	2.236	1.710	.5	3.536	2.321	27.	5.196
1	26.01	132.7	2.258	1.721	.6	3.550	2.327	.2	5.215
2	27.04	140.6	2.280	1.732	.7	3.564	2.333	.4	5.235
3	28.09	148.9	2.302	1.744	.8	3.578	2 339	.6	5.254
4	29.16	157.5	2.324	1.754	.9	3.592	2.345	.8	5.273
5	30.25	166.4	2.345	1.765	13.	3.606	2.351	28.	5.292
6	31.36	175.6	2.366	1.776	.2	3.633	2.363	.2	5.310

BLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000.

ARK ON THE FOLLOWING TABLE. Wherever the effect of a fifth decimal in the roots would be to to the fourth and final decimal in the table, the addition has been made. No errors.

Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
1	1	1.0000	1.0000	61	3721	226981	7.8102	3.9365
4	8	1.4142	1.2599	62	3844	238328	7.8740	3.9579
9	27	1.7321	1.4422	63	3969	250047	7.9373	3.9791
16	64	2.0000	1.5874	64	4096	262144	8.0000	4.
25	125	2.2361	1.7100	65	4225	274625	8.0623	4.0207
36	216	2.4495	1.8171	66	4356	287496	8.1240	4.0412
49	343	2.6458	1.9129	67	4489	300763	8.1854	4.0615
64	512	2.8284	2.0000	68	4624	314432	8.2462	4.0817
81	729	3.0000	2.0801	69	4761	328509	8.3066	4.1016
100	1000	3.1623	2.1544	70	4900	343000	8.3666	4.1213
121	1331	3.3166	2.2240	71	5041	357911	8.4261	4.1408
144	1728	3.4641	2.2894	72	5184	373248	8.4853	4.1602
169	2197	3.6056	2.3513	73	5329	389017	8.5440	4.1793
196	2744	3.7417	2.4101	74	5476	405224	8.6023	4.1983
225	3375	3.8730	2.4662	75	5625	421875	8.6603	4.2172
256	4096	4.0000	2.5198	76	5776	438976	8.7178	4.2358
289	4913	4.1231	2.5713	77	5929	456533	8.7750	4.2543
324	5832	4.2426	2.6207	78	6084	474552	8.8318	4.2727
361	6859	4.3589	2.6684	79	6241	493039	8.8882	4.2908
400	8000	4.4721	2.7144	80	6400	512000	8.9443	4.3089
441	9261	4.5826	2.7589	81	6561	531441	9.	4.3267
484	10648	4.6904	2.8020	82	6724	551368	9.0554	4.3445
529	12167	4.7958	2.8439	83	6889	571787	9.1104	4.3621
576	13824	4.8990	2.8845	84	7056	592704	9.1652	4.3795
625	15625	5.0000	2.9240	85	7225	614125	9.2195	4.3968
676	17576	5.0990	2.9625	86	7396	636056	9.2736	4.4140
729	19683	5.1962	3.0000	87	7569	658503	9.3274	4.4310
784	21952	5.2915	3.0366	88	7744	681472	9.3808	4.4480
841	24389	5.3852	3.0723	89	7921	704969	9.4340	4.4647
900	27000	5.4772	3.1072	90	8100	729000	9.4868	4.4814
961	29791	5.5678	3.1414	91	8281	753571	9.5394	4.4979
1024	32768	5.6569	3.1748	92	8464	778688	9.5917	4.5144
1089	35937	5.7446	3.2075	93	8649	804357	9.6437	4.5307
1156	39304	5.8310	3.2396	94	8836	830584	9.6954	4.5468
1225	42875	5.9161	3.2711	95	9025	857375	9.7468	4.5629
1296	46656	6.0000	3.3019	96	9216	884736	9.7980	4.5789
1369	50653	6.0828	3.3322	97	9409	912673	9.8489	4.5947
1444	54872	6.1644	3.3620	98	9604	941192	9.8995	4.6104
1521	59319	6.2450	3.3912	99	9801	970299	9.9499	4.6261
1600	64000	6.3246	3.4200	100	10000	1000000	10.	4.6416
1681	68921	6.4031	3.4482	101	10201	1030301	10.0499	4.6570
1764	74088	6.4807	3.4760	102	10404	1061208	10.0995	4.6723
1849	79507	6.5574	3.5034	103	10609	1092727	10.1489	4.6875
1936	85184	6.6332	3.5303	104	10816	1124864	10.1980	4.7027
2025	91125	6.7082	3.5569	105	11025	1157625	10.2470	4.7177
2116	97336	6.7823	3.5830	106	11236	1191016	10.2956	4.7326
2209	103823	6.8557	3.6088	107	11449	1225043	10.3441	4.7475
2304	110592	6.9282	3.6342	108	11664	1259712	10.3923	4.7622
2401	117649	7.0000	3.6593	109	11881	1295029	10.4403	4.7769
2500	125000	7.0711	3.6840	110	12100	1331000	10.4881	4.7914
2601	132651	7.1414	3.7084	111	12321	1367631	10.5357	4.8059
2704	140608	7.2111	3.7325	112	12544	1404928	10.5830	4.8203
2809	148877	7.2801	3.7563	113	12769	1442897	10.6301	4.8346
2916	157464	7.3485	3.7798	114	12996	1481544	10.6771	4.8488
3025	166375	7.4162	3.8030	115	13225	1520875	10.7238	4.8629
3136	175616	7.4833	3.8259	116	13456	1560896	10.7703	4.8770
3249	185193	7.5498	3.8485	117	13689	1601613	10.8167	4.8910
3364	195112	7.6158	3.8709	118	13924	1643032	10.8628	4.9049
3481	205379	7.6811	3.8930	119	14161	1685159	10.9087	4.9187
3600	216000	7.7460	3.9149	120	14400	1728000	10.9545	4.9324

TABLE of Squares, Cubes, Square Roots, and Cube Ro of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.
121	14641	1771561	11.	4.9461	186	34596	6434856	13.6382
122	14884	1815848	11.0454	4.9597	187	34969	6539203	13.6748
123	15129	1860867	11.0905	4.9732	188	35344	6644672	13.7113
124	15376	1906624	11.1355	4.9866	189	35721	6751269	13.7477
125	15625	1953125	11.1803	5.	190	36100	6859000	13.7840
126	15876	2000376	11.2250	5.0133	191	36481	6967871	13.8203
127	16129	2048383	11.2694	5.0265	192	36864	7077888	13.8564
128	16384	2097152	11.3137	5.0397	193	37249	7189057	13.8924
129	16641	2146689	11.3578	5.0528	194	37636	7301384	13.9284
130	16900	2197000	11.4018	5.0658	195	38025	7414875	13.9642
131	17161	2248091	11.4455	5.0788	196	38416	7529536	14.
132	17424	2299968	11.4891	5.0916	197	38809	7645373	14.0357
133	17689	2352637	11.5326	5.1045	198	39204	7762392	14.0712
134	17956	2406104	11.5758	5.1172	199	39601	7880599	14.1067
135	18225	2460375	11.6190	5.1299	200	40000	8000000	14.1421
136	18496	2515456	11.6619	5.1426	201	40401	8120601	14.1774
137	18769	2571353	11.7047	5.1551	202	40804	8242408	14.2127
138	19044	2628072	11.7473	5.1676	203	41209	8365427	14.2478
139	19321	2685619	11.7898	5.1801	204	41616	8489664	14.2829
140	19600	2744000	11.8322	5.1925	205	42025	8615125	14.3178
141	19881	2803221	11.8743	5.2048	206	42436	8741816	14.3527
142	20164	2863288	11.9164	5.2171	207	42849	8869743	14.3875
143	20449	2924207	11.9583	5.2293	208	43264	8998912	14.4222
144	20736	2985984	12.	5.2415	209	43681	9129329	14.4568
145	21025	3048625	12.0416	5.2536	210	44100	9261000	14.4914
146	21316	3112136	12.0830	5.2656	211	44521	9393931	14.5258
147	21609	3176523	12.1244	5.2776	212	449 4	9528128	14.5602
148	21904	3241792	12.1655	5.2896	213	45369	9663597	14.5945
149	22201	3307949	12.2066	5.3015	214	45796	9800344	14.6287
150	22500	3375000	12.2474	5.3133	215	46225	9938375	14.6629
151	22801	3442951	12.2882	5.3251	216	46656	10077696	14.6969
152	23104	3511808	12.3288	5.3368	217	47089	10218313	14.7309
153	23409	3581577	12.3693	5.3485	218	47524	10360232	14.7648
154	23716	3652264	12.4097	5.3601	219	47961	10503459	14.7986
155	24025	3723875	12.4499	5.3717	220	48400	10648000	14.8324
156	24336	3796416	12.4900	5.3832	221	48841	10793861	14.8661
157	24649	3869893	12.5300	5.3947	222	49284	10941048	14.8997
158	24964	3944312	12.5698	5.4061	223	49729	11089567	14.9332
159	25281	4019679	12.6095	5.4175	224	50176	11239424	14.9666
160	25600	4096000	12.6491	5.4288	225	50625	11390625	15.
161	25921	4173281	12.6886	5.4401	226	51076	11543176	15.0333
162	26244	4251528	12.7279	5.4514	227	51529	11697083	15.0665
163	26569	4330747	12.7671	5.4626	228	51984	11852352	15.0997
164	26896	4410944	12.8062	5.4737	229	52441	12008989	15.1327
165	27225	4492125	12.8452	5.4848	230	52900	12167000	15.1658
166	27556	4574296	12.8841	5.4959	231	53361	12326391	15.1987
167	27889	4657463	12.9228	5.5069	232	53824	12487168	15.2315
168	28224	4741632	12.9615	5.5178	233	54289	12649337	15.2643
169	28561	4826809	13.	5.5288	234	54756	12812904	15.2971
170	28900	4913000	13.0384	5.5397	235	55225	12977875	15.3297
171	29241	5000211	13.0767	5.5505	236	55696	13144256	15.3623
172	29584	5088448	13.1149	5.5613	237	56169	13312053	15.3948
173	29929	5177717	13.1529	5.5721	238	56644	13481272	15.4272
174	30276	5268024	13.1909	5.5828	239	57121	13651919	15.4596
175	30625	5359375	13.2288	5.5934	240	57600	13824000	15.4919
176	30976	5451776	13.2665	5.6041	241	58081	13997521	15.5242
177	31329	5545233	13.3041	5.6147	242	58564	14172488	15.5563
178	31684	5639752	13.3417	5.6252	243	59049	14348907	15.5885
179	32041	5735339	13.3791	5.6357	244	59536	14526784	15.6205
180	32400	5832000	13.4164	5.6462	245	60025	14706125	15.6525
181	32761	5929741	13.4536	5.6567	246	60516	14886936	15.6844
182	33124	6028568	13.4907	5.6671	247	61009	15069223	15.7162
183	33489	6128487	13.5277	5.6774	248	61504	15252992	15.7480
184	33856	6229504	13.5647	5.6877	249	62001	15438249	15.7797
85	34225	6331625	13.6015	5.6980	250	62500	15625000	15.8114

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000 (CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
251	63001	15813251	15.8430	6.3080	316	99856	31554496	17.7764	6.8113
252	63504	16003008	15.8745	6.3164	317	100489	31855013	17.8045	6.8185
253	64009	16194277	15.9060	6.3247	318	101124	32157432	17.8326	6.8256
254	64516	16387064	15.9374	6.3330	319	101761	32461759	17.8606	6.8328
255	65025	16581375	15.9687	6.3413	320	102400	32768000	17.8885	6.8399
256	65536	16777216	16.	6.3496	321	103041	33076161	17.9165	6.8470
257	66049	16974593	16.0312	6.3579	322	103684	33386248	17.9444	6.8541
258	66564	17173512	16.0624	6.3661	323	104329	33698267	17.9722	6.8612
259	67081	17373979	16.0935	6.3743	324	104976	34012224	18.	6.8683
260	67600	17576000	16.1245	6.3825	325	105625	34328125	18.0278	6.8753
261	68121	17779581	16.1555	6.3907	326	106276	34645976	18.0555	6.8824
262	68644	17984728	16.1864	6.3988	327	106929	34965783	18.0831	6.8894
263	69169	18191447	16.2173	6.4070	328	107584	35287552	18.1108	6.8964
264	69696	18399744	16.2481	6.4151	329	108241	35611289	18.1384	6.9034
265	70225	18609625	16.2788	6.4232	330	108900	35937000	18.1659	6.9104
266	70756	18821096	16.3095	6.4312	331	109561	36264691	18.1934	6.9174
267	71289	19034163	16.3401	6.4393	332	110224	36594368	18.2209	6.9244
268	71824	19248832	16.3707	6.4473	333	110889	36926037	18.2483	6.9313
269	72361	19465109	16.4012	6.4553	334	111556	37259704	18.2757	6.9382
270	72900	19683000	16.4317	6.4633	335	112225	37595375	18.3030	6.9451
271	73441	19902511	16.4621	6.4713	336	112896	37933056	18.3303	6.9521
272	73984	20123648	16.4924	6.4792	337	113569	38272753	18.3576	6.9589
273	74529	20346417	16.5227	6.4872	338	114244	38614472	18.3848	6.9658
274	75076	20570824	16.5529	6.4951	339	114921	38958219	18.4120	6.9727
275	75625	20796875	16.5831	6.5030	340	115600	39304000	18.4391	6.9795
276	76176	21024576	16.6132	6.5108	341	116281	39651821	18.4662	6.9864
277	76729	21253933	16.6433	6.5187	342	116964	40001688	18.4932	6.9932
278	77284	21484952	16.6733	6.5265	343	117649	40353607	18.5203	7.
279	77841	21717639	16.7033	6.5343	344	118336	40707584	18.5472	7.0068
280	78400	21952000	16.7332	6.5421	345	119025	41063625	18.5742	7.0136
281	78961	22188041	16.7631	6.5499	346	119716	41421736	18.6011	7.0203
282	79524	22425768	16.7929	6.5577	347	120409	41781923	18.6279	7.0271
283	80089	22665187	16.8226	6.5654	348	121104	42144192	18.6548	7.0338
284	80656	22906304	16.8523	6.5731	349	121801	42508549	18.6815	7.0406
285	81225	23149125	16.8819	6.5808	350	122500	42875000	18.7083	7.0473
286	81796	23393656	16.9115	6.5885	351	123201	43243551	18.7350	7.0540
287	82369	23639903	16.9411	6.5962	352	123904	43614208	18.7617	7.0607
288	82944	23887872	16.9706	6.6039	353	124609	43986977	18.7883	7.0674
289	83521	24137569	17.	6.6115	354	125316	44361864	18.8149	7.0740
290	84100	24389000	17.0294	6.6191	355	126025	44738875	18.8414	7.0807
291	84681	24642171	17.0587	6.6267	356	126736	45118016	18.8680	7.0873
292	85264	24897088	17.0880	6.6343	357	127449	45499293	18.8944	7.0940
293	85849	25153757	17.1172	6.6419	358	128164	45882712	18.9209	7.1006
294	86436	25412184	17.1464	6.6494	359	128881	46268279	18.9473	7.1072
295	87025	25672375	17.1756	6.6569	360	129600	46656000	18.9737	7.1138
296	87616	25934336	17.2047	6.6644	361	130321	47045881	19.	7.1204
297	88209	26198073	17.2337	6.6719	362	131044	47437928	19.0263	7.1269
298	88804	26463592	17.2627	6.6794	363	131769	47832147	19.0526	7.1335
299	89401	26730899	17.2916	6.6869	364	132496	48228544	19.0788	7.1400
300	90000	27000000	17.3205	6.6943	365	133225	48627125	19.1050	7.1466
301	90601	27270901	17.3494	6.7018	366	133956	49027896	19.1311	7.1531
302	91204	27543608	17.3781	6.7092	367	134689	49430863	19.1572	7.1596
303	91809	27818127	17.4069	6.7166	368	135424	49836032	19.1833	7.1661
304	92416	28094464	17.4356	6.7240	369	136161	50243409	19.2094	7.1726
305	93025	28372625	17.4642	6.7313	370	136900	50653000	19.2354	7.1791
306	93636	28652616	17.4929	6.7387	371	137641	51064811	19.2614	7.1855
307	94249	28934443	17.5214	6.7460	372	138384	51478848	19.2873	7.1920
308	94864	29218112	17.5499	6.7533	373	139129	51895117	19.3132	7.1984
309	95481	29503629	17.5784	6.7606	374	139876	52313624	19.3391	7.2048
310	96100	29791000	17.6068	6.7679	375	140625	52734375	19.3649	7.2112
311	96721	30080231	17.6352	6.7752	376	141376	53157376	19.3907	7.2177
312	97344	30371328	17.6635	6.7824	377	142129	53582633	19.4165	7.2240
313	97969	30664297	17.6918	6.7897	378	142884	54010152	19.4422	7.2304
314	98596	30959144	17.7200	6.7969	379	143641	54439939	19.4679	7.2368
315	99225	31255875	17.7482	6.8041	380	144400	54872000	19.4936	7.2

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
381	145161	55306341	19.5192	7.2495	446	198916	88716536	21.1187	7.6403
382	145924	55742968	19.5448	7.2558	447	199809	89314623	21.1424	7.6460
383	146689	56181887	19.5704	7.2622	448	200704	89915392	21.1660	7.6517
384	147456	56623104	19.5959	7.2685	449	201601	90518849	21.1896	7.6574
385	148225	57066625	19.6214	7.2748	450	202500	91125000	21.2132	7.6631
386	148996	57512456	19.6469	7.2811	451	203401	91733851	21.2368	7.6688
387	149769	57960603	19.6723	7.2874	452	204304	92345408	21.2603	7.6744
388	150544	58411072	19.6977	7.2936	453	205209	92959677	21.2838	7.6801
389	151321	58863869	19.7231	7.2999	454	206116	93576664	21.3073	7.6857
390	152100	59319000	19.7484	7.3061	455	207025	94196375	21.3307	7.6914
391	152881	59776471	19.7737	7.3124	456	207936	94818816	21.3542	7.6970
392	153664	60236288	19.7990	7.3186	457	208849	95443993	21.3776	7.7026
393	154449	60698457	19.8242	7.3248	458	209764	96071912	21.4009	7.7082
394	155236	61162984	19.8494	7.3310	459	210681	96702579	21.4243	7.7138
395	156025	61629875	19.8746	7.3372	460	211600	97336000	21.4476	7.7194
396	156816	62099136	19.8997	7.3434	461	212521	97972181	21.4709	7.7250
397	157609	62570773	19.9249	7.3496	462	213444	98611128	21.4942	7.7305
398	158404	63044792	19.9499	7.3558	463	214369	99252847	21.5174	7.7362
399	159201	63521199	19.9750	7.3619	464	215296	99897344	21.5407	7.7418
400	160000	64000000	20.	7.3681	465	216225	100544625	21.5639	7.7473
401	160801	64481201	20.0250	7.3742	466	217156	101194696	21.5870	7.7529
402	161604	64964808	20.0499	7.3803	467	218089	101847563	21.6102	7.7584
403	162409	65450827	20.0749	7.3864	468	219024	102503232	21.6333	7.7639
404	163216	65939264	20.0998	7.3925	469	219961	103161709	21.6564	7.7695
405	164025	66430125	20.1246	7.3986	470	220900	103823000	21.6795	7.7750
406	164836	66923416	20.1494	7.4047	471	221841	104487111	21.7025	7.7805
407	165649	67419143	20.1742	7.4108	472	222784	105154048	21.7256	7.7860
408	166464	67917312	20.1990	7.4169	473	223729	105823817	21.7486	7.7915
409	167281	68417929	20.2237	7.4229	474	224676	106496424	21.7715	7.7970
410	168100	68921000	20.2485	7.4290	475	225625	107171875	21.7945	7.8025
411	168921	69426531	20.2731	7.4350	476	226576	107850176	21.8174	7.8079
412	169744	69934528	20.2978	7.4410	477	227529	108531333	21.8403	7.8134
413	170569	70444997	20.3224	7.4470	478	228484	109215352	21.8632	7.8188
414	171396	70957944	20.3470	7.4530	479	229441	109902239	21.8861	7.8243
415	172225	71473375	20.3715	7.4590	480	230400	110592000	21.9089	7.8297
416	173056	71991296	20.3961	7.4650	481	231361	111284641	21.9317	7.8352
417	173889	72511713	20.4206	7.4710	482	232324	111980168	21.9545	7.8406
418	174724	73034632	20.4450	7.4770	483	233289	112678587	21.9773	7.8460
419	175561	73560059	20.4693	7.4829	484	234256	113379904	22.	7.8514
420	176400	74088000	20.4939	7.4889	485	235225	114084125	22.0227	7.8568
421	177241	74618461	20.5183	7.4948	486	236196	114791256	22.0454	7.8622
422	178084	75151448	20.5426	7.5007	487	237169	115501303	22.0681	7.8676
423	178929	75686967	20.5670	7.5067	488	238144	116214272	22.0907	7.8730
424	179776	76225024	20.5913	7.5126	489	239121	116930169	22.1133	7.8784
425	180625	76765625	20.6155	7.5185	490	240100	117649000	22.1359	7.8837
426	181476	77308776	20.6398	7.5244	491	241081	118370771	22.1585	7.8891
427	182329	77854483	20.6640	7.5302	492	242064	119095488	22.1811	7.8944
428	183184	78402752	20.6882	7.5361	493	243049	119823157	22.2036	7.8998
429	184041	78953589	20.7123	7.5420	494	244036	120553784	22.2261	7.9051
430	184900	79507000	20.7364	7.5478	495	245025	121287375	22.2486	7.9105
431	185761	80062991	20.7605	7.5537	496	246016	122023936	22.2711	7.9158
432	186624	80621568	20.7846	7.5595	497	247009	122763473	22.2935	7.9211
433	187489	81182737	20.8087	7.5654	498	248004	123505992	22.3159	7.9264
434	188356	81746504	20.8327	7.5712	499	249001	124251499	22.3383	7.9317
435	189225	82312875	20.8567	7.5770	500	250000	125000000	22.3607	7.9370
436	190096	82881856	20.8806	7.5828	501	251001	125751501	22.3830	7.9423
437	190969	83453453	20.9045	7.5886	502	252004	126506008	22.4054	7.9476
438	191844	84027672	20.9284	7.5944	503	253009	127263527	22.4277	7.9528
439	192721	84604519	20.9523	7.6001	504	254016	128024064	22.4499	7.9581
440	193600	85184000	20.9762	7.6059	505	255025	128787625	22.4722	7.9634
441	194481	85766121	21.	7.6117	506	256036	129554216	22.4944	7.9686
442	195364	86350888	21.0238	7.6174	507	257049	130323843	22.5167	7.9739
443	196249	86938307	21.0476	7.6232	508	258064	131096512	22.5389	7.9791
444	197136	87528384	21.0713	7.6289	509	259081	131872229	22.5610	7.9843
445	198025	88121125	21.0950	7.6346	510	260100	132651000	22.5832	7.9896

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000 (CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
511	261121	133432831	22.6053	7.9948	576	331776	191102976	24.	8.3203
512	262144	134217728	22.6274	8.	577	332929	192100033	24.0208	8.3251
513	263169	135005697	22.6495	8.0052	578	334084	193100552	24.0416	8.3300
514	264196	135796744	22.6716	8.0104	579	335241	194104539	24.0624	8.3348
515	265225	136590875	22.6936	8.0156	580	336400	195112000	24.0832	8.3396
516	266256	137388096	22.7156	8.0208	581	337561	196122941	24.1039	8.3443
517	267289	138188413	22.7376	8.0260	582	338724	197137368	24.1247	8.3491
518	268324	138991832	22.7596	8.0311	583	339889	198155287	24.1454	8.3539
519	269361	139798359	22.7816	8.0363	584	341056	199176704	24.1661	8.3587
520	270400	140608000	22.8035	8.0415	585	342225	200201625	24.1868	8.3634
521	271441	141420761	22.8254	8.0466	586	343396	201230056	24.2074	8.3682
522	272484	142236648	22.8473	8.0517	587	344569	202262003	24.2281	8.3730
523	273529	143055667	22.8692	8.0569	588	345744	203297472	24.2487	8.3777
524	274576	143877824	22.8910	8.0620	589	346921	204336469	24.2693	8.3825
525	275625	144703125	22.9129	8.0671	590	348100	205379000	24.2899	8.3872
526	276676	145531576	22.9347	8.0723	591	349281	206425071	24.3105	8.3919
527	277729	146363183	22.9565	8.0774	592	350464	207474688	24.3311	8.3967
528	278784	147197952	22.9783	8.0825	593	351649	208527857	24.3516	8.4014
529	279841	148035889	23.	8.0876	594	352836	209584584	24.3721	8.4061
530	280900	148877000	23.0217	8.0927	595	354025	210644875	24.3926	8.4108
531	281961	149721291	23.0434	8.0978	596	355216	211708736	24.4131	8.4155
532	283024	150568768	23.0651	8.1028	597	356409	212776173	24.4336	8.4202
533	284089	151419437	23.0868	8.1079	598	357604	213847192	24.4540	8.4249
534	285156	152273304	23.1084	8.1130	599	358801	214921799	24.4745	8.4296
535	286225	153130375	23.1301	8.1180	600	360000	216000000	24.4949	8.4343
536	287296	153990656	23.1517	8.1231	601	361201	217081801	24.5153	8.4390
537	288369	154854153	23.1733	8.1281	602	362404	218167208	24.5357	8.4437
538	289444	155720872	23.1948	8.1332	603	363609	219256227	24.5561	8.4484
539	290521	156590819	23.2164	8.1382	604	364816	220348864	24.5764	8.4530
540	291600	157464000	23.2379	8.1433	605	366025	221445125	24.5967	8.4577
541	292681	158340421	23.2594	8.1483	606	367236	222545016	24.6171	8.4623
542	293764	159220088	23.2809	8.1533	607	368449	223648543	24.6374	8.4670
543	294849	160103007	23.3024	8.1583	608	369664	224755712	24.6577	8.4716
544	295936	160989184	23.3238	8.1633	609	370881	225866529	24.6779	8.4763
545	297025	161878625	23.3452	8.1683	610	372100	226981000	24.6982	8.4809
546	298116	162771336	23.3666	8.1733	611	373321	228099131	24.7184	8.4856
547	299209	163667323	23.3880	8.1783	612	374544	229220928	24.7386	8.4902
548	300304	164566592	23.4094	8.1833	613	375769	230346397	24.7588	8.4948
549	301401	165469149	23.4307	8.1882	614	376996	231475544	24.7790	8.4994
550	302500	166375000	23.4521	8.1932	615	378225	232608375	24.7992	8.5040
551	303601	167284151	23.4734	8.1982	616	379456	233744896	24.8193	8.5086
552	304704	168196608	23.4947	8.2031	617	380689	234885113	24.8395	8.5132
553	305809	169112377	23.5160	8.2081	618	381924	236029032	24.8596	8.5178
554	306916	170031464	23.5372	8.2130	619	383161	237176659	24.8797	8.5224
555	308025	170953875	23.5584	8.2180	620	384400	238328000	24.8998	8.5270
556	309136	171879616	23.5797	8.2229	621	385641	239483061	24.9199	8.5316
557	310249	172808693	23.6008	8.2278	622	386884	240641848	24.9399	8.5362
558	311364	173741112	23.6220	8.2327	623	388129	241804367	24.9600	8.5408
559	312481	174676879	23.6432	8.2377	624	389376	242970624	24.9800	8.5453
560	313600	175616000	23.6643	8.2426	625	390625	244140625	25.	8.5499
561	314721	176558481	23.6854	8.2475	626	391876	245314376	25.0200	8.5544
562	315844	177504328	23.7065	8.2524	627	393129	246491883	25.0400	8.5590
563	316969	178453547	23.7276	8.2573	628	394384	247673152	25.0599	8.5635
564	318096	179406144	23.7487	8.2621	629	395641	248858189	25.0799	8.5681
565	319225	180362125	23.7697	8.2670	630	396900	250047000	25.0998	8.5726
566	320356	181321496	23.7908	8.2719	631	398161	251239591	25.1197	8.5772
567	321489	182284263	23.8118	8.2768	632	399424	252435968	25.1396	8.5817
568	322624	183250432	23.8328	8.2816	633	400689	253636137	25.1595	8.5862
569	323761	184220009	23.8537	8.2865	634	401956	254840104	25.1794	8.5907
570	324900	185193000	23.8747	8.2913	635	403225	256047875	25.1992	8.5952
571	326041	186169411	23.8956	8.2962	636	404496	257259456	25.2190	8.5997
572	327184	187149248	23.9165	8.3010	637	405769	258474853	25.2389	8.6042
573	328329	188132517	23.9374	8.3059	638	407044	259694072	25.2587	8.6086
574	329476	189119224	23.9583	8.3107	639	408321	260917119	25.2784	8.6131
575	330625	190109375	23.9792	8.3155	640	409600	262144000	25.2982	8.6175

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
641	410881	263374721	25.3180	8.6222	706	498436	351895816	26.5707	8.9043
642	412164	264609288	25.3377	8.6267	707	499849	353393243	26.5895	8.9085
643	413449	265847707	25.3574	8.6312	708	501264	354894912	26.6083	8.9127
644	414736	267089984	25.3772	8.6357	709	502681	356400829	26.6271	8.9169
645	416025	268336125	25.3969	8.6401	710	504100	357911000	26.6458	8.9211
646	417316	269586136	25.4165	8.6446	711	505521	359425431	26.6646	8.9253
647	418609	270840023	25.4362	8.6490	712	506944	360944128	26.6833	8.9295
648	419904	272097792	25.4558	8.6535	713	508369	362467097	26.7021	8.9337
649	421201	273359449	25.4755	8.6579	714	509796	363994344	26.7208	8.9378
650	422500	274625000	25.4951	8.6624	715	511225	365525875	26.7395	8.9420
651	423801	275894451	25.5147	8.6668	716	512656	367061696	26.7582	8.9462
652	425104	277167808	25.5343	8.6713	717	514089	368601813	26.7769	8.9503
653	426409	278445077	25.5539	8.6757	718	515524	370146232	26.7955	8.9545
654	427716	279726264	25.5734	8.6801	719	516961	371694959	26.8142	8.9587
655	429025	281011375	25.5930	8.6845	720	518400	373248000	26.8328	8.9628
656	430336	282300416	25.6125	8.6890	721	519841	374805361	26.8514	8.9670
657	431649	283593393	25.6320	8.6934	722	521284	376367048	26.8701	8.9711
658	432964	284890312	25.6515	8.6978	723	522729	377933067	26.8887	8.9752
659	434281	286191179	25.6710	8.7022	724	524176	379503424	26.9072	8.9794
660	435600	287496000	25.6905	8.7066	725	525625	381078125	26.9258	8.9835
661	436921	288804781	25.7099	8.7110	726	527076	382657176	26.9444	8.9876
662	438244	290117528	25.7294	8.7154	727	528529	384240583	26.9629	8.9918
663	439569	291434247	25.7488	8.7198	728	529984	385828352	26.9815	8.9959
664	440896	292754944	25.7682	8.7241	729	531441	387420489	27.	9.
665	442225	294079625	25.7876	8.7285	730	532900	389017000	27.0185	9.0041
666	443556	295408296	25.8070	8.7329	731	534361	390617891	27.0370	9.0082
667	444889	296740963	25.8263	8.7373	732	535824	392223168	27.0555	9.0123
668	446224	298077632	25.8457	8.7416	733	537289	393832837	27.0740	9.0164
669	447561	299418309	25.8650	8.7460	734	538756	395446904	27.0924	9.0205
670	448900	300763000	25.8844	8.7503	735	540225	397065375	27.1109	9.0246
671	450241	302111711	25.9037	8.7547	736	541696	398688256	27.1293	9.0287
672	451584	303464448	25.9230	8.7590	737	543169	400315553	27.1477	9.0328
673	452929	304821217	25.9422	8.7634	738	544644	401947272	27.1662	9.0369
674	454276	306182024	25.9615	8.7677	739	546121	403583419	27.1846	9.0410
675	455625	307546875	25.9808	8.7721	740	547600	405224000	27.2029	9.0450
676	456976	308915776	26.	8.7764	741	549081	406869021	27.2213	9.0491
677	458329	310288733	26.0192	8.7807	742	550564	408518488	27.2397	9.0532
678	459684	311665752	26.0384	8.7850	743	552049	410172407	27.2580	9.0572
679	461041	313046839	26.0576	8.7893	744	553536	411830784	27.2764	9.0613
680	462400	314432000	26.0768	8.7937	745	555025	413493625	27.2947	9.0654
681	463761	315821241	26.0960	8.7980	746	556516	415160936	27.3130	9.0694
682	465124	317214568	26.1151	8.8023	747	558009	416832723	27.3313	9.0735
683	466489	318611987	26.1343	8.8066	748	559504	418508992	27.3496	9.0775
684	467856	320013504	26.1534	8.8109	749	561001	420189749	27.3679	9.0816
685	469225	321419125	26.1725	8.8152	750	562500	421875000	27.3861	9.0856
686	470596	322828856	26.1916	8.8194	751	564001	423564751	27.4044	9.0896
687	471969	324242703	26.2107	8.8237	752	565504	425259008	27.4226	9.0937
688	473344	325660672	26.2298	8.8280	753	567009	426957777	27.4408	9.0977
689	474721	327082769	26.2488	8.8323	754	568516	428661064	27.4591	9.1017
690	476100	328509000	26.2679	8.8366	755	570025	430368875	27.4773	9.1057
691	477481	329939371	26.2869	8.8408	756	571536	432081216	27.4955	9.1098
692	478864	331373888	26.3059	8.8451	757	573049	433798093	27.5136	9.1138
693	480249	332812557	26.3249	8.8493	758	574564	435519512	27.5318	9.1178
694	481636	334255384	26.3439	8.8536	759	576081	437245479	27.5500	9.1218
695	483025	335702375	26.3629	8.8578	760	577600	438976000	27.5681	9.1258
696	484416	337153536	26.3818	8.8621	761	579121	440711081	27.5862	9.1298
697	485809	338608873	26.4008	8.8663	762	580644	442450728	27.6043	9.1338
698	487204	340068392	26.4197	8.8706	763	582169	444194947	27.6225	9.1378
699	488601	341532099	26.4386	8.8748	764	583696	445943744	27.6405	9.1418
700	490000	343000000	26.4575	8.8790	765	585225	447697125	27.6586	9.1458
701	491401	344472101	26.4764	8.8833	766	586756	449455096	27.6767	9.1498
702	492804	345948408	26.4953	8.8875	767	588289	451217663	27.6948	9.1537
703	494209	347428927	26.5141	8.8917	768	589824	452984832	27.7128	9.1577
704	495616	348913664	26.5330	8.8959	769	591361	454756609	27.7308	9.1617
705	497025	350402625	26.5518	8.9001	770	592900	456533000	27.7488	9.1657

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000 — (CONTINUED.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
771	594441	458314011	27.7669	9.1696	836	698896	584277056	28.9137	9.4204
772	595984	460099648	27.7849	9.1736	837	700569	586376253	28.9310	9.4241
773	597529	461889917	27.8029	9.1775	838	702244	588480472	28.9482	9.4279
774	599076	463684824	27.8209	9.1815	839	703921	590589719	28.9655	9.4316
775	600625	465484375	27.8388	9.1855	840	705600	592704000	28.9828	9.4354
776	602176	467288576	27.8568	9.1894	841	707281	594823321	29.	9.4391
777	603729	469097433	27.8747	9.1933	842	708964	596947688	29.0172	9.4429
778	605284	470910952	27.8927	9.1973	843	710649	599077107	29.0345	9.4466
779	606841	472729139	27.9106	9.2012	844	712336	601211584	29.0517	9.4503
780	608400	474552000	27.9285	9.2052	845	714025	603351125	29.0689	9.4541
781	609961	476379541	27.9464	9.2091	846	715716	605495736	29.0861	9.4578
782	611524	478211768	27.9643	9.2130	847	717409	607645423	29.1033	9.4615
783	613089	480048687	27.9821	9.2170	848	719104	609800192	29.1204	9.4652
784	614656	481890304	28.	9.2209	849	720801	611960049	29.1376	9.4690
785	616225	483736625	28.0179	9.2248	850	722500	614125000	29.1548	9.4727
786	617796	485587656	28.0357	9.2287	851	724201	616295051	29.1719	9.4764
787	619369	487443403	28.0535	9.2326	852	725904	618470208	29.1890	9.4801
788	620944	489303872	28.0713	9.2365	853	727609	620650477	29.2062	9.4838
789	622521	491169069	28.0891	9.2404	854	729316	622835864	29.2233	9.4875
790	624100	493039000	28.1069	9.2443	855	731025	625026375	29.2404	9.4912
791	625681	494913671	28.1247	9.2482	856	732736	627222016	29.2575	9.4949
792	627264	496793088	28.1425	9.2521	857	734449	629422793	29.2746	9.4986
793	628849	498677257	28.1603	9.2560	858	736164	631628712	29.2916	9.5023
794	630436	500566184	28.1780	9.2599	859	737881	633839779	29.3087	9.5060
795	632025	502459875	28.1957	9.2638	860	739600	636056000	29.3258	9.5097
796	633616	504358336	28.2135	9.2677	861	741321	638277381	29.3428	9.5134
797	635209	506261573	28.2312	9.2716	862	743044	640503928	29.3598	9.5171
798	636804	508169592	28.2489	9.2754	863	744769	642735647	29.3769	9.5207
799	638401	510082399	28.2666	9.2793	864	746496	644972544	29.3939	9.5244
800	640000	512000000	28.2843	9.2832	865	748225	647214625	29.4109	9.5281
801	641601	513922401	28.3019	9.2870	866	749956	649461896	29.4279	9.5317
802	643204	515849608	28.3196	9.2909	867	751689	651714363	29.4449	9.5354
803	644809	517781627	28.3373	9.2948	868	753424	653972032	29.4618	9.5391
804	646416	519718464	28.3549	9.2986	869	755161	656234909	29.4788	9.5427
805	648025	521660125	28.3725	9.3025	870	756900	658503000	29.4958	9.5464
806	649636	523606616	28.3901	9.3063	871	758641	660776311	29.5127	9.5501
807	651249	525557943	28.4077	9.3102	872	760384	663054848	29.5296	9.5537
808	652864	527514112	28.4253	9.3140	873	762129	665338617	29.5466	9.5574
809	654481	529475129	28.4429	9.3179	874	763876	667627624	29.5635	9.5610
810	656100	531441000	28.4605	9.3217	875	765625	669921875	29.5804	9.5647
811	657721	533411731	28.4781	9.3255	876	767376	672221376	29.5973	9.5683
812	659344	535387328	28.4956	9.3294	877	769129	674526133	29.6142	9.5719
813	660969	537367797	28.5132	9.3332	878	770884	676836152	29.6311	9.5756
814	662596	539353144	28.5307	9.3370	879	772641	679151439	29.6479	9.5792
815	664225	541343375	28.5482	9.3408	880	774400	681472000	29.6648	9.5828
816	665856	543338496	28.5657	9.3447	881	776161	683797841	29.6816	9.5865
817	667489	545338513	28.5832	9.3485	882	777924	686128968	29.6985	9.5901
818	669124	547343432	28.6007	9.3523	883	779689	688465387	29.7153	9.5937
819	670761	549353259	28.6182	9.3561	884	781456	690807104	29.7321	9.5973
820	672400	551368000	28.6356	9.3599	885	783225	693154125	29.7489	9.6010
821	674041	553387661	28.6531	9.3637	886	784996	695506456	29.7658	9.6046
822	675684	555412248	28.6705	9.3675	887	786769	697864103	29.7825	9.6082
823	677329	557441767	28.6880	9.3713	888	788544	700227072	29.7993	9.6118
824	678976	559476224	28.7054	9.3751	889	790321	702595369	29.8161	9.6154
825	680625	561515625	28.7228	9.3789	890	792100	704969000	29.8329	9.6190
826	682276	563559976	28.7402	9.3827	891	793881	707347971	29.8496	9.6226
827	683929	565609283	28.7576	9.3865	892	795664	709732288	29.8664	9.6262
828	685584	567663552	28.7750	9.3902	893	797449	712121957	29.8831	9.6298
829	687241	569722789	28.7924	9.3940	894	799236	714516984	29.8998	9.6334
830	688900	571787000	28.8097	9.3978	895	801025	716917375	29.9166	9.6370
831	690561	573856191	28.8271	9.4016	896	802816	719325136	29.9333	9.6406
832	692224	575930368	28.8444	9.4053	897	804609	721734273	29.9500	9.6442
833	693889	578009537	28.8617	9.4091	898	806404	724150792	29.9666	9.6477
834	695556	580093704	28.8791	9.4129	899	808201	726572699	29.9833	9.6513
835	697225	582182875	28.8964	9.4166	900	810000	729000000	30.	9.6549

TABLE of Squares, Cubes, Square Roots, and Cube Roots, of Numbers from 1 to 1000—(Continued.)

No.	Square.	Cube.	Sq. Rt.	C. Rt.	No.	Square.	Cube.	Sq. Rt.	C. Rt.
901	811801	731432701	30.0167	9.6585	951	904401	860085351	30.8383	9.8[illegible]
902	813604	733870808	30.0333	9.6620	952	906304	862601408	30.8545	9.8374
903	815409	736314327	30.0500	9.6656	953	908209	865128177	30.8707	9.8[illegible]
904	817216	738763264	30.0666	9.6692	954	910116	867665664	30.8869	9.8[illegible]
905	819025	741217625	30.0832	9.6727	955	912025	870203875	30.9031	9.8537
906	820836	743677416	30.0998	9.6763	956	913936	872752616	30.9192	9.8571
907	822649	746142643	30.1164	9.6799	957	915849	875467493	30.9354	9.8605
908	824464	748613312	30.1330	9.6834	958	917764	879217912	30.9516	9.8639
909	826281	751089429	30.1496	9.6870	959	919681	881974079	30.9677	9.8674
910	828100	753571000	30.1662	9.6905	960	921600	884736000	30.9839	9.8709
911	829921	756058031	30.1828	9.6941	961	923521	887503681	31.	9.8743
912	831744	758550528	30.1993	9.6976	962	925444	890277128	31.0161	9.8777
913	833569	761048497	30.2159	9.7012	963	927369	893056347	31.0322	9.8811
914	835396	763551944	30.2324	9.7047	964	929296	895841344	31.0483	9.8845
915	837225	766060875	30.2490	9.7082	965	931225	898632125	31.0644	9.8879
916	839056	768575296	30.2655	9.7118	966	933156	901428696	31.0805	9.8914
917	840889	771095213	30.2820	9.7153	967	935089	904231063	31.0966	9.8948
918	842724	773620632	30.2985	9.7188	968	937024	907039232	31.1127	9.8982
919	844561	776151559	30.3150	9.7224	969	938961	909853209	31.1288	9.9016
920	846400	778688000	30.3315	9.7259	970	940900	912673000	31.1448	9.9050
921	848241	781229961	30.3480	9.7294	971	942841	915498611	31.1609	9.9084
922	850084	783777448	30.3645	9.7329	972	944784	918330048	31.1769	9.9118
923	851929	786330467	30.3809	9.7364	973	946729	921167317	31.1929	9.9152
924	853776	788889024	30.3974	9.7400	974	948676	924010424	31.2090	9.9186
925	855625	791453125	30.4138	9.7435	975	950625	926859375	31.2250	9.9220
926	857476	794022776	30.4302	9.7470	976	952576	929714176	31.2410	9.9254
927	859329	796597983	30.4467	9.7505	977	954529	932574833	31.2570	9.9287
928	861184	799178752	30.4631	9.7540	978	956484	935441352	31.2730	9.9321
929	863041	801765089	30.4795	9.7575	979	958441	938313739	31.2890	9.9355
930	864900	804357000	30.4959	9.7610	980	960400	941192000	31.3050	9.9389
931	866761	806954491	30.5123	9.7645	981	962361	944076141	31.3209	9.9423
932	868624	809557568	30.5287	9.7680	982	964324	946966168	31.3369	9.9456
933	870489	812166237	30.5450	9.7715	983	966289	949862087	31.3528	9.9490
934	872356	814780504	30.5614	9.7750	984	968256	952763904	31.3688	9.9523
935	874225	817400375	30.5778	9.7785	985	970225	955671625	31.3847	9.9557
936	876096	820025856	30.5941	9.7819	986	972196	958585256	31.4006	9.9591
937	877969	822656953	30.6105	9.7854	987	974169	961504803	31.4166	9.9624
938	879844	825293672	30.6268	9.7889	988	976144	964430272	31.4325	9.9658
939	881721	827936019	30.6431	9.7924	989	978121	967361669	31.4484	9.9691
940	883600	830584000	30.6594	9.7959	990	980100	970299000	31.4642	9.9725
941	885481	833237621	30.6757	9.7993	991	982081	973242271	31.4802	9.9758
942	887364	835896888	30.6920	9.8028	992	984064	976191488	31.4960	9.9791
943	889249	838561807	30.7083	9.8063	993	986049	979146657	31.5119	9.9824
944	891136	841232384	30.7246	9.8097	994	988036	982107784	31.5278	9.9857
945	893025	843908625	30.7409	9.8132	995	990025	985074875	31.5436	9.9890
946	894916	846590536	30.7571	9.8167	996	992016	988047936	31.5595	9.9923
947	896809	849278123	30.7734	9.8201	997	994009	991026973	31.5753	9.9956
948	898704	851971392	30.7896	9.8236	998	996004	994011992	31.5911	9.9989
949	900601	854670349	30.8058	9.8270	999	998001	997002999	31.6070	9.9967
950	902500	857375000	30.8221	9.8305	1000	1000000	1000000000	31.6228	10.

Square Roots and Cube Roots of Numbers from 1000 to 10000.

No errors.

Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.
1005	31.70	10.02	1405	37.48	11.20	1805	42.49	12.18	2205	46.96	13.02
1010	31.78	10.03	1410	37.55	11.21	1810	42.54	12.19	2210	47.01	13.03
1015	31.86	10.05	1415	37.62	11.23	1815	42.60	12.20	2215	47.06	13.04
1020	31.94	10.07	1420	37.68	11.24	1820	42.66	12.21	2220	47.12	13.05
1025	32.02	10.08	1425	37.75	11.25	1825	42.72	12.22	2225	47.17	13.05
1030	32.09	10.10	1430	37.82	11.27	1830	42.78	12.23	2230	47.22	13.06
1035	32.17	10.12	1435	37.88	11.28	1835	42.84	12.24	2235	47.28	13.07
1040	32.25	10.13	1440	37.95	11.29	1840	42.90	12.25	2240	47.33	13.08
1045	32.33	10.15	1445	38.01	11.31	1845	42.95	12.26	2245	47.38	13.09
1050	32.40	10.16	1450	38.08	11.32	1850	43.01	12.28	2250	47.43	13.10
1055	32.48	10.18	1455	38.14	11.33	1855	43.07	12.29	2255	47.49	13.11
1060	32.56	10.20	1460	38.21	11.34	1860	43.13	12.30	2260	47.54	13.12
1065	32.63	10.21	1465	38.28	11.36	1865	43.19	12.31	2265	47.59	13.13
1070	32.71	10.23	1470	38.34	11.37	1870	43.24	12.32	2270	47.64	13.14
1075	32.79	10.24	1475	38.41	11.38	1875	43.30	12.33	2275	47.70	13.15
1080	32.86	10.26	1480	38.47	11.40	1880	43.36	12.34	2280	47.75	13.16
1085	32.94	10.28	1485	38.54	11.41	1885	43.42	12.35	2285	47.80	13.17
1090	33.02	10.29	1490	38.60	11.42	1890	43.47	12.36	2290	47.85	13.18
1095	33.09	10.31	1495	38.67	11.43	1895	43.53	12.37	2295	47.91	13.19
1100	33.17	10.32	1500	38.73	11.45	1900	43.59	12.39	2300	47.96	13.20
1105	33.24	10.34	1505	38.79	11.46	1905	43.65	12.40	2305	48.01	13.21
1110	33.32	10.35	1510	38.86	11.47	1910	43.70	12.41	2310	48.06	13.22
1115	33.39	10.37	1515	38.92	11.49	1915	43.76	12.42	2315	48.11	13.23
1120	33.47	10.38	1520	38.99	11.50	1920	43.82	12.43	2320	48.17	13.24
1125	33.54	10.40	1525	39.05	11.51	1925	43.87	12.44	2325	48.22	13.25
1130	33.62	10.42	1530	39.12	11.52	1930	43.93	12.45	2330	48.27	13.26
1135	33.69	10.43	1535	39.18	11.54	1935	43.99	12.46	2335	48.32	13.27
1140	33.76	10.45	1540	39.24	11.55	1940	44.05	12.47	2340	48.37	13.28
1145	33.84	10.46	1545	39.31	11.56	1945	44.10	12.48	2345	48.43	13.29
1150	33.91	10.48	1550	39.37	11.57	1950	44.16	12.49	2350	48.48	13.30
1155	33.99	10.49	1555	39.43	11.59	1955	44.22	12.50	2355	48.53	13.30
1160	34.06	10.51	1560	39.50	11.60	1960	44.27	12.51	2360	48.58	13.31
1165	34.13	10.52	1565	39.56	11.61	1965	44.33	12.53	2365	48.63	13.32
1170	34.21	10.54	1570	39.62	11.62	1970	44.38	12.54	2370	48.68	13.33
1175	34.28	10.55	1575	39.69	11.63	1975	44.44	12.55	2375	48.73	13.34
1180	34.35	10.57	1580	39.75	11.65	1980	44.50	12.56	2380	48.79	13.35
1185	34.42	10.58	1585	39.81	11.66	1985	44.55	12.57	2385	48.84	13.36
1190	34.50	10.60	1590	39.87	11.67	1990	44.61	12.58	2390	48.89	13.37
1195	34.57	10.61	1595	39.94	11.68	1995	44.67	12.59	2395	48.94	13.38
1200	34.64	10.63	1600	40.00	11.70	2000	44.72	12.60	2400	48.99	13.39
1205	34.71	10.64	1605	40.06	11.71	2005	44.78	12.61	2405	49.04	13.40
1210	34.79	10.66	1610	40.12	11.72	2010	44.83	12.62	2410	49.09	13.41
1215	34.86	10.67	1615	40.19	11.73	2015	44.89	12.63	2415	49.14	13.42
1220	34.93	10.69	1620	40.25	11.74	2020	44.94	12.64	2420	49.19	13.43
1225	35.00	10.70	1625	40.31	11.76	2025	45.00	12.65	2425	49.24	13.43
1230	35.07	10.71	1630	40.37	11.77	2030	45.06	12.66	2430	49.30	13.44
1235	35.14	10.73	1635	40.44	11.78	2035	45.11	12.67	2435	49.35	13.45
1240	35.21	10.74	1640	40.50	11.79	2040	45.17	12.68	2440	49.40	13.46
1245	35.28	10.76	1645	40.56	11.80	2045	45.22	12.69	2445	49.45	13.47
1250	35.36	10.77	1650	40.62	11.82	2050	45.28	12.70	2450	49.50	13.48
1255	35.43	10.79	1655	40.68	11.83	2055	45.33	12.71	2460	49.60	13.50
1260	35.50	10.80	1660	40.74	11.84	2060	45.39	12.72	2470	49.70	13.52
1265	35.57	10.82	1665	40.80	11.85	2065	45.44	12.73	2480	49.80	13.54
1270	35.64	10.83	1670	40.87	11.86	2070	45.50	12.74	2490	49.90	13.55
1275	35.71	10.84	1675	40.93	11.88	2075	45.55	12.75	2500	50.00	13.57
1280	35.78	10.86	1680	40.99	11.89	2080	45.61	12.77	2510	50.10	13.59
1285	35.85	10.87	1685	41.05	11.90	2085	45.66	12.78	2520	50.20	13.61
1290	35.92	10.89	1690	41.11	11.91	2090	45.72	12.79	2530	50.30	13.63
1295	35.99	10.90	1695	41.17	11.92	2095	45.77	12.80	2540	50.40	13.64
1300	36.06	10.91	1700	41.23	11.93	2100	45.83	12.81	2550	50.50	13.66
1305	36.12	10.93	1705	41.29	11.95	2105	45.88	12.82	2560	50.60	13.68
1310	36.19	10.94	1710	41.35	11.96	2110	45.93	12.83	2570	50.70	13.70
1315	36.26	10.96	1715	41.41	11.97	2115	45.99	12.84	2580	50.79	13.72
1320	36.33	10.97	1720	41.47	11.98	2120	46.04	12.85	2590	50.89	13.73
1325	36.40	10.98	1725	41.53	11.99	2125	46.10	12.86	2600	50.99	13.75
1330	36.47	11.00	1730	41.59	12.00	2130	46.15	12.87	2610	51.09	13.77
1335	36.54	11.01	1735	41.65	12.02	2135	46.21	12.88	2620	51.19	13.79
1340	36.61	11.02	1740	41.71	12.03	2140	46.26	12.89	2630	51.28	13.80
1345	36.67	11.04	1745	41.77	12.04	2145	46.31	12.90	2640	51.38	13.82
1350	36.74	11.05	1750	41.83	12.05	2150	46.37	12.91	2650	51.48	13.84
1355	36.81	11.07	1755	41.89	12.06	2155	46.42	12.92	2660	51.58	13.86
1360	36.88	11.08	1760	41.95	12.07	2160	46.48	12.93	2670	51.67	13.87
1365	36.95	11.09	1765	42.01	12.09	2165	46.53	12.94	2680	51.77	13.89
1370	37.01	11.11	1770	42.07	12.10	2170	46.58	12.95	2690	51.87	13.91
1375	37.08	11.12	1775	42.13	12.11	2175	46.64	12.96	2700	51.96	13.93
1380	37.15	11.13	1780	42.19	12.12	2180	46.69	12.97	2710	52.06	13.94
1385	37.22	11.15	1785	42.25	12.13	2185	46.74	12.98	2720	52.15	13.96
1390	37.28	11.16	1790	42.31	12.14	2190	46.80	12.99	2730	52.25	13.98
1395	37.35	11.17	1795	42.37	12.15	2195	46.85	13.00	2740	52.35	14.00
1400	37.42	11.19	1800	42.43	12.16	2200	46.90	13.01	2750	52.44	14.01

Square Roots and Cube Roots of Numbers from 1000 to 10000
—(CONTINUED.)

Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.
2760	52.54	14.03	3550	59.58	15.25	4340	65.88	16.31	5130	71.62	17.25
2770	52.63	14.04	3560	59.67	15.27	4350	65.95	16.32	5140	71.69	17.26
2780	52.73	14.06	3570	59.75	15.28	4360	66.03	16.34	5150	71.76	17.27
2790	52.82	14.08	3580	59.83	15.30	4370	66.11	16.35	5160	71.83	17.28
2800	52.92	14.09	3590	59.92	15.31	4380	66.18	16.36	5170	71.90	17.29
2810	53.01	14.11	3600	60.00	15.33	4390	66.26	16.37	5180	71.97	17.30
2820	53.10	14.13	3610	60.08	15.34	4400	66.33	16.39	5190	72.04	17.31
2830	53.20	14.14	3620	60.17	15.35	4410	66.41	16.40	5200	72.11	17.32
2840	53.29	14.16	3630	60.25	15.37	4420	66.48	16.41	5210	72.18	17.34
2850	53.39	14.18	3640	60.33	15.38	4430	66.56	16.42	5220	72.25	17.35
2860	53.48	14.19	3650	60.42	15.40	4440	66.63	16.44	5230	72.32	17.36
2870	53.57	14.21	3660	60.50	15.41	4450	66.71	16.45	5240	72.39	17.37
2880	53.67	14.23	3670	60.58	15.42	4460	66.78	16.46	5250	72.46	17.38
2890	53.76	14.24	3680	60.66	15.44	4470	66.86	16.47	5260	72.53	17.39
2900	53.85	14.26	3690	60.75	15.45	4480	66.93	16.49	5270	72.59	17.40
2910	53.94	14.28	3700	60.83	15.47	4490	67.01	16.50	5280	72.66	17.41
2920	54.04	14.29	3710	60.91	15.48	4500	67.08	16.51	5290	72.73	17.42
2930	54.13	14.31	3720	60.99	15.49	4510	67.16	16.52	5300	72.80	17.44
2940	54.22	14.33	3730	61.07	15.51	4520	67.23	16.53	5310	72.87	17.45
2950	54.31	14.34	3740	61.16	15.52	4530	67.31	16.55	5320	72.94	17.46
2960	54.41	14.36	3750	61.24	15.54	4540	67.38	16.56	5330	73.01	17.47
2970	54.50	14.37	3760	61.32	15.55	4550	67.45	16.57	5340	73.08	17.48
2980	54.59	14.39	3770	61.40	15.56	4560	67.53	16.58	5350	73.14	17.49
2990	54.68	14.41	3780	61.48	15.58	4570	67.60	16.59	5360	73.21	17.50
3000	54.77	14.42	3790	61.56	15.59	4580	67.68	16.61	5370	73.28	17.51
3010	54.86	14.44	3800	61.64	15.60	4590	67.75	16.62	5380	73.35	17.52
3020	54.95	14.45	3810	61.73	15.62	4600	67.82	16.63	5390	73.42	17.53
3030	55.05	14.47	3820	61.81	15.63	4610	67.90	16.64	5400	73.48	17.54
3040	55.14	14.49	3830	61.89	15.65	4620	67.97	16.66	5410	73.55	17.55
3050	55.23	14.50	3840	61.97	15.66	4630	68.04	16.67	5420	73.62	17.57
3060	55.32	14.52	3850	62.05	15.67	4640	68.12	16.68	5430	73.69	17.58
3070	55.41	14.53	3860	62.13	15.69	4650	68.19	16.69	5440	73.76	17.59
3080	55.50	14.55	3870	62.21	15.70	4660	68.26	16.70	5450	73.82	17.60
3090	55.59	14.57	3880	62.29	15.71	4670	68.34	16.71	5460	73.89	17.61
3100	55.68	14.58	3890	62.37	15.73	4680	68.41	16.73	5470	73.96	17.62
3110	55.77	14.60	3900	62.45	15.74	4690	68.48	16.74	5480	74.03	17.63
3120	55.86	14.61	3910	62.53	15.75	4700	68.56	16.75	5490	74.09	17.64
3130	55.95	14.63	3920	62.61	15.77	4710	68.63	16.76	5500	74.16	17.65
3140	56.04	14.64	3930	62.69	15.78	4720	68.70	16.77	5510	74.23	17.66
3150	56.12	14.66	3940	62.77	15.79	4730	68.77	16.79	5520	74.30	17.67
3160	56.21	14.67	3950	62.85	15.81	4740	68.85	16.80	5530	74.36	17.68
3170	56.30	14.69	3960	62.93	15.82	4750	68.92	16.81	5540	74.43	17.69
3180	56.39	14.71	3970	63.01	15.83	4760	68.99	16.82	5550	74.50	17.71
3190	56.48	14.72	3980	63.09	15.85	4770	69.07	16.83	5560	74.57	17.72
3200	56.57	14.74	3990	63.17	15.86	4780	69.14	16.85	5570	74.63	17.73
3210	56.66	14.75	4000	63.25	15.87	4790	69.21	16.86	5580	74.70	17.74
3220	56.75	14.77	4010	63.32	15.89	4800	69.28	16.87	5590	74.77	17.75
3230	56.83	14.78	4020	63.40	15.90	4810	69.35	16.88	5600	74.83	17.76
3240	56.92	14.80	4030	63.48	15.91	4820	69.43	16.89	5610	74.90	17.77
3250	57.01	14.81	4040	63.56	15.93	4830	69.50	16.90	5620	74.97	17.78
3260	57.10	14.83	4050	63.64	15.94	4840	69.57	16.92	5630	75.03	17.79
3270	57.18	14.84	4060	63.72	15.95	4850	69.64	16.93	5640	75.10	17.80
3280	57.27	14.86	4070	63.80	15.97	4860	69.71	16.94	5650	75.17	17.81
3290	57.36	14.87	4080	63.87	15.98	4870	69.79	16.95	5660	75.23	17.82
3300	57.45	14.89	4090	63.95	15.99	4880	69.86	16.96	5670	75.30	17.83
3310	57.53	14.90	4100	64.03	16.01	4890	69.93	16.97	5680	75.37	17.84
3320	57.62	14.92	4110	64.11	16.02	4900	70.00	16.98	5690	75.43	17.85
3330	57.71	14.93	4120	64.19	16.03	4910	70.07	17.00	5700	75.50	17.86
3340	57.79	14.95	4130	64.27	16.04	4920	70.14	17.01	5710	75.56	17.87
3350	57.88	14.96	4140	64.34	16.06	4930	70.21	17.02	5720	75.63	17.88
3360	57.97	14.98	4150	64.42	16.07	4940	70.29	17.03	5730	75.70	17.89
3370	58.05	14.99	4160	64.50	16.08	4950	70.36	17.04	5740	75.76	17.90
3380	58.14	15.01	4170	64.58	16.10	4960	70.43	17.05	5750	75.83	17.92
3390	58.22	15.02	4180	64.65	16.11	4970	70.50	17.07	5760	75.89	17.93
3400	58.31	15.04	4190	64.73	16.12	4980	70.57	17.08	5770	75.96	17.94
3410	58.40	15.05	4200	64.81	16.13	4990	70.64	17.09	5780	76.03	17.95
3420	58.48	15.07	4210	64.88	16.15	5000	70.71	17.10	5790	76.09	17.96
3430	58.57	15.08	4220	64.96	16.16	5010	70.78	17.11	5800	76.16	17.97
3440	58.65	15.10	4230	65.04	16.17	5020	70.85	17.12	5810	76.22	17.98
3450	58.74	15.11	4240	65.12	16.19	5030	70.92	17.13	5820	76.29	17.99
3460	58.82	15.12	4250	65.19	16.20	5040	70.99	17.15	5830	76.35	18.00
3470	58.91	15.14	4260	65.27	16.21	5050	71.06	17.16	5840	76.42	18.01
3480	58.99	15.15	4270	65.35	16.22	5060	71.13	17.17	5850	76.49	18.02
3490	59.08	15.17	4280	65.42	16.24	5070	71.20	17.18	5860	76.55	18.03
3500	59.16	15.18	4290	65.50	16.25	5080	71.27	17.19	5870	76.62	18.04
3510	59.25	15.20	4300	65.57	16.26	5090	71.34	17.20	5880	76.68	18.05
3520	59.33	15.21	4310	65.65	16.27	5100	71.41	17.21	5890	76.75	18.06
3530	59.41	15.23	4320	65.73	16.29	5110	71.48	17.22	5900	76.81	18.07
3540	59.50	15.24	4330	65.80	16.30	5120	71.55	17.24	5910	76.88	18.08

Square Roots and Cube Roots of Numbers from 1000 to 10000

(CONTINUED.)

Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.
5920	76.94	18.09	6710	81.91	18.86	7500	86.60	19.57	8290	91.05	20.24
5930	77.01	18.10	6720	81.98	18.87	7510	86.66	19.58	8300	91.10	20.25
5940	77.07	18.11	6730	82.04	18.88	7520	86.72	19.59	8310	91.16	20.26
5950	77.14	18.12	6740	82.10	18.89	7530	86.78	19.60	8320	91.21	20.26
5960	77.20	18.13	6750	82.16	18.90	7540	86.83	19.61	8330	91.27	20.27
5970	77.27	18.14	6760	82.22	18.91	7550	86.89	19.62	8340	91.32	20.28
5980	77.33	18.15	6770	82.28	18.92	7560	86.95	19.63	8350	91.38	20.29
5990	77.40	18.16	6780	82.34	18.93	7570	87.01	19.64	8360	91.43	20.30
6000	77.46	18.17	6790	82.40	18.94	7580	87.06	19.64	8370	91.49	20.30
6010	77.52	18.18	6800	82.46	18.95	7590	87.12	19.65	8380	91.54	20.31
6020	77.59	18.19	6810	82.52	18.95	7600	87.18	19.66	8390	91.60	20.32
6030	77.65	18.20	6820	82.58	18.96	7610	87.24	19.67	8400	91.65	20.33
6040	77.72	18.21	6830	82.64	18.97	7620	87.29	19.68	8410	91.71	20.34
6050	77.78	18.22	6840	82.70	18.98	7630	87.35	19.69	8420	91.76	20.34
6060	77.85	18.23	6850	82.76	18.99	7640	87.41	19.70	8430	91.82	20.35
6070	77.91	18.24	6860	82.83	19.00	7650	87.46	19.70	8440	91.87	20.36
6080	77.97	18.25	6870	82.89	19.01	7660	87.52	19.71	8450	91.92	20.37
6090	78.04	18.26	6880	82.95	19.02	7670	87.58	19.72	8460	91.98	20.38
6100	78.10	18.27	6890	83.01	19.03	7680	87.64	19.73	8470	92.03	20.38
6110	78.17	18.28	6900	83.07	19.04	7690	87.69	19.74	8480	92.09	20.39
6120	78.23	18.29	6910	83.13	19.05	7700	87.75	19.75	8490	92.14	20.40
6130	78.29	18.30	6920	83.19	19.06	7710	87.81	19.76	8500	92.20	20.41
6140	78.36	18.31	6930	83.25	19.07	7720	87.86	19.76	8510	92.25	20.42
6150	78.42	18.32	6940	83.31	19.07	7730	87.92	19.77	8520	92.30	20.42
6160	78.49	18.33	6950	83.37	19.08	7740	87.98	19.78	8530	92.36	20.43
6170	78.55	18.34	6960	83.43	19.09	7750	88.03	19.79	8540	92.41	20.44
6180	78.61	18.35	6970	83.49	19.10	7760	88.09	19.80	8550	92.47	20.45
6190	78.68	18.36	6980	83.55	19.11	7770	88.15	19.81	8560	92.52	20.46
6200	78.74	18.37	6990	83.61	19.12	7780	88.20	19.81	8570	92.57	20.46
6210	78.80	18.38	7000	83.67	19.13	7790	88.26	19.82	8580	92.63	20.47
6220	78.87	18.39	7010	83.73	19.14	7800	88.32	19.83	8590	92.68	20.48
6230	78.93	18.40	7020	83.79	19.15	7810	88.37	19.84	8600	92.74	20.49
6240	78.99	18.41	7030	83.85	19.16	7820	88.43	19.85	8610	92.79	20.50
6250	79.06	18.42	7040	83.90	19.17	7830	88.49	19.86	8620	92.84	20.50
6260	79.12	18.43	7050	83.96	19.17	7840	88.54	19.87	8630	92.90	20.51
6270	79.18	18.44	7060	84.02	19.18	7850	88.60	19.87	8640	92.95	20.52
6280	79.25	18.45	7070	84.08	19.19	7860	88.66	19.88	8650	93.01	20.53
6290	79.31	18.46	7080	84.14	19.20	7870	88.71	19.89	8660	93.06	20.54
6300	79.37	18.47	7090	84.20	19.21	7880	88.77	19.90	8670	93.11	20.54
6310	79.44	18.48	7100	84.26	19.22	7890	88.83	19.91	8680	93.17	20.55
6320	79.50	18.49	7110	84.32	19.23	7900	88.88	19.92	8690	93.22	20.56
6330	79.56	18.50	7120	84.38	19.24	7910	88.94	19.92	8700	93.27	20.57
6340	79.62	18.51	7130	84.44	19.25	7920	88.99	19.93	8710	93.33	20.57
6350	79.69	18.52	7140	84.50	19.26	7930	89.05	19.94	8720	93.38	20.58
6360	79.75	18.53	7150	84.56	19.26	7940	89.11	19.95	8730	93.43	20.59
6370	79.81	18.54	7160	84.62	19.27	7950	89.16	19.96	8740	93.49	20.60
6380	79.87	18.55	7170	84.68	19.28	7960	89.22	19.97	8750	93.54	20.61
6390	79.94	18.56	7180	84.73	19.29	7970	89.27	19.97	8760	93.59	20.61
6400	80.00	18.57	7190	84.79	19.30	7980	89.33	19.98	8770	93.65	20.62
6410	80.06	18.58	7200	84.85	19.31	7990	89.39	19.99	8780	93.70	20.63
6420	80.12	18.59	7210	84.91	19.32	8000	89.44	20.00	8790	93.75	20.64
6430	80.19	18.60	7220	84.97	19.33	8010	89.50	20.01	8800	93.81	20.65
6440	80.25	18.60	7230	85.03	19.34	8020	89.55	20.02	8810	93.86	20.65
6450	80.31	18.61	7240	85.09	19.35	8030	89.61	20.02	8820	93.91	20.66
6460	80.37	18.62	7250	85.15	19.35	8040	89.67	20.03	8830	93.97	20.67
6470	80.44	18.63	7260	85.21	19.36	8050	89.72	20.04	8840	94.02	20.68
6480	80.50	18.64	7270	85.26	19.37	8060	89.78	20.05	8850	94.07	20.68
6490	80.56	18.65	7280	85.32	19.38	8070	89.83	20.06	8860	94.13	20.69
6500	80.62	18.66	7290	85.38	19.39	8080	89.89	20.07	8870	94.18	20.70
6510	80.68	18.67	7300	85.44	19.40	8090	89.94	20.07	8880	94.23	20.71
6520	80.75	18.68	7310	85.50	19.41	8100	90.00	20.08	8890	94.29	20.72
6530	80.81	18.69	7320	85.56	19.42	8110	90.06	20.09	8900	94.34	20.72
6540	80.87	18.70	7330	85.62	19.43	8120	90.11	20.10	8910	94.39	20.73
6550	80.93	18.71	7340	85.67	19.43	8130	90.17	20.11	8920	94.45	20.74
6560	80.99	18.72	7350	85.73	19.44	8140	90.22	20.12	8930	94.50	20.75
6570	81.06	18.73	7360	85.79	19.45	8150	90.28	20.12	8940	94.55	20.75
6580	81.12	18.74	7370	85.85	19.46	8160	90.33	20.13	8950	94.60	20.76
6590	81.18	18.75	7380	85.91	19.47	8170	90.39	20.14	8960	94.66	20.77
6600	81.24	18.76	7390	85.97	19.48	8180	90.44	20.15	8970	94.71	20.78
6610	81.30	18.77	7400	86.02	19.49	8190	90.50	20.16	8980	94.76	20.79
6620	81.36	18.78	7410	86.08	19.50	8200	90.55	20.17	8990	94.82	20.79
6630	81.42	18.79	7420	86.14	19.50	8210	90.61	20.17	9000	94.87	20.80
6640	81.49	18.80	7430	86.20	19.51	8220	90.66	20.18	9010	94.92	20.81
6650	81.55	18.81	7440	86.26	19.52	8230	90.72	20.19	9020	94.97	20.82
6660	81.61	18.81	7450	86.31	19.53	8240	90.77	20.20	9030	95.03	20.82
6670	81.67	18.82	7460	86.37	19.54	8250	90.83	20.21	9040	95.08	20.8
6680	81.73	18.83	7470	86.43	19.55	8260	90.88	20.21	9050	95.13	20.
6690	81.79	18.84	7480	86.49	19.56	8270	90.94	20.22	9060	95.18	2
6700	81.85	18.85	7490	86.54	19.57	8280	90.99	20.23	9070	95.24	

Square Roots and Cube Roots of Numbers from 1000

—(CONTINUED.)

Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	Sq. Rt.	Cu. Rt.	Num.	S
9080	95.29	20.86	9320	96.54	21.04	9550	97.72	21.22	9780	
9090	95.34	20.87	9330	96.59	21.05	9560	97.78	21.22	9790	
9100	95.39	20.88	9340	96.64	21.06	9570	97.83	21.23	9800	
9110	95.45	20.89	9350	96.70	21.07	9580	97.88	21.24	9810	
9120	95.50	20.89	9360	96.75	21.07	9590	97.93	21.25	9820	
9130	95.55	20.90	9370	96.80	21.08	9600	97.98	21.25	9830	
9140	95.60	20.91	9380	96.85	21.09	9610	98.03	21.26	9840	
9150	95.66	20.92	9390	96.90	21.10	9620	98.08	21.27	9850	
9160	95.71	20.92	9400	96.95	21.10	9630	98.13	21.28	9860	
9170	95.76	20.93	9410	97.01	21.11	9640	98.18	21.28	9870	
9180	95.81	20.94	9420	97.06	21.12	9650	98.23	21.29	9880	
9190	95.86	20.95	9430	97.11	21.13	9660	98.29	21.30	9890	
9200	95.92	20.95	9440	97.16	21.13	9670	98.34	21.30	9900	
9210	95.97	20.96	9450	97.21	21.14	9680	98.39	21.31	9910	
9220	96.02	20.97	9460	97.26	21.15	9690	98.44	21.32	9920	
9230	96.07	20.98	9470	97.31	21.16	9700	98.49	21.33	9930	
9240	96.12	20.98	9480	97.37	21.16	9710	98.54	21.33	9940	
9250	96.18	20.99	9490	97.42	21.17	9720	98.59	21.34	9950	
9260	96.23	21.00	9500	97.47	21.18	9730	98.64	21.35	9960	
9270	96.28	21.01	9510	97.52	21.19	9740	98.69	21.36	9970	
9280	96.33	21.01	9520	97.57	21.19	9750	98.74	21.36	9980	
9290	96.38	21.02	9530	97.62	21.20	9760	98.79	21.37	9990	
9300	96.44	21.03	9540	97.67	21.21	9770	98.84	21.38	10000	1
9310	96.49	21.04								

To find Square, or Cube Roots of large numbers n tained in the column of numbers of the tab

Such roots may sometimes be taken at once from the table, by merely regarding t powers as being columns of numbers; and those of numbers as being those of roots. *sq rt* of 25281 is reqd, first find that number in the column of *squares;* and opposi column of numbers, is its sq rt 159. For the *cube rt* of 857375, find that number in *cubes;* and opposite to it, in the col of numbers, is its cube rt 95. When the exact num tained in the column of squares, or cubes, as the case may be, we may use instead the n to it, if no great accuracy is reqd. But when a considerable degree of accuracy is following very correct methods may be used.

For the square root.

This rule applies both to whole numbers, and to those which are *partly* (not wholly) d in the foregoing manner, take out the tabular number, which is nearest to the given on tabular sq rt. Mult this tabular number by 3; to the prod add the given number. C Then mult the given number by 3; to the prod add the tabular number. Call the sum

A : B : : Tabular root : Reqd root.

Ex. Let the given number be 946.53. Here we find the nearest tabular number to tabular sq rt 30.7734. Hence,

```
947 = tab num                946.53 = given num.
  3                                3
----                         -------
2841                         2839.59
946.53 = given num.          947    = tab num.
-------                      -------
3787.53 = A.                 3786.59 = B.
```

and

	A.		B.		Tab root.		Reqd root.
Then	3787.53	:	3786.59	: :	30.7734	:	30.7657 +.

The root as found by actual mathematical process is also 30.7657 +.

For the cube root.

This rule applies both to whole numbers, and to those which are *partly* decimal. Fi tabular number which is nearest to the given one; and also its tabular cube rt. Mu number by 2; and to the prod add the given number. Call the sum A. Then mult the by 2; and to the prod add the tabular number. Call the sum B. Then

A : B : : Tabular root : Reqd root.

Ex. Let the given number be 7368. Here we find the nearest tabular number (in *cubes*) to be 6859; and its tabular cube rt 19. Hence,

```
6859 = tab num.              7368 = given num.
   2                            2
-----                        -----
13718                        14736
 7368 = given num.           6859 = tab num.
-------                      -------
21086 = A.                   21595 = B.
```

and

	A.		B.		Tab Root.		Reqd Rt.
Then, as	*21086*	:	*21595*	: :	19	:	19.4585

The root as found by correct mathematical process is 19.4585. The engineer ra

his degree of accuracy; for his purposes, therefore, this process is greatly preferable to the ordinary aborious one.

To find the square root of a number which is wholly decimal.

Very simple, and correct to the third numeral figure inclusive. If the number does not contain at least five figures, *counting from the first numeral, and including it*, add one or more ciphers to make five. If, after that, the *whole number* is not separable into twos, add another cipher to make it so. Then beginning at the first numeral figure, and including it, assume the number to be a whole one. In the table find the number nearest to this assumed one; take out its tabular sq rt; move the decimal point of this tabular root to the left, *half* as many places as the finally modified *decimal* number has figures.

Ex. What is the sq rt of the decimal .002? Here, in order to have at least five decimal figures, counting from the first numeral (2), and including it, add ciphers thus, .00,20,00,0. But, as it is not now separable into twos, add another cipher, thus, .00,20,00,00. Then beginning at the first numeral (2), assume this decimal to be the whole number 200000. The nearest to this in the table is 199809; and the sq rt of this is 447. Now, the decimal number as finally modified, namely, .00,20,00,00, has eight figures; one-half of which is 4; therefore, move the decimal point of the root 447, four places to the left; making it .0447. This is the reqd sq rt of .002, correct to the third numeral 7 included.

To find the cube root of a number which is wholly decimal.

Very simple, and correct to the third numeral inclusive.

If the number does not contain at least five figures, counting from the first numeral, and including it, add one or more ciphers to make five. If, after that, the number is not separable into threes, add one or more ciphers to make it so. Then beginning at the first numeral, and including it, assume the number to be a whole one. In the table find the number nearest to this assumed one, and take out its tabular cub rt. Move the decimal point of this rt to the left, one-third as many places as the finally modified *decimal* number has figures.

Ex. What is the cube rt of the decimal .002? Here, in order to have at least five figures, counting from the first numeral (2), and including it, add ciphers thus, .002.000,0. But as it is not now separable into threes, add two more ciphers to make it so; thus, .002,000,000. Then beginning with the first numeral (2), assume the decimal to be the whole number 2000000. The nearest cube to this in the table in the column of cubes, is 2000376; and its tabular cube rt as found in the col of numbers, is 126. Now, the decimal number as finally modified, namely, .002 000 000, has nine figures; one-third of which is 3; therefore, move the decimal point of the root 126, three places to the left, making it .126. This is the reqd cube rt of the decimal .002, correct to the third numeral 6 included.

To find roots by logarithms, see p. 614.

GEOMETRY.

A geometrical line

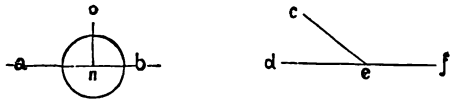
is simply length, or distance. The lines we draw on paper have not only length, but breadth and thickness; still they are the most convenient symbol we can employ for denoting a geometrical line. Straight lines are also called *right* lines. A PLANE FIGURE is merely any flat surface or area entirely inclosed by lines either straight or curved; which are called its outline, boundary, circumf, or periphery. We often confound the outline with the fig itself as when we speak of drawing circles, squares, &c; for we actually draw only their outlines. Geometrically speaking, a fig has length and breadth only; no thickness. A SOLID is any body; it has length, breadth, and thickness.

Geometrically similar

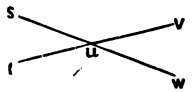
Figs or solids, are not necessarily of the same *size*; but only of precisely the same *shape*. Thus, any two squares are, scientifically speaking, similar to each other; so also any two circles, cubes, &c, no matter how different they may be in size. When they are not only of the same shape, but of the same size, they are said to be similar, and equal.

The *quantities* of *lines* are to each other simply as their lengths; but the quantities, or areas, or surfaces of similar *figures*, are as, or in proportion to, the *squares* of any one of the corresponding lines or sides which enclose the figures, or which may be drawn upon them; and the quantities, or solidities of similar *solids*, are as the *cubes* of any of the corresponding lines which form their edges, or the figures by which they are enclosed.

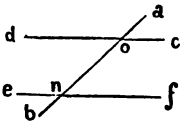
When two straight, or right lines meet each other at any inclination, the inclination is called an ANGLE; and is measured by the degrees contained in the arc of a circle described from the point of meeting as a center. Since all circles, whether large or small, are supposed to be divided into 360 degrees, it follows that any number of degrees of a small circle will measure the same degree of inclination as will the same number of a large one.



When two straight lines, as *o n* and *a b*, meet in such a manner that the inclination *o n a* is equal to the inclination *o n b*, then the two *lines are* said to be perpendicular to each other; and the angles *o n a*, and *o n b*, are *called* RIGHT ANGLES; and are each measd by, or are equal to, 90°, or one-fourth part of the *circumf of a circle. Any angle, as c e d*, smaller than a right angle, is called ACUTE, or sharp; and *c e f larger than a right angle*, is called OBTUSE, or blunt. When one line meets another without crossing it, as in the two foregoing figures, the two angles *o n a*, and *o n b*, are called CONTIGUOUS OR ADJACENT; so also *c e d*, and *c e f*. Two adjacent angles are together always equal to two right angles; or to 180°. Therefore, if we know the number of degrees contained in one of them, as [illegible] to 180°, we obtain the other.

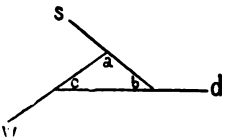


When two straight lines *cross* each other, forming fo either pair of those angles which point in exactly oppo tions are called OPPOSITE, or sometimes, improperly, ANGLES; thus, the pair *s u t*, and *v u w* are opposite an the pair *s u v* and *t u w*. The opposite angles of an always equal to each other.



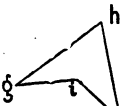
When a straight line *a b* crosses two parallel lines *c d* ALTERNATE angles, or those which form a kind of *Z*, are equ other. Thus, the angles *d o n*, and *o n f*, are equal; as are and *o n e*. Also the two internal angles on the same side o equal to two right angles, or 180°; thus, the two angles *o n f* = 180°; as do also *d o n* and *o n e*.

An interior angle,

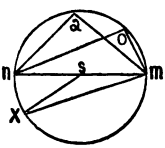


In any fig. is any angle formed *inside* of that fig, by ing of two of its sides, as the angles *c a b*, *a b c*, *b c* triangle. All the interior angles of any straight-lined any number of sides whatever, are together equal to many right angles minus four, as the figure has sides. triangle has 3 sides; twice that number is 6; and 6 rig or 6 × 90° = 540°; from which take 4 right angles, or 3 there remain 180°, which is the number of degrees plane, or straight-lined triangle. This principle furn easy means of testing our measurements of the angl fig; for if the sum of all our measurements does not a the sum given by the rule, it is a proof that we have committed some error.

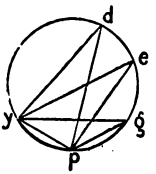
An exterior angle



Of any straight-lined figure, is any angle, as *a b d*, formed OUTSIDE of by the meeting of any side, as *a b*, with the prolongation of an adjac as *c b*; so likewise the angles *c a s*, and *b c w*. All the exterior angl straight-lined fig. no matter how many sides it may have, amoun right angles, or 360°. Thus, in the foregoing fig, the three exterio *a b d*, *c a s*, and *b c w*, amount to 360°. In any fig, those angles, and *j*, which point outward, are called SALIENT; and those which point as *t*, are called RE-ENTERING.



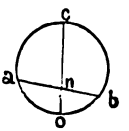
All angles, as *n a m*, *n o m*, at the circumf of a semicircle, an ing on its diam *n m*, are right angles; or, as it is usually expre *angles in a semicircle* are right angles. An angle *n s x* at t of a circle, is twice as great as an angle *n m x* at the circumf, w stand upon the same arc *n x*.



All angles, as *y d p*, *y e p*, *y g p*, at the circumf of a circle, and upon the same arc, as *y p*, are equal to each other; or, as usually ex *all angles in the same segment of a circle* are equal.

The complement of an angle

Is what it wants of being 90°; and its *supplement* is what it wants 180°.*



If any chord, as *a b*, be drawn across any diam *c o*, of a circle, then of the two parts of the chord will be equal to that of the two parts of t that is, *a n* × *n b* = *o n* × *n c*.

Sines, Tangents, &c.

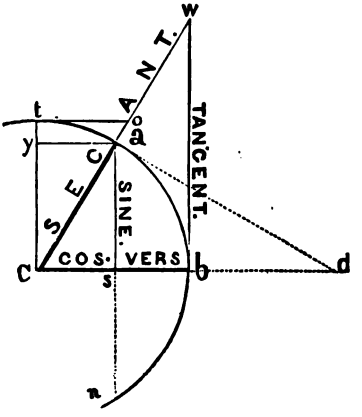
The SINE, *a s*, *of any angle*, *a c b*, or which is the same thing, the sine of any circular which subtends or measures the angle, is a straight line drawn from one end, as *a*, of the arc, angles to, and terminating at, the rad *c b*, drawn to the other end *b* of the arc. It is, therefo half the chord *a n*, of the arc *a b n*, which is equal to twice the arc *a b*; or, the sine of an

* This distinction can be remembered by saying that a small angle receives more complem ge one.

al to half the chord of twice that angle; and vice versa, the chord of an angle is always
ies the sine of half the angle.
: of an angle *t c b*, or of an arc
, is equal to the rad of the arc
ircle; and this sine of 90° is
n that of any other angle.

IE *c s* of an angle *a c b*, is that
rad which lies between the
e center of the circle. It is
al to the sine *y a* of the com-
a of *a c b*; or of what *a c b*
ing 90°. The prefix *co* before
means complement; thus, co-
sine of the complement.

ED SINE *s b* of any angle *a c b*,
of the rad which lies between
d the outer end *b*. It is very
it erroneous, when speaking
&c, to call the rise or height
cular arch *a b n*, its versed
It is actually the versed sine
f the arch. This absurdity
s; for the word rise or height
more expressive, but is correct.
ET *b w*, or *a d*, of any angle
line drawn from, and at right
e end *b* or *a* of either rad *c b*,
h forms one of the legs of the
terminating as at *w*, or *d*,
ongation of the rad which
her leg.* This last rad thus
hat is, *c w*, or *c d*, as the case
alled the SECANT of the angle
two tangents are equal; so

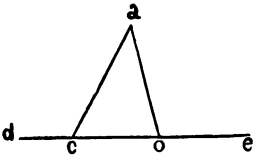


two secants. The angle *t c b* being supposed to be 90°, the angle *t c a* becomes the com-
he angle *a c b*, or what *a c b* wants of being 90°; and the sine *y a* of this complement; its
t y; its tangent *t o*; and its secant *c o*, are respectively the co-sine, co-versed sine; co-
d co-secant, of the angle *a c b*. Or, vice versa, the sine, *&c.* of *a c b*, are the cosine, *&c.*
cause the angle *a c b* is the complement of the angle *t c a*. When the rad *c b*, *c a*, or *c t*,
o be equal to unity, or 1, the corresponding sines, tangents, *&c*, are called *natural* ones;
veral lengths for diff angles, for said rad of unity, have been calculated; constituting the
tables of nat sines, *&c*. In any circle whose rad is either larger or smaller than 1, the
the angles will be in the same proportion larger or smaller than those in the tables, and
antly found by mult the sine, *&c*, of the table, by said larger or smaller rad.

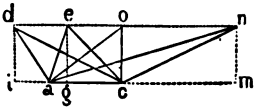
* This is the *trigonometrical* tang. For others see Remark p 101.

Triangles.

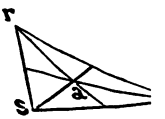
e is called *equilateral* when it has three equal
sides, when only two of its sides are equal; *sca-*
he three sides are unequal. A *right-angled* tri-
me right angle. Those which have no right
lled *oblique*. If all the angles are less than right
riangle is *acute*. If one of the angles is greater
angle, *obtuse*.

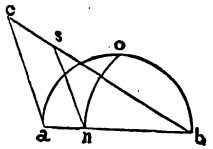


ane (or straight-sided) triangle, as a o c, *the*
hree angles is always equal to two right angles,
erefore, if we know two of the angles we can
d by subtracting the sum of the two from 180°.
sides are always opposite the greater angles;
aller sides opposite the smaller angles; but the
as, or in proportion to, the opposite angles, but to their *sines*. If any side, as *o c* of a
produced or extended, as from *o* to *d*, then the exterior angle *a c d* is equal to the two
osite angles *c a o*, and *c o a*. And in like manner, if the side *c o* be extended toward *e*,
equal to *c a o*, and *a c o*.



f a triangle may be called its base; as *a c*; and
om that side (extended if necessary) to the op-
, is called its *perpendicular height*; or simply
See Mensuration.
les, as *a c d*, *a c e*, *a c o*, *a c n*, which have equal
d equal perp heights *m n*, *o c*, *e g*, and *d t*, have
reas; and the area of each is equal to half the
rectangle whatever which has the same base,
ight. If lines be drawn from each angle *r s t*, of a triangle, to
f the opposite side, they will cross each other at one point *a*;
s part of each of the lines will be equal to ⅓d part of its entire
e the point *a* is the cen of grav of the triangle. If, instead of
opposite sides, they bisect the angles from which they are
rill meet at a point which is perpendicularly equidistant from
d is therefore the cen of an inscribed circle.

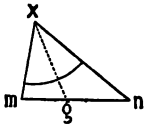




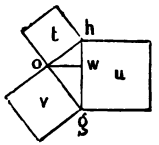
If a line *s n* be drawn in any triangle, parallel to sides, as *c a*, then the small triangle *s b n* will be simi large one, *a b c*.

To divide any triangle, as *a b c* two equal parts,

By a line *s n* parallel to any one of its sides, as *a c*; one (as *a b*) of the other sides, as a diam. describe a *a o b*; from the middle, *o*, of this semicircle, and angle *b* (opposite the side *a c*) as a center, describe th from *n* draw *n s* parallel to *a c*.

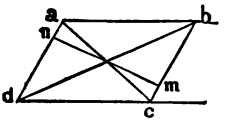


If any angle *x* of any triangle be bisected by a line *x g*, the two and *n g* of the opposite side *m n* will have to each other the same as the other two sides *x m*, and *x n*.



A right-angled triangle, as *o h g*, has one right angle *o*. If right angle a line *o w* be drawn perp to the long side, or hypoth then the two small triangles and the large triangle will be sim a line drawn from *o* to the center of the long side, will be half that side. If, on the three sides of any right-angled triangle, w three squares, as *t, u, v*; or three circles; or any three figs that a to each other, then the area of the fig on the long side will be ec areas of the other two.

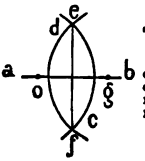
PARALLELOGRAMS.



A parallelogram is any four-sided straight-lined fig opposite sides are equal, as *a b c d*; or a square, &c. suration.) Any line drawn across a parallelogram opposite angles, is called a *diagonal*, as *a c*, or *b d*. A di a parallelogram into two equal parts; as does also a drawn through the center of either diag; and moreov *m n* itself is div into two equal parts, or is bisected. bisect each other; they also divide the parallelogra triangles of equal areas. *Any two adjacent angles of lelogram are equal to two right angles*, or 180°; as *d a b* and *a b c*; or *a b c* and *b c d*; and the four always equal to four right angles, or 360°.

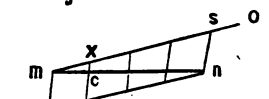
The sum of the squares of the four sides, is equal to the sum of the squares of the two d

Simple as many of the following operations appear, it is only by care, and good instrun they are made to give accurate results. Several of them can be much better performed by metallic triangle having one perfectly accurate right angle.* In the field, the tape-line, measuring-rod will take the place of the dividers and ruler used indoors.



To divide a given line, *a b*, into two equal

From its ends *a* and *b* as centers, and with any rad greater than one-describe the arcs *c* and *d*, and join *e f*. If the line *a b* is very long, equal dists *a e* and *b g*, each way from the ends, so as to approach co near to each other; and then proceed as if *o g* were the line to be di measure *a b* by a scale, and thus ascertain its center.



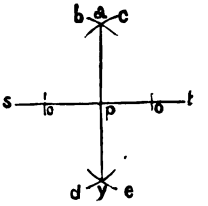
To divide a given line, *m n*, in given number of equal pa

From *m* and *n* draw any two parallel lines *m* to an indefinite dist; and on them, from *m* and *n* reqd number of equal parts of any convenient le ly, join the corresponding points thus stepped o one line, as *m o*. may be drawn and stepped c then join *s n*; and draw the other short lines pa

To divide a given line, *m n*, into two parts which shal *a given* proportion to each other.

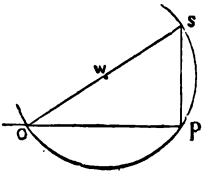
This is done on the same principle as the last; thus, let the proportion be as 1 to 3. *any line m o*; and with any convenient opening of the dividers, make *m x* equal to one st *equal to three steps. Join s n*; and parallel to it draw *x c*. Then *m c* is to *c n* as 1 is to

* *German silver is good for rulers and triangles.* It does not rust, like ste



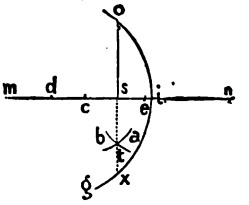
From any given point, *p*, on a line *s t*, to draw a perp, *p a*.

From *p*, with any convenient opening of the dividers, step off the equals *p s*, *p a*. From *s* and *a* as centers, with any opening greater than half *s a*, describe the two short arcs *b* and *c*; and join *a p*. Or still better, describe four arcs, and join *a y*.



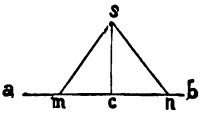
If the point *p* is at one end of the line, or very near it,

Extend the line, if possible, and proceed as above. But if this cannot be done, then from any convenient point, *w*, open the dividers to *p*, and describe the semicircle, *s p o*; through *o w* draw *o w s*; join *p a*.



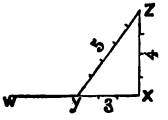
From a given point, *o*, to let fall a perp *o s*, to a given line, *m n*.

From *o*, measure to the line *m n*, any two equal dists, *o c*, *o e*; and from *c* and *e* as centers, with any opening greater than half of *c e*, describe the two arcs *a* and *b*; join *o t*. Or from any point, as *d* on the line, open the dividers to *o*, and describe the arc *o g*; make *t x* equal to *t o*; and join *o x*.



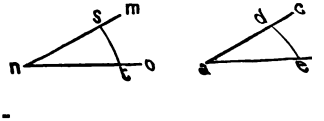
If the line, *a b*, is on the ground,

And a perp is reqd to be drawn from *c*, first measure off any two equal dists, *c m*, *c n*. At *m* and *n*, hold the ends of a piece of string, tape-line, or chain, *m s n*; then tighten out the string, &c, as shewn by *m s n*; *s* being its center. Then will *s c* be the reqd perp. Or if the perp *x x* is to be drawn from the *end* of the line *w x*, first measure *x y* upon the line, and equal to three feet; then holding the end of a tape-line at *x*, and its nine feet mark at *y*, hold the four feet mark at *z*, keeping *z x* and *z y* equally stretched. Then *z x* will be the reqd perp, because 3, 4, and 5, make the sides of a right-angled triangle. Instead of 3, 4, and 5, any multiples of those numbers may be used, such as 6, 8, and 10; or 9, 12, 15, &c: also instead of feet, we may use yards, chains, &c.



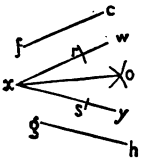
Through a given point, *a*, to draw a line, *a c*, parallel to another line, *e f*.

With the perp dist, *a e*, from any point, *n*, in *e f*, describe an arc, *t*; draw *a c* just touching the arc.



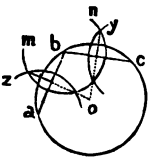
At any point, *a*, in a line *a b*, to make an angle *c a b*, equal to a given angle, *m n o*.

From n and a, with any convenient rad, describe the arcs s t, d s; measure s t, and make e d equal to t; through a d draw a c.



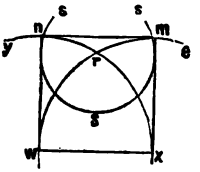
To bisect, or divide any angle, *w x y* two equal parts.

From *x* set off any two equal dists, *x r*, *x s*. From *r* and *s* wi describe two arcs intersecting, as at *o*; and join *o x*. If the t the angle do not meet, as *c f* and *g h*, either first extend them do meet; or else draw lines *x w*, and *x y*, parallel to them, a dists from them, so as to meet; then proceed as before.



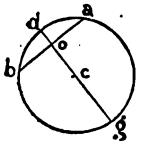
To describe a circle through any tl points, *a b c*, not in a straight lin

Join the points by the lines *a b*, *b c*; from the centers of these the dotted perps meeting, as at *o*, which will be the center of Or from *b*, with any convenient rad, draw the arc *m n*; and fro with the same rad, draw arcs *y* and *z*; then two lines drawn tl intersections of these arcs, will meet at the center *o*.



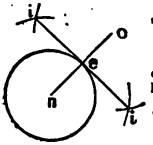
On a given line *w x*, to draw a s *w x n m*.

From *w* and *x*, with rad *w x*, describe the arcs *x r y* From their intersection *r*, and with rad equal to ½ of *w s s*. From *w* and *x* draw *w n* and *x m* tangential to ending at the other arcs; join *n m*.



To find the center *c*, of a given cir

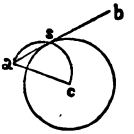
Draw any chord *a b*; and from the middle of it *o*, draw at righ it, a diam *d g*; find the center *c* of this diam.



To draw a tangent, *i e i*, to a circle, fro given point, *e*, in its circumf.

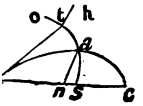
Through the center *n*, and the given point *e*, draw *n o*; make *e e n*; from *n* and *o*, with any rad greater than half of *o n*, descri pairs of arc *i i*; join their intersections *i i*.

Here, and in the following three figs, the *tangents* are *ordina metrical* ones; and may end where we please. But the *trigo* tangent of a given *angle*, must end in a secant, as in the top fig of



To draw a tang, *a s b*, to a circle, from a *a*, which is outside of the circle.

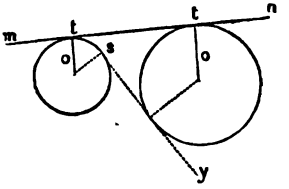
Draw *a c*, and on it describe a semicircle; through the intersectic *a s b*. Here *c* is the center of the circle.



To draw a tang, *g h*, from a circular arc

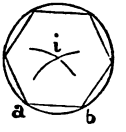
Of which n a is the rise. With rad *g a*, describe an arc, *s a o*. *equal to s a*. Through *t* draw *g h*.

To draw a tang to two circles.



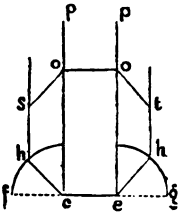
First draw the line *m n*, just touching the two circles; this gives the *direction* of the tang. Then from the centers of the circles draw the radii, *o o*, perp to *m n*. The points *s t* are the tang points. If the tang is in the position of the dotted line, *s y*, the operation is the same.

To draw a hexagon, each side of which shall be equal to a given line, *a b*.



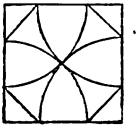
From *a* and *b*, with rad *a b*, describe the two arcs; from their intersection, *i*, with the same rad, describe a circle; around the circumf of which, step off the same rad.

To draw an octagon, with each side equal to a given line, *c e*.



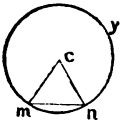
From *c* and *e* draw two perps, *c p*, *e p*. Also prolong *c s* toward *f* and *g*; and from *c* and *e*, with rad equal *c e*, draw the two quadrants; and find their centers *h h*; join *c h*, and *e h*; draw *h s* and *h t* parallel to *c p*; and make each of them equal to *c e*; make *c o*, and *e o*, each equal to *h h*; join *o o*, *o s*, and *o t*.

To draw an octagon in a given square.



From each corner of the square, and with a rad equal to half its diag, describe the four arcs; and join the points at which they cut the sides of the square.

To draw any regular polygon, with each side equal to *m n*.

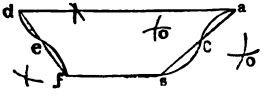


Div 360 degrees by the number of sides; take the quot from 180°; div the rem by 2. This will give the angle *c m n*, or *c n m*. At *m* and *n* lay down these angles by a protractor: the sides of these angles will meet at a point, *c*, from which describe the circle *m n y*; and around its circumf step off dists equal to *m n*.

In any circle, *m n y*, to draw any regular polygon.

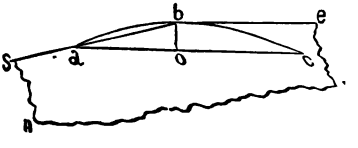
Div 360° by the number of sides; the quot will be the angle *m c n*, at the center. Lay off this angle by a protractor; and its chord *m n* will be one side; which step off around the circumf.

On the given line, *a s*, to draw a cyma recta, *a c s*.



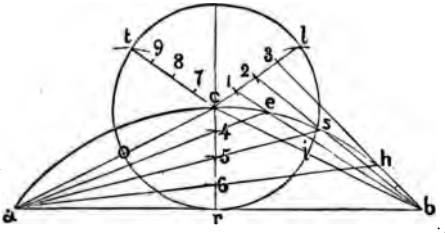
Find the center *c*, of *a s*. From *a*, *c*, and *s*, with one-half of *a s* as rad, draw the four small arcs at *o*, *o*. The intersections *e*, *e*, are the centers for drawing the cyma, with the same rad. By reversing the position of the arcs, we obtain the *cyma reversa*, or *a g e c*, *d e f*.

To describe the arc of a circle too large for the dividers.



Let *a c* be the chord; and *o b* the height of the reqd arc, as laid down *on the drawing*. On a separate strip of paper, *s m n s*, draw *a c*, *o b*, and *a b*; also *b e*, parallel to the chord *a c*. It is well to make *b s*, and *b n*, each a little longer than *a b*. Then *cut off the paper carefully along the lines s b and b n, so as to leave remaining only the strip s a b o m n. Now*, if the *straight sides a b and b o be applied to the drawing, so that any parts of them shall touch at*

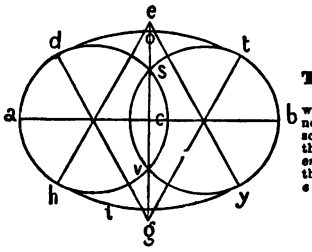
the same time the points *a* and *b*, or *b* and *c*, the point *b* on the strip will be in th
of the arc, and may be pricked off. Thus, any number of points in the arc may be found,



ward united to form t

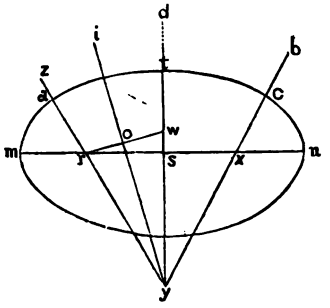
Or thus: draw the
the rise *r o*; and *a c*,
c with rad *c r* describ
Make each of the ar
t l equal to *r o* or *r l*;
a t, *c l*. Div *c t*, *c l*, *c r*
half as many equal p
curve is to be divi
Draw the lines *b* 1, *b*
a 4, *a* 5, *a* 6, extende
the first ones at *c*, *a*,
e, *s*, *h*, are points in co
curve. Then for the
draw similar lines fr
8, 9; and others from
them, as before.
curve by hand.

Also see Remark
suration," page 17,
find the length of
arc."



To draw an oval, or false e

When only the long diam *a b* is given, the
will give agreeable curves, of which the sp
not exceed about three times the rise *c o*.
scribe two intersecting circles of any rad
their intersections *s*, *v*, draw *s g*; make *s*
each equal to the diam of one of the circles.
the centers of the circles, draw *s y*, *s h*, *g d*,
s describe *h i y*; and from *g* describe *d o t*.



When the span, *m n*, a rise, *s t*, are both gi

Make any *t w* and *m r*, equal to e
but each less than *t s*. Draw *r w*; a
its center *o* draw the perp *t o y*. D
Make *n x* equal *m r*, and draw *y x b*. F
r describe *n c* and *m a*; and from
a t c. By making *s d* equal to *s y*,
the center for the other side of the o

The beauty of the curve will de
what portion of *t s* is taken for *m*
When an oval is very flat, more than
ters are required for drawing a grac
but the finding of these centers is qui
blesome as to draw the correct ellips

For drawing the ellipse, parabola, a
bola, see "Mensuration."

To reduce any polygon, as *a b c d e f a*, to a triangle o same area.

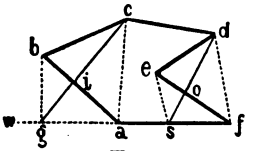


Fig 1

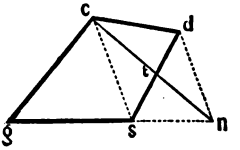
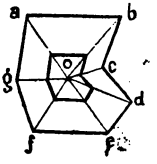


Fig 2

If we produce the side f a toward *w*; and draw *b g* parallel to *a c*, and join *g c*, we ha
angles a c b, and *a c g*, both on the same base *a c*; and both of the same perp betw... l
y are between the two parallels a c and *g b*. But the part *a c i* forms a portion of b

in other words, *is common to both*. Therefore, if it be taken away from both triangles, ing parts, *i c b* of one of them, and *i g a* of the other, are also equal. Therefore, if the e left off from the polygon, and the part *i g a* be taken into it, the polygon *g f e d c i g* will ame area as *a f e d c b a*; but it will have but five sides, while the other has six. Again, awn parallel to *d f*, and *d s* joined, we have upon the same base *e s*, and between the e and *d f*, the two equal triangles *e s d*, and *e s f*, with the part *s o s* common to both; and ly the remaining part *e o d* of one, and *o s f* of the other, are equal. Therefore, if *o s f* be the polygon, and *s o d* be taken into it, the new polygon *g e d c g*, Fig 2, will have the same *e d c g*; but it has but four sides, while the other has five. Finally, if *g s*, Fig 2, be ward *n*; and *d n* drawn parallel to *c s*; and *c n* joined, we have on the same base *c s*, and e same parallels *c s* and *d n*, the two equal triangles *c s n*, and *c s d*, with the part *c s t* both. Therefore, if we leave out *c d t*, and take in *s t n*, we have the triangle *g n* c equal ron *g s d c g*, Fig 2; or to *a f e d c b a*, Fig 1.
ple method is applicable to polygons of any number of sides.

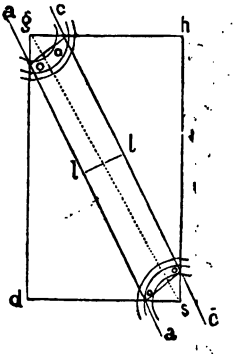


uce a large fig, *a b c d e f g*, to a smaller similar one.

r interior point *o*, which had better be near the center, draw lines ngles *a*, *b*, *c*, &c. Join these lines by others parallel to the sides If it should be reqd to enlarge a small fig, draw, from any point lines extending beyond its angles; and join these lines by others the sides of the small fig.

To reduce a map to one on a smaller scale.

method is by dividing the large map into squares by faint lines, with a very soft lead- l then drawing the reduced map upon a sheet of uares. A pair of proportional dividers will assist xing points intermediate of the sides of the squares. e map would be injured by drawing, and rubbing uares, threads may be stretched across it to form the



lans, and drawings of all kinds, are now copied, larged, and multiplied, cheaply and expeditiously, by y. For this purpose they should be prepared only in t and white; shading should be done in *lines*; not in and care be taken to make the lettering and every part e strong and distinct than for ordinary drawings.

n a rectangular fig, *g h s d*,

ng an open panel, to find the points *o o o o* in its at equal dists from the angles *g*, and *s*; for inserting ose *o o o o*, of a given width *l l*, measured at right its length. From *g* and *s* as centers, describe several arcs, as in the Fig. Draw upon transparent paper, l lines *a a*, *c c*, at a distance apart equal to *l l*; and ese lines on top of the panel, move them about until it by the arcs that the four dists *g o*, *g o*, *s o*, *s o*, are stead of the transparent paper, a strip of common he width *l l* may be used.
any problems which would otherwise be very difficult, us solved with an accuracy sufficient for practical by means of transparent paper.

ARITHMETIC.

subject we shall merely give a few examples for refreshing the memory of those who for nstant practice cannot always recall the processes at the moment.

Subtraction of Vulgar Fractions.

$= 0.$ $\quad \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}.$ $\quad \frac{6}{8} - \frac{2}{8} = \frac{4}{8} = \frac{1}{2}.$ $\quad \frac{3}{4} - \frac{5}{9} = \frac{27}{36} - \frac{20}{36} = \frac{7}{36}.$

$$3\frac{4}{7} - 1\frac{2}{3} = \frac{25}{7} - \frac{5}{3} = \frac{75}{21} - \frac{35}{21} = \frac{40}{21} = 1\frac{19}{21}.$$

Addition of Vulgar Fractions.

$\frac{2}{2} = 1.$ $\quad \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1.$ $\quad \frac{6}{8} + \frac{2}{8} = \frac{8}{8} = 1.$ $\quad \frac{3}{4} + \frac{5}{9} = \frac{27}{36} + \frac{20}{36} = \frac{47}{36} = 1\frac{11}{36}.$

$$3\frac{4}{7} + 1\frac{2}{3} = \frac{25}{7} + \frac{5}{3} = \frac{75}{21} + \frac{35}{21} = \frac{110}{21} = 5\frac{5}{21}.$$

$$\frac{2}{3} + \frac{3}{5} + \frac{5}{4} = \frac{40}{60} + \frac{36}{60} + \frac{120}{60} = \frac{196}{60} = \frac{49}{15} = 3\frac{4}{15}.$$

* *Not* absolutely necessary.

Multiplication of Vulgar Fractions.

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. $\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$. $\frac{6}{8} \times \frac{2}{8} = \frac{12}{64} = \frac{3}{16}$. $\frac{3}{4} \times \frac{5}{9} = \frac{15}{36} = \frac{5}{12}$.

$3\frac{4}{7} \times 1\frac{2}{3} = \frac{25}{7} \times \frac{5}{3} = \frac{125}{21}$. $\frac{2}{3} \times \frac{3}{5} \times \frac{5}{4} = \frac{48}{60} = \frac{4}{5}$.

$\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{5}{3}$ of $\frac{7}{5} = \frac{3}{4} \times \frac{1}{2} \times \frac{5}{3} \times \frac{7}{5} = \frac{105}{120} = \frac{21}{24} = \frac{7}{8}$.

Division of Vulgar Fractions.

$\frac{1}{2} \div \frac{1}{2} = \frac{2}{2} = 1$. $\frac{3}{4} \div \frac{1}{4} = \frac{12}{4} = \frac{3}{1} = 3$. $\frac{6}{8} \div \frac{2}{8} = \frac{48}{16} = \frac{24}{8} = \frac{12}{4} = \frac{3}{1} = 3$. $\frac{3}{4} \div \frac{5}{9} = \frac{27}{20} = 1$

$3\frac{4}{7} \div 1\frac{2}{3} = \frac{25}{7} \div \frac{5}{3} = \frac{75}{35} = \frac{15}{7} = 2\frac{1}{7}$. $5 \div \frac{7}{8} = \frac{5}{1} \div \frac{7}{8} = \frac{40}{7} = 5\frac{5}{7}$.

To find the greatest common divisor of a Vulgar Fracti

Ex. 1. Of $\frac{70}{175}$.

```
70)175(2
   140
   ---
    35)70(2
       70    Ans 35.
```

Ex. 2. Of $\frac{84}{20}$.

```
20)84(4
   80
   --
    4)20(5
      20    Ans 4.
```

To reduce a Vulgar Fraction to its lowest terms.

First find the greatest common divisor; then divide both the numerator and denominator b
Thus, in the preceding example $\frac{70}{175} = \frac{2}{5}$ Ans. And $\frac{84}{20} = \frac{21}{5}$ Ans.

To reduce a Vulgar Fraction to a decimal form.

Divide the numerator by the denominator. Thus,

```
1/2 = 2 )1.0(0.5 Ans.
         1 0

1 3/4 = 4)13(3.25 Ans.
          12
          --
          10
           8
          --
          20
          20

32/40 = 40)32.0(0.8 Ans.
           32 0
```

Reduce 3 inches to the decimal of a foot. There are 12 ins in a foot; therefore, the questi
to reduce $\frac{3}{12}$ to a decimal. Therefore,

```
12)3.0(0.25 of a foot. Ans.
   2 4
   ---
    60
    60
```

Reduce 2 ft 3 ins to the decimal of a yard. There are 36 ins in a yard; and 27 ins in 2 ft 3
therefore, $\frac{27}{36}$ of a yard = 36)27.0(0.75 of a yd. Ans.

```
   25 2
   ----
    180
    180
```

How many feet and ins are there in .75 of a yard? Here

```
.75
  3 ft in a yd.
----
Ft 2).25
     12  ins in a ft.
-------
Ins 3.00    Ans 2 ft 3 ins.
```

How many feet and ins are there in .0625 of a yard?

```
.0625
    3 feet in a yd.
------
No feet, .1875
        12 ins in a ft.
        -------
Ins 2.2500          Ans.  ft.  ins.
                          0    2.25.
```

How many cubic feet are there in .314 of a cub yard? And cub ins in .46 of a cub ft?

```
.314
  27 cub ft in a yd.
----
2198
628
----
8.478 cub ft.  Ans.

.46
1728 cub ins a cub ft.
----
368
92
322
46
----
794.88 cub ins.  Ans.
```

Decimals.

ADDITION. Add together .25 and .75; also .006, 1.3472, and 43.

```
 .25                                  .006
 .75                                 1.3472
 ---                                 43.
1.00  Ans.                           -------
                                     44.3532  Ans.
```

SUBTRACTION. Subtract .25 from .75; also .0001 from 1; also 6.30 from 9.01.

```
 .75                 1.                  9.01
 .25                 .0001               6.30
 ---                 -----               ----
 .50 Ans.            .9999  Ans.         2.71  Ans.
```

MULTIPLICATION. Mult 3 × .3; also .3 × .3; also .3 × .03; also 4.326 × .003.

```
 3          .3          .3          4.326
 .3         .3          .03         .003
 --         --          ----        -----
 .9 Ans.    .09 Ans.    .009 Ans.   .012978 Ans.
```

DIVISION. Divide 3. by .3; also .3 by .3; also .3 by .03; also 4.326 by .0003.

```
.3)3.0(10. Ans.   .3).3(1. Ans.   .03).30(10. Ans.   .0003)4.3260(14420. Ans.
   3                 3                 3                 3
   --                                  --                --
   0                                   0                 18
                                                         12
Divide 62 by 87.042.   87.042)62.0000(0.712, &c. Ans.    --
                              60.9294                    12
                              -------                    12
                               107060                    --
                                87042                     6
                               ------                     6
                               200180                     --
                                                          0
Divide .006 by 20.   20.000).0060000(0.0003 Ans.
                             60000
                             -----
```

Duodecimals.

Duodecimals refer to square *feet* of 144 sq ins; to twelfths of a square or duodecimal foot; each ch twelfth being called an *inch;* and being equal to 12 square inches; and to *twelfths*, each equal e the 12th part of a decimal inch, or to one square inch. The dimensions of the thing to be measd re supposed to be taken in common feet, ins, and 12ths of an inch; but as ordinary measuring ules are divided into 8ths of an inch, it is usually guess-work to some extent. Duodecimals are ry properly going out of use, in favor of decimals; we shall therefore give no rule for them. By eans of our table of "Inches reduced to Decimals of a Foot," page 75, all dimensions in feet, ins, nd 8ths, &c, can be at once taken out in ft and decimals of a foot.

Single Rule of Three; or, Simple Proportion.

If 3 men lay 10000 bricks in a certain time, how many could 6 men lay in the same time? They ill evidently lay more; therefore, the second term of the proportion must be greater than the first.

```
3 : 6 : : 10000 : 20000 Ans.
              6
          3)60000
          -------
          20000 Ans.
```

If 3 men require 10 hours to lay a certain number of bricks, how many hours would 6 men require? They will evidently require less time; therefore, the second term of the proportion must e less than the first.

```
6 : 3 : : 10 : 5 Ans.
           3
          --
        6)30
          --
           5 Ans
```

Double Rule of Three; or, Compound Proportion.

If three men can lay 4000 bricks in 2 days, how many men can lay 12000 in 3 days? Here we see at 4000 bricks require 3 × 2 = 6 *days' work;* therefore 12000 will require,

4000 : 12000 : : 6 : 18 days' work.

But there are only 3 days to do the 18 days work in; therefore the number of men must be $\frac{18}{3} = 6$ n. Ans.

A moment's reflection will suffice to reduce any case of double rule of three to this simple form.

Arithmetical Progression,

a series of numbers, is a progressive increase or decrease in each successive number, by the addi- n or subtraction of the same amount at each step; as in 1, 2, 3, 4, 5, &c., in which 1 is added at h step; or 10, 8, 6, 4, &c., in which 2 is subtracted at each step; or ¼, ½, ¾, 1, 1¼, &c. In any h series the numbers are *called its terms;* and the equal increase or decrease at each step its com- n *difference.*

And the com diff, knowing the first and last terms; and the number of terms. Find the dif- *ween the first and last terms.* From the number of terms subtract ' Div th diff jun found, y nus.

Multiplication of Vulgar Fractions.

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. $\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$. $\frac{6}{8} \times \frac{2}{8} = \frac{12}{64} = \frac{3}{16}$. $\frac{3}{4} \times \frac{5}{9} = \frac{15}{36} = \frac{5}{12}$.

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$3\frac{4}{7} \div 1\frac{2}{3} = \frac{25}{7} \div \frac{5}{3} = \frac{75}{35} = \frac{15}{7} = 2\frac{1}{7}$. $5 \div \frac{7}{8} = \frac{5}{1} \div \frac{7}{8} = \frac{40}{7} = 5\frac{5}{7}$.

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   80
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To reduce a Vulgar Fraction to its lowest terms.

First find the greatest common divisor; then divide both the numerator and denominator by it. Thus, in the preceding example $\frac{70}{175} = \frac{2}{5}$ Ans. And $\frac{84}{20} = \frac{21}{5}$ Ans.

To reduce a Vulgar Fraction to a decimal form.

Divide the numerator by the denominator. Thus,

```
1/2 = 2 )1.0(0.5 Ans.
         1 0

13/4 = 4)13(3.25 Ans.
         12
         --
         10
          8
         --
         20
         20

32/40 = 40)32.0(0.8 Ans.
           32 0
```

Reduce 3 inches to the decimal of a foot. There are 12 ins in a foot; therefore, the question is to reduce $\frac{3}{12}$ to a decimal. Therefore,

```
12)3.0(0.25 of a foot. Ans.
   24
   --
   60
   60
```

Reduce 2 ft 3 ins to the decimal of a yard. There are 36 ins in a yard; and 27 ins in 2 ft 3 ins; therefore, $\frac{27}{36}$ of a yard = 36)27.0(0.75 of a yd. Ans.

```
   25 2
   ----
    180
    180
```

How many feet and ins are there in .75 of a yard? Here

```
      .75
        3 ft in a yd.
      ----
Ft 2).25
      12  ins in a ft.
      ----
Ins 3.00    Ans 2 ft 3 ins.
```

How many feet and ins are there in .0625 of a yard?

```
      .0625
          3 feet in a yd.
      -----
No feet, .1875
         12 ins in a ft.
      ------
Ins 2.2500          Ans.  ft.  ins.
                          0    2.25.
```

How many cubic feet are there in .314 of a cub yard? And cub ins in .46 of a cub ft?

```
 .314
   27 cub ft in a yd.
 ----
 2198
 628
 ----
8.478 cub ft.  Ans.

  .46
 1728 cub ins a cub ft.
 ----
  368
   92
 322
  46
 ----
794.88 cub ins.  Ans.
```

Decimals.

ADDITION. Add together .25 and .75; also .006, 1.3472, and 43.

```
 .25                                          .006
 .75                                         1.3472
 ---                                         43.
1.00  Ans.                                   --------
                                             44.3532  Ans.
```

SUBTRACTION. Subtract .25 from .75; also .0001 from 1; also 6.30 from 9.01.

```
 .75                  1.                      9.01
 .25                  .0001                   6.30
 ---                  ------                  ----
 .50 Ans.             .9999  Ans.             2.71  Ans.
```

MULTIPLICATION. Mult 3 × .3; also .3 × .3; also .3 × .03; also 4.326 × .003.

```
 3          .3          .3          4.326
 .3         .3          .03         .003
 --         --          ----        -----
 .9 Ans.    .09 Ans.    .009 Ans.   .012978 Ans.
```

DIVISION. Divide 3. by .3; also .3 by .3; also .3 by .03; also 4.326 by .0003.

```
.3)3.0(10. Ans.   .3).3(1. Ans.   .03).30(10. Ans.   .0003)4.3260(14420. Ans.
   3                 3                 3                 3
   --                --                --                --
   0                 0                 0                 13
                                                        12
                                                        ---
                                                        12
                                                        12
                                                        ---
                                                         6
                                                         6
                                                         --
                                                         0
```

Divide 62 by 87.042.

```
87.042)62.0000(0.712, &c. Ans.
       60.9294
       -------
        107060
         87042
        ------
         200180
```

Divide .006 by 20.

```
20.000).0060000(0.0003 Ans.
        60000
        -----
```

Duodecimals.

Duodecimals refer to square *feet* of 144 sq ins; to twelfths of a square or duodecimal foot; each such twelfth being called an *inch;* and being equal to 12 square inches; and to *twelfths*, each equal to the 12th part of a decimal inch, or to one square inch. The dimensions of the thing to be measd are supposed to be taken in common feet, ins, and 12ths of an inch; but as ordinary measuring rules are divided into 8ths of an inch, it is usually guess-work to some extent. Duodecimals are very properly going out of use, in favor of decimals; we shall therefore give no rule for them. By means of our table of "Inches reduced to Decimals of a Foot," page 75, all dimensions in feet, ins, and 8ths, &c, can be at once taken out in ft and decimals of a foot.

Single Rule of Three; or, Simple Proportion.

If 3 men lay 10000 bricks in a certain time, how many could 6 men lay in the same time? They will evidently lay more; therefore, the second term of the proportion must be greater than the first.

3 : 6 : : 10000 : 20000 Ans.

```
      6
   3)60000
   -------
   20000 Ans.
```

If 3 men require 10 hours to lay a certain number of bricks, how many hours would 6 men require? They will evidently require less time; therefore, the second term of the proportion must be less than the first.

6 : 3 : : 10 : 5 Ans.

```
      3
   ----
   6)30
   ----
    5 Ans
```

Double Rule of Three; or, Compound Proportion.

If three men can lay 4000 bricks in 2 days, how many men can lay 12000 in 3 days? Here we see that 4000 bricks require 3 × 2 = 6 *days' work;* therefore 12000 will require,

4000 : 12000 : : 6 : 18 days' work.

But there are only 3 days to do the 18 days work in; therefore the number of men must be $\frac{18}{3} = 6$ men. Ans.

A moment's reflection will suffice to reduce any case of double rule of three to this simple form.

Arithmetical Progression,

is a series of numbers, is a progressive increase or decrease in each successive number, by the addition or subtraction of the same amount at each step; as in 1, 2, 3, 4, 5, &c., in which 1 is added at each step; or 10, 8, 6, 4, &c., in which 2 is subtracted at each step; or ¼, ½, ¾, 1, 1¼, &c. In any such series the numbers are *called its terms;* and the equal increase or decrease at each step its common *difference.*

To find the com diff, knowing the first and last terms; and the number of terms. Find the difference *between the first and last terms.* From the number of terms subtract 1. Div the diff just found, by the rem.

To find the last term, knowing the first term; the com diff; and the number of terms. number of terms take 1. Mult the rem by the com diff. To the prod add the first term.

To find the number of terms, having the first and last ones; and the com diff. Ta between the first and last terms. Div this diff by the com diff. To the quot add 1.

To find the sum of all the terms, having the first and last ones; and the number of t together the first and last terms. Div their sum by 2. Mult the quot by the number of ter

Geometrical Progression,

In a series of numbers, is a progressive increase or decrease in each successive number, t multiplier or divisor at each step; as 3, 9, 27, 81, &c, where each succeeding term is increa the preceding one by 3. Or 48, 24, 12, 6, &c, or 27, 13½, 6¾, 3⅜, &c, where each succeed found by dividing the preceding one by 2. The multiplier or divisor is called the *common* series, or progression.

To find the last term, knowing the first one; the ratio; and the number of terms. Rai to a power 1 less than the number of terms. Mult this power by the first term.

Ex. First term 10; ratio 3; number of terms 8; what is the last term? Here the numb being 8, the ratio 3 must be raised to the 7th power; thus:

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2187, = \text{7th power. And } 2187 \times 10 = 21870 \text{ last term.}$$

A man agreed to buy 8 fine horses; paying $10 for the first; $30 for the second; $90 fo &c; how much will the last one cost him? Ans, $21870, as before.

To find the sum of all the terms, knowing the first one; the ratio; and the number of te the ratio to a power equal to the whole number of terms. From this power subtract 1. I by 1 less than the ratio. Mult the quot by the first term.

Ex. As before. What is the sum of all the terms? Here the ratio must be raised power: thus, $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 6561 =$ 8th pow. And 6560 div by 1 less th 3, $= \frac{6560}{2} = 3280$. And 3280×10 (or number of terms) $= 32800 =$ sum. Ans.

In the foregoing case, the 8 horses would cost $32800.

Permutation

Shows in how many positions any number of things can be arranged in a row. To d together all the numbers used in counting the things. Thus, in how many positions in things be placed? Here,

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880 \text{ positions. Ans.}$$

Combination

Shows how many combinations of a few things can be made out of a greater number of do this, first set down that number which indicates the greater number of things; and aft of numbers, diminishing by 1, until there are *in all* as many as the number of the few are to form each combination. Then beginning under the last one, set down said nu things; and going backward, set down another series, also diminishing by 1, until arrivin first of the upper numbers. Mult together all the upper numbers to form one prod; and s ones to form another. Div the upper prod by the lower one.

Ex. How many combinations of 4 figures each, can be made from the 9 figs 1, 2, 3, 4, or from 9 any things?

$$\frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} = \frac{3024}{24} = 126 \text{ combinations. Ans.}$$

Alligation

Shows the value of a mixture of different ingredients, when the quantity and value of e last is known.

Ex. What is the value of a pound of a mixture of 20 lbs of sugar worth 15 cts per lb; worth 25 cts per lb?

lbs.		cts.		cts.
20	×	15	=	300
30	×	25	=	750
50 lbs.				1050 cts.

Therefore, $\frac{1050}{50} = 21$ cts. Ans.

Equation of Payments.

A owes B $1200; of which $400 are to be paid in 3 months; $500 in 4 months; a months; all bearing interest until paid; but it has been agreed to pay all at once. Now, a must this payment be made so that neither party shall lose any interest?

$		months.		
400	×	3	=	1200
500	×	4	=	2000
300	×	6	=	1800
1200				5000

Therefore, $\frac{5000}{1200} = 4.16$, &c, months. Ans.

A owes B $1000 to be paid in 12 days; and $500 to be paid in 3 months. What would *for paying all at once?*

$		days.		
1000	×	*12*	=	12000
500	×	*90*	=	45000
1500				57000

Therefore, $\frac{57000}{1500} = 38$ days. Ans.

Simple Interest.

is the simple interest on $865.32 cts for one year, at 6 per ct per annum?

Principal.		Interest.		Principal.		Interest.
$100	:	$6	::	$865.32	:	$51.9192

6

100)5191.92(51.9192 Ans. = $51.91 $\frac{92}{100}$ cts.

is the interest on $865.32 cts for 1 year, 3 months, and 10 days, at 7 per cent per annum?

alculate the interest for 1 year only; thus:

Prin.		Int.		Prin.		Int.
$100	:	$7	::	$865.32	:	$60.5724

7

100)6057.24(60.5724

ay, If 1 year or 365 days give $60.5724 int, what will 465 days give? or

Days.		Int.		Days.		Int.
365	:	$60.5724	::	465	:	$77.16, &c. Ans.

er ct simple interest, money doubles itself in 20 years; at 6 per ct, in 16⅔ years; and at 7 a 14$\frac{2}{7}$ years. Simple Interest is Single Rule of Three.

Compound Interest.

money is borrowed for more than a year at compound interest, find the simple interest at the e first year, and add it to the principal, for a second principal. Find the simple interest on nd enlarged principal for the next year, and add it to the enlarged principal for a third prin- d so on for each successive year.

r ct compound interest, money doubles itself in about 14$\frac{1}{5}$ years; at 6 per ct, in about 11.9 nd at 7 per ct, in about 10¼ years.

Discount

action of a part of the interest, when money at interest is paid before it is due. Or it is a of the whole of the interest in advance, at the time the money is lent. In the first case, if $100 for 1 year at 8 per ct. I must at the end of the year pay back $108; but if I pay at the months, I must add only $2, or the interest for those 3 months, paying back $102; and the is the discount. Therefore, to find the discount in such cases, first find the interest for me; then that for the short time; and take the diff.

second case, if I borrow $100 from a bank for one year, at 6 per ct, I receive but 100 — 6 = $94; e end of the year I must pay back $100. By discounting in this manner, the bank actually rs than 6 per ct: for it gains $6 for the use of $94 for 1 year. In the United States, the banks iscount for 3 days more than the time stipulated in the note; these are called "days of

Commission, or Brokerage,

centage (or so much per each $100) paid to commission merchants for selling our goods; or s, or other kinds of agents, for transacting business for us. It is Single Rule of Three.

a broker makes purchases for me to the amount of $9362, at 2 per ct, what is his brokerage?

Purchase.		Brokerage.		Purchase.		Brokerage.
$100	:	$2	::	$9362	:	$187.24

Insurance

centage (called a *premium*) paid to a company for insuring our property against fire, &c. pany, or insurers, (called also underwriters,) deliver to the person insured, a paper bearing , &c, and called the *Policy of Insurance*; which contains the conditions of the transaction. e is calculated like Commissions, &c.; being merely Single Rule of Three.

Fellowship.

$6000 into a business in partnership with B, who puts in $9000. At the end of a year they e $2400; how much is each one's share? Here, $6000 + $9000 = $15000 joint capital; then say,

Joint cap.		Total gain.		A's cap.		A's share.
$15000	:	$2400	::	$6000	:	$960

				B's cap.		B's share.
$15000	:	$2400	::	$9000	:	$1440

WEIGHTS AND MEASURES.

States and British measures of length and weight, of the same denomination, may, *for all purposes*, be considered as equal; but the *liquid* and *dry* measures of the same denomina- widely in the two countries. The standard measure of length of both countries is theo- that of a pendulum vibrating seconds at the level of the sea, in the lat of London, in a with Fahr's thermom at 62°. The length of such a pendulum is supposed to be divided into ual parts, called inches; and 36 of these inches were adopted as the standard yard of both . But the Parliamentary standard having been destroyed by fire, in 1834, it was found to ible to restore it by measurement of a pendulum; and the present Brit standard yard is, in es, shorter *than that of the U. S.* by about 1 part in 17230, which is equal to about $\frac{1}{4}$ of *100 feet, or in 3⅜ ins* in a mile. This will account in great measure for the slight discrep- *ently observed between the estimations of length of the metre, of a degree of longitude al mile, &c, &c, as given by the highest scientific authorities of the two countries.* *ot, yard, &c, = 1.000058 British ones*, and a Brit one = .9999420 U. S.

G

Troy Weight.

U. S. AND BRITISH.

24 grains..... 1 pennyweight, dwt.
20 pennyweights ..... 1 ounce = 480 grains.
12 ounces..... 1 pound = 240 dwts = 5760 grains.

Troy weight is used for gold and silver.*

A carat of the jewellers, for precious stones, is in the U. S. = 3.2 grs; in London, 3.17 grs; in Paris, 3.18 grs, divided into 4 jewellers' grs. In troy, apothecaries', and avoirdupois, the grain is the same. In the U. S. the standard for coins is, 9 parts by weight of gold or silver, to 1 part of alloy.

Perfectly pure gold is worth		$20.67183	per ounce troy;	or	$18.84151	per oz avoir.	
" "	silver (1873)	1.33333	"	"	1.21528	"	"
Standard gold	"	18.60465	"	"	16.95736	"	"
" silver	"	1.20000	"	"	1.09375	"	"

The U. S. silver dollar weighs 384 grains; or $15 = 1 lb troy; and $18.23 = 1 lb avoir. The U. S. $20 gold piece weighs 516 grains; the $10 = 258 grs; the $1 = 25.8 grs. 17 gold dollars, or one $10, one $5, and two $1 pieces, weigh 438.6 grains, or very little more than an avoir ounce of 437½ grs. The $20 gold piece contains 464 grains, or .9666 + troy ounces; or 1.0606 commercial or avoir ounces, of perfectly pure gold; the $10 piece half as much, and so on; the dollar containing 23.2 grs.

The troy ounce is greater than the avoir; but the troy lb is less than the avoir.

Apothecaries' Weight.

U. S. AND BRITISH.

20 grains ..... 1 scruple.
3 scruples ..... 1 dram = 60 grs.
8 drams ..... 1 ounce = 24 scruples = 480 grs.
12 ounces ..... 1 pound = 96 drams = 288 scruples = 5760 grains.

In troy and apoth weights, the grain, ounce, and pound are the same.

Avoirdupois, or ordinary Commercial Weight.

U. S. AND BRITISH.

27.34375 grains ..... 1 dram.
16 drams ..... 1 ounce = 437½ grains.
16 ounces ..... 1 pound = 256 drams = 7000 grains.
28 pounds ..... 1 quarter = 448 ounces.
4 quarters ..... 1 hundredweight = 112 lbs.
20 hundredweights ..... 1 ton = 80 quarters = 2240 lbs.

A stone = 14 pounds. A quintal = 100 pounds.

The standard of the avoir pound, which is the one in common commercial use, is the weight of 27.7015 cub ins of pure distilled water, at its maximum density at about 39°.2 Fahr, in latitude of London, at the level of the sea; barom at 30 ins.

A cube of 3.0258 ins on each edge, contains 27.7015 cub ins; or 1 pound of pure water.
A troy lb = .82286 avoir lb. An avoir lb = 1.21528 troy lb.
A troy oz = 1.09714 avoir oz. An avoir oz = .911458 troy oz.

Long Measure.

U. S. AND BRITISH.

These are usually considered to be the same, but in fact the British measures are shorter than those of the U. S. by about 1 part in 17230, or 3.677 inches in a mile; or $\frac{1}{14}$ inch in 100 feet.

12 inches..... 1 foot = .3048121 metre.
3 feet..... 1 yard = 36 ins.
5½ yards..... 1 rod, pole, or perch = 16½ feet = 198 ins.
40 rods..... 1 furlong = 220 yards = 660 feet.
8 furlongs..... 1 statute, or land mile = 320 rods = 1760 yds = 5280 ft = 63360 ins.
3 miles..... 1 league = 24 furlongs = 960 rods = 5280 yds = 15840 ft.

A point = $\frac{1}{72}$ inch. A line = 6 points = $\frac{1}{12}$ inch. A palm = 3 ins. A hand = 4 ins. A span = 9 ins. A fathom = 6 feet. A cable's length = 120 fathoms = 720 feet. A Gunter's surveying chain is 66 feet, or 4 rods long. It is divided into 100 links of 7.92 ins long. 80 Gunter's chains = 1 mile. One U. S. inch = .025401 metre. One Brit inch = .0253994 metre. A U. S. foot = .304801 metre. One Brit foot = .304793 metre. A yard, U. S. = .91443; Brit = .91438 metre.

A nautical *knot*, or sea mile, is the length of 1 minute of longitude of the earth at the equator, at the level of the sea, or the $\frac{1}{21600}$ part of the earth's equatorial circumf. By the U. S. standard and as used by the Coast Survey, its length is 1.152664 common statute or land miles: 1855.11 metres, 2028.69 yards; or 6086.07 feet; consequently one degree of long at the equator = 69.160 land miles; and a land mile = 86755 of a nautical mile. By Brit standard the sea mile is about 4 ins longer than the U. S.‡ Sometimes 1 minute of mean *latitude* is taken as a nautical mile. A min of lat at the equator is about 6040 ft; and at the poles about 6107; the mean of which is 6076½ ft.

---

* Perfectly pure gold is called fine, or 24 carat gold. When alloyed, it is assumed to be divided into 24 parts. If 15, 18, 20, &c., of these parts are pure gold, the alloy is said to be 15, 18, 20, &c. carats fine. By the Cyclopedia of Commerce (1858), a cubic foot of pure gold is worth $362660 U. S. standard; and a cubic inch, $209.84. A cubic ft of pure silver is worth $12338 U. S. standard; and *a cubic inch, $7.14.* In 1873 the value of pure gold by weight is about 15.50† times that of silver. *The average fineness of California native gold*, by some thousands of assays at the Philadelphia mint, *is 88½ parts gold, and 11½ silver*, in 100 parts.

‡ *Bowditch's Navigation* assumes 1.150091 mile; or 6120 ft.

engths of a Degree of Longitude in different Latitudes, and at the level of the Sea.

These lengths are in common land or statute miles, of 5280 ft. Since the figure of the earth has ver been *precisely* ascertained, these are but close approximations.

Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.	Deg of Lat.	Miles.
0	69.16	14	67.12	28	61.11	42	51.47	56	38.76	70	23.72
2	69.12	16	66.50	30	59.94	44	49.83	58	36.74	72	21.43
4	68.99	18	65.80	32	58.70	46	48.12	60	34.67	74	19.12
6	68.78	20	65.02	34	57.39	48	46.36	62	32.55	76	16.78
8	68.49	22	64.15	36	56.01	50	44.54	64	30.40	78	14.42
10	68.12	24	63.21	38	54.56	52	42.67	66	28.21	80	12.05
12	67.66	26	62.20	40	53.05	54	40.74	68	25.98	82	9.66

ermediate ones may be found correctly by simple proportion.

Inches reduced to Decimals of a Foot. No errors.

ns.	Foot.	Ins.	Foot.	Ins.	Foot.	Ins.	Foot.	Ins.	Foot.	Ins.	Foot.
	.0000	2	.1667	4	.3333	6	.5000	8	.6667	10	.8333
32	.0026		.1693		.3359		.5026		.6693		.8359
16	.0052		.1719		.3385		.5052		.6719		.8385
32	.0078		.1745		.3411		.5078		.6745		.8411
⅛	.0104	⅛	.1771	⅛	.3438	⅛	.5104	⅛	.6771	⅛	.8438
32	.0130		.1797		.3464		.5130		.6797		.8464
16	.0156		.1823		.3490		.5156		.6823		.8490
32	.0182		.1849		.3516		.5182		.6849		.8516
¼	.0208	¼	.1875	¼	.3542	¼	.5208	¼	.6875	¼	.8542
32	.0234		.1901		.3568		.5234		.6901		.8568
16	.0260		.1927		.3594		.5260		.6927		.8594
32	.0286		.1953		.3620		.5286		.6953		.8620
⅜	.0313	⅜	.1979	⅜	.3646	⅜	.5313	⅜	.6979	⅜	.8646
32	.0339		.2005		.3672		.5339		.7005		.8672
16	.0365		.2031		.3698		.5365		.7031		.8698
32	.0391		.2057		.3724		.5391		.7057		.8724
½	.0417	½	.2083	½	.3750	½	.5417	½	.7083	½	.8750
32	.0443		.2109		.3776		.5443		.7109		.8776
16	.0469		.2135		.3802		.5469		.7135		.8802
32	.0495		.2161		.3828		.5495		.7161		.8828
⅝	.0521	⅝	.2188	⅝	.3854	⅝	.5521	⅝	.7188	⅝	.8854
32	.0547		.2214		.3880		.5547		.7214		.8880
16	.0573		.2240		.3906		.5573		.7240		.8906
32	.0599		.2266		.3932		.5599		.7266		.8932
¾	.0625	¾	.2292	¾	.3958	¾	.5625	¾	.7292	¾	.8958
32	.0651		.2318		.3984		.5651		.7318		.8984
16	.0677		.2344		.4010		.5677		.7344		.9010
32	.0703		.2370		.4036		.5703		.7370		.9036
⅞	.0729	⅞	.2396	⅞	.4063	⅞	.5729	⅞	.7396	⅞	.9063
32	.0755		.2422		.4089		.5755		.7422		.9089
16	.0781		.2448		.4115		.5781		.7448		.9115
32	.0807		.2474		.4141		.5807		.7474		.9141
	.0833	3	.2500	5	.4167	7	.5833	9	.7500	11	.9167
	.0859		.2526		.4193		.5859		.7526		.9193
	.0885		.2552		.4219		.5885		.7552		.9219
	.0911		.2578		.4245		.5911		.7578		.9245
⅛	.0938	⅛	.2604	⅛	.4271	⅛	.5938	⅛	.7604	⅛	.9271
	.0964		.2630		.4297		.5964		.7630		.9297
	.0990		.2656		.4323		.5990		.7656		.9323
	.1016		.2682		.4349		.6016		.7682		.9349
¼	.1042	¼	.2708	¼	.4375	¼	.6042	¼	.7708	¼	.9375
	.1068		.2734		.4401		.6068		.7734		.9401
	.1094		.2760		.4427		.6094		.7760		.9427
	.1120		.2786		.4453		.6120		.7786		.9453
⅜	.1146	⅜	.2813	⅜	.4479	⅜	.6146	⅜	.7813	⅜	.9479
	.1172		.2839		.4505		.6172		.7839		.9505
	.1198		.2865		.4531		.6198		.7865		.9531
	.1224		.2891		.4557		.6224		.7891		.9557
½	.1250	½	.2917	½	.4583	½	.6250	½	.7917	½	.9583
	.1276		.2943		.4609		.6276		.7943		.9609
	.1302		.2969		.4635		.6302		.7969		.9635
	.1328		.2995		.4661		.6328		.7995		.9661
	.1354	⅝	.3021	⅝	.4688	⅝	.6354	⅝	.8021	⅝	.9688
	.1380		.3047		.4714		.6380		.8047		.9714
	.1406		.3073		.4740		.6406		.8073		.9740
	.1432		.3099		.4766		.6432		.8099		.9766
	.1458	¾	.3125	¾	.4792	¾	.6458	¾	.8125	¾	.9792
	.1484		.3151		.4818		.6484		.8151		.9818
	.1510		.3177		.4844		.6510		.8177		.9844
	.1536		.3203		.4870		.6536		.8203		.9870
	.1563	⅞	.3229	⅞	.4896	⅞	.6563	⅞	.8229	⅞	.9896
	.1589		.3255		.4922		.6589		.8255		.9922
	.1615		.3281		.4948		.6615		.8281		.9948
	.1641		.3307		.4974		.6641		.8307		.9974

Square, or Land Measure.

U. S. AND BRITISH.

144 square inches..... 1 sq foot.
9 sq feet..... 1 sq yard = 1296 sq ins.
30¼ sq yards..... 1 sq rod = 272¼ sq feet.
40 sq rods..... 1 rood = 1210 sq yds = 10890 sq feet.
4 roods..... 1 acre = 160 rods = 4840 sq yds = 43560 sq feet.

A *section* of land is 1 mile sq, and contains 27878400 sq feet; or 3097600 sq yds; or 640 acres. An acre contains 10 sq Gunter's chains. A sq acre is 208.710 feet; a sq half acre, 147.581 ft; and a sq quarter acre, 104.355 ft on each side. A circular acre is 235.504 feet; a circular half acre = 166.537 ft; and a circular quarter acre = 117.752 ft diam. A circular inch is a circle of 1 inch diam; a sq ft = 183.346 cir ins.

Cubic, or Solid Measure.

U. S. AND BRITISH.

1728 cubic inches..... 1 cubic, or solid foot.
27 cubic feet..... 1 cubic, or solid yard.

A cord of wood = 128 cub ft; being 4 ft × 4 ft × 8 ft. A perch of masonry actually contains 24¾ cub ft; being 16½ ft × 1½ ft × 1 ft. It is generally taken at 25 cub ft; but by some at 22, &c; and there is every probability that a payer will be cheated unless the number of cubic ft be distinctly agreed upon in his contract.* It is gradually falling into disuse among engineers; and the cub yd is very properly taking its place. To reduce cub yds to perches of 25 cub ft, mult by 1.080; and to reduce perches to cub yds, mult by .926. The Brit *rod* of brickwork, of house-builders, is 16½ feet square, by 14 inches (1½ English bricks) thick, = 272¼ sq ft of 14 inch wall. It is conventionally taken at 272 sq ft; which gives 317¾ cub ft. In Brit engineering works the rod is 306 cub ft, or 11⅓ cub yds. The Montreal, (Canada) *toise* = 261⅓ cub ft; or 9.6852 cub yds, or 10.46 perches of 25 cub ft. The Canadian chaldron = 58.64 cub ft. A ton (2240 lbs) of Pennsylvania anthracite, when broken for domestic use, occupies from 41 to 43 cub ft of space; the meau of which is equal to 1.556 cub yds; or a cube of 3.476 ft on each edge. Bituminous coal 44 to 48 cub ft; mean equal to 1.704 cub yd; or a cube of 3.583 ft on each edge. Coke 80 cub ft.

A cubic foot is equal to

1728 cub ins.
.037037 cub yard.
.002832 myriolitre, or decastere.
.028320 kilolitre, or cubic metre, or stere.
.283202 hectolitre, or decistere.
2.83202 decalitres, or centisteres.
28.3202 litres, or cub decimetres.
283.202 decilitres.
2832.02 centilitres.
28320.2 millilitres, or cub centimetres.
.803564 U. S. struck bushel of 2150.42 cub ins, or 1.24445 cub ft.
.779013 Brit bushel of 2218.191 cub ins, or 1.28368 cub ft.
3.21426 U. S. pecks.
3.11605 Brit pecks.
7.48052 U. S. liquid galls of 231 cub ins.
6.42851 U. S. dry galls.
6.23210 Brit galls of 277.274 cub ins.
29.92208 U. S. liquid quarts.
25.71405 U. S. dry quarts.
24.92842 Brit quarts.
59.84416 U. S. liquid pints.
51.42809 U. S. dry pints.
49.85684 Brit pints.
239.37662 U. S. gills.
199.42737 Brit gills.
.26667 flour barrel of 3 struck bushels.
.23748 U. S. liquid barrel of 31½ galls.

In this table the Brit cub ft is used for the Brit capacities. For ordinary purposes it may be considered as being the same as that of the U. S.

A cubic inch is equal to

16.38901 millilitres; or 1.638901 centilitres; or .1638901 decilitre; or .01638901 litre.

A cubic yard is equal to

27 cub feet.
46656 cub ins.
.0764646 myriolitre.
.764646 kilolitre, or cub metre.
7.64646 hecatolitres.
76.4646 decalitres.
764.646 litres, or cub decimetres.
7646.46 decilitres.
21.69623 U. S. bushels (struck).
21.93336 Brit bushels.

Liquid Measure.

U. S. ONLY.

The basis of this measure in the U. S. is the old Brit wine gallon of 231 cub ins; or 8.33888 lbs avoir of pure water, at its max density at about 39°.2 Fahr; the barom at 30 ins. A cylinder 7 ins diam, and 6 ins high, contains 230.904 cub ins, or almost precisely a gallon; as does also a cube of 6.1538 ins on an edge. Also a gallon = .13368 of a cub ft; and a cub ft contains 7.48052 galls; very nearly 7½ galls.

		cub ins.
4 gills	1 pint	= 28.875
2 pints.....	1 quart	= 57.750 = 8 gills.
4 quarts	1 gallon	= 231. = 8 pints = 32 gills.

63 gallons..... 1 hogshead.
2 hogsheads..... 1 pipe, or butt.
2 pipes..... 1 tun.

In the U. S. and Great Brit, 1 barrel of wine or brandy = 31½ galls; in Pennsylvania, a half barrel, 16 galls; a double barrel, 64 galls; a puncheon, 84 galls; a tierce, 42 galls. A liquid measure barrel of 31½ galls, contains 4.211 cub ft = a cube of 1.615 ft on an edge; or 3.384 U. S. struck bushels. A gill = 7.21875 cub ins. The following cylinders contain some of these measures very approximately.

	Diam. Ins.	Height. Ins.		Diam. Ins.	Height. Ins.
Gill	1¾	3	Gallon.....	7	6
½ pint	2¼	3¾	2 gallons	7	12
Pint	3½	3	8 gallons	14	12
Quart	3½	6	10 gallons	14	15

* *In Philadelphia*, 22 cub ft are called a perch in rubble walls in buildings.

To reduce U. S. liquid measures to Brit ones of the same denomination, divide by 1.20032; or near enough for common use, by 1.2; or to reduce Brit to U. S. multiply by 1.2.

Dry Measure.

U. S. ONLY.

The basis of this measure is the old British Winchester struck bushel of 2150.42 cub ins; or 77.627413 pounds avoir of pure water at its max density. Its dimensions by law are 18½ ins inner diam; 19½ ins outer diam; and 8 ins deep; and when heaped, the cone is not to be less than 6 ins high; which makes a heaped bushel equal to 1¼ struck ones; or to 1.55556 cub ft.

		Edge of a cube of equal capacity.
2 pints	1 quart, = 67.2006 cub ins, = 1.16365 liquid qt	4.066 ins.
4 quarts	1 gallon, = 8 pints, = 268.8025 cub ins, = 1.16365 liq gal	6.454 "
2 gallons	1 peck, = 16 pints, = 8 quarts, = 537.6050 cub ins	8.131 "
4 pecks	1 struck bushel, = 64 pints, = 32 quarts, = 8 gals, = 2150.4200 cub ins.	12.903 "

A struck bushel = 1.24445 cub ft. A cub ft = .80356 of a struck bushel. The dry flour barrel = 2.75 cub ft; = 3 struck bushels. The dry barrel is not, however, a legalized measure; and no great attention is given to its capacity; consequently, barrels vary considerably. A barrel of flour contains by law, 196 lbs. In ordering by the barrel, the amount of its contents should be specified in pounds or gals. By a barrel of U. S. hyd cement, is usually understood about 300 to 310 lbs; it should, however, always be contracted for by weight. A barrel of genuine Portland averages 425 lbs.

To reduce U. S. dry measures to Brit imp ones of the same name, div by 1.031516; and to reduce Brit ones to U. S. mult by 1.031516; or for common purposes use 1.032.

British Imperial Measure, both liquid and dry.

GREAT BRITAIN ONLY.

This system is established throughout Great Britain, to the exclusion of the old ones. Its basis is the imperial gallon of 277.274 cub ins, or 10 lbs avoir of pure water at the temp of 62° Fahr, when the barom is at 30 ins.

	Avoir lbs. of water.		Cub. ins.	Cub. ft.	Edge of a cube of equal capacity. Inches.
4 gills 1 pint	1.25		34.6592	.....	3.2605
2 pints 1 quart	2.50		69.3185	.....	4.1079
2 quarts 1 pottle	5.		138.637	.....	5.1756
2 pottles 1 gallon	10.		277.274	.....	6.5208
2 gallons 1 peck	20.	Dry meas.	554.548	.....	8.2157
4 pecks 1 bushel	80.		2218.192	1.2837	13.0417
4 bushels 1 coomb	320.		8872.768	5.1347	
2 coombs 1 quarter	640.		17745.536	10.2694	

The imp gall = .16046 cub ft; and 1 cub ft = 6.23210 galls. The imp gal = 1.20032, or very nearly $1\frac{1}{5}$ U. S. liquid galls. To reduce imp *liquid* measures to U. S. ones of the same name, mult by 1.20032; or add $\frac{1}{5}$ part. To reduce imp *dry* measures to U. S. ones, mult by 1.031515.

A cylinder 1 foot in diameter, and 1 foot high, contains

.02909 cub yard.
.7854 cub foot.
1357.1712 cub inches.
.63112 U. S. dry bushels.
2.5245 U. S. dry pecks.
20.1958 U. S. dry quarts.
40.3916 U. S. dry pints.
5.8752 U. S. liquid gallons.
23.5008 U. S. liquid quarts.
47.0016 U. S. liquid pints.
188.0064 U. S. liquid gills.
4.8947 Brit imp gallons.
19.5788 Brit imp quarts.
39.1575 Brit imp pints.
156.6302 Brit imp gills.
222.427 decilitres.
22.2427 litres.
2.22427 decalitres.
.222427 hectolitre.

A cylinder 1 inch in diameter, and 1 foot high, contains

.005454 cub foot.
9.4248 cub inches.
.2805 U. S. dry pint.
.3264 liquid pint.
1.3056 U. S. gill.
.2719 Brit imp pint.
1.0877 Brit imp gill.
15.4463 centilitres.
1 54463 decilitres.
.154463 litres.

A sphere 1 foot in diameter, contains

.01939 cub yard.
.5236 cub foot.
904.781 cub inches.
.42075 U. S. bushel.
1.6830 U. S. pecks.
13.4639 U. S. dry quarts.
26.9278 U. S. dry pints.
3.9168 U. S. liquid gallons.
15.6672 U. S. liquid quarts.
31.3344 U. S. liquid pints.
125.3376 U. S. liquid gills.
3.2631 Brit imp gallons.
13.0525 Brit imp quarts.
26.1050 Brit imp pints.
104.4201 Brit imp gills.
14.8285 litres.
1.48285 decalitres.
14.8285 hectolitres.

A sphere 1 inch in diameter, contains

.000303 cub foot.
.5236 cub inch.
.07253 U. S. gill.
.06043 Brit gill.
8.581 millilitre.
.8581 centilitre.
.08581 decilitre.

French Measures of Length.

ACCORDING TO UNITED STATES STANDARD.

	U. S. Ins.	U. S. Ft.	U. S. Yds
Millimetre *	.039368	.003281	.....
Centimetre †	.393685	.032807	.....
Decimetre	3.93685	.328071	.109357
Metre ‡	39.3685	3.28071	1.09357
Decametre	393.685	32.8071	10.9357
Hectometre	Road	328.071	109.357
Kilometre	measures.	3280.71	1093.57
Myriametre		32807.1	10935.7

* Nearly the $\frac{1}{25}$ part of an inch. † Full ⅜ inch.

‡ Very nearly 3 ft, 3⅜ ins, which is too long by only 1 part in 6062.

French Square Measure.

ACCORDING TO U. S. STANDARD.

	U. S. Sq. Ins.	U. S. Sq. Feet.	U. S. Sq. Yds
Sq Millimetre	.001549	.00001076	.0000012
Sq Centimetre	.154988	.00107631	.0001196
Sq Decimetre	15.4988	.10763058	.0119589
Sq Metre, or Centiare	1549.88	10.763058	1.195895
Sq Decametre, or Are	154988.	1076.3058	119.5895
Decare (not used)	.....	10763.058	1195.895
Hectare	.....	107630.58	11958.95
Sq Kilometre	.3860716 sq miles.	10763058.	1195895.
Sq Myriametre	38.60716 sq miles.	.....	.....

French Cubic, or Solid Measures.

ACCORDING TO U. S. STANDARD.

Only those marked "Brit" are British.

	Cub Ins.	
Millilitre, or cub Centimetre	.0610165	Liquid. .0084525 gill. " .0070420 Brit gill. Dry. .001816 dry pint.
Centilitre	.610165	Liquid. .084525 gill. " .070420 Brit gill. Dry. .01816 dry pint.
Decilitre	6.10165	Liquid. .84525 gill = .21131 pint. " .70420 Brit gill = .17605 Brit pi Dry. .1816 dry pint.
Litre, or cubic Decimetre	61.0165	Liquid. 1.05656 quart = 2.1131 pints. " .88025 Brit quart = 1.7605 Brit Dry. .1135 peck = .908 dry quart = 1
Decalitre, or Centistere	610.165 cub ft .353105	Liquid. 2.64141 U. S. liquid gal. " 2.20059 Brit gal. Dry. .283742 bush = 1.135 peck = 9.0
Hectolitre, or Decistere	3.53105	Liquid. 26.4141 U. S. liquid gal. " 22.0059 Brit gal. Dry. 2.83742 bush.
Kilolitre, or Cubic *Metre*, or Stere	35.3105	Liquid. 264.141 U. S. liquid gal. " 220.059 Brit gal. Dry. 28.3742 bush. } Cub yd
Myriolitre, or Decastere	353.105	Liquid. 2641.41 U. S. liquid gal. Dry. 283.742 bush. } Cub y

The French Metre.

The French metre was intended to be the one ten-millionth part of the dist from either pole of the earth to the equator: but after it had been introduced into use, errors were discovered in the calculations employed for ascertaining that dist; so that the French metre, like the Brit standard yard, is not what it was intended to be. By the Brit standard, the length of the metre is, approximately, 1.093633 yard; or 3.280899 feet; or 39.37079 ins. By the U. S. standard it is, very approximately, 1.093697 yd; or 3.280709 ft; 39.368505 ins; the Brit yard, foot, and inch being about 1 part in 17230 shorter than the U. S., which is equal to 3.677 inches in a mile; or to 1-14th inch in 100 ft.

French Weights, reduced to common Commercial or Avoir Weight, of 1 pound = 16 ounces, or 7000 grains.

[From Alexander's "Weights and Measures."]

Scarcely any two writers give *precisely* the same results. Professor Rankine, however, a very high British authority, gives for the kilogramme 2.20462 lbs; or less than Mr. Alexander but about 1 part in 19000; so that for practical purposes the best American and Brit authorities may be considered as the same. The mean is 2.20468 lbs.

	Grains.	
Milligramme	.01543316	.....
Centigramme	.1543316	.....
Decigramme	1.543316	.....
Gramme	15.43316	.0352758 ounce avoir.
(1 U. S. gold dol weighs 1.671725 grammes.)	Pounds av.	
Decagramme	.02204737	.352758 " "
Hectogramme	.2204737	3.52758 " "
Kilogramme	2.204737	35.2758 " "
Myriogramme	22.04737	.00984258 ton of 2240 lbs.
Quintal	220.4737	.0984258 " "
Tonneau; Millier; or Tonne	2204.737	.984258 " "

The gramme is the basis of French weights; and is the weight of a cub centimetre of distilled water at its max density, at sea level, in lat of Paris; barom 29.922 ins.

French Measures of the "Systeme Usuel."

This system was in use from about 1812 to 1840, when it was forbidden by law to use even its names. This was done in order to expedite the general use of the tables which we have before given. But as the Systeme Usuel appears in books published during the above interval, we add a table of some of its values.

Measures of Length.

	Yards.	Feet.	Inches.
Ligne usuel, or line	.....	.....	.09113
Pouce usuel, or inch, = 12 lignes	.....	.09113	1.09362
Pied usuel, or foot, = 12 pouces	.36454	1.09362	13.12344
Aune usuel, or ell	1.31236	3.93708	47.245
Toise usuel, = 6 pieds	2.18727	6.56181	78.74172

Weights, Usuel.		Cubic, or Solid, Usuel.	
Grain usuel	.8375 grains.	Litron usuel, or 1 litre	= 1.7608 British pint.
Gros usuel	60.297 "		
Once usuel	1.10258 avoir oz.		
Marc usuel	.55129 avoir lb.	Boisseau usuel	2.7512 British gals.
Livre usuel, or pound,	1.10258 avoir lb.		

Before 1812, or before the "Systeme usuel," the Old System, "Systeme Ancien," was in use.

French Measures of the "Systeme Ancien."

Lineal.	Square.			Cubic.		
	Sq. ins.	Sq. ft.	Sq. yds.	C. ins.	C. ft.	C. yds.
Point ancien, .0148 ins						
Ligne ancien, .0888 ins	.00789	.....	.....	.0007		
Pouce ancien, 1.06577 ins = .0888 ft	1.1359	.....	.....	1.2106		
Pied ancien, 12.7892 ins = 1.06577 ft	.....	1.1359	.....	.....	1.2106	
Aune ancien, 46.8939 ins = 3.90782 ft = 1.30261 yds						
Toise ancien, = 6.3946 ft = 2.1315 yds	.....	40.8908	4.5434	.....	261.482	9.6943
League = 2282 toises = 2.7637 miles						

There is, however, much confusion about these old measures. Different measures had the same in different provinces.

French Measures of the "Systeme Ancien"—(Continued.)

Weights, Ancien.		Capacity, Ancien.			Land, Ancien.	
Grain	.8203 grs.	Pint	1.6766	Brit pints.	Perche des eaux et forets	549.753 sq. ft
Gros	59.062 grs.	Litron	1.073	Brit gallons.	Arpent des eaux et forets	1.262 acres
Once	1.08 avoir oz.	Boisseau	4.29191	" "	Perche de Paris	368.03 sq. ft.
Marc	.54 avoir lb.	Minot	1.37737	cub feet.	Arpent de Paris	.84488 acres
Livre	1.08 avoir lb	Mine	2.75474	cub feet.		
		Septier	5.50947	cub feet.		
		Muid	2.4486	cub yards.		

Spanish.

The *castellano* of Spain and New Granada, for weighing gold, is variously estimated, from 71.0 to 71.04 grains. At 71.055 grs. (the mean between the two,) an avoir. or common commercial ounce contains 6.1572 castel; and a lb avoir contains 98.515. Also a troy ounce = 6.7553 castel; and a troy lb = 81.064 castel. Three U. S. gold dollars weigh about 1.1 castel.

The Spanish *mark*, or marco, for precious metals, in South America, may be taken in practice, as .5065 of a lb avoir. In Spain, .5076 lb. In other parts of Europe, it has a great number of values; most of them, however, being between .5 and .54 of a pound avoir. The .5065 of a lb = 3545½ grs; and .5076 lb = 3553.2 grs. 1 Marco = 50 castellanos = 400 tomine = 4800 *Spanish gold*-grs.

The *arroba* has various values in different parts of Spain. That of Castile, or Madrid, is 25.4025 lbs avoir; the *tonelada* of Castile = 2032.2 lbs avoir; the *quintal* = 101.61 lbs avoir; the *libra* = 1.0161 lbs avoir.

The *vara* of Castile = 32.8748 ins, or almost precisely 32¾ ins; or 2 ft 8¾ ins. The *fanegada* of land since 1801 = 1.5871 acres = 69134.08 sq ft. The *fanega* of corn, &c. = 1.59914 U. S. struck bushels.

The *cantara* of wine, &c, of Castile, = 4.263 U. S. galls; that of Havana = 4.1 galls.

Civil, or Common Clock Time.

60 thirds, marked '''	1 second, marked ''.
60 seconds	1 minute, '.
60 minutes	1 hour, = 3600 sec.
24 hours	1 civil day, = 1440 min. = 86400 sec.
7 days	1 week, = 168 hours = 10080 min.
4 weeks	1 civil month, = 28 days = 672 hours.

13 civil months, (or 52 weeks,) 1 day, 5 hours, 48 min, 49 $\frac{7}{10}$ sec; or 365 days, 5 hours, 48 min, 49 $\frac{7}{10}$ sec. = 1 civil year. A *solar* day is the time that elapses between two successive solar noons, or transits of the sun over the meridian of a place. These intervals are not of equal lengths all the year round. The average length of all the solar days is called the *mean solar day;* and is the same as the common civil day of 24 hours of clock time. Civil noon is at 12 o'clock; but solar, or apparent noon, may be about 14½ minutes before; or 16¼ minutes after 12 of correct clock time. A *sidereal day* is the interval between two passages of the same star past the range of two fixed objects; and is the precise time reqd for one complete rev of the earth on its axis. The sidereal day never varies; but is always equal to 23 hours, 56 min, 4.09 sec;* so that a star will on any night appear to set, or to pass the range of any two fixed objects, 3 min, 55.91 sec earlier by the clock, than it did on the night before; so that the number of sidereal days in a civil year is 1 greater than that of the civil days.†

* This may be made the means of regulating a clock or watch with quite satisfactory accuracy and by a very simple process. The writer, after having regulated his chronometer watch for a year by this method only, differed but a few seconds from the actual time as deduced from careful solar observations. Even a person not accustomed to ranging objects very accurately, need scarcely err a minute in a period of any number of years. It having occurred to him that the motion of a star in a second or two might be visible to the naked eye, he stuck a pin horizontally into a window-jamb; and placing his eye close to it, sighted along one side of it, at a large star setting behind the top of a roof about 100 feet distant, and found that his conjecture was correct. Those stars which are farthest from the poles appear to move the fastest, and are therefore the best. Those less than of the second magnitude are not satisfactory. If the first observations of a given star be made as late as midnight, that same star will answer for about three months, until at last it will begin to pass the range in daylight. Before this happens, the observer must transfer the time to another star which sets later; if near midnight, the better, as it will serve for a longer time. A window looking west is the best. The longer the range, the greater will be the apparent motion of the star; and, consequently, the observations will be more correct. If such a range can be secured as will strike the heavens at an angle of at least 40° above the horizon, the error from refraction will not appreciably affect an observation; at a much less angle it may do so to the extent of three or four seconds. A candle must be so placed as to render the pin and the watch visible at the same time. A little practice will render the process very easy, and supersede the necessity for more remarks on the subject. Of course, a memorandum must be made and preserved of the date, hour, minute, and (approximately) second, at which the first passage of the star took place. Subsequent passages will occur earlier, as shown in the following table. The watch must be previously known to be right, when taking the first observation, if we require afterward to keep the correct time. Any person who will take the trouble thus to observe and note down throughout a year, about half a dozen stars following each other at tolerably equal intervals of time, will on almost any clear night afterward be able, after a short calculation, to ascertain the correct clock time. The writer observed the passages of two or three stars behind different ranges, on the same nights, in order to obtain a mean of several observations; his object being to ascertain how pocket chronometers of the best makers would keep time under the vicissitudes of temperature, railroad travelling, &c, &c, to which they are ordinarily exposed. He used two of the best for this purpose, and the result was that their changes of rate were at times as great as from three to eight seconds per day. For ordinary purposes, therefore, they are of but little, if any, more service than a good common watch, of one-fourth the cost.

† More accurately 3 min, 55.90944 sec.

TABLE showing how much earlier a star passes a given range, on each succeeding night.—(Original.)

Nights.	Min.	Sec.	Nights.	H.	Min.	Sec.	Nights.	H.	Min.	Sec.
1	3	55.91	11		43	15.01	21	1	22	34.11
2	7	51.82	12		47	10.92	22	1	26	30.02
3	11	47.73	13		51	6.83	23	1	30	25.93
4	15	43.64	14		55	2.74	24	1	34	21.84
5	19	39.55	15		58	58.65	25	1	38	17.75
6	23	35.46	16	1	2	54.56	26	1	42	13.66
7	27	31.37	17	1	6	50.47	27	1	46	9.57
8	31	27.28	18	1	10	46.38	28	1	50	5.48
9	35	23.19	19	1	14	42.29	29	1	54	1.39
10	39	19.10	20	1	18	38.20	30	1	57	57.30
							31	2	1	53.21

After an entire year, the same star will pass the same range, on the same day of the month, 4 min, 57 sec, and $\frac{94}{100}$ of a sec, *later* than in the previous year; unless, during the interval, a 29th of February (of leap year) has occurred; in which case it will pass 0 min, 57 sec, and $\frac{97}{100}$ of a sec, *later* than in the previous year.

Approximate Values of Foreign Coins, in U. S. Currency.

It is difficult to obtain positive information respecting these; and it is probable that many of the values in this table are in error from 1 to 5 per cent. Since, however, many of the coins appear in statements of costs of engineering works abroad, it is convenient to have even such an approximation.

	Dolls.	Cts.
Augustus. Saxony	3	98
Carlin. Sardinia	8	21
Carolin. Bavaria	4	93
Crown. Great Britain	1	13
" Spain (Half Pistole)	1	95
" Baden, Bavaria, N. Germany	1	6
" Sicily		96
" Sardinia		92
" Portugal	1	12
Copeck. Russia		¾
Dollar. Chili, Mexico, Peru, Bolivia, New Granada	1	
" Central America, average		95
" Norway, Sweden, Spain	1	5
" Rigsbank Doll, Denmark		52
" Species Dollar, Denmark	1	5
Doubloon. Spain, Mexico	15	65
" Central America	14 to 15	50 to 65
" New Granada	15	34
Drachm. Greece		18
Ducat. Austria, Bohemia, Hamburg, Hanover	2	28
" Sweden	2	20
" Denmark	1	81
Ducatoon. Holland	1	32
Franc. France, Belgium		19
5 Franc piece		95
Florin. Austria, Silesia		48
" Holland, Netherlands, S. Germany		40
" (gold) Hanover	1	66
" (silver) Hanover		56
" Prussia		55
Guilder. Netherland		40
" British Guiana		26
Guilden. Baden		40
Guinea. Great Britain, 21 shillings	5	8
Groschen. Poland, Prussia		$\frac{2\frac{4}{10}}{12}$
Groschen. " "		1
Grote. Bremen		
Imperial. Russia	7	92
Kreutzer. Bavaria		⅜
" 10. Austria		8
" 60. " or florin		48
Livre. France, Sardinia (Franc)		18½
" Tuscany, Venice		16
Lira. Milan		14
Marc. Denmark		9
Maximilian. Bavaria	3	30
Milreis. Portugal	1	12
Moidore. "	6	50

	Dolls.	Cts.
Mohur. Bombay	7	28
" Bengal	8	15
Napoleon. 20 francs. France	3	84
Ounce. Sicily	2	50
Pistola. Rome	3	37
Pistole. Spain	3	90
Paolo. Rome		10
Peseta. Spain		20
Pistareen. Spain		20
Piastre. Turkey. Old		42
" " New		$4\frac{3}{10}$
" " other authorities 19 cts; 11 cts, &c		
" Spain	1	4
" Egypt		5
Para		$\frac{9}{10}$
Pound. Great Britain	4	$84\frac{}{10}$
" Canada, N. Scotia, New Bruns, Newfoundland	4	
Reale plate. Spain		10
" vellon. "		5
" Central America, average		5½
2 Reales. Equador		18½
Rix dollar. Hamburg, Hanover	1	10
" " Sweden, Holland	1	5
" " Baden, Brunswick	1	
" " Bavaria, Austria, Hungary		97
" 10 thalers. Prussia	8	
" thaler. Prussia, Poland, N. Germany, Bremen, Saxony, Hanover,		69
" species thaler. Saxony		98
Rupee. Bombay, Madras		45
" Sicca		53
Reis (1200). Brazil		99
Rouble. Russia		79
5 roubles. Russia	3	95
Schilling. Hamburg		2
Shilling. Great Britain		23
Skilling. Denmark		¾
Scudo. Piedmont	1	36
" Naples, Sicily		95
" Sardinia		92
" Rome	1	
" Genoa	1	28
Souversin. Austria, Bohemia	3	57
Sovereign. Gr Britain, or pound	4	86
Sous. France		1
Star Pagoda. Madras	1	81
Stiver. Holland		2
Teston. Rome		30
Testoon. Portugal		12
Zecchin. Turkey	1	40
Zecchino. Rome	2	27

Distance = 1.

Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
49	88°0′	4°0′	.9976	.0698	86°0′
55	58	2	.9975	.0703	58
61	56	4	.9975	.0709	56
66	54	6	.9974	.0715	54
72	52	8	.9974	.0721	52
78	50	10	.9974	.0727	50
84	48	12	.9973	.0732	48
90	46	14	.9973	.0738	46
96	44	16	.9972	.0744	44
01	42	18	.9972	.0750	42
07	40	20	.9971	.0756	40
13	38	22	.9971	.0761	38
19	36	24	.9971	.0767	36
25	34	26	.9970	.0773	34
30	32	28	.9970	.0779	32
36	30	30	.9969	.0785	30
42	28	32	.9969	.0790	28
48	26	34	.9968	.0796	26
54	24	36	.9968	.0802	24
59	22	38	.9967	.0808	22
65	20	40	.9967	.0814	20
71	18	42	.9966	.0819	18
77	16	44	.9966	.0825	16
83	14	46	.9965	.0831	14
88	12	48	.9965	.0837	12
94	10	50	.9964	.0843	10
00	8	52	.9964	.0848	8
06	6	54	.9963	.0854	6
12	4	56	.9963	.0860	4
18	2	58	.9962	.0866	2
23	87°0′	5°0′	.9962	.0872	85°0′
29	58	2	.9961	.0877	58
35	56	4	.9961	.0883	56
41	54	6	.9960	.0889	54
47	52	8	.9960	.0895	52
52	50	10	.9959	.0901	50
58	48	12	.9959	.0906	48
64	46	14	.9958	.0912	46
70	44	16	.9958	.0918	44
76	42	18	.9957	.0924	42
81	40	20	.9957	.0929	40
87	38	22	.9956	.0935	38
93	36	24	.9956	.0941	36
99	34	26	.9955	.0947	34
05	32	28	.9955	.0953	32
10	30	30	.9954	.0958	30
16	28	32	.9953	.0964	28
22	26	34	.9953	.0970	26
28	24	36	.9952	.0976	24
34	22	38	.9952	.0982	22
39	20	40	.9951	.0987	20
45	18	42	.9951	.0993	18
51	16	44	.9950	.0999	16
57	14	46	.9949	.1005	14
63	12	48	.9949	.1011	12
69	10	50	.9948	.1016	10
74	8	52	.9948	.1022	8
80	6	54	.9947	.1028	6
86	4	56	.9946	.1034	4
92	2	58	.9946	.1039	2
98	86°0′	6°0′	.9945	.1045	84°0′
			Dep. or E. W.	Lat. or N. S.	

Traverse Table for a D

Dep. or E. W.			Lat. or N. S.	
.1045	84°0′	8°0′	.9903	.139
.1051	58	2	.9902	.139
.1057	56	4	.9901	.1403
.1063	54	6	.9900	.1409
.1068	52	8	.9899	.1415
.1074	50	10	.9899	.1421
.1080	48	12	.9898	.1426
.1086	46	14	.9897	.1432
.1092	44	16	.9896	.1438
.1097	42	18	.9895	.1444
.1103	40	20	.9894	.1449
.1109	38	22	.9893	.1455
.1115	36	24	.9892	.1461
.1120	34	26	.9891	.1467
.1126	32	28	.9890	.1472
.1132	30	30	.9889	.1478
.1138	28	32	.9888	.1484
.1144	26	34	.9888	.1490
.1149	24	36	.9887	.1495
.1155	22	38	.9886	.1501
.1161	20	40	.9885	.1507
.1167	18	42	.9884	.1513
.1172	16	44	.9883	.1518
.1178	14	46	.9882	.1524
.1184	12	48	.9881	.1530
.1190	10	50	.9880	.1536
.1196	8	52	.9880	.1541
.1201	6	54	.9879	.1547
.1207	4	56	.9878	.1553
.1213	2	58	.9877	.1559
.1219	83°0′	9°0′	.9876	.1564
.1224	58	2	.9875	.1570
.1230	56	4	.9874	.1576
.1236	54	6	.9873	.1582
.1242	52	8	.9872	.1587
.1248	50	10	.9871	.159
.1253	48	12	.9870	.159
.1259	46	14	.9869	.160
.1265	44	16	.9869	.16
.1271	42	18	.9868	.1
.1276	40	20	.9867	
.1282	38	22	.9866	
.1288	36	24	.9865	
.1294	34	26	.9864	
.1299	32	28	.9863	
.1305	30	30	.9862	
.1311	28	32	.9861	
.1317	26	34	.9860	
.1322	24	36	.985	
.1328	22	38	.98	
.1334	20	40	.98	
.1340	18	42		
.1346	16	44		
.1351	14	46		
.1357	12	48		
.1363	10	50		
.1369	8	52		
.1374	6	54		
.1380	4	56		
.1386	2	58		
.1392	89°0′	10°0′		
			Lat. or N. S.	

Traverse Table for a Distance = 1. (CONTINUED.)

	Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
0	.9945	.1045	84°0′	8°0′	.9903	.1392	82°0′	10°0′	.9848	.1736	80°0′
1	.9945	.1051	58	2	.9902	.1397	58	2	.9847	.1742	58
4	.9944	.1057	56	4	.9901	.1403	56	4	.9846	.1748	56
6	.9943	.1063	54	6	.9900	.1409	54	6	.9845	.1754	54
8	.9943	.1068	52	8	.9899	.1415	52	8	.9844	.1759	52
10	.9942	.1074	50	10	.9899	.1421	50	10	.9843	.1765	50
2	.9942	.1080	48	12	.9898	.1426	48	12	.9842	.1771	48
4	.9941	.1086	46	14	.9897	.1432	46	14	.9841	.1777	46
6	.9940	.1092	44	16	.9896	.1438	44	16	.9840	.1782	44
8	.9940	.1097	42	18	.9895	.1444	42	18	.9839	.1788	42
10	.9939	.1103	40	20	.9894	.1449	40	20	.9838	.1794	40
2	.9938	.1109	38	22	.9894	.1455	38	22	.9837	.1799	38
4	.9938	.1115	36	24	.9893	.1461	36	24	.9836	.1805	36
6	.9937	.1120	34	26	.9892	.1467	34	26	.9835	.1811	34
8	.9936	.1126	32	28	.9891	.1472	32	28	.9834	.1817	32
10	.9936	.1132	30	30	.9890	.1478	30	30	.9833	.1822	30
2	.9935	.1138	28	32	.9889	.1484	28	32	.9831	.1828	28
4	.9934	.1144	26	34	.9888	.1490	26	34	.9830	.1834	26
6	.9934	.1149	24	36	.9888	.1495	24	36	.9829	.1840	24
8	.9933	.1155	22	38	.9887	.1501	22	38	.9828	.1845	22
10	.9932	.1161	20	40	.9886	.1507	20	40	.9827	.1851	20
2	.9932	.1167	18	42	.9885	.1513	18	42	.9826	.1857	18
4	.9931	.1172	16	44	.9884	.1518	16	44	.9825	.1862	16
6	.9930	.1178	14	46	.9883	.1524	14	46	.9824	.1868	14
8	.9930	.1184	12	48	.9882	.1530	12	48	.9823	.1874	12
10	.9929	.1190	10	50	.9881	.1536	10	50	.9822	.1880	10
2	.9928	.1196	8	52	.9880	.1541	8	52	.9821	.1885	8
4	.9928	.1201	6	54	.9880	.1547	6	54	.9820	.1891	6
6	.9927	.1207	4	56	.9879	.1553	4	56	.9818	.1897	4
8	.9926	.1213	2	58	.9878	.1559	2	58	.9817	.1902	2
9	.9925	.1219	83°0′	9°0	.9877	.1564	81°0′	11°0′	.9816	.1908	79°0′
1	.9925	.1224	58	2	.9876	.1570	58	2	.9815	.1914	58
4	.9924	.1230	56	4	.9875	.1576	56	4	.9814	.1920	56
6	.9923	.1236	54	6	.9874	.1582	54	6	.9813	.1925	54
8	.9923	.1242	52	8	.9873	.1587	52	8	.9812	.1931	52
10	.9922	.1248	50	10	.9872	.1593	50	10	.9811	.1937	50
2	.9921	.1253	48	12	.9871	.1599	48	12	.9810	.1942	48
4	.9920	.1259	46	14	.9870	.1605	46	14	.9808	.1948	46
6	.9920	.1265	44	16	.9869	.1610	44	16	.9807	1954	44
8	.9919	.1271	42	18	.9869	.1616	42	18	.9806	.1959	42
10	.9918	.1276	40	20	.9868	.1622	40	20	.9805	.1965	40
2	.9917	.1282	38	22	.9867	.1628	38	22	.9804	.1971	38
4	.9917	.1288	36	24	.9866	.1633	36	24	.9803	.1977	36
6	.9916	.1294	34	26	.9865	.1639	34	26	.9802	.1982	34
8	.9915	.1299	32	28	.9864	.1645	32	28	.9800	.1988	32
10	.9914	.1305	30	30	.9863	.1650	30	30	.9799	.1994	30
2	.9914	.1311	28	32	.9862	.1656	28	32	.9798	.1999	28
4	.9913	.1317	26	34	.9861	.1662	26	34	.9797	.2005	26
6	.9912	.1323	24	36	.9860	.1668	24	36	.9796	.2011	24
8	.9911	.1328	22	38	.9859	.1673	22	38	.9795	.2016	22
10	.9911	.1334	20	40	.9858	.1679	20	40	.9793	.2022	20
2	.9910	.1340	18	42	.9857	.1685	18	42	.9792	.2028	18
4	.9909	.1346	16	44	.9856	.1691	16	44	.9791	.2034	16
6	.9908	.1351	14	46	.9855	.1696	14	46	.9790	.2039	14
8	.9907	.1357	12	48	.9854	.1702	12	48	.9789	.2045	12
10	.9907	.1363	10	50	.9853	.1708	10	50	.9787	.2051	10
2	.9906	.1369	8	52	.9852	.1714	8	52	.9786	.2056	8
4	.9905	.1374	6	54	.9851	.1719	6	54	.9785	.2062	6
6	.9904	.1380	4	56	.9850	.1725	4	56	.9784	.2068	4
8	.9903	.1386	2	58	.9849	.1731	2	58	.9783	.2073	2
10	.9903	.1392	82°0′	10°0′	.9848	.1736	80°0′	12°0′	.9781	.2079	78°0′
	Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.	

Traverse Table for a Distance = 1. (Continued.)

Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
.9781	.2079	78°0′	14°0′	.9703	.2419	76°0′	16°0′	.9613	.2756	74°0′
.9780	.2084	58	2	.9702	.2425	58	2	.9611	.2762	58
.9779	.2090	56	4	.9700	.2431	56	4	.9609	.2768	56
.9778	.2096	54	6	.9699	.2436	54	6	.9608	.2773	54
.9777	.2102	52	8	.9697	.2442	52	8	.9606	.2779	52
.9775	.2108	50	10	.9696	.2447	50	10	.9605	.2784	50
.9774	.2113	48	12	.9694	.2453	48	12	.9603	.2790	48
.9773	.2119	46	14	.9693	.2459	46	14	.9601	.2795	46
.9772	.2125	44	16	.9692	.2464	44	16	.9600	.2801	44
.9770	.2130	42	18	.9690	.2470	42	18	.9598	.2807	42
.9769	.2136	40	20	.9689	.2476	40	20	.9596	.2812	40
.9768	.2142	38	22	.9687	.2481	38	22	.9595	.2818	38
.9767	.2147	36	24	.9686	.2487	36	24	.9593	.2823	36
.9765	.2153	34	26	.9684	.2493	34	26	.9591	.2829	34
.9764	.2159	32	28	.9683	.2498	32	28	.9590	.2835	32
.9763	.2164	30	30	.9681	.2504	30	30	.9588	.2840	30
.9762	.2170	28	32	.9680	.2509	28	32	.9587	.2846	28
.9760	.2176	26	34	.9679	.2515	26	34	.9585	.2851	26
.9759	.2181	24	36	.9677	.2521	24	36	.9583	.2857	24
.9758	.2187	22	38	.9676	.2526	22	38	.9582	.2862	22
.9757	.2193	20	40	.9674	.2532	20	40	.9580	.2868	20
.9755	.2198	18	42	.9673	.2538	18	42	.9578	.2874	18
.9754	.2204	16	44	.9671	.2543	16	44	.9577	.2879	16
.9753	.2210	14	46	.9670	.2549	14	46	.9575	.2885	14
.9751	.2215	12	48	.9668	.2554	12	48	.9573	.2890	12
.9750	.2221	10	50	.9667	.2560	10	50	.9572	.2896	10
.9749	.2227	8	52	.9665	.2566	8	52	.9570	.2901	8
.9748	.2232	6	54	.9664	.2571	6	54	.9568	.2907	6
.9746	.2238	4	56	.9662	.2577	4	56	.9566	.2913	4
.9745	.2244	2	58	.9661	.2583	2	58	.9565	.2918	2
.9744	.2250	77°0′	15°0′	.9659	.2588	75°0′	17°0′	.9563	.2924	73°0′
.9742	.2255	58	2	.9658	.2594	58	2	.9561	.2929	58
.9741	.2261	56	4	.9656	.2599	56	4	.9560	.2935	56
.9740	.2267	54	6	.9655	.2605	54	6	.9558	.2940	54
.9738	.2272	52	8	.9653	.2611	52	8	.9556	.2946	52
.9737	.2278	50	10	.9652	.2616	50	10	.9555	.2952	50
.9736	.2284	48	12	.9650	.2622	48	12	.9553	.2957	48
.9734	.2289	46	14	.9649	.2628	46	14	.9551	.2963	46
.9733	.2295	44	16	.9647	.2633	44	16	.9549	.2968	44
.9732	.2300	42	18	.9646	.2639	42	18	.9548	.2974	42
.9730	.2306	40	20	.9644	.2644	40	20	.9546	.2979	40
.9729	.2312	38	22	.9642	.2650	38	22	.9544	.2985	38
.9728	.2317	36	24	.9641	.2656	36	24	.9542	.2990	36
.9726	.2323	34	26	.9639	.2661	34	26	.9541	.2996	34
.9725	.2329	32	28	.9638	.2667	32	28	.9539	.3002	32
.9724	.2334	30	30	.9636	.2672	30	30	.9537	.3007	30
.9722	.2340	28	32	.9635	.2678	28	32	.9535	.3013	28
.9721	.2346	26	34	.9633	.2684	26	34	.9534	.3018	26
.9720	.2351	24	36	.9632	.2689	24	36	.9532	.3024	24
.9718	.2357	22	38	.9630	.2695	22	38	.9530	.3029	22
.9717	.2363	20	40	.9628	.2700	20	40	.9528	.3035	20
.9715	.2368	18	42	.9627	.2706	18	42	.9527	.3040	18
.9714	.2374	16	44	.9625	.2712	16	44	.9525	.3046	16
.9713	.2380	14	46	.9624	.2717	14	46	.9523	.3051	14
.9711	.2385	12	48	.9622	.2723	12	48	.9521	.3057	12
.9710	.2391	10	50	.9621	.2728	10	50	.9520	.3062	10
.9709	.2397	8	52	.9619	.2734	8	52	.9518	.3068	8
.9707	.2402	6	54	.9617	.2740	6	54	.9516	.3074	6
.9706	.2408	4	56	.9616	.2745	4	56	.9514	.3079	4
.9704	.2414	2	58	.9614	.2751	2	58	.9512	.3085	2
.9703	.2419	76°0′	16°0′	.9613	.2756	74°0′	18°0′	.9511	.3090	72°0′
Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.	

Traverse Table for a Distance = 1. (Con

	Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.		
18°0′	.9511	.3090	72°0′	20°0′	.9397	.3420	70°0′	22°0′
2	.9509	.3096	58	2	.9395	.3426	58	2
4	.9507	.3101	56	4	.9393	.3431	56	4
6	.9505	.3107	54	6	.9391	.3437	54	6
8	.9503	.3112	52	8	.9389	.3442	52	8
10	.9502	.3118	50	10	.9387	.3448	50	10
12	.9500	.3123	48	12	.9385	.3453	48	12
14	.9498	.3129	46	14	.9383	.3458	46	14
16	.9496	.3134	44	16	.9381	.3464	44	16
18	.9494	.3140	42	18	.9379	.3469	42	18
20	.9492	.3145	40	20	.9377	.3475	40	20
22	.9491	.3151	38	22	.9375	.3480	38	22
24	.9489	.3156	36	24	.9373	.3486	36	24
26	.9487	.3162	34	26	.9371	.3491	34	26
28	.9485	.3168	32	28	.9369	.3497	32	28
30	.9483	.3173	30	30	.9367	.3502	30	30
32	.9481	.3179	28	32	.9365	.3508	28	32
34	.9480	.3184	26	34	.9363	.3513	26	34
36	.9478	.3190	24	36	.9361	.3518	24	36
38	.9476	.3195	22	38	.9359	.3524	22	38
40	.9474	.3201	20	40	.9356	.3529	20	40
42	.9472	.3206	18	42	.9354	.3535	18	42
44	.9470	.3212	16	44	.9352	.3540	16	44
46	.9468	.3217	14	46	.9350	.3546	14	46
48	.9466	.3223	12	48	.9348	.3551	12	48
50	.9465	.3228	10	50	.9346	.3557	10	50
52	.9463	.3234	8	52	.9344	.3562	8	52
54	.9461	.3239	6	54	.9342	.3567	6	54
56	.9459	.3245	4	56	.9340	.3573	4	56
58	.9457	.3250	2	58	.9338	.3578	2	58
19°0′	.9455	.3256	71°0′	21°0′	.9336	.3584	69°0′	23°0′
2	.9453	.3261	58	2	.9334	.3589	58	2
4	.9451	.3267	56	4	.9332	.3595	56	4
6	.9449	.3272	54	6	.9330	.3600	54	6
8	.9448	.3278	52	8	.9327	.3605	52	8
10	.9446	.3283	50	10	.9325	.3611	50	10
12	.9444	.3289	48	12	.9323	.3616	48	12
14	.9442	.3294	46	14	.9321	.3622	46	14
16	.9440	.3300	44	16	.9319	.3627	44	16
18	.9438	.3305	42	18	.9317	.3633	42	18
20	.9436	.3311	40	20	.9315	.3638	40	20
22	.9434	.3316	38	22	.9313	.3643	38	22
24	.9432	.3322	36	24	.9311	.3649	36	24
26	.9430	.3327	34	26	.9308	.3654	34	26
28	.9428	.3333	32	28	.9306	.3660	32	28
30	.9426	.3338	30	30	.9304	.3665	30	30
32	.9424	.3344	28	32	.9302	.3670	28	32
34	.9423	.3349	26	34	.9300	.3676	26	34
36	.9421	.3355	24	36	.9298	.3681	24	36
38	.9419	.3360	22	38	.9296	.3687	22	38
40	.9417	.3365	20	40	.9293	.3692	20	40
42	.9415	.3371	18	42	.9291	.3697	18	42
44	.9413	.3376	16	44	.9289	.3703	16	44
46	.9411	.3382	14	46	.9287	.3708	14	46
48	.9409	.3387	12	48	.9285	.3714	12	48
50	.9407	.3393	10	50	.9283	.3719	10	50
52	.9405	.3398	8	52	.9281	.3724	8	52
54	.9403	.3404	6	54	.9278	.3730	6	54
56	.9401	.3409	4	56	.9276	.3735	4	56
58	.9399	.3415	2	58	.9274	.3741	2	58
20°0′	.9397	.3420	70°0′	22°0′	.9272	.3746	68°0′	24°0′
	Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.		

Traverse Table for a Distance = 1. (Continued.)

	Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
24°0′	.9135	.4067	66°0′	26°0′	.8988	.4384	64°0′	28°0′	.8829	.4695	62°0′
2	.9133	.4073	58	2	.8985	.4389	58	2	.8827	.4700	58
4	.9131	.4078	56	4	.8983	.4394	56	4	.8824	.4705	56
6	.9128	.4083	54	6	.8980	.4399	54	6	.8821	.4710	54
8	.9126	.4089	52	8	.8978	.4405	52	8	.8819	.4715	52
10	.9124	.4094	50	10	.8975	.4410	50	10	.8816	.4720	50
12	.9121	.4099	48	12	.8973	.4415	48	12	.8813	.4726	48
14	.9119	.4105	46	14	.8970	.4420	46	14	.8810	.4731	46
16	.9116	.4110	44	16	.8967	.4425	44	16	.8808	.4736	44
18	.9114	.4115	42	18	.8965	.4431	42	18	.8805	.4741	42
20	.9112	.4120	40	20	.8962	.4436	40	20	.8802	.4746	40
22	.9109	.4126	38	22	.8960	.4441	38	22	.8799	.4751	38
24	.9107	.4131	36	24	.8957	.4446	36	24	.8796	.4756	36
26	.9104	.4136	34	26	.8955	.4452	34	26	.8794	.4761	34
28	.9102	.4142	32	28	.8952	.4457	32	28	.8791	.4766	32
30	.9100	.4147	30	30	.8949	.4462	30	30	.8788	.4772	30
32	.9097	.4152	28	32	.8947	.4467	28	32	.8785	.4777	28
34	.9095	.4158	26	34	.8944	.4472	26	34	.8783	.4782	26
36	.9092	.4163	24	36	.8942	.4478	24	36	.8780	.4787	24
38	.9090	.4168	22	38	.8939	.4483	22	38	.8777	.4792	22
40	.9088	.4173	20	40	.8936	.4488	20	40	.8774	.4797	20
42	.9085	.4179	18	42	.8934	.4493	18	42	.8771	.4802	18
44	.9083	.4184	16	44	.8931	.4498	16	44	.8769	.4807	16
46	.9080	.4189	14	46	.8928	.4504	14	46	.8766	.4812	14
48	.9078	.4195	12	48	.8926	.4509	12	48	.8763	.4818	12
50	.9075	.4200	10	50	.8923	.4514	10	50	.8760	.4823	10
52	.9073	.4205	8	52	.8921	.4519	8	52	.8757	.4828	8
54	.9070	.4210	6	54	.8918	.4524	6	54	.8755	.4833	6
56	.9068	.4216	4	56	.8915	.4530	4	56	.8752	.4838	4
58	.9066	.4221	2	58	.8913	.4535	2	58	.8749	.4843	2
25°0′	.9063	.4226	65°0′	27°0′	.8910	.4540	63°0′	29°0′	.8746	.4848	61°0′
2	.9061	.4231	58	2	.8907	.4545	58	2	.8743	.4853	58
4	.9058	.4237	56	4	.8905	.4550	56	4	.8741	.4858	56
6	.9056	.4242	54	6	.8902	.4555	54	6	.8738	.4863	54
8	.9053	.4247	52	8	.8899	.4561	52	8	.8735	.4868	52
10	.9051	.4253	50	10	.8897	.4566	50	10	.8732	.4874	50
12	.9048	.4258	48	12	.8894	.4571	48	12	.8729	.4879	48
14	.9046	.4263	46	14	.8892	.4576	46	14	.8726	.4884	46
16	.9043	.4268	44	16	.8889	.4581	44	16	.8724	.4889	44
18	.9041	.4274	42	18	.8886	.4586	42	18	.8721	.4894	42
20	.9038	.4279	40	20	.8884	.4592	40	20	.8718	.4899	40
22	.9036	.4284	38	22	.8881	.4597	38	22	.8715	.4904	38
24	.9033	.4289	36	24	.8878	.4602	36	24	.8712	.4909	36
26	.9031	.4295	34	26	.8875	.4607	34	26	.8709	.4914	34
28	.9028	.4300	32	28	.8873	.4612	32	28	.8706	.4919	32
30	.9026	.4305	30	30	.8870	.4617	30	30	.8704	.4924	30
32	.9023	.4310	28	32	.8867	.4623	28	32	.8701	.4929	28
34	.9021	.4316	26	34	.8865	.4628	26	34	.8698	.4934	26
36	.9018	.4321	24	36	.8862	.4633	24	36	.8695	.4939	24
38	.9016	.4326	22	38	.8859	.4638	22	38	.8692	.4944	22
40	.9013	.4331	20	40	.8857	.4643	20	40	.8689	.4950	20
42	.9011	.4337	18	42	.8854	.4648	18	42	.8686	.4955	18
44	.9008	.4342	16	44	.8851	.4654	16	44	.8683	.4960	16
46	.9006	.4347	14	46	.8849	.4659	14	46	.8681	.4965	14
48	.9003	.4352	12	48	.8846	.4664	12	48	.8678	.4970	12
50	.9001	.4358	10	50	.8843	.4669	10	50	.8675	.4975	10
52	.8998	.4363	8	52	.8840	.4674	8	52	.8672	.4980	8
54	.8996	.4368	6	54	.8838	.4679	6	54	.8669	.4985	6
56	.8993	.4373	4	56	.8835	.4684	4	56	.8666	.4990	4
58	.8990	.4378	2	58	.8832	.4690	2	58	.8663	.4995	2
26°0′	.8988	.4384	64°0′	28°0′	.8829	.4695	62°0′	30°0′	.8660	.5000	60°0′
	Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.	

Traverse Table for a Distance = 1. (Continued.)

	Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
30°0′	.8660	.5000	60°0′	32°0′	.8480	.5299	58°0′	34°0′	.8290	.5592	56°0′
2	.8657	.5005	58	2	.8477	.5304	58	2	.8287	.5597	58
4	.8654	.5010	56	4	.8474	.5309	56	4	.8284	.5602	56
6	.8652	.5015	54	6	.8471	.5314	54	6	.8281	.5606	54
8	.8649	.5020	52	8	.8468	.5319	52	8	.8277	.5611	52
10	.8646	.5025	50	10	.8465	.5324	50	10	.8274	.5616	50
12	.8643	.5030	48	12	.8462	.5329	48	12	.8271	.5621	48
14	.8640	.5035	46	14	.8459	.5334	46	14	.8268	.5626	46
16	.8637	.5040	44	16	.8456	.5339	44	16	.8264	.5630	44
18	.8634	.5045	42	18	.8453	.5344	42	18	.8261	.5635	42
20	.8631	.5050	40	20	.8450	.5348	40	20	.8258	.5640	40
22	.8628	.5055	38	22	.8446	.5353	38	22	.8254	.5645	38
24	.8625	.5060	36	24	.8443	.5358	36	24	.8251	.5650	36
26	.8622	.5065	34	26	.8440	.5363	34	26	.8248	.5654	34
28	.8619	.5070	32	28	.8437	.5368	32	28	.8245	.5659	32
30	.8616	.5075	30	30	.8434	.5373	30	30	.8241	.5664	30
32	.8613	.5080	28	32	.8431	.5378	28	32	.8238	.5669	28
34	.8610	.5085	26	34	.8428	.5383	26	34	.8235	.5674	26
36	.8607	.5090	24	36	.8425	.5388	24	36	.8231	.5678	24
38	.8604	.5095	22	38	.8421	.5393	22	38	.8228	.5683	22
40	.8601	.5100	20	40	.8418	.5398	20	40	.8225	.5688	20
42	.8599	.5105	18	42	.8415	.5402	18	42	.8221	.5693	18
44	.8596	.5110	16	44	.8412	.5407	16	44	.8218	.5698	16
46	.8593	.5115	14	46	.8409	.5412	14	46	.8215	.5702	14
48	.8590	.5120	12	48	.8406	.5417	12	48	.8211	.5707	12
50	.8587	.5125	10	50	.8403	.5422	10	50	.8208	.5712	10
52	.8584	.5130	8	52	.8399	.5427	8	52	.8205	.5717	8
54	.8581	.5135	6	54	.8396	.5432	6	54	.8202	.5721	6
56	.8578	.5140	4	56	.8393	.5437	4	56	.8198	.5726	4
58	.8575	.5145	2	58	.8390	.5442	2	58	.8195	.5731	2
31°0′	.8572	.5150	59°0′	33°0′	.8387	.5446	57°0′	35°0′	.8192	.5736	55°0′
2	.8569	.5155	58	2	.8384	.5451	58	2	.8188	.5741	58
4	.8566	.5160	56	4	.8380	.5456	56	4	.8185	.5745	56
6	.8563	.5165	54	6	.8377	.5461	54	6	.8181	.5750	54
8	.8560	.5170	52	8	.8374	.5466	52	8	.8178	.5755	52
10	.8557	.5175	50	10	.8371	.5471	50	10	.8175	.5760	50
12	.8554	.5180	48	12	.8368	.5476	48	12	.8171	.5764	48
14	.8551	.5185	46	14	.8364	.5480	46	14	.8168	.5769	46
16	.8548	.5190	44	16	.8361	.5485	44	16	.8165	.5774	44
18	.8545	.5195	42	18	.8358	.5490	42	18	.8161	.5779	42
20	.8542	.5200	40	20	.8355	.5495	40	20	.8158	.5783	40
22	.8539	.5205	38	22	.8352	.5500	38	22	.8155	.5788	38
24	.8536	.5210	36	24	.8348	.5505	36	24	.8151	.5793	36
26	.8532	.5215	34	26	.8345	.5510	34	26	.8148	.5798	34
28	.8529	.5220	32	28	.8342	.5515	32	28	.8145	.5802	32
30	.8526	.5225	30	30	.8339	.5519	30	30	.8141	.5807	30
32	.8523	.5230	28	32	.8336	.5524	28	32	.8138	.5812	28
34	.8520	.5235	26	34	.8332	.5529	26	34	.8134	.5816	26
36	.8517	.5240	24	36	.8329	.5534	24	36	.8131	.5821	24
38	.8514	.5245	22	38	.8326	.5539	22	38	.8128	.5826	22
40	.8511	.5250	20	40	.8323	.5544	20	40	.8124	.5831	20
42	.8508	.5255	18	42	.8320	.5548	18	42	.8121	.5835	18
44	.8505	.5260	16	44	.8316	.5553	16	44	.8117	.5840	16
46	.8502	.5265	14	46	.8313	.5558	14	46	.8114	.5845	14
48	.8499	.5270	12	48	.8310	.5563	12	48	.8111	.5850	12
50	.8496	.5275	10	50	.8307	.5568	10	50	.8107	.5854	10
52	.8493	.5279	8	52	.8303	.5573	8	52	.8104	.5859	8
54	.8490	.5284	6	54	.8300	.5577	6	54	.8100	.5864	6
56	.8487	.5289	4	56	.8297	.5582	4	56	.8097	.5868	4
58	.8484	.5294	2	58	.8294	.5587	2	58	.8094	.5873	2
32°0′	.8480	.5299	58°0′	34°0′	.8290	.5592	56°0′	36°0′	.8090	.5878	54°0′
	Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.	

Traverse Table for a Distance = 1. (Continued.)

	Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	[illegible]	[illegible]	[illegible]	1	[illegible]	[illegible]	[illegible]	1	[illegible]	[illegible]
2	[illegible]	[illegible]	[illegible]	2	[illegible]	[illegible]	[illegible]	2	[illegible]	[illegible]
3	[illegible]	[illegible]	[illegible]	3	[illegible]	[illegible]	[illegible]	3	[illegible]	[illegible]
4	[illegible]	[illegible]	[illegible]	4	[illegible]	[illegible]	[illegible]	4	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	5	[illegible]	[illegible]	[illegible]	5	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	6	[illegible]	[illegible]	[illegible]	6	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	7	[illegible]	[illegible]	[illegible]	7	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]	8	[illegible]	[illegible]	[illegible]	8	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]	9	[illegible]	[illegible]	[illegible]	9	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]	[illegible]	10	[illegible]	[illegible]
11	[illegible]	[illegible]	[illegible]	11	[illegible]	[illegible]	[illegible]	11	[illegible]	[illegible]
12	[illegible]	[illegible]	[illegible]	12	[illegible]	[illegible]	[illegible]	12	[illegible]	[illegible]
13	[illegible]	[illegible]	[illegible]	13	[illegible]	[illegible]	[illegible]	13	[illegible]	[illegible]
14	[illegible]	[illegible]	[illegible]	14	[illegible]	[illegible]	[illegible]	14	[illegible]	[illegible]
15	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]	[illegible]	15	[illegible]	[illegible]
16	[illegible]	[illegible]	[illegible]	16	[illegible]	[illegible]	[illegible]	16	[illegible]	[illegible]
17	[illegible]	[illegible]	[illegible]	17	[illegible]	[illegible]	[illegible]	17	[illegible]	[illegible]
18	[illegible]	[illegible]	[illegible]	18	[illegible]	[illegible]	[illegible]	18	[illegible]	[illegible]
19	[illegible]	[illegible]	[illegible]	19	[illegible]	[illegible]	[illegible]	19	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	20	[illegible]	[illegible]	[illegible]	20	[illegible]	[illegible]
21	[illegible]	[illegible]	[illegible]	21	[illegible]	[illegible]	[illegible]	21	[illegible]	[illegible]
22	[illegible]	[illegible]	[illegible]	22	[illegible]	[illegible]	[illegible]	22	[illegible]	[illegible]
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60° 0′	.5000	.8660	30° 0′	60° 0′	.7500	.6428	50° 0′	40° 0′	.7431	.6691
	Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.

Traverse Table for a Distance—1. (CONCLUDED.)

Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.			Lat. or N. S.	Dep. or E. W.	
.7431	.6691	48°0'	43°0'	.7314	.6820	47°0'	44°0'	.7193	.6947	46°0'
.7428	.6696	58	2	.7310	.6824	58	2	.7189	.6951	58
.7424	.6700	56	4	.7306	.6828	56	4	.7185	.6955	56
.7420	.6704	54	6	.7302	.6833	54	6	.7181	.6959	54
.7416	.6709	52	8	.7298	.6837	52	8	.7177	.6963	52
.7412	.6713	50	10	.7294	.6841	50	10	.7173	.6967	50
.7408	.6717	48	12	.7290	.6845	48	12	.7169	.6972	48
.7404	.6722	46	14	.7286	.6850	46	14	.7165	.6976	46
.7400	.6726	44	16	.7282	.6854	44	16	.7161	.6980	44
.7396	.6730	42	18	.7278	.6858	42	18	.7157	.6984	42
.7392	.6734	40	20	.7274	.6862	40	20	.7153	.6988	40
.7388	.6739	38	22	.7270	.6867	38	22	.7149	.6992	38
.7385	.6743	36	24	.7266	.6871	36	24	.7145	.6997	36
.7381	.6747	34	26	.7262	.6875	34	26	.7141	.7001	34
.7377	.6752	32	28	.7258	.6879	32	28	.7137	.7005	32
.7373	.6756	30	30	.7254	.6884	30	30	.7133	.7009	30
.7369	.6760	28	32	.7250	.6888	28	32	.7128	.7013	28
.7365	.6764	26	34	.7246	.6892	26	34	.7124	.7017	26
.7361	.6769	24	36	.7242	.6896	24	36	.7120	.7021	24
.7357	.6773	22	38	.7238	.6900	22	38	.7116	.7026	22
.7353	.6777	20	40	.7234	.6905	20	40	.7112	.7030	20
.7349	.6782	18	42	.7230	.6909	18	42	.7108	.7034	18
.7345	.6786	16	44	.7226	.6913	16	44	.7104	.7038	16
.7341	.6790	14	46	.7222	.6917	14	46	.7100	.7042	14
.7337	.6794	12	48	.7218	.6921	12	48	.7096	.7046	12
.7333	.6799	10	50	.7214	.6926	10	50	.7092	.7050	10
.7329	.6803	8	52	.7210	.6930	8	52	.7088	.7055	8
.7325	.6807	6	54	.7206	.6934	6	54	.7083	.7059	6
.7321	.6811	4	56	.7201	.6938	4	56	.7079	.7063	4
.7318	.6816	2	58	.7197	.6942	2	58	.7075	.7067	2
.7314	.6820	47°0'	44°0'	.7193	.6947	46°0'	45°0'	.7071	.7071	45°0'
Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.			Dep. or E. W.	Lat. or N. S.	

the angle exceeds 45°, the lats and deps are read upward from the bottom.

Since these lats and deps are for a dist 1, we may proceed as follows for greater dists. t the dist be 856.1. Add together 800 times, 50 times, 6 times, and $\frac{1}{10}$ time the correspond- and deps of the table.

What is the lat and dep for 856.1 feet; the angle being 43°?

r 43° we have from the table, lat .7314; dep .6820.

Hence, .7314 × 800 = 585.12; and .6820 × 800 = 545.60
.7314 × 50 = 36.57; and .6820 × 50 = 34.10
.7314 × 6 = 4.39; and .6820 × 6 = 4.09
.7314 × .1 = .07; and .6820 × .1 = .07

Lat 626.15 Dep 583.86

multiplications may be made mentally. Or we may, with a little more trouble, mult the lat of the table by the given dist. Thus,

.7314 × 856.1 = 626.15 lat; and .6820 × 856.1 = 583.86 dep.*

* uch as the engineer but rarely needs a traverse table, we have thought it best to give a se, rather than the common one for ¼ degrees. The first involves more trouble in using it; at is entirely unfit for other than the rude calculations for common surveying with compass aken to the nearest ¼ degree.

LAND SURVEYING.

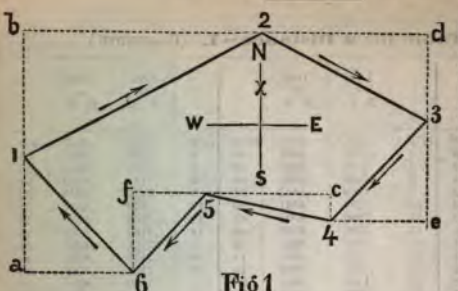

Fig 1

In surveying a tract of ground, the sides which compose its outline are designated by numbers in the order in which they occur. That end of each side which first presents itself in the course of the survey, may be called its *near* end; and the other its *far* end. The number of each side is placed at its far end. Thus, in Fig 1, the survey being supposed to commence at the corner 6, and to follow the direction of the arrows, the first side is 6, 1; and its number is placed at its far end at 1; and so of the rest. Let N S be a meridian line, that is, a north and south line; and E W an east and west line. Then in any side which runs northwardly, whether due north, or northeast, as side 2; or northwest, as sides 5 and 1, the dist in a due north direction between its near end and its far end, is called its *northing;* thus, *a* 1 is the northing of side 1; 1 *b* the northing of side 2; 4 c of side 5. In like manner, if any side runs in a southwardly direction, whether due south, or southeastwardly, as side 3; or southwestwardly, as sides 4 and 6, the corresponding dist in a due south direction between its near end and its far end, is called its *southing;* thus, *d* 3 is the southing of side 3; 3 *e* of side 4; *f* 6 of side 6. Both northings and southings are included in the general term *Difference of Latitude* of a side; or more commonly, but erroneously, its *latitude*. The dist due east, or due west, between the near and the far end of *any* side, is in like manner called the *easting*, or *westing* of that side, as the case may be; thus, 6 *a* is the westing of side 1; 5 *f* of side 6; c 5 of side 5; *e* 4 of side 4; and *b* 2 is the easting of side 2; 2 *d* of side 3. Both eastings and westings are included in the general term *Departure* of a side; implying that the side *departs* so far from a north or south direction. We may employ the directions (or courses, or bearings, as they are usually called) of sides, as verbs; and say that a side norths, wests, southeasts, &c. We shall call the northings, southings, &c, the Ns, Ss, Es, and Ws; the latitudes, lats; and the departures, deps. The preceding Traverse Table consists of the lats, (or Ns and Ss;) and the deps, (or Es and Ws,) corresponding to diff angles or courses, for a side whose length is 1; therefore, to obtain the actual lat and dep of any given side, those taken from the table must be mult by the length of the side. Beyond 44°, the lats and deps of this table must be read upward from the bottom of the page. The angles in the table are those which the course or bearing of any side would make with a meridian line drawn through either end of said side; but it is self-evident that what would be N from one end, would be S from the other; and so of E and W; in other words, the *angle* is the same at both ends; but the *direction* is reversed.

Perfect accuracy is unattainable in any operation involving the measurements of angles and dists. That work is accurate enough, which cannot be made more so without an expenditure more than commensurate with the object to be gained. The writer conceives that the accuracy essential to constitute practically fair surveying is purely a matter of dollars and cents. In the purchase and sale of tracts of land, such as farms, &c, an uncertainty of about 1 part in 200 respecting the content, and consequently respecting the price, probably never prevents a transfer; and on this principle we assume that a survey which proves itself within that limit, may *ordinarily* be regarded as accurate enough. There is no great difficulty in attaining this limit, which, if exceeded, is the result of bad work. Many circumstances combine to render trifling errors absolutely unavoidable;* they always become apparent when we come to *work out* the field notes; and since the map or plot of the survey, and the calculations for ascertaining the content, should be consistent within themselves, we do what is usually called *correcting* the errors, but what in fact is simply *humoring them in*, no matter how scientific the process may appear. We distribute them all around the survey. Two methods are used for this purpose, both based upon precisely the same principle; one of them mechanical, by means of drawing; the other more exact, but much more troublesome, by calculation. We shall describe both; but will state now, that by proportioning the scale of the plot in the following manner, the mechanical method becomes, in the hands of a correct draftsman, sufficiently exact for all ordinary purposes. Add all the sides in feet together; and div the sum by their number, for the average length. Div this average by 8; the quot will be the proper scale in feet per inch. In other words, take about 8 ins to represent an average side.† We shall take it for granted, that an engineer does not consider it accurate work to

* A 100 ft measuring-chain may vary its length 5 feet per mile, between winter and summer, by mere change of temperature; and by this alone we shall make a difference of about $1\frac{1}{5}$ acres in a lot 1 mile square, which contains 640 acres. Even this error amounts to 1 acre in 533. Not one farmer in a hundred would dream of paying for a scrupulously correct survey and plot of his property.

† It will seldom happen that precisely this quotient will be adopted for a scale. For instance, if the quot should be 738 feet, or 83 feet, per inch, we should adopt the most convenient near number, if smaller the better, as 700, or 80; or rather more than 8 inches to a side. With a scale so proportioned, and with good drawing instruments, the error in *protracting* (excluding of course errors of the field work) will rarely exceed about $\frac{1}{400}$ part of the periphery of the plot; and the area may be found mechanically by dividing the plot into triangles, within $\frac{1}{200}$ part of the truth. This remark applies *particularly to such plots as* may be protracted, and computed within a period so short as not *to allow the paper to contract* or expand appreciably by atmospheric changes. A larger scale *will insure proportionally* greater accuracy. The young assistant should practise plotting *from perfectly accurate data;* as, for instance, from the example given in the table, p. 97, or,

measure his angles to the nearest quarter of a degree, which is the usual practice among land-surveyors. They can, by means of the engineer's transit, now in universal use on our public works, be readily measured within a minute or two; and being thus much more accurate than the compass courses, (which cannot be read off so closely, and which are moreover subject to many sources of error,) they serve to correct the latter in the office. The noting of the courses, however, should not be confined to the nearest quarters of a degree, but should be read as closely as the observer can guess at the minutes. The back courses also should be taken at every corner, as an additional check, and for the detection of local attraction. It is well in taking the compass bearings, to adopt as a rule, always to point the north of the compass-box toward the object whose bearing is to be taken, and to read off from the *north* end of the needle. A person who uses indifferently the N and the S of the box, and of the needle, will be very liable to make mistakes. It is best to measure the least angle (shown by dotted arcs, Fig 2,) at the corners; whether it be exterior, as that at corner 5; or interior, as all the others; because it is always less than 180°; so that there is less danger of reading it off incorrectly, than if it exceeded 180°; taking it for granted that the transit instrument is graduated from the same zero to 180° each way; if it is graduated from zero to 360° the precaution is useless. When the small angle is exterior, subtract it from 360° for the interior one.

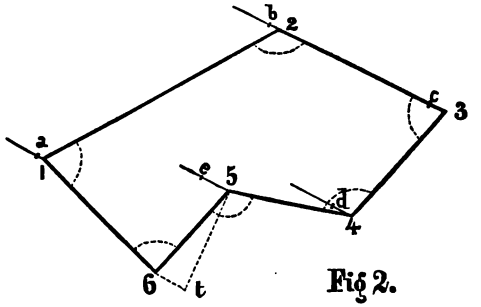

Fig 2.

Supposing the field work to be finished, and that we require a plot from which the contents may be obtained mechanically, by dividing it into triangles, (the bases and heights of which may be measured by scale, and their areas calculated one by one,) a protraction of it may be made at once from the field notes, either by using the *angles*, or by first correcting the *bearings* by means of the angles, and then using them. The last is the best, because in the first the protractor must be moved to each angle; whereas in the last it will remain stationary while all the bearings are being pricked off. Every movement of it increases the liability to errors. The manner of correcting the bearings is explained on the next page.

In either case the protracted plot will certainly not close precisely; not only in consequence of errors in the field work, but also in the protracting itself. Thus the last side, No 6, Fig 2, instead of closing in at corner 6, will end somewhere else, say, for instance, at *t*; the dist *t* 6 being the *closing error*, which, however, as represented in Fig 2, is more than ten times as great, proportionally to the size of the survey, as would be allowable in practice. Now to humor-in this error, rule through every corner a short line parallel to *t* 6; and, in all cases, in the direction *from t* (wherever it may be) *to* the starting point 6. Add all the sides together; and measure *t* 6 by the scale of the plot. Then beginning at corner 1, at the far end of side 1, say, as the

Sum of all the sides	:	Total closing error *t* 6	: :	Side 1	:	Error for side 1.

Lay off this error from 1 to *a*. Then at corner 2, say, as the

Sum of all the sides	:	Total closing error *t* 6	: :	Sum of sides 1 and 2	:	Error for side 2.

Which error lay off from 2 to *b*; and so at each of the corners; always using, as the third term, the sum of the sides between the starting point and the given corner. Finally, join the points *a*, *b*, *c*, *d*, *e*; and the plot is finished.

The correction has evidently changed the length of every side; lengthening some and shortening others. It has also changed the angles. The new lengths and angles may with tolerable accuracy be found by means of the scale and protractor; and be marked on the plot instead of the old ones.

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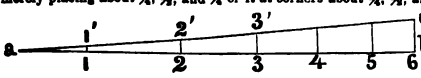
from those to be found in books on surveying. This is the only way in which he can learn what is meant by accurate work. His semicircular protractor should be about 9 to 12 ins in diam and graduated to 10 min. His straight edge and triangle should be of metal; we prefer German silver, which does not rust as steel does; and they should be made with *scrupulous accuracy* by a skilful instrument-maker. A very fine needle, with a sealing-wax head, should be used for pricking off dists and angles; it must be held vertically; and the eye of the draftsman must be directly over it. The lead pencil should be hard (Faber's No. 4 is good for protracting), and must be kept to a sharp point by rubbing on a fine file, after using a knife for removing the wood. The scale should be at least as long as the longest side of the plot, and should be made at the edge of a strip of the same paper as the plot is drawn on. This will obviate to a considerable extent, errors arising from contraction and expansion. Unfortunately, a sheet of paper does not contract and expand in the same proportion lengthwise and crosswise, thus preventing the paper scale from being a perfect corrective. In plots of common farm surveys, &c, however, the errors from this source may be neglected. For such plots as may be protracted, divided, and computed within a time too short to admit of appreciable change, the ordinary scales of wood, ivory, or metal may be used; but satisfactory accuracy cannot be obtained with them on plots requiring *several days*, if the air be meanwhile alternately moist and dry, or subject to *considerable variations in temperature*. What is called *parchment* paper is worse in this respect than *good ordinary drawing-paper*.

With the foregoing precautions we may work from a drawing, with as much accuracy as is obtained in the field work.

When the plot has many sides, this calculating the error for each of them beoc since, in a well-performed survey and protraction, the entire error will be but a ve (it should not *exceed* about $\frac{1}{400}$ part of the periphery,) it may usually be divided a merely placing about ¼, ½, and ¾ of it at corners about ¼, ½, and ¾ way aroun



intermediate tion it by ey may be ave drawing a li equal to th of all the s

into distances *a*, 1; 1, 2: &c, equal to the respective sides. Make *b c* equal to the en join *a c*; and draw 1, 1'; 2, 2', &c, which will give the error at each corner.

When the plot is thus completed, it may be divided by fine pencil lines into bases and heights may be measured by the scale, in order to compute the conten both the survey and the drawing, the error should not *exceed* about $\frac{1}{200}$ part of t least two distinct sets of triangles should be drawn and computed, as a guard agains the two sets differ in calculated contents more than about $\frac{1}{200}$ part, they have no prepared as they should have been. The closing error due to imperfect field-work, calculated, as we shall show, and laid down on the paper before beginning the plo a perfect test of the accuracy of the protraction work, which, if correctly done, w point of beginning, but at the point which indicates the error. But this *calculatio* a little additional trouble, furnishes data also for dividing it by calculation amo besides the means of drawing the plot *correctly* at once, without the use of a pro bling us to make the subsequent measurements and computations of the triangl tainty.

We shall now describe this process, but would recommend that even when it especially in complicated surveys, a rough plot should first be made and corrected two mechanical methods already alluded to. It will prove to be of great service in by calculation, inasmuch as it furnishes an eye check to vexatious mistakes which to occur; for, although the principles involved are extremely simple, and easily once understood, yet the continual changes in the directions of the sides will, w cause us to use Ns instead of Ss; Es instead of Ws, &c.

We suppose, then, that such a rough plot has been prepared, and that the ang distances, as taken from the field book, are figured upon it in *lead pencil*.

Add together the *interior* angles formed at all the corners: call their sum *a*. M sides by 180°; from the pred subtract 360°: if the remainder is equal to the sum *a* the angles have been correctly measured.* This, however, will rarely if ever always be some discrepancy; but if the field work has been performed with mode not *exceed* about two min for each angle. In this case div it *in equal parts* am adding or subtracting, as the case may be, unless it amounts to less than a min to it may be entirely disregarded in common farm surveys. The corrected angles m on the plot in ink, and the pencilled figures erased. We will suppose the correc shown in Fig 3.

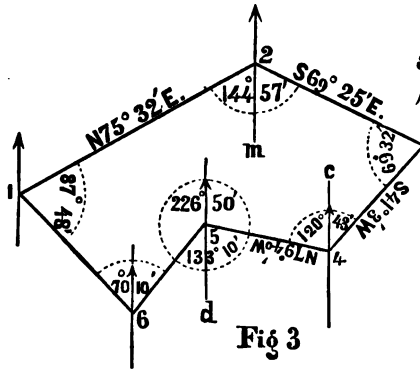


Fig 3

Next, b corrected bearings Select son the better of which t reverse be showing was proba by local a 2 be the o sume its b as taken o correct; t of it, as at the short r sliel to w through e having th N 75° 32' that of sid the revers ner 2 is that there *m*, is 75° 3 take 75° 3 corrected 57', the the angle the bearin

S 69° 25' E. For finding the bearing of side 4, we now have the angle 23 *a* of the side 3, also equal to 69° 25'; and if we add this to the entire corrected angle 234, or the angle *a* 34 = 69° 25' + 69° 32' = 138° 57'; which taken from 180°, leaves the a consequently the bearing of side 4 *must* be S 41° 3' W. For the bearing of side 5 *angle 34 c = 41° 3', which* taken from the corrected angle 345, or 120° 43', leaves t *40'; consequently the bearing of side 5 must* be N 79° 40' W. At corner 5, for the *we have the angle 45 d = 79° 40'*, which taken from 133° 10', leaves the angle *d* 5 *quently the bearing of side 6 must* be S 53° 30' W. And so with each of the

* *Because in every straight-lined figure the sum of all its interior angles is equ ht angles as the figure has sides,* minus 4 right angles, or 360°.

useful observation is necessary to see how the several angles are to be employed at each corner. Rules are sometimes given for this purpose, but unless frequently used, they are soon forgotten. The plot mechanically prepared obviates the necessity for such rules, inasmuch as the principle of proceeding thereby becomes merely a matter of sight, and tends greatly to prevent error from using the wrong bearings; while the protractor will at once detect any serious mistakes as to the angles, and thus prevent their being carried farther along. After having obtained all the corrected bearings, they may be figured on the plot instead of those taken in the field. They will, however, require a still farther correction after a while, since they will be affected by the adjustment of the closing error.

We now proceed to calculate the closing error § 6 of Fig 2, which is done on the principle that in a correct survey the northings will be equal to the southings, and the eastings to the westings. Prepare a table of 7 columns, as below, and in the first 3 cols place the numbers of the sides, and their corrected courses; also the dists or lengths of the sides, as measured on the rough plot, if such a one has been prepared; but if not, then as measured on the ground. Let them be as follows:

Side.	Bearing.	Dist. Ft.	Latitudes.		Departures.	
			N.	S.	E.	W.
1	N 16° 40′ W	1060	1015.5			304.
2	N 75° 32′ E	1202	300.3		1163.9	
3	S 69° 25′ E	1110		390.2	1039.2	
4	S 41° 3′ W	850		641.		558.2
5	N 79° 40′ W	802	143.9			789.
6	S 53° 30′ W	705		419.3		566.7
			1459.7	1450.5	2203.1	2217.9
			1450.5			2203.1
			9.2	Error in Lat.	Error in Dep.	14.8

Now find the N, S, E, W, of the several sides, and place them in the corresponding four columns, thus: By means of the Traverse Table find out the lat and dep for the angle of each course. Mult each of them by the length of the side; and place the prod in the corresponding col of N, S, E, W. Thus, for side 1, which is 1060 feet long, the latitude from the traverse table for 16° 40′ is .9580; and the departure is .2868: and .9580 × 1060 = 1015.5 lat; which, since the side norths, we put in the N col. Again, .2868 × 1060 = 304 dep; which, since the side wests, we put in the W col. Proceed thus with all. Add up the four cols; find the diff between the N and S cols; and also between the E and W ones. In this instance we find that the Ns are 9.2 feet greater than the Ss; and that the Ws are 14.8 ft greater than the Es; in other words, there is a closing error which would cause a correct protraction of our first three cols, to terminate 9.2 feet too far north of the starting point; and 14.8 feet too far west of it. So that by placing this error upon the paper before beginning to protract, we should have a test for the accuracy of the protracting work; but, as before remarked, a little more trouble will now enable us to div the error proportionally among all the Ns, Ss, Es, and Ws, and thereby give us data for drawing the plot correctly at once, without using a protractor at all.

To divide the errors, prepare a table precisely the same as the foregoing, except that the hor spaces are farther apart; and that the addings-up of the *old* N, S, E, W columns are omitted. The additions here noticed are made subsequently.

The new table is on the next page.

of the survey.* The corrected northings and southings we have already found; as also the and westings. The middle dists are found by means of the latter, by employing their halves half eastings, and subtracting half westings. Thus it is evident that the middle dist 2' of equal to half the easting of side 2. To this add the other half easting of side 2, and a half of side 3; and the sum is plainly equal to the middle dist 3' of side 3. To this add the ea easting of side 3, and subtract a half westing of side 4, for the middle dist 4' of side 4. F subtract the other half westing of side 4, and a half westing of side 5, for the middle dist 5; and so on. The actual calculation may be made thus:

$$\text{Half easting of side 2} = \frac{1167}{2} = \begin{array}{r} 583.5\ \text{E} \\ 583.5\ \text{E} \\ \hline 1167.0\ \text{E} \\ 521.0\ \text{E} \\ \hline 1688.0\ \text{E} \\ 521.0\ \text{E} \end{array} = \text{mid dist of side 2.}$$

$$\text{Half easting of side 3} = \frac{1042.1}{2} = \begin{array}{r} 2209.0\ \text{E} \\ 278.0\ \text{W} \\ \hline 1931.0\ \text{E} \\ 278.0\ \text{W} \end{array} = \text{mid dist of side 4.}$$

$$\text{Half westing of side 4} = \frac{556}{2} = \begin{array}{r} 1653.0\ \text{E} \\ 393.5\ \text{W} \\ \hline 1259.5\ \text{E} \\ 393.5\ \text{W} \end{array} = \text{mid dist of side 5.}$$

$$\text{Half westing of side 5} = \frac{786.9}{2} = \begin{array}{r} 866.0\ \text{E} \\ 282.4\ \text{W} \\ \hline 583.6\ \text{E} \\ 282.4\ \text{W} \end{array} = \text{mid dist of side 6.}$$

$$\text{Half westing of side 6} = \frac{564.9}{2} = \begin{array}{r} 301.2\ \text{E} \\ 150.6\ \text{W} \\ \hline 150.6\ \text{E} \end{array} = \text{mid dist of side 1.}$$

The work always proves itself by the last two results being equal.

Next make a table like the following, in the first 4 cols of which place the numbers of t the middle dists, the northings, and southings. Mult each middle dist by its corresponding or southing, and place the products in their proper col. Add up each col; subtract the least

Side.	Middle dist.	Northing.	Southing.	North prod.	South prod.
1	150.6	1013.8		152678	
2	583.5	298.4		174116	
3	1688		392		661696
4	1931		642.3		1240281
5	1259.5	142.6		179605	
6	583.6		420.4		245345
				506399	2147322
					506399
					43560)1640923(37.67 A

* *Proof.* To illustrate the principle upon which this rule is based, let ab, bc, and ca, Fig 6, represent in order the 3 sides of the triangular plot of a survey, with a meridian line df drawn through the extreme west corner, a. Let lines bd and cf be drawn from each corner, perp to the meridian line; also from the middle of each side draw lines ws, mn, so, also perp to meridian; and representing the middle dists of the sides. Then since the sides are regarded in the order ab, bc, ca, it is plain that ad represents the northing of the side ab; fa the northing of ca; and df the southing of bc. Now if we mult the *northing* ad of the side ab, by its *mid dist* sw, the prod is the area of the triangle abd. *In like manner the northing* fa of the side ca, mult by *its mid dist* so, *gives the* area of the triangle acf. Again, *the southing* df *of the side* bc, mult by its mid dist mn, *gives the area of the entire fig* $dbcfd$. If from this *area we subtract the areas of the two triangles* abd, *and* acf, *the rem is evidently the area of the plot* abc. *with any other plot, however complicated.*

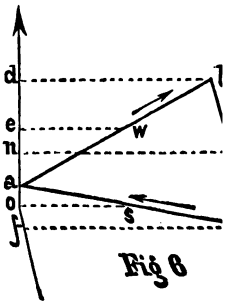


Fig 6

ment. The rem will be the area of the survey in sq ft; which, div by 43560, (the number of sq ft an acre,) will be the area in acres; in this instance, 37.67 ac.

It now remains only to calculate the corrected bearings and lengths of the sides of the survey, all which are necessarily changed by the adoption of the corrected lats and deps. To find the bearing any side, div its departure (E or W) by its lat (N or S); in the table of nat tang, find the quot; s angle opposite it is the reqd angle of bearing. Thus, for the course of side 1, we have $\frac{301.3\ W}{1013.8\ N}$ .2972 = nat tang; opposite which in the table is the reqd angle, 16° 33'; the bearing, therefore, is 16° 33' W.

Again: for the dist or length of any side, from the table of nat secants* take the sec opposite to the gle of the corrected bearing; mult it by the corrected lat (N or S) of the side. Thus, for the dist side 1, we find opposite 16° 33', the sec 1.0432. And

Sec. Lat.
1.0432 × 1013.8 = 1057.6 the reqd dist.

The following table contains all the corrections of the foregoing survey; consequently, if the bear-

Side.	Bearing.	Dist. Ft.
1	N 16° 33′ W	1057.6
2	N 75° 39′ E	1204.0
3	S 69° 23′ E	1113.3
4	S 40° 53′ W	849.6
5	N 79° 44′ W	100.1
6	S 55° 21′ W	704.3

gs and dists are correctly plotted, they will close perfectly. The young assistant is advised to ractise doing this, as well as dividing the plot into triangles, and computing the content. In this anner he will soon learn what degree of care is necessary to insure accurate results.

Under the heads Mensuration, Geometry, and Trigonometry will be found much pertaining to land urveying. See Remark after Parallelograms. The following hints in addition may be of service.

1st. Avoid taking bearings and lats along a circuitous boundary line like a b c, Fig 7; but run be straight line a c; and at ight angles to it, measure offsets to the crooked line. 2d. Wishing to survey a straight line from a to c, but being unable to direct the instrument readily toward c, on account f intervening woods, or other obstacles; first run a trial line, m a m, as nearly in the proper direction as can be guessed at.

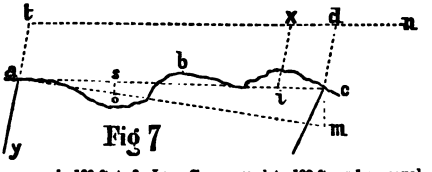


Fig 7

Measure m c, and say, as a m is to m c, so is 100 ft to ? Lay off a e equal to 100 ft, and o s equal to ?; and run the final line a s c. Or, if m c is quite small, calculate offsets like o s for every 100 ft along a m, and thus avoid the necessity for running a second line. 3d. When c is visible from a, but the intervening ground difficult to measure along, on account of marshes, &c. extend the side y a to good ground at t: then, making the angle y t d equal to y a c, run the line t w to that point d at which the angle w d c is found by trial to be equal to the angle a t d. It will rarely be necessary to make more than one trial for this point d: for, suppose it to be made at x, see where it strikes a c at i; measure i c, and continue from x, making x d = i c. 4th. In case of a very irregular piece of land, or a lake, Fig 8, surround it by straight lines. Survey these and at right angles to them, measure offsets to the crooked boundary. 5th. Surveying a straight line from w toward y, Fig 9,


Fig 8

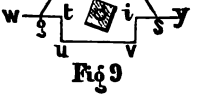


Fig 9

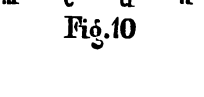


Fig. 10

a obstacle, o, is met with. To pass it, lay off a right angle w t u; measure any t u; make t u v = t; measure u v; make u v i = 90°; make v i = t u; make v i y = 90°. Then is t i = u v; and y is in the straight line. Or, with less trouble, at g make t g a = 60; measure any g a; make a s = 60°; and a s = g a: make a s i = 60°. Then is g s = g a or a s; and i s, continued toward is in the straight line. 6th. Being between two objects, m and n, and wishing to place myself in nge with them, *I lay a straight rod c b on* the ground, and point it to one of the objects m; then *ing to the end c. I find that it does* not point to the other object. By successive trials, I find the *ition e d, in which it points to both* objects, and consequently is in range with them. If no re

Our table does not contain nat secants; but the nat sec of any angle is readily found, thus: *I lay the nat cosine of the angle.*

is at hand, two stones will answer, or two chain-pins. A plumb line (a pebble tied to a pie thread) will add to the accuracy of ranging the rod, or stones, &c.

7th. *Deductions must be made in chaining over ground which is not perfectly level.* Even wl is so nearly level that the eye cannot detect the slope, an over-measurement of an inch or two i ft may readily occur. It is plain, that, if we measure all the undulations of the ground, we sha greater totals than if we measure hor, as is *supposed* always to be done; but since few surv pretend to measure hor until the slope becomes apparent to the eye, their lines are usually too by from one to two ins in 100 feet. To counteract this to some extent, chains are frequently from one to two ins longer than 100 feet; and for ordinary purposes the precaution is a good Where greater accuracy is required the chainmen should be attended by a third person, with and slope-level, for taking the inclinations of the ground. These deductions being made, the re der will be the actual hor dist. But if, while measuring up or down hill, we wish to place stal correspond with this actual hor dist when the excavation or embankment is finished, we mus instead of deducting; and must place the stakes so much *in advance* of our sloped chain dist This dist in advance should be measured *horizontally*. It is the natural versed sine of the an slope, multiplied by 100. When the ground is very sloping, all this calculation may be av where great accuracy is not required, by actually holding the chain horizontal, as nearly as c judged by eye and finding, by means of a plumb-line, where its raised end would strike the gr A whole chain at a time cannot be measured in this way; but shorter distances must be tak the ground requires; at times, on very steep ground, not more than 5 or 10 feet.

Table of Deductions to be made per 100 feet, in chain over sloping ground.

IN ORDER TO REDUCE THE INCLINED MEASUREMENTS TO HORIZONTAL ONES.

Slope in Deg.	Deduct Feet.	Rise in 100 ft hor.	Slope in Deg.	Deduct Feet.	Rise in 100 ft hor.	Slope in Deg.	Deduct Feet.	Rise in 100 ft hor.	Slope in Deg.	Deduct Feet.	R
¼	.001	.436	¼	.420	9.189	¼	1.596	18 08	¼	3.521	
½	.004	.873	½	.460	9.629	½	1.675	18.53	½	3.637	
¾	.009	1.309	¾	.503	10.07	¾	1.755	18.99	¾	3.754	
1	.015	1.746	6	.548	10.51	1r	1.837	19.44	16	3.874	
¼	.024	2.182	¼	.594	10.95	¼	1.921	19.89	¼	3.995	
½	.034	2.619	½	.643	11.39	½	2.008	20 35	½	4.118	
¾	.047	3.055	¾	.693	11.84	¾	2 095	20.80	¾	4 243	
2	.061	3.492	7	.745	12.28	12	2.185	21.26	17	4.370	
¼	.077	3.929	¼	.800	12.72	¼	2.277	21.71	¼	4.498	
½	.095	4.366	½	.856	13 17	½	2.370	22.17	½	4.628	
¾	.115	4.803	¾	.913	13.61	¾	2.466	22.63	¾	4.760	
3	.137	5.241	8	.973	14.05	13	2.563	23.09	18	4.894	
¼	.161	5.678	¼	1.035	14.50	¼	2.662	23 55	¼	5.030	
½	.187	6.116	½	1.098	14 95	½	2.763	24.01	½	5.168	
¾	.214	6.554	¾	1.164	15.39	¾	2.866	24.47	¾	5.307	
4	.244	6.993	9	1.231	15.84	14	2.970	24.93	19	5.448	
¼	.275	7.431	¼	1.300	16.29	¼	3.077	25.40	¼	5.591	
½	.308	7.870	½	1.371	16.73	½	3.185	25.86	½	5.736	
¾	.343	8.309	¾	1.444	17.18	¾	3.295	26.33	¾	5.882	
5	.381	8.749	10	1.519	17 63	15	3 407	26.79	20	6.0[illegible]1	

Engineers have abandoned the Gunter's chain of 66 ft, div into 100 links of 7.92 ins in lengtl use one of 100 ft, with links 1 ft long; and calculate areas in sq ft; which, div by 43560, reduce to acres and decimal parts, instead of roods and perches. Both the chain and the chain pins. F should be of good strong steel; and there should be a stout leather bag for carrying them. To bea mering into hard ground, the pins may be of thi and size, 11 or 12 ins long, ¼ inch thick, ⅝ wid 2½ wide, with a circular hole of 1½ diam. Ea should have a strip of bright red flannel tied to i that it may be readily found among grass, &c. hind chainman. The length of the chain sho tested every few days; and the target-rod may b for this purpose. While locating, it is well to have the chain one or two ins longer than K Steel wire, No 11 or 12, is a good size for a 100 ft chain. This is scant ¼ inch diam.

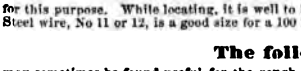


Fig. 11

The following Table

may sometimes be found useful for the rough measurement *of angles*, either on a drawing, tween distant objects in the field. If the inner edges of a common two-foot rule be opened to tent shown in the column of inches, its edges will be inclined at the angles shown in the colu angles. Since an opening of ¼ of an inch up to 19 inches or about 105°, corresponds to from ½ to 1°, no great accuracy is to be expected; and beyond 105° still less; the liability to er creasing very rapidly as the opening becomes greater. Thus, the last ¼ inch corresponds to abo

As to the table itself, angles for openings intermediate of those therein given, may be calcul the nearest minute or two, by simple proportion, up to 23 inches of opening, or about 147°.

The four fingers of the hand, held at right angles to the arm, a *arms-length from the eye*, cover about 7 degrees. And 7° corresponds to about 12.2 ft in 100 ft *36.6 ft in 100 yds; or to 645 ft in a mile*; or in the same proportion as the distance.

ble of Angles corresponding to openings of a 2-foot rule.

(Original.)

D, degrees; M, minutes. Correct.

D.	M.	Ins.	D.	M.	Ins.	D.	M.	Ins.	D.	M.	Ins.	D.	M.	Ins.	D.	M.
1	12	4¼	20	24	8¼	40	13	12¼	61	23	16¼	85	14	20¼	115	5
1	48		21			40	51		62	5		86	3		116	12
2	24	½	21	37	½	41	29	½	62	47	½	86	52	½	117	20
3	00		22	13		42	7		63	28		87	41		118	30
3	36	¾	22	50	¾	42	46	¾	64	11	¾	88	31	¾	119	40
4	11		23	27		43	24		64	53		89	21		120	52
4	47	5	24	3	9	44	3	13	65	35	17	90	12	21	122	6
5	23		24	39		44	42		66	18		91	3		123	20
5	58	¼	25	16	¼	45	21	¼	67	1	¼	91	54	¼	124	36
6	34		25	53		45	59		67	44		92	46		125	54
7	10	½	26	30	½	46	38	½	68	28	½	93	38	½	127	14
7	46		27	7		47	17		69	12		94	31		128	35
8	22	¾	27	44	¾	47	56	¾	69	55	¾	95	24	¾	129	59
8	58		28	21		48	35		70	38		96	17		131	25
9	34	6	28	58	10	49	15	14	71	22	18	97	11	22	132	53
10	10		29	35		49	54		72	6		98	5		134	24
10	46	¼	30	11	¼	50	34	¼	72	51	¼	99	00	¼	135	58
11	22		30	49		51	13		73	36		99	55		137	35
11	58	½	31	26	½	51	53	½	74	21	½	100	51	½	139	16
12	34		32	3		52	33		75	6		101	48		141	1
13	10	¾	32	40	¾	53	13	¾	75	51	¾	102	45	¾	142	51
13	46		33	17		53	53		76	36		103	43		144	46
14	22	7	33	54	11	54	34	15	77	22	19	104	41	23	146	48
14	58		34	33		55	14		78	8		105	40		148	58
15	34	¼	35	10	¼	55	55	¼	78	54	¼	106	39	¼	151	17
16	10		35	47		56	35		79	40		107	40		153	48
16	46	½	36	25	½	57	16	½	80	27	½	108	41	½	156	54
17	22		37	3		57	57		81	14		109	43		159	43
17	59	¾	37	41	¾	58	38	¾	82	2	¾	110	46	¾	163	27
18	35		38	19		59	19		82	49		111	49		168	18
19	12	8	38	57	12	60	00	16	83	37	20	112	53	24	180	00
19	48		39	35		60	41		84	26		113	58			

r this table may be of use thus: From any point *a*, Fig 6, measure *a b*, 12 ft. toward one of the nts; and *a c*, also 12 ft, toward the other; and measure *b c* in ft. Then look in the first col of the le for this last measure, as if the first col contained ft instead of ins: and the angle opposite to it l evidently be the angle *b a c*. Thus, if *b c* is 3¼ ft, or 3 ft 1½ ins, the angle *b a c* is 14° 58′. Or may measure *a b* and *a c* each 100 ft. (or any other number of ft;) also measure *b c*. Then, as 100 or any other number actually used) is to 1, so is *half* of *b c* to the nat sine of the angle. Having n the nat sine, the angle itself may be found in the table of nat sines.

o find a Meridian Line (a true North and South line) by means of the North Star.

The north star appears to describe a small circle, *n n′*, &c, Fig 14, around the true north point, or rth pole, as a centre. The rad of this circle is estimated by the angle between the star and the h, as measured from the earth; and is called the *polar distance* of the star. This polar dist be- ms $19\frac{3}{10}$ seconds, or very nearly ¼ of a minute less every year. On Jan 1, 1868, it is, approxi- ately, 1 deg 23½ min.* On the first of 1871 it will be 1° 22½′; 1874, 1° 21½′; 1877, 1° 20½′, &c. hen, in its revolution, the star is farthest east or west from the pole, as at *n* or *n″*, it is said to be its *greatest east or west elongation*. At that time its apparent motion for several min is nearly t, and consequently affords the best opportunity for an observation in the simple manner here ribed. The arrows in Fig 14 show the direction in which the stars appear to move from east to st when the spectator faces the north.

The latitude of the place must be known approximately. If within a quarter of a degree, however, error in the meridian line will not exceed about 1½ minutes in lat 60°; 1 min in lat 50°; ½ min at 40°: and less as we approach the equator.

Set 3 ft above ground fix firmly, and as nearly level, and as nearly east and west as may be, a rech narrow piece of board, about 3 ft long, to serve as a kind of lt. Also prepare another piece *a a*, about a foot long; and fasten t, at right angles, a compass sight, or a strip of thin metal, with traight slit, (shown by a black line in the fig.) about 6 ins long $\frac{1}{16}$ inch wide. This piece of board is to be slid along the de, as the observer follows the motions of the star toward the t or west: looking at it through the vert slit. Plant a stout e, about 20 ft long, firmly in the ground, with its top as nearly th as possible from the middle of the table. Its top should n 2 or 3 ft toward either the east or the west; and a plumb-

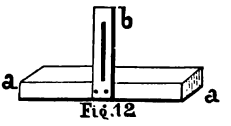


Fig.12

t must be suspended *from its top, with* a bob weighing one or two lbs, which may swing in a ket of water *placed on the ground*. This is to prevent the line from being so easily moved by *ht currents of air; and for further steadiness*, the pole itself should be well braced from within.

474826 *1° 22′ 37″.8.

fe at hand, two stones will answer, or two chain-pins. A plumb line (a pebble tied to a piece of thread) will add to the accuracy of ranging the rod, or stones, &c.

7th. *Deductions must be made in chaining over ground which is not perfectly level.* Even when it is so nearly level that the eye cannot detect the slope, an over-measurement of an inch or two in 100 ft may readily occur. It is plain, that, if we measure all the undulations of the ground, we shall get greater totals than if we measure hor, as is *supposed* always to be done; but since few surveyors pretend to measure hor until the slope becomes apparent to the eye, their lines are usually too long by from one to two ins in 100 feet. To counteract this to some extent, chains are frequently made from one to two ins longer than 100 feet; and for ordinary purposes the precaution is a good one. Where greater accuracy is required the chainmen should be attended by a third person, with a rod and slope-level, for taking the inclinations of the ground. These deductions being made, the remainder will be the actual hor dist. But if, while measuring up or down hill, we wish to place stakes to correspond with this actual hor dist when the excavation or embankment is finished, we must *add* instead of deducting; and must place the stakes so much in *advance of* our sloped chain distances. This dist in advance should be measured *horizontally*. It is the natural versed sine of the angle of slope, multiplied by 100. When the ground is very sloping, all this calculation may be avoided where great accuracy is not required, by actually holding the chain horizontal, as nearly as can be judged by eye and finding, by means of a plumb-line, where its raised end would strike the ground. A whole chain at a time cannot be measured in this way; but shorter distances must be taken as the ground requires; at times, on very steep ground, not more than 5 or 10 feet.

Table of Deductions to be made per 100 feet, in chaining over sloping ground.

IN ORDER TO REDUCE THE INCLINED MEASUREMENTS TO HORIZONTAL ONES.

Slope in Deg.	Deduct Feet.	Rise in 100 ft hor.	Slope in Deg.	Deduct Feet.	Rise in 100 ft hor.	Slope in Deg.	Deduct Feet.	Rise in 100 ft hor.	Slope in Deg.	Deduct Feet.	Rise in 100 ft hor.
¼	.001	.436	¼	.420	9.189	¼	1.596	18.08	¼	3.521	27.26
½	.004	.873	½	.460	9.629	½	1.675	18.53	½	3.637	27.73
¾	.009	1.309	¾	.503	10.07	¾	1.755	18.99	¾	3.754	28.20
1	.015	1.746	6	.548	10.51	17	1.837	19.44	16	3.874	28.67
¼	.024	2.182	¼	.594	10.95	¼	1.921	19.89	¼	3.995	29.15
½	.034	2.619	½	.643	11.39	½	2.008	20.35	½	4.118	29.63
¾	.047	3.055	¾	.693	11.84	¾	2.095	20.80	¾	4.243	30.10
2	.061	3.492	7	.745	12.28	12	2.185	21.26	17	4.370	30.57
¼	.077	3.929	¼	.800	12.72	¼	2.277	21.71	¼	4.498	31.05
½	.095	4.366	½	.856	13.17	½	2.370	22.17	½	4.628	31.53
¾	.115	4.803	¾	.913	13.61	¾	2.465	22.63	¾	4.760	32.01
3	.137	5.241	8	.973	14.05	13	2.563	23.09	18	4.894	32.49
¼	.161	5.678	¼	1.035	14.50	¼	2.662	23.55	¼	5.030	32.98
½	.187	6.116	½	1.098	14.95	½	2.763	24.01	½	5.168	33.46
¾	.214	6.554	¾	1.164	15.39	¾	2.866	24.47	¾	5.307	33.95
4	.244	6.993	9	1.231	15.84	14	2.970	24.93	19	5.448	34.43
¼	.275	7.431	¼	1.300	16.29	¼	3.077	25.40	¼	5.591	34.92
½	.308	7.870	½	1.371	16.73	½	3.185	25.86	½	5.736	35.41
¾	.343	8.309	¾	1.444	17.18	¾	3.295	26.33	¾	5.882	35.90
5	.381	8.749	10	1.519	17.63	15	3.407	26.79	20	6.031	36.40

Engineers have abandoned the Gunter's chain of 66 ft, div into 100 links of 7.92 ins in length; and use one of 100 ft, with links 1 ft long; and calculate areas in sq ft; which, div by 43560, reduces them to acres and decimal parts, instead of roods and perches. Both the chain and the chain pins, Fig. 11, should be of good strong steel; and there should be a stout leather bag for carrying them. To bear hammering into hard ground, the pins may be of this shape and size, 11 or 12 ins long, ¾ inch thick, ¾ wide, head 2½ wide, with a circular hole of 1½ diam. Each pin should have a strip of bright red flannel tied to its top, that it may be readily found among grass, &c. by the hind chainman. The length of the chain should be tested every few days; and the target-rod may be used for this purpose. While locating, it is well to have the chain one or two ins longer than 100 feet. Steel wire, No 11 or 12, is a good size for a 100 ft chain. This is scant ¼ inch diam.

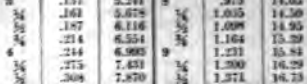


Fig. 11

The following Table

may sometimes be found useful for the rough measurement *of angles*, either on a drawing, or between distant objects in the field. If the inner edges of a common two-foot rule be opened to the extent shown in the column of inches, its edges will be inclined at the angles shown in the columns of angles. Since an opening of ¾ of an inch up to 19 inches or about 105°, corresponds to from about ½ to 1°, no great accuracy is to be expected; and beyond 105° still less; the liability to error increasing very rapidly as the opening becomes greater. Thus, the last ¾ inch corresponds to about 12°.

As to the table itself, angles for openings intermediate of those therein given, may be calculated to the nearest minute or two, by simple proportion, up to 23 inches of opening, or about 147°.

The four fingers of the hand, held at right angles to the arm, and at arms-length from the eye, cover about 7 degrees. And 7° corresponds to about 12.2 ft in 100 ft; or to 36.6 ft in 100 yds; or to 645 ft in a mile; or in the same proportion as the distance.

a be no difficulty in finding Alioth, as it is one of the 7 bright stars in the fine constellation wn as the Great Bear, or the Wagon and Horses. Alioth is the horse nearest to the four the wagon. The two hind-wheels *t t* are known to every schoolboy as the "Pointers," ey point nearly in the direction to the north star. The relative positions of these 7 stars, n *Fig* 14, is tolerably correct.

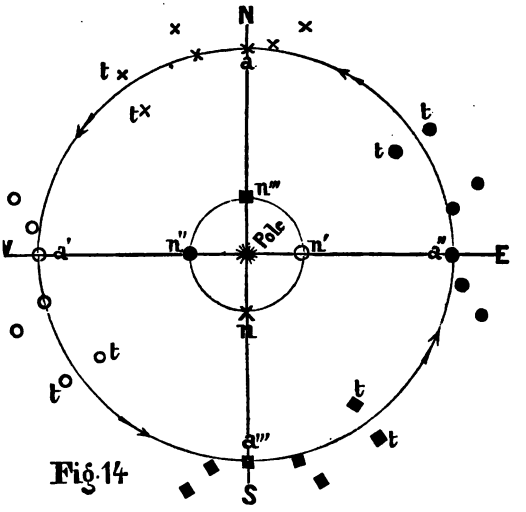


Fig. 14

said that Alioth is *nearly* in a vert line with the north star when the latter is on the me- refore the stakes fixed when these stars are *exactly* in line cannot be *precisely* in a meridian asmuch as the error is less than half a min. and consequently too small to be appreciated surveying instruments, it may be disregarded.

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ARK ON THE FOLLOWING TABLE OF NATURAL SINES, &c.

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e field *practice of railroad curves,* two trigonometrical tan- *minate where they meet each other*; and the corresponding secants would *terminate at that same point.*

few feet below its top. The proper dist a o, of the pole p o, from the table t d, may thus: Make an angle n m s, equal approximately to the lat of the place. Open a pair to equal, by any convenient scale, the height t a of the table; and draw t a. Then ta

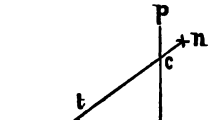

Fig 13

same scale, the height, p o, of the pole above ground; it upon the sketch, so that the top p shall be by so two above m n. Then a o, by the same scale, will be dist reqd; probably from 3 to 5 yards. A deviation of from this will not be important.

The correct clock time at any place, for the elongatio found within a few min from the following table. T ones take place about 6 hours before, and the weste hours after the times in the table; which are those, 3 min, at which the star is exactly above the true The observer should be at his station at least ½ of advance of the time. Placing the board a o, upon the in range with the plumb-line and star, he will watch b through the vert slit; sliding the board along the tab keep the slit in the range as long as the star continu toward the east or west, as the case may be. An assis hold a candle, or lantern, on a pole near the plumb-lin the observer to see the latter. As the star approaches tion, it will appear to move nearly vert for several min, so that it can be seen without slit. When certain by this that the star has reached its elongation, confine the sliding b table by sticking a few tacks around its edges. Then let a third person, with another ca some dist, (a hundred yards or more if convenient,) in a direction toward the star; and a stake as directed by the observer, who will take care that it is exactly in range with t plumb-line. Another stake must then be driven exactly under either the slit or the Having thus placed the two stakes in the *range* of the elongation, defer the remainder of th until morning. From the table of nat sines, &c. take out the sine of the polar dist, a secant of the lat.* Mult these together. The prod will be the nat sine of a certain angle *azimuth* of the star. Find the sine in the table, and take out the nat tang which corresp This is the tang of the azimuth, and is to be used in the final operation. Measure carefu between the two stakes. Mult this dist by the tangent of azimuth just found. The p a dist in ft, which must be laid off from the northern stake, at right angles to the mea and in the direction to the east if the elongation is west, and vice versa. At that point dr stake, and remove the old one. The two remaining stakes are in a true north and south lin be supposed that after driving the first two stakes, a true meridian could be had by merel the polar dist, by means of a compass or transit; but this is not so. Place the compa south stake, and take sight to the north one. If, then, the north end of the needle points line, the variation of the compass is east, and vice versa.

Approximate Time, by a correct clock, at any place, North Star's passing the meridian above the pol

Correct until the year 1880, within a few min. It will pass the meridian below the pole time, very nearly; but at A. M. instead of P. M., and vice versa.

	DAY OF THE MONTH.		
	1st Day.	11th Day.	21st Day.
Jan.	6 26 P. M.	5.46 P. M.	5. 7 P. M.
Feb.	4.23 "	3.44 "	3. 5 "
Mar.	2.33 "	1.54 "	1.14 "
April	12.31 "	11.52 A. M.	11.13 A. M.
May	10.33 A. M.	9.54 "	9 15 "
June	8.32 "	7.53 "	7.13 "
July	6.34 "	5.55 "	5.16 "
Aug.	4.33 "	3.54 "	3.14 "
Sept.	2.31 "	1.52 "	1.12 "
Oct.	12.29 "	11.50 P. M.	11.11 P. M.
Nov.	10.27 P. M.	9.48 "	9. 9 "
Dec.	8.29 "	7.50 "	7.11 "

For days of the month intermediate of those in the table, it will be near enough to mal 4 min earlier each succeeding day.

During nearly all of the four months, March, April, September, and October, the elong place in daylight; so that this method cannot then be used. Nor can it be used at any tin south of about 4° north of the equator, because there the north star is not visible. But time a meridian may be found by recollecting that when the north star n is *on the merid* other words, is *truly* north from us, then the star Alioth, a, is very nearly vertically *abo* north star n is on the meridian *below* the pole: or below it, as at a''', if the north star the meridian *above* the pole. When the north star n' is at its east elongation, Alioth *tally west of it, as at* a': and when the north star, n'', is at its west elongation, Alioth is h *east of it, as at* a''. All that is necessary, therefore, is to prepare an arrangement of *plumb line, &c. precisely* as before; except that the plumb-line must be nearer the obse *will have to watch Alioth* above the north star. Watch through the movable slit until *the same vert line with the* north star. Then put in two stakes as before, and they w *in a true north and south line*, without requiring any alteration for azimuth.

* See foot-note page 97.

can be no difficulty in finding Alioth, as it is one of the 7 bright stars in the fine constellation known as the Great Bear, or the Wagon and Horses. Alioth is the horse nearest to the four of the wagon. The two hind-wheels *t t* are known to every schoolboy as the "Pointers," they point nearly in the direction to the north star. The relative positions of these 7 stars, n in Fig 14, is tolerably correct.

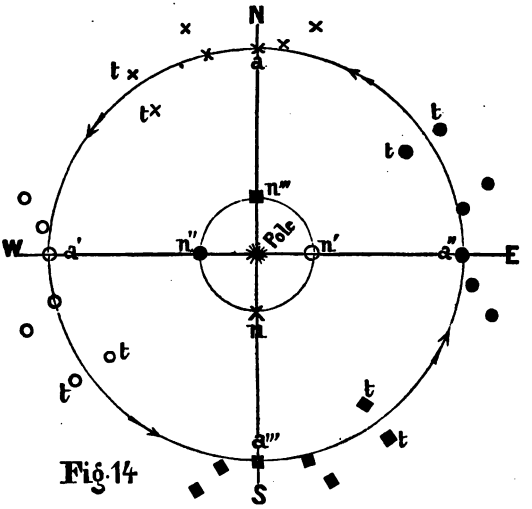

Fig. 14

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NATURAL SINES AND TANGENTS TO A RADIUS 1.

0 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0000000	·000000	Infinite.	1·000000	60	21	·0061086	·006108	163·7001	·9999813	39	41	·0119261	·011927	83·84350	·9999289	19
1	·0002909	·000291	3437·746	1·000000	59	22	·0063995	·006399	156·2590	·9999795	38	42	·0122170	·012217	81·84704	·9999254	18
2	·0005818	·000582	1718·873	·9999998	58	23	·0066904	·006690	149·4650	·9999776	37	43	·0125079	·012508	79·94343	·9999218	17
3	·0008727	·000872	1145·915	·9999996	57	24	·0069813	·006981	143·2371	·9999756	36	44	·0127987	·012799	78·12634	·9999181	16
4	·0011636	·001163	859·4363	·9999993	56	25	·0072721	·007272	137·5075	·9999736	35	45	·0130896	·013090	76·39000	·9999143	15
5	·0014544	·001454	687·5488	·9999989	55	26	·0075630	·007563	132·2185	·9999714	34	46	·0133805	·013381	74·72916	·9999105	14
6	·0017453	·001745	572·9572	·9999985	54	27	·0078539	·007854	127·3213	·9999692	33	47	·0136713	·013672	73·13899	·9999065	13
7	·0020362	·002036	491·1060	·9999979	53	28	·0081448	·008145	122·7739	·9999668	32	48	·0139622	·013963	71·61507	·9999025	12
8	·0023271	·002327	429·7175	·9999973	52	29	·0084357	·008436	118·5401	·9999644	31	49	·0142530	·014254	70·15334	·9998984	11
9	·0026180	·002618	381·9709	·9999966	51	30	·0087265	·008726	114·5886	·9999619	30	50	·0145439	·014545	68·75008	·9998942	10
10	·0029089	·002908	343·7737	·9999958	50	31	·0090174	·009017	110·8920	·9999593	29	51	·0148348	·014836	67·40185	·9998900	9
11	·0031998	·003199	312·5213	·9999949	49	32	·0093083	·009308	107·4264	·9999567	28	52	·0151256	·015127	66·10547	·9998856	8
12	·0034907	·003490	286.4777	·9999939	48	33	·0095992	·009599	104·1709	·9999539	27	53	·0154165	·015418	64·85800	·9998812	7
13	·0037815	·003781	264·4408	·9999928	47	34	·0098900	·009890	101·1069	·9999511	26	54	·0157073	·015709	63·65674	·9998766	6
14	·0040724	·004072	245.5519	·9999917	46	35	·0101809	·010181	98·21794	·9999482	25	55	·0159982	·016000	62·49915	·9998720	5
15	·0043633	·004363	229·1816	·9999905	45	36	·0104718	·010472	95·48947	·9999452	24	56	·0162890	·016291	61·38290	·9998673	4
16	·0046542	·004654	214·8576	·9999892	44	37	·0107627	·010763	92·90848	·9999421	23	57	·0165799	·016582	60·30582	·9998625	3
17	·0049451	·004945	202·2187	·9999878	43	38	·0110535	·011054	90·46333	·9999389	22	58	·0168707	·016873	59·26587	·9998577	2
18	·0052360	·005236	190·9841	·9999863	42	39	·0113444	·011345	88·14357	·9999357	21	59	·0171616	·017164	58·26117	·9998527	1
19	·0055268	·005526	180·9322	·9999847	41	40	·0116353	·011636	85·93979	·9999323	20	60	·0174524	·017455	57·28996	·9998477	0
20	·0058177	·005817	171·8854	·9999831	40												
′	Cosine.	Cot n.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′

0 Deg.

Deg. 89

0 Deg.

Deg. 89.

Deg. 89

′	1 Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	1 Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	1 Deg. Sine.	Tang.	Cotang.	Cosine.	′
0	·0174524	·017455	57·28996	·9998477	60	21	·0235598	·023566	42·43346	·9997224	39	41	·0293755	·029388	34·02730	·9995684	19
1	·0177432	·017746	56·35059	·9998426	59	22	·0238506	·023857	41·91579	·9997156	38	42	·0296662	·029679	33·69350	·9995599	18
2	·0180341	·018037	55·44151	·9998374	58	23	·0241414	·024148	41·41058	·9997086	37	43	·0299570	·029970	33·36619	·9995512	17
3	·0183249	·018328	54·56130	·9998321	57	24	·0244322	·024439	40·91741	·9997015	36	44	·0302478	·030261	33·04517	·9995424	16
4	·0186158	·018619	53·70858	·9998267	56	25	·0247230	·024730	40·43583	·9996943	35	45	·0305385	·030552	32·73026	·9995336	15
5	·0189066	·018910	52·88211	·9998213	55	26	·0250138	·025021	39·96546	·9996871	34	46	·0308293	·030843	32·42129	·9995247	14
6	·0191974	·019201	52·08067	·9998157	54	27	·0253046	·025312	39·50589	·9996798	33	47	·0311200	·031135	32·11809	·9995157	13
7	·0194883	·019492	51·30315	·9998101	53	28	·0255954	·025603	39·05677	·9996724	32	48	·0314108	·031426	31·82051	·9995066	12
8	·0197791	·019783	50·54850	·9998044	52	29	·0258862	·025894	38·61773	·9996649	31	49	·0317015	·031717	31·52839	·9994974	11
9	·0200699	·020074	49·81572	·9997986	51	30	·0261769	·026185	38·18845	·9996573	30	50	·0319922	·032008	31·24157	·9994881	10
10	·0203608	·020365	49·10388	·9997927	50	31	·0264677	·026477	37·76861	·9996497	29	51	·0322830	·032299	30·95992	·9994788	9
11	·0206516	·020656	48·41208	·9997867	49	32	·0267585	·026768	37·35789	·9996419	28	52	·0325737	·032591	30·68330	·9994693	8
12	·0209424	·020947	47·73950	·9997807	48	33	·0270493	·027059	36·95600	·9996341	27	53	·0328644	·032882	30·41158	·9994598	7
13	·0212332	·021238	47·08534	·9997745	47	34	·0273401	·027350	36·56265	·9996262	26	54	·0331552	·033173	30·14461	·9994502	6
14	·0215241	·021529	46·44886	·9997683	46	35	·0276309	·027641	36·17759	·9996182	25	55	·0334459	·033464	29·88229	·9994405	5
15	·0218149	·021820	45·82935	·9997620	45	36	·0279216	·027932	35·80055	·9996101	24	56	·0337366	·033755	29·62449	·9994308	4
16	·0221057	·022111	45·22614	·9997556	44	37	·0282124	·028223	35·43128	·9996020	23	57	·0340274	·034047	29·37110	·9994209	3
17	·0223965	·022402	44·63859	·9997492	43	38	·0285032	·028514	35·06954	·9995937	22	58	·0343181	·034338	29·12200	·9994110	2
18	·0226873	·022693	44·06611	·9997426	42	39	·0287940	·028805	34·71511	·9995854	21	59	·0346088	·034629	28·87708	·9994009	1
19	·0229781	·022984	43·50812	·9997360	41	40	·0290847	·029097	34·36777	·9995770	20	60	·0348995	·034920	28·63625	·9993908	0
20	·0232690	·023275	42·96407	·9997292	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′
			Deg 88.						Deg. 88.							Deg. 88	

NATURAL SINES AND TANGENTS TO A RADIUS 1.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
							2 Deg.						2 Deg.				
0	·0348995	·034920	28·63625	·9993908	60	21	·0410037	·041038	24·36750	·9991590	39	41	·0468159	·046867	21.33685	·9989035	19
1	·0351902	·035212	28·39939	·9993806	59	22	·0412944	·041329	24·19571	·9991470	38	42	·0471065	·047158	21·20494	·9988899	18
2	·0354809	·035503	28·16642	·9993704	58	23	·0415850	·041621	24·02632	·9991350	37	43	·0473970	·047450	21·07466	·9988761	17
3	·0357716	·035794	27·93723	·9993600	57	24	·0418757	·041912	23·85927	·9991228	36	44	·0476876	·047741	20·94596	·9988623	16
4	·0360623	·036085	27·71174	·9993495	56	25	·0421663	·042203	23·69453	·9991106	35	45	·0479781	·048033	20·81882	·9988484	15
5	·0363530	·036377	27·48985	·9993390	55	26	·0424569	·042495	23·53205	·9990983	34	46	·0482687	·048325	20·69322	·9988344	14
6	·0366437	·036668	27·27148	·9993284	54	27	·0427475	·042786	23·37177	·9990859	33	47	·0485592	·048616	20·56911	·9988203	13
7	·0369344	·036959	27·05655	·9993177	53	28	·0430382	·043078	23·21366	·9990734	32	48	·0488498	·048908	20·44648	·9988061	12
8	·0372251	·037250	26·84498	·9993069	52	29	·0433288	·043369	23·05767	·9990609	31	49	·0491403	·049199	20·32530	·9987919	11
9	·0375158	·037542	26·63669	·9992960	51	30	·0436194	·043660	22·90376	·9990482	30	50	·0494308	·049491	20·20555	·9987775	10
10	·0378065	·037833	26·43160	·9992851	50	31	·0439100	·043952	22·75189	·9990355	29	51	·0497214	·049782	20·08719	·9987631	9
11	·0380971	·038124	26·22963	·9992740	49	32	·0442006	·044243	22·60201	·9990227	28	52	·0500119	·050074	19·97021	·9987486	8
12	·0383878	·038416	26·03073	·9992629	48	33	·0444912	·044535	22·45409	·9990098	27	53	·0503024	·050366	19·85459	·9987340	7
13	·0386785	·038707	25·83482	·9992517	47	34	·0447818	·044826	22·30809	·9989968	26	54	·0505929	·050657	19·74029	·9987194	6
14	·0389692	·038998	25·64183	·9992404	46	35	·0450724	·045118	22·16398	·9989837	25	55	·0508835	·050949	19·62729	·9987046	5
15	·0392598	·039290	25·45170	·9992290	45	36	·0453630	·045409	22·02171	·9989706	24	56	·0511740	·051241	19·51558	·9986898	4
16	·0395505	·039581	25·26436	·9992176	44	37	·0456536	·045701	21·88125	·9989573	23	57	·0514645	·051532	19·40513	·9986748	3
17	·0398411	·039872	25·07975	·9992060	43	38	·0459442	·045992	21·74256	·9989440	22	58	·0517550	·051824	19·29592	·9986598	2
18	·0401318	·040164	24·89782	·9991944	42	39	·0462347	·046284	21·60563	·9989306	21	59	·0520455	·052116	19·18793	·9986447	1
19	·0404224	·040455	24·71851	·9991827	41	40	·0465253	·046575	21·47040	·9989171	20	60	·0523360	·052407	19·08113	·9986295	0
20	·0407131	·040746	24·54175	·9991709	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′
				Deg. 87.						Deg. 87.						Deg. 87.	

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0523360	·052407	19·08113	·9986295	60
1	·0526261	·052699	18·97552	·9986143	59
2	·0529169	·052991	18·87106	·9985989	58
3	·0532074	·053282	18·76775	·9985835	57
4	·0534979	·053574	18·66556	·9985680	56
5	·0537883	·053866	18·56447	·9985524	55
6	·0540788	·054158	18·46447	·9985367	54
7	·0543693	·054449	18·36553	·9985209	53
8	·0546597	·054741	18·26765	·9985050	52
9	·0549502	·055033	18·17080	·9984891	51
10	·0552406	·055325	18·07497	·9984731	50
11	·0555311	·055616	17·98015	·9984570	49
12	·0558215	·055908	17.88631	·9984408	48
13	·0561119	·056200	17·79344	·9984245	47
14	·0564024	·056492	17·70152	·9984081	46
15	·0566928	·056784	17·61055	·9983917	45
16	·0569832	·057075	17·52051	·9983751	44
17	·0572736	·057367	17·43138	·9983585	43
18	·0575640	·057659	17·34315	·9983418	42
19	·0578544	·057951	17·25580	·9983250	41
20	·0581448	·058243	17·16933	·9983082	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0584352	·058535	17·08372	·9982912	39
22	·0587256	·058827	16·99895	·9982742	38
23	·0590160	·059119	16·91502	·9982570	37
24	·0593064	·059410	16·83191	·9982398	36
25	·0595967	·059702	16·74961	·9982225	35
26	·0598871	·059994	16·66811	·9982052	34
27	·0601775	·060286	16·58739	·9981877	33
28	·0604678	·060578	16·50745	·9981701	32
29	·0607582	·060870	16·42827	·9981525	31
30	·0610485	·061162	16·34985	·9981348	30
31	·0613389	·061454	16·27217	·9981170	29
32	·0616292	·061746	16·19522	·9980991	28
33	·0619196	·062038	16·11899	·9980811	27
34	·0622099	·062330	16·04348	·9980631	26
35	·0625002	·062622	15·96866	·9980450	25
36	·0627905	·062914	15·89454	·9980267	24
37	·0630808	·063206	15·82110	·9980084	23
38	·0633711	·063498	15·74833	·9979900	22
39	·0636614	·063790	15·67623	·9979716	21
40	·0639517	·064082	15·60478	·9979530	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0642420	·064375	15·53398	·9979343	19
42	·0645323	·064667	15·46381	·9979156	18
43	·0648226	·064959	15·39427	·9978968	17
44	·0651129	·065251	15·32535	·9978779	16
45	·0654031	·065543	15·25705	·9978589	15
46	·0656934	·065835	15·18934	·9978399	14
47	·0659836	·066127	15·12224	·9978207	13
48	·0662739	·066419	15·05572	·9978015	12
49	·0665641	·066712	14·98978	·9977821	11
50	·0668544	·067004	14·92441	·9977627	10
51	·0671446	·067296	14·85961	·9977433	9
52	·0674349	·067588	14·79537	·9977237	8
53	·0677251	·067880	14·73167	·9977040	7
54	·0680153	·068173	14·66852	·9976843	6
55	·0683055	·068465	14·60591	·9976645	5
56	·0685957	·068757	14·54383	·9976445	4
57	·0688859	·069049	14·48227	·9976245	3
58	·0691761	·069342	14·42123	·9976045	2
59	·0694663	·069634	14·36069	·9975843	1
60	·0697565	·069926	14·30066	·9975641	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
					4 Deg.						4 Deg.						
0	·0697565	·069926	14·30066	·9975641	60	21	·0758489	·076068	13·14612	·9971193	39	41	·0816486	·081922	12·20671	·9966612	19
1	·0700467	·070219	14·24113	·9975437	59	22	·0761390	·076360	13·09575	·9970972	38	42	·0819385	·082215	12·16323	·9966371	18
2	·0703368	·070511	14·18209	·9975233	58	23	·0764290	·076653	13.04576	·9970750	37	43	·0822284	·082507	12·12006	·9966135	17
3	·0706270	·070803	14·12353	·9975028	57	24	·0767190	·076945	12·99616	·9970528	36	44	·0825183	·082800	12·07719	·9965895	16
4	·0709171	·071096	14·06545	·9974822	56	25	·0770091	·077238	12·94692	·9970304	35	45	·0828082	·083093	12·03462	·9965655	15
5	·0712073	·071388	14·00785	·9974615	55	26	·0772991	·077531	12·89805	·9970080	34	46	·0830981	·083386	11·99234	·9965414	14
6	·0714974	·071680	13·95071	·9974408	54	27	·0775891	·077823	12·84955	·9969854	33	47	·0833880	·083679	11·95037	·9965172	13
7	·0717876	·071973	13·89404	·9974199	53	28	·0778791	·078116	12·80141	·9969628	32	48	·0836778	·083972	11·90868	·9964929	12
8	·0720777	·072265	13·83782	·9973990	52	29	·0781691	·078409	12·75363	·9969401	31	49	·0839677	·084265	11·86728	·9964685	11
9	·0723678	·072558	13·78206	·9973780	51	30	·0784591	·078701	12·70620	·9969173	30	50	·0842576	·084558	11·82616	·9964440	10
10	·0726580	·072850	13·72673	·9973569	50	31	·0787491	·078994	12·65912	·9968945	29	51	·0845474	·084851	11·78533	·9964195	9
11	·0729481	·073143	13·67185	·9973357	49	32	·0790391	·079287	12·61239	·9968715	28	52	·0848373	·085144	11·74477	·9963948	8
12	·0732382	·073435	13·61740	·9973145	48	33	·0793290	·079579	12·56599	·9968485	27	53	·0851271	·085437	11·70450	·9963701	7
13	·0735283	·073727	13·56339	·9972931	47	34	·0796190	·079872	12·51994	·9968254	26	54	·0854169	·085730	11·66449	·9963453	6
14	·0738184	·074020	13·50979	·9972717	46	35	·0799090	.080165	12·47422	·9968022	25	55	·0857067	·086023	11·62476	·9963204	5
15	·0741085	·074312	13·45662	·9972502	45	36	·0801989	·080458	12·42883	·9967789	24	56	·0859966	·086316	11·58529	·9962954	4
16	·0743986	·074605	13·40386	·9972286	44	37	·0804889	·080750	12·38376	·9967555	23	57	·0862864	·086609	11·54609	·9962704	3
17	·0746887	·074897	13·35151	·9972069	43	38	·0807788	·081043	12·33902	·9967321	22	58	·0865762	·086902	11·50715	·9962452	2
18	·0749787	·075190	13·29957	·9971851	42	39	·0810687	·081336	12·29460	·9967085	21	59	·0868660	·087195	11·46847	·9962200	1
19	·0752688	·075482	13·24803	·9971633	41	40	·0813587	·081629	12·25050	·9966849	20	60	·0871557	·087488	11·43005	·9961947	0
	·0755589	·075775	13·19688	·9971413	40												

Deg.	5 Deg.											5 Deg.				
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0871557	·087488	11·43005	·9961947	60	21	·0932395	·093647	10·67834	·9956437	39	41	·0990303	·099519	10·04828	·9950844	19
1	·0874455	·087781	11·39188	·9961693	59	22	·0935291	·093940	10·64499	·9956165	38	42	·0993197	·099813	10·01871	·9950556	18
2	·0877353	·088074	11·35397	·9961438	58	23	·0938187	·094234	10·61184	·9955892	37	43	·0996092	·100107	9·989305	·9950266	17
3	·0880251	·088368	11·31630	·9961182	57	24	·0941083	·094527	10·57889	·9955620	36	44	·0998986	·100400	9·960072	·9949976	16
4	·0883148	·088661	11·27888	·9960926	56	25	·0943979	·094821	10·54615	·9955345	35	45	·1001881	·100694	9·931008	·9949685	15
5	·0886046	·088954	11·24171	·9960669	55	26	·0946875	·095114	10·51360	·9955070	34	46	·1004775	·100988	9·902112	·9949393	14
6	·0888943	·089247	11·20478	·9960411	54	27	·0949771	·095408	10·48126	·9954794	33	47	·1007669	·101282	9·873382	·9949101	13
7	·0891840	·089540	11·16808	·9960152	53	28	·0952666	·095701	10·44911	·9954517	32	48	·1010563	·101576	9·844816	·9948807	12
8	·0894738	089834	11·13163	·9959892	52	29	·0955562	·095995	10·41715	·9954240	31	49	·1013457	·101870	9·816414	·9948513	11
9	·0897635	·090127	11·09541	·9959631	51	30	·0958458	·096289	10·38539	·9953962	30	50	·1016351	·102164	9·788173	·9948217	10
10	·0900532	·090420	11·05943	·9959370	50	31	·0961353	·096582	10·35382	·9953683	29	51	·1019245	·102458	9·760092	·9947921	9
11	·0903429	·090713	11·02367	·9959107	49	32	·0964248	·096876	10·32244	·9953403	28	52	·1022138	·102752	9·732171	·9947625	8
12	·0906326	·091007	10·98815	·9958844	48	33	·0967144	·097169	10·29125	·9953122	27	53	·1025032	·103046	9·704407	·9947327	7
13	·0909223	·091300	10·95285	·9958580	47	34	·0970039	·097463	10·26024	·9952840	26	54	·1027925	·103339	9·676800	·9947028	6
14	·0912119	·091593	10·91777	·9958315	46	35	·0972934	·097757	10·22942	·9952557	25	55	·1030819	·103634	9·649347	·9946729	5
15	·0915016	·091887	10·88292	·9958049	45	36	·0975829	·098050	10·19878	·9952274	24	56	·1033712	·103928	9·622048	·9946428	4
16	·0917913	·092180	10·84828	·9957783	44	37	·0978724	·098344	10·16833	·9951990	23	57	·1036605	·104222	9·594902	·9946127	3
17	·0920809	·092473	10·81387	·9957515	43	38	·0981619	·098638	10·13805	·9951705	22	58	·1039499	·104516	9·567906	·9945825	2
18	·0923706	·092767	10·77967	·9957247	42	39	·0984514	·098932	10·10795	·9951419	21	59	·1042392	·104810	9·541061	·9945523	1
19	·0926602	·093060	10·74568	·9956978	41	40	·0987408	·099225	10·07803	·9951132	20	60	·1045285	·105104	9·514364	·9945219	0
20	·0929199	·093354	10·71191	·9956708	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′
				Deg. 84.						Deg. 84.						Deg. 84.	

NATURAL SINES AND TANGENTS TO A RADIUS 1.

′	Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	6 Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	6 Deg. Sine.	Tang.	Cotang.	Cosine.	′
0	·1045285	·105104	9·514364	·9945219	60	21	·1106017	·111284	8·985984	·9938648	39	11	·1163818	·117178	8·534017	·9932045	19
1	·1048178	·105398	9·487814	·9944914	59	22	·1108908	·111578	8·962266	·9938326	38	12	·1166707	·117473	8·512594	·9931706	18
2	·1051070	·105692	9·461411	·9944609	58	23	·1111799	·111873	8·938672	·9938003	37	13	·1169596	·117767	8·491277	·9931367	17
3	·1053963	·105986	9·435153	·9944303	57	24	·1114689	·112168	8·915200	·9937679	36	14	·1172485	·118062	8·470065	·9931026	16
4	·1056856	·106280	9·409038	·9943996	56	25	·1117580	·112462	8·891850	·9937355	35	15	·1175374	·118357	8·448957	·9930685	15
5	·1059748	·106575	9·383066	·9943688	55	26	·1120471	·112757	8·868620	·9937029	34	16	·1178263	·118652	8·427953	·9930342	14
6	·1062641	·106869	9·357235	·9943379	54	27	·1123361	·113051	8·845510	·9936703	33	17	·1181151	·118947	8·407051	·9929999	13
7	·1065533	·107163	9·331545	·9943070	53	28	·1126252	·113346	8·822518	·9936375	32	18	·1184040	·119242	8·386251	·9929655	12
8	·1068425	·107457	9·305993	·9942760	52	29	·1129142	·113641	8·799644	·9936047	31	19	·1186928	·119537	8·365553	·9929310	11
9	·1071318	·107751	9·280580	·9942448	51	30	·1132032	·113935	8·776887	·9935719	30	50	·1189816	·119832	8·344955	·9928965	10
10	·1074210	·108046	9·255303	·9942136	50	31	·1134922	·114230	8·754246	·9935389	29	51	·1192704	·120127	8·324457	·9928618	9
11	·1077102	·108340	9·230162	·9941823	49	32	·1137812	·114525	8·731719	·9935058	28	52	·1195593	·120423	8·304058	·9928271	8
12	·1079994	·108634	9·205156	·9941510	48	33	·1140702	·114819	8·709307	·9934727	27	53	·1198481	·120718	8·283757	·9927922	7
13	·1082885	·108929	9·180283	·9941195	47	34	·1143592	·115114	8·687008	·9934395	26	54	·1201368	·121013	8·263554	·9927573	6
14	·1085777	·109223	9·155543	·9940880	46	35	·1146482	·115409	8·664822	·9934062	25	55	·1204256	·121308	8·243448	·9927224	5
15	·1088669	·109517	9·130934	·9940563	45	36	·1149372	·115703	8·642747	·9933728	24	56	·1207144	·121603	8·223488	·9926873	4
16	·1091560	·109812	9·106456	·9940246	44	37	·1152261	·115998	8·620783	·9933393	23	57	·1210031	·121898	8·203523	·9926521	3
17	·1094452	·110106	9·082107	·9939928	43	38	·1155151	·116293	8·598929	·9933057	22	58	·1212919	·122194	8·183704	·9926169	2
18	·1097343	·110401	9·057886	·9939610	42	39	·1158040	·116588	8·577183	·9932721	21	59	·1215806	·122489	8·163978	·9925816	1
19	·1100234	·110695	9·033793	·9939290	41	40	·1160929	·116883	8·555546	·9932384	20	60	·1218693	·122784	8·144346	·9925462	0
20	·1103126	·110989	9·009826	·9938969	40												

Deg.							7 Deg.						7 Deg.					
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1218693	·122784	8·144346	·9925462	60	21	·1279302	·128990	7·752536	·9917832	39	41	·1336979	·134909	7·412397	·9910221	19
1	·1221581	·123079	8·124807	·9925107	59	22	·1282186	·129285	7·734802	·9917459	38	42	·1339862	·135205	7·396159	·9909832	18
2	·1224468	·123375	8·105359	·9924751	58	23	·1285071	·129581	7·717148	·9917086	37	43	·1342744	·135501	7·379990	·9909442	17
3	·1227355	·123670	8·086004	·9924394	57	24	·1287956	·129877	7·699573	·9916712	36	44	·1345627	·135797	7·363891	·9909051	16
4	·1230241	·123965	8·066739	·9924037	56	25	·1290841	·130173	7·682076	·9916337	35	45	·1348509	·136094	7·347861	·9908659	15
5	·1233128	·124261	8·047564	·9923679	55	26	·1293725	·130469	7·664658	·9915961	34	46	·1351392	·136390	7·331898	·9908266	14
6	·1236015	·124556	8·028479	·9923319	54	27	·1296609	·130764	7·647317	·9915584	33	47	·1354274	·136686	7·316004	·9907873	13
7	·1238901	·124852	8·009483	·9922959	53	28	·1299494	·131060	7·630053	·9915206	32	48	·1357156	·136983	7·300178	·9907478	12
8	·1241788	·125147	7·990575	·9922599	52	29	·1302378	·131356	7·612865	·9914828	31	49	·1360038	·137279	7·284418	·9907083	11
9	·1244674	·125442	7·971755	·9922237	51	30	·1305262	·131652	7·595754	·9914449	30	50	·1362919	·137575	7·268725	·9906687	10
10	·1247560	·125738	7·953022	·9921874	50	31	·1308146	·131948	7·578717	·9914069	29	51	·1365801	·137872	7·253098	·9906290	9
11	·1250446	·126033	7·934375	·9921511	49	32	·1311030	·132244	7·561756	·9913688	28	52	·1368683	·138168	7·237537	·9905893	8
12	·1253332	·126329	7·915815	·9921147	48	33	·1313913	·132540	7·544869	·9913306	27	53	·1371564	·138465	7·222042	·9905494	7
13	·1256218	·126624	7·897339	·9920782	47	34	·1316797	·132836	7·528057	·9912923	26	54	·1374445	·138761	7·206611	·9905095	6
14	·1259104	·126920	7·878948	·9920416	46	35	·1319681	·133132	7·511317	·9912540	25	55	·1377327	·139058	7·191245	·9904694	5
15	·1261990	·127216	7·860642	·9920049	45	36	·1322564	·133428	7·494651	·9912155	24	56	·1380208	·139354	7·175943	·9904293	4
16	·1264875	·127511	7·842419	·9919682	44	37	·1325447	·133724	7·478057	·9911770	23	57	·1383089	·139651	7·160705	·9903891	3
17	·1267761	·127807	7·824279	·9919314	43	38	·1328330	·134020	7·461535	·9911384	22	58	·1385970	·139947	7·145530	·9903489	2
18	·1270646	·128103	7·806221	·9918944	42	39	·1331213	·134316	7·445085	·9910997	21	59	·1388850	·140244	7·130419	·9903085	1
19	·1273531	·128398	7·788245	·9918574	41	40	·1334096	·134612	7·428706	·9910610	20	60	·1391731	·140540	7·115369	·9902681	0
20	·1276416	·128694	7·770350	·9918204	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′
				Deg. 82.						Deg. 82.						Deg. 82.	

8 Deg.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

8 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1391731	·140540	7·115369	·9902681	60	21	·1452197	·146775	6·813122	·9893994	39	41	·1509733	·152723	6·547767	·9885378	19
1	·1394612	·140837	7·100382	·9902275	59	22	·1455075	·147072	6·799356	·9893572	38	42	·1512608	·153021	6·535029	·9884939	18
2	·1397492	·141134	7·085457	·9901869	58	23	·1457953	·147369	6·785644	·9893148	37	43	·1515484	·153319	6·522339	·9884498	17
3	·1400372	·141430	7·070593	·9901462	57	24	·1460830	·147667	6·771986	·9892723	36	44	·1518359	·153617	6·509698	·9884057	16
4	·1403252	·141727	7·055790	·9901055	56	25	·1463708	·147964	6·758382	·9892298	35	45	·1521234	·153914	6·497104	·9883615	15
5	·1406132	·142024	7·041048	·9900646	55	26	·1466585	·148261	6·744831	·9891872	34	46	·1524109	·154212	6·484558	·9883172	14
6	·1409012	·142321	7·026366	·9900237	54	27	·1469463	·148559	6·731334	·9891445	33	47	·1526984	·154510	6·472059	·9882728	13
7	·1411892	·142617	7·011744	·9899826	53	28	·1472340	·148856	6·717889	·9891017	32	48	·1529858	·154808	6·459607	·9882284	12
8	·1414772	·142914	6·997180	·9899415	52	29	·1475217	·149153	6·704496	·9890588	31	49	·1532733	·155106	6·447201	·9881838	11
9	·1417651	·143211	6·982678	·9899003	51	30	·1478094	·149451	6·691156	·9890159	30	50	·1535607	·155404	6·434842	·9881392	10
10	·1420531	·143508	6·968233	·9898590	50	31	·1480971	·149748	6·677867	·9889728	29	51	·1538482	·155701	6·422530	·9880945	9
11	·1423410	·143805	6·953847	·9898177	49	32	·1483848	·150045	6·664630	·9889297	28	52	·1541356	·155999	6·410263	·9880497	8
12	·1426289	·144102	6·939519	·9897762	48	33	·1486724	·150343	6·651444	·9888865	27	53	·1544230	·156297	6·398042	·9880048	7
13	·1429168	·144399	6·925248	·9897347	47	34	·1489601	·150640	6·638310	·9888432	26	54	·1547104	·156595	6·385866	·9879599	6
14	·1432047	·144696	6·911035	·9896931	46	35	·1492477	·150938	6·625225	·9887998	25	55	·1549978	·156893	6·373735	·9879148	5
15	·1434926	·144993	6·896879	·9896514	45	36	·1495353	·151235	6·612191	·9887564	24	56	·1552851	·157191	6·361650	·9878697	4
16	·1437805	·145290	6·882780	·9896096	44	37	·1498230	·151533	6·599208	·9887128	23	57	·1555725	·157490	6·349609	·9878245	3
17	·1440684	·145587	6·868737	·9895677	43	38	·1501106	·151830	6·586273	·9886692	22	58	·1558598	·157788	6·337612	·9877792	2
18	·1443562	·145884	6·854750	·9895258	42	39	·1503981	·152128	6·573389	·9886255	21	59	·1561472	·158086	6·325660	·9877338	1
19	·1446440	·146181	6·840819	·9894838	41	40	·1506857	·152426	6·560553	·9885817	20	60	·1564345	·158384	6·313751	·9876883	0
20	·1449319	·146478	6·826943	·9894416	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 81 Deg. 81 Deg. 81

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	Deg.					9 Deg.						9 Deg.					
0	·1564345	·158384	6·313751	·9876883	60	21	·1624650	·164652	6·073397	·9867143	39	41	·1682026	·170633	5·860505	·9857524	19
1	·1567218	·158682	6·301886	·9876428	59	22	·1627520	·164951	6·062396	·9866670	38	42	·1684894	·170933	5·850241	·9857035	18
2	·1570091	·158980	6·290065	·9875972	58	23	·1630390	·165250	6·051434	·9866196	37	43	·1687761	·171232	5·840011	·9856544	17
3	·1572963	·159279	6·278286	·9875514	57	24	·1633260	·165548	6·040510	·9865722	36	44	·1690628	·171532	5·829817	·9856053	16
4	·1575836	·159577	6·266551	·9875057	56	25	·1636129	·165847	6·029624	·9865246	35	45	·1693495	·171831	5·819657	·9855561	15
5	·1578708	·159875	6·254858	·9874598	55	26	·1638999	·166146	6·018777	·9864770	34	46	·1696362	·172130	5·809531	·9855068	14
6	·1581581	·160174	6·243208	·9874138	54	27	·1641868	·166445	6·007967	·9864293	33	47	·1699228	·172430	5·799440	·9854574	13
7	·1584453	·160472	6·231600	·9873678	53	28	·1644738	·166744	5·997195	·9863815	32	48	·1702095	·172730	5·789382	·9854079	12
8	·1587325	·160770	6·220034	·9873216	52	29	·1647607	·167043	5·986461	·9863336	31	49	·1704961	·173029	5·779358	·9853583	11
9	·1590197	·161069	6·208510	·9872754	51	30	·1650476	·167342	5·975764	·9862856	30	50	·1707828	·173329	5·769368	·9853087	10
10	·1593069	·161367	6·197027	·9872291	50	31	·1653345	·167641	5·965104	·9862375	29	51	·1710694	·173628	5·759412	·9852590	9
11	·1595940	·161666	6·185586	·9871827	49	32	·1656214	·167940	5·954481	·9861894	28	52	·1713560	·173928	5·749488	·9852092	8
12	·1598812	·161964	6·174186	·9871363	48	33	·1659082	·168239	5·943895	·9861412	27	53	·1716425	·174228	5·739598	·9851593	7
13	·1601683	·162263	6·162827	·9870897	47	34	·1661951	·168539	5·933345	·9860929	26	54	·1719291	·174527	5·729741	·9851093	6
14	·1604555	·162561	6·151508	·9870431	46	35	·1664819	·168838	5·922832	·9860445	25	55	·1722156	·174827	5·719917	·9850593	5
15	·1607426	·162860	6·140230	·9869964	45	36	·1667687	·169137	5·912355	·9859960	24	56	·1725022	·175127	5·710125	·9850091	4
16	·1610297	·163159	6·128992	·9869496	44	37	·1670556	·169436	5·901913	·9859475	23	57	·1727887	·175427	5·700366	·9849589	3
17	·1613167	·163457	6·117794	·9869027	43	38	·1673423	·169735	5·891508	·9858988	22	58	·1730752	·175727	5·690639	·9849086	2
18	·1616038	·163756	6·106636	·9868557	42	39	·1676291	·170035	5·881138	·9858501	21	59	·1733617	·176027	5·680944	·9848582	1
19	·1618909	·164055	6·095517	·9868087	41	40	·1679159	·170334	5·870804	·9858013	20	60	·1736482	·176327	5·671281	·9848078	0
20	·1621779	·164353	6·084438	·9867615	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′
			Deg. 80.						Deg. 80.							Deg. 80.	

10 Deg.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

10 Deg.

10 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1736482	·176327	5·671281	·9848078	60	21	·1796607	·182632	5·475478	·9837286	39	41	·1853808	·188650	5·300801	·9826668	19
1	·1739346	·176626	5·661650	·9847572	59	22	·1799469	·182933	5·466481	·9836763	38	42	·1856666	·188952	5·292350	·9826128	18
2	·1742211	·176926	5·652051	·9847066	58	23	·1802330	·183233	5·457512	·9836239	37	43	·1859524	·189253	5·283925	·9825587	17
3	·1745075	·177226	5·642483	·9846558	57	24	·1805191	·183534	5·448571	·9835715	36	44	·1862382	·189554	5·275525	·9825046	16
4	·1747939	·177527	5·632947	·9846050	56	25	·1808052	·183835	5·439659	·9835189	35	45	·1865240	·189855	5·267151	·9824504	15
5	·1750803	·177827	5·623442	·9845542	55	26	·1810913	·184135	5·430775	·9834663	34	46	·1868098	·190157	5·258803	·9823961	14
6	·1753667	·178127	5·613968	·9845032	54	27	·1813774	·184436	5·421918	·9834136	33	47	·1870956	·190458	5·250480	·9823417	13
7	·1756531	·178427	5·604524	·9844521	53	28	·1816635	·184737	5·413090	·9833608	32	48	·1873813	·190760	5·242183	·9822873	12
8	·1759395	·178727	5·595112	·9844010	52	29	·1819495	·185038	5·404290	·9833079	31	49	·1876670	·191061	5·233911	·9822327	11
9	·1762258	·179027	5·585730	·9843498	51	30	·1822355	·185339	5·395517	·9832549	30	50	·1879528	·191363	5·225664	·9821781	10
10	·1765121	·179327	5·576378	·9842985	50	31	·1825215	·185639	5·386771	·9832019	29	51	·1882385	·191664	5·217442	·9821234	9
11	·1767984	·179628	5·567057	·9842471	49	32	·1828075	·185940	5·378053	·9831487	28	52	·1885241	·191966	5·209245	·9820686	8
12	·1770847	·179928	5·557766	·9841956	48	33	·1830935	·186241	5·369363	·9830955	27	53	·1888098	·192268	5·201073	·9820137	7
13	·1773710	·180228	5·548505	·9841441	47	34	·1833795	·186542	5·360699	·9830422	26	54	·1890954	·192569	5·192926	·9819587	6
14	·1776573	·180529	5·539274	·9840921	46	35	·1836654	·186843	5·352062	·9829888	25	55	·1893811	·192871	5·184803	·9819037	5
15	·1779435	·180829	5·530072	·9840407	45	36	·1839514	·187144	5·343452	·9829353	24	56	·1896667	·193173	5·176705	·9818485	4
16	·1782298	·181129	5·520900	·9839889	44	37	·1842373	·187446	5·334869	·9828818	23	57	·1899523	·193474	5·168631	·9817933	3
17	·1785160	·181430	5·511757	·9839370	43	38	·1845232	·187747	5·326313	·9828282	22	58	·1902379	·193776	5·160581	·9817380	2
18	·1788022	·181730	5·502644	·9838850	42	39	·1848091	·188048	5·317783	·9827744	21	59	·1905234	·194078	5·152555	·9816826	1
19	·1790884	·182031	5·493560	·9838330	41	40	·1850949	·188349	5·309279	·9827206	20	60	·1908090	·194380	5·144554	·9816272	0
20	·1793746	·182331	5·484505	·9837808	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 79.

Deg. 79.

Deg. 79.

′	Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	11 Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	11 Deg. Sine.	Tang.	Cotang.	Cosine.	′
0	·1908090	·194380	5·144554	·9816272	60	21	·1968018	·200727	4·981881	·9804433	39	11	·2025024	·206786	4·835901	·9792818	19
1	·1910945	·194682	5·136576	·9815716	59	22	·1970870	·201030	4·974381	·9803860	38	12	·2027873	·207090	4·828817	·9792228	18
2	·1913801	·194984	5·128622	·9815160	58	23	·1973722	·201332	4·966903	·9803286	37	13	·2030721	·207393	4·821753	·9791638	17
3	·1916656	·195286	5·120692	·9814603	57	24	·1976573	·201635	4·959447	·9802712	36	14	·2033569	·207696	4·814709	·9791047	16
4	·1919510	·195588	5·112785	·9814045	56	25	·1979425	·201938	4·952012	·9802136	35	15	·2036418	·208000	4·807685	·9790455	15
5	·1922365	·195890	5·104902	·9813486	55	26	·1982276	·202240	4·944599	·9801560	34	16	·2039265	·208303	4·800680	·9789862	14
6	·1925220	·196192	5·097042	·9812927	54	27	·1985127	·202543	4·937206	·9800983	33	17	·2042113	·208607	4·793695	·9789268	13
7	·1928074	·196494	5·089206	·9812366	53	28	·1987978	·202846	4·929835	·9800405	32	18	·2044961	·208910	4·786730	·9788674	12
8	·1930928	·196796	5·081392	·9811805	52	29	·1990829	·203149	4·922485	·9799827	31	19	·2047808	·209214	4·779783	·9788079	11
9	·1933782	·197098	5·073602	·9811243	51	30	·1993679	·203452	4·915157	·9799247	30	50	·2050655	·209518	4·772856	·9787483	10
10	·1936636	·197400	5·065835	·9810680	50	31	·1996530	·203755	4·907849	·9798667	29	51	·2053502	·209821	4·765949	·9786886	9
11	·1939490	·197703	5·058090	·9810116	49	32	·1999380	·204058	4·900562	·9798086	28	52	·2056349	·210125	4·759060	·9786288	8
12	·1942344	·198005	5·050369	·9809552	48	33	·2002230	·204361	4·893295	·9797504	27	53	·2059195	·210429	4·752190	·9785689	7
13	·1945197	·198307	5·042670	·9808986	47	34	·2005080	·204664	4·886049	·9796921	26	54	·2062042	·210733	4·745340	·9785090	6
14	·1948050	·198610	5·034993	·9808420	46	35	·2007930	·204967	4·878824	·9796337	25	55	·2064888	·211036	4·738508	·9784490	5
15	·1950903	·198912	5·027339	·9807853	45	36	·2010779	·205270	4·871620	·9795752	24	56	·2067734	·211340	4·73.695	·9783889	4
16	·1953756	·199214	5·019707	·9807285	44	37	·2013629	·205573	4·864435	·9795167	23	57	·2070580	·211644	4·724901	·9783287	3
17	·1956609	·199517	5·012098	·9806716	43	38	·2016478	·205876	4·857271	·9794581	22	58	·2073426	·211948	4·718125	·9782684	2
18	·1959461	·199819	5·004511	·9806147	42	39	·2019327	·206180	4·850128	·9793994	21	59	·2076272	·212252	4·711368	·9782080	1
19	·1962314	·200122	4·996945	·9805576	41	40	·2022176	·206483	4·843004	·9793406	20	60	·2079117	·212556	4·704630	·9781476	0
20	·1965166	·200424	4·989402	·9805005	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cot ng.	Tang.	Sine.	′
			Deg. 78.						Deg. 78.							Deg. 78.	

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
		12 Deg.						12 Deg.									
0	·2079117	·212550	4·704630	·9781476	60	21	·2138829	·218949	4·567261	·9768593	39	41	·2195624	·225054	4·443376	·9755985	19
1	·2081962	·212860	4·697910	·9780871	59	22	·2141671	·219254	4·560911	·9767970	38	42	·2198462	·225359	4·437350	·9755345	18
2	·2084807	·213164	4·691208	·9780265	58	23	·2144512	·219559	4·554577	·9767347	37	43	·2201300	·225665	4·431339	·9754706	17
3	·2087652	·213468	4·684524	·9779658	57	24	·2147353	·219864	4·548260	·9766723	36	44	·2204137	·225971	4·425343	·9754065	16
4	·2090497	·213773	4·677859	·9779050	56	25	·2150194	·220169	4·541960	·9766098	35	45	·2206974	·226276	4·419364	·9753423	15
5	·2093341	·214077	4·671212	·9778441	55	26	·2153035	·220474	4·535677	·9765472	34	46	·2209811	·226582	4·413399	·9752781	14
6	·2096186	·214381	4·664583	·9777832	54	27	·2155876	·220779	4·529410	·9764845	33	47	·2212648	·226888	4·407450	·9752138	13
7	·2099030	·214685	4·657972	·9777222	53	28	·2158716	·221084	4·523160	·9764217	32	48	·2215485	·227194	4·401516	·9751494	12
8	·2101874	·214990	4·651378	·9776611	52	29	·2161556	·221389	4·516926	·9763589	31	49	·2218321	·227500	4·395597	·9750849	11
9	·2104718	·215294	4·644803	·9775999	51	30	·2164396	·221694	4·510708	·9762960	30	50	·2221158	·227806	4·389694	·9750203	10
10	·2107561	·215598	4·638245	·9775386	50	31	·2167236	·221999	4·504507	·9762330	29	51	·2223994	·228112	4·383805	·9749556	9
11	·2110405	·215903	4·631705	·9774773	49	32	·2170076	·222305	4·498322	·9761699	28	52	·2226830	·228418	4·377931	·9748909	8
12	·2113248	·216207	4·625183	·9774159	48	33	·2172915	·222610	4·492153	·9761067	27	53	·2229666	·228724	4·372073	·9748261	7
13	·2116091	·216512	4·618678	·9773544	47	34	·2175754	·222915	4·486000	·9760435	26	54	·2232501	·229030	4·366229	·9747612	6
14	·2118934	·216816	4·612190	·9772928	46	35	·2178593	·223221	4·479863	·9759802	25	55	·2235337	·229336	4·360400	·9746962	5
15	·2121777	·217121	4·605720	·9772311	45	36	·2181432	·223526	4·473742	·9759168	24	56	·2238172	·229642	4·354586	·9746311	4
16	·2124619	·217425	4·599268	·9771693	44	37	·2184271	·223831	4·467637	·9758533	23	57	·2241007	229949	4·348786	·9745660	3
17	·2127462	·217730	4·592832	·9771075	43	38	·2187110	·224137	4·461548	·9757897	22	58	·2243842	·230255	4·343001	·9745008	2
18	·2130304	·218035	4·586414	·9770456	42	39	·2189948	·224442	4·455475	·9757260	21	59	·2246676	·230561	4·337231	·9744355	1
19	·2133146	·218340	4·580012	·9769836	41	40	·2192786	·224748	4·449418	·9756623	20	60	·2249511	·230868	4·331475	·9743701	0
	·2135988	·218644	4·573628	·9769215	40												

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	13 Deg.						13 Deg.										
0	·2249511	·230868	4·331475	·9743701	60	21	·2308989	·237311	4·213869	·9729777	39	41	·2365555	·243465	4·107356	·9716180	19
1	·2252345	·231174	4·325734	·9743046	59	22	·2311819	·237618	4·208419	·9729105	38	42	·2368381	·243773	4·102164	·9715491	18
2	·2255179	·231481	4·320007	·9742390	58	23	·2314649	·237926	4·202983	·9728432	37	43	·2371207	·244081	4·096985	·9714802	17
3	·2258013	·231787	4·314295	·9741734	57	24	·2317479	·238233	4·197560	·9727759	36	44	·2374033	·244390	4·091817	·9714112	16
4	·2260846	·232094	4·308597	·9741077	56	25	·2320309	·238541	4·192151	·9727084	35	45	·2376859	·244698	4·086662	·9713421	15
5	·2263680	·232400	4·302913	·9740419	55	26	·2323138	·238848	4·186754	·9726409	34	46	·2379684	·245006	4·081519	·9712729	14
6	·2266513	·232707	4·297244	·9739760	54	27	·2325967	·239156	4·181371	·9725733	33	47	·2382510	·245315	4·076389	·9712036	13
7	·2269346	·233014	4·291588	·9739100	53	28	·2328796	·239463	4·176001	·9725056	32	48	·2385335	·245623	4·071270	·9711343	12
8	·2272179	·233320	4·285947	·9738439	52	29	·2331625	·239771	4·170644	·9724378	31	49	·2388159	·245932	4·066164	·9710649	11
9	·2275012	·233627	4·280319	·9737778	51	30	·2334454	·240078	4·165299	·9723699	30	50	·2390984	·246240	4·061070	·9709953	10
10	·2277844	·233934	4·274706	·9737116	50	31	·2337282	·240386	4·159968	·9723020	29	51	·2393808	·246549	4·055987	·9709258	9
11	·2280677	·234241	4·269107	·9736453	49	32	·2340110	·240694	4·154650	·9722339	28	52	·2396633	·246857	4·050917	·9708561	8
12	·2283509	·234547	4·263521	·9735789	48	33	·2342938	·241001	4·149344	·9721658	27	53	·2399457	·247166	4·045859	·9707863	7
13	·2286341	·234854	4·257950	·9735124	47	34	·2345766	·241309	4·144051	·9720976	26	54	·2402280	·247475	4·040812	·9707165	6
14	·2289172	·235161	4·252392	·9734458	46	35	·2348594	·241617	4·138771	·9720294	25	55	·2405104	·247783	4·035777	·9706466	5
15	·2292004	·235468	4·246848	·9733792	45	36	·2351421	·241925	4·133504	·9719610	24	56	·2407927	·248092	4·030755	·9705766	4
16	·2294835	·235775	4·241317	·9733125	44	37	·2354248	·242233	4·128249	·9718926	23	57	·2410751	·248401	4·025744	·9705065	3
17	·2297666	·236082	4·235800	·973 457	43	38	·2357075	·242541	4·123007	·9718240	22	58	·2413574	·248710	4·020744	·9704363	2
18	·2300497	·236390	4·230297	·9731789	42	39	·2359902	·242849	4·117778	·9717554	21	59	·2416396	·249019	4·015757	·9703660	1
19	·2303328	·236697	4·224808	·9731119	41	40	·2362729	·243157	4·112561	·9716867	20	60	·2419219	·249328	4·010780	·9702957	0
20	·2306159	·237004	4·219331	·9730449	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
		Deg. 76.						Deg. 76.								Deg. 76	

NATURAL SINES AND TANGENTS TO A RADIUS 1.

14 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	14 Deg.						14 Deg.						14 Deg.				
0	·2419219	·249328	4.010780	·9702957	60	21	·2478445	·255826	3·908901	·9687998	39	41	·2534766	·262034	3·816295	·9673415	19
1	·2422041	·249637	4·005816	·9702253	59	22	·2481263	·256136	3·904171	·9687277	38	42	·2537579	·262345	3·811773	·9672678	18
2	·2424863	·249946	4·000863	·9701548	58	23	·2484081	·256446	3·899451	·9686555	37	43	·2540393	·262656	3·807260	·9671939	17
3	·2427685	·250255	3·995922	·9700842	57	24	·2486899	·256756	3·894742	·9685832	36	44	·2543206	·262967	3·802758	·9671200	16
4	·2430507	·250564	3·990992	·9700135	56	25	·2489716	·257066	3·890044	·9685108	35	45	·2546019	·263278	3·798266	·9670459	15
5	·2433329	·250873	3·986073	·9699428	55	26	·2492533	·257376	3·885357	·9684383	34	46	·2548832	·263589	3·793783	·9669718	14
6	·2436150	·251182	3·981166	·9698720	54	27	·2495350	·257686	3·880680	·9683658	33	47	·2551645	·263900	3·789310	·9668977	13
7	·2438971	·251491	3·976271	·9698011	53	28	·2498167	·257997	3·876014	·9682931	32	48	·2554458	·264211	3·784848	·9668234	12
8	·2441792	·251801	3·971386	·9697301	52	29	·2500984	·258307	3·871358	·9682204	31	49	·2557270	·264522	3·780395	·9667490	11
9	·2444613	·252110	3·966513	·9696591	51	30	·2503800	·258617	3·866713	·9681476	30	50	·2560082	·264833	3·775951	·9666746	10
10	·2447433	·252420	3·961651	·9695879	50	31	·2506616	·258928	3·862078	·9680748	29	51	·2562894	·265145	3·771518	·9666001	9
11	·2450254	·252729	3·956801	·9695167	49	32	·2509432	·259238	3·857453	·9680018	28	52	·2565705	·265456	3·767094	·9665255	8
12	·2453074	·253038	3·951961	·9694453	48	33	·2512248	·259548	3·852839	·9679288	27	53	·2568517	·265768	3·762680	·9664508	7
13	·2455894	·253348	3·947133	·9693740	47	34	·2515063	·259859	3·848235	·9678557	26	54	·2571328	·266079	3·758276	·9663761	6
14	·2458713	·253658	3·942315	·9693025	46	35	·2517879	·260169	3·843642	·9677825	25	55	·2574139	·266390	3·753881	·9663012	5
15	·2461533	·253967	3·937509	·9692309	45	36	·2520694	·260480	3·839059	·9677092	24	56	·2576950	·266702	3·749496	·9662263	4
16	·2464352	·254277	3·932714	·9691593	44	37	·2523508	·260791	3·834486	·9676358	23	57	·2579760	·267014	3·745120	·9661513	3
17	·2467171	·254587	3·927929	·9690875	43	38	·2526323	·261101	3·829923	·9675624	22	58	·2582570	·267325	3·740754	·9660762	2
18	·2469990	·254896	3·923156	·9690157	42	39	·2529137	·261412	3·825370	·9674888	21	59	·2585381	·267637	3·736398	·9660011	1
19	·2472809	·255206	3·918393	·9689438	41	40	·2531952	·261723	3·820828	·9674152	20	60	·2588190	·267949	3·732050	·9659258	0
20	·2475627	·255516	3·913642	·9688719	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
			Deg. 75.						Deg. 75.							Deg. 75.	

Deg.	15 Deg.						15 Deg.									
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2588190	·267949	3·732050	·9659258	60	21	·2647147	·274507	3·642891	·9643268	39	41	·2703204	·280773	3·561590	·9627704	19
1	·2591000	·268261	3·727713	·9658505	59	22	·2649952	·274820	3·638744	·9642497	38	42	·2706004	·281087	3·557613	·9626917	18
2	·2593810	·268572	3·723384	·9657751	58	23	·2652757	·275133	3·634606	·9641726	37	43	·2708805	·281401	3·553644	·9626130	17
3	·2596619	·268884	3·719065	·9656996	57	24	·2655561	·275445	3·630477	·9640954	36	44	·2711605	·281715	3·549684	·9625342	16
4	·2599428	·269196	3·714756	·9656240	56	25	·2658366	·275758	3·626356	·9640181	35	45	·2714404	·282029	3·545732	·9624552	15
5	·2602237	·269508	3·710455	·9655484	55	26	·2661170	·276071	3·622244	·9639407	34	46	·2717204	·282343	3·541788	·9623762	14
6	·2605045	·269820	3·706164	·9654726	54	27	·2663973	·276385	3·618141	·9638633	33	47	·2720003	·282657	3·537852	·9622972	13
7	·2607853	·270132	3·701883	·9653968	53	28	·2666777	·276698	3·614046	·9637858	32	48	·2722802	·282971	3·533925	·9622180	12
8	·2610662	·270444	3·697610	·9653209	52	29	·2669581	·277011	3·609960	·9637081	31	49	·2725601	·283285	3·530005	·9621387	11
9	·2613469	·270757	3·693346	·9652449	51	30	·2672384	·277324	3·605883	·9636305	30	50	·2728400	·283599	3·526093	·9620594	10
10	·2616277	·271069	3·689092	·9651689	50	31	·2675187	·277637	3·601814	·9635527	29	51	·2731198	·283914	3·522190	·9619800	9
11	·2619085	·271381	3·684847	·9650927	49	32	·2677989	·277951	3·597754	·9634748	28	52	·2733997	·284228	3·518294	·9619005	8
12	·2621892	·271694	3·680611	·9650165	48	33	·2680792	·278264	3·593702	·9633969	27	53	·2736794	·284543	3·514407	·9618210	7
13	·2624699	·272006	3·676384	·9649402	47	34	·2683594	·278578	3·589659	·9633189	26	54	·2739592	·284857	3·510527	·9617413	6
14	·2627506	·272318	3·672166	·9648638	46	35	·2686396	·278891	3·585624	·9632408	25	55	·2742390	·285172	3·506655	·9616616	5
15	·2630312	·272631	3·667957	·9647873	45	36	·2689198	·279205	3·581597	·9631626	24	56	·2745187	·285486	3·502791	·9615818	4
16	·2633118	·272943	3·663757	·9647108	44	37	·2692000	·279518	3·577579	·9630843	23	57	·2747984	·285801	3·498935	·9615019	3
17	·2635925	·273256	3·659566	·9646341	43	38	·2694801	·279832	3·573569	·9630060	22	58	·2750781	·286115	3·495087	·9614219	2
18	·2638730	·273569	3·655384	·9645574	42	39	·2697602	·280145	3·569568	·9629275	21	59	·2753577	·286430	3·491247	·9613418	1
19	·2641536	·273881	3·651211	·9644806	41	40	·2700403	·280459	3·565574	·9628490	20	60	·2756374	·286745	3·487414	·9612617	0
20	·2644342	·274194	3·647046	·9644037	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
		Deg. 74.						Deg. 74.							Deg. 74.		

16 Deg.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	16 Deg.						16 Deg.						16 Deg.				
0	·2756374	·286745	3·487414	·9612617	60	21	·2815042	·293368	3·408688	·9595600	39	41	·2870819	·299697	3·336699	·9579060	19
1	·2759170	·287060	3·483589	·9611815	59	22	·2817833	·293683	3·405021	·9594781	38	42	·2873605	·300014	3·333173	·9578225	18
2	·2761965	·287375	3·479772	9611012	58	23	·2820624	·293999	3·401361	·9593961	37	43	·2876391	·300331	3·329654	·9577389	17
3	·2764761	·287690	3·475963	·9610208	57	24	·2823415	·294316	3·397708	·9593140	36	44	·2879177	·300648	3·326141	·9576552	16
4	·2767556	·288005	3·472161	·9609403	56	25	·2826205	·294632	3·394063	·9592318	35	45	·2881963	·300965	3·322636	·9575714	15
5	·2770352	·288320	3·468367	·9608598	55	26	·2828995	·294948	3·390424	·9591496	34	46	·2884748	·301283	3·319137	·9574875	14
6	·2773147	·288635	3·464581	·9607792	54	27	·2831785	·295264	3·386793	·9590672	33	47	·2887533	·301600	3·315645	·9574035	13
7	·2775941	·288950	3·460802	·9606984	53	28	·2834575	·295580	3·383169	·9589848	32	48	·2890318	·301917	3·312159	·9573195	12
8	·2778736	·289265	3·457031	·9606177	52	29	·2837364	·295897	3·379553	·9589023	31	49	·2893103	·302235	3·308681	·9572354	11
9	·2781530	·289580	3·453267	·9605368	51	30	·2840153	·296213	3·375943	·9588197	30	50	·2895887	·302552	3·305209	·9571512	10
10	·2784324	·289896	3·449512	·9604558	50	31	·2842942	·296529	3·372340	·9587371	29	51	·2898671	·302870	3·301743	·9570669	9
11	·2787118	·290211	3·445763	·9603748	49	32	·2845731	·296846	3·368745	·9586543	28	52	·2901455	·303187	3·298285	·9569825	8
12	·2789911	·290526	3·442022	·9602937	48	33	·2848520	·297163	3 365156	·9585715	27	53	·2904239	·303505	3·294833	·9568981	7
13	·2792704	·290842	3·438289	·9602125	47	34	·2851308	·297479	3·361575	·9584886	26	54	·2907022	·303823	3·291387	·9568136	6
14	·2795497	·291157	3·434563	·9601312	46	35	·2854096	·297796	3·358000	·9584056	25	55	·2909805	·304141	3·287948	·9567290	5
15	·2798290	·291473	3·430844	·9600499	45	36	·2856884	·298112	3·354433	·9583226	24	56	·2912588	·304458	3·284516	·9566443	4
16	·2801083	·291789	3·427133	·9599684	44	37	·2859671	·298429	3·350872	·9582394	23	57	·2915371	·304776	3·281090	·9565595	3
17	·2803875	·292104	3·423429	·9598869	43	38	·2862458	·298746	3·347319	·9581562	22	58	·2918153	·305094	3·277671	·9564747	2
18	·2806667	·292420	3·419733	·9598053	42	39	·2865246	·299063	3·343772	·9580729	21	59	·2920935	·305412	3·274258	·9563898	1
19	·2809459	·292736	3·416044	·9597236	41	40	·2868032	·299380	3·340232	·9579895	20	60	·2923717	·305730	3·270852	·9563048	0
20	·2812251	·293052	3·412362	·9596418	40												

′	Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	17 Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	17 Deg. Sine.	Tang.	Cotang.	Cosine.	′
0	·2923717	·305730	3·270852	·9563048	60	21	·2982079	·312422	3·200789	·9545009	39	11	·3037559	·318820	3·136563	·9527499	19
1	·2926499	·306048	3·267452	·9562197	59	22	·2984856	·312742	3·197521	·9544141	38	12	·3040331	·319140	3·133414	·9526615	18
2	·2929280	·306367	3·264059	·9561345	58	23	·2987632	·313061	3·194259	·9543273	37	13	·3043102	·319461	3·130270	·9525730	17
3	·2932061	·306685	3·260672	·9560492	57	24	·2990408	·313381	3·191003	·9542403	36	14	·3045872	·319781	3·127131	·9524844	16
4	·2934842	·307003	3·257292	·9559639	56	25	·2993184	·313700	3·187754	·9541533	35	15	·3048643	·320102	3·123999	·9523958	15
5	·2937623	·307321	3·253918	·9558785	55	26	·2995959	·314020	3·184510	·9540662	34	16	·3051413	·320423	3·120872	·9523071	14
6	·2940403	·307640	3·250550	·9557930	54	27	·2998734	·314339	3·181272	·9539790	33	17	·3054183	·320744	3·117750	·9522183	13
7	·2943183	·307958	3·247189	·9557074	53	28	·3001509	·314659	3·178040	·9538917	32	18	·3056953	·321064	3·114635	·9521294	12
8	·2945963	·308277	3·243834	·9556218	52	29	·3004284	·314979	3·174814	·9538044	31	19	·3059723	·321385	3·111525	·9520404	11
9	·2948743	·308595	3·240486	·9555361	51	30	·3007058	·315298	3·171594	·9537170	30	20	·3062492	·321706	3·108421	·9519514	10
10	·2951522	·308914	3·237143	·9554502	50	31	·3009832	·315618	3·168380	·9536294	29	21	·3065261	·322027	3·105322	·9518623	9
11	·2954302	·309233	3·233807	·9553643	49	32	·3012606	·315938	3·165172	·9535418	28	22	·3068030	·322348	3·102229	·9517731	8
12	·2957081	·309551	3·230478	·9552784	48	33	·3015380	·316258	3·161970	·9534542	27	23	·3070798	·322670	3·099141	·9516838	7
13	·2959859	·309870	3·227154	·9551923	47	34	·3018153	·316578	3·158774	·9533664	26	24	·3073566	·322991	3·096059	·9515944	6
14	·2962638	·310189	3·223837	·9551062	46	35	·3020926	·316898	3·155584	·9532786	25	25	·3076334	·323312	3·092983	·9515050	5
15	·2965416	·310508	3·220526	·9550199	45	36	·3023699	·317218	3·152399	·9531907	24	26	·3079102	·323633	3·089912	·9514154	4
16	·2968194	·310827	3·217221	·9549336	44	37	·3026471	·317538	3·149220	·9531027	23	27	·3081869	·323955	3·086846	·9513258	3
17	·2970971	·311146	3·213922	·9548473	43	38	·3029244	·317859	3·146047	·9530146	22	28	·3084636	·324276	3·083786	·9512361	2
18	·2973749	·311465	3·210630	·9547608	42	39	·3032016	·318179	3·142880	·9529264	21	29	·3087403	·324598	3·080732	·9511464	1
19	·2976526	·311784	3·207344	·9546743	41	40	·3034788	·318499	3·139719	·9528382	20	30	·3090170	·324919	3·077683	·9510565	0
20	·2979303	·312103	3·204063	·9545876	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
				Deg. 72.						Deg. 72.						Deg. 72.	

NATURAL SINES AND TANGENTS TO A RADIUS 1.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	18 Deg.						18 Deg.						18 Deg.				
0	·3090170	·324919	3·077683	·9510565	60	21	·3148209	·331686	3·014892	·9491511	39	41	·3203374	·338157	2·957205	·9473035	19
1	·3092936	·325241	3·074640	·9509666	59	22	·3150969	·332009	3·011960	·9490595	38	42	·3206130	·338481	2·954372	·9472103	18
2	·3095702	·325563	3·071602	·9508766	58	23	·3153730	·332332	3·009033	·9489678	37	43	·3208885	·338805	2·951545	·9471170	17
3	·3098468	·325884	3·068569	·9507865	57	24	·3156490	·332655	3·006110	·9488760	36	44	·3211640	·339129	2·948722	·9470236	16
4	·3101234	·326206	3·065542	·9506963	56	25	·3159250	·332978	3·003193	·9487842	35	45	·3214395	·339454	2·945905	·9469301	15
5	·3103999	·326528	3·062520	·9506061	55	26	·3162010	·333302	3·000282	·9486922	34	46	·3217149	·339778	2·943092	·9468366	14
6	·3106764	·326850	3·059503	·9505157	54	27	·3164770	·333625	2·997375	·9486002	33	47	·3219903	·340103	2·940284	·9467430	13
7	·3109529	·327172	3·056492	·9504253	53	28	·3167529	·333948	2·994473	·9485081	32	48	·3222657	·340427	2·937480	·9466493	12
8	·3112294	·327494	3·053487	·9503348	52	29	·3170288	·334271	2·991576	·9484159	31	49	·3225411	·340752	2·934682	·9465555	11
9	·3115058	·327816	3·050486	·9502443	51	30	·3173047	·334595	2·988685	·9483237	30	50	·3228164	·341077	2·931888	·9464616	10
10	·3117822	·328138	3·047491	·9501536	50	31	·3175805	·334918	2·985798	·9482313	29	51	·3230917	·341401	2·929099	·9463677	9
11	·3120586	·328461	3·044501	·9500629	49	32	·3178563	·335242	2·982916	·9481389	28	52	·3233670	·341726	2·926315	·9462736	8
12	·3123349	·328783	3·041517	·9499721	48	33	·3181321	·335566	2·980040	·9480464	27	53	·3236422	·342051	2·923535	·9461795	7
13	·3126112	·329105	3·038538	·9498812	47	34	·3184079	·335889	2·977168	·9479538	26	54	·3239174	·342376	2·920761	·9460854	6
14	·3128875	·329428	3·035564	·9497902	46	35	·3186836	·336213	2·974301	·9478612	25	55	·3241926	·342701	2·917990	·9459911	5
15	·3131638	·329750	3·032595	·9496991	45	36	·3189593	·336537	2·971439	·9477684	24	56	·3244678	·343026	2·915225	·9458968	4
16	·3134400	·330073	3·029632	·9496080	44	37	·3192350	·336861	2·968583	·9476756	23	57	·3247429	·343351	2·912464	·9458023	3
17	·3137163	·330395	3·026673	·9495168	43	38	·3195106	·337185	2·965731	·9475827	22	58	·3250180	·343677	2·909708	·9457078	2
18	·3139925	·330718	3·023720	·9494255	42	39	·3197863	·337509	2·962884	·9474897	21	59	·3252931	·344002	2·906957	·9456132	1
19	·3142686	·331041	3·020772	·9493341	41	40	·3200619	·337833	2·960042	·9473966	20	60	·3255682	·344327	2·904210	·9455186	0
20	·3145448	·331363	3·017830	·9492426	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
		Deg. 71.						Deg. 71.								Deg. 71.	

′	Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	19 Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	19 Deg. Sine.	Tang.	Cotang.	Cosine.	′
0	·3255682	·344327	2·904210	·9455186	60	21	·3313379	·351175	2·847583	·9435122	39	41	·3368214	·357723	2·795453	·9415686	19
1	·3258432	·344653	2·901468	·9454238	59	22	·3316123	·351501	2·844935	·9434157	38	42	·3370953	·358051	2·792891	·9414705	18
2	·3261182	·344978	2·898731	·9453290	58	23	·3318867	·351828	2·842292	·9433192	37	43	·3373691	·358380	2·790333	·9413724	17
3	·3263932	·345304	2·895998	·9452341	57	24	·3321611	·352155	2·839653	·9432227	36	44	·3376429	·358708	2·787780	·9412743	16
4	·3266681	·345629	2·893270	·9451391	56	25	·3324355	·352482	2·837019	·9431260	35	45	·3379167	·359036	2·785230	·9411760	15
5	·3269430	·345955	2·890546	·9450441	55	26	·3327098	·352809	2·834389	·9430293	34	46	·3381905	·359365	2·782685	·9410777	14
6	·3272179	·346281	2·887827	·9449489	54	27	·3329841	·353136	2·831763	·9429324	33	47	·3384642	·359693	2·780144	·9409793	13
7	·3274928	·346606	2·885113	·9448537	53	28	·3332584	·353464	2·829142	·9428355	32	48	·3387379	·360022	2·777606	·9408808	12
8	·3277676	·346932	2·882403	·9447584	52	29	·3335326	·353791	2·826525	·9427386	31	49	·3390116	·360350	2·775073	·9407822	11
9	·3280424	·347258	2·879697	·9446630	51	30	·3338069	·354118	2·823912	·9426415	30	50	·3392852	·360679	2·772544	·9406835	10
10	·3283172	·347584	2·876997	·9445675	50	31	·3340810	·354446	2·821304	·9425444	29	51	·3395589	·361008	2·770019	·9405848	9
11	·3285919	·347910	2·874300	·9444720	49	32	·3343552	·354773	2·818700	·9424471	28	52	·3398325	·361337	2·767499	·9404860	8
12	·3288666	·348236	2·871608	·9443764	48	33	·3346293	·355101	2·816100	·9423198	27	53	·3401060	·361666	2·764982	·9403871	7
13	·3291413	·348563	2·868921	·9442807	47	34	·3349034	·355428	2·813504	·9422525	26	54	·3403796	·361994	2·762469	·9402881	6
14	·3294160	·348889	2·866238	·9441849	46	35	·3351775	·355756	2·810913	·9421550	25	55	·3406531	·362324	2·759960	·9401891	5
15	·3296906	·349215	2·863560	·9440890	45	36	·3354516	·356084	2·808326	·9420575	24	56	·3409265	·362653	2·757456	·9400899	4
16	·3299653	·349542	2·860886	·9439931	44	37	·3357256	·356411	2·805743	·9419598	23	57	·3412000	·362982	2·754955	·9399907	3
17	·3302398	·349868	2·858216	·9438971	43	38	·3359996	·356739	2·803164	·9418621	22	58	·3414734	·363311	2·752458	·9398914	2
18	·3305144	·350195	2·855551	·9438010	42	39	·3362735	·357067	2·800590	·9417644	21	59	·3417468	·363640	2·749966	·9397921	1
19	·3307889	·350521	2·852891	·9437048	41	40	·3365475	·357395	2·798019	·9416665	20	60	·3420201	·363970	2·747477	·9396926	0
20	·3310634	·350848	2·850234	·9436085	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
				Deg. 70.						Deg. 70.						Deg. 70	

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
							20 Deg.						20 Deg.				
0	·3420201	·363970	2·747477	·9396926	60	21	·3477540	·370903	2·696118	·9375858	39	41	·3532027	·377536	2·648753	·9355468	19
1	·3422935	·364299	2·744992	·9395931	59	22	·3480267	·371234	2·693714	·9374846	38	42	·3534748	·377868	2·646423	·9354440	18
2	·3425668	·364629	2·742512	·9394935	58	23	·3482994	·371565	2·691314	·9373833	37	43	·3537469	·378201	2·644096	·9353412	17
3	·3428400	·364958	2·740035	·9393938	57	24	·3485720	·371896	2·688919	·9372820	36	44	·3540190	·378533	2·641774	·9352382	16
4	·3431133	·365288	2·737562	·9392940	56	25	·3488447	·372227	2·686526	·9371806	35	45	·3542910	·378866	2·639454	·9351352	15
5	·3433865	·365618	2·735093	·9391942	55	26	·3491173	·372559	2·684138	·9370790	34	46	·3545630	·379198	2·637139	·9350321	14
6	·3436597	·365948	2·732628	·9390943	54	27	·3493898	·372890	2·681753	·9369774	33	47	·3548350	·379531	2·634827	·9349289	13
7	·3439329	·366277	2·730167	·9389943	53	28	·3496624	·373221	2·679372	·9368758	32	48	·3551070	·379864	2·632518	·9348257	12
8	·3442060	·366607	2·727710	·9388942	52	29	·3499349	·373553	2·676995	·9367740	31	49	·3553789	·380197	2·630213	·9347223	11
9	·3444791	·366937	2·725256	·9387940	51	30	·3502074	·373884	2·674621	·9366722	30	50	·3556508	·380530	2·627912	·9346189	10
10	·3447521	·367268	2·722807	·9386938	50	31	·3504798	·374216	2·672251	·9365703	29	51	·3559226	·380863	2·625614	·9345154	9
11	·3450252	·367598	2·720362	·9385934	49	32	·3507523	·374547	2·669885	·9364683	28	52	·3561944	·381196	2·623319	·9344119	8
12	·3452982	·367928	2·717920	·9384930	48	33	·3510246	·374879	2·667522	·9363662	27	53	·3564662	·381529	2·621028	·9343082	7
13	·3455712	·368258	2·715482	·9383925	47	34	·3512970	·375211	2·665163	·9362641	26	54	·3567380	·381862	2·618741	·9342045	6
14	·3458441	·368589	2·713048	·9382920	46	35	·3515693	·375543	2·662808	·9361618	25	55	·3570097	·382196	2·616457	·9341007	5
15	·3461171	·368919	2·710618	·9381913	45	36	·3518416	·375875	2·660456	·9360595	24	56	·3572814	·382529	2·614176	·9339968	4
16	·3463900	·369250	2·708192	·9380906	44	37	·3521139	·376207	2·658108	·9359571	23	57	·3575531	·382863	2·611899	·9338928	3
17	·3466628	·369580	2·705769	·9379898	43	38	·3523862	·376539	2·655764	·9358547	22	58	·3578248	·383196	2·609625	·9337888	2
18	·3469357	·369911	2·703351	·9378889	42	39	·3526584	·376871	2·653423	·9357521	21	59	·3580964	·383530	2·607355	·9336846	1
19	·3472085	·370242	2·700936	·9377880	41	40	·3529306	·377203	2·651086	·9356495	20	60	·3583679	·383864	2·605089	·9335804	0
20	·3474812	·370572	2·698525	·9376869	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
				Deg. 69.						Deg. 69.						Deg. 69.	

′	Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	21 Deg. Sine.	Tang.	Cotang.	Cosine	′	′	21 Deg. Sine.	Tang.	Cotang.	Cosine.	′
0	·3583679	·383864	2·605089	·9335804	60	21	·3640641	·390889	2·558268	·93[illegible]37[illegible]9	39	41	·3694765	·397611	2·5[illegible]501	·9292401	19
1	·3586395	·384197	2·602825	·9334761	59	22	·3643351	·391221	2·556075	·9312679	38	42	·3697468	·397948	2·512889	·9291326	18
2	·3589110	·384531	2·600565	·9333718	58	23	·3646059	·391560	2·553885	·9311619	37	43	·3700170	·398285	2·510762	·9290250	17
3	·3591825	·384865	2·598309	·9332673	57	24	·3648768	·391895	2·551699	·9310558	36	44	·3702872	·398622	2·508639	·9289173	16
4	·3594540	·385199	2·596056	·9331628	56	25	·3651476	·392231	2·549516	·9309496	35	45	·3705574	·398959	2·506519	·9288096	15
5	·3597254	·385533	2·593806	·9330582	55	26	·3654184	·392567	2·547335	·9308434	34	46	·3708276	·399296	2·50440[illegible]	·9287017	14
6	·3599968	·385867	2·591560	·9329535	54	27	·3656891	·392902	2·545159	·9307370	33	47	·3710977	·399634	2·502289	·9285938	13
7	·3602682	·386202	2·589317	·9328488	53	28	·3659599	·393238	2·542985	·9306306	32	48	·3713678	·399971	2·500178	·9284858	12
8	·3605395	·386536	2·587078	·9327439	52	29	·3662306	·393574	2·540815	·9305241	31	49	·3716379	·400308	2·498070	·9283778	11
9	·3608108	·386870	2·584842	·9326390	51	30	·3665012	·393910	2·538647	·9304176	30	50	·3719079	·400646	2·495966	·9282696	10
10	·3610821	·387205	2·582609	·9325340	50	31	·3667719	·394246	2·536483	·9303109	29	51	·3721780	·400984	2·493864	·9281614	9
11	·3613534	·387539	2·580380	·9324290	49	32	·3670425	·394582	2·534323	·9302042	28	52	·3724479	·401321	2·491766	·9280531	8
12	·3616246	·387874	2·578153	·9323238	48	33	·3673130	·394918	2·532165	·9300974	27	53	·3727179	·401659	2·489670	·9279447	7
13	·3618958	·388209	2·575931	·9322186	47	34	·3675836	·395255	2·530011	·9299905	26	54	·3729878	·401997	2·487578	·9278363	6
14	·3621669	·388543	2·573711	·9321133	46	35	·3678541	·395591	2·527859	·9298835	25	55	·3732577	·402335	2·485488	·9277277	5
15	·3624380	·388878	2·571495	·9320079	45	36	·3681246	·395928	2·525711	·9297765	24	56	·3735275	·402673	2·483402	·9276191	4
16	·3627091	·389213	2·569283	·9319024	44	37	·3683950	·396264	2·523566	·9296694	23	57	·3737973	·403011	2·481319	·9275104	3
17	·3629802	·389548	2·567073	·9317969	43	38	·3686654	·396601	2·521424	·9295622	22	58	·3740671	·403349	2·479238	·9274016	2
18	·3632512	·389883	2·564867	·9316912	42	39	·3689358	·396937	2·519286	·9294549	21	59	·3743369	·403687	2·477161	·9272928	1
19	·3635222	·390218	2·562664	·9315855	41	40	·3692061	·397274	2·517150	·9293475	20	60	·3746066	·404026	2·475086	·9271839	0
20	·3637932	·390554	2·560464	·9314797	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang	Sine.	′
				Deg. 68.						Deg. 68.						Deg. 68.	

NATURAL SINES AND TANGENTS TO A RADIUS 1.

′	22 Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	22 Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3746066	·404026	2·475086	·9271839	60	21	·3802634	·411149	2·432204	·9248782	39	41	·3856377	·417967	2·392531	·9226503	19
1	·3748763	·404364	2·473015	·9270748	59	22	·3805324	·411489	2·430193	·9247676	38	42	·3859060	·418309	2·390576	·9225381	18
2	·3751459	·404703	2·470947	·9269658	58	23	·3808014	·411830	2·428186	·9246568	37	43	·3861744	·418650	2·388625	·9224258	17
3	·3754156	·405041	2·468881	·9268566	57	24	·3810704	·412170	2·426181	·9245460	36	44	·3864427	·418992	2·386675	·9223134	16
4	·3756852	·405380	2·466819	·9267474	56	25	·3813393	·412510	2·424180	·9244351	35	45	·3867116	·419334	2·384729	·9222010	15
5	·3759547	·405719	2·464759	·9266380	55	26	·3816082	·412851	2·422181	·9243242	34	46	·3869792	·419676	2·382785	·9220884	14
6	·3762243	·406057	2·462703	·9265286	54	27	·3818770	·413191	2·420185	·9242131	33	47	·3872474	·420019	2·380844	·9219758	13
7	·3764938	·406396	2·460649	·9264192	53	28	·3821459	·413532	2·418191	·9241020	32	48	·3875156	·420361	2·378906	·9218632	12
8	·3767632	·406735	2·458598	·9263096	52	29	·3824147	·413872	2·416201	·9239908	31	49	·3877837	·420703	2·376970	·9217504	11
9	·3770327	·407074	2·456551	·9262000	51	30	·3826834	·414213	2·414213	·9238795	30	50	·3880518	·421046	2·375037	·9216375	10
0	·3773021	·407413	2·454506	·9260902	50	31	·3829522	·414554	2·412228	·9237682	29	51	·3883199	·421388	2·373106	·9215246	9
1	·3775714	·407753	2·452464	·9259805	49	32	·3832209	·414895	2·410246	·9236567	28	52	·3885880	·421731	2·371179	·9214116	8
2	·3778408	·408092	2·450425	·9258706	48	33	·3834895	·415236	2·408267	·9235452	27	53	·3888560	·422073	2·369254	·9212986	7
3	·3781101	·408431	2·448389	·9257606	47	34	·3837582	·415577	2·406290	·9234336	26	54	·3891240	·422416	2·367331	·9211854	6
4	·3783794	·408771	2·446355	·9256506	46	35	·3840268	·415918	2·404316	·9233220	25	55	·3893919	·422759	2·365411	·9210722	5
5	·3786486	·409110	2·444325	·9255405	45	36	·3842953	·416259	2·402345	·9232102	24	56	·3896598	·423102	2·363494	·9209589	4
6	·3789178	·409450	2·442298	·9254303	44	37	·3845639	·416601	2·400377	·9230984	23	57	·3899277	·423445	2·361580	·9208455	3
7	·3791870	·409790	2·440273	·9253201	43	38	·3848324	·416942	2·398411	·9229865	22	58	·3901955	·423788	2·359668	·9207320	2
8	·3794562	·410129	2·438251	·9252097	42	39	·3851008	·417284	2·396449	·9228745	21	59	·3904633	·424131	2·357759	·9206185	1
9	·3797253	·410469	2·436233	·9250993	41	40	·3853693	·417625	2·394488	·9227624	20	60	·3907311	·424474	2·355852	·9205049	0
0	·3799944	·410809	2·434217	·9249888	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
				Deg. 67.						Deg. 67.						Deg. 67.	

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	23 Deg.						23 Deg.						23 Deg.				
0	·3907311	·424474	2·355852	·9205019	60	21	·3963468	·431703	2·316407	·9181009	39	41	·4016814	·438622	2·279865	·9157795	19
1	·3909989	·424818	2·353948	·9203912	59	22	·3966139	·432048	2·314557	·9179855	38	42	·4019478	·438969	2·278063	·9156626	18
2	·3912666	·425161	2·352046	·9202774	58	23	·3968809	·432393	2·312709	·9178701	37	43	·4022141	·439316	2·276264	·9155456	17
3	·3915343	·425505	2·350148	·9201635	57	24	·3971479	·432738	2·310863	·9177546	36	44	·4024804	·439663	2·274467	·9154286	16
4	·3918019	·425848	2·348251	·9200496	56	25	·3974148	·433084	2·309020	·9176391	35	45	·4027467	·440010	2·272672	·9153115	15
5	·3920695	·426192	2·346358	·9199356	55	26	·3976818	·433429	2·307180	·9175234	34	46	·4030129	·440357	2·270880	·9151943	14
6	·3923371	·426536	2·344467	·9198215	54	27	·3979486	·433775	2·305342	·9174077	33	47	·4032791	·440705	2·269090	·9150770	13
7	·3926047	·426880	2·342578	·9197073	53	28	·3982155	·434120	2·303506	·9172919	32	48	·4035453	·441052	2·267303	·9149597	12
8	·3928722	·427223	2·340692	·9195931	52	29	·3984823	·434466	2·301673	·9171760	31	49	·4038114	·441400	2·265518	·9148422	11
9	·3931397	·427568	2·338809	·9194788	51	30	·3987491	·434812	2·299842	·9170601	30	50	·4040775	·441747	2·263735	·9147247	10
10	·3934071	·427912	2·336928	·9193644	50	31	·3990158	·435158	2·298014	·9169440	29	51	·4043436	·442095	2·261955	·9146072	9
11	·3936745	·428256	2·335050	·9192499	49	32	·3992825	·435504	2·296188	·9168279	28	52	·4046096	·442443	2·260177	·9144895	8
12	·3939419	·428600	2·333174	·9191353	48	33	·3995492	·435850	2·294365	·9167118	27	53	·4048756	·442791	2·258401	·9143718	7
13	·3942093	·428944	2·331301	·9190207	47	34	·3998158	·436196	2·292544	·9165955	26	54	·4051416	·443139	2·256628	·9142540	6
14	·3944766	·429289	2·329431	·9189060	46	35	·4000825	·436542	2·290725	·9164791	25	55	·4054075	·443487	2·254857	·9141361	5
15	·3947439	·429633	2·327563	·9187912	45	36	·4003490	·436889	2·288909	·9163627	24	56	·4056734	·443835	2·253088	·9140181	4
16	·3950111	·429978	2·325697	·9186763	44	37	·4006156	·437235	2·287095	·9162462	23	57	·4059393	·444183	2·251322	·9139001	3
17	·3952783	·430323	2·323834	·9185614	43	38	·4008821	·437582	2·285284	·9161297	22	58	·4062051	·444531	2·249558	·9137819	2
18	·3955455	·430668	2·321974	·9184464	42	39	·4011486	·437928	2·283475	·9160130	21	59	·4064709	·444880	2·247796	·9136637	1
19	·3958127	·431012	2·320116	·9183313	41	40	·4014150	·438275	2·281669	·9158963	20	60	·4067366	·445228	2·246036	·9135455	0
20	·3960798	·431357	2·318260	·9182161	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
		Deg. 66.						Deg. 66.								Deg. 66.	

24 Deg.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

24 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4067366	·445228	2·246036	·9135455	60	21	·4123096	·452568	2·209611	·9110438	39	41	·4176028	·459596	2·175822	·9086297	19
1	·4070024	·445577	2·244279	·9134271	59	22	·4125745	·452918	2·207901	·9109238	38	42	·4178671	·459948	2·174155	·9085082	18
2	·4072681	·445926	2·242524	·9133087	58	23	·4128395	·453269	2·206193	·9108038	37	43	·4181313	·460301	2·172491	·9083866	17
3	·4075337	·446274	2·240772	·9131902	57	24	·4131044	·453620	2·204487	·9106837	36	44	·4183956	·460653	2·170828	·9082649	16
4	·4077993	·446623	2·239021	·9130716	56	25	·4133693	·453970	2·202784	·9105635	35	45	·4186597	·461006	2·169167	·9081432	15
5	·4080649	·446972	2·237273	·9129529	55	26	·4136342	·454321	2·201083	·9104432	34	46	·4189239	·461359	2·167509	·9080214	14
6	·4083305	·447321	2·235528	·9128342	54	27	·4138990	·454672	2·199384	·9103228	33	47	·4191880	·461711	2·165852	·9078995	13
7	·4085960	·447670	2·233784	·9127154	53	28	·4141638	·455023	2·197687	·9102024	32	48	·4194521	·462064	2·164198	·9077775	12
8	·4088615	·448020	2·232043	·9125965	52	29	·4144285	·455375	2·195992	·9100819	31	49	·4197161	·462417	2·162546	·9076554	11
9	·4091269	·448369	2·230304	·9124775	51	30	·4146932	·455726	2·194299	·9099613	30	50	·4199801	·462771	2·160895	·9075333	10
10	·4093923	·448718	2·228567	·9123584	50	31	·4149579	·456077	2·192609	·9098406	29	51	·4202441	·463124	2·159247	·9074111	9
11	·4096577	·449068	2·226833	·9122393	49	32	·4152226	·456429	2·190921	·9097199	28	52	·4205080	·463477	2·157601	·9072888	8
12	·4099230	·449417	2·225100	·9121201	48	33	·4154872	·456780	2·189234	·9095990	27	53	·4207719	·463831	2·155957	·9071665	7
13	·4101883	·449767	2·223370	·9120008	47	34	·4157517	·457132	2·187551	·9094781	26	54	·4210358	·464184	2·154315	·9070440	6
14	·4104536	·450117	2·221643	·9118815	46	35	·4160163	·457483	2·185869	·9093572	25	55	·4212996	·464538	2·152675	·9069215	5
15	·4107189	·450467	2·219917	·9117620	45	36	·4162808	·457835	2·184189	·9092361	24	56	·4215634	·464891	2·151037	·9067989	4
16	·4109841	·450817	2·218194	·9116425	44	37	·4165453	·458187	2·182511	·9091150	23	57	·4218272	·465245	2·149402	·9066762	3
17	·4112492	·451167	2·216473	·9115229	43	38	·4168097	·458539	2·180836	·9089938	22	58	·4220909	·465599	2·147768	·9065535	2
18	·4115144	·451517	2·214754	·9114033	42	39	·4170741	·458891	2·179163	·9088725	21	59	·4223546	·465953	2·146136	·9064307	1
19	·4117795	·451867	2·213037	·9112835	41	40	·4173385	·459243	2·177492	·9087511	20	60	·4226183	·466307	2·144506	·9063078	0
20	·4120445	·452217	2·211323	·9111637	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 65. Deg. 65. Deg. 65

′	Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	25 Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	25 Deg. Sine.	Tang.	Cotang.	Cosine.	
0	·4226183	·466307	2·144506	·9063078	60	21	·4281467	·473765	2·110747	·9037093	39	41	·4333970	·480909	2·079394	·9012031	19
1	·4228819	·466661	2·142879	·9061848	59	22	·4284095	·474122	2·109161	·9035847	38	42	·4336591	·481267	2·077846	·9010770	18
2	·4231455	·467016	2·141253	·9060618	58	23	·4286723	·474478	2·107577	·9034600	37	43	·4339212	·481625	2·076300	·9009508	17
3	·4234090	467370	2·139630	·9059386	57	24	·4289351	·474834	2·105995	·9033353	36	44	·4341832	·481984	2·074756	·9008246	16
4	·4236725	·467725	2·138008	·9058154	56	25	·4291979	·475191	2·104415	·9032105	35	45	·4344453	·482342	2·073214	·9006982	15
5	·4239360	·468079	2·136389	·9056922	55	26	·4294606	·475548	2·102836	·9030856	34	46	·4347072	·482701	2·071674	·9005718	14
6	·4241994	·468434	2·134771	·9055688	54	27	·4297233	·475904	2·101260	·9029606	33	47	·4349692	·483060	2·070135	·9004453	13
7	·4244628	·468789	2·133155	·9054454	53	28	·4299859	·476261	2·099686	·9028356	32	48	·4352311	·483418	2·068599	·9003188	12
8	·4247262	·469143	2·131542	·9053219	52	29	·4302485	·476618	2·098114	·9027105	31	49	·4354930	·483777	2·067064	·9001921	11
9	·4249895	·469498	2·129930	·9051983	51	30	·4305111	·476975	2·096543	·9025853	30	50	·4357548	·484136	2·065531	·9000654	10
10	·4252528	·469853	2·128321	·9050746	50	31	·4307736	·477332	2·094975	·9024600	29	51	·4360166	·484495	2·064000	·8999386	9
11	·4255161	·470209	2·126713	·9049509	49	32	·4310361	477689	2·093408	·9023347	28	52	·4362784	·484855	2·062471	·8998117	8
12	·4257793	·470564	2·125108	·9048271	48	33	·4312986	·478047	2·091843	·9022092	27	53	·4365401	·485214	2·060944	·8996848	7
13	·4260425	·470919	2·123504	·9047032	47	34	·4315610	478404	2·090280	·9020838	26	54	·4368018	·485573	2·059418	·8995578	6
14	·4263056	·471275	2·121903	·9045792	46	35	·4318234	·478762	2·088720	·9019582	25	55	·4370634	·485933	2·057895	·8994307	5
15	·4265687	·471630	2·120303	·9044551	45	36	·4320857	·479119	2·087161	·9018325	24	56	·4373251	·486293	2·056373	·8993035	4
16	·4268318	·471986	2·118705	·9043310	44	37	·4323481	·479477	2·085603	·9017068	23	57	·4375866	·486652	2·054853	·8991763	3
17	·4270949	·472342	2·117110	·9042068	43	38	·4326103	·479835	2·084048	·9015810	22	58	·4378482	·487012	2·053334	·8990489	2
18	·4273579	·472697	2·115516	·9040825	42	39	·4328726	·480193	2·082495	·9014551	21	59	·4381097	·487372	2·051818	·8989215	1
19	·4276208	·473053	2·113924	·9039582	41	40	·4331348	·480551	2·080943	·9013292	20	60	·4383711	·487732	2·050303	·8987940	0
20	·4278838	·473409	2· 12334	·9038338	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine	′
		Deg. 64.						Deg. 64.								Deg. 64.	

26 Deg.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

26 Deg.

26 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4383711	·487732	2·050303	·8987940	60	21	·4438534	·495317	2·018908	·8960994	39	11	·4490591	·502583	1·989720	·8935021	19
1	·4386326	·488092	2·048791	·8986665	59	22	·4441140	·495679	2·017433	·8959703	38	12	·4493190	·502947	1·988278	·8933714	18
2	·4388940	·488453	2·047280	·8985389	58	23	·4443746	·496041	2·015959	·8958411	37	13	·4495789	·503312	1·986838	·8932406	17
3	·4391553	·488813	2·045770	·8984112	57	24	·4446352	·496404	2·014486	·8957118	36	14	·4498387	·503676	1·985400	·8931098	16
4	·4394166	·489173	2·044263	·8982834	56	25	·4448957	·496766	2·013016	·8955824	35	15	·4500984	·504041	1·983963	·8929789	15
5	·4396779	·489534	2·042757	·8981555	55	26	·4451562	·497129	2·011547	·8954529	34	16	·4503582	·504406	1·982528	·8928480	14
6	·4399392	·489894	2·041254	·8980276	54	27	·4454167	·497492	2·010080	·8953234	33	17	·4506179	·504771	1·981095	·8927169	13
7	·4402004	·490255	2·039751	·8978996	53	28	·4456771	·497855	2·008615	·8951938	32	18	·4508775	·505136	1·979663	·8925858	12
8	·4404615	·490616	2·038251	·8977715	52	29	·4459375	·498218	2·007151	·8950641	31	19	·4511372	·505501	1·978233	·8924546	11
9	·4407227	·490977	2·036753	·8976433	51	30	·4461978	·498581	2·005689	·8949344	30	50	·4513967	·505866	1·976805	·8923234	10
10	·4409838	·491338	2·035256	·8975151	50	31	·4464581	·498944	2·004229	·8948045	29	51	·4516563	·506232	1·975378	·8921920	9
11	·4412448	·491699	2·033761	·8973868	49	32	·4467184	·499308	2·002771	·8946746	28	52	·4519158	·506597	1·973953	·8920606	8
12	·4415059	·492061	2·032268	·8972584	48	33	·4469786	·499671	2·001314	·8945446	27	53	·4521753	·506963	1·972529	·8919291	7
13	·4417668	·492422	2·030776	·8971299	47	34	·4472388	·500035	1·999859	·8944146	26	54	·4524347	·507329	1·971107	·8917975	6
14	·4420278	·492783	2·029287	·8970014	46	35	·4474990	·500398	1·998405	·8942844	25	55	·4526941	·507694	1·969687	·8916659	5
15	·4422887	·493145	2·027799	·8968727	45	36	·4477591	·500762	1·996953	·8941542	24	56	·4529535	·508060	1·968268	·8915342	4
16	·4425496	·493507	2·026313	·8967440	44	37	·4480192	·501126	1·995503	·8940240	23	57	·4532128	·508426	1·966851	·8914024	3
17	·4428104	·493868	2·024828	·8966153	43	38	·4482792	·501490	1·994055	·8938936	22	58	·4534721	·508792	1·965436	·8912705	2
18	·4430712	·494230	2·023346	·8964864	42	39	·4485392	·501854	1·992608	·8937632	21	59	·4537313	·509159	1·964022	·8911385	1
19	·4433319	·494592	2·021865	·8963575	41	40	·4487992	·502218	1·991163	·8936326	20	60	·4539905	509525	1·962610	·8910065	0
20	·4435927	·494954	2·020386	·8962285	40												

′	Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	27 Deg. Sine.	Tang.	Cotang.	Cosine	′	′	27 Deg. Sine.	Tang.	Cotang.	Cosine.	′
0	·4539905	·509525	1·962610	·8910065	60	21	·4594248	517244	1·933323	·8882166	39	41	·4645845	·524640	1·906066	·8855288	19
1	·4542497	·509891	1·961200	·8908744	59	22	·4596832	·517612	1·931945	·8880830	38	42	·4648420	·525011	1.904719	·8853936	18
2	·4545088	·510258	1·959791	·8907423	58	23	·4599415	·517981	1·930569	·8879492	37	43	·4650996	·525382	1.903373	·8852584	17
3	·4547679	·510625	1·958383	·8906100	57	24	·4601998	·518350	1·929195	·8878154	36	44	·4653571	·525754	1·902029	·8851230	16
4	·4550269	·510991	1·956978	·8904777	56	25	·4604580	·518719	1·927822	·8876815	35	45	·4656145	·526125	1 900687	·8849876	15
5	·4552859	·511358	1·955573	·8903453	55	26	·4607162	·519089	1·926451	·8875475	34	46	·4658719	·526496	1·899346	·8848522	14
6	·4555449	·511725	1·954171	·8902128	54	27	·4609744	·519458	1·925081	·8874134	33	47	·4661293	·526868	1·898006	·8847166	13
7	·4558038	·512093	1·952770	·8900803	53	28	·4612325	·519827	1·923713	·8872793	32	48	·4663866	·527240	1·896668	·8845810	12
8	·4560627	·512460	1·951371	·8899476	52	29	·4614906	·520197	1·922347	·8871451	31	49	·4666439	·527612	1·895332	·8844453	11
9	·4563216	·512827	1·949973	·8898149	51	30	·4617486	·520567	1·920982	·8870108	30	50	·4669012	·527983	1·893997	·8843095	10
10	·4565804	·513195	1·948577	·8896822	50	31	·4620066	·520936	1·919618	·8868765	29	51	·4671584	·528356	1·892663	·8841736	9
11	·4568392	·513562	1·947182	·8895493	49	32	·4622646	·521306	1·918256	·8867420	28	52	·4674156	·528728	1·891331	·8840377	8
12	·4570979	·513930	1·945789	·8894164	48	33	·4625225	·521676	1·916896	·8866075	27	53	·4676727	·529100	1·890000	·8839017	7
13	·4573566	·514298	1·944398	·8892834	47	34	·4627804	·522046	1·915537	·8864730	26	54	·4679298	·529472	1·888671	·8837656	6
14	·4576153	·514665	1·943008	·8891503	46	35	·4630382	·522117	1·914179	·8863383	25	55	·4681869	·529845	1·887343	·8836295	5
15	·4578739	·515033	1·941620	·8890171	45	36	·4632960	·522787	1·912823	·8862036	24	56	·4684439	·530217	1·886017	·8834933	4
16	·4581325	·515401	1·940233	·8888839	44	37	·4635538	·523157	1·911469	·8860688	23	57	·4687009	·530590	1·884692	·8833569	3
17	·4583910	·515770	1·938848	·8887506	43	38	·4638115	·523528	1·910116	·8859339	22	58	·4689578	·530963	1·883369	·8832206	2
18	·4586496	·516138	1·937464	·8886172	42	39	·4640692	·523899	1·908764	8857989	21	59	·4692147	·531336	1·882047	·8830841	1
19	·4589080	·516506	1·936082	·8884838	11	40	·4643269	·524269	1·907414	·8856639	20	60	·4694716	·531709	1·880726	·8829476	0
20	·4591665	·516875	1·934702	·8883503	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine	′
			Deg. 62.						Deg. 62.							Deg. 62.	

NATURAL SINES AND TANGENTS TO A RADIUS 1.

8 Deg.						28 Deg.						28 Deg.				
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4694716	·531709	1·880726	·8829476	60	21	·4748564	·539570	1·853325	·8800633	39	41	·4799683	·547106	1·827799	·8772858	19
1	·4697284	·532082	1·879407	·8828110	59	22	·4751124	·539946	1·852035	·8799251	38	42	·4802235	·547484	1·826537	·8771462	18
2	·4699852	·532455	1·878089	·8826743	58	23	·4753683	·540322	1·850747	·8797869	37	43	·4804786	·547862	1·825276	·8770064	17
3	·4702419	·532829	1·876773	·8825376	57	24	·4756242	·540698	1·849461	·8796486	36	44	·4807337	·548240	1·824017	·8768666	16
4	·4704986	·533202	1·875458	·8824007	56	25	·4758801	·541074	1·848176	·8795102	35	45	·4809888	·548618	1·822759	·8767268	15
5	·4707553	533576	1·874145	·8822638	55	26	·4761359	·541450	1·846892	·8793717	34	46	·4812438	·548997	1·821502	·8765868	14
6	·4710119	·533950	1·872833	·8821269	54	27	·4763917	·541826	1·845609	·8792332	33	47	·4814987	·549375	1·820247	·8764468	13
7	·4712685	·534324	1·871523	·8819898	53	28	·4766474	·542202	1·844328	·8790946	32	48	·4817537	·549754	1·818993	·8763067	12
8	·4715250	·534698	1·870214	·8818527	52	29	·4769031	·542579	1·843049	·8789559	31	49	·4820086	·550133	1·817740	·8761665	11
9	·4717815	·535072	1·868906	·8817155	51	30	·4771588	·542955	1·841770	·8788171	30	50	·4822634	·550512	1·816489	·8760263	10
10	·4720380	·535446	1·867600	·8815782	50	31	·4774144	·543332	1·840494	·8786783	29	51	·4825182	·550891	1·815239	·8758859	9
11	·4722944	·535820	1·866295	·8814409	49	32	·4776700	·543709	1·839218	·8785394	28	52	·4827730	·551270	1·813990	·8757455	8
12	·4725508	·536195	1·864992	·8813035	48	33	·4779255	·544086	1·837944	·8784004	27	53	·4830277	·551650	1·812743	·8756051	7
13	·4728071	·536569	1·863690	·8811660	47	34	·4781810	·544463	1·836671	·8782613	26	54	·4832824	·552029	1·811496	·8754645	6
14	·4730634	·536944	1·862389	·8810284	46	35	·4784364	·544840	1·835399	·8781222	25	55	·4835370	·552409	1·810252	·8753239	5
15	·4733197	·537319	1·861090	·8808907	45	36	·4786919	·545217	1·834129	·8779830	24	56	·4837916	·552789	1·809008	·8751832	4
16	·4735759	·537694	1·859792	·8807530	44	37	·4789472	·545595	1·832861	·8778437	23	57	·4840462	·553168	1·807766	·8750425	3
17	·4738321	·538069	1·858496	·8806152	43	38	·4792026	·545972	1·831593	·8777043	22	58	·4843007	·553548	1·806525	·8749016	2
18	·4740882	·538444	1·857201	·8804774	42	39	·4794579	·546350	1·830327	·8775649	21	59	·4845552	·553928	1·805286	·8747607	1
19	·4743443	·538819	1·855908	·8803394	41	40	·4797131	·546728	1·829062	·8774254	20	60	·4848096	·554309	1·804047	·8746197	0
20	·4746004	·539195	1·854615	·8802014	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
																	Deg. 61.
					Deg. 61.						Deg. 61.						

Deg. ′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
							29 Deg.						29 Deg.				
0	·4848096	·554309	1·804047	·8746197	60	21	·4901433	·562321	1·778340	·8716419	39	41	·4952060	·570004	1·754372	·8687756	19
1	·4850640	·554689	1·802810	·8744786	59	22	·4903968	·562704	1·777130	·8714993	38	42	·4954587	·570389	1·753186	·8686315	18
2	·4853184	·555069	1·801575	·8743375	58	23	·4906503	·563087	1·775921	·8713566	37	43	·4957113	·570775	1·752002	·8684874	17
3	·4855727	·555450	1·800340	·8741963	57	24	·4909038	·563471	1·774714	·8712138	36	44	·4959639	·571161	1·750819	·8683431	16
4	·4858270	·555831	1·799107	·8740550	56	25	·4911572	·563854	1·773507	·8710710	35	45	·4962165	·571547	1·749637	·8681988	15
5	·4860812	·556211	1·797875	·8739137	55	26	·4914105	·564237	1·772302	·8709281	34	46	·4964690	·571933	1·748456	·8680544	14
6	·4863354	·556592	1·796645	·8737722	54	27	·4916638	·564621	1·771098	·8707851	33	47	·4967215	·572319	1·747276	·8679100	13
7	·4865895	·556973	1·795416	·8736307	53	28	.4919171	·565005	1·769895	·8706420	32	48	·4969740	·572705	1·746098	·8677655	12
8	·4868436	·557355	1·794188	·8734891	52	29	·4921704	·565388	1·768694	·8704989	31	49	·4972264	·573091	1·744921	·8676209	11
9	·4870977	·557736	1·792961	·8733475	51	30	·4924236	·565772	1·767494	·8703557	30	50	·4974787	·573478	1·743745	·8674762	10
10	·4873517	·558117	1·791736	·8732058	50	31	·4926767	·566156	1·766295	·8702124	29	51	·4977310	·573864	1·742570	·8673314	9
11	·4876057	·558499	1·790512	·8730640	49	32	·4929298	·566541	1·765097	·8700691	28	52	·4979833	.574251	1·741396	·8671866	8
12	·4878597	·558881	1·789289	·8729221	48	33	·4931829	·566925	1·763900	·8699256	27	53	·4982355	·574638	1·740224	·8670417	7
13	·4881136	·559262	1·788067	·8727801	47	34	·4934359	·567309	1·762705	·8697821	26	54	·4984877	·575025	1·739053	·8668967	6
14	·4883674	·559644	1·786847	·8726381	46	35	·4936889	·567694	1·761511	·8696386	25	55	·4987399	·575412	1·737883	·8667517	5
15	·4886212	·560026	1·785628	·8724960	45	36	·4939419	·568079	1·760318	·8694949	24	56	·4989920	·575799	1·736714	·8666066	4
16	·4888750	·560409	1·784410	·8723538	44	37	·4941948	·568463	1·759126	·8693512	23	57	·4992441	·576187	1·735546	·8664614	3
17	·4891288	·560791	1·783194	·8722116	43	38	·4944476	·568848	1·757936	·8692074	22	58	·4994961	·576574	1·734380	·8663161	2
18	·4893825	·561173	1·781979	·8720693	42	39	·4947005	·569233	1·756747	·8690636	21	59	4997481	·576962	1·733214	·8661708	1
19	·4896361	·561556	1·780765	·8719269	41	40	·4949532	·569619	1·755559	·8689196	20	60	·5000000	·577350	1·732050	·8660254	0
20	·4898897	·561939	1·779552	·8717844	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
				Deg. 60.						Deg. 60.						Deg. 60.	

NATURAL SINES AND TANGENTS TO A RADIUS 1.

30 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5000000	·577350	1·732050	·8660254	60	21	·5052809	·585524	1·707871	·8629549	39	41	·5102928	·593363	1·685308	·8600007	19
1	·5002519	·577738	1·730887	·8658799	59	22	·5055319	·585914	1·706732	·8628079	38	42	·5105429	·593756	1·684191	·8598523	18
2	·5005037	·578126	1·729726	·8657344	58	23	·5057828	·586305	1·705595	·8626608	37	43	·5107930	·594150	1·683076	·8597037	17
3	·5007556	·578514	1·728565	·8655887	57	24	·5060338	·586696	1·704458	·8625137	36	44	·5110431	·594543	1·681962	·8595551	16
4	·5010073	·578902	1·727406	·8654430	56	25	·5062846	·587087	1·703323	·8623664	35	45	·5112931	·594937	1·680848	·8594064	15
5	·5012591	·579291	1·726247	·8652973	55	26	·5065355	·587478	1·702189	·8622191	34	46	·5115431	·595331	1·679736	·8592576	14
6	·5015107	·579679	1·725090	·8651514	54	27	·5067863	·587870	1·701055	·8620717	33	47	·5117930	·595725	1·678625	·8591088	13
7	·5017624	·580068	1·723934	·8650055	53	28	·5070370	·588261	1·699923	·8619243	32	48	·5120429	·596119	1.677515	·8589599	12
8	·5020140	·580457	1·722779	·8648595	52	29	·5072877	·588653	1·698792	·8617768	31	49	·5122927	·596514	1·676406	·8588109	11
9	·5022655	·580846	1·721626	·8647134	51	30	·5075384	·589045	1·697663	.8616292	30	50	·5125425	·596908	1·675298	·8586619	10
10	·5025170	·581235	1·720473	·8645673	50	31	·5077890	·589436	1·696534	·8614815	29	51	·5127923	·597303	1·674192	·8585127	9
11	·5027685	·581624	1·719322	·8644211	49	32	·5080396	·589828	1·695406	·8613337	28	52	·5130420	·597697	1·673086	·8583635	8
12	·5030199	·582013	1·718172	·8642748	48	33	·5082901	·590221	1·694280	·8611859	27	53	·5132916	·598092	1·671981	·8582143	7
13	·5032713	·582403	1·717023	·8641284	47	34	·5085406	·590613	1·693155	·8610380	26	54	·5135413	·598487	1·670878	·8580649	6
14	·5035227	·582793	1·715875	·8639820	46	35	·5087910	·591005	1·692030	·8608901	25	55	·5137908	·598882	1·669775	·8579155	5
15	·5037740	·583182	1·714728	·8638355	45	36	·5090414	·591398	1·690907	·8607420	24	56	·5140404	·599278	1·668674	·8577660	4
16	·5040252	·583572	1·713582	·8636889	44	37	·5092918	·591791	1·689785	·8605939	23	57	·5142899	·599673	1·667574	·8576164	3
17	·5042765	·583962	1·712438	·8635423	43	38	·5095421	·592183	1·688664	·8604457	22	58	·5145393	·600069	1·666474	·8574668	2
18	·5045276	·584352	1·711294	·8633956	42	39	·5097924	·592576	1·687544	·8602975	21	59	·5147887	·600464	1·665376	·8573171	1
19	·5047788	·584743	1·710152	·8632488	41	40	·5100426	·592969	1·686426	·8601491	20	60	·5150381	·600860	1·664279	·8571673	0
	·5050298	·585133	1·709011	·8631019	40												

31 Deg.						31 Deg.						31 Deg.				
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5150381	·600860	1·664279	·8571673	60	21	·5202646	·609205	1·641482	·8540051	39	41	·5252241	·617210	1·620192	·8509639	19
1	·5152874	·601256	1·663183	·8570174	59	22	·5205130	·609604	1·640408	·8538538	38	42	·5254717	·617612	1·619138	·8508111	18
2	·5155367	·601652	1·662088	·8568675	58	23	·5207613	·610003	1·639335	·8537023	37	43	·5257191	·618014	1·618085	·8506582	17
3	·5157859	·602049	1·660994	·8567175	57	24	·5210096	·610402	1·638263	·8535508	36	44	·5259665	·618416	1·617033	·8505053	16
4	·5160351	·602445	1·659901	·8565674	56	25	·5212579	·610801	1·637191	·8533992	35	45	·5262139	·618818	1·615982	·8503522	15
5	·5162842	·602841	1·658809	·8564173	55	26	·5215061	·611201	1·636121	·8532475	34	46	·5264613	·619221	1·614932	·8501991	14
6	·5165333	·603238	1·657718	·8562671	54	27	·5217543	·611601	1·635052	·8530958	33	47	·5267085	·619623	1·613882	·8500459	13
7	·5167824	·603635	1·656629	·8561168	53	28	·5220024	·612000	1·633984	·8529440	32	48	·5269558	·620026	1·612834	·8498927	12
8	·5170314	·604032	1·655540	·8559664	52	29	·5222505	·612400	1·632917	·8527921	31	49	·5272030	·620429	1·611787	·8497394	11
9	·5172804	·604429	1·654452	·8558160	51	30	·5224986	·612800	1·631851	·8526402	30	50	·5274502	·620832	1·610741	·8495860	10
10	·5175293	·604826	1·653366	·8556655	50	31	·5227466	·613201	1·630786	·8524881	29	51	·5276973	·621235	1·609696	·8494325	9
11	·5177782	·605224	1·652280	·8555149	49	32	·5229945	·613601	1·629722	·8523360	28	52	·5279443	·621638	1·608652	·8492790	8
12	·5180270	·605621	1·651196	·8553643	48	33	·5232424	·614001	1·628659	·8521839	27	53	·5281914	·622041	1·607609	·8491254	7
13	·5182758	·606019	1·650112	·8552135	47	34	·5234903	·614402	1·627597	·8520316	26	54	·5284383	·622445	1·606567	·8489717	6
14	·5185246	·606417	1·649030	·8550627	46	35	·5237381	·614803	1·626536	·8518793	25	55	·5286853	·622848	1·605526	·8488179	5
15	·5187733	·606814	1·647949	·8549119	45	36	·5239859	·615204	1·625476	·8517269	24	56	·5289322	·623252	1·604485	·8486641	4
16	·5190219	·607213	1·646868	·8547609	44	37	·5242336	·615605	1·624417	·8515745	23	57	·5291790	·623656	1·603446	·8485102	3
17	·5192705	·607611	1·645789	·8546099	43	38	·5244813	·616006	1·623359	·8514219	22	58	·5294258	·624060	1·602408	·8483562	2
18	·5195191	·608009	1·644711	·8544588	42	39	·5247290	·616407	1·622302	·8512693	21	59	·5296726	·624465	1·601370	·8482022	1
19	·5197676	·608408	1·643633	·8543077	41	40	·5249766	·616809	1·621246	·8511167	20	60	·5299193	·624869	1·600334	·8480481	0
20	·5200161	·608806	1·642557	·8541564	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
		Deg. 58·						Deg. 58.								Deg. 58.	

32 Deg. 32 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5299193	·624869	1·600334	·8480481	60	21	·5350898	·633395	1·578791	·8447952	39	41	·5399955	·641577	1·558657	·8416679	19
1	·5301659	·625273	1·599299	·8478939	59	22	·5353355	·633803	1·577776	·8446395	38	42	·5402403	·641988	1·557660	·8415108	18
2	·5304125	·625678	1·598264	·8477397	58	23	·5355812	·634211	1·576761	·8444838	37	43	·5404851	·642399	1·556663	·8413536	17
3	·5306591	·626083	1·597231	·8475853	57	24	·5358268	·634619	1·575747	·8443279	36	44	·5407298	·642810	1·555668	·8411963	16
4	·5309057	·626488	1·596198	·8474309	56	25	·5360724	·635027	1·574735	·8441720	35	45	·5409745	·643221	1·554674	·8410390	15
5	·5311521	·626893	1·595167	·8472765	55	26	·5363179	·635435	1·573723	·8440161	34	46	·5412191	·643632	1·553680	·8408816	14
6	·5313986	·627298	1·594136	·8471219	54	27	·5365634	·635844	1·572712	·8438600	33	47	·5414637	·644044	1·552688	·8407241	13
7	·5316450	·627704	1·593107	·8469673	53	28	·5368089	·636252	1·571702	·8437039	32	48	5417082	·644456	1·551696	·8405666	12
8	·5318913	·628109	1·592078	·8468126	52	29	·5370543	·636661	1·570693	·8435477	31	49	.5419527	·644867	1·550705	·8404090	11
9	·5321376	·628515	1·591050	·8466579	51	30	·5372996	·637070	1·569685	·8433914	30	50	·5421971	·645279	1·549715	·8402513	10
10	·5323839	·628921	1·590023	·8465030	50	31	·5375449	·637479	1·568678	·8432351	29	51	·5424415	·645691	1·548726	·8400936	9
11	·5326301	·629327	1·588997	8463481	49	32	·5377902	·637888	1·567672	·8430787	28	52	·5426859	·646104	1·547738	·8399357	8
12	·5328763	·629733	1·587973	·8461932	48	33	·5380354	·638297	1·566666	·8429222	27	53	·5429302	·646516	1·546751	·8397778	7
13	·5331224	·630139	1·586949	·8460381	47	34	·5382806	·638707	1·565662	·8427657	26	54	·5431744	·646929	1·545764	·8396199	6
14	·5333685	·630546	1·585926	·8458830	46	35	·5385257	·639116	1·564659	·8426091	25	55	·5434187	·647341	1·544779	·8394618	5
15	·5336145	·630953	1·584904	·8457278	45	36	·5387708	·639526	1·563656	·8424524	24	56	·5436628	·647754	1·543794	·8393037	4
16	·5338605	·631359	1·583883	8455726	44	37	·5390158	·639936	1·562654	·8422956	23	57	·5439069	·648167	1·542810	·8391455	3
17	·5341065	·631766	1·582862	8454172	43	38	·5392608	·640346	1·561654	·8421388	22	58	·5441510	·648580	1·541828	·8389873	2
18	·5343523	·632173	1·581843	·8452618	42	39	·5395058	·640756	1·560654	·8419819	21	59	·5443951	·648994	1·540846	·8388290	1
19	·5345982	·632581	1·580825	·8451064	41	40	·5397507	·641167	1·559655	·8418249	20	60	·5446390	·649407	1·539865	·8386706	0
	·5348440	·632988	1·579807	·8449508	40												

′	Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	33 Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	33 Deg. Sine.	Tang.	Cotang.	Cosine.	′
0	·5446390	·649407	1·539865	·8386706	60	21	·5497520	·658127	1·519463	·8353279	39	41	·5546024	·666496	1·500382	·8321155	19
1	·5448830	·649821	1·538884	·8385121	59	22	·5499950	·658544	1·518501	·8351680	38	42	5548444	·666917	1·499436	·8319541	18
2	·5451269	·650235	1·537905	·8383536	58	23	·5502379	·658961	1 517540	·8350085	37	43	·5550864	·667337	1·498492	·8317927	17
3	·5453707	·650649	1·536927	·8381950	57	24	·5504807	·659378	1·516579	·8348479	36	44	·5553283	667758	1·497548	·8316312	16
4	·5456145	·651063	1·535949	·8380363	56	25	·5507236	·659796	1·515620	·8346877	35	45	·5555702	·668178	1·496605	·8314696	15
5	·5458583	·651477	1·534972	·8378775	55	26	·5509663	·660213	1·514661	·8345275	34	46	·5558121	·668599	1·495663	·8313080	14
6	·5461020	·651891	1·533996	·8377187	54	27	·5512091	·660631	1·513703	·8343672	33	47	5560539	·669020	1·494722	·8311463	13
7	·5463456	·652306	1·533021	·8375598	53	28	·5514518	·661049	1·512746	·8342068	32	48	·5562956	·669441	1·493782	·8309845	12
8	·5465892	·652721	1·532047	·8374009	52	29	·5516944	·661467	1·511790	·8340463	31	49	·5565373	·669863	1·492842	·8308226	11
9	·5468328	·653136	1·531074	·8372418	51	30	·5519370	·661885	1·510835	·8338858	30	50	·5567790	·670284	1·491903	8306607	10
10	·5470763	·653551	1·530102	·8370827	50	31	·5521795	·662304	1·509880	·8337252	29	51	5570206	·670706	1·490965	·8304987	9
11	·5473198	·653966	1·529130	·8369236	49	32	·5524220	·662722	1·508927	·8335646	28	52	5572621	·671128	1·490028	·8303366	8
12	·5475632	·654381	1·528160	·8367643	48	33	·5526645	·663141	1·507974	·8334038	27	53	5575036	·671550	1·489092	·8301745	7
13	·5478066	·654797	1·527190	·8366050	47	34	·5529069	·663560	1·507022	·8332430	26	54	·5577451	·671972	1·488157	·8300123	6
14	·5480499	·655212	1·526221	·8364456	46	35	·5531492	·663979	1·506071	8330822	25	55	·5579865	·672394	1·487222	·8298500	5
15	·5482932	·655628	1·525253	·8362862	45	36	·5533915	·664398	1·505121	·8329212	24	56	·5582279	·672816	1·486288	·8296877	4
16	·5485365	·656044	1·524286	·8361266	44	37	·5536338	·664817	1·504171	·8327602	23	57	·5584692	·673239	1·485355	·8295252	3
17	·5487797	·656460	1·523320	·8359670	43	38	·5538760	·665237	1·503222	·8325991	22	58	5587105	·673662	1·484423	·8293628	2
18	·5490228	·656877	1·522354	·8358074	42	39	·5541182	·665657	1·502275	·8324380	21	59	·5589517	·674085	1·483491	·8292002	1
19	·5492659	·657293	1·521389	·8356476	41	40	·5543603	·666076	1·501328	·8322768	20	60	·5591929	·6745.8	1·482561	·8290376	0
20	·5495090	·657710	1·520426	·8354878	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
				Deg. 56.						Deg. 56.						Deg. 56.	

34 Deg.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	34 Deg.						34 Deg.						34 Deg.				
0	·5591929	·674508	1·482561	·8290376	60	21	·5642467	·683433	1·463200	·8256062	39	41	·5690403	·692002	1·445081	·8223096	19
1	·5594340	·674931	1·481631	·8288749	59	22	·5644869	·683860	1·462287	·8254420	38	42	·5692795	·692432	1·444183	·8221440	18
2	·5596751	·675355	1·480702	·8287121	58	23	·5647270	·684287	1·461374	·8252778	37	43	·5695187	·692863	1·443286	·8219784	17
3	·5599162	·675779	1·479773	·8285493	57	24	·5649670	·684714	1·460463	·8251135	36	44	·5697577	·693293	1·442389	·8218127	16
4	·5601572	·676202	1·478846	·8283864	56	25	·5652070	·685141	1·459552	·8249491	35	45	·5699968	·693724	1·441494	·8216469	15
5	·5603981	·676626	1·477919	·8282234	55	26	·5654469	·685569	1·458642	·8247847	34	46	·5702357	·694155	1·440599	·8214811	14
6	·5606390	·677050	1·476993	·8280603	54	27	·5656868	·685996	1·457732	·8246202	33	47	·5704747	·694586	1·439704	·8213152	13
7	·5608798	·677475	1·476068	·8278972	53	28	·5659267	·686424	1·456824	·8244556	32	48	·5707136	·695018	1·438811	·8211492	12
8	·5611206	·677899	1·475144	·8277340	52	29	·5661665	·686852	1·455916	·8242909	31	49	·5709524	·695449	1·437918	·8209832	11
9	·5613614	·678324	1·474221	·8275708	51	30	·5664062	·687281	1·455009	·8241262	30	50	·5711912	·695881	1·437026	·8208170	10
10	·5616021	·678749	1·473298	·8274074	50	31	·5666459	·687709	1·454102	·8239614	29	51	·5714299	·696313	1·436135	·8206509	9
11	·5618428	·679174	1·472376	·8272440	49	32	·5668856	·688137	1·453197	·8237965	28	52	·5716686	·696745	1·435245	·8204846	8
12	·5620834	·679599	1·471455	·8270806	48	33	·5671252	·688566	1·452292	·8236316	27	53	·5719073	·697177	1·434355	·8203183	7
13	·5623239	·680024	1·470535	·8269170	47	34	·5673648	·688995	1·451388	·8234666	26	54	·5721459	·697609	1·433466	·8201519	6
14	·5625645	·680450	1·469615	·8267534	46	35	·5676043	·689424	1·450485	·8233015	25	55	·5723844	.698042	1·432578	·8199854	5
15	·5628049	·680875	1·468696	·8265897	45	36	·5678437	·689853	1·449582	·8231364	24	56	·5726229	·698474	1·431690	·8198189	4
16	·5630453	·681301	1·467778	·8264260	44	37	·5680832	·690283	1·448680	·8229712	23	57	·5728614	·698907	1.430803	·8196523	3
17	·5632857	·681727	1·466861	·8262622	43	38	·5683225	·690712	1·447779	·8228059	22	58	·5730998	·699340	1.429917	·8194856	2
18	·5635260	·682153	1·465945	·8260983	42	39	·5685619	·691142	1·446879	·8226405	21	59	·5733381	·699774	1.429032	·8193189	1
19	·5637663	·682580	1·465029	·8259343	41	40	·5688011	·691572	1·445980	·8224751	20	60	·5735764	·700207	1.428148	·8191520	0
20	·5640066	·683006	1·464114	·8257703	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
				Deg. 55.					Deg. 55.							Deg. 56.	

′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
			Deg. 54.						Deg. 54.						Deg. 54.		
1	·5735184	·700207	1·428148	·8191520	60	21	·5785690	·709350	1·409740	·8156330	39	41	·5833050	·718131	1·392501	·8122532	19
2	·5738147	·700641	1·427264	·8189852	59	22	·5788069	·709787	1·408871	·8154647	38	42	·5835412	·718572	1·391647	·8120835	18
3	·5740529	·701074	1·426381	·8188182	58	23	·5790440	·710225	1·408003	·8152963	37	43	·5837774	·719014	1·390793	·8119137	17
4	·5742911	·701508	1·425498	·8186512	57	24	·5792812	·710663	1·407136	·8151278	36	44	·5840136	·719455	1·389940	·8117439	16
5	·5745292	·701943	1·424617	·8184841	56	25	·5795183	·711100	1·406270	·8149593	35	45	·5842497	·719897	1·389087	·8115740	15
6	·5747672	·702377	1·423736	·8183169	55	26	·5797553	·711539	1·405404	·8147906	34	46	·5844857	·720338	1·388235	·8114040	14
7	·5750053	·702811	1·422856	·8181497	54	27	·5799923	·711977	1·404539	·8146220	33	47	·5847217	·720780	1·387384	·8112339	13
8	·5752432	·703246	1·421976	·8179824	53	28	·5802292	·712415	1·403674	·8144532	32	48	·5849577	·721222	1·386534	·8110638	12
9	·5754811	·703681	1·421097	·8178151	52	29	·5804661	·712854	1·402811	·8142844	31	49	·5851936	·721665	1·385684	·8108936	11
10	·5757190	·704116	1·420220	·8176476	51	30	·5807030	·713293	1·401948	·8141155	30	50	·5854294	·722107	1·384835	·8107234	10
11	·5759568	·704551	1·419342	·8174801	50	31	·5809397	·713732	1·401086	·8139466	29	51	·5856652	·722550	1·383986	·8105530	9
12	·5761946	·704986	1·418466	·8173125	49	32	·5811765	·714171	1·400224	·8137775	28	52	·5859010	·722993	1·383139	·8103826	8
13	·5764323	·705422	1·417590	·8171449	48	33	·5814132	·714610	1·399363	·8136084	27	53	·5861367	·723436	1·382292	·8102122	7
14	·5766700	·705858	1·416715	·8169772	47	34	·5816498	·715050	1·398503	·8134393	26	54	·5863724	·723879	1·381445	·8100416	6
15	·5769076	·706294	1·415840	·8168094	46	35	·5818864	·715489	1·397644	·8132701	25	55	·5866080	·724322	1·380600	·8098710	5
16	·5771452	·706730	1·414967	·8166416	45	36	·5821230	·715929	1·396785	·8131008	24	56	·5868435	·724766	1·379755	·8097004	4
17	·5773827	·707166	1·414094	·8164736	44	37	·5823595	·716369	1·395927	·8129314	23	57	·5870790	·725210	1·378910	·8095296	3
18	·5776202	·707602	1·413222	·8163056	43	38	·5825959	·716810	1·395069	·8127620	22	58	·5873145	·725654	1·378067	·8093588	2
19	·5778576	·708039	1·412350	·8161376	42	39	·5828323	·717250	1·394213	·8125925	21	59	·5875499	726098	1·377224	·8091879	1
20	·5780950	·708476	1·411479	8159695	41	40	·5830687	·717691	393357	·8124229	20	60	·5877853	726542	1·376381	·8090170	0
	·5783323	·708913	1·410609	8158013	40												

NATURAL SINES AND TANGENTS TO A RADIUS 1.

36 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5877853	·726542	1·376381	·8090170	60
1	·5880206	·726987	1·375540	·8088460	59
2	·5882558	·727431	1·374699	·8086749	58
3	·5884910	·727876	1·373859	·8085037	57
4	·5887262	·728321	1·373019	·8083325	56
5	5889613	·728767	1·372180	·8081612	55
6	·5891964	·729212	1·371342	·8079899	54
7	·5894314	·729658	1·370504	·8078185	53
8	·5896663	·730104	1·369667	·8076470	52
9	·5899012	·730550	1·368831	·8074754	51
10	·5901361	·730996	1·367995	·8073038	50
11	·5903709	·731442	1·367161	·8071321	49
12	·5906057	·731889	1·366326	·8069603	48
13	·5908404	·732336	1·365493	·8067885	47
14	·5910750	·732783	1·364660	·8066166	46
15	·5913096	·733230	1·363827	·8064446	45
16	·5915442	·733677	1·362996	·8062726	44
17	·5917787	·734125	1·362165	·8061005	43
18	·5920132	·734573	1·361335	·8059283	42
19	·5922476	·735021	1·360505	·8057560	41
20	·5924819	735469	1·359676	·8055837	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 53.

36 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·5927163	·735917	1·358848	·8054113	39
22	·5929505	·736366	1·358020	·8052389	38
23	·5931847	·736814	1·357193	·8050664	37
24	·5934189	·737263	1·356367	·8048938	36
25	·5936530	·737712	1·355541	·8047211	35
26	·5938871	·738162	1·354716	·8045484	34
27	·5941211	·738611	1·353891	·8043756	33
28	·5943550	·739061	1·353068	·8042028	32
29	·5945889	·739511	1·352244	·8040299	31
30	·5948228	·739961	1·351422	·8038569	30
31	·5950566	·740411	1·350600	·8036838	29
32	·5952904	·740861	1·349779	·8035107	28
33	·5955241	·741312	1·348958	·8033375	27
34	·5957577	·741763	1·348139	·8031642	26
35	·5959913	·742214	1·347319	·8029909	25
36	·5962249	·742665	1·346501	·8028175	24
37	·5964584	·743117	1.345683	·8026440	23
38	·5966918	·743568	1.344865	·8024705	22
39	·5969252	·744020	1.344049	·8022969	21
40	·5971586	·744472	1.343233	·8021232	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 53.

36 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5973919	·744924	1·342417	·8019495	19
42	·5976251	·745377	1·341602	·8017756	18
43	·5978583	·745829	1·340788	·8016018	17
44	·5980915	·746282	1·339975	·8014278	16
45	·5983246	·746735	1·339162	·8012538	15
46	·5985577	·747188	1·338350	·8010797	14
47	·5987906	·747642	1·337538	·8009056	13
48	·5990236	·748095	1·336727	·8007314	12
49	·5992565	·748549	1·335917	·8005571	11
50	·5994893	·749003	1·335107	·8003827	10
51	·5997221	·749457	1·334298	·8002083	9
52	·5999549	·749911	1·333490	·8000338	8
53	·6001876	·750366	1·332682	·7998593	7
54	·6004202	·750821	1·331875	·7996847	6
55	·6006528	·751276	1·331068	·7995100	5
56	·6008854	·751731	1·330262	·7993352	4
57	·6011179	·752186	1·329457	·7991604	3
58	·6013503	·752642	1·328652	·7989855	2
59	·6015827	·753098	1·327848	·7988105	1
60	·6018150	·753554	1·327044	·7986355	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 53.

37 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6018150	·753554	1·327044	·7986355	60	21	·6066824	·763175	1·310314	·7949444	39	11	·6112969	·772423	1·294627	·7914014	19
1	·6020473	·754010	1·326242	·7984604	59	22	·6069136	·763636	1·309523	·7947678	38	42	·6115270	·772887	1·293848	·7912235	18
2	·6022795	·754466	1·325439	·7982853	58	23	·6071447	·764096	1·308734	·7945913	37	43	·6117572	·773352	1·293071	·7910456	17
3	·6025117	·754923	1·324638	·7981100	57	24	·6073758	·764557	1·307945	·7944146	36	44	·6119873	·773817	1·292294	·7908676	16
4	·6027439	·755379	1·32?837	·7979347	56	25	·6076069	·765018	1·307157	·7942379	35	45	·6122173	·774282	1·291517	·7906896	15
5	·6029760	·755836	1·323036	·7977594	55	26	·6078379	·765480	1·306369	·7940611	34	46	·6124473	·774748	1·290742	·7905115	14
6	·6032080	·756294	1·322237	·7975839	54	27	·6080689	·765941	1·305582	·7938843	33	47	·6126772	·775213	1·289966	·7903333	13
7	·6034400	·756751	1·321437	·7974084	53	28	·6082998	·766403	1·304796	·7937074	32	48	·6129071	·775679	1·289192	·7901550	12
8	·6036719	·757209	1·320639	·7972329	52	29	·6085306	·766864	1·304010	·7935304	31	49	·6131369	·776145	1·288418	·7899767	11
9	·6039038	·757666	1·319841	·7970572	51	30	·6087614	·767327	1·303225	·7933533	30	50	·6133666	·776611	1·287644	·7897983	10
10	·6041356	758124	1·319044	·7968815	50	31	·6089922	·767789	1·302440	·7931762	29	51	·6135964	·777078	1·286871	·7896198	9
11	·6043674	·758582	1·318247	·7967058	49	32	·6092229	·768251	1·301656	·7929990	28	52	·6138260	·777544	1·286099	·7894413	8
12	·6045991	·759041	1·317451	·7965299	48	33	·6094535	·768714	1·300873	·7928218	27	53	·6140556	·778011	1·285327	·7892627	7
13	·6048308	·759499	1·316655	·7963540	47	34	·6096841	·769177	1·300090	·7926445	26	54	·6142852	·778478	1·284556	·7890841	6
14	·6050624	·759958	1·315861	·7961780	46	35	·6099147	·769640	1·299308	·7924671	25	55	·6145147	·778946	1·283786	·7889054	5
15	·6052940	·760417	1.315066	·7960020	45	36	·6101452	·770103	1·298526	·7922896	24	56	·6147442	·779413	1·283016	·7887266	4
16	·6055255	·760876	1·314273	·7958259	44	37	·6103756	·770567	1·297745	·7921121	23	57	·6149736	·779881	1·282246	·7885477	3
17	·6057570	·761336	1.313480	·7956497	43	38	·6106060	·771030	1·296964	·7919345	22	58	·6152029	·780349	1·281477	·7883688	2
18	·6059884	·761795	1.312687	·7954735	42	39	·6108363	·771494	1·296185	·7917569	21	59	·6154322	·780817	1·280709	·7881898	1
19	·6062198	·762255	1.311895	.7952972	41	40	·6110666	·771958	1·295405	·7915792	20	60	·6156615	·781285	1·279941	·7880108	0
20	·6064511	·762715	1.311104	·7951208	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

37 Deg. — Deg. 52. — 37 Deg. — Deg. 52. — 37 Deg. — Deg. 52.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
							38 Deg.						38 Deg.				
0	·6156615	·781285	1·279941	·7880108	60	21	·6204636	·791170	1·263950	·7842352	39	41	·6250156	·800673	1·248948	·7806123	19
1	·6158907	·781754	1·279174	·7878316	59	22	·6206917	·791643	1·263195	·7840547	38	42	·6252427	·801151	1·248204	·7804304	18
2	·6161198	·782222	1·278407	·7876524	58	23	·6209198	·792116	1·262440	·7838741	37	43	·6254696	·801628	1·247460	·7802485	17
3	·6163489	·782691	1·277641	·7874732	57	24	·6211478	·792590	1·261686	·7836935	36	44	·6256966	·802106	1·246716	·7800665	16
4	·6165780	·783161	1·276876	·7872939	56	25	·6213757	·793064	1·260932	·7835127	35	45	·6259235	·802584	1·245974	·7798845	15
5	·6168069	·783630	1·276111	·7871145	55	26	·6216036	·793537	1·260179	·7833320	34	46	·6261503	·803063	1·245232	·7797024	14
6	·6170359	·784100	1·275347	·7869350	54	27	·6218314	·794012	1·259426	·7831511	33	47	·6263771	·803541	1·244490	·7795202	13
7	·6172648	·784570	1·274583	·7867555	53	28	·6220592	·794486	1·258674	·7829702	32	48	·6266038	·804020	1·243749	·7793380	12
8	·6174936	·785040	1·273820	·7865759	52	29	·6222870	·79 1961	1·257923	·7827892	31	49	·6268305	·804499	1·243008	·7791557	11
9	·6177224	·785510	1·273057	·7863963	51	30	·6225146	·795435	1·257172	·7826082	30	50	·6270571	·804979	1·242268	·7789733	10
10	·6179511	·785980	1·272295	·7862165	50	31	·6227423	·795911	1·256421	·7824270	29	51	·6272837	805458	1·241529	·7787909	9
11	·6181798	·786451	1·271534	·7860367	49	32	·6229698	·796386	1·255672	·7822459	28	52	·6275102	·805938	1·240790	·7786084	8
12	·6184084	·786922	1·270773	·7858569	48	33	·6231974	·796861	1·254922	·7820646	27	53	·6277366	·806418	1·240051	·7784258	7
13	·6186370	·787393	1·270013	·7856770	47	34	·6234248	·797337	1·254174	·7818833	26	54	·6279631	·806898	1·239313	·7782431	6
14	·6188655	·787864	1·269253	·7854970	46	35	·6236522	·797813	1·253426	·7817019	25	55	·6281894	·807378	1·238576	·7780604	5
15	·6190939	·788336	1·268494	·7853169	45	36	·6238796	·798289	1·252678	·7815205	24	56	·6284157	·807859	1·237839	·7778777	4
16	·6193224	788808	1·267735	·7851368	44	37	·6241069	·798765	1·251931	·7813390	23	57	·6286420	·808340	1·237103	·7776949	3
17	·6195507	·789280	1·266977	·7849566	43	38	·6243342	·799242	1·251184	·7811574	22	58	·6288682	·808821	1·236367	·7775120	2
18	·6197790	·789752	1·266219	·7847764	42	39	·6245614	·799719	1·250438	·7809757	21	59	·6290943	·809302	1·235631	·7773290	1
19	·6200073	·790224	1·265462	·7845961	41	40	·6247885	·800196	1·249693	·7807940	20	60	·6293204	·809784	1·234897	·7771460	0
20	·6202355	·790697	1·264706	·7844157	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
				Deg. 51						Deg. 51.						Deg. 51.	

Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	39 Deg.						39 Deg.									
6293204	·809784	1·234897	·7771460	60	21	·6340559	·819948	1·219588	·7732872	39	41	·6385440	·829724	1·205219	·7695853	19
6295464	·810265	1·234162	·7769629	59	22	·6342808	·820435	1·218865	·7731027	38	42	·6387678	·830216	1·204505	·7693996	18
6297724	·810747	1·233429	·7767797	58	23	·6345057	·820922	1·218142	·7729182	37	43	·6389916	·830707	1·203793	·7692137	17
6299983	·811230	1·232696	·7765965	57	24	·6347305	·821409	1·217419	·7727336	36	44	·6392153	·831199	1·203081	·7690278	16
6302242	·811712	1·231963	·7764132	56	25	·6349553	·821896	1·216698	·7725489	35	45	·6394390	·831691	1·202369	·7688418	15
6304500	·812195	1·231231	·7762298	55	26	·6351800	·822384	1·215976	·7723642	34	46	·6396626	·832183	1·201658	·7686558	14
6306758	·812678	1·230499	·7760464	54	27	·6354046	·822871	1·215256	·7721794	33	47	·6398862	·832675	1·200947	·7684697	13
6309015	·813161	1·229768	·7758629	53	28	·6356292	·823359	1·214535	·7719945	32	48	·6401097	·833168	1·200237	·7682835	12
6311272	·813644	1·229038	·7756794	52	29	·6358537	·823847	1·213816	·7718096	31	49	·6403332	·833661	1·199527	·7680973	11
6313528	·814128	1·228308	·7754957	51	30	·6360782	·824336	1·213097	·7716246	30	50	·6405566	·834154	1·198818	·7679110	10
6315784	·814611	1·227578	·7753121	50	31	·6363026	·824825	1·212378	·7714395	29	51	·6407799	·834648	1·198109	·7677246	9
6318039	·815095	1·226849	·7751283	19	32	·6365270	·825314	1·211660	·7712544	28	52	·6410032	·835141	1·197401	·7675382	8
6320293	·815580	1·226121	·7749445	18	33	·6367513	·825803	1·210942	·7710692	27	53	·6412264	·835635	1·196693	·7673517	7
6322547	·816064	1·225393	·7747606	47	34	·6369756	·826292	1·210225	·7708840	26	54	·6414496	·836129	1·195986	·7671652	6
6324800	·816549	1·224665	·7745767	46	35	·6371998	·826782	1·209508	·7706986	25	55	·6416728	·836624	1·195279	·7669785	5
6327053	·817034	1·223938	·7743926	45	36	·6374240	·827271	1·208792	·7705132	24	56	·6418958	·837118	1·194573	·7667918	4
6329306	·817519	1·223212	·7742086	44	37	·6376481	·827762	1·208076	·7703278	23	57	·6421189	·837613	1·193867	·7666051	3
6331557	·818004	1·222486	·7740244	43	38	·6378721	·828252	1·207361	·7701423	22	58	·6423418	·838108	1·193162	·7664183	2
6333809	·818490	1·221761	·7738402	42	39	·6380961	·828742	1·206646	·7699567	21	59	·6425647	·838604	1·192457	·7662314	1
6336059	·818976	1·221036	·7736559	41	40	·6383201	·829233	1·205932	·7697710	20	60	·6427876	·839099	1·191753	·7660444	0
6338310	·819462	1·220312	·7734716	40												
Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
			Deg. 50.						Deg. 50.						Deg. 50.	

40 Deg.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
						40 Deg.						40 Deg.					
0	·6427876	·839099	1·191753	·7660444	60	21	·6474551	·849563	1·177075	·7621036	39	41	·6518778	·859629	1·163291	·7583240	19
1	·6430104	·839595	1·191049	·7658574	59	22	·6476767	·850064	1·176382	·7619152	38	42	·6520984	·860135	1·162607	·7581343	18
2	·6432332	·840091	1.190346	·7656704	58	23	·6478984	·850565	1·175688	·7617268	37	43	6523189	·860641	1·161923	·7579446	17
3	·6434559	·840587	1·189643	·7654832	57	24	·6481199	·851066	1·174996	·7615383	36	44	·6525394	·861148	1·161240	·7577548	16
4	·6436785	·841084	1·188941	·7652960	56	25	·6483414	·851568	1·174303	·7613497	35	45	·6527598	·861655	1·160557	·7575650	15
5	·6439011	·841581	1·188239	·7651087	55	26	·6485628	·852070	1·173612	·7611611	34	46	·6529801	·862162	1·159874	·7573751	14
6	·6441236	·842078	1·187538	·7649214	54	27	·6487842	·852572	1·172920	·7609724	33	47	·6532004	862669	1·159192	7571851	13
7	·6443461	·842575	1·186837	·7647340	53	28	·6490056	·853075	1·172229	·7607837	32	48	·6534206	·863176	1·158511	·7569951	12
8	·6445685	·843073	1·186136	·7645465	52	29	·6492268	·853577	1·171539	·7605949	31	49	·6536408	·863684	1·157830	·7568050	11
9	·6447909	·843570	1·185437	·7643590	51	30	·6494480	·854080	1·170849	·7604060	30	50	·6538609	·864192	1·157149	·7566148	10
10	·6450132	·844068	1·184737	·7641714	50	31	·6496692	·854583	1·170160	·7602170	29	51	·6540810	·864700	1·156469	·7564246	9
11	·6452355	·844567	1·184038	·7639838	49	32	·6498903	·855087	1·169471	·7600280	28	52	·6543010	·865209	1·155789	·7562343	8
12	·6454577	·845065	1·183340	·7637960	48	33	·6501114	·855591	1·168782	·7598389	27	53	·6545209	·865718	1·155110	·7560439	7
13	·6456798	·845564	1·182642	·7636082	47	34	·6503324	·856095	1·168094	·7596498	26	54	·6547408	·866227	1·154431	·7558535	6
14	·6459019	·846063	1·181944	·7634204	46	35	·6505533	·856599	1·167407	·7594606	25	55	·6549607	·866736	1·153753	·7556630	5
15	·6461240	·846562	1·181247	·7632325	45	36	·6507742	·857103	1·166720	·7592713	24	56	·6551804	·867246	1·153075	·7554724	4
16	·6463460	·847062	1·180551	·7630445	44	37	·6509951	·857608	1·166033	·7590820	23	57	·6554002	·867755	1·152397	·7552818	3
17	·6465679	·847561	1·179855	·7628564	43	38	·6512158	·858113	1·165347	·7588926	22	58	·6556198	·868265	1·151721	·7550911	2
18	·6467898	·848061	1·179159	·7626683	42	39	·6514366	·858618	1·164661	·7587031	21	59	·6558395	·868776	1·151044	·7549004	1
19	·6470116	·848561	1·178464	·7624802	41	40	·6516572	·859124	1·163976	·7585136	20	60	·6560590	·869286	1·150368	·7547096	0
20	·6472334	·849062	1·177769	·7622919	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
				Deg. 49.						Deg. 49.						Deg. 49.	

′	Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	41 Deg. Sine.	Tang.	Cotang.	Cosine.	′	′	41 Deg. Sine.	Tang.	Cotang.	Cosine.	′
0	·6560590	·869286	1·150368	·7547096	60	21	·6606570	·880068	1·136274	·7506879	39	41	·6650131	·890445	1·123032	·7468317	19
1	·6562785	·869797	1·149692	·7545187	59	22	·6608754	·880585	1·135608	·7504957	38	42	·6652304	·890967	1·122375	·7466382	18
2	·6564980	·870308	1·149017	·7543278	58	23	·6610936	·881101	1·134942	·7503034	37	43	·6654475	·891489	1·121718	·7464446	17
3	·6567174	·870820	1·148342	·7541368	57	24	·6613119	·881618	1·134277	·7501111	36	44	·6656646	·892011	1·121061	·7462510	16
4	·6569367	·871331	1·147668	·7539457	56	25	·6615300	·882135	1·133612	·7499187	35	45	·6658817	·892534	1·120405	·7460574	15
5	·6571560	·871843	1·146994	·7537546	55	26	·6617482	·882653	1·132947	·7497262	34	46	·6660987	·893056	1·119749	·7458636	14
6	·6573752	·872355	1·146321	·7535634	54	27	·6619662	·883170	1·132283	·7495337	33	47	·6663156	·893579	1·119094	·7456699	13
7	·6575944	·872868	1·145648	·7533721	53	28	·6621842	·883688	1·131620	·7493411	32	48	·6665325	·894103	1·118439	·7454760	12
8	·6578135	·873380	1·144976	·7531808	52	29	·6624022	·884206	1·130957	·7491484	31	49	·6667493	·894626	1·117784	·7452821	11
9	·6580326	·873893	1·144304	·7529894	51	30	·6626200	·884725	1·130294	·7489557	30	50	·6669661	·895150	1·117130	·7450881	10
10	·6582516	·874406	1·143632	·7527980	50	31	·6628379	·885244	1·129632	·7487629	29	51	·6671828	·895674	1·116476	·7448941	9
11	·6584706	·874920	1·142961	·7526065	49	32	·6630557	·885763	1·128970	·7485701	28	52	·6673994	·896199	1·115823	·7446999	8
12	·6586895	·875433	1·142290	·7524149	48	33	·6632734	·886282	1·128308	·7483772	27	53	·6676160	·896723	1·115170	·7445058	7
13	·6589083	·875947	1·141620	·7522233	47	34	·6634910	·886801	1·127647	·7481842	26	54	·6678326	·897248	1·114518	·7443115	6
14	·6591271	·876462	1·140950	·7520316	46	35	·6637087	·887321	1·126987	·7479912	25	55	·6680490	·897773	1·113866	·7441173	5
15	·6593458	·876976	1·140281	·7518398	45	36	·6639262	·887841	1·126327	·7477981	24	56	·6682655	·898299	1·113214	·7439229	4
16	·6595645	·877491	1·139612	·7516480	44	37	·6641437	·888361	1·125667	·7476049	23	57	·6684818	·898825	1·112563	·7437285	3
17	·6597831	·878006	1·138944	·7514561	43	38	·6643612	·888882	1·125008	·7474117	22	58	·6686981	·899351	1·111912	·7435340	2
18	·6600017	·878521	1·138276	·7512641	42	39	·6645785	·889403	1·124349	·7472184	21	59	·6689144	·899877	1·111262	·7433394	1
19	·6602202	·879037	1·137608	·7510721	41	40	·6647959	·889924	1·123690	·7470251	20	60	·6691306	·900404	1·110612	·7431448	0
20	·6604386	·879552	1·136941	·7508800	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
				Deg. 48.						Deg. 48.						Deg. 48.	

NATURAL SINES AND TANGENTS TO A RADIUS 1.

42 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6691306	·900404	1·110612	·7431448	60
1	·6693468	·900930	1·109963	·7429502	59
2	·6695628	·901458	1·109314	·7427554	58
3	·6697789	·901985	1·108665	·7425606	57
4	·6699948	·902513	1·108017	·7423658	56
5	·6702108	·903041	1·107369	·7421708	55
6	·6704266	·903569	1·106721	·7419758	54
7	·6706424	904097	1·106075	·7417808	53
8	·6708582	·904626	1·105428	·7415857	52
9	·6710739	·905155	1·104782	·7413905	51
10	·6712895	·905685	1·104136	·7411953	50
11	·6715051	·906214	1·103491	·7410000	49
12	·6717206	·906744	1·102846	·7408046	48
13	·6719361	·907274	1·102201	·7406092	47
14	·6721515	·907805	1·101557	·7404137	46
15	·6723668	·908336	1·100914	·7402181	45
16	·6725821	·908867	1·100270	·7400225	44
17	·6727973	·909398	1·099628	·7398268	43
18	·6730125	·909930	1·098985	·7396311	42
19	·6732276	·910461	1·098343	·7394353	41
20	·6734427	·910994	1·097702	·7392394	40

42 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6736577	·911526	1·097060	·7390435	39
22	·6738727	·912059	1·096420	·7388475	38
23	·6740876	·912592	1·095779	·7386515	37
24	·6743024	·913125	1·095139	·7384553	36
25	·6745172	·913659	1·094500	·7382592	35
26	·6747319	·914192	1·093861	·7380629	34
27	·6749466	·914727	1·093222	·7378666	33
28	·6751612	·915261	1·092584	·7376703	32
29	·6753757	·915796	1·091946	·7374738	31
30	·6755902	·916331	1·091308	·7372773	30
31	·6758046	·916866	1·090671	·7370808	29
32	·6760190	·917402	1·090034	·7368842	28
33	·6762333	·917937	1·089398	·7366875	27
34	·6764476	·918474	1·088762	·7364908	26
35	·6766618	·919010	1·088126	·7362940	25
36	·6768760	·919547	1·087491	·7360971	24
37	·6770901	·920084	1·086857	·7359002	23
38	·6773041	·920621	1·086222	·.357032	22
39	·6775181	·921159	1·085588	·7355061	21
40	·6777320	·921696	1·084955	·7353090	20

42 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6779459	·922235	1.084322	·7351118	19
42	·6781597	·922773	1·083689	·7349146	18
43	·6783734	·923312	1·083057	·7347173	17
44	·6785871	·923851	1·082425	·7345199	16
45	·6788007	·924390	1·081793	·7343225	15
46	·6790143	·924930	1·081162	·7341250	14
47	·6792278	·925470	1·080532	·7339275	13
48	·6794413	·926010	1·079901	·7337299	12
49	·6796547	·926550	1·079271	·7335322	11
50	·6798681	·927091	1·078642	·7333345	10
51	·6800813	·927632	1·078013	·7331367	9
52	·6802946	·928173	1·077384	·7329388	8
53	·6805078	·928715	1·076756	·7327409	7
54	·6807209	·929257	1·076128	·7325429	6
55	·6809339	·929799	1·075500	·7323449	5
56	·6811469	·930342	1·074873	·7321467	4
57	·6813599	·930884	1·074246	·7319486	3
58	·6815728	·931428	1·073620	·7317503	2
59	·6817856	·931971	1·072994	·7315521	1
60	·6819984	·932515	1·072368	·7313537	0

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
	43 Deg.						43 Deg.						43 Deg.				
0	·6819984	·932515	1·072368	·7313537	60	21	·6864532	·944001	1·059320	·7271740	39	41	·6906721	·955064	1·047049	·7231681	19
1	·6822111	·933059	1·071743	·7311553	59	22	·6866647	·944551	1·058703	·7269743	38	42	·6908824	·955620	1·046440	·7229671	18
2	·6824237	·933603	1·071118	·7309568	58	23	·6868761	·945102	1·058086	·7267745	37	43	·6910927	·956177	1·045831	·7227661	17
3	·6826363	·934147	1·070494	·7307583	57	24	·6870875	·945653	1·057470	·7265747	36	44	·6913029	·956734	1·045222	·7225651	16
4	·6828489	·934692	1·069870	·7305597	56	25	·6872988	·946204	1·056854	·7263748	35	45	·6915131	·957291	1·044613	·7223640	15
5	·6830613	·935238	1·069246	·7303610	55	26	·6875101	·946755	1·056238	·7261748	34	46	·6917232	·957849	1·044005	·7221628	14
6	·6832738	·935783	1·068623	·7301623	54	27	·6877213	·947307	1·055623	·7259748	33	47	·6919332	·958407	1·043397	·7219615	13
7	·6834861	·936329	1·068000	·7299635	53	28	·6879325	·947859	1·055008	·7257747	32	48	·6921432	·958965	1·042790	·7217602	12
8	·6836984	·936875	1·067377	·7297646	52	29	·6881435	·948411	1·054394	·7255746	31	49	·6923531	·959524	1·042183	·7215589	11
9	·6839107	·937421	1·066755	·7295657	51	30	·6883546	·948964	1·053780	·7253744	30	50	·6925630	·960082	1·041576	·7213574	10
10	·6841229	·937968	1·066134	·7293668	50	31	·6885655	·949517	1·053166	·7251741	29	51	·6927728	·960642	1·040970	·7211559	9
11	·6843350	·938515	1·065512	·7291677	49	32	·6887765	·950070	1·052553	·7249738	28	52	·6929825	·961201	1·040364	·7209544	8
12	·6845471	·939062	1·064891	·7289686	48	33	·6889873	·950624	1·051940	·7247734	27	53	·6931922	·961761	1·039758	·7207528	7
13	·6847591	·939610	1·064271	·7287695	47	34	·6891981	·951178	1·051327	·7245729	26	54	·6934018	·962321	1·039153	·7205511	6
14	·6849711	·940157	1·063651	·7285703	46	35	·6894089	·951732	1·050715	·7243724	25	55	·6936114	·962881	1·038548	·7203494	5
15	·6851830	·940706	1·063031	·7283710	45	36	·6896195	·952287	1·050103	·7241719	24	56	·6938209	·963442	1·037944	·7201476	4
16	·6853948	·941254	1·062411	·7281716	44	37	·6898302	·952842	1·049492	·7239712	23	57	·6940304	·964003	1·037340	·7199457	3
17	·6856066	·941803	1·061792	·7279722	43	38	·6900407	·953397	1·048880	·7237705	22	58	·6942398	·964565	1·036736	·7197438	2
18	·6858184	·942352	1·061174	·7277728	42	39	·6902512	·953952	1·048270	·7235698	21	59	·6944491	·965126	1·036133	·7195418	1
19	·6860300	·942901	1·060556	·7275732	41	40	·6904617	·954508	1·047659	·7233690	20	60	·6946584	·965688	1·035530	·7193398	0
20	·6862416	·943451	1·059938	·7273736	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
		Deg. 16.						Deg. 46.								Deg. 46.	

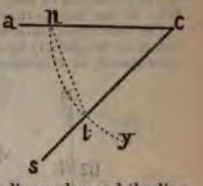
NATURAL SINES AND TANGENTS TO A RADIUS 1.

44 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6946584	·965688	1·035530	·7193398	60	21	·6990396	·977564	1·022950	·7150830	39	41	·7031879	·989066	1·011115	·7110941	19
1	·6948676	·966251	1·034927	·7191377	59	22	·6992476	·978133	1·022355	·7148796	38	42	·7033947	·989582	1·010527	·7107995	18
2	·6950767	·966813	1·034325	·7189355	58	23	·6994555	·978702	1·021760	·7146762	37	43	·7036014	·990158	1·009939	·7105049	17
3	·6952858	·967376	1·033723	·7187333	57	24	·6996633	·979272	1·021166	·7144727	36	44	·7038081	·990734	1·009352	·7102101	16
4	·6954949	·967939	1·033122	·7185310	56	25	·6998711	·979842	1·020572	·7142691	35	45	·7040147	·991311	1·008764	·7101854	15
5	·6957039	·968503	1·032520	·7183287	55	26	·7000789	·980412	1·019978	·7140655	34	46	·7042213	·991888	1·008178	·7099806	14
6	·6959128	·969067	1·031919	·7181263	54	27	·7002866	·980983	1·019385	·7138618	33	47	·7044278	·992465	1·007591	·7097757	13
7	·6961217	·969631	1·031319	·7179238	53	28	·7004942	·981554	1·018792	·7136581	32	48	·7046342	·993042	1·007005	·7095707	12
8	·6963305	·970196	1·030719	·7177213	52	29	·7007018	·982125	1·018199	·7134543	31	49	·7048406	·993620	1·006420	·7093657	11
9	·6965392	·970761	1·030119	·7175187	51	30	·7009093	·982697	1·017607	·7132504	30	50	·7050469	·994199	1·005834	·7091607	10
10	·6967479	·971326	1·029520	·7173161	50	31	·7011167	·983269	1·017015	·7130465	29	51	·7052532	·994777	1·005249	·7089556	9
11	·6969565	·971891	1·028921	·7171134	49	32	·7013241	·983841	1·016423	·7128426	28	52	·7054594	·995356	1·004665	·7087504	8
12	·6971651	·972457	1·028322	·7169106	48	33	·7015314	·984414	1·015832	·7126385	27	53	·7056655	·995935	1·004080	·7085451	7
13	·6973736	·973023	1·027724	·7167078	47	34	·7017387	·984987	1·015241	·7124344	26	54	·7058716	·996515	1·003496	·7083398	6
14	·6975821	·973590	1·027126	·7165049	46	35	·7019459	·985560	1·014651	·7122303	25	55	·7060776	·997095	1·002913	·7081345	5
15	·6977905	·974156	1·026528	·7163019	45	36	·7021531	·986133	1·014061	·7120260	24	56	·7062835	·997676	1·002329	·7079291	4
16	·6979988	·974724	1·025931	·7160989	44	37	·7023601	·986707	1·013471	·7118218	23	57	·7064894	·998256	1·001746	·7077236	3
17	·6982071	·975291	1·025334	·7158959	43	38	·7025672	·987282	1·012881	·7116174	22	58	·7066953	·998837	1·001164	·7075180	2
18	·6984153	·975859	1·024738	·7156927	42	39	·7027741	·987856	1·012292	·7114130	21	59	·7069011	·999418	1·000581	·7073124	1
19	·6986234	·976427	1·024141	·7154895	41	40	·7029811	·988431	1·011703	·7112086	20	60	·7071068	1·00000	1·000000	·7071068	0
20	·6988315	·976995	1·023546	·7152863	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 45. Deg. 45. Deg. 45.

REM. **The table,** pp. 608–612, is to furnish the means of laying down angles on paper more accurately than by an ordinary protractor. To do this, after having drawn and measured the first side (say $a\,c$) of the figure that is to be plotted; from its end c as a center, describe an arc $n\,y$ of a circle of sufficient extent to subtend the angle at that point. The rad $c\,n$ with which the arc is described should be as great as convenience will permit; and it is to be assumed as unity or 1; and must be decimally divided, and subdivided, to be used as a scale for laying down the chords taken from the table, in which their lengths are given in parts of said rad 1. Having described the arc, find in the table the length of the chord $n\,t$ corresponding to the angle $a\,c\,t$. Let us suppose this angle to be 45°; then we find that the tabular chord is .7654 of our rad 1. Therefore from n we lay off the chord $n\,t$, equal to .7654 of our radius-scale; and the line $c\,t$ drawn through the point t will form the reqd angle $a\,c\,t$ of 45°. And so at each angle. The degree of accuracy attained will evidently depend on the length of the rad, and the neatness of the drafting. The method becomes preferable to the common protractor in proportion as the lengths of the sides of the angles exceed the rad of the protractor. With a protractor of 4 to 6 ins rad, and with sides of angles not much exceeding the same limits, the protractor will usually be preferable. The dividers in boxes of instruments are rarely fit for accurate arcs of more than about 6 ins diam. In practice it is not necessary to actually describe the whole arc, but merely the portion near t, as well as can be judged by eye. We thus avoid much use of the India-rubber, and dulling of the pencil-point. For larger radii we may dispense with the dividers, and use a straight strip of paper with the length of the rad marked on one edge; and by laying it from c toward s, and at the same time placing another strip (with one edge divided to a radius-scale) from n toward t, we can by trial find their exact point of intersection at the required point t. In such matters, practice and some ingenuity are very essential to satisfactory results. We cannot devote more space to the subject.



CONTOUR LINES.

A CONTOUR LINE is a curved hor one, every point in which represents the same level; thus each of the contour lines 88c, 91c, 94c, &c, Fig 1, indicates that every point in the ground through which it is traced is at the same level; and that that level or height is everywhere 88, 91, or 94 ft above a certain other level or height called *datum*; to which all others are referred.

Frequently the level of the starting point of a survey is taken as being O, or zero, or datum; and if we are sure of meeting with no points lower than it, this answers every purpose. But if there is a probability of many lower points, it is better to assume the starting point to be so far above a certain supposed datum, that none of these lower points shall become minus quantities, or *below* said supposed datum or zero. The only object in this is to avoid the liability to error which arises when some of the levels are +, or plus; and some −, or minus. Hence we may assume the level of the starting point to be 10, 100, 1000, &c, ft above datum, according to circumstances.

The vert dists between each two contour lines are supposed to be equal; and in railroad surveys through well-known districts, where the engineer knows that his actual line of survey will not require to be much changed, the dist may be 1 or 2 ft only; and the lines need not be laid down for widths greater than 100 or 200 ft on each side of his center-stakes. But in regions of which the topography is comparatively unknown; and where consequently unexpected obstacles may occur which require the line to be materially changed for a considerable dist back, the observations should extend to greater widths; and for expedition the vertical dists apart may be increased to 3, 5, or even 10 ft, depending on the character of the country, &c. Also, when a survey is made for a topographical map of a State, or of a county, vert dists of 5 or 10 ft will generally suffice.

Let the line A B, Fig 1, starting from O, represent three stations (S 1, S 2, S 3,) of the center line of a railroad survey; and let the numbers 100, 103, 101, 104, along that line denote the heights at the stakes above datum, as determined by levelling. Then the use of the contour lines is to show in the office what would be the effect of changing the surveyed center line A B, by moving any part of it to the right

left hand.* Thus, if it should be moved 100 ft to the left, the starting point 0 wo be on ground about 6 ft higher than at present; inasmuch as its level would t be about 106 ft above datum, instead of 100. Station 1 would be about 7 ft hig or 110 ft instead of 103. Station 2 would be about 7 ft higher, or 108 ft instead 101. If the line be thrown to the right, it will plainly be on lower ground.

The field observations for contour lines are sometimes made with the spirit-lev but more frequently by a slope-man, with a straight 12-ft graduated rod, and a sl instrument, or clinometer. At each station he lays his rod upon the ground,

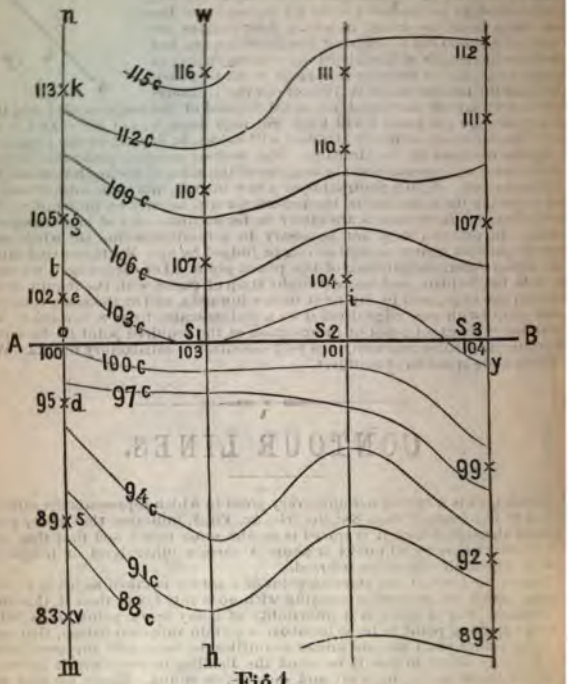


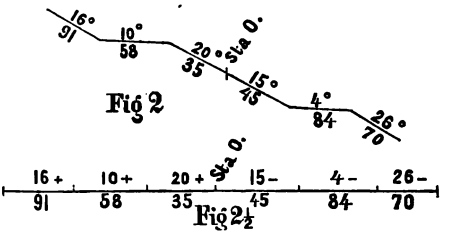
Fig 1.

nearly at right angles to the center line A B as he can judge by eye; and plac the slope instrument upon it, he takes the angle of the slope of the ground to nearest ¼ of a degree. He also observes how far beyond the rod the slope contin the same; and with the rod he measures the dist. Then laying down the rod at t point also, he takes the next slope, and measures its length; and so on as far as n be judged necessary. His notes are entered in his field-book as shown in Fig 2; angles of the slopes being written above the lines, and their lengths below; should be accompanied by such remarks as the locality suggests; such as wo rocks, marsh, sand, field, garden, across small run, &c, &c.

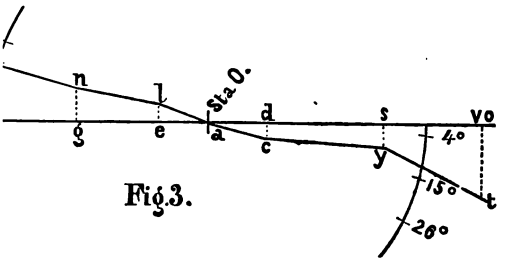
* In thus using the words right and left we are supposed to have our backs turned to the start point of the survey. In a river, the right bank or shore is that which is on the right hand as we *ascend it; that is, in speaking of its right or left bank, we are supposed to have our backs turned ward its head, or origin;* and so with a survey.

olutely necessary to represent the slopes roughly in the field-book, as by using the sign + to signify "up;" —"down;" and = "level," y be writ- ight line,

ving been eparation : lines by m, is of ork; and ie at the the draw- , &c. The ns at each ien sepa- y protrac- as shown e starting



scale should not be less than about $\frac{1}{2}$ inch to a ft, if anything like ied at. Suppose that at said station the slopes to the right, taken in , as in Fig 2, 15°, 4°, and 26°; and those to the left, 20°, 10°, and 16°; ihs as in the same Fig. Draw a hor line *h o*, Fig 3; and consider the be the station-stake. From this point as a center, lay off these angles tor, as shown on the arcs in Fig 3. Then beginning say on the right arallel ruler draw the first dist *a c*, at its proper slope of 15°; and of th, 45 ft, by scale. Then the same with *c y* and *y t*. Do the same with ft hand. We then have a *cross-section* of the ground at Sta O. Then in Fig 1, draw a line as *m n*, or *h w*, at right angles to the line of road, rough the station-stake. On this line lay down the *hor* dists *a d*, *d s*, *s v*, :king them with a small star, as is done and lettered in Fig 1, at Sta O. ne accuracy is pretended to, these hor dists must be found by measure as a general rule it will be near enough, when the slopes do not ex- ume them to be the same as the sloping dists measured in the field. how high each of the points *c y t l n i* is above datum. Thus, measure rt dist *a c*. Suppose it is found to be 5 ft; or in other words, that *c* ation-stake O. Then since the level at stake O is 100 ft above datum, be 5 ft *less*, or 100 − 5 = 95 ft above datum; which may be marked in il figures on the map, as at *d*, Fig 1. Next for the point *y*, suppose e 11 ft, or *y* to be 11 ft *below* stake O; then its height above datum 11 = 89; which also write in pencil, as at *s*. Proceed in the same Text going to the left hand of the station-stake, we find *e l* to be say *bove* the level of the station-stake, therefore its height above datum is



ft, as figured at *e* on the map. Let *n g* be 5 ft; then is *n*, 100 + 5 = atum, as marked at *g*; and so on at each station. When this has been l stations, we may draw in the contour lines of that portion by hand they are to represent vert heights of 3 ft. Beginning at Station O height above datum is 100 ft) to lay down a contour line 103 ft above *at once that the height* of 103 ft must be at t, or at ⅓ the dist from *ight lead-pencil dot* at *t*; and then go to the next Station 1. Her *height of 103 ft* coincides with the station-stake itself; place a *Sta 2. The level* at this stake is 101; therefore the contour for

ft must evidently be 2 ft higher, or at i, $\frac{2}{3}$ of the dist from Sta 2 to +104; there make a dot at i. Then go to Sta 3. Here the level being 104 above datum, the tour of 103 must be at y, or $\frac{1}{3}$ of the dist from Sta 3 to +99; put a dot at y. Fin draw by hand a curving line through t, S 1, i, and y; and the contour line of 1 is done. All the others are prepared in the same way, one by one. The level of must be figured upon it at short intervals along the map, as at 103 c, 106 c, &c.

Or, instead of first placing the + points on the map, to denote the slope dists ally measured upon the ground, we may at once, and with less trouble, find and those only which represent the points t, S 1, i, y, &c, of the contours themse Thus, say that at any given station-stake, Fig 4, the level is 104; that the cross tion $c\,s$ of the ground has been prepared as before; and that we want the hor from the stake, to contour lines for 94, 97, 100 ft, &c, 3 ft apart vert.

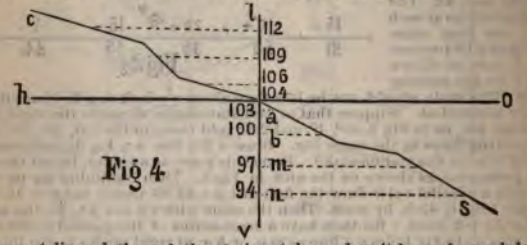


Fig 4

Draw a vert line $v\,l$, through the station-stake, and on it by scale mark lev 94, 97, 100, &c. ft. This is readily done, inasmuch as we have the level 104 stake already given. Through these levels draw the hor lines a, b, m, n, &c. ground-slopes. Then these lines, measured by the scale, plainly give the re dists.

When the ground is very irregular transversely, the cross-sections must be in the field nearer together than 100 ft. The preparation of contour lines w greatly facilitated by the use of paper ruled into small squares of not less than $\frac{1}{12}$ inch to a side, for drawing the cross-sections upon.

When the ground is very steep, it is usual to shade such portions of the m represent hill-side. The closer together the contours come, the steeper of co the ground between them; and the shading should be proportionally darker a portions. But for *working* maps it is best to omit the shading.

In surveys of wide districts, the theodolite, or the transit instrument, is us measuring the angles of slope, instead of the common slope-instrument.*

DIALLING.

To make a horizontal Sun-dial,

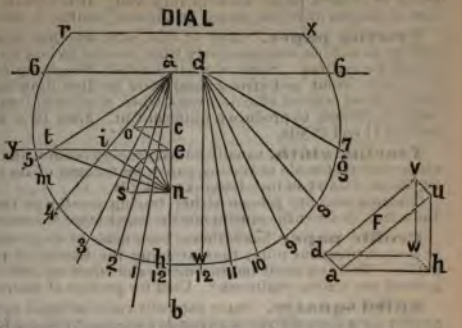
DRAW a line $a\,b$; and at right angles to it, draw 66. From any convenient poin in $a\,b$, draw the perp $c\,o$. Make the angle $c\,a\,o$ equal to the lat of the place

* The preparing of contour lines is a slow and tedious office-work; and the writer consider of but little value in many cases; as when they are taken for only about 100 ft on each sid line, with reference to slight changes of direction. He conceives that, ordinarily, every useful is fulfilled if the leveller or the topographer enters into his field-book at each station, notes to the following:

Sta 60	− 3. 1 R.	+ 2. 1 L.
61	+ 2. 2 R.	− 1. 3 L.
62	= 1. R.	+ 4. 2 L.
63	"	"

Which means that at station 60, the slope of the ground on the right, as nearly as he can ju eye, or by his hand-level, is about 3 ft downward, for 1 chain, or 100 ft; and on the left, ab *upward in 1 chain.* At 61, 2 ft. up, in 2 chains to the right; and 1 ft down in 3 chains to t *At 62, level for 1 chain to the right*; and ascending 4 ft in 2 chains to the left. At 63, the san *62. At some spots it will be well to add a sketch of a cross-section, like Fig 2; only, instea angles, use ft of rise or fall, to indicate the slopes, as judged by eye, or by a hand-level. method, the result at every station will be somewhat in error; but these small errors wil each other so nearly that the total may be regarded as sufficiently correct for all the pur preliminary estimate of the cost of a road. When the final stakes for guiding the workmen the slopes should be carefully taken, in order to calculate the quantity of excavation ac payment.*

e equal to the same; join *o e*. Make *e n* equal to *o e*; and from *n* as a
he rad *e n*, describe a quadrant *e s*; and div it into 6 equal parts. Draw *e*
6, 6; and
ugh the 5
quadrant,
t, *n i*, &c,
n *e y*. From
5, *a* 4, &c,
gh *t*, *i*, &c.
onvenient
escribe an
kind of fin-
to half the
lines may
ed, except
s *a* 6, *a* 5,
12, or *a h*;
generally
dial is to
o quarters
least. In
h of the
the quad-



t be subdivided into 4 equal parts; and lines drawn from *n*, through
subdivision, terminating in *e y*. The quarter-hour lines must be drawn
ere the hour lines. Subdivisions of 5 min may be made in the same
se, as well as single min, may usually be laid off around the border, by
8 or 10 times the size of our Fig will be a convenient one for an ordi-
o draw the other half of the Fig, make *a d* equal to the intended thick-
omon, or style, of the dial; and draw *d* 12, parallel, and equal to *a* 12; and
g w, precisely similar to the arc *r m h*. Between *x* and *w*, on the arc *x g w*,
ions equal to those on the arc *r m h*; and number them for the hours,
g. The style F, of metal or stone, (wood is too liable to warp,) will be
ts thickness must throughout be equal to *a d* or *h w*; its base must
ce *a d h w*; its point will be at *a d*; and its perp height *h u*, over *h w*,
that lines *v d*, *u a*, drawn from its top, down to *a* and *d*, will make the
v d w, each equal to the lat of the place. Its thickness, if of metal, may
be from ⅛ to ¼ inch; or if of stone, an inch or two, or more, according
the dial. Usually, for neatness of appearance, the back *h u v w* of the
ved inward. The upper edges, *u a*, *v d*, which cast the shadows, must
straight. The dial must be fixed in place hor, or perfectly level; *a h*
be placed truly north and south; *a d* being south, and *h w* north. The
y sun or solar time; but clock time can be found by means of the "fast
e sun," as given by all almanacs. If by the almanac the sun is 5 min,
dial will be the same; and the clock or watch, to be correct, must be 5
an it; and vice versa.

PAPER.

24 sheets 1 quire. 20 quires 1 ream.

Sizes of drawing papers.

	Ins.		Ins.		Ins.		Ins.
arian.....	52½	×	30½	Imperial.....	29½	×	21½
Elephant	39½	×	26½	Super Royal.....	27¼	×	19¼
.....	33	×	26	Royal.....	23½	×	19
ler	34½	×	23	Medium.....	22½	×	17½
at	28	×	23	Demy	19½	×	15¼

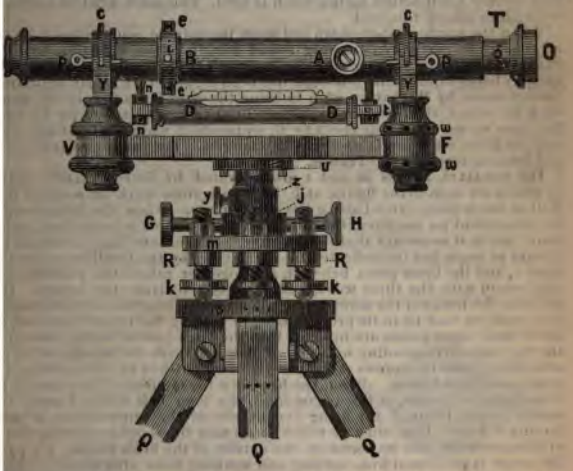
h drawing-papers are stronger and superior to the American. Those
have a high reputation; they are, however, of different qualities. When
d on muslin, the difference in quality is not so important.

ite and tinted papers, for the use of engineers, are made in
lls, without seams. Widths 40, 54, and 60 ins; usual lengths up to 40
e had to order to 400 yds or more. These may also be purchased ready
in, in rolls up to 10 yds long. This last, on account of its strength

[illegible]

ut of this position by a partial revolution of the telescope, when carrying or when setting the tripod down suddenly upon the ground; but since, in the *intersection* of the hairs is directed to the target-rod, this derangement affect the accuracy of the work. Still it is well to keep them nearly vert by keeping the BUBBLE-TUBE D D as nearly directly over the bar V F as can l by eye. This enables the leveller to see that the rod-man holds his rod rt, which is absolutely essential for correct levelling. If perfect verticality , as is sometimes the case, when staking out work, it may be obtained (*if ment is in perfect adjustment, and levelled*) by sighting at a plumb-line, or t object, and then turning the telescope a little in its Ys, so as to bring the rrespond. When this is done, a short continuous scratch may be made on and its Y, to save that trouble in future. Also lift the telescope out of its t end for end, and replace it; then sight again at the plumb-line, and make on the other Y, to correspond with that previously made on the telescope. can after that be made vert in a moment, whichever way the telescope may Ys, by merely bringing together the scratches on the telescope and Ys. tches, or some other indicator, should always be added by the maker; but know that it is ever done. Should the hairs be broken by accident, they placed as directed hereafter.

ll holes around the heads of the 4 small capstan-screws *e*, *i*, just referred to,



nitting the end of a small steel pin, or lever, for turning them. If first screw *e* be loosened, and then the lower one tightened, the interior ring sed, and the horizontal hair with it. But on looking through the telescope *appear* to be lowered. If first the lower one be loosened, and the upper ned, the hor hair will be actually lowered, but apparently raised. This is e glasses in the eye-piece E reverse the apparent position of objects *inside* scope; which effect is obviated, as regards *exterior* objects, by means of -glass O. This must be remembered when adjusting the cross-hairs; for if a ars to strike too high, it must be raised still higher; if it appears to be o far to the right or left, it must be actually moved still more in the same

mark, however, does not apply to telescopes which make objects appear

no danger of injuring the hairs by these motions, inasmuch as the four *against the ring only*, and do not come in contact with the hairs them-

e telescope is the BUBBLE-TUBE D D. One end of this tube can be raised o *ghtly by means of the two* capstan-headed nuts *n n*, one of which mu *before the other is tightened.* On top of the bubble-tube are scratc

for showing when the bubble is central in the tube. Frequently these scratches or marks, are made on a strip of brass placed above the tube, as in our fig. There are several of them, to allow for the lengthening or shortening of the bubble by change of temperature. At the other end of the bubble-tube are two small capstan-screws placed on opposite sides horizontally. The circular head of one of them is shown near *t*. By means of these two screws, that end of the tube can be slightly moved hor, or to right or left. Under the bubble-tube is the BAR Y F; at one end of which, as at F, are two large capstan-nuts *w w*, which operate upon a stout interior screw which forms a prolongation of the Y. The holes in these nuts are larger than the others, as they require a larger lever for turning them. If the lower nut is loosened and the upper one tightened, the Y above is raised; and that end of the telescope becomes farther removed from the bar; and vice versa. Some makers place a similar screw and nuts under both Ys; while others dispense with the nuts entirely, and substitute beneath one end of the bar a large circular milled head, to be turned by the fingers. This, however, is exposed to accidental alteration, which should be avoided. Frequently a compass is placed on top of the bar, below the bubble-tube. Under the bar is the SUPPORTING-PLATE *v*, which is cast in one piece with the hollow socket *z*. This socket fits upon an interior spindle; and in putting it upon the spindle, a slit in one side slips past the screw *y*, which is then tightened, to keep the level from falling off when carried upon the shoulder. Sometimes, instead of this screw *y*, a self-acting spring-catch is used. This catch must be drawn back when we wish to lift off the level.

When the portions above *j* are put upon the spindle, and fastened by the screw *y*, all the upper part may be swung round hor, in either direction, by loosening the CLAMP-SCREW H; or such motion may be prevented by tightening that screw. It frequently happens, after the telescope has been sighted very nearly upon an object, and then clamped by H, that we wish to bring the cross-hairs to coincide more precisely with the object than we can readily do by turning the telescope *by hand;* and in this case we use the TANGENT-SCREW G, by means of which a slight but steady motion may be given after the instrument is clamped. For fuller remarks on the clamp and tangent screws, see "Transit."

The PARALLEL PLATES *m* and *s* are operated by four LEVELLING-SCREWS; three of which are seen in the figure, at *k*, *k*. The screws work in sockets R; which, as well as the screws, extend above the upper plate. When the instrument is placed on the ground for levelling, the lower parallel plate *s* is never hor, unless by accident; nor is it necessary that it should be. But for levelling, the upper one *m* must always be made hor (as indicated by the bubble) by the levelling-screws. The lower plate *s*, and the brass parts below it, are together called the TRIPOD-HEAD; and, in connection with the three wooden legs Q Q Q, constitute the tripod. In the figure are seen the heads of the screws which confine the legs to the tripod-head. In order that each leg may be in its proper place, in putting the tripod together, one, two, or three small brass points are inserted into the back part of each, (see middle leg in the fig.) and corresponding numbers of dots are made in the edge of the lower parallel plate; also the screws are generally distinguished by the same number of lines scratched around them. When the head of a leg is inserted between its appropriate brass jaws, or cheeks, in the tripod-head, the long bolt with a female screw at one end is passed through it; using the handle of the screw-driver, if necessary, to force it home. Care must be taken to see that the short projecting pin at the head of this screw fits into its recess on the outside of the brass cheek; for by this means the screw is prevented from turning and working loose afterward. The male screw is then put in by the screw-driver. Under the centre of the tripod-head should always be placed a small ring, from which a plumb-bob may be suspended. This is not needed in ordinary levelling, but becomes useful when ranging centre-stakes, &c.

To adjust a Level.

This is a quite simple operation, but requires a little patience. Be careful to avoid *straining* any of the screws. The large Y nuts *w w* sometimes require some force to *start* them; but it should be applied by pressure, and not by blows. Before beginning to adjust, attend to the object-glass, as directed in the first sentence under "To adjust a plain transit," p. 159.

Three adjustments are necessary; *and must be made in the following order:*

First, that of the cross-hairs; to secure that their intersection shall continue to strike the same point of a distant object, while the telescope is being *turned round a complete* revolution in its Ys. This is called adjusting the **line of collimation,** or sometimes, the *line of sight*; but it is not strictly the line *of sight until all the* adjustments are finished; for until then, the line of collimation *will not serve for taking* levelling sights.

Second, that of the bubble-tube D D, to place it parallel to the li

previously adjusted; so that when the bubble stands at the centre of
ating that it is level, we know that our sight through the telescope

at of the Ys, by which the telescope and bubble-tube are supported;
ble-tube, and line of sight, shall be perp to the vert axis of the instru-
remain hor while the telescope is pointed to objects in diff directions,
back and fore sights.

the first adjustment, or that of the cross-hairs, plant the
pon the ground. In this adjustment it is not necessary to level the
pen the clips of the Ys; unclamp; draw out the eye-glass E, until
are seen *perfectly clear;* sight the telescope toward some clear dis-
n object; or still better, toward some straight line, whether vert or
object-glass O, by means of the milled head A, so that the object shall
n, **without parallax,** that is, without any apparent dancing
ross-hairs, if the eye is moved a little up or down or sideways. To
e object-glass alone is moved to suit different distances; the eye-glass
anged after it is once properly fixed upon the cross-hairs. The neglect
a source of frequent errors in levelling. Clamp; and, by means of the
G, bring either one of the cross-hairs to coincide *precisely* with the
gently, and without jarring, revolve the telescope half-way round in
this is done, if the hair still coincides precisely with the object, it is
and we proceed to try the other hair. But if it does not coincide,
of the 4 screws *e, i,* move the ring which carries the hairs, so as to
rly as can be judged by eye, only *one-half* of the error; remembering
must be moved in the direction opposite to what *appears* to be the
ess the telescope is an inverting one. Then turn the telescope back
mer position: and again by the tangent-screw bring the cross-hair to
he object. Then again turn the telescope half-way round as before.
ow be found to be more nearly in its right place, but, in all probabil-
ly so; inasmuch as it is difficult to estimate one-half the error accu-
Therefore a little more alteration of the ring must be made; and it
ry to repeat the operation several times, before the adjustment is
ward treat the other hair in precisely the same manner. When both
eir intersection will strike the same precise spot while the telescope
entirely round in its Ys. This must be tried before the adjustment
ced perfect; because at times the adjustment of the second hair,
es that of the first one; especially if both were much *out* in the be-

the second adjustment, or to place the bubble-tube parallel
ollimation. This consists of two dis-
ts, one hor, and one vert. The first
ted by the two hor screws at the end
and the second by means of the two
e vert screw at the other end of the
at the bubble-tube endwise, from *t* in
ig, its two hor adjusting-screws *t t* are
s sketch. The larger capstan-headed
nothing to do with the adjustments;
the end of the tube in its place, and
any levels.

hor adjustment of the bubble-tube, place the telescope over two of
crews *k k* which stand *diagonally* to each other; clamp it; and by the
ling-screws bring the bubble to the centre; at the same time seeing
e-tube appears, as nearly as may be, to be directly under the telescope,
. Then gently revolve the telescope a little to one side in its Ys, say
so that the bubble-tube shall stand out on that side, from over the
ar, or from under the telescope. If the bubble then remains at the
tube, the hor adjustment is correct; but if it runs toward one end,
high; and *one-half* of the error of the bubble must be corrected by the
raising the end that is lowest, or lowering that which is highest,
ay require. This done, turn the telescope back again, until the
over the bar; and bring the bubble again to the center of its
of the levelling-screws *k*. Then again turn the telescope as before, so
e bubble-tube stand out from over the bar. As it is only by chance
tment is perfected at the first trial, the bubble will most probably
centre of the tube, but not as much as before. If so, again rectify
means of the screws t t; turn the telescope back again; re-level the
eat until the bubble remains at the center, both when under th

bar, and when standing out from it a short dist. This hor adjustment ready made in new instruments; and is not at all liable to derangement, accidental blows. To make the vert adjustment of the bubble-tube, by me two nuts *n n*, place the telescope over a diagonal pair of the levelling-screw clamp it there. Open the clips of the Ys; and by means of the levelling sc the bubble to the center of its tube. Lift the telescope gently out of the end for end, and put it back again in its reversed position. This being d bubble still remains at the center of its tube, this adjustment is in order moves toward one end, that end is too high, and must be lowered; or els end must be raised. This must be done by means of the two small capstan-h *n n*. If the end *n n* is to be raised, the upper nut must first be loosene lower one tightened, and vice versa; but (as when adjusting the cross indeed in all the adjustments) only *one-half* of the error is to be correc *first* process. Then correct the other half by means of the levelling Having thus brought the bubble to the middle again, again lift the teles its Ys; turn it end for end, and replace it. The bubble will now settle center than it did before, but will probably require still further adjustm correct *half* the remaining error by the nuts as before; and half by th screws; and so continue to repeat the operation until the bubble rem center in both positions. That part also of its adjustment will then be and the bubble will remain at the center (after the instrument is levelled telescope is pointed in any direction. *Make this adjustment before the hor*

To make the third adjustment, or to adjust the heights of as to make the line of collimation parallel to the bar V F, or perp to th of the instrument. The other adjustments being made, fasten down the Ys. Make the instrument nearly level by means of all four of the level *k*. Place the telescope over two of the levelling-screws which stand and leave it there unclamped. Then bring the bubble to the center of the two levelling-screws. Swing the upper part of the instrument half-w so that the telescope shall again stand over the same two screws; but e This done, if the bubble leaves the center, bring it *half-way* back by the stan nuts *w, w*; and the other half by the two levelling-screws. Remem raise the Y, and the end of the bubble over *w, w*, the lower *w* must be loo the upper one tightened; and vice versa. Now place the telescope over diagonal pair of levelling-screws; and repeat the whole operation with th ing completed it, again try with the first pair; and so keep on until the mains at the center of its tube, in every position of the telescope.

Correct levelling may be performed even if all the foregoing adjus out of order; provided each fore-sight be taken *at precisely the same dis the instrument as the back-sight is.* But a good leveller will keep his instrun in adjustment; and will test the adjustments at least once a day when at much, however, depends upon the rodman, or target-man, as upon the level man who is careless about holding the rod vert, or about reading the sight should be discharged without mercy.

Forms for level note-books. When the distance is short, s require two sets of books, the following is perhaps as good as any.

No. of Station.	Back sights.	Fore sights.	Diff.	Level.	Grade.	Cut.	Fil

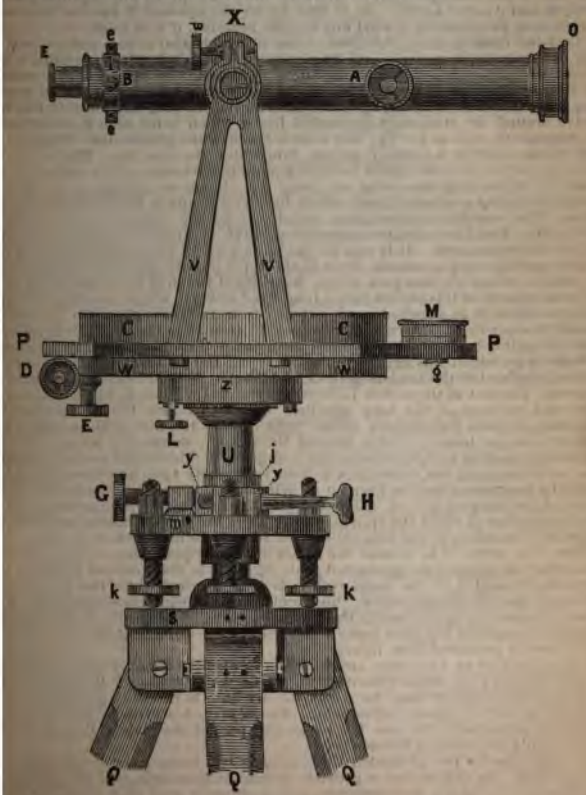
But on public works generally the original field-books have only the fir After the grades have been determined by means of the profile drawn f the results are placed in another book, which has only the first col and th In both cases, the right-hand page is reserved for memoranda. The write it best, both with the level and with the transit, to consider the term "S apply to the whole dist between two consecutive stakes; and that its nu be that written on the last stake. Thus, with the transit, Station 6 mea from stake 5 to stake 6; that it has a bearing or course of so and so; and is so and so. And with the level, Station 6 also means the dist from stake 6; the back-sight for that dist being taken at stake 5, and the fore-sigh 6; and that the level, grade, cut, or fill is that at stake 6. The starting-p survey, whether a stake, or any thing else, we call and mark simply 0. gineers call this 0, No. 1 for the compass, and 0 for the level; but this is to error when comparing together the compass and the level books. It *surable that one system* should become general; and with that view we *the foregoing one as* being perfectly simple, and free from liability to mi *does not interfere with* designating any *stake* by its number also, as stak

Note. The levelling-screws may become very hard to turn if dirty. Clean wi *tooth-brush. Don't use oil on field instruments.*

THE ENGINEER'S TRANSIT.

Like the level, its details are differently arranged by diff makers, and to suit par-
cular purposes. Still its more essential parts so nearly resemble each other in all,
at they may be understood from the following fig, which represents it in its most
uple form, as generally used on the public works of Pennsylvania.

An addition is sometimes made of either a graduated full vert circle, or of a seg-
ent of one, for measuring vert angles; in which case a bubble-tube is also at-
ched below the telescope. The instrument is thus virtually converted into a theo-
dite; but for ordinary railroad and canal purposes, the more simple form shown
our Fig (or the *plain* transit) is generally to be preferred.



he parts below *j* are similar to those already described for the level; except that
ead of a single tangent-screw G, two screws similar to G are employed, being so
ed that their ends butt against each other. When so arranged, they are called
crews. Only one of these, G, is shown in the drawing; the other being supposed
hidden behind the clamp-screw H. These set-screws are used by unscrewin

O

one, while screwing the other moderately tightly up against it. more steady motion, and less liability to disturbance, are secured, screw is used; especially after the latter becomes somewhat worn

The telescope E O, which is essentially similar to that of a lev is usually from 9 to 12 inches long. It is sometimes made to show but more generally upright. It revolves vert around a transverse ported by a STANDARD at each end. Only one of these standar shown in the drawing. The transverse axis is not fixed at the ce of the telescope, but nearer to one or the other of its ends. Thi of a longer and more powerful telescope; but at the same time ing vert in both directions; which, however, is a matter of little o or importance. X is a stout ring surrounding the telescope, fo axis; and *w* is a clamp-screw, for holding the telescope in any For E, B, and A, see "Level." It is best to let the eye-end E r because then the *shade* on O need not be first removed, if it is in

C C is the COMPASS BOX; the glass cover of which also cover LIMB, or graduated hor circle and its verniers; of which last it i at least two, for securing greater accuracy, by taking the mear But for merely tracing curves, this precaution is unnecessary. being measured, the telescope, standards, and all the showing port revolve around the stationary graduated limb; which is not see the instrument, such as our fig; but is seen through the glass co fined to the limb by a circular groove, into which the limb fits. stationary (while the outer parts revolve around it) by means of t which, by tightening the collar or clamp *y y*, clamps everythi together with the graduated limb, which lies upon Z, and is firm screws. The movable circular plate which carries the verniers NIER-PLATE. Also Z is called the SUPPORTING-PLATE, because it sup parts of the instrument. It is cast in one piece with U, which la forming a SOCKET, which extends down to the upper parallel-plate the parallel-plates all that part of the instrument which is abov this socket inside of U is placed over a long fixed SPINDLE, which f plate, (to which it is attached,) and extends from it up nearly Around this spindle the instrument revolves. Surrounding the foo distant from it only the full thickness of the metal of the socket, is or CLAMP *y y*; which lies loosely upon the top of the plate *m*; s by being confined to the tangent-screws; which may be said to c its screw. The foot of the socket U, from *j*, down to the plate *m*, collar and the foot of the long spindle. When, therefore, the tightened, it closes the circular collar tight around the foot of therefore, can no longer revolve around the spindle, unless the give a slight motion to it by moving the clamp itself. This slight when, after bringing the cross-hairs to bear *very nearly* upon an clamp the instrument for the purpose of bringing them *precisely* means of the tangent-screws. The clamping does not clamp the spindle, nor does it affect the spindle in any way; but merely un socket and the collar, to the tangent-screws, so as to be moved on

When it is required to confine the upper or vernier-plate also, from revolving around the graduated limb, tighten the clamp-sc this a slight motion of the vernier-plate is still required while stationary, use the tangent-screw, the circular head of which is s screw for loosening or tightening the compass-needle. The needl tightened up against the glass cover when not in use; as the poin pin is thereby preserved from injury.

M is a circular or UNIVERSAL LEVEL, and is frequently the only the more simple form of the transit. It is confined to the plate P by unscrewing which it may be removed. Its base is carefully plate, so as not to require adjustment. Frequently, however, screws for adjusting it, in case the projecting part of the plate stands should accidentally become bent.

In many transits the graduated limb extends out beyond the vernier-plate. By this means a greater circumf is obtained; ar may therefore be larger, more accurate, and more easily read off.

Instead of one universal level, two long bubble-tubes, set at ri *other, are frequently used.* These, being more sensitive, are to be

In the plain transit, for ordinary railroad work, the difference *but for very accurate* work, and especially when a vertical circl *bubble-tubes* become essential.

e bottom of the socket U, and around its inside circumf, is a shallow
h it is prevented from falling off, (when being carried on the shoulder,)
ng-catch, or a screw, like that marked *y* in the figure of the level.
ate S, and the brass parts below it, constitute the "TRIPOD-HEAD,"
and the three wooden legs Q Q Q are the TRIPOD.

an additional plate has been introduced, somewhat smaller than S,
nediately upon it, with a clamp for confining it to S, or for loosening
asure. The screws *k k*, &c, foot upon this plate, which may be called
-plate, inasmuch as it can be shifted to some extent in any direction
loing, all the upper part of the transit shifts with it. Below the cen-
ing-plate hangs the usual plumb-bob for placing the instrument pre-
center of a stake. The plumb-line passes through an opening about
the lower plate S, and covered by the shifting-plate.

the shifting-plate is to facilitate placing the instrument *precisely*
of a stake. Hitherto the instrument has been first planted pretty
ver the center, and afterward brought directly over, by pushing one
ipod feet farther into the ground, or by spreading them more or less.
is sometimes difficult, especially on very steep rocky slopes; and is
shifting-plate, which allows the whole transit to be slid to the *pre-*
the tripod has been firmly planted *nearly* right. The clamp of the
then tightened, and all goes on as with the other kind of transit.
s are precisely the same in both instruments.*

To adjust a plain Transit.

a level or a transit is purchased, it is a good precaution (but one
r has never seen alluded to) to first screw the object-glass firmly home
then make a short continuous scratch upon the ring of the glass, and
o as to be able to see at any time when at work, that the glass is
me position with regard to the slide. For if, after all the adjustments
he position of the glass should become changed, (as it is apt to be if
afterward not screwed up to the same precise spot,) the adjustments
come materially deranged; especially if the object-glass is eccentric,
and, which is often the case. Such scratches should be prepared by
making adjustments, as well as when using a transit or level, be
eye-glass and object-glass are so drawn out that there shall be no
eye-glass must first be drawn out so as to obtain perfect distinctness
s; it must not be disturbed afterward; but the object-glass must
ferent distances.

djust the bubble-tubes. If only a single circular or uni-
attached to the instrument, and confined to it by a single screw, it
ed by the maker, and needs nothing more. If the universal bubble
; and the plate to which they are affixed becomes accidentally some-
bubble may be adjusted by first bringing it to the center by means
screws *k*, *k*; then turn the instrument half-way round on its spin-
telescope points in the opposite direction; and if the bubble leaves
g it *half-way* back by the bubble-screws; and the other half by the
Repeat the trial, until the bubble remains in the center while the
rned completely round.

ubbles are used, as is frequently the case, first bring both to the cen-
ion of the instrument; then, as before, turn the instrument half-way
one-half the error (if any) by means of the two horizontal screws,
gure of the level; and the other half, by the levelling-screws *k*, *k*.
until both bubbles remain in the center while the instrument is
tirely around on its spindle.

e that the cross-hairs *are truly vertical and horizontal, when*
level. When the telescope *inverts*, the cross-hairs are nearer the eye
gure) than when it shows objects erect. The maker takes care to
airs at right angles to each other in their ring, or diaphragm; and
places the ring in the telescope, that when levelled, they shall be vert
times, however, this is neglected; or if attended to, the ring may by
turned a little; especially if the novice permits all the screws to be
time. To be certain that one hair is vert, (in which case the other
ction, be hor,) after having adjusted the bubble-tubes, level the in-

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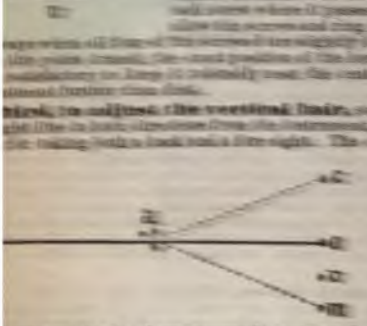
rst-class plain transit, of Philadelphia make, with shifting-plate and plumb-bob,
with vertical arc, &c, about $200; as per price-list of Knox & Shain. Instru-
firm, or by Mr. Draper, Mr. Young, or Heller & Brightly, No. 33 North 7th St.,
may be relied on as first-class.

[illegible] or other vert [illegible] in adjustment; [illegible] *a little*; [illegible] against the screw-[illegible] until the hair be-[illegible] of the screws; and in [illegible] the absence of a plumb-[illegible] a very small distant point; and [illegible] the telescope is raised or lowered by re-[illegible]

The [illegible] of [illegible] the Diagram will be readily [illegible] from this fig. which represents a section across [illegible] of the telescope, and of the cross-hairs. The [illegible] vert hair, *v*; telescope tube, *g*; [illegible] of telescope tube, *d*; is one of the four cap-[illegible] screws, which hold the hair-ring, *a*, in its [illegible] to adjust it. The lower ends of [illegible] screws, work in the thickness of the hair-ring; so [illegible] they do not bee[illegible] ring. Small brass washers, *c*, are placed [illegible] of the screws. A square *y y* is left around [illegible] through the telescope tube, to [illegible] ring, together to be moved slightly [illegible]

[illegible] of the hor hair is of no importance; but [illegible] to keep it nearly over the center of the vert one. It needs no [illegible]

Third, to adjust the vertical hair, so that it shall strike in the same [illegible] when the telescope is revolved [illegible] the other adjustments having been [illegible] made, plant the tripod firmly upon the ground, as shown by the three dots at *a*. Level the instrument; clamp it; and direct the vert hair by means of the tangent-screw 6 (see figure of transit) upon some convenient object [illegible] or if there is none such, drive a thin stake, or a chalk-pin. Then revolving the tele-[illegible]



[illegible] vert on its axis, observe some object, as *c*, where the vert hair now strikes; if [illegible] place a second pin. Then, unclamp the instrument by the clamp-[illegible] and turn the whole upper part of it around until the vert hair again strikes [illegible] clamp again; and again revolve the telescope vertically on its axis. If the vert [illegible] strikes *c*, as it did before, it shows that *c* is really at *c*; and that *b*, *a*, *c* are [illegible] straight line; and therefore this adjustment is in order. But if it is not [illegible] where it does strike when the telescope is revolved the second time, [illegible] (the dist. *a m* being taken equal to *a c*, as near as may be,) and place a pin [illegible] Measure the dist. *m c*; and place a pin at *n*, in the line *m c*, making *m n* [illegible] one-fourth part of *m c*. Also put a pin at *o*, half-way between *m* and *c*, or [illegible] with *a* and *b*. Then, by means of the two hor screws that move the vert hair, [illegible] that hair until it cuts *n*; in doing which, remember that the hair must be [illegible] in the direction opposite to what appears to be the right one, unless the tele[illegible] is an inverting one. Now repeat the entire operation; and persevere until [illegible] telescope, after being directed to *b*, shall strike the same object *c*, *both times*, [illegible] revolved on its axis.

[illegible] it is seen that a straight line may be run, even if the hairs are out of al[illegible] ment; but with somewhat more trouble. For at each station, as at *a*, two back-[illegible] and two fore-sights, as at *a m*, may be taken, as when making the adjust[illegible] it; and the point *o*, half-way between *a* and *m*, will be in the straight line. The [illegible] ument may then be moved to the station *O*; and the two back-sights be taken to [illegible] and so on.

[illegible] measured by the transit, whether vert or hor, will evidently not be affected [illegible] hairs being out of adjustment, provided either that the vert hair is truly vert, [illegible] we use the intersection of the hairs when measuring.

[illegible]urth, see whether the vert hair will travel vertically; [illegible] whether it remains vert while the telescope is revolved vertically through [illegible] arc. *This will always* be the case if the instrument is well made, (supposing

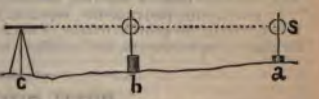
is adjustments to be correct,) providing it has met with no accident by standards V V have become bent. It is only requisite that the standards of the same length, and perp to the plate to which they are affixed; and ways in this condition when they leave the maker's hands. If they should identally deranged, the instrument must be returned to the maker. To hether any accidental derangement has taken place, first level the in- erfectly; then direct the intersection of the hairs to some point of a high h subtends a large vert angle; then clamp the instrument; and lower pe until the intersection strikes some point of a low object; or if there h. place a chain-pin or stake. Then unclamp; and turn all that part of ent above *j*, half way round; then fix the wires again upon the high ap; lower the telescope to the low point. If the hair now strikes the he instrument is in order; if not, it should be sent to the maker to be ome instruments, however, are furnished with the means of adjusting for derangements of the standards, by means of a small block under one end of the telescope. This block is moved by a screw placed beneath it. One e diff must be corrected by this; after which the trial must be repeated resetting the chainpin. Same principle as at opposite figure.

may be used *for running straight lines*, even if the standards become t, by the process described at the end of the third adjustment.

re frequently made with a **graduated vert circle** attached to one telescope axis, for measuring vert angles; in which case a long bubble- etimes attached to the telescope. When this latter is not added, **the ent of the vernier of the vert circle is made thus:** djustments being in order, bring the zero of the circle, and that of its oincide perfectly. Bring the bubble-tubes level; and then, without sep- zeros, sight the hor hair to some small object. Then turn all the in- ove *j*, half way round; then revolve the telescope vert; and again sight upon the same small object. Now observe whether the two zeros again f they do, the adjustment is in order; but if not, then loosen the screws ier, and move the vernier itself, to the extent of *one-half* the diff. Then lescope and circle the other half. Again sight the hor hair to some other t, because it cannot now strike the former one; and perform the whole gain; or until the zeros coincide after the telescope has been revolved nd directed to the last small object selected. When this is effected, the is level with the line of sight; and we know that whenever the zeros coincide afterward, the line of sight of the telescope will be hor, or led the bubbles are levelled by the levelling-screws *k*.

addition to the vert circle, a long bubble-tube is attached to the telescope, to secure the horizontality of the telescope more accurately than can done by merely bringing the two zeros to coincide. It also enables us ransit as a level. It is by no means as well adapted as the latter to this t by making each fore-sight of the same length as its back-sight, it may confidence. Perfect horizontality in the line of sight is essential chiefly omes necessary to measure a vert angle, above or below an assumed or r line, as in examples 2, 3, and 7 of "Trigonometry;" in which case, the e incorrect, unless the line of sight be truly hor.

ust the long bubble-tube, we must first place the line of sight cope in a hor or level position; and then make the tube parallel to it. first drive two pegs *a* and eir tops at precisely the and not less than about rt; 300 feet or more will Then plant the instrument ange with them, as at *c*. the four levelling-screws; eated trials, by moving the

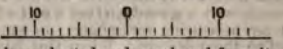


rom stake to stake, altering the height of the target, and revolving the or down a little to correspond, find that position in which the hairs isely the same heights on the rod at both stakes. The line of sight *telescope* is then evidently level, or hor. Keep the target at the same ding on the farthest peg *a*, as a guide for now adjusting the bubble- l to the line of sight. This is done by moving the two small screws, like figure of the level, until the bubble stands at the center of the tube; ne of sight at the same time strikes the target at *a*. The zeros of the and of its *vernier*, may now be adjusted, if they require it, by loosening *screws; and then moving* the vernier until the two coincide.

It is at hand for previously levelling the two pegs *a* and *b*, it may be *transit itself, thus:* Carefully level the two short bubbles, by means o

the levelling-screws k. Drive a peg m, from 100 to 300 feet from the instrument n. Then placing a target-rod on m, clamp the target tight at whatever height, as can the hor hair happens to cut it; it being of no importance whether the telescope is level or not; although it might as well be as nearly so as can conveniently be guessed at. Clamp the telescope in its position, by the clamp-screw w. Then revolve all the instrument above j (see the figure of the transit) a considerable way round; say nearly or quite half way. Place another peg n, *at precisely the same dist* from the instrument that m is; and continue to drive it until the hor hair cuts the target placed on it, and still kept clamped to the rod, at the same height as when it was on m. When this is done, the tops of the two pegs are on a level with each other, and are ready to be used as before directed.

When a transit is intended to be used for surveying farms, &c, or for retracing lines of old surveys, it is very useful to set the compass so as to allow for the "variation" during the interval between the two surveys. For this purpose a "VARIATION-VERNIER" is added to such transits; and also to the common compass.

When the graduations of a transit are figured, or numbered, so as to read both ways from zero, thus, 

the vernier also is made double; that is, it also is graduated and numbered from its zero both ways. In this case, if the angle is measured from zero toward the right hand, the reading must be made from the right hand half of the vernier; and vice versa. If the figuring is single, or only in one direction, from zero to 360°, then only the single vernier is necessary, as the angles are then measured only in the direction that the figuring counts. Engineers differ in their preferences for various manners of figuring the graduations. The writer prefers from zero each way to 180°, with two double verniers.

To replace cross-hairs in a level, or transit. Take out the tube from the eye end of the telescope. Looking in, notice which side of the cross-hair diaphragm is turned toward the eye end. Then loosen the four screws which hold the diaphragm, so as to let the latter fall out of the telescope. Fasten on new hairs with beeswax, varnish, glue, or gum-arabic water, &c. This requires care. Then, to return the diaphragm to its place, press firmly into one of the screw-holes on the circumf of the diaphragm itself, the end of a piece of stick, long enough to reach easily into the telescope as far as to where the diaphragm belongs. By this stick, as a handle, insert the diaphragm edgewise to its place in the telescope, and hold it there until two *opposite* screws are put in place and screwed. Then draw the stick out of the hole in the diaphragm; and with it turn the diaphragm until the same side presents itself toward the eye end as before; then put in the other two screws.

The so-called cross-hairs are actually spider-web, so fine as to be barely visible to the naked eye. Heller & Brightly use very fine platina wire, which is much better. Human hair is entirely too coarse.

To replace a spirit-level, or bubble-glass. Detach the level from the instrument; draw off its sliding ends; push out the broken glass vial, and the cement which held it; insert the new one, with the proper side up; wrapping some paper around its ends, if it fits loosely. Finally, put a little putty, or melted beeswax, over the ends of the vial, to secure it against moving in its tube.

In purchasing instruments, especially when they are to be used far from a maker, it is advisable to provide extras of such parts as may be easily broken or lost; such as glass compass-covers, and needles; adjusting pins; level vials; magnifiers, &c.

THE THEODOLITE.

THE adjustments are performed like those of the level and transit,

1st. That of the cross-hairs: the same as in the level.

2d. The long bubble-tube of the telescope; also as in the level.

3d. The two short bubble-tubes; as in the transit.

4th. The vernier of the vert limb; as in the transit with a vert circle.

5th. To see that the vert hair travels vertically; as in the fourth adjustment of the *transit. In some theodolites*, as in most transits, no adjustment is provided for this; *but in large ones there is.* It is accomplished by screws under the feet of the *standards.*

Sometimes a second telescope is added; it is placed below the hor limb, and

It has its own clamp, and tangent-screw. Its use is to ascertain
o of that limb has moved during the measurement of hor angles.
to beginning the measurement, the zero and upper telescope are
the first object, point the lower telescope to any small distant
clamp it. During the subsequent measurement, look through it,
s, to be sure that it still strikes that object; thus proving that no
rred.

E BOX OR POCKET SEXTANT.

of the pocket sextant, and the fact that it reads to single minutes,
very useful to the engineer.* By it, angles can be measured while
horseback: and in many situations which preclude the use of a
ful for obtaining latitudes, by aid of an artificial horizon. When
les a cylindrical brass box, about 3 inches in diameter, and 1½
is box is in two parts;
which, then inverting
en screwing them to-
lower part becomes a
ling the instrument.
pon its top when thus
e, as in this figure, a
C, called the INDEX,
center at C, and car-
V at its other end. G
ed arc or LIMB; which
s about 73°, but is di-
t 146°. Its zero is at
luations are not shown

e index is a small mov-
hown in the figure,)
g around C, for read-
ions of the limb. When
gle, the index is moved
milled-head P of a
rks in a rack placed within the box. The eye is applied to a cir-
side of the box, near A. A small telescope, about 3 inches long,
instrument; but may generally be dispensed with. When so, the
uld be partially closed by a slide which has a very small eye-hole
is moved by the pin *h*, moving in the curved slot. Another slide,
box, carries a dark glass for covering the eye-hole when observing
the telescope is used, it is fastened on by the milled-head screw T.
wn in our figure, can be separated from the cylindrical part, by
small screws around its edge; and the interior can then be exam-
d if necessary. Like nautical, and other sextants, this one has
asses, both of which are mirrors. One of these, called the INDEX-
l to the under side of the index, at C: its upper edge being indicated
d lines. The other, called the HORIZON-GLASS, (because, when meas-
ngles of celestial bodies, it is directed toward the horizon,) is also
the position of its upper edge being shown by the dotted lines at
-glass is silvered only half-way down; so that one of the observed
seen directly through its lower half, while the image of the other
the upper half, reflected from the index-glass. That the instrument
ment, ready for use, these two glasses must be at right angles to the
rument; that is, to the under side of the top of the box, to which they
l must also be parallel to each other, when the zeros of the vernier
coincide. The index-glass is already permanently fixed by the
ires no other adjustment. But the horizon-glass has two adjust-
e made by a key like that of a watch, and having a milled-head K.
o the top of the box, so as to be always at hand for use. When
rewed. This key fits upon two small square-heads, (like that for

scope, about $50. Made by Stackpole & Bro., 41 Fulton St., New York.

The mounted sextant, used on ships, is constructed on the same pri
[illegible] and its adjustments are very similar. In it, also, the
[illegible] the horizon-glass has the two
[illegible] for looking at the sun
[illegible] is used when the microscope is dispensed with.

THE COMPASS.

To adjust a Compass.

The first adjustment is that of the bubbles. Plant firmly; and let
instrument, in *any position*; that is, bring the bubbles to the centers of their
Then turn the instrument half-way round. If the bubbles then remain at th
[illegible] they are in adjustment; but if not, correct *one-half* the diff in each b
by means of the adjusting-screws of the tubes. Level the instrument again
it half round; and if the bubbles still do not remain at the center, the adj
screws must be again moved a little, so as to rectify half the remaining diff.

lly several trials must be thus made, until the bubbles will remain at the center, hile the compass is being turned entirely around.

Second adjustment. Level the compass, and then see that the needle is ar; and if not, make it so by means of the small piece of wire which is wrapped round it; sliding the wire toward the high end. A needle thus horizontally adusted at one place, will not remain so if removed far north or south from that place. carried to the north, the north end will dip down; and if to the south, the south d will do so. The sliding wire is intended to counteract this.

Third adjustment. This is always fixed right at first by the maker; that the sights, or slits for sighting through, are placed at right angles to the compass xte; so that when the latter is levelled by the bubbles, the sights e vert. To test whether they are so, hang up a plumb-line; and ving levelled the compass, take sight at the line, and see if the its coincide with it. If one or both slits should prove to be t of plumb, as shown to an exaggerated extent in this sketch, should be unscrewed from the compass, and a portion of its foot the high side be filed or ground off, as per the dotted line; or a temporary expedient, a small wedge may be placed under the w side, so as to raise it.

Fourth adjustment, to straighten the needle, if it should become bent. e compass being levelled, and the needle hor, and loose on its pivot, see whether two ends continue to point to exactly opposite graduations, (that is, graduations 90° apart;) while the compass is turned completely around. If it does, the needle straight; and its pin is in the center of the graduated circle; but if it does not, en one or both of these require adjusting. First level the compass. Then turn it ntil some graduation (say 90°) comes precisely to the north end of the needle. If e south end does not then point precisely to the opposite 90° division, lift off the edle, and bend the *pivot-point* until it does; remembering that every time said int is bent, the compass must be turned a hairsbreadth so as to keep the *north* end the needle at its 90° mark. Then turn the compass half-way round, or until the posite 90° mark comes precisely to the north end of the needle. Make a fine pen- mark where the *south* end of the needle now points. Then take off the needle, d bend it until its south end points *half-way between* its 90° mark and the pencil ark, while its north end is kept at 90° by moving the compass round a hairsbreadth. e needle will then be straight, and must not be altered in making the following djustment, although it will not yet cut opposite degrees.

Fifth adjustment, of the pivot-pin. After being certain that the needle is aight, turn the compass around until a part is arrived at where the two ends of the edle happen to cut opposite degrees. Then turn the compass *quarter way* around, through 90°. If the needle then cuts opposite degrees, the pivot-point is already adjustment; but if the needle does not so cut, bend the *pivot-point* until it does. epeat, if necessary, until the needle cuts opposite degrees while being turned entirely ound.

Care and nicety of observation are necessary in making these adjustments properly; cause the entire error to be rectified is, in itself, a minute quantity; and the novice very apt to increase his trouble by not knowing how to use his MAGNIFYING GLASS, en looking at the end of the needle and the corresponding graduations. The magier must always be held with its *center directly over* the point to be examined; and must be held parallel to the graduated circle. Otherwise annoying errors of eral minutes will be made in a single observation; and the accumulation of two three such errors, arising from a cause unknown to him, may compel him to ndon the adjustments in despair. This suggestion applies also to the reading of gles taken by the transit, &c; although the errors are not then likely to be so at as in the case of the compass. In purchasing a magnifier for a compass, see t no part of it, as hinges, or rivets, are made of iron; for such would change the ection of the needle.

f the sight-slits of a compass are not fixed by the maker in line with the two osite zeros, the engineer cannot remedy the defect. This can be ascertained by ing a piece of fine thread through the slits, and observing whether it stands cisely over the zeros.

Variation of the Compass.

he *numerous disturbing influences* to which the compass is subject, render its

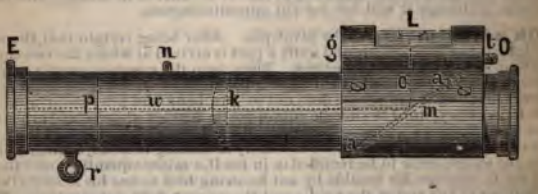
indications of bearings or courses very unreliable. The daily variation itself some times amounts to ¼ of a degree; and always to at least several minutes. It is almo incessantly changing the direction of the needle, to one side or the other, at the rat of 1 or 2 minutes per hour, especially in summer. Local attraction, from iron in th soil, ferruginous gravel, trap rocks, &c, is another source of inaccuracy; as are als the annual and the secular variations. Electricity, either atmospheric, or excite by rubbing the glass cover, sometimes gives trouble. It may be removed by touch ing the glass with the moist tongue, or finger. It is plain that none of these caus (except the last) will affect the measurement of *angles* by the compass.

A straight line drawn across the United States, passing through Cleveland, Ohi and Wilmington, North Carolina, is nearly a line of no variation, at present; that a compass placed anywhere in the vicinity of that line, will point nearly due nor and south. To the eastward of this line, the variation is westward; and vice vers becoming greater, the farther the place is from the line; until in some parts of Mai and along the Pacific coast it is as great as 18° to 21°. This line is moving westwa at an average rate of about 3 or 4 minutes per year.

The needle, if of *soft* metal, sometimes loses part of its magnetism, and consequent does not work well. It may be restored by simply drawing the north pole of common magnet (either straight or horseshoe) about a dozen times, from the cen to the end of the south half of the needle; and the south pole, in the same way, alo the north half; pressing the magnet gently upon the needle. After each stro remove the magnet several inches from the needle, while bringing it back to center for making another stroke. Each half of the needle in turn, while being th operated on, should be held flat upon a smooth hard surface. Sluggish action of needle is, however, more generally produced by the dulling or other injury of point of the pivot. Remagnetizing will throw the needle out of balance; which m be counteracted by the sliding wire.

In order to prevent mistakes by reading sometimes from one e and sometimes from the other end of the needle, it is best to ALWAYS point the N the compass-box toward the object whose bearing is to be taken; and to read from the north end of the needle. This is also more accurate.

THE HAND-LEVEL.



THIS very useful little instrument, as arranged by Professor Locke, of Cincinnat but about five or six inches long. Simply holding it in one hand, and looking thro it in any direction, we can ascertain at once, approximately, what objects are at same level with the eye. E is the eye end: and O the object end. L is a sm level, enclosed in a kind of brass boxing *t g*, the bottom of which is open, with a responding opening under it, through the top of the main tube E O. Immediat at the bottom of the small level L, is a cross-wire, stretched across said opening, carried by a small plate, which, for adjusting the wire, can be pushed backwar trifle by tightening the screw *t*, or pushed forward by a small spring within the b ing, near *g*, when the screw *t* is loosened. At *m* is a small semicircular mirror silvered on the back *m*. This is placed at an angle of 45°, and occupies one-half width of the tube E O. Through the forementioned openings, the images of cross-wire and of the level-bubble are reflected down on the unsilvered face *a* the mirror, and thence to the eye, as shown by the single dotted lines *c* and *w*: when the instrument is adjusted, and held level, the wire will appear to be at *center of the bubble*. At *k* is one-half of a plano-convex lens, at the inner end *short tube k p, which* may be moved backward or forward by a pin *n*, project *through a short slit in* the main tube. By this means the image of the cross-wir *rendered distinct*; and the half lens must be moved until, when viewing an ob *the wire shall show* no parallax; but appear steady against the object when th

ghtly moved up or down. At each end of the tube E O is a circular piece of glass for excluding dust.

adjust the hand-level. first fix two precisely level marks, say from t to 100 yards apart. This being done, rest the instrument against one of the marks, and take sight at the other. If, then, the wire does not appear to be ly at the center of the bubble, move it slightly backward or forward, as the may be, by the screw *t*, until it does so appear. wo level marks may be fixed by means of the level itself, even if it is entirely out of adjust- thus: First, by the pin *n* arrange the half lens s to show the wire distinctly and without paral- Then holding the level steadily, at any selected , as *a*, so that the wire appears to cut the center

bubble, see where it cuts any other convenient object, as *b*. Then go to *b*, om it, in like manner, sight back toward *a*. If the instrument is in adjust- the wire will cut *a*; but if not, it will strike either above it or below it, as at *c*. her case, make a mark *m*, half-way between *c* and *a*. Then *b* and *m* will be the vel marks required. With care, these adjustments, when once made, will n in order for years. The instrument generally has a small ring *r*, for hanging nd the neck: it is not adapted to very accurate work, but admirably so for ing a route. The height of a bare hill can be found by beginning at the foot, ghting ahead at any little chance object which the cross-wire may strike, as a , twig, &c; then going forward, stand at that object, and fix the wire on r one still farther on, and so to the top. At each observation we plainly rise ht equal to that of the eye, say 5¼ feet, or whatever it may be. Whether up or down it, if the hill is covered with grass, bushes, &c, a target rod must d for the fore-sights; and the constant height of the eye may be regarded as ck-sight at each station. An attachment may be made for screwing the level mall ball and socket on top of a cane, or of a longer stick, for occasional use, rather more accuracy is desired. The hand-level deserves much more notice the profession than it has received, especially for preliminary examinations of , or for taking topography, &c. No assistant engineer, or leader of a field should be without one.*

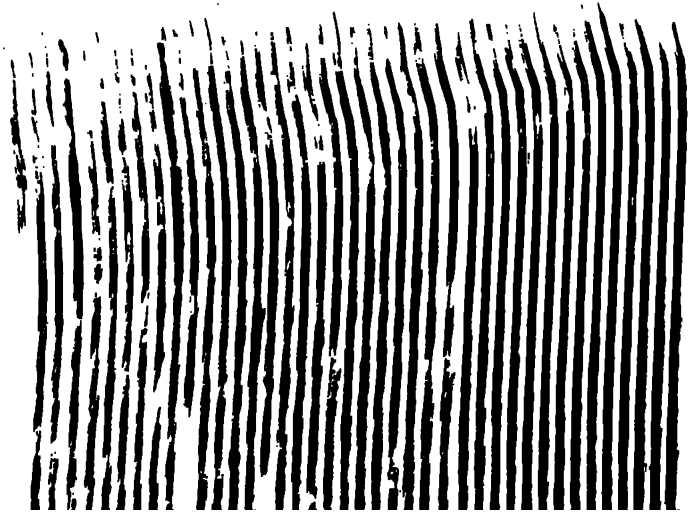
adjust a builder's plumb-l, *t b d*; stand it upon any two sup- *m* and *n*, and mark where the plumb- uts at *o*. Then reverse it, placing the upon *n*, and *d* upon *m*, and mark where he now cuts at *c*. Half-way between *o* make the permanent mark. Whenever he cuts this, the feet *t* and *d* are on a

adjust a slope-instrument, or clinometer. As usually made, ubble-tube is attached to the movable bar by a screw near each end, and the f one of the screws conceals a small slot in the bar, which allows a slight vert n to the screw when loose, and with it to that end of the tube. Therefore, in to adjust the bubble, this screw is first loosened a little, and then moved up wn a trifle, as may be reqd. It is then tightened again.

LEVELLING BY THE BAROMETER.

Many circumstances combine to render the results of this kind of levelling un- le where great accuracy is required. This fact was most conclusively proved observations made by Captain T. J. Cram, of the U. S. Coast Survey. See t of U. S. C. S., vol. for 1854. It is difficult to read off from an aneroid (the f barom generally employed for engineering purposes) to within from two to r six ft, depending on its size. The moisture or dryness of the air affects the ; also winds, the vicinity of mountains, and the daily atmospheric tides, cause incessant and irregular fluctuations in the barom. A barom hanging y in a room will often vary $\frac{1}{10}$ of an inch within a few hours, corresponding iff of elevation of nearly 100 ft. No formula can possibly be devised that shall ce these sources of error. The variations dependent upon temperature, lati-

** Price, about $10 or $12; with about $3 more for attachable ball and socket.*



ascertain the diff of height between two points.

Observe the heights, or readings of the barom and thermom (Fah) at both Add together the two readings of the barom, and div their sum by 2, for ; which call b. Do the same with the two readings of the thermom, and an t. Subtract the least reading of the barom from the greatest; and call Then mult together this diff d; the number from the next Table No. 1, ; and the constant number 30. Div the prod by b. Or

$$\text{eight feet} = \frac{\text{Diff } (d) \text{ of barom} \times \text{Tabular number opposite mean } (t) \text{ of thermom} \times \text{Constant } 30.}{\text{mean } (b) \text{ of barom.}}$$

. Reading of the barom at lower station, 26.64 ins; and at the upper ns. Thermom at lowest sta, 70°; at upper sta, 40°. What is the diff in the two stations? Here,

Barom, 26.64
" 20.82
2)47.46
23.73 mean of bar, or b.

Also,

Therm, 70°
" 40°
2)110
55° mean of therm, or t.

ular number opposite 55°, is 917.2.

Again, 26.64 (Bar.) − 20.82 (Bar.) = 5.82, diff of bar; or d. Hence,

$$\text{Height in feet} = \frac{\overset{d,}{5.82} \times \overset{\text{Tab No. Con.}}{917.2} \times 30}{23.73 \text{ (or } b)} = \frac{160143.12}{23.73} = 6748.5 \text{ ft; answer.}$$

rrect for latitude, if more accuracy is reqd, by rule on next page.

w at the back of an aneroid is for adjusting the index by means of a stand- ; and after this has been done it must by no means be meddled with. In ruments specially made to order with that intention, this screw may be or turning the index back, after having risen to an elevation so great that has reached the extreme limit of the graduated arc. After thus turning e indications of the index at greater heights must be added to that at- en it was turned back.

TABLE 1. For Rule 1.

Mean of Ther.	No.	Mean of Ther.	No.	Mean of Ther.	No.	Mean of Ther.	No.
0°	801.1	30°	864.4	60°	927.7	90°	991.0
1	803.2	31	866.5	61	929.8	91	993.1
2	805.3	32	868.6	62	931.9	92	995.2
3	807.4	33	870.7	63	934.0	93	997.3
4	809.5	34	872.8	64	936.1	94	999.4
5	811.7	35	874.9	65	938.2	95	1001.6
6	813.8	36	877.0	66	940.3	96	1003.7
7	815.9	37	879.2	67	942.4	97	1005.8
8	818.0	38	881.3	68	944.5	98	1007.9
9	820.1	39	883.4	69	946.7	99	1001.0
10	822.2	40	885.4	70	948.8	100	1012.1
11	824.3	41	887.5	71	950.9	101	1014.2
12	826.4	42	889.6	72	953.0	102	1016.3
13	828.5	43	891.7	73	955.1	103	1018.4
14	830.6	44	893.8	74	957.2	104	1020.5
15	832.8	45	896.0	75	959.3	105	1022.7
16	834.9	46	898.1	76	961.4	106	1024.8
17	837.0	47	900.2	77	963.5	107	1026.9
18	839.1	48	902.3	78	965.6	108	1029.0
19	841.2	49	904.5	79	967.7	109	1031.1
20	843.3	50	906.6	80	969.9	110	1033.2
21	845.4	51	908.7	81	972.0	111	1035.3
22	847.5	52	910.8	82	974.1	112	1037.4
23	849.6	53	913.0	83	976.2	113	1039.5
24	851.8	54	915.1	84	978.3	114	1041.6
25	853.9	55	917.2	85	980.4	115	1043.8
26	856.0	56	919.3	86	982.6	116	1045.9
27	858.1	57	921.4	87	984.7	117	1048.0
28	860.2	58	923.5	88	986.8	118	1050.1
29	862.3	59	925.6	89	988.9	119	1052.2

* both the barom and the thermom must be **in the shade.**

RULE 2. **Belville's short approx rule** is the one best adapted to rapid field use, namely, add together the two readings of the barom only. Also find the diff between said two readings; then, **as the sum of the two readings is to their diff, so is 55000 feet to the reqd altitude.**

EX. Same as before. Here the sum = 26.64 + 20.82 = 47.46 ins; and the diff = 26.64 — 20.82 = 5.82 ins.

Sum. Diff. Feet. Feet.
Hence, as 47.46 : 5.82 : : 55000 : 6744.6 reqd alt; instead of the 6768.5 of Rule 1.

Correction for latitude. The lat of the place affects the result to some extent; but is usually omitted when great accuracy is not reqd. To apply the correction, first find the altitude by the rule, as before. Then divide it by the number in the following table opp the lat of the place. *Add* the quotient to the alt if the lat is *less* than 45°; or *subtract* it if the lat is more than 45°. If the two places are in diff lats, use their mean.

Table of corrections for latitude.

Lat.		Lat.		Lat.		Lat.		Lat.		Lat.	
0°	352	14°	399	28°	630	42°	3367	54°	1140	68°	[illegible]
2	354	16	416	30	705	44	10101	56	941	70	[illegible]
4	356	18	436	32	804	45	0	58	804	72	[illegible]
6	360	20	460	34	941	46	10101	60	705	74	[illegible]
8	367	22	490	36	1140	48	3367	62	630	76	[illegible]
10	375	24	527	38	1458	50	2028	64	572	78	[illegible]
12	386	26	572	40	2028	52	1458	66	527	80	[illegible]

Levelling by Barometer; or by the boiling point.

RULE 3. The following table, No. 2, enables us to measure heights either by means of boiling water, or by the barom. The third column shows the approximate altitude above sea-level corresponding to diff heights, or readings of the barom; and to the diff degrees of Fahrenheit's thermom, at which water boils in the open air. Thus when the barom, under undisturbed conditions of the atmosphere, stands at 24.08 inches, or when pure rain or distilled water boils at the temp of 201° Fah; the place is about 5764 ft above the level of the sea, as shown by the table. It is therefore very easy to find the *diff* of altitude of two places. Thus: take out from table No 2, the altitudes opposite to the two boiling temperatures; or to the two barom readings. Subtract the one opposite the lower reading, from that opposite the upper reading. The rem will be the reqd height, as a rough approximation. To correct this, add together the two therm readings; and div the sum by 2, for their mean. From table 3, of corrections for temp, take out the number opposite this mean. Mult the approximate height just found, by this tabular number. Then correct for lat if reqd.

EX. The same as preceding; namely, barom at lower sta, 26.64; and at upper sta, 20.82. Thermom at lower sta, 70° Fah; and at the upper one, 40°. What is the diff of height of the two stations?

	Alt.
Here the tabular altitudes are, for 20.82.....	9579
and for 26.64.....	3115
	6464 ft, approx height.

To correct this, we have $\frac{70° + 40°}{2} = \frac{110°}{2} = 55°$ mean; and in table 3, opposite to 55°, we find 1.048. Therefore 6464 × 1.048 = 6774 ft, the reqd height.

This is about 26 ft more than by Rule 1; or nearly .4 of a ft in each 100 ft.

At 70° Fah, pure water will boil at 1° less of temp, for an average of about 550 ft of elevation above sea-level, up to a height of ½ a mile. At the height of 1 mile, 1° of boiling temp will correspond to about 560 ft of elevation. In table No. 3, the mean of the temps at the two stations is assumed to be 32° Fah; at which no correction for temp is necessary in using the table; hence the tabular number opposite 32° in table 3, is 1.

This diff produced in the temp of the *boiling point*, by change of elevation, must not be confounded with that of the *atmosphere*, due to the same cause. The air becomes cooler as we ascend above sea-level, at the rate (very roughly) of about 1° Fah *for every* 200 ft near sea-level, to 350 ft at the height of 1 mile. See "Atmosphere."

The following table, No. 2, (so far as it relates to the barom,) was *deduced by the writer from* the standard work on the barom by Lieut.-Col. R. S. Williamson, *U. S. army.**

* *Published by permission of Government in 1868 by Van Nostrand, N. Y.*

TABLE 2.

ling by Barometer; or by the boiling point.

ned temperature 32° Fah. Barom deduced from Williamson.

ltitude above ea level Feet.	Boil point in deg Fah.	Barom. Ins.	Altitude above sea level Feet.	Boil point in deg Fah.	Barom. Ins.	Altitude above sea level Feet.	Boil point in deg Fah.	Barom. Ins.	Altitude above sea level Feet.
15221	.3	19.66	11063	.6	22.93	7048	.9	26.59	3164
15159	.4	19.70	11029	.7	22.98	6991	206	26.64	3115
15112	.5	19.74	10976	.8	23.02	6945	.1	26.69	3066
15050	.6	19.78	10923	.9	23.07	6888	.2	26.75	3007
15003	.7	19.82	10870	199	23.11	6843	.3	26.80	2958
14941	.8	19.87	10804	.1	23.16	6786	.4	26.86	2899
14895	.9	19.92	10738	.2	23.21	6729	.5	26.91	2850
14833	192	19.96	10685	.3	23.26	6673	.6	26.97	2792
14772	.1	20.00	10633	.4	23.31	6617	.7	27.02	2743
14710	.2	20.05	10567	.5	23.36	6560	.8	27.08	2685
14649	.3	20.10	10502	.6	23.40	6516	.9	27.13	2637
14588	.4	20.14	10450	.7	23.45	6460	207	27.18	2589
14543	.5	20.18	10398	.8	23.49	6415	.1	27.23	2540
14482	.6	20.22	10346	.9	23.54	6359	.2	27.29	2483
14421	.7	20.27	10281	200	23.59	6304	.3	27.34	2435
14361	.8	20.31	10230	.1	23.64	6248	.4	27.40	2377
14315	.9	20.35	10178	.2	23.69	6193	.5	27.45	2329
14255	193	20.39	10127	.3	23.74	6137	.6	27.51	2272
14195	.1	20.43	10075	.4	23.79	6082	.7	27.56	2224
14135	.2	20.48	10011	.5	23.84	6027	.8	27.62	2167
14075	.3	20.53	9947	.6	23.89	5972	.9	27.67	2120
14015	.4	20.57	9896	.7	23.94	5917	208	27.73	2063
13956	.5	20.61	9845	.8	23.98	5874	.1	27.78	2016
13896	.6	20.65	9794	.9	24.03	5819	.2	27.84	1959
13837	.7	20.69	9743	201	24.08	5764	.3	27.89	1912
13778	.8	20.73	9693	.1	24.13	5710	.4	27.95	1856
13718	.9	20.77	9642	.2	24.18	5656	.5	28.00	1809
13660	194	20.82	9579	.3	24.23	5602	.6	28.06	1753
13601	.1	20.87	9516	.4	24.28	5547	.7	28.11	1706
13542	.2	20.91	9466	.5	24.33	5494	.8	28.17	1650
13498	.3	20.96	9403	.6	24.38	5440	.9	28.23	1595
13440	.4	21.00	9353	.7	24.43	5386	209	28.29	1539
13396	.5	21.05	9291	.8	24.48	5332	.1	28.35	1483
13338	.6	21.09	9241	.9	24.53	5279	.2	28.40	1437
13280	.7	21.14	9179	202	24.58	5225	.3	28.45	1391
13222	.8	21.18	9130	.1	24.63	5172	.4	28.51	1336
13164	.9	21.22	9080	.2	24.68	5119	.5	28.56	1290
13106	195	21.26	9031	.3	24.73	5066	.6	28.62	1235
13049	.1	21.31	8969	.4	24.78	5013	.7	28.67	1189
12991	.2	21.35	8920	.5	24.83	4960	.8	28.73	1134
12934	.3	21.40	8859	.6	24.88	4907	.9	28.79	1079
12877	.4	21.44	8810	.7	24.93	4855	210	28.85	1025
12820	.5	21.49	8749	.8	24.98	4802	.1	28.91	970
12763	.6	21.53	8700	.9	25.03	4750	.2	28.97	916
12706	.7	21.58	8639	203	25.08	4697	.3	29.03	862
12649	.8	21.62	8590	.1	25.13	4645	.4	29.09	808
12593	.9	21.67	8530	.2	25.18	4593	.5	29.15	754
12536	196	21.71	8481	.3	25.23	4541	.6	29.20	709
12480	.1	21.76	8421	.4	25.28	4489	.7	29.25	664
12424	.2	21.81	8361	.5	25.33	4437	.8	29.31	610
12367	.3	21.86	8301	.6	25.38	4386	.9	29.36	565
12311	.4	21.90	8253	.7	25.43	4334	211	29.42	512
12256	.5	21.95	8193	.8	25.49	4272	.1	29.48	458
12200	.6	21.99	8145	.9	25.54	4221	.2	29.54	405
12144	.7	22.04	8086	204	25.59	4169	.3	29.60	352
12089	.8	22.08	8038	.1	25.64	4118	.4	29.65	308
12033	.9	22.13	7979	.2	25.70	4057	.5	29.71	255
11978	197	22.17	7932	.3	25.76	3996	.6	29.77	202
11923	.1	22.22	7873	.4	25.81	3945	.7	29.83	149
11868	.2	22.27	7814	.5	25.86	3894	.8	29.88	105
11799	.3	22.32	7755	.6	25.91	3844	.9	29.94	52
11745	.4	22.36	7708	.7	25.96	3793	212	30.00	sea lev=0
11690	.5	22.41	7649	.8	26.01	3742		Below sea level.	
11635	.6	22.45	7602	.9	26.06	3692	.1	30.06	— 52
11581	.7	22.50	7544	205	26.11	3642	.2	30.12	—104
11527	.8	22.54	7498	.1	26.17	3582	.3	30.18	—156
11472	.9	22.59	7439	.2	26.22	3532	.4	30.24	—209
11418	198	22.64	7381	.3	26.28	3472	.5	30.30	—261
11364	.1	22.69	7324	.4	26.33	3422	.6	30.35	—304
11310	.2	22.74	7266	.5	26.38	3372	.7	30.41	—356
11243	.3	22.79	7208	.6	26.43	3322	.8	30.47	—408
11190	.4	22.84	7151	.7	26.48	3273	.9	30.53	—459
11136	.5	22.89	7093	.8	26.54	3213	213	30.59	—51[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Mult. by
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.112
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.117
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.121
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	.11	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	.19	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.154
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

PENDULUMS.

The numbers of vibrations which [illegible] will make in any given plac [illegible] time, are [illegible] as the square roots of their lengths; thus, if one of t [illegible] times as long as the other, its [illegible] will be [illegible] or 4 times as great; [illegible] as great. The times in which [illegible] will make a vibration, are [illegible] of their lengths. T [illegible] times as long as the other, its [illegible] will be 2, 3, or 4 time [illegible] the time occupied in one of its vibrations.

The length of a pendulum vibrating seconds at the level of the sea, in a vacu [illegible] London [illegible] North, is 39.1393 ins; and in the lat of N. York ([illegible] N [illegible] ins. At the equator about [illegible] inch shorter; and at the poles, abou [illegible] Approximately enough for experiments which occupy but a few [illegible] at any place call the length of a seconds pendulum in the open air, 39 [illegible] 3/4 ins; and may assume that long and short vibrations of the same [illegible] made in the same time, which they actually are, very nearly. For m [illegible] by sound, a sufficiently good sec pendulum may be made [illegible] a small piece of metal is better; and a piece of thread, suspended fro [illegible] The length of 39 ins should be measured from the centre of the [illegible] of the

the vibrations, the pebble, or *bob*, must not be *thrown* into motion, but *drop*, after extending the string at the proper height.

d the length of a pendulum reqd to make a given number of in a min, divide 375 by said reqd number. The square of the quot will be in ins, near enough for such temporary purposes as the foregoing. Thus, ulum to make 100 vibrations per min, we have $\frac{375}{100} = 3.75$; and the square 14.06 ins, the reqd length.

d the number of vibrations per min for a pendulum of th, in ins, take the sq rt of said length, and div 375 by said sq rt. Thus, ulum 14.06 ins long, the sq rt is 3.75; and $\frac{375}{3.75} = 100$, the reqd number.

By practising before the sec pendulum of a clock, or one prepared as just erson will soon learn to count 5 in a sec, for a few sec in succession; and will le to divide a sec into 5 equal parts; and this may at times be useful for estimating when he has no pendulum.

Centre of Oscillation and Percussion.

When a pendulum, or any other suspended body, is vibrating or oscillating and forward, it is plain that those particles of it which are far from the spension move faster than those which are near it. But there is always oint in the body, such that if *all* the particles were concentrated at it, so ould move with the same actual vel, neither the number of oscillations, ngular vel,* would be changed. This point is called the *center of oscilla-* not the same as the cen of grav, and is always farther than it from the spension. It is also the *centre of percussion* of the suspended vibrating e dist of this point from the point of susp is found thus: Suppose the body ed into many (the more the better) small parts; the smaller the better. eight of each part. Also find the cen of grav of each part; also the dist such cen of grav to the point of susp. Square each of these dists, and square by the wt of the corresponding small part of the body. Add the ogether, and call their sum p. Next mult the weight of the entire body of its cen of grav from the point of susp. Call the prod g. Divide p by g.

SOUND.

CITY OF SOUND, in quiet open air, has been experimentally determined to proximately 1090 ft per sec, when the temp is at freezing point, or 32°. that it increases about $1\frac{1}{4}$ ft per sec for every degree above 32°, or, by orities, 1 ft for every 2°. The first would make it

1090 ft at 32°	1150 ft at 80°
1100 " 40°	1162½ " 90°
1112½ " 50°	1175 " 100°
1125 " 60°	1187½ " 110°
1137½ " 70°	1200 " 120°

nd would travel a mile in 4.84 sec; at 80°, in 4.6; and at 120°, in 4.4 sec.
is calm, fogs, or rain do not appreciably affect the result; but winds do. There is some lieve that very loud sounds travel somewhat faster than low ones. The watchword of s been heard across still water, on a calm night, 10½ miles; and a cannon 20 miles. nds, at intervals of $\frac{1}{10}$ of a sec, cannot be distinguished, but appear to be connected. which a speaker can be understood, in front, on one side, and behind him, are about as 4,

M. Cresson informs the writer that, by repeated trials, he found that in a Philadelphia ins diam, and 16000 ft long, laid and covered in the earth, but empty of gas, and having al bend of 90°, and of 40 ft radius, the sound of a pistol-shot travelled 16000 ft in pre- , or 1000 ft per sec. The arrival of the sound was barely audible; but was rendered very the eye by its blowing off a diaphragm of tissue-paper placed over the end of the main.

anchored at a considerable dist apart may serve as a base line for triangulating objects st: the dist between them being first found by firing guns on board one of them.

he vel of sound is about 4708 ft per sec, or about 4 times as great as in the air. ALONG it is from 10 to 16 times; and ALONG METALS, from 4 to 16 times greater than in air, some authorities.

* *For angular vel, see foot-note* page 447, "Force in Rigid Bodies."

STRENGTH OF MATERIALS.

Art. 1. Table of average ultimate crushing loads square inch, for wood. in pieces whose height does not exceed 2 or 3 the diam. or the least side. Being the results of experiments by Eaton Hodgki on cylinders 1 inch in diameter, and 2 inches high, with flat ends. It is pro that larger pieces would have given greater results; for the resistance to cru certainly increases more rapidly than the area, but in what ratio is not known.

	Weight per cub. ft.	Pounds per sq. inch.		Weight per cub. ft.	Po p sq.
Alder	.....	6900	Mahogany, Spanish	.....	8
Ash	43 to 53	8600	Oak, Quebec, unseas.	54	4
Baywood	51	7200	" " seas	.....	8
Beech, unseasoned	53	7700	" English	58	6
" seasoned	43	9300	" " well seas	.....	9
Birch, Amer, unseas	.....	6000	" Dantzic, very dry	.....	7
" " seas	41	11600	Pine, pitch	.....	6
" English, unseas	.....	4500	" Amer yel'w, uns	.....	8
" " seas	.....	6500	" " " seas	.....	5
Box, dry	.....	10000	" red, unseas	.....	5
Cedar, unseas	56	5700	" " seas	.....	7
" seas	.....	6500	Pear	.....	7
Crab-tree, green	.....	6500	Poplar, unseas.	.....	3
" " dry	48	7000	" seas	.....	5
Deal,* red, unseas	47	5700	Plum, wet.	.....	3
" " seas.	.....	6500	" dry.	.....	9
" white, unseas	.....	6700	Sycamore or *Planetree*	.....	7
" " seas	.....	7300	Spruce, or Fir, unseas	.....	6
Elder, unseas	43	7400	" " " seas	.....	6
" seas	37	9900	" " " Riga	.....	6
Elm, seas	37	10000	Teak	.....	12
Fir,* or Spruce, uns	.....	6500	Larch, unseas	.....	3
" " " seas	.....	6800	" seas	.....	5
" Riga	.....	6000	Walnut, unseas	.....	6
Hornbeam, unseas	47	4500	" seas	.....	7
" seas	50	7300	Willow, unseas	.....	2
Larch, green	.....	3200	" seas	.....	6
" dry	.....	5500			

Hence it appears that SEASONED white and yellow pines, spruce, and ordinary which are the woods most employed in the United States for bridges, roofs, &c, with from 5000 to 7000 lbs per sq inch. WHEN IN SHORT BLOCKS, (short in pr tion to their diam;) the average being 6000 lbs.† In practice, however, the cru load rapidly becomes less AS THE PIECE BECOMES LONGER; and when the length is 30 times the diam, 1200 lbs per sq inch will probably crush it; and the safe loa be reduced to from 150 to 350 lbs per sq inch; or even much less, if the piece is same time exposed to cross-strains, as in the case of rafters, &c. See "Wooden Pil Mistakes are sometimes made by employing 800 or 1000 lbs as the safe cru resistance of wood, without regard to length, or even to transverse strains.

The tensile or cohesive strengths of these woods average about 10000 lbs p inch, or ½ as much as average cast-iron; or nearly double their resistance to c ing. The tensile strength does not change with the length of the piece; so th practice we may take its safe strain at from 1000 to 2000 lbs per sq inch, depe upon the character of the structure, &c. without regard to the length, except this is so great that two or more pieces have to be spliced together to make it; weakening the piece very much. It will be seen from the table that SEASONED resist crushing much better than green ones; in many cases, twice as well. *must be* taken into consideration when building bridges, &c, of timber recently

* DEAL *is not the name of any particular species of timber, but is a local English term fo kind of pine boards more than 7 ins wide. When less than 7 ins wide they are called BATTENS is another English name for pine. When used alone, it generally refers to white spruce pine, ve call simply spruce. Yellow fir is yellow pine.*

† *C. Shaler Smith* assumes 5000 for moderately seasoned. See p 223.

found that a pressure of 900 lbs per sq inch against the *side* of a piece ine barely makes a PERCEPTIBLE, and one of 1000 lbs a DISTINCT IMPRES- hat 1400 lbs produced on English oak about the same results. The writer 1000 lbs per sq inch indents the side of *well-seasoned* white pine about $\frac{1}{80}$; common yellow pine about the thickness of writing-paper; and ordi- oak, cherry, or mahogany, not at all.

Ultimate average crushing loads in tons, per square stones, &c. The stones are supposed to be ON BED, and the height ore than 2 or 3 times the least side. Stones generally begin to crack or about one-half of their crushing loads. In practice, neither stone nor should be trusted with more than $\frac{1}{6}$ to $\frac{1}{10}$ th of the crushing load, ac- circumstances.

	Tons per sq. ft.	Mean. Tons.		Tons per sq. ft.	Mean. Tons.
d Syenites..	300 to 1200	750	Concrete (lime 1 part, gravel 3 parts,) 3 weeks old.....	30 to 50	40
.....	.....	700	Portland hydraulic Cement, English, pure, 6 weeks old*	80 to 160	120
and Mar-	250 to 1000	625	" 1 part cement, 1 part sand. "	.....	80
d	100 to 250	175	" 1 part cement, 3 parts sand. "	.....	40
fit for build-	150 to 700	425	" 1 part cement, 10 p'ts shingle."	.....	18
red, of Con- and N. Jer- ack	.....	200	Roman hydraulic Cement, English, pure. "	.....	50
.....	50 to 300	175	" 1 part cement, 2 parts sand. "	.....	5
ordinary, th	20 to 30	25	Sheppy Cement, and Atkinson's Cement, pure.....	.....	80
good, in ce-	30 to 40	35	Glass, green, crown and flint	1300to2300	1800
first-rate, t	50 to 70	60	Ice, firm.....	12 to 18	15
.....	400 to 800	600			
.....	100 to 200	150			
to crack.....	.....	70			
l.....	20 to 30	25			
Paris, 1 day	.....	40			

or 3 times that of granite.

Crushing height of Brick and Stone.

ume the wt of ordinary brickwork at 112 lbs per cub ft, and that it would r 30 tons per sq ft, then a vert uniform column of it 600 ft high, would base, under its own wt. Caen stone, weighing 130 lbs per cub ft, would olumn 1376 ft high to crush it. Average sandstones at 145 lbs per cub ft, uire one of 4158 ft high; and average granites, at 165 lbs per cub ft, one . But stones begin to crack and splinter at about half their ultimate ad; and in practice it is not considered expedient to trust them with more to $\frac{1}{10}$th part of it, especially in important works; inasmuch as settle- imperfect workmanship, often cause undue strains to be thrown on cer-

hants' shot-tower at Baltimore is 246 ft high; and its base sustains 6½ ft. The base of the granite pier of Saltash bridge, (by Brunel,) of solid the height of 96 ft, and supporting the ends of two iron spans of 455 ft ns 9½ tons per sq ft The base of a brick chimney at Glasgow, Scotland, bears 9 tons per sq ft; and Professor Rankine considers that in a high d, its leeward side may have to bear 15 tons. The highest pier of Rocque- e aqueduct, Marseilles, is 305 ft, and sustains a pressure at base of 13½ ft. For greater pressures on arch stones, see p 342.

at St. Louis bridge by order of Capt James B. Eads, C. E., showed nagnesian limestone did not yield under less than 1100 tons per sq ft. A s high and 2 *ins diam* shortened ¼ inch under pressure; and recovered ed.

*See "Mortar, Bricks, &c." Also, "Concrete, Cement, &c."

Ultimate average tensile or cohesive strength of Timber,

q inch; being the weights which, if attached to the lower end of a vert
ı square, firmly upheld at its upper end, would break it by tearing it
large timbers we recommend to reduce these constants ¼ to ⅓ part.

	Lbs per sq. inch.		Lbs per sq. inch.
.....	14000	Mahogany, Honduras.....	8000
ıh.....	16000	" Spanish.....	16000
ıcan (author) abt.....	16500	Mangrove, white, Bermuda....	10000
.....	15000	Mulberry.....	12000
n black.....	7000	Oak, Amer'n white...	
.....	12000	" " basket.....	
lish.....	11500	" " red.....	
.....	6000	" Dantzic, seasoned	10000
.....	20000	" Riga.....	
ıuda.....	7600	" English..	
laloupe	9500	" live, Amer'n	
.....	13000	Pear.....	10000
rse.....	10000	Pine, Amer'n, white, red, and Pitch, Memel, Riga...	10000
.....	6000		
.....	10000	Plane.....	11000
.....	6000	Plum.....	11000
a.....	13000	Poplar.....	7000
ıce.	10000	Quince..	7000
.....	10000	Spruce, or Fir.....	10000
.....	18000	Sycamore.....	12000
.....	16000	Teak.....	15000
.....	20000	Walnut	8000
mer'n.....	11000	Yew.....	8000
æ, Amer'n.....	11000		
.....	23000	**Across the grain,** Oak..	2300
ch.....	7000	" " " Poplar.....	1800
.....	18000	" " " Larch, 900 to	1700
.....	10000	" " " Fir, & Pines	550

E AVERAGES. The strengths vary much with the age of the tree; the its growth; whether the piece is from the center, or from the outer por- tree; the degree of seasoning; straightness of grain; knots, &c, &c. Also, ıs the constants are deduced from experiments with good specimens of whereas large beams are almost invariably more or less defective from kedness of fibre, &c, it is advisable in practice to reduce these constants ınded above.

ıate tensile strength of such woods as are usually employed in bridges, ı commonly assumed at 10000 lbs, or say 4½ tons per sq inch; and the from 1000 to 2500 lbs, depending on the character of the structure. Thus uld probably be quite as safe in a roof, as 1000 or 1500 in a bridge subject ɜration.

ɔdulus of Elasticity, and its use. Within the limit of elasticity, a f given material lengthens or shortens equally under equal additions of load. If this case beyond said limit, it is plain that there would be some load which would stretch a ɔ twice its original length, or shorten it to zero or 0. And this load *in lbs*, for a bar of *re* cross section, is the mod of elas for the given material; or is the E of authors on aterials. For example, a one-inch square bar of wrought iron will, within the limit ı on an average about 1 part in 12000 of its length under each additional load of 2240 lbs. , if the same rate of stretching continued beyond the limit of elas, it is evident that r 26880000 lbs, would stretch the bar to twice its original length. Hence these 26880000 ıd of elas for average bar iron. And so with any other material. Hence the mod of which bears the same proportion to the original length of a uniform bar, as the load ɔduce any given amount of stretch, is to the length of said stretch. This fact facilitates ations; thus,

$$\left.\begin{array}{l}\text{Load in lbs reqd}\\ \text{to produce a given}\\ \text{stretch within}\\ \textit{elas limit, in ins,}\end{array}\right\} = \frac{\text{Reqd stretch in inches}}{\text{orig length in inches}} \times \text{mod of elas.} \times \text{cross sec in sq ins}$$

$$\left.\begin{array}{l}\text{ur}\\ \text{s}\end{array}\right\} = \frac{\textit{Load in lbs} \times \textit{orig length in ins}}{\textit{mod of elas} \times \textit{cross sec in sq ins}}$$

See Table, p 652.

ge ultimate tensile, or cohesive strength of Metals per square inch. (CONTINUED.)

	Pounds per sq. inch.	Tons. per sq. in.
mes B. Eads found that forged steel bolts for the St. Louis bridge, 5¾ nd 22 to 36 ft. long, broke *short* with only 30000 lbs per sq inch; while t ¾ inch section, *cut from the large ones*, in no instance broke with per sq inch, but stretched considerably.		
st, Bessemer ingots, average	63000	28 1
best American Bessemer ingots	86600	38.6
" " rolled and hammered, 120000 to 130000	125000	55.8
homogeneous, Cammell & Co, England, No 1	58240	26
" " " No 2	71680	32
" " " No 3	76160	34
iddled bars, rolled and hammered, 65000 to 135000	100000	44.6
xperiments by Lieut. W. S. Shock, U. S. N., at Washington, on steel from the Black Diamond Steel-Works, Pittsburg, Pa. All the pieces were cut from the same bar, three pieces for each exp. They were turned down to a diam of 62 of an inch at the intended point of fracture, by a groove, in shape of a circular segment, with a chord of about 1 inch:		
e bar in its original condition, 109500 to 131900	120700	53.9
ated to light cherry-red, then plunged into oil of 82° Fah, 201300 to 227500	214400	95.7
ated to light cherry-red, then plunged into water of 79° Fah. Then tempered on a heated plate, 152500 to 176100	164300	73.3
ated to light cherry-red, then plunged into water of 79° Fah, 132700 to 150500	141600	63.2
mpering in oil usually increases the strength from 40 to 80 per cent.*		
rome, made at Brooklyn, N. Y., and tested at West Point Foundry, N.Y, (specific gr 7.816 to 7.956,) 163000 to 199000. Average of 12 specimens	180000	80
ade from very pure Swedish iron, but containing different proportions of carbon. The bars were 21½ ins long, with 14 ins of this length turned down to a uniform diam of 1 inch. The breaking wts, however, in the table, are per sq inch:		
ark No. 2, carbon .33 per ct, stretched 1.37 ins	.....	30.4
" No. 4 " .43 " " 1.37 "	.....	34
" No. 5 " .48 " " 1.25 "	.....	37.5
" No. 6 " .53 " " 1.12 "	.....	42.5
" No. 7 " .58 " " 0.81 "	.....	41.5
" No. 8 " .63 " " 1.00 "	.....	45
" No. 10 " .74 " " 0.69 "	.....	45.5
" No. 12 " .84 " " 1.12 "	.....	55
" No. 15 " 1.00 " " 1.00 "	.....	60
" No. 20 " 1.25 " " 0.62 "	.....	69
re than about 1.5 per ct of carbon the tensile strength of iminishes. A bar of the above No. 15, which broke at s per sq inch, when turned down for 14 ins of its length; with 79½ tons per sq inch when turned down at one only. This is owing to the fact that the last could not as much as the first, and therefore its diam could diminished as much before breaking. All its fibres more unitedly. It will be observed that the steel of st strength stretched the least before breaking. This er steel would break under a *suddenly* applied force, or se, more easily than a weaker one would; because the r one, by its stretching, gradually breaks the force of the se, on the same principle as a spring. Hence the steel, ch, which is strongest against a gradually applied force in, may be unfit for uses where the strain comes upon it ily.†		

ring is hardening by sudden cooling.

SHOWS ULTIMATE TENSILE STRENGTH OF STEEL is about twice as great as that of wrou *ost 7 times that of cast* iron. ITS DEFLECTION AS A BEAM, within the elastic limi

Average ultimate tensile or cohesive strength of M per square inch. (CONTINUED.)

	Pounds per sq. inch.
Silver, cast	41000
Tin, English block	4600
" wire	7000
Zinc, cast	7500
" sheet	16000
" wire	22000

Art. 6. Average ultimate tensile or cohesive streng various materials.

	Pounds per sq. inch.	Tons per sq. ft.		Pounds per sq. inch
Brick, 150 to 300	225	14.4	Horn, ox	9000
Caen stone, 100 to 200	150	9.7	Ivory	16000
Cement, hydraulic, English Portland, pure, 7 days old	100	6.4	Mortar, hydraulic, 100 to 200	150
" 6 months old	350	22.5	" common, 6 months old, 6 to 34	20
" 1 year old	450	29	" does not often exceed	15
" English Roman, after 1 year	200	12.8	Marble, white Italian	84
Pure hyd cements adhere to brick and stone with from 15 to 35 when only 1 month old	25	1.6	" black	140
At end of 1 year, 3 times as much	75	4.8	Oolites, 100 to 200	150
See art on "Cements," &c.			Plaster of Paris, well set	70
Glass	9500	611	Rope, Manilla	12000
Glue holds wood together with from 300 to 800	550	35	" hemp	15000
			Sandstones, 100 to 200	150
			Slate	11000
			Stone, Ransome's artificial	300
			Whalebone	7600

To reduce lbs per sq inch to tons per sq ft, mult them by .0643.
To reduce tons per sq ft to lbs per sq inch, mult them by 15.55.

REM. In practice it is difficult to secure that either a tensile or a com force shall act precisely *in the direction of the axis* of the piece strained; a does not, a less degree of force will produce fracture, because the strain, whicl be diffused among all the particles, is thrown chiefly upon only a portion c This is one of the many reasons why it becomes necessary to make a large all for safety.

The idea was at one time prevalent, that the tensile strength of a bar of i became greater by the bars being broken; for it was found that a piece c broken by tension would bear more than the entire bar did; and that if a a piece were broken in the same way, it would in turn support more than But this proved nothing more than that the bar broke at its weakest part fi at the next weakest in succession.

Iron is weakened by extreme cold.

The belief (originating with Styff of Sweden,) is gaining ground that i steel are not rendered *more brittle by intense cold*, but that the great nu

about ⅔ that of wrought, or ⅓ that of cast iron. GOOD AVERAGE STEEL STRUTS WILL about t or 1 part in 12500, for every ton per square inch of load, up to its elastic limit, which *ranges at between ⅓ and ⅔ of its breaking strength*; the latter being for the harder, stro *less stretchy kinds. A uniform bar of rolled steel, gradually loaded, will stretch from of its length before breaking*; or from $\frac{3}{10}$ of an inch to 2 ins per foot, according to qual *mean of these is nearly* $\frac{1}{10}$ of the length, or $1\frac{2}{10}$ inch to a foot. When steel, especially *o be heated to softness* in order to give it a required shape, it is thereby weakened.

of rails, wheels, axles, &c, in winter, is owing to the more severe blows the frozen and unyielding nature of the earth at that period of the year. erg's experiments show conclusively that although these metals may per- as much *steady* force, *gradually* applied, in winter as in summer, yet their to *impulse*, or *sudden force*, is not more than ½ or ⅓ as great in severe h renders them less flexible and less stretchy. It is probable that this ot receive as much attention as it should, in proportioning iron bridges, &c. periments with good wrought iron showed that even at 23° Fah, or only han freezing point, there was a loss of strength of from 2½ to 4 per

Breaking by shearing. Let l, represent a beam, with its ends supports S S; with a load *l*, so heavy it by forcing its entire central part, y from the two end parts *a d g* and *b c o*; hile the two latter remain in their central part *slides* out; or is, as it *shed* clean out from between them. ar mode of fracture is called *shearing*, *n*. The force required to produce it, tance which the beam opposes to such y practically be assumed to be in pro- the area of the sheared section. Thus, area of cross section of a beam 1 ft sq

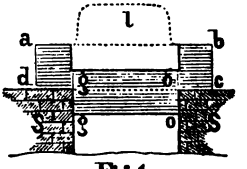


Fig. 1

s great as that of a beam 6 ins sq, the former will present 4 times as great to shearing; or will, in other words, require 4 times as great a load, or ar it across. In Fig 1 the total sheared area is equal to twice the trans- of the beam See Remark 2, p 293. of "Trusses." Bridge chords are ex- great shearing force where they rest on the abuts, but it becomes less center of the span; and so with every equally loaded beam.

very little experimental data on this subject.

earing strength of white pine, spruce, k, *parallel to the fibres*, by the author, 250 to 500 lbs ; oak 400 to 700; and is of use in estimating the along the line *c c*, Fig. 2, at the end of a tie-beam; ead of a queen-post, &c. *Across the fibres*, their however considerably greater. In very firm oaks other woods it is stated at 2000 to 5000 lbs per sq the fibres.

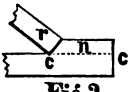


Fig. 2.

ght iron is stated at 40000 to 45000 lbs per sq iron 30000 to 40000; rivet-steel 64000; fine cast ; copper 33000.

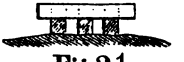


Fig. 2½

ring strength of steel rivets is said to be about ¼ han the tensile. The punching of rivet-holes in l plates, is an example of shearing. The rivets in dges are frequently sheared in two, in time, by the motion of the plates ich they are driven. In punching holes, the area of section is evidently ult the circumf of the hole by the thickness of the plate in which it is If a piece of material be supported as shown in Fig 2½, its resistance to ill be 3 times as great as in Fig 1, where it is sheared across in 2 places eas in Fig 2½, shearing would have to occur at 6 places, as per the 6 s.

Breaking by torsion, or twisting. Let *n*, Fig 3, be a vert rod of any material, 1 inch diam, the lower ch is *immovably* fixed; and let *c* be a lever rage (see levers) *a b*, measd from the axis of ical rod, is 1 ft. Suppose that with a spring ached to the end *b* of the lever, we apply force y, and around the axis of the rod as a center, d breaks by being twisted. Then if we mult e leverage *a b* in feet, and the amount of force he spring balance in lbs, and div the prod by the diam of the rod in ins the quot will be a ber of foot-pounds; and will be what is called

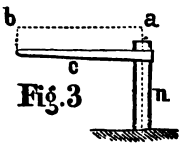


Fig. 3

t, or *coefficient for torsion*, for all cylindrical bars of that material. If we e bar, we *shall get the coef* for square bars; and so with any other shape. *ith any other bar, or shaft*, we mult the cube of its diam in inches by

[illegible] the leverage in feet, the cost will be the force in [illegible] in shape of [illegible].

$$\text{And } \frac{\text{Cube of diam in feet} \times \text{Constant}}{\text{Leverage in feet}} = \text{Breaking force in lbs.} \quad \text{Also}$$

$$\text{And } \frac{\text{Leverage in feet} \times \text{Brkg force in lbs}}{\text{Constant}} = \text{Cube of diam in ins.}$$

[illegible] of any material increases directly as the [illegible] and diminishes as either the leverage

[illegible] iron, by diff experimenters, is from 430 to [illegible]. The average for each of them is nearly [illegible] middled steel about 5[illegible]; as averages—per[illegible] in woods may probably be taken at about 40

[illegible] diam of a rod to have a safety of 3, 4, 5, &c, [illegible] mult said force by 3, 4, 5, &c, as the case [illegible] force. The diam will then be the *safe* one.

[illegible] when made to revolve around the axis of [illegible] is called the ANGLE OF TORSION. The [illegible] affect the amount of force reqd to produce [illegible] will be the angle of torsion; or in other [illegible] which the force must revolve around the [illegible] say that a working shaft should not

[illegible] reqd for shearing any material, see pre[illegible] a cylinder of that material by torsion,

$$\text{[illegible] shearing} \times 3.1416 \times \frac{\text{Cube of rad of cylinder in ins}}{\text{[illegible] leverage in inches.}}$$

[illegible] shaft is about $1\frac{1}{3}$ times that of a round one [illegible] or about $\frac{1}{3}$ less than that of a round [illegible] reqd to fracture *any* solid rectangular

$$\text{[illegible] shearing} \times \frac{\text{The square of one side} \times \text{The square of the other side.}}{\text{[illegible] of the sum of the two squares}} \times \text{Leverage in inches.}$$

[illegible] much better than solid ones of the same area of

[illegible] iron shafting. The shafting used for the [illegible] parts of machine-shops, many manufacturing re[illegible] twisting strains. It is usually made cylindrical, [illegible] shows that we may safely use the following for [illegible] bearing but little weight, and well supported at proper [illegible] adjusting ball and socket hangers.

$$\text{[illegible]} = \frac{\text{Horse-powers}}{\text{Number of revs per min.}} \times 125.$$

[illegible] the diam in ins, div the number of horse-powers [illegible] the shaft, by the number of revs which the shaft is [illegible] the quot by 125. The prod will be the cube of the [illegible] be the diam itself, at the thinnest part of the shaft;

[illegible] the faster a shaft revolves under the same number [illegible] the torsional strain upon it. This may at first seem [illegible] that a horse-power is made up of *pres* and *dist*; [illegible] the less is its pressure. Hence many horse-powers re-

lving rapidly will require a less diam than a small number revolving slower in oportion than its number.

Art. 9. Transverse (or arees) strength of mate-ials; or that by which they re-it breaking, when employed as ams; as, for instance, in Fig 4. Experiment has shown that in ams of the same material, and actly alike, except in their eadths, *n d*, the strengths vary the same proportion as those eadths; that is, if one is 2, 3, or times broader than the other, its strength will be 2, 3, or 10 times as great. If ey are alike, except in their clear lengths or spans, *a a*, between the points of sup-rt, their strengths will be *inversely* as those lengths; that is, if one is 2, 3, or 10 mes longer than the other, it will be but ½, ⅓, or $\frac{1}{10}$ part as strong. If they are like, except in point of depth, *o d*, measured vert, their strengths will be *directly* as e *squares* of their depths; that is, if one is 2, 3, or 10 times as deep as the other, will also be 4, 9, or 100 times as strong; or in other words, will require 4, 9, or 100 mes as great a load to break it. See Art. 11. It must be remembered that we are w speaking only of *strength*, or resistance to *breaking*; and not of *stiffness*, or resist-ce to *bending*, or *deflecting*. STIFFNESS FOLLOWS LAWS VERY DIFF FROM THOSE OF RENGTH, and will be alluded to further on, at Art. 26, &c.

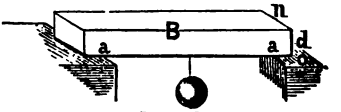


Fig. 4

Now, if we combine all the three foregoing elements of size, namely, length, eadth, and depth, we have the fact, that the strength of any beam, of any size, of ty given material, is in proportion to its $\frac{\text{breadth} \times \text{the square of its depth.}}{\text{its length}}$ There-re, if we find by actual trial, what center load will break any beam of known size; d then find what is the proportion between its $\frac{\text{breadth} \times \text{sq of depth}}{\text{length}}$, and its akg load, said proportion (or, more strictly speaking, *ratio*) will also be that ich any similar beam has to *its* breakg load, and will therefore serve to calcu-e the breakg load of any other similar beam of the same material. For instance, we take any piece of average good white pine, say 6 ins broad, 10 ins deep, and 12 t clear span, we find that its $\frac{\text{breadth} \times \text{sq of its depth}}{\text{length}}$ is equal to $\frac{6 \times 100}{12} = 50$. d if we gradually load this at its center until it breaks, we shall find that the akg load, including half the wt of the clear span of the beam itself, amounts to 00 lbs. Therefore, the proportion between the $\frac{\text{breadth in ins} \times \text{sq of depth in ins}}{\text{length in feet}}$, and the akg load, is as 50 to 22500; which is the same as 1 to 450; that is, the breakg d of the beam, including half its own weight, may be found by mult its adth $\times$ sq of depth in ins / length in feet, by 450. And in this same manner may be found the total ter breakg load of any rectangular beam of average quality of white pine. For *neat* load one-half the wt of the clear span must be deducted. It is self-evident t the weight of the beam assists to break it, as well as the neat load; and the ent to which it does so, is the same as if *one-half* of its unsupported wt were centrated at its center. Hence the rule. On this principle the rule in Art 12 is ed. The ratio thus found for any material, is called the *coef for center breakg ts*, for that material; or, frequently, its *constant for*, &c, because it does not vary h the size of the beam. If we take a piece, all of whose dimensions are 1, as 1 h wide, 1 inch deep and 1 foot span; then the $\frac{\text{breadth in ins} \times \text{sq of depth in ins}}{\text{length in feet}}$ will be $\frac{\text{sq of 1}}{1} = 1$; and the breakg load (including half its own weight) of such a piece it once, the constant reqd. See Remarks 1 to 4. In an average piece of white , this load will be found to be about 450 lbs; or the same as the constant obtained the large beam. *In this* manner the student may readily find constants for *self; and if he uses materials* not included in our table, Art 10, it will be well to *ify the deficiency, by inserting* his own results.

e *foregoing directions* for finding coefficients may be more briefly express

[illegible]

[illegible] materials, which [illegible] with small [illegible] proportion that [illegible] material, more time [illegible] of which, there is a wan[illegible] and this conduces to w[illegible] judgment and caution in making a[illegible]

[illegible] constants or coefficients for the quie[illegible] rectangular beams, supported hor[illegible] and loaded at the center: [illegible] the aver[illegible] one half the weight of the beams the[illegible] length between the supports. [illegible] of these constants [illegible] be employ[illegible] structure, its temperature [illegible] which it will be exposed [illegible] Even with a [illegible] 96.

[illegible] mult by [illegible] side of the square [illegible] diagonal vertical

hird part of any of these constants (except those for wrought eel), may be taken in ordinary practice as about the average **constant greatest center load within the elastic limit.** The loads for wrought iron and steel, are already the greatest within elastic limits.

sverse Strength.	Lbs.
WOODS.	
ish	650
r White	650
ap	400
t.	300
e, Amer	250
anada	350
glish	500
er White	450
er Red	550
er Black	450
er Yellow	450
muda	400
dalouPe	600
er White, Arbor Vitæ	250
	450
ish	350
, Canada	600
	400
Amer	700
" Bitter nut	500
, Canada	600
	600
itæ	650
	400
	450
White	650
Black	550
ck	550
ft.	550
ish	550
r White	600
Red, Black, Basket	550
	600
r White	450
Yellow	500
Pitch	550
el	450
	550
	700
	450
ck	350
	500
	400
	750
	450
	350
METALS.	
	850
1500 to 2700....average	2100
common pig	2000
castings from pig	2300
employed in our tables	2025
for castings 2½ or 3 ins thick	1800?
, 2200 to 2800.....av	2500
on does not break;	

Transverse Strength.	Lbs.
but at about the average of 2500 lbs its elasticity is destroyed. So also with the loads for steel.	
Steel, hammered, 5000 to 7000, av	
" rolled " "	6000
STONES, ETC.	
Blue stone flagging, Hudson River	125
Brick, common, 10 to 30..average	20
" good Amer pressed, 30 to 50 .....average	40
Caen Stone	25
Cement, Hydraulic, English Portland, artificial 4 months old	20
" " Roman, of Scotland	8
" " U. States, about	12
" *Trass*, sand, slacked lime in powder; 1 measure of each, 1 year old	18
" *Puzzuolana*, instead of Trass in the above	18
The following hydraulic cements were made into prisms, in vertical moulds, under a pressure of 32 lbs per cub inch, and were kept in sea water for 1 year.	
Portland Cement, English, pure, 1 year old	32
Roman Cement, Scotch, pure	10
American Cements, pure, av about	15
Granite, 50 to 150.....average	100
" Quincy	100
Glass, Millville, N. Jersey, thick flooring	170
Mortar, of lime alone, 60 days old	10
" 1 measure of slacked lime in powder, 1 sand	8
" 1 measure of slacked lime in powder, 2 sand	7
" 1 measure of slacked lime in powder, 4 sand	5
These are of course rude averages, as indeed all such constants must be; for much depends on the quality of the lime and sand, and the care with which they are mixed.	
Oolites, or Roe stones. These do not occur in the United States. 20 to 50.....av	35
Sandstones, 20 to 70 .....average	45
" Red of Connecticut and New Jersey	
Slate, laid on its bed, 200 to 450, av	

by a formula. After finding the neat center breakg load, by experiment, add to
one-half the wt of the clear span of the beam, for a total center load; then the

$$\left.\begin{array}{c}\text{Coef for breakg}\\ \text{strength}\end{array}\right\} = \frac{\text{Span in feet} \times \text{Total load in lbs}}{\text{Breadth in ins} \times \text{Square of depth in inches.}}$$

In a cylinder, inasmuch as the breadth and the depth are each equal to the dia
it is plain that $B \times D^2$, amounts to the same thing as diam^3; and it is always
expressed. See Remark, Art 29. p 201.

Rem. 1. The variation in strength of equal beams of the same material is so gre
that it is necessary to experiment with several pieces, in order to find an average
a constant. The loads given in the preceding tables are also constants, but for cru
ing and tension. They are averages of the strengths of the materials, derived fr
experiment. The actual strength of any particular specimen, if of superior qua
may be considerably greater than the average; or on the other hand, if of very p
quality, it may fall as much below it. We should always keep this in mind wh
referring to any table of constants; and if we have doubts as to the quality of t
piece of material which we are about to employ, we should make a correspondi
deduction from the constant in the table.

Rem. 2. If, instead of white pine, we had experimented with oak, iron, stone,
the process for finding the constant would have been precisely the same. If, inste
of a square, or rectangular beam, we use cylindrical, or triangular ones, or any oth
shape, such as hollow cylinders, H,T, or U beams, &c, we shall in the same w
establish constants for either larger or smaller beams of those *shapes*, and of precis
the same *proportions* in every part. See Remark, Art 29. Or if, instead of suppo
ing the beam at both ends, we secure it firmly at one end, and load it at the oth
end until it breaks, we shall obtain the *constant for beams fixed at one end, and load
at the other*, &c. It must also be remembered that the constants are for loads at re
or quiescent. If they are liable to jars, jolts, vibrations, &c, a large margin must
left for safety. Moreover, the constants given in tables are generally deduced fro
small specimens free from important defects; whereas large beams of any kind
material usually contain irregularities, which diminish their strength; and on th
account larger allowances for safety should be made as the dimensions of the bea
increase.

Rem. 3. It is not *necessary* that the breadth and depth be taken in ins, and th
lengths in ft. They may all be in ins, ft, yds, or any other measure; but since
every-day practice we usually speak of the breadths and depths of beams in ins, an
of their lengths in ft, it becomes more *convenient* so to consider them. If other mea
ures be used, the constant will of course be diff; but it will still be such that if th
same measure be used for calculating the strength of another beam, the final resu
will be the same as before. In like manner, the loads may all be taken in tons,
instead of lbs; but in giving the rule, it must be stated what measures have be
employed. See Remark, Art 29. Frequently all the dimensions are given in ins,
which case the resulting constant will be twelve times as great.

Rem. 4. There are peculiarities in the nature of some materials, which dimin
materially the reliability of constants derived from experimenting with small piec
Thus, a large beam of cast iron will break with a less load in proportion than a sma
one; because, in the interior of thick masses of that material, more time to coo
required than in the outer surfaces; in consequence of which, there is a want of u
formity in the arrangement of the particles of iron, and this conduces to weakne
All we can do in such cases, is to exercise judgment and caution in making sufficie
allowance for safety.

**Art. 10. Table of constants or coefficients for the quiesce
breaking loads of rectangular beams, supported horizo
tally at both ends, and loaded at the center**; being the average q
escent breaking loads in lbs (including one-half the weight of the beams themselv
for beams 1 inch square, and 1 foot *clear* length between the supports. For *saf
in practice*, not more than about $\frac{1}{3}$ to $\frac{1}{6}$ of these constants should be employed;
pending upon the importance of the structure, its temporary or permanent char
ter, and the degree of vibration to which it will be exposed. Thus a roof will pr
ably be as safe at $\frac{1}{3}$, as a bridge at $\frac{1}{6}$. Even with a perfectly *safe* load, a beam m
bend too much. See Art 26. p 196.

If any of these coefficients be mult by .589 (or say .6) it will give that for a cyl
drical beam whose diam = side of the square. Or if mult by .71 it will give th
for a square beam with its diagonal vertical.

One Third part of any of these constants (except those for wrought n and steel), may be taken in ordinary practice as about the average **constant r the greatest center load within the elastic limit.** The loads re given for wrought iron and steel, are already the greatest within elastic limits. e p 198.

Transverse Strength.	Lbs.
WOODS.	
ch, English	650
" Amer White	650
" Swamp	400
" Black	300
rbor *Vitæ*, Amer	250
lsam, Canada	350
ech, English	500
" Amer White	450
" Amer Red	550
Birch, Amer Black	450
" Amer Yellow	450
Cedar, Bermuda	400
" Guadaloupe	600
" Amer White, or Arbor Vitæ	250
Chestnut	450
Elm, English	350
" Rock, Canada	600
Hemlock	400
Hickory, Amer	700
" " Bitter nut	500
ron *Wood*, Canada	600
ocust	600
ignum Vitæ	650
arch	400
ahogany	450
angrove, White	650
" Black	550
aple, Black	550
" Soft	550
ak, English	550
" Amer White	600
" " Red, Black, Basket	550
" Live	600
ine, Amer White	450
" " Yellow	500
" " Pitch	550
" Memel	450
plar	550
ron	700
pruce	450
" Black	350
ycamore	500
amarack	400
alk	750
Walnut	450
Willow	350
METALS.	
rass	850
ron, cast, 1500 to 2700....average	2100
" " common pig	2000
" " castings from pig	2300
" " employed in our tables	2025
" " for castings 2½ or 3 *ins thick*	1800?
ron, wrought, 2200 to 2600....av	2500
Wrought iron does not break;	

Transverse Strength.	Lbs.
but at about the average of 2500 lbs its elasticity is destroyed. So also with the loads for steel.	
Steel, hammered, 5000 to 7000, av	
" rolled " "	6000
STONES, ETC.	
Blue stone flagging, Hudson River	125
Brick, common, 10 to 30..average	20
" good Amer pressed, 30 to 50 ....average	40
Caen Stone	25
Cement, Hydraulic, English Portland, artificial 4 months old	20
" " Roman, of Scotland	8
" " U. States, about	12
" *Trass*, sand, slacked lime in powder; 1 measure of each, 1 year old	18
" *Puzzuolana*, instead of Trass in the above	18
The following hydraulic cements were made into prisms, in vertical moulds, under a pressure of 32 lbs per cub inch, and were kept in sea water for 1 year.	
Portland Cement, English, pure, 1 year old	32
Roman Cement, Scotch, pure	10
American Cements, pure, av about	15
Granite, 50 to 150....average	100
" Quincy	100
Glass, Millville, N. Jersey, thick flooring	170
Mortar, of lime alone, 60 days old	10
" 1 measure of slacked lime in powder, 1 sand	8
" 1 measure of slacked lime in powder, 2 sand	7
" 1 measure of slacked lime in powder, 4 sand	5
These are of course rude averages, as indeed all such constants must be; for much depends on the quality of the lime and sand, and the care with which they are mixed.	
Oolites, or Roe stones. These do not occur in the United States. 20 to 50....av	35
Sandstones, 20 to 70 ....average	45
" Red of Connecticut and New Jersey	45
Slate, laid on its bed, 200 to 450, av	325

Art. 11. The following are general facts respecting the breakg loads of a uniform beam of any form of section. Calling the breakg load,

When the beam is firmly fixed at one end, and loaded at the other 1
Then when so fixed, and uniformly loaded, it will be..... 2
When so fixed, and loaded at the center, ... 4
When merely *supported* at both ends, and loaded at the center .. 8
" " " " " " and uniformly loaded 8
Firmly *fixed*, or tightly *confined* at both ends, and loaded at the center .. 16
" " " " " " and uniformly loaded. 16

By some authorities the last two are given at 6 and 12. The term "fixed" in such cases is rather indefinite Prof. De Volson Wood gives 12 for the last.

Rem. 1. When one beam of any form of section is 2, 3, or 4, &c, times as long, broad and deep, as another, its weight will be 8, 27, or 64, &c, times as great, or as the cubes of the linear dimensions; but its breaking load will be only 4, 9, or 16, &c, times as great; that is, the strengths of similar beams are to each other as the *squares* of their respective linear dimensions. But in these breakg loads are included the weights of the beams themselves; one-half of which must be deducted when load is all at the middle of the beam; or the whole of it when equally distributed. When beams are of the moderate dimensions usually employed in buildings, their own weight is usually so small in comparison with their loads, that at times it may safely be neglected; but as they become longer, since their weight increases more rapidly than their strength, it at last constitutes too important an item to be overlooked; for the beam may become so great as to break under its own weight although proportioned precisely like a small one which may safely bear many times its own weight.

When a square beam is supported on its edge, instead of its flat side (or in other words has its diag vert), it will bear but about 7/10ths as much breakg load.

The deflections or bendings of beams (see p 196) are directly in proportion to the load, and to the cube of the length; and inversely to the breadth and to the cube of the depth.

Rem. 2. It is very important to remember that a beam will bear a much load placed upon it in small amounts at a time; or (in case it is all applied at once) if its pres be allowed to come upon the beam very gradually, than if it come upon it suddenly, even without any jarring, and without any previous pressure. When applied in small amounts, or gradually, the bending takes place with slight momentum; but when applied all at once, the great load immediately and rapidly to the full extent of the bending due to the load. In so doing acquires a momentum which carries it still further; thus producing which authorities maintain to be just twice as great as in the former case. A train coming very rapidly upon a bridge, presents a condition intermediate the two. The tables of constants always suppose the load to be applied gradually, but as this is frequently not the case in practice, an allowance must accordingly.

Art. 12. To find the quiescent breakg load of a square or rectangular beam B, Fig 4, of any [illegible] at both ends, and loaded at the cen[illegible]

Rule. Mult together the square of its depth [illegible] the coef from the table p. 185. Div the [illegible] supports, in ft. The quot will be the req[illegible] half the wt of that portion of the beam [illegible] must be deducted in order to get the [illegible]

Ex. What will be the cer[illegible] 15 ins wide, 10 ins deep, a[illegible]

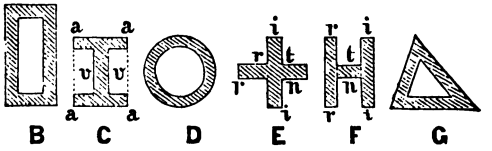
$$\frac{\text{Depth}^2 \text{ in ins} \times \text{Breadth in ins} \times [illegible]}{\text{Clear leng[illegible]}}$$

load reqd.

But we mu[illegible]

S OVAL, OR ELLIPTIC, first find the load for a rectangular beam, whose
:ively equal to the two diams, and mult it by .6
S OF WOOD, AND TRIANGULAR, and its base (whether up or down) hor,
akg load for a rectangular beam, whose breadth is equal to the base;
ual to the perp height of the triangle; and take one-third of the
roximation. When the edge is down, the ends must rest in trian-
n the supports: otherwise, they will be crushed when loaded. See

th such cross-sections as Figs A to G, the following rude rules of
ı be preferred to more intricate ones, being sufficiently approximate
poses, and for any material. See near end of Art 24.
Figs A. B, D, G, or for an ellipse, (each one supposed to be of equal
ghout,) first find the load for a solid beam of the same size and shape.



f a beam of the size of the hollow part. Subtract the last from the
f the remainder.
and bottom being of equal size,) first find for a rectangular beam
r two beams corresponding to the two hollows *v v*. Subtract these
t. Take ¾ of the remainder.
d for three separate beams *r r*, *i i*, *t n*, and add them together. Take
nany other forms of beams in common use, do not admit of any
roximate rules. The writer knows of no alternative in such cases
nent with a model made of the given material, and thus find the
nt, as directed in Art 9. See Remark, Art 29; also Art 24, &c.
Phœnix, and for Cooper & Hewitt's beams, see Arts 37, 38; and for
st-iron ones, Art 35. p 208.

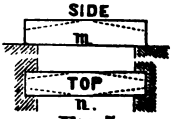


Fig 5

this case we may remove ¼ part of the solid square, or rectangular beam, without reaking strength, although it will bend more. remain uniform, and the deduction be made he depth, as shown by the dotted lines at *m*, es should, theoretically, be curves, but the best in practice. Or the depth may remain breadth be reduced, as shown by the dots at view of the beam. Theoretically, the dotted meet at the ends of the beam; but in prac-ot generally leave sufficient material at the ends for the beam to ly.

s of beams are rarely made when they are iron ones much expense may be saved

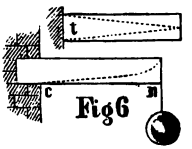


Fig 6

WHEN THE LOAD IS ALL AT ONE END OF THE HOR
WHILE THE OTHER END IS IMMOVABLY FIXED,
to be at the center, and calculate it by the
Then div the result by 4.
e lower side of the beam may be cut away
parabola, as shown by the dots. To draw
Parabola." Or the depth may be left uni-
es be cut away, as shown by the dots at *t*, which is a top view.

When the load is equally distributed along the length of a horizontal beam, t both ends, as in Fig 7, instead of be-
t the center, assume it to be at the center,
cisely as in the foregoing rule, Art 12.
load by 2. But in this case the wt of the
th of the beam *is to be* deducted for the

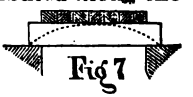


Fig 7

l be the equally distributed breakg load of the beam of sandston
le? *Here the center* breakg load has already been found to

5625 lbs; and 5625 × 2 = 11250 lbs, the reqd distributed load. From this subtract the wt of the entire 12 feet clear length of beam, or 1750 lbs; and the rem, 9500 lbs is the neat extraneous breakg load. About $\frac{1}{10}$ part of this, or 950 lbs, is quite as much as should be trusted upon so variable and treacherous a material as red sandstone.

REM. 1. A beam requires twice as much breaking load, *equally distributed*, as it will at its center. In this case the breakg strength of the beam will not be diminished if the top be cut away in the form of a true semi-ellipse, as shown by the dots in Fig 7. Or if the depth must be kept uniform, the sides may be trimmed to two parabolas *o c o*, *o t o*, Fig 8. The mode of drawing these figs to any span and height will be found under their respective heads.

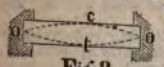


Fig 8

REM. 2. WHEN THE LOAD IS UNIFORMLY DISTRIBUTED ALONG THE ENTIRE CLEAR LENGTH, *y g*, Fig 9, OF A HOR RECTANGULAR BEAM FIRMLY FIXED AT ONE END ONLY, assume it to be at the center, as in Art 12, and calculate it by the rule in that Art. Then div the result by 2.

Fig 9

In this case, theoretically, we may cut off one-half the projecting part *y o* of the beam, as by the dotted line *y o*, without diminishing its breakg strength. But in practice it will merely be advisable to reduce it to a mere thin edge at *o*. Or the depth *c s* of the beam may be left uniform, and the sides be cut away as shown by the two semi-parabolas *a c*, *a c*, at *t*, which is the top of the beam. If *a c*, *a c*, be even made straight, instead of parabolas, it is plain that there would still be a considerable saving of expense, if the beam is of iron.

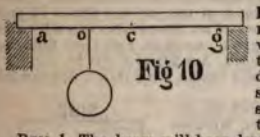


Fig 10

Art. 14. When the entire breakg load is applied at any point *o*, Fig 10, not at the center; first find by Art 12, what would be the center breakg load; and deduct half the wt of the beam. Then mult together the two dists *o a* and *o g*, in feet. Call the prod *a*. Also square half the clear length, *c g* or *c a*. Call this square *b*. Then mult the center load by *b*, and div the prod by *a*; the quot will be the reqd load.

REM. 1. The beam will bear less at its center than at any other point; and the breakg load at the center, is to that at any other point, *inversely* as the square of half the length is to the rectangle of the two parts *o a* and *o g*.

REM. 2. This beam will bear to be reduced, as at *m* and *n*, Fig 5; except, that instead of reducing from its center, as in Fig 5, we must do so from where the load is applied.

Art. 15. When the breakg load is applied at diff points between *a* and *g*, find the cen of grav of the load, and consider that as the single point at which it is all applied; then proceed as in the preceding Art.

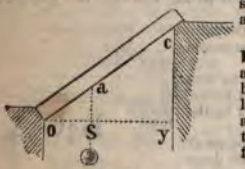


Fig 11

Art. 16. When the beam, instead of being hor, is inclined, as in Fig 11, in any of the foregoing cases, the *hor* dist *o y* must be taken as its span, instead of the actual clear length *o c*; and *s o*, *s y* instead of *a o* and *a c*. This applies also to beams fixed at one end, and whether the inclination is upward or downward from the fixed end.

NOTE. The quantity of material in inclined beams may be reduced, in the same manner as in hor ones.

Art. 17. Triangular beams of wood, according to Barlow's experiments with pine, require about $\frac{1}{6}$ greater breakg loads with the base up, than when it is down. Or with the base down, about $\frac{1}{6}$ less than when up. Tredgold considers them about equally strong in either position; and that to find the center breakg *load, we may first calculate* it by Art 12, as if the beam were a rectangular one with *the same base and perp height* as the triangle; and take $\frac{1}{4}$ of the result. Hence *the triangle is not an economical shape for a beam*; for with only $\frac{1}{3}$ the strength *a rectangular one, it has half as much material.*

Hodgkinson's experiments with CAST-IRON beams of triangular section, base

made the breakg loads equal to $\frac{1}{2}$ of those of rectangular bars, as in wood. Rennie's experiments give about the same proportion, with the base up; but with the base down, he made the strength nearly twice as great, or about $\frac{6}{10}$ that of a rectangular beam of the same width and vertical height. The comparative strengths in the two positions will vary in diff materials, inasmuch as it is affected by the comparative resistances which any given material presents to tension and compression. Within the limit of elasticity the beam will be equally strong, whether the edge or base be up; and will bend equally in either case; so also with the Hodgkinson, or any other form of beam.

Art. 18. To find the side of a square hor beam supported at both ends, and reqd to break under a given quiescent center load.

RULE. Mult the clear bearing in ft. by the given breakg load in pounds. Div the prod by the corresponding constant p 185. Take the cube root of the quot. This cube root will be the reqd depth or breadth of the beam, approximately, in ins. When the size of the beam is so great that its wt must be taken into consideration, increase either its breadth, as directed, in Remark, Art 21; or its depth, as per Art 22.

The breakg, or the safe load of a square beam, if mult by .6, will give that of a solid cylinder, whose diam is equal to a side of the square one.

Art. 19. When the beam is reqd to bear its center load safely, mult the given safe load by the number of times it is exceeded by the breakg load. Then find, by Art 18, the side of a square beam to *break* under this increased load. The beam thus found will evidently be approximately the *safe* one for the actual load; exclusive, however, of the wt of the beam. When this must be included, increase the breadth, by Remark, Art 21.

If the load is equally distributed, first div it by 2, then proceed precisely as before.

Art. 20. When the beam is cylindrical, and reqd to break under its center load, to find its diam, mult the load by 1.7, and by Art 18 find the side of a square beam, to break under this increased load. The side thus found will also be approximately the reqd diam.

If the load is to be borne safely, first mult it by the number of times it is to be exceeded by the breakg load. Then mult the prod by 1.7, and proceed precisely as before.

REM. 1. In neither case, however, is the wt of the beam itself included. When this is necessary, first find the approximate diam as before. Then calculate the wt of a beam having this diam. Add this wt to the given center load, in either case; and with this increased center load, repeat the whole calculation. The resulting diam will be the required one *very* approximately, but still a mere trifle too small.

REM. 2. IF IN EITHER CASE THE LOAD IS EQUALLY DISTRIBUTED, first take one-half of it as being a center load, and with this proceed precisely as before.

Art. 21. To find the breadth of a hor rectangular beam, supported at both ends, to break under a given quiescent center load; mult the center load in lbs by the span in feet. Mult the square of the depth in ins by the constant p 185. Div the first prod by the last. The quot will be the breadth approximately. Calculate the wt of a beam having this breadth. Then say, as the center load is to *half* this wt, so is the breadth found, to a new breadth to be added to it. It will still be somewhat too small, owing to the neglect of the wt of the breadth last added. This may readily be found, and its corresponding breadth added.

REM. 1. IF THE GIVEN LOAD IS TO BE SUPPORTED SAFELY, (without any regard to the amount of deflection,) first mult it by the number of times it is exceeded by the breakg load.

REM. 2. If in either case the load is EQUALLY DISTRIBUTED, take half of it as if a center load, and proceed precisely as before.

Art. 22. To find the depth, when the breadth is given, mult the load in lbs by the span in feet. Mult the breadth in ins by the constant p 185. Div the first prod by the last; take the sq rt of the quot for an approximate depth. Calculate the wt of a beam having the depth just found; add *half* of it to the given center load, and with this new load repeat the whole calculation; for a more approximate depth, but still somewhat too small, owing to the neglect of the wt of the depth last added. We may *find this*, and repeat the whole calculation, or we may merely *increase the breadth by Art 21.*

REM. *If the load is to be borne safely*, or if it is equally distributed, see Remarks Art 21.

Art. 22½. To find the safe dimensions to be given to a re ngular beam of given span, supported at both ends, a

which is at the same time exposed both to a transverse strain and to a longitudinal tensile or pulling one. This is the case of the lower chords of a bridge, when they support floor-beams intermediate of the web members; or the tie of a roof, &c, if loaded. See footnote p 191.

RULE. If the transverse strain is distributed either equally or unequally along the beam, reduce it to the equivalent load at the center. Assume any probable convenient *depth* for the beam in inches. Then,

$$\text{Reqd safe area of beam in sq ins} = \left(\text{Total tensile strain in lbs} + \left(\frac{\text{6 times the center load in lbs} \times \text{Half span of beam in ins}}{\text{Twice the depth in ins}}\right)\right) \div \text{Safe tensile strength in lbs per sq in.}$$

Or in words, mult the center load in lbs by 6. Mult the prod by half the span in ins. Div this last prod by twice the depth in ins. To the quot add the total tensile strain or pull in lbs. Div the sum by the safe tensile strength of the material in lbs per sq inch.

Ex. A wrought-iron beam of 10 ft, or 120 ins span, sustaining a longitudinal pull of 200000 lbs, and at the same time an equally-distributed transverse load of 100000 lbs. This last will be equivalent to 50000 lbs at the center of the beam. Assume say 12 ins as the *depth* of the beam; then if we also assume say 10000 lbs per sq inch as the safe tensile strength of the iron, we have

$$\text{Area of beam in sq in} = \left(200000 + \left(\frac{6 \times 50000 \times 60}{12 \times 2}\right)\right) \div 10000 = \frac{200000 + 750000}{10000} = \frac{950000}{10000} =$$

$= 95$ sq ins. Since the area is 95, and the depth 12, it is plain that the breadth must be $\frac{95}{12} = 7.92$ ins.

REM. The safe tensile strains per sq inch can be deduced from the breaking ones given in Arts 4, 5, and 6.

Art. 22½. To find the safe dimensions to be given to a rectangular beam of given span, supported at both ends, and which is at the same time exposed to a transverse strain, and to a longitudinal compressive one. This is the case of a rafter supporting purlins; or of the upper chords of a bridge, when they support floor-beams placed along them between the web members. See footnote p 191.

RULE. Reduce the transverse load, if distributed, to an equivalent center one. Assume any probable *depth* for the beam in ins. Then first regarding the beam only as a rectangular pillar, reqd to sustain safely the compressive strain alone, consider its depth to be its *least* side; and find by the formula for rectangular pillars, in Rule 10 of "Strength of Iron Pillars," (if of wrought or cast iron; or by Mr. Smith's rule, page 238 of "Wooden Pillars," if of wood,) how much would be its *safe* load per sq *inch*, and call this safe load C. In using said formula, the *area* must now be omitted from it, being as yet unknown. Having found C, then

$$\text{Reqd safe area of beam in sq ins, approx} = \left(\frac{\text{Total compressive strain in lbs}}{C}\right) + \left(\frac{\text{6 times center load in lbs} \times \text{Half span in ins}}{\text{Twice the depth in ins} \times \left(\frac{C + \text{safe tons strain per sq in}}{2}\right)}\right)$$

And the area thus found, when div by the assumed depth, will give the breadth of the beam approximately enough for practice, unless said breadth turns out to be less than the assumed depth. If this should be the case, it indicates that C has not been found correctly; because, if the depth which was merely *assumed* to be the least side is not *actually* so, the formula for pillars will give C too great a value. In this case we must use the breadth thus found as being the least side of the pillar, and with it go over the calculation for another C. With this new C, proceed again as per above formula.

Ex. A wrought-iron beam of 10 ft, or 120 ins, clear span, is to sustain a longitudinal pressure of 200000 lbs, besides a uniformly distributed transverse load of 100000 lbs. This last will be equivalent to 50000 lbs at the center. Now assume say 12 ins for the depth of the beam. Then, first considering the beam as only a rectangular pillar, and its depth 12 ins as its least side; find by Rule 10 of "Strength of Iron Pillars" its safe load in lbs per sq inch, thus,

$$\text{Breaking load as a pillar, in lbs per sq inch} = \frac{36000}{1 + \left(\frac{120^2}{36000 \times \frac{12^2}{12}}\right)} = \frac{36000}{1 + \left(\frac{14400}{36000 \times 12}\right)} = \frac{36000}{1.0333} = 34839 \text{ lbs.}$$

If say $\frac{1}{5}$th part of this is considered safe, we have $C = \frac{34839}{5} = 6968$ lbs. Hence, *according to the above formula, we have*

afe
im $=\left(\frac{200000}{6968}\right)+\left(\frac{6\times 50000\times 60}{24\times\left(\frac{6968+10000}{2}\right)}\right)=\frac{200000}{6968}+\frac{18000000}{202616}=28.7+88.4=117.1$ sq ins.

e area being 117.1 sq ins; and the assumed depth 12 ins; the breadth must e $\frac{117.1}{12}=9.76$ ins. But we see that this breadth is *less* than our assumed 12 ins; therefore we have got our pillar, or C, too great; and must recal- another C, using 9.76 ins as an approximate least side of the rectangular tead of 12 ins. By this new process we get

g load of s per sq in $=\frac{36000}{1+\left(\frac{12^2}{36000\times\frac{9.76^2}{12}}\right)}=\frac{36000}{1+\left(\frac{14400}{36000\times 7.94}\right)}=\frac{36000}{1.0504}=34273$ lbs;

of this, (as before,) or $\frac{34273}{5}=6855$ lbs = the new C. Hence, by formula,

fe area sq ins $=\left(\frac{200000}{6855}\right)+\left(\frac{6\times 50000\times 60}{24\times\left(\frac{6855+10000}{2}\right)}\right)=\frac{200000}{6855}+\frac{18000000}{202200}=29.2+89=118.2$ sq ins.

breadth $=\frac{118.2}{12}=9.85$ ins, approximately enough for practice. If, how- beam itself is very heavy, the breadth should be increased, thus: Say as the istributed transverse load, is to the wt of the beam, so is the breadth last a new breadth to be added to it. This, however, does not prevent the rule g merely an approximate one, not based upon strict theoretical principle.*

. Table of safe quiescent loads for horizontal rec- ular beams of white pine or spruce, one inch broad, orted at both ends, and loaded at the center.

e load is here assumed at $\frac{1}{6}$ of the breakg one; and the last at 450 lbs at the a beam 1 inch square, and 1 foot clear length between its supports. For porary purposes, $\frac{1}{2}$ part may be added to the loads in the table, thus hem equal to the $\frac{1}{4}$ of the breakg load. But in important structures, sub- ration, one $\frac{1}{4}$ part should be deducted from the tabular loads, thus reducing

eat loads, deduct $\frac{1}{2}$ the wt of the beam itself. See Rule, Art 12, p 186. The e however are the actual ones; the wts of the beams having been introcalculating them, by the rule in art 27, p 199. (Original.)

an 4 ft.		Span 6 ft.		Span 8 ft.		Span 10 ft.		Span 12 ft.		Span 14 ft.		Span 16 ft.		Wt. of 10 ft of
ad be.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def. ins.	load lbs.	def ins.	load lbs.	def. ins.	load lbs.	def. ins.	beam. lbs.
19	.39	13	.92	10	1.8	8	3.0	6	4.4	.....	.....	.....	.....	2
75	.22	50	.45	38	.82	30	1.3	25	1.9	21	2.7	19	3.7	4
170	.13	114	.30	85	.53	67	.84	57	1.3	48	1.7	42	2.3	6
300	.10	200	.22	150	.39	120	.63	100	.92	86	1.3	75	1.7	8
469	.08	312	.18	234	.31	187	.50	156	.72	134	1.0	117	1.3	10
675	.06	450	.15	337	.26	270	.41	225	.60	193	.83	168	1.1	12
919	.06	612	.12	460	.22	367	.35	306	.51	262	.70	230	.93	14
200	.05	800	.11	600	.19	480	.31	400	.45	343	.61	300	.81	16
520	.04	1014	.10	760	.17	607	.27	507	.40	434	.54	380	.72	18
875	.04	1250	.09	937	.16	750	.24	625	.35	536	.49	468	.64	20
270	.04	1514	.08	1135	.14	907	.22	757	.32	648	.44	567	.58	22
700	.03	1800	.07	1350	.13	1080	.20	900	.29	772	.40	675	.53	24
375	.03	2450	.06	1837	.11	1470	.17	1225	.25	1050	.34	918	.45	28
800	.02	3200	.05	2400	.10	1920	.15	1600	.22	1372	.30	1200	.40	32
075	.02	4050	.05	3037	.09	2430	.14	2025	.20	1736	.27	1518	.35	36
000	.02	5000	.04	3750	.08	3000	.12	2500	.18	2145	.24	1875	.31	40
075	.02	6050	.04	4537	.07	3630	.11	3025	.16	2593	.22	2268	.29	44
800	.02	7200	.04	5400	.06	4320	.10	3600	.15	3088	.20	2700	.26	48

(Continued on next page.)

gard to this rule, and that in Art 22¼, we would state that the 7.92 ins of breadth found r, is not enough for the beam *alone* with a safety of 6, *without* the pull. We suggest in- ose rules, *to use the following.* It will always give results more or less in excess, and *%. Namely: First find the dimensions* for a beam alone. Then those for a pillar, or for a *case may be*) *and add the two areas together.* This gives 8.68 ins, instead of the 7.92 of *22¼; and this cannot be much* in excess. For the 9.85 ins. of Art 22½, it gives show *panel-length of a bridge chord, if* loaded, is also a beam

Table, continued. (Original.)

Depth of beam. Ins.	Span 18 ft. load lbs.	Span 18 ft. def. ins.	Span 20 ft. load lbs.	Span 20 ft. def. ins.	Span 25 ft. load lbs.	Span 25 ft. def. ins.	Span 30 ft. load lbs.	Span 30 ft. def. ins.	Span 35 ft. load lbs.	Span 35 ft. def. ins.	Span 40 ft. load lbs.	Span 40 ft. def. ins.	Wt. of 10 ft. of beam. lbs.
6	150	1.4	135	1.8	108	2.9	90	4.5	77	6.5	67	9.2	12
7	204	1.2	184	1.5	147	2.5	122	3.9	105	5.8	92	7.6	14
8	267	1.0	240	1.3	192	2.1	160	3.2	137	4.6	120	6.4	16
9	338	.92	304	1.2	243	1.9	202	2.8	174	4.0	152	5.5	18
10	417	.82	375	1.0	300	1.7	250	2.5	214	3.5	188	4.9	20
11	505	.74	454	.93	363	1.5	302	2.2	259	3.2	227	4.3	22
12	600	.68	540	.85	432	1.4	360	2.0	308	2.9	270	3.9	24
14	817	.58	735	.72	588	1.2	490	1.7	420	2.4	367	3.2	28
16	1067	.50	960	.63	768	1.0	640	1.5	548	2.1	480	2.8	32
18	1350	.45	1215	.56	972	.90	810	1.3	694	1.8	607	2.5	36
20	1666	.40	1500	.50	1200	.79	1000	1.2	857	1.6	750	2.2	40
22	2017	.37	1815	.45	1452	.72	1210	1.1	1037	1.5	907	2.0	44
24	2400	.33	2160	.41	1728	.65	1440	.96	1234	1.3	1080	1.8	48
26	2817	.31	2526	.38	2018	.60	1684	.88	1449	1.2	1263	1.6	52
28	3267	.28	2940	.35	2352	.55	1960	.81	1680	1.1	1470	1.5	56
30	3750	.26	3375	.33	2700	.50	2250	.76	1928	1.1	1687	1.4	60
32	4267	.25	3840	.30	3072	.45	2560	.71	2194	1.0	1920	1.3	64
34	4817	.23	4335	.29	3468	.44	2890	.67	2477	.92	2167	1.2	68
36	5400	.22	4860	.27	3888	.43	3240	.63	2777	.86	2430	1.1	72

them to ⅓ of the breakg loads. This is especially necessary if the timber is not well seasoned. With the safe loads in this table, a beam may bend too much for many practical building purposes. When much bending is objectionable, see Table, p 204.

White oak and best Southern pitch pine, will bear loads ¼ greater.

For cast iron, mult the **loads** in the table by 4.5; and for **wrought** by 5.3. For these new loads, mult the **defs** by .4 for cast; and by .3 for wrought.

If the load is equally distributed over the entire clear length, or span, it will be twice as great as the center one. When more accuracy is reqd, half the wt of the beam itself must be deducted from the center load; and the whole of it from an equally distributed load. The wt of the beam, in the last column, supposes the wood to be but moderately seasoned, and therefore to weigh 28.8 lbs per cub ft.

Examples of the use of the foregoing Table—Ex. 1. What must be the breadth of a hor rect beam of wh pine, 18 ins deep, supported at both ends, and of 20 ft clear length between its supports, to bear safely a load of 5 tons, or 11200 lbs at its center? Here, opposite the depth of 18 ins in the table, and in the column of 20 feet lengths, we find that a beam 1 inch thick will bear 1215 lbs; consequently, $\frac{11200}{1215} = 9.22$ ins, the reqd breadth; for the strength is in the same proportion as the breadth.

Ex. 2. What will be the safe load at the center of a joist of white pine, 18 ft long, 3 ins broad, and 12 ins deep? Here, in the col for 18 ft, and opposite 12 ins in depth, we find the safe load for a breadth of 1 inch to be 600 lbs; consequently, $600 \times 3 =$ 1800 lbs, the load reqd.

Rem. Certain caution must be used with regard to this table. For instance, in placing very heavy loads upon short, but deep and strong beams, we must take care that the beams rest for a sufficient dist on their supports to prevent all danger from *crushing* at the ends. Thus, if we place a load of 6075 lbs at the center of a beam of 4 feet span, 18 ins deep, and only 1 inch thick, each end of the beam sustains a vert crushing force of $\frac{6075}{2} = 3037$ lbs. Now, ordinary timbers crush under 6000 lbs per sq inch; therefore, if this beam, only 1 inch thick, rested but one inch on each support, its base at each end would be but one sq inch in area; consequently, it would have a safety of but 2 against crushing; with a safety of 6 against breaking at the center under its load. To make it as safe against crushing at the ends, as it is against breakg under its center load, it must rest at least 3 ins at each end upon its supports. One end would not then crush under less than 18000 lbs, or 6 times what it has to sustain. To assure a safety of 6 against crushing at the ends, therefore, div the whole load by 2, to get what is supported at each end. Then div this half load by *1000; the quot* will be the least number of sq ins of base that should be allowed for *each end to rest* on. There is, however, very little probability of a beam being so *much loaded as* to endanger either crushing or shearing at its ends.

Again, although we speak of a center load, it is plain that in practice we cannot

.e term in its utmost strictness; otherwise the load would have to be nere knife-edge, at the *very* center of the beam. Now, in the instance attempted to sustain the center load of 6075 lbs upon such a knife- .t once cut the beam in two. If we even applied it along one inch of breadth of the top of the beam, it would probably cut into the wood; not be sufficiently safe unless we applied it along at least 6 ins of the ring it a base of 6 sq inches (the beam being but 1 inch thick) to rest prevent it from crushing into the top of the beam; as we before gave two ends, to prevent crushing at those points. If the load were ited along the whole beam, it might safely be twice as great as when n which case it would require the area of base at the ends to be dou- l require no precaution whatever against crushing the top of the ld then be applied along its entire 48 inches of length. These are lerations which the student must bear in mind in using such tables.

trength of hollow Beams.

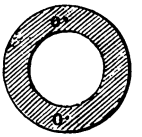


Fig.12

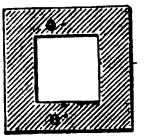


Fig.13

preliminary investigations construction of the Menai , a few experiments were :rength of hollow cast-iron ar, oval, square, and rectan- :tions, supported at both d at the center. The clear the supports was in every :hickness of metal in each area of *solid* cross-section ins. The mean depth *o o*, Fig 12, of the circular tube, 3½ ins; of , Fig 13, 2¾; of the oval, 4⅔; breadth, 2⅓; and of the rectangular n, 3⅔; breadth, 1.833 ins. From these experiments Mr. Edwin Clark, eer in charge, deduced the following constants, and rules for center

circular tubes, .95; oval, 1; square, 1.14; rectangle, .91. Then, first ı of the solid part of the cross-section in sq ins,

$$\textbf{breaking in tons} = \frac{\text{Area of solid in sq ins} \times \text{Mean depth, } o\ o, \text{ in ins} \times \text{Corresponding constant.}}{\text{Clear span in feet.}}$$

r beam, mean depth *o o*, 3½ ins; area of solid ring, 4.12 sq ins; clear re,

$$\frac{\text{Mean depth.} \times 3.5 \times \text{Const.} .95}{6 \text{ (length.)}} = 2.28 \text{ tons, or 5107 lbs, breakg load.}$$

s of the cylinder or tube is about $\frac{1}{10}$ of the diam; and as a mean roke with a center load of 2.287 tons, or 5122 lbs; span 6 ft. Hence milar tubes, the constant 475, to be used in the rule, Art 12; that is, load in lbs of cast-iron tubes with a thickness of $\frac{1}{10}$ of the outer $\frac{\text{(in ins)} \times 475}{\text{span in feet.}}$; supposing Mr. Clark's iron to have been of average

breakg load of 3 square beams was 2.152 tons, or 4820 lbs; of the rec- 2.3 tons, or 5152 lbs; and of the 6 elliptic ones, 3.207 tons, or 7183 lbs. :oing extraneous loads must be added half the wt of the beam itself.

ıumb, on p 187, agrees sufficiently well with all Mr. Clark's results, oval beams, in which said rule gives a breakg load of but ¾ that n.

eams of thin wrought iron were experimented on at the l for these Mr. Clark deduced the following constants, to be used with ıle for cast-iron ones:

or thin riveted tubes, circular, 1.74; oval, 1.85; rectangular, 1.96.
" welded tubes, " 1.09; " 1.27; " 1.51.

he following experiments on **riveted sheet-iron cylindri-** are *by Fairbairn*. *1st*. Cylinder 18 ft long; 1 ft outer diam; clear :*ness of iron* .037, or $\frac{1}{27}$ of an inch; wt of tube 107 lbs.

Center load. Lbs.	Def. Ins.	Center load. Lbs.
1360	.32	2368
1920	.41	2480
2114	.46	2592
2256	.60	2704

After bearing 2704 lbs. for 1½ minutes, failed by crushing at top.

2d. Cyl 16 ft 10 ins long; 12.4 ins outer diam; clear span 15 ft 7½ ins; iron .113, or full $\frac{1}{9}$ inch; wt of tube 392 lbs.

Center load. Lbs.	Def. Ins.	Center load. Lbs.
2000	.17	8000
4000	.34	10000
6000	.52	11440

With 11440 broke by the tearing of the bottom across the shackle-hol the load was suspended.

3d. Cyl 25 ft long; 17.68 ins outer diam; clear span 23 ft 5 ins; thick full $\frac{1}{16}$ inch; weight of tube 346 lbs.

Center load. Lbs.	Def. Ins.	Center load. Lbs.
1000	.12	5000
2000	.21	5280
3000	.30	5840
4000	.40	6120

With 6400 broke at bottom; 25 ins from center, by tearing through th

4th. Cyl 25 ft long; 18.18 ins outer diam; clear span 23 ft 5 ins; thic scant ⅛ inch; wt of tube 777 lbs.

Center load. Lbs.	Def. Ins.	Center load. Lbs.
2000	.15	10000
4000	.30	12000
6000	.43	13000
8000	.59	14240

Broke through the rivet-holes 3 ft 3 ins from center, after sustainin half a min.

5th. Cyl 32½ ft long: 24.2 ins outer diam; clear span 31 ft 3 ins; thick scant $\frac{1}{16}$ inch; wt of tube 1005 lbs.

Center load. Lbs.	Def. Ins.	Center load. Lbs.
2000	.05	8000
4000	.20	10000
6000	.36	10900

Br
r

Fig 14

The tubes were composed of sheets about 2½ ft wide that a single sheet sufficed to form the entire circumf They were united by double-riveted lap-joints, Fig. 32 were placed on a platform, supported by a rod *r*, Fi passed through a hole *h* in the bottom of the tube *s*. attached at its upper end to a block of wood *w*, ro lower surface, so as to fit the tube.

Circular blocks of wood were fitted into the ends of prevent them from crushing at those parts under the the ends rested upon blocks hollowed out to correspon cylindrical shape, to a depth equal to about ¼ part of

REM. **The writer understands Prof. Moseley to say** lowing rule is applicable to solid beams of any form of cross-section wl it certainly does not agree with experiment in all cases.

From the load so found, one-half the wt of the beam itself must be c the whole of it, if the load is equally distributed.

$$\textit{Center Breakg load, in lbs.} = \frac{\text{Moment of inertia of the cross-section of the beam, with respect to a neutral axis } G\,G\text{, passing hor through its cen of grav } O. \times \text{Constant for rupture o of which the beam}}{\text{Dist o } g \text{ from neutral axis; (and at right angles to it) to the fibre } g\text{, the farthest one from the axis, in ins.} \times \text{Clear span of beam, in ins}}$$

either 1, ½, ¼, ⅛, or $\frac{1}{16}$, according to the arrangement of the d, as referred to in Art 11.

T FOR RUPTURE," of Rankine, is 18 ants for center ven p 185, with he states, that *st-iron beams* it e constant in p 9 times, the wri- ow; nor where between solid commences.

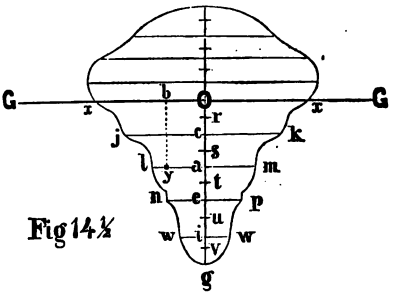


Fig 14½

ion of the axis G G, may ng out a correct long, of the sec- ick paper or tin, over a straight at which it bal- en this has been

y, in ins, to the farthest fibre, (which may be either above or below o the shape of the Fig,) can be measured in ins.

OF INERTIA" of the cross-section, with respect to the neutral axis G G, hich results from adding together the prods found by mult together all area of each fibre, as *y*, of the section, by the square of its dist right angles to G G. Such multiplication cannot be performed by etic, but an approximation, sufficiently close for all practical pur- ly be made thus: Both above and below G G, and parallel to it, *m*, &c, dividing the section into narrow strips. If these lines are subsequent calculations will in some cases be easier; but otherwise whether they are so or not. If they are drawn no closer together, *the size of the figure*, than in Fig 14½, the approximation will be practical purposes. The closer they are the more accurate will be however close they may be, it will always be a *trifle too small*. Begin ea in sq ins, of the first strip *x x j k*, below G G. Mult this area by dist, *O r*, to the center of the strip, in ins. Then proceed to the next d its area; and mult it by the square of the dist *o s* to its center. Do ach strip. Add all the prods together, and if the section has the , and position above G G as below it, (as would be the case with a , or circle,) mult their sum by 2. The prod will be the reqd moment. ; 14½, the section above G G differs from the portion below it, we into strips, and proceed as with the lower part. The sum of *all* the sides of the neutral axis will then be the moment. We add modes alculation, the moments of inertia of a few well-known figures. The each separate fibre, as *y*, for instance, of the cross-section, opposes a load *within the limits of elasticity*, is in proportion to its distance right angles to the hor line G G passing across the section, and er of gravity O. Having first found the moment of inertia for a fig hat of another precisely similar, whether larger or smaller, may be lt together the breadth, and the cube of the depth, of the one found. n, and cube of the depth, of the one to be found. Then as these two ne another, so are the respective moments.

Moment of inertia

a hor axis through the center of gravity of the cross-section.

l square, her a side or nal vertical, ... $\frac{\text{The square of the square of one side}}{12}$

id rectangle, sides vertical, ... $\frac{\text{Breadth} \times \text{Cube of depth}}{12}$

w square, *er a side or al vertical,* ..... $\frac{\text{The square of the square of an outer side} - \text{The square of the square of an inner side}}{12}$

A hollow rectangle, any 2 sides vertical, ... $\dfrac{\text{The outer breadth mult by the cube of the outer depth} - \text{The inner breadth mult by the cube of the inner depth}}{12}$

A solid circle, ... Square of square of rad $\times$ .7854.

A solid semicircle, approximately. ..... Sq of sq of rad $\times$.11.

The dist of the center of gravity of a semicircle, from the centre of the circle, equal to $\dfrac{\text{rad}}{2.3562}$

A hollow circle, approximately, ... $\left(\text{Sq of sq of outer rad} - \text{Sq of sq of inner rad}\right) \times .7854.$

A solid triangle, calling any side the base, ... $\dfrac{\text{Base} \times \text{cube of perp height}}{36}$

The formula for this last is, if we mistake not, given 4 times too great by Mosel[illegible] in at least two places.

Ex. What is the center breakg load of a solid cast-iron beam 4 ins square, and [illegible] ft, or 72 ins, clear span, supported at both ends?

Here the moment of inertia is $\dfrac{4^3 \times 4^3}{12} = \dfrac{16 \times 16}{12} = \dfrac{256}{12} = 21.333$. The co[illegible] stant for rupture, or 18 times our constant for cast iron on p 185 is 2025×18 [illegible] 36450. Since the beam is supported at both ends, and loaded at the middle, m, (see Art 11) is ¼. The dist *o g* of the farthest fibre from the neutral axis must in a square be equal to ½ of one side; consequently it is here 2 ins. The clear span is 72 ins. Hence,

$$\text{Breakg load} = \frac{\text{Mom of In} \times \text{Con for Rup}}{m \times \text{Dist } o\,g \text{ of farthest fibre} \times \text{span}} = \frac{21.333 \times 36450}{\frac{1}{4} \times 2 \times 72} = \frac{777588}{36} = 21600 \text{ lbs} = 9.64 \text{ tons.}$$

By our table, p 191 a beam of average cast iron, 4 ins deep, 1 inch broad, and 6 [illegible] span, breaks with 2.41 tons; consequently, four such, or one 4 ins square, would break with $2.41 \times 4 = 9.64$ tons; thus confirming the accuracy of the foregoing. Applied in the same way to solid cylinders, the result corresponds equally well with ex[illegible] periment and with our table on p 207. But with respect to Mr. Clark's *hollow* squares and cylinders, p 193, we find that this method gives breakg loads about 50 per ct *greater* than his actual ones; and with regard to the beam *l*, in p 210, it gives a breakg load of about 40½ tons, or about 20 per ct *less* than the actual ones of 50 [illegible] 54 tons. If it were correct, a Hodgkinson beam should be equally strong whether the large or the small flange were uppermost *

Art. 26. Deflections, or bendings of beams, under their loads. The foregoing relates to the *strength* of beams, or their resistance to *breakg;* the following to their *stiffness*, or resistance to *bendg*. The two follow very diff laws.

It is with the defs *within the elastic limit* that the engineer is chiefly interested. When the load exceeds that limit, the beam may be said to begin to break. Within that limit,

If the deflection of a beam *supported* at both ends and loaded at the center be called	1.
Then that of the same beam, with the same load uniformly distributed, will be.....	.625, or ⅝
Firmly *fixed* at both ends, and loaded at the center, by Moseley	.2, or ⅕
" " " " " uniformly loaded.....	.125, or ⅛

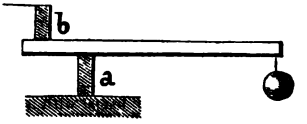
Other writers make these last two defs much greater, or about .66, and .417.

Fixed at one end, and loaded at the other.....	32.
" " " uniformly loaded.....	12.
Partially fixed at one end, and loaded at the other, by the writer's experiments with various woods, about.....	24.

The pieces were 6 feet long, and 1 inch square. Unsupported lengths, 5 ft. The loads, exclusive of the weights of the beams, (which he did not take into consid-

* *It is possible that we have misunderstood Mr. Moseley: indeed, in view of the obscure style of expressing themselves, and the innumerable typographical errors with which their learned hieroglyphics abound, it is almost impossible for a merely practical man to assure himself that he understands anything in the leading authors on mechanics and engineering. To advertise such with such claims to being adapted to the use of practical men, is sheer swindling. In the writer's opinion, superficial tions can furnish satisfactory practical rules on this and kindred subjects.*

10 lbs. The fixed end of the beam merely rested against the two pieces *a* hich were 1 inch apart vert. Wt of beams 1 to 1⅛ lbs. The term "fixed," ed to the ends of beams, is very in- and theoretical fixing can rarely be in practice.



ttent to which a beam may bend ven a perfectly safe load, may be at for many purposes in every- ctice. Tredgold and others as- at is order not to be observed, it may not cause the plaster of ceilings to crack, &c, a beam should not it its center more than the $\frac{1}{480}$th part of its span, or $\frac{1}{40}$th of an inch per s, if its span be 20 ft, it should not bend more than $\frac{20}{40}$ths, or ½ of an inch, also $\frac{1}{480}$th of 20 ft.

afts of wheels in machinery should not deflect more than half of this, nor a ore than about ⅓ of it, or say $\frac{1}{1500}$ of its span, or $\frac{1}{125}$ inch per foot, under est load.

all allude first to defs within the limits of safety, or of the elasticity of the nd afterward of those not exceeding $\frac{1}{480}$ of the span. After the elastic assed, the defs increase irregularly, and more rapidly than before; and the comes unsafe. As a general rule, the elasticity of a wooden beam is not for practical purposes, if the quiescent load does not *exceed* about ⅓ of the one. Within the elastic limits, the defs theoretically vary *directly* in pro- to the *load*, and also to the *cube* of the span, or clear length; and *inversely* rtion to the breadth, and to the *cube* of the depth. That is,

$$\left.\begin{array}{c}\text{Deflection within}\\ \text{elastic limits}\end{array}\right\} \begin{array}{c}\text{is in proportion to}\\ \text{(not equal to)}\end{array} \left\{\frac{\text{Load} \times \text{cube of span}}{\text{breadth} \times \text{cube of depth.}}\right.$$

ore, constants or coefs for the bendings of beams of diff shapes and mate- hin the limit of elasticity, may, like those of transverse strength, (see Art 9,) ly found by experiment. Thus, at the center of any rectangular beam, or upon supports at each end, place any load that is within its elastic limit, sure the resulting def in ins. Mult the wt of the span of the beam by l the prod to the neat load, for a total load. Mult together the total load d the *cube* of the span in feet. Also mult together the breadth in ins, and of the depth in ins. Div the first prod by the last one. Div the def by the he last quot will be the reqd constant for any rectangular beam of the same l quality of material, whether wood, metal, stone, &c. That is,

$$\left.\begin{array}{c}\text{The constant for}\\ \text{def within}\\ \text{elastic, or safe}\\ \text{limits of beam.}\end{array}\right\} = \begin{array}{c}\text{Def in ins}\\ \text{divided by}\end{array} \left\{\frac{\begin{array}{c}\text{Total load}\\ \text{in lbs}\end{array} \times \begin{array}{c}\text{Cube of span}\\ \text{in feet}\end{array}}{\begin{array}{c}\text{Breadth}\\ \text{in ins}\end{array} \times \begin{array}{c}\text{Cube of depth}\\ \text{in ins.}\end{array}}\right.$$

d to the experimental neat load. the .625, or ⅝ of the wt of the clear span am itself, because the wt of the beam equally distributed throughout its o aids in producing the def; and it does so to the same extent that ⅝ of it , if collected at the center of an imaginary beam having the same strength ont as the real one. but of no wt except at its very center. Therefore, in the constants for def to beams intended for actual use, we must not omit ⅝ of the wt of the span, to the intended center load, for an equivalent total ad, before making the calculations for def. The weights of *similar* beams beams *proportioned* exactly alike in every part, but of diff sizes) increase so ore rapidly than their clear spans, that although a small one may safely bear f many times its own wt, a much larger one will break down without any laving by experiment found the constant of def for any given material, the y similar beam of the same material, whether larger or smaller, and loaded nter, may be found thus:

$$\begin{array}{c}\text{Def}\\ \text{within safe}\\ \text{or elastic}\\ \text{limit in ins}\end{array} = \frac{\begin{array}{c}\text{Total equivalent}\\ \text{center load in lbs}\end{array} \times \begin{array}{c}\text{Cube of span}\\ \text{in feet}\end{array} \times \text{Constant}}{\begin{array}{c}\text{Breadth}\\ \text{in ins}\end{array} \times \begin{array}{c}\text{Cube of depth}\\ \text{in ins.}\end{array}}$$

limit of elasticity of a beam of any particular form, aterial, is determined by experiment with a similar beam, as in the case ants for breakg loads. *&c.* Thus, load a beam at the center, by the careful *addition of small equal loads*; carefully note down the def that takes place *ne mins (the more the better)* after each load has been applied; in order t *when the defs begin to increase* more rapidly than the loads; for when th *s, the load for elastic limit* has been reached. See Remarks, Art 9 and

It is not the defs of the *whole* beam that are to be noted, but those of its clear span only. Several beams should be tried when possible, in order to get an average constant; for even in rolled iron beams of the same pattern, and same iron, there is a very appreciable diff of strengths and defs.

Then, to get the constant, so as to apply it to similar beams, using the *total* load applied during the equal defs, including ½ wt of beam,

$$\text{Constant for greatest center loads within elas limit} = \frac{\text{Span in feet} \times \text{Total load in lbs}}{\text{Breadth in ins} \times \text{Square of depth in ins}}$$

Constants for greatest center loads within limits of elasticity, may be had near enough for common practice by taking one-third of the breaking constants in the table on p 185; except those for rolled iron and steel. It is assumed always that the load is not subject to jars or vibrations; these would increase the defs.

REM. Within the limits of elasticity, a beam of irregular shape, such as a T, or Hodgkinson beam, a triangle, &c, will bend to the same extent, whether its top or its bottom be uppermost.

To find the greatest center load that a given beam, supported at both ends, can sustain without exceeding its elastic limit,

$$\text{Greatest center load within elas limit of beam in lbs} = \frac{\text{Breadth in ins} \times \text{Square of depth in ins} \times \tfrac{1}{3}\text{ of the constant on page 185.}}{\text{Span in feet}}$$

In this manner we have prepared the following ones for the rolled iron Phœnix beams. We will remark that, in practice, it is frequently difficult to ascertain with precision when, or under what load, the defs actually do begin to increase more rapidly than the successive loads. For although *theoretically* the defs are equal for equal loads, until the elastic limit is reached, yet in *practice* they are only nearly equal, up to that point. This is owing to the fact that no material composing a beam is perfectly uniform throughout in texture and strength; so that instead of perfect equality of defs, we shall have an alternation of larger and smaller ones. Therefore some judgment is reqd to determine the final point; in doing which, it is better, in case of doubt, to lean to the side of *safety*, as we have done in the following instances.

Constants for greatest center loads within elastic limit, of rolled iron Phœnix, &c. beams, deduced from single experiments only. An allowance has been made for the wt of the beam.

7 inch, 20 lbs,	Phœnix, A, Fig 17. p 210	678
9 inch, 30 lbs,	"	617
9 inch, 50 lbs,	" B, Fig 17	503
12 inch, 41⅔ lbs,	"	574
15 inch, 51⅔ lbs,	"	390
15 inch, 66⅔ lbs,	"	464

Ex. What is the greatest center load for a 7-inch, 20 lbs Phœnix beam of 20 ft clear span, without exceeding its elasticity? Here,

$$\frac{B \times D^2 \times \text{const}}{\text{span in feet}} = \frac{3.25 \times 49 \times 678}{20} = \frac{107971}{20} = 5398 \text{ lbs, including about 200 lbs}$$

half wt of beam: or say 5200 lbs of neat load; or about 10400 lbs equally distributed. The deflection under the safe loads will be too great for some building purposes. See beginning of Art.

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* We have taken pains to render the method of finding all these constants perfectly plain to practical men. There is a great want of data on this subject, especially now that cast and wrought iron are coming into general use as beams, &c, of shapes whose strength can only be satisfactorily determined by experiment. Scientific authors so mystify even the most simple facts, as to render them entirely unintelligible to that class; and thus prevent them from doing good service to both science and art, by experimenting for themselves in cases of novelty or doubt. We do not know of a single book in the English language in which even these simple processes are explained in language that *can be understood* by one artizan in a thousand. The same remark applies to every subject treated *on by the leading authorities* in theoretical mechanics of the present day. Their books are mere *waste paper to any one not* well versed in the higher mathematics. Let us hope that one day a scholar *may appear with sufficient* brains to write on mechanics, as Tyndall has written on heat; one that *shall write, not for schools,* but for scholars.

Table of constants for the deflections, within the safe, or elastic limits, of hor rectangular beams, supported at both ends and loaded at the center. The timbers are supposed to be well seasoned; if not, the constant should be increased.

White oak	.00023 *	White pine	.00032*
Best southern pitch pine, and white ash	.00027 *	Ordinary yellow pine	
Hickory	.00016*	Spruce	
		Good straight-grained hemlock.	
		Ordinary oaks	

Average bar iron ..... .000015 *
Average cast-iron ..... .000025
Steel, rolled ..... .000010 *

Full and reliable experiments on the strength and deflections of the various steels are much needed.

It is evident that the stiffer the material is, the smaller will be its constant for *bending*. All these constants vary somewhat with the quality of the metal. The defs also of timber of the same kind, vary so much with the degree of seasoning, the age of the tree, the part it is cut from, &c, that the writer considers it mere affectation to pretend to assign constants for practical use, more nearly approximate than he has here done. They are averages deduced from his own experiments on good pieces, well seasoned; and the loads were allowed to remain on for months, instead of minutes, as usual. Every structure is more or less exposed to vibrations and jars, which in time increase the deflections. In several instances, our experimental timbers bore their breakg loads for months before they actually gave way. And in all kinds, less than $\frac{1}{50}$ of the breakg load produced in a few months a permanent set, or def.

The following are deduced from single experiments only. An allowance is made for the weight of the beam.

Rolled iron beams proportioned exactly as the 7-inch Phœnix beam,						A, Fig 17,	.0000303†
"	"	"	"	30 lbs, 9 inch,	"	"	.0000321†
"	"	"	"	50 " heavy 9 inch	"	" B, Fig 17,	.0000264
"	"	"	"	$41\frac{2}{3}$ lbs, 12 inch	"	"	.0000313†
"	"	"	"	$51\frac{2}{3}$ lbs, 15 inch	"	"	.0000365†
"	"	"	"	$66\frac{2}{3}$ lbs, 15 inch	"	"	.0000438

Art. 27. To find the def in inches, of a hor rectangular beam, supported at both ends, and loaded at its center, with any given load within its elasticity; mult the weight of the clear beam itself, in lbs, by the decimal .625. Add the prod to the given center load in lbs. Call the sum the total load. Mult together this total load, the cube of the span in ft, and the constant from the foregoing table. Also mult together the breadth in ins, and the cube of the depth in ins. Div the first prod by the last one.

Ex. What will be the def of such a beam of average white pine, 9 ins broad, 12 ins deep, 21 feet clear span, and weighing 450 lbs; with a neat center load of 1218.75 lbs?

Here first, $450 \times .625 = 281.25$ lbs. And $281.25 + 1218.75 = 1500$ lbs total load. Hence,

$$\frac{1500 \times 21^3 \times .\text{Const.}}{9 \times 12^3} = \frac{1500 \times 9261 \times .00032}{9 \times 1728} = \frac{4445.2}{15552} = .286 \text{ inch; reqd def.}$$

Rem 1. **When the load is all at one point not at the center,** as at *o*, Fig 10, mult together the two dists *o a*, *o g*, from the load to the points of support. Mult the prod by 4. Div the result by the clear span. Use the quot as if it were the span, in the last rule. The wt of the beam is not here taken into account; it will of course somewhat increase the def.

* Averages near enough for ordinary practice by the writer's own trials. Calling the def of a steel beam 1, that of a similar wrought-iron one will be 1.5, and that of a cast-iron one 2.5; calling that of a cast-iron beam 1, that of a wrought-iron one will be .6, and of a steel one .4. Or if that of a wrought-iron one be 1, cast-iron will be 1.67, and steel .67; all as average approximations.

† We believe that these four beams have the same proportions, as nearly as the process of making them will admit of; so that .000033 may be taken as a near enough average for all four. As before remarked, extreme accuracy must never be expected in such matters. Two halves of the same chemical beam will often *give differences greater* than this.

REM. 2. **When the neat load is equally distributed along the span,** instead of all being at the center, then for an equivalent total center load, add together the neat load, and the entire wt of the clear span of beam; and mult the sum by the dec .625. With the resulting equivalent center load, proceed precisely as in the foregoing example.

EX. The def of the foregoing beam of white pine, 9 ins broad, 12 ins deep, 21 feet span, weighing 450 lbs, and bearing an equally distributed load of 1218.75 lbs?

Here first $450 + 1218.75 = 1668.75$. And $1668.75 \times .625 = 1042.97$ lbs = equivalent center load. Hence

$$\frac{1042.97 \times 21^3 \times .00032}{9 \times 12^3} = \frac{3090.862}{15552} = .1987 \text{ ins, reqd def.}$$

REM. 3. With an **equally distributed** load, including the wt of the beam, the def is only $\frac{5}{8}$, or the .625 part as great as it would be if the same total load, including the entire wt of the beam, were all applied at the center.

REM. 4. If the beam in any of these, or the following cases, **is inclined,** as in Fig 11, use the hor dist $o\,y$, instead of the actual span $o\,c$.

Art. 28. RULE 1. **To find the neat center load which will (together with the wt of the beam itself) produce any given def within the elastic limit of the beam;** find the cube of the clear length in feet; mult this cube by the constant from the table in Art 26. Also mult the breadth in ins, by the cube of the depth in ins. Div the first prod by the last one. Div the given def in ins, by the quot, for the total reqd load in lbs. Mult the wt of the clear length of the beam in lbs by .625, and deduct the prod from the load so obtained, for the *neat* load. By formula,

$$\text{Total load, including wt of beam} = \text{Deflect in ins} \div \frac{\text{Cube of length in feet} \times \text{Constant in p 199}}{\text{Breadth in ins} \times \text{Cube of depth in ins.}}$$

EX. What center load in lbs will (together with the wt of the beam itself) produce a def of .286 of an inch, in a beam of white pine, 21 ft span, 9 ins broad, 12 ins deep, and which weighs 450 lbs?

Here $\overset{\text{Cube of 21.}}{9261} \times \overset{\text{Const.}}{.00032} = 2.9635$. And $\overset{\text{Breadth.}}{9} \times \overset{\text{Cube of 12.}}{1728} = 15552$.

And $\frac{2.9635}{15552} = .0001906$. And $\frac{.286}{.0001906} = 1500$ lbs.

For the neat load we must deduct .625 of the wt of the beam; or 450 lbs $\times$ .625 = 281.25 lbs; so that the neat load is $1500 - 281.25 = 1218.75$ lbs, as in Ex 1, Art 27.

If the load is **uniformly distributed,** use precisely the same rule for getting the total load. Then mult this load by 1.6. Deduct the *entire* wt of the clear length of beam.

EX. What equally distributed load will deflect the foregoing beam .1987 ins?

Here, proceeding as before, the only diff is that instead of .286 def, we have .1987 def to be div by .0001906. And $\frac{.1987}{.0001906} = 1042.5$ lbs, as the equivalent *center* load. And $1042.5 \times 1.6 = 1668$ lbs for the total distributed load, including the entire wt of the beam, or 450 lbs. Hence $1668 - 450 = 1218$ lbs, the neat distributed load reqd; agreeing with the preceding example within $\frac{3}{4}$ of a lb; the diff being owing to a neglect of small decimals in the calculation.

RULE 2. **The length, depth, neat center load, and def being given, to find the breadth.**

$$\frac{\text{Neat cen load in lbs} \times \text{Cube of length in feet} \times \text{Constant in Art 26}}{\text{Cube of depth in ins} \times \text{Def in ins}} = \text{Breadth in ins approx.}$$

Or sufficient for the neat load alone.

Now calculate the wt of a beam with the breadth already found. Mult this wt by .625, then say, as

Neat center load : Breadth first found :: .625 of the weight of the beam : Additional breadth reqd.

Add these two breadths together, and their sum will be the total breadth reqd, more *approximately; but still* somewhat too small, inasmuch as it provides only for the

beam of the breadth first found, and not for that having the additional This may readily be calculated and added.

The length, breadth, neat center load, and def, iven, to find the depth.

$$\frac{\text{eat cen load in lbs} \times \text{Cube of length in feet} \times \text{Constant in Art 26}}{\text{Breadth in ins} \times \text{Def in ins}} = \text{Cube of depth in ins approx.}$$

a cube root of this for the depth itself, approximately. REM. This, eadth given by the preceding formula, is too small, inasmuch as it does for the wt of the beam. Therefore, when greater accuracy is required, us: Calculate the wt of a beam having the depth just found. Mult this . Add the prod to the neat center load. Consider the sum as a new neat l; and using it instead of the neat center load first given, go through the ulation again, to obtain a new cube of depth. The cube root of this will arly correct; but still a trifle too small, for the same reason as in the fore-

9. Deflection not to exceed $\frac{1}{40}$ of an inch for each foot length, or $\frac{1}{480}$ part of the clear length. To obtain constants for such center of any hor rectangular beam supported at each end, place any s entirely within its elastic limit. Also, measure the def in ins produced ter load. Then,

$$\frac{\times \text{Breadth in ins} \times \text{Cube of depth in ins} \times \text{Deflection in ins}}{\text{Center load in lbs} \times \text{Cube of clear length in feet}} = \text{Constant for def of } \tfrac{1}{480} \text{ of the span.}$$

ore exact, .625 of the wt of the beam itself should be added to the actual See Rem. Art 26.

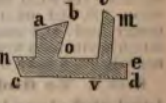
ame way, constants may be found for beams of any form whatever. If, $\frac{1}{40}$, the def must not exceed, $\frac{1}{30}$, $\frac{1}{60}$, &c, of an inch per foot, then sub- &c, for the 40 of the formula.

f constants or coefficients for deflections of $\frac{1}{40}$ inch ot span, of horizontal rectangular beams supported h ends, and loaded at the center.

ry	.0064	Best Southern Pitch Pine	.0108
Oak	.0092	White Ash	"
ardy Poplar	.0230	White Pine, or Common Yellow	.0128
	.0080	Spruce, good Hemlock	
Chestnut	.0170	Red and Black Oak	
ge Cast Iron	.0011	Average Wrought Iron	.00066
ge Steel	.00044		

on beams, proportioned exactly as 7 inch Phœnix beam,						A, Fig 17	.00121
"	"	"	30 lb, 9 in	"	"	"	.00129
"	"	"	heavy 9 in	"	"	B, Fig 17	.00106
"	"	"	$41\frac{2}{3}$ lbs, 12 in	"	"		.00125
"	"	"	$51\frac{2}{3}$ lbs, 15 in	"	"		.00131
"	"	"	$66\frac{2}{3}$ lbs, 15 in	"	"		.00175

In experimenting for constants of with beams of irregular cross-sections, this, for in- is quite immaterial which breadths and depths are us, for the breadth we may take *a b*, *l m*, *c d*, or *o e*, r the depth, either *n c*, *l v*, *m d*, *b o*, &c. It is only to state what parts actually have been taken, so rresponding ones may be measd in any other beam to be calculated by the constant derived from the t. This remark applies to all constants involving h and the depth. The constant itself will of course vary according to ensions are taken in the experiment; but the results derived from it when other beams of similar forms, will not be affected thereby, if the corre- *parts be measd in both cases.**



ven take any single oblique measurement, as *a b*, *l m*, *n c*, *a d*, &c, and call it both the *e depth. This applies to* rectangular, or to any other shaped beams.

Art. 30. Rule 1. **To find the greatest center load which an given hor rectangular beam, supported at both ends, ca sustain without bending more than $\frac{1}{480}$ of its span.**

$$\frac{\text{Breadth in ins} \times \text{Cube of depth in ins}}{\text{*Square of span in feet} \times \text{Constant in Art 29}} = \text{Reqd load in lbs, approx,}$$

and generally near enough for practice.

To be more accurate, a deduction must be made for the wt of the beam itself. get the neat load. To do this, mult the wt of the span of the beam by .625, and d duct the prod from the approx load just found.

Rem. 1. The load may also be found by tables in Arts 31, 32.

Rem. 2. If the load is **uniformly distributed,** use the same rule formula; but mult the resulting load by 1.6; and then, if an allowance is reqd the wt of the beam, deduct the wt of the *entire* span for the neat load.

Rule 2. **The neat center load, length, and depth being give to find the breadth, such that the beam shall not defle more than $\frac{1}{480}$ of its span.**

$$\frac{\text{Square of length in feet} \times \text{Neat center load in lbs} \times \text{Constant in Art 29}}{\text{Cube of depth in ins}} = \text{Breadth in ins, approx,}$$

and generally near enough for practice.

Then say, as Center load : this first breadth :: .625 of the weight of the clear span of beam : a second breadth.

Add together these two breadths for the reqd breadth, very approx, but a me trifle too small. See Rule 2, Art 28.

If the load is **uniformly distributed,** first mult it by .625. Consid the prod as a center load; then, by the foregoing formula, find the first breadt Now find the wt of the whole span, having this first breadth, and mult it by .6 Then say, as

.625 of the uniform load : first breadth :: .625 of the weight of clear span of the beam : a second breadth.

Add these two breadths together, for the reqd one very approx, but a mere trif too small. See Remark after Rule 3, Art 28.

Rule 3. **The neat center load, length, and breadth, bein given, to find the depth, such that the beam shall not deflec more than $\frac{1}{480}$ of its clear length.**

$$\frac{\text{Square of length in feet} \times \text{Neat cen load in lbs} \times \text{Constant in Art 29}}{\text{Breadth in ins}} = \text{Cube of depth in ins, approx.}$$

Take the cube root of this for the approx depth itself, generally near enough f practice. Then calculate the wt of the entire clear span of a beam having th depth, mult it by .625, and add the prod to the neat center load. Consider the su as a new neat center load; and using it instead of the one first given, go through t whole calculation again, for a new cube of depth. The cube root of this will be t reqd depth more approx, but a little too small, for the reason given at Rule 3, Art 2

If the neat load is **uniformly distributed,** first mult it by .625. Use t prod as a center load, and by the foregoing formula find the first approx depth The calculate the wt of the entire clear length of a beam having that depth. Mult th wt by .625, and add it to the prod used as a center load. Consider the sum as a ne center load; and using it instead of the one first used, go through the whole calc lation again, for a new cube of depth. The cube root of this will be the reqd dept approx, but a mere trifle too small, for the reason given at Rule 3, Art 28.

Rule 4. **To find the side of a square beam, when its cente load and clear length are given,** not to deflect more than $\frac{1}{480}$ of its spa

$$\text{Square of length in feet} \times \text{Center load in lbs} \times \text{Constant in Art 29} = \text{A certain fourth power.}$$

Next, find the fourth root of this fourth power; that is, take its sq rt, then tak the sq rt of said sq rt. This last sq rt will be one side of a square beam, general

* *We must now employ the square of the length; not its cube, as in the preceding arts*

: practice; but which will answer truly only for the given center load, ıe wt of the clear beam; which wt can now be calculated.
ım is to answer for the given center load, *plus* the wt of the beam, must now be repeated, thus: Find the wt of the beam just calculated; ; add the prod to the given center load. Then, with this increased ploy the formula as before, obtaining a new fourth power. The sq rt this will be the reqd side very approx, but a mere trifle too small. Rule 3, Art 28.

find the diam of a solid cylinder, when its cen- ıd clear length are given. Mult the load by 1.7, and with oad proceed, as in the foregoing case of a square beam, to find a cer- Take the sq rt of the sq rt of this, for a diam sufficient for the given *ıus* .625 of the wt of the beam. For a larger diam, sufficient for the also, first mult said wt just found by .625. Add the prod to the ori- ter load. Mult the sum by 1.7 for a new load; then, with this new formula as before, for a new 4th power. The sq rt of the sq rt of reqd diam more approx, but a trifle too small. See Rem, after Rule 3,

stiffness of a cylinder is to that of a square beam, whose breadth ach equal to the diam of the cyl, as .589 to 1; or that of the square f the cyl as 1 to .589, or as 1.698 to 1; in practice we may use .6 and cylinder will bend 1.7 times more than the square one, under the

en, in any of the foregoing cases, **the beam is inclined,** as he hor dist o *y* for the span, instead of o e.

[illegible] 81. Table of greatest center loads for horizontal rectangular beams of white or yellow [illegible] of spruce, 1 inch broad, supported at both ends, and required not to bend more than $\frac{1}{10}$ inch per foot of clear span, or $\frac{1}{10}$ part of the entire clear span. [illegible] and steel, see page of table.

This table was calculated with a constant 1/14. Instead of the 1/14 of one foot [illegible] The loads in this table include the weight of the beam itself, [illegible] ½ of which must be deducted from the tabular loads to get the net load, which the beam is loaded at its center. [illegible] shown in this table, but in that case the weight of the entire clear beam must be deducted [illegible] in practice this deduction [illegible] be made

CLEAR BEAMS IN FEET. (Spruce.)

Depths in Inches	4	4	5	6	7	8	9	10	12	14	16	18	20	30	40	50	60	70	[illegible]	[illegible]
1	lbs. 8.4	lbs. 4.8	lbs. 3.0	lbs. 2.1	lbs. [illegible]	lbs. [illegible]														
2	28.7	16.2	10.4	7.3	5.5															
3	60.4	38.4	24.4	17.3	12.4	9.6														
4		70	48	35	26	19	14													
5		130	88	66	47	37	28	22												
6			131	99	77	60	49	39	27											
7			196	137	102	80	66	53	37	26										
8				192	144	106	86	70	49	36	27									
9				257	196	150	115	93	66	49	37	29								
10					260	200	156	126	89	67	51	40	32							
11					336	262	204	165	117	87	67	53	43							
12						332	260	207	148	110	85	68	55	27						
13						417	324	263	188	140	108	86	70	34						
14							404	324	232	173	133	106	86	42	27					
15							485	394	282	210	162	129	105	51	33					
16							572	466	334	249	192	153	124	61	39	28				
17								540	388	289	223	177	144	71	46	33				
18								618	444	331	255	203	165	81	53	38	28			
19								700	503	375	289	230	187	92	60	43	32			
20									566	422	325	259	211	104	68	49	36	27		
21									632	471	363	289	235	116	76	55	40	30		
22									702	523	403	321	261	129	84	61	45	34		
23									775	577	444	354	288	142	93	67	49	37		
24									850	633	487	388	316	156	102	74	54	41		
25									929	692	532	424	345	170	111	80	59	45		
26										754	579	461	375	185	121	87	64	49		
27										818	628	500	407	201	131	94	69	53		
28										884	679	540	440	217	142	102	75	57		
29										953	732	582	474	234	153	110	81	62		
30										1025	787	626	510	252	165	118	87	67		
32											1000	796	648	320	209	150	111	85		
34											1230	979	797	394	258	185	137	105		
36												1175	956	472	309	222	164	126		
38												1395	1135	560	367	263	195	150		
40													1330	656	429	308	228	175		
42														760	497	357	264	203		
44														873	571	410	303	233		
46														994	650	467	345	265		
48														1121	733	526	389	299		
50														1256	821	589	436	335		
52															914	656	485	373		
54															1014	727	538	413		
56															1121	804	595	457		
58															1234	885	655	503		
60																970	718	551		
62																1060	784	602		
64																1155	854	656		
66																1255	928	712		
68																	1006	772		
70																	1088	835		
72																	1175	901		
74																		970		
76																		1043		
78																		1120		
80																		1200		

. Table of greatest center loads of square beams of ron, supported at both ends, and reqd not to bend than $\frac{1}{40}$ of an inch per foot of clear length, or $\frac{1}{480}$ of the entire clear length.

is are about $\frac{1}{12}$ part greater than would be given by our constant .0011 for ast iron, Art 29. Wrought iron will bear about $\frac{2}{3}$ more than cast, with the deflection. But .625 (or $\frac{5}{8}$) of the wt of the beam itself must be deducted e center loads. If the load is equally distributed, it will be 1.6 times as ese tabular center loads; but in this case the wt of the entire clear length m is to be deducted. These deductions are rarely reqd in practice.

Clear Spans, in Feet. (From Tredgold)

4	6	8	10	12	14	16	18	20	22	24	26	28	30
Depth	Depth	Depth	Depth	Depth	Depth	Depth	Depth	Depth	Depth	Depth	Depth	Depth	Depth
In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
1.2	1.4	1.7	1.9	2.0	2.2	2.4	2.5	2.6	2.7	2.9	3.0	3.1	3.2
1.4	1.7	2.0	2.2	2.4	2.6	2.8	3.0	3.1	3.3	3.4	3.6	3.7	3.8
1.6	1.9	2.2	2.4	2.7	2.9	3.1	3.3	3.4	3.6	3.8	3.9	4.1	4.2
1.7	2.0	2.4	2.6	2.9	3.1	3.3	3.5	3.7	3.9	4.0	4.2	4.3	4.5
1.8	2.2	2.5	2.8	3.0	3.3	3.5	3.7	3.9	4.1	4.3	4.4	4.6	4.8
1.8	2.2	2.6	2.9	3.2	3.4	3.7	3.9	4.1	4.3	4.5	4.6	4.8	5.0
1.9	2.3	2.7	3.0	3.3	3.6	3.8	4.1	4.2	4.4	4.6	4.8	5.0	5.2
2.0	2.4	2.8	3.1	3.4	3.7	3.9	4.2	4.4	4.6	4.8	5.0	5.2	5.4
2.0	2.5	2.9	3.2	3.5	3.8	4.0	4.3	4.5	4.7	4.9	5.1	5.3	5.5
2.1	2.6	3.0	3.3	3.6	3.9	4.2	4.4	4.7	4.9	5.2	5.3	5.4	5.7
2.1	2.6	3.0	3.4	3.7	4.0	4.3	4.5	4.8	5.0	5.3	5.4	5.6	5.8
2.2	2.7	3.1	3.5	3.8	4.1	4.4	4.7	4.9	5.1	5.3	5.5	5.7	5.9
2.2	2.7	3.1	3.5	3.8	4.2	4.4	4.7	4.9	5.2	5.4	5.6	5.9	6.0
2.3	2.8	3.2	3.6	3.9	4.2	4.5	4.8	5.0	5.3	5.5	5.7	6.0	6.1
2.3	2.8	3.2	3.6	4.0	4.3	4.6	4.9	5.2	5.4	5.6	5.8	6.1	6.2
2.4	2.9	3.3	3.7	4.0	4.4	4.7	5.0	5.2	5.5	5.7	5.9	6.2	6.4
2.4	2.9	3.4	3.8	4.1	4.4	4.7	5.0	5.3	5.5	5.8	6.0	6.2	6.5
2.4	3.0	3.4	3.8	4.2	4.5	4.8	5.1	5.4	5.6	5.9	6.1	6.4	6.6
2.5	3.0	3.5	3.9	4.2	4.6	4.9	5.2	5.4	5.7	6.0	6.2	6.5	6.7
2.5	3.0	3.5	3.9	4.3	4.6	4.9	5.2	5.5	5.8	6.0	6.3	6.5	6.8
2.6	3.2	3.7	4.1	4.5	4.9	5.2	5.5	5.8	6.1	6.4	6.6	6.9	7.2
2.8	3.4	3.9	4.3	4.7	5.1	5.5	5.8	6.1	6.4	6.7	7.0	7.2	7.5
2.9	3.5	4.0	4.5	4.9	5.3	5.7	6.0	6.3	6.7	6.9	7.2	7.5	7.7
2.9	3.5	4.1	4.7	5.1	5.5	5.9	6.2	6.5	6.8	7.2	7.6	7.7	8.0
3.1	3.8	4.4	4.9	5.4	5.8	6.2	6.6	6.9	7.3	7.6	7.9	8.2	8.5
3.3	4.0	4.6	5.1	5.7	6.1	6.5	6.9	7.3	7.6	7.9	8.3	8.6	8.9
3.4	4.1	4.8	5.3	5.8	6.3	6.7	7.1	7.5	7.9	8.2	8.6	8.9	9.2
3.5	4.3	4.9	5.5	6.0	6.5	7.0	7.4	7.8	8.2	8.5	8.9	9.2	9.5
.....	4.4	5.1	5.7	6.2	6.7	7.2	7.6	8.0	8.4	8.8	9.1	9.5	9.8
.....	4.5	5.2	5.8	6.4	6.9	7.4	7.8	8.2	8.6	9.0	9.4	9.7	10.1
.....	.....	5.5	6.3	6.7	7.2	7.7	8.2	8.6	9.0	9.4	9.8	10.2	10.5
.....	.....	5.7	6.3	6.9	7.5	8.0	8.5	8.9	9.4	9.8	10.2	10.6	11.0
.....	.....	5.9	6.6	7.2	7.8	8.3	8.8	9.3	9.7	10.1	10.6	10.9	11.3
.....	.....	6.0	6.8	7.4	8.0	8.5	9.0	9.5	10.0	10.4	10.9	11.3	11.7
.....	.....	.....	6.9	7.6	8.2	8.8	9.3	9.8	10.3	10.7	11.2	11.6	12.0
.....	.....	.....	7.1	7.8	8.4	9.0	9.5	10.0	10.5	11.0	11.5	11.9	12.3
.....	.....	.....	7.2	7.9	8.6	9.2	9.7	10.2	10.8	11.2	11.7	12.1	12.5
.....	.....	.....	7.4	8.1	8.8	9.4	9.9	10.4	11.0	11.5	11.9	12.4	12.8
.....	.....	.....	7.5	8.3	8.9	9.5	10.1	10.6	11.1	11.7	12.1	12.6	13.0
.....	.....	.....	7.7	8.4	9.1	9.7	10.3	10.8	11.4	11.9	12.3	12.8	13.2
.....	.....	.....	7.8	8.5	9.2	9.8	10.4	11.0	11.5	12.0	12.5	13.0	13.5
.....	.....	.....	7.9	8.7	9.4	10.0	10.6	11.2	11.7	12.2	12.7	13.2	13.7
.....	.....	.....	8.0	8.8	9.5	10.1	10.8	11.3	11.9	12.4	12.9	13.4	13.9
.....	.....	.....	8.1	8.9	9.6	10.3	10.9	11.5	12.2	12.6	13.1	13.6	14.1
.....	.....	.....	.....	9.0	9.7	10.4	11.0	11.6	12.5	12.7	13.2	13.8	14.3
.....	.....	.....	.....	9.2	10.0	10.7	11.3	11.9	12.8	13.0	13.6	14.1	14
.....	.....	.....	.....	9.4	10.2	10.9	11.5	12.2	13.0	13.4	13.9	14.4	1
.....	.....	.....	.....	9.6	10.4	11.1	11.8	12.4	13.3	13.6	14.2	14.7	1
.....	.....	.....	.....	9.8	10.6	11.4	12.0	12.7	13.5	13.9	14.4	15.0	

Art. 32. Table of breaking loads, in tons, at the center of horizontal rectangular cast-iron beams, one inch thick; supported at both ends: including the weight of the beam itself; one-*half* of which must be deducted from the loads in the table, to obtain the neat breaking load. To do this use the last column. If equally distributed over the clear bearing, the load will be twice as great; including the weight of the beam itself; all of which must be deducted in that case, for the neat load. *Rolled iron* of average quality, is ruined as a beam, by about $1\frac{1}{4}$ times the breaking load of a cast iron one. For the breaking loads of white pine, spruce, and ordinary oaks, divide those in the table by 4.5; and for white oak, white ash, and best Southern yellow pine, by 4. To find the center breaking weight of a *square* beam, multiply the load in the tables, by a breadth equal to the depth in inches.

Depths of Beams, in ins.	CLEAR SPANS BETWEEN THE ENDS OF SUPPORTS, IN FEET. (ORIGINAL.)																		Wt. of 10 ft. of Beam, in tons.
	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	8 ft.	10 ft.	12 ft.	14 ft.	16 ft.	18 ft.	20 ft.	24 ft.	28 ft.	32 ft.	36 ft.	40 ft.	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	.904	.452	.301	.226	.181	.151	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.014
15/16	1.41	.706	.471	.353	.283	.235	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.017
7/8	2.03	1.02	.678	.508	.407	.339	.254	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.021
3/4	2.77	1.38	9.3	.693	.554	.461	.346	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.024
2	3.62	1.81	1.21	.904	.723	.603	.452	.362	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.028
15/16	[redacted]	2.29	1.53	1.14	.915	.763	.573	.458	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.031
7/8	.....	2.82	1.88	1.41	1.13	.942	.706	.565	.471	.....	.....	.....	.....	.....	.....	.....	.....	.....	.034
3/4	.....	3.42	2.28	1.71	1.37	1.14	.854	.684	.570	.....	.....	.....	.....	.....	.....	.....	.....	.....	.038
3	.....	4.07	2.71	2.03	1.63	1.36	1.02	.814	.678	.581	.....	.....	.....	.....	.....	.....	.....	.....	.042
15/16	.....	[redacted]	3.18	2.39	1.91	1.59	1.19	.955	.796	.682	.....	.....	.....	.....	.....	.....	.....	.....	.045
7/8	.....	.....	3.69	2.77	2.21	1.85	1.38	1.11	.923	.791	.692	.615	.....	.....	.....	.....	.....	.....	.049
3/4	.....	.....	4.24	3.18	2.54	2.12	1.59	1.27	1.06	.908	.794	.706	.....	.....	.....	.....	.....	.....	.052
4	.....	.....	[redacted]	3.62	2.89	2.41	1.81	1.45	1.20	1.03	.904	.803	.....	.....	.....	.....	.....	.....	.056
15/16	.....	.....	.....	4.08	3.27	2.72	2.04	1.63	1.36	1.17	1.02	.907	.816	.....	.....	.....	.....	.....	.059
7/8	.....	.....	.....	4.58	3.66	3.05	2.29	1.83	1.53	1.31	1.14	1.02	.915	.....	.....	.....	.....	.....	.063
3/4	.....	.....	.....	5.10	4.08	3.40	2.55	2.04	1.70	1.46	1.27	1.13	1.02	.....	.....	.....	.....	.....	.066
5	.....	.....	.....	5.65	4.52	3.77	2.82	2.26	1.88	1.61	1.41	1.26	1.13	.942	.....	.....	.....	.....	.069
15/16	.....	.....	.....	[redacted]	4.98	4.15	3.11	2.49	2.08	1.78	1.56	1.38	1.25	1.04	.....	.....	.....	.....	.073
7/8	.....	.....	.....	.....	5.47	4.56	3.42	2.73	2.28	1.95	1.71	1.52	1.37	1.14	.....	.....	.....	.....	.077
3/4	.....	.....	.....	.....	5.98	4.98	3.74	2.99	2.49	2.13	1.87	1.66	1.49	1.25	.....	.....	.....	.....	.080
6	.....	.....	.....	.....	6.51	5.42	4.07	3.25	2.71	2.32	2.03	1.81	1.63	1.36	1.16	.....	.....	.....	.084
15/16	.....	.....	.....	.....	[redacted]	6.37	4.77	3.82	3.18	2.73	2.39	2.12	1.91	1.59	1.36	.....	.....	.....	.090
7	.....	.....	.....	.....	.....	7.38	5.54	4.43	3.69	3.16	2.77	2.46	2.21	1.85	1.58	.....	.....	.....	.097
7/8	.....	.....	.....	.....	.....	8.48	6.36	5.09	4.24	3.63	3.18	2.82	2.54	2.12	1.82	.....	.....	.....	.104
8	.....	.....	.....	.....	.....	9.64	7.23	5.79	4.82	4.13	3.62	3.21	2.89	2.41	2.07	.....	.....	.....	.111
9/8	.....	.....	.....	.....	.....	[redacted]	8.16	6.53	5.44	4.67	4.08	3.63	3.27	2.72	2.33	.....	.....	.....	.118
9	.....	.....	.....	.....	.....	.....	9.15	7.32	6.10	5.23	4.58	4.07	3.66	3.05	2.62	2.29	.....	.....	.125
9/8	.....	.....	.....	.....	.....	.....	10.2	8.16	6.80	5.83	5.10	4.53	4.08	3.40	2.91	2.55	.....	.....	.132

[illegible] the entire clear length must be deducted. Wrought-iron cylinders will bear about 1 fourth more than cast. For white pine, spruce, or ordinary oaks, divide [illegible] in the table by 4.5; and for white oak, white ash, or best Southern yellow pine, by 4. For the breaking load of a square beam, whose breadth and depth each equals [illegible] diameter of the cylinder, multiply the tabular loads by 1.7.

(ORIGINAL.)

Diam. in Inches.	Clear Spans between the End Supports, in Feet																	Wt. of 10 ft. of Beams in Tons.
	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	8 ft.	10 ft.	12 ft.	14 ft.	16 ft.	18 ft.	20 ft.	24 ft.	28 ft.	32 ft.	36 ft.	40 ft.	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1.	.533																		.01
¼	1.04	.266	.177	.133	.106														.02
½	1.80	.520	.347	.260	.208														.03
¾	2.85	.899	.600	.449	.360	.300													.03
2.	4.26	1.43	.951	.713	.571	.476													.04
¼	6.07	2.13	1.42	1.06	.852	.710	.533												.06
½	8.32	3.03	2.02	1.52	1.21	1.01	.758												.07
¾	11.1	4.16	2.77	2.08	1.66	1.39	1.04	.832											.08
3.	14.4	5.54	3.69	2.77	2.22	1.85	1.38	1.11											.10
¼		7.19	4.79	3.59	2.88	2.40	1.80	1.44	1.20										.11
½		9.14	6.09	4.57	3.66	3.05	2.29	1.83	1.52										.13
¾		11.4	7.61	5.71	4.57	3.81	2.85	2.28	1.90	1.63									.15
4.		14.0	9.36	7.02	5.62	4.68	3.51	2.81	2.34	2.01									.18
¼		17.0	11.4	8.52	6.82	5.68	4.26	3.41	2.84	2.43	2.13	1.89							.20
½			13.6	10.2	8.18	6.81	5.11	4.09	3.41	2.92	2.56	2.27							.22
¾			16.2	12.1	9.71	8.09	6.07	4.85	4.04	3.47	3.03	2.70							.25
5.			19.0	14.3	11.4	9.51	7.14	5.71	4.76	4.08	3.57	3.17	2.85						.28
¼			22.2	16.6	13.3	11.1	8.32	6.66	5.55	4.76	4.16	3.70	3.33						.31
½				19.3	15.4	12.8	9.63	7.71	6.42	5.50	4.82	4.28	3.85	3.21					.33
¾				22.2	17.7	14.8	11.1	8.86	7.38	6.33	5.54	4.92	4.43	3.69					.37
6.				25.3	20.3	16.9	12.7	10.1	8.44	7.23	6.33	5.63	5.06	4.22	3.62				.40
¼				28.8	23.0	19.2	14.4	11.5	9.59	8.22	7.19	6.39	5.75	4.79	4.11				.45
½					26.2	24.3	18.2	14.6	12.1	10.4	9.11	8.10	7.29	6.08	5.21	4.56			.54
¾					36.5	30.4	22.8	18.3	15.2	13.0	11.4	10.1	9.13	7.61	6.52	5.71	5.07	4.57	.62
7.					44.9	37.4	28.1	22.5	18.7	16.0	14.0	12.5	11.2	9.36	8.02	7.02	6.24	5.62	.70
¼						45.4	34.1	27.3	22.7	19.5	17.0	15.1	13.6	11.4	9.74	8.52	7.57	6.82	.80
½						54.5	40.9	32.7	27.3	23.4	20.4	18.2	16.4	13.6	11.7	10.2	9.08	8.18	.89
¾						64.7	48.5	38.8	32.4	27.7	24.3	21.6	19.4	16.2	13.9	12.1	10.8	9.71	.99
8.							57.1	45.7	38.1	32.6	28.5	25.4	22.8	19.0	17.3	14.3	12.7	11.4	1.1
¼							66.6	53.3	44.4	38.0	33.3	29.6	26.6	22.2	19.0	16.6	14.8	13.3	1.2
½							77.1	61.7	51.4	44.0	38.5	34.3	30.8	25.7	22.0	19.3	17.1	15.4	1.3
¾								70.9	59.1	50.6	44.3	39.4	35.4	29.5	25.3	22.1	19.7	17.7	1.5
9.								81.0	67.5	57.9	50.6	45.0	40.5	33.8	28.9	25.3	22.5	20.2	1.6
¼									76.7	65.7	57.5	51.1	46.0	38.3	32.9	28.8	25.6	23.0	2.2
½									122	104	91.3	81.2	73.0	60.9	52.2	45.7	40.6	36.5	2.8
¾										156	136	121	109	90.9	77.9	68.1	60.6	54.5	3.6
10.										222	194	173	155	129	111	97.1	86.3	77.7	
¼																			
½																			
¾																			
11.																			
¼																			
½																			
¾																			
12.																			

Art. 35. Hodgkinson's beams have nearly 1¾ times the stre beam of equal wt whose top and bottom are equal. Mr. Hodgkinson, having found an average, cast iron reqd about 6½ times force to crush it as it did to pull it apart, c the beam (of which Fig 15 is a cross-secti center) in which the upper or compress flange, *u*, has but ⅙ of the area of the extended one, *b*. The top flange he the sumed to be safe; inasmuch as its area what greater than the proportion of 1 to hence, breaking will take place from the of the bottom flange by extension. As t of his experiments, he gives the follow when the load is applied at top, or equally sides of the beam.

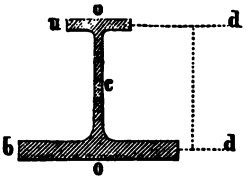


Fig 15

$$\text{Breakg load in tons} = \frac{\text{Area of bot flange in sq ins} \times \text{depth } oo \text{ in ins} \times \text{Constant } 2.166}{\text{Clear length in feet.}}$$

When the lower flange is as much as about 2½ inches thick, experime that part of the breakg load thus obtained should be deducted; because thick are proportionally weaker than thin ones. Half the wt of the beam itself deducted, for the neat breakg load; this, however, is necessary only when t are very long; for such as are used for ordinary building purposes, it m glected. If the load is equally distributed, it will be twice as great; but t wt of the beam must then be deducted.

Ex. The upper rib u = 3 ins × 1 inch = 3 sq ins area; bottom rib b = × 12 ins = 18 sq ins area; total depth oo, 15 ins; clear span, 20 ft. Here,

$$\frac{18 \times 15 \times 2.166}{20} = \frac{584.82}{20} = 29.241 \text{ tons, the reqd load, including ½ t}$$

Now to find the wt of half the beam, we may proceed th the entire area of its cross-section in sq ins, by the clear span in ins. This the cub ins of iron contained in the beam; and these div by 8600, give the beam in tons; because 8600 cub ins of cast iron weigh about 1 ton; or nea ins 1 lb.* Thus, if the vert rib contains 12 sq ins, then since the two flan tain 21, the entire section is 33 sq ins; and the span being 240 ins, we have

$= 7920$ cub ins of iron. And $\frac{7920}{8600} = .92$ of a ton, the wt of the beam.

of this, or .46 ton, taken from the breakg load 29.241 tons, leaves 28.78 tons as breakg load; showing that in such cases as this it is scarcely worth while in to make the deduction. These beams are not always made of the same throughout, (see Fig 16,) but diminish toward the ends; this method is ther always strictly correct, but no great accuracy is needed in such cases.

To find the size of a Hodgkinson beam, reqd to break a given center load, having the depth. Mult the given loa by the clear span in feet. Mult the constant 2.166 by the total depth, oo Div the first prod by the last; the quot will be the area of the bottom rib i This, div by 6, will be the area of the top rib. The bottom rib is usually m 6 to 8 times as wide as it is thick; and the top one from 3 to 6 times. The t of the stem is usually a little greater at bottom than at top; the average t being from $\frac{1}{14}$ to $\frac{1}{24}$ of the depth of the beam.

To save material, the *width* of the bottom flange, and of the top one also if proper, may be reduced l to about ½ as great at of the beam as at its c shown by the middle s Fig 16, of which the upp is a side view. Or, lea dimensions of those fla altered, the *depth* of the rib may be reduced to ends, as shown by th sketch. The theoretical here an ellipse. When t

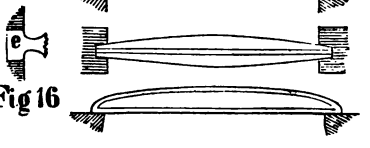


Fig 16

is reduced, the very ends may, for stability, be widened out, as at *e*, which is a

The vert rib is generally strengthened by casting b

* A cub inch of wrought iron may be taken at .28 lb; and of cast iron .3

as in the upper sketch. These should not extend entirely to the then expose the beam to crack as it cools. To prevent this tend- attached alternately to the top and bottom ribs. The upper ones, y needed.

ıse beams, as well as in all other castings, it is important to avoid from thin to thick parts; and to keep all parts as nearly as possi- nickness. Otherwise the castings are apt to warp and crack in ır in mind that the resistance or strength per sq inch is *considera-* stings than in thin ones.

e rule for breakg loads is safe when the load is equally disposed side of the vert web; and when said web and the flanges are pro- other about the same as those used in Mr. Hodgkinson's experi- quent investigators have found his beams to break with but little ; loads given by the rule, when applied to only one side, as *b o*, or op or bottom flange. W. H. Barlow, C. E., London, experimenting finds that when a cast-iron beam is liable to be loaded on only one the top flange should have an area equal to ⅓ that of the entire e beam; and for beams so proportioned, he gives the following:

$$\left(\begin{matrix}\text{ı bot flange}\\ \text{n sq ins}\end{matrix}\right)+\left(\begin{matrix}\text{Half area of}\\ \text{vert rib}\end{matrix}\right)\times\left(\begin{matrix}\text{Depth in ins.*from}\\ \text{center to center of}\\ \text{top and bot flanges}\end{matrix}\right)\times\frac{\text{Constant}\ 2.333}{\text{Clear span in feet.}}$$

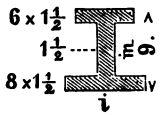
ters recommend that even for loads pressing vertically through the wer flange should have but about 3 instead of 6 times the area of

mate resistance of steel to compression being about twice that to tinson beam of that metal should have its lower flange of twice er one.

r **the purpose of ready reference,** we give a few ex- with cast-iron beams of various shapes: being the actual center ons of sound beams. Some beams of Sterling's toughened cast ıll ⅓ higher than those of common iron.

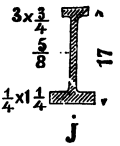
er breakg loads in tons, of cast-iron beams. Clear adths and depths in inches.

an 4¼ ft.
load 2 tons.



Span 11⅔ ft †
Br load 20 tons.

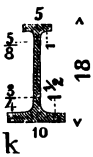
bove inverted.
ıd 2.3 to 2.9 tons.



Span 18 ft.
Br load 22 to 28 tons.

1·36 Span 4¼ ft.
Br load .125 ton.

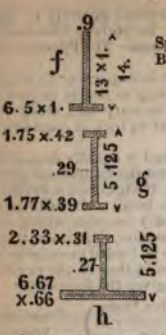
above inverted.
load .4 ton.



Span 27⅓ ft.‡
Br load 29¼ tons.

Span 4½ ft.
Br load { 3.7 tons to 4.2.

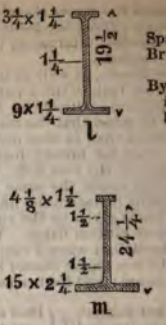
'ig 15.
as, the beam was unloaded, and its elasticity appeared to be but little if at all
f tons. .15 inch; 8½ tons, .3 inch; 17 tons, 1.1 inches.
f these beams were tested by center loads of 12 tons. Def ⅜ to ⅝ inch.



Span 15 ft.
Br load 12½ tons.

Span 4½ ft.
Br load 3 tons.

Span 4½ ft.
Br load 10.5 to 11.6 tons.



Span 19 ft.
Br load 50

By Hodgki
it should
but 25.*

Span 3
Br load

In describing such beams, it is better to give the *entire* depth of th when the depth of the *web* is given, doubts arise whether it is meant to thicknesses of the two flanges, or not. Every writer, almost, that w leaves us in this doubt.

REM.. **In beams either larger or smaller than these** cross-sections are proportioned exactly as these are, and whose spans that these have, the center breakg loads will be as the cubes of their lines. Thus, in a beam which is $\frac{1}{10}$, $\frac{1}{3}$, $\frac{1}{2}$, 2, 3, or 10 times as way, except in span, the breakg load will be $\frac{1}{1000}$, $\frac{1}{27}$, $\frac{1}{8}$, 8, 27, or 1000 times

If the spans also differ, first find the load as above, as if t same; then say, as the new span, is to the span given in our Figs, so i load thus found, to the actual breakg load for the new beam. Thus, sup to make a cast-iron beam, 4 times as large every way as the dimensions first of these Figs; except its span, which is to be, say 10 ft, instead of 4 the first breakg load is found to be $4 \times 4 \times 4 = 64$ times as great; or = 128 tons. Next,

New Span.		Span in Fig.		First load.		Actual load.
10	:	4.5	::	128	:	57.6 tons.

In such cases we must, however, have regard to "Rem," Art 11. T process applies equally to beams of any other shapes, such as the follow whether solid or hollow, &c; and of any other materials; so that if we dimensions, and the breakg load of any beam whatever, we may find that one of the same material, and of the same proportions of cross-secti

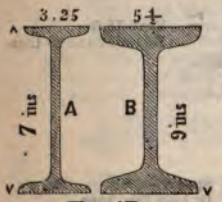


Fig 17

become advisable in important cases, to eve or more model beams of some hitherto u and to break them, in order to find the br of the actual beam of the same material. I the defs should also be measd, in order to those of the actual beam may not be too Art 26, &c; and Art 46.

Art. 37. Phœnix rolled bea Phœnix Iron Co, of Philadelphia. See Re

The general form of these beams is show The upper and lower flanges are of the sa the connecting stem, or vert web, is of u ness. They are called 6, 9, 12, or 15 inch be ing to their total depth.

* *His rule, however,* was intended only for beams proportioned by it. By the Ru *would be 40½ tons.* Two beams were broken.

† *This iron was "Sterling's toughened,"* having about 16 per cent of wrought scra *Each of the 89 beams was tested by a center load of 20 tons,* which produced defs of fr *Entire length, 84½ ft.*

Experiments with these beams have given the following results:

The 7-inch beam. A, Fig 17. Weight 20 lbs per foot run. Flanges $3\frac{1}{4}$ inches wide. Stem $\frac{4}{10}$ thick. Area of cross-section of beam, 6 sq ins. Clear span 20 ft.

Center Load, in Lbs.	Defln. Ins.	Increase. Ins.	Center Load, in Lbs.	Defln. Ins.	Increase. Ins.
2000	.500				
2357	.575	.075	6232	1.450	.140
3732	.860	.285	6732	1.650	.200
4232	.990	.130	7232	1.770	.120
4732	1.090	.100	7732	1.900	.130
5232	1.190	.100	8232	2.050	.150
5732	1.310	.120	9032		

With 9032 lbs, the beam yielded by bending, sideways, without any sign of fracture. With 2050 lbs, its def equals $\frac{1}{480}$ of its span. See Arts 26 and 29.

A 9-inch beam. Weight 30 lbs per foot run. Flanges 4 ins wide. Stem $\frac{1}{2}$ inch thick. Area of cross-section of beam, 9 sq ins. Clear span 20 ft.

Center Load, in Lbs.	Defln. Ins.	Increase. Ins.	Center Load, in Lbs.	Defln. Ins.	Increase. Ins.
3732	.350				
4732	.440	.090	10732	.980	.090
5377	.500	.060	11732	1.065	.085
5732	.525	.025	12732	1.170	.105
6732	.630	.105	13732	1.270	.100
7732	.710	.080	14732	1.400	.130
8732	.805	.095	15732	1.630	.230
9732	.890	.085	16732	2.340	.710

With 16732 lbs the beam bent sideways, and yielded, without any sign of fracture. With 5377 lbs, it deflected $\frac{1}{480}$ of its span. See Arts 26 and 29.

A heavier 9-inch beam. B, Fig 17. Weight 50 lbs per foot run. Flanges $4\frac{1}{2}$ ins wide. Stem $\frac{3}{4}$ inch thick. Area of cross-section of beam, 15 sq inches. Clear span 14 feet.

Center Load in Lbs.	Tons.	Defln. Ins.	Increase. Ins.	Center Load in Lbs.	Tons.	Defln. Ins.	Increase. Ins.
6720	3	.126		19040	$8\frac{1}{2}$	.382	.023
7840	$3\frac{1}{2}$	.148	.022	20160	9	.409	.027
8960	4	.170	.022	21280	$9\frac{1}{2}$	.435	.026
10080	$4\frac{1}{2}$	.192	.022	22400	10	.458	.023
11200	5	.214	.022	23520	$10\frac{1}{2}$	.487	.029
12320	$5\frac{1}{2}$	.239	.025	24640	11	.516	.029
13440	6	.261	.022	25760	$11\frac{1}{2}$	.543	.027
14560	$6\frac{1}{2}$	.287	.026	26880	12	.572	.029
15680	7	.310	.023	28000	$12\frac{1}{2}$	.600	.028
16800	$7\frac{1}{2}$	.336	.026	29120	13	.633	.033
17920	8	.359	.023	" in $\frac{3}{4}$ hour		.682	.049

With 17470 lbs, it deflected $\frac{1}{480}$ *of its span.* See Arts 26 and 29.

They also make a 25 lb 9 inch beam. Their 6 inch beam, weighs $13\frac{1}{3}$ *lbs per ft. Flanges* $2\frac{3}{4}$ *ins wide. Stem* $\frac{3}{10}$ *inch thick.*

Price of Phœnix beams, Philadelphia, 1873, about $5\frac{3}{4}$ *to* $6\frac{1}{2}$ *cts. per lb; depending on size of quantity ordered.*

A 12-inch beam. Weight 41⅔ lbs per foot run. Flanges 4 ins wide. 5 inch thick. Area of cross-section of beam, 12½ sq ins. Clear span 25 ft.

Center Load in Lbs.	Tons.	Deflection. Ins.	Increase. Ins.	Center Load in Lbs.	Tons.	Deflection. Ins.	In
6720	3	.691	.....	15680	7	1.800	
7840	3½	.821	.130	16800	7½	1.976	
8960	4	.948	.127	17920	8	2.228	
10080	4½	1.061	.113	19040	8½	2.455	
11200	5	1.186	.125	20160	9	2.742	
12320	5½	1.328	.142	20720	9¼	2.900	
13440	6	1.466	.138	" in 15 min		2.965	
14560	6½	1.630	.164	With 6800 lbs, def = $\frac{1}{480}$ span.			

A 15-inch beam. Weight 51⅔ lbs per foot run. Flanges 4 ins wide $\frac{9}{16}$ inch thick. Area of cross-section of beam, 15½ sq ins. Clear span 27 ft.

Center Load in Lbs.	Tons.	Deflection. Ins.	Increase. Ins.	Center Load in Lbs.	Tons.	Deflection. Ins.	In
6720	3	.342	.....	15680	7	.845	
7840	3½	.402	.060	16800	7½	.913	
8960	4	.462	.060	17920	8	.992	
10080	4½	.523	.061	19040	8½	1.063	
11200	5	.580	.057	20160	9	1.149	
12320	5½	.639	.059	22400	10	1.309	
13440	6	.707	.068	24640	11	1.505	
14560	6½	.778	.071	25760	11½	1.603	

In half an hour after removing the load, the 1.603 def became reduced to .26
With 12900 lbs, the def = $\frac{1}{480}$ span.

A heavier 15-inch beam. Weight 66⅔ lbs per foot run. Flanges wide. Stem $\frac{11}{16}$ inch thick. Area of cross-section of beam, 20 sq ins. Cle 14 ft.

Center Load in Lbs.	Tons.	Deflection. Ins.	Increase. Ins.	Center Load in Lbs.	Tons.	Deflection. Ins.	In
6720	3	.048	.....	33600	15	.222	
13440	6	.090	.042	40320	18	.258	
20160	9	.134	.044	47040	21	.305	
26880	12	.178	.044				

At 21 tons the apparatus gave way, and prevented further experiment
An hour after removing the load the deflection was .222 inch.

The Phœnix beams are used not only in buildings, but for bridges of smal Singly, and by themselves, or with a stout longitudinal strip of iron riveted flange they are analogous to Fairbairn's plate beam, (see Art 41:) and placed i with broader strips riveted to the two upper and two lower flanges, they re his box beam. See Art 31 of Trusses, p 304.

For these beams as pillars, see page 237; and as bridges, p 304.
A deck beam, is a rolled one with the section Fig 18.

Fig

. 38. Rolled Iron beams of the New Jersey Steel and Iron Company.

Made at the Trenton, N. J., Iron Works, Cooper, Hewitt & Co.
Morris, Wheeler & Co., Market & Sixteenth Sts., Philadelphia, agents.*
Similar to the Phœnix beams, Fig. 17, p 210. ‡

Depth of beam.	Thicks. of web.	Flange width.	Weight per yd.	Coef for ⅓ of the breakg load.	
Ins.	Ins.	Ins.	Lbs.	Lbs.	Tons.
15⅛	.6	5¾	200	748000	334
15 8/16	½	5	150	551000	246
12 5/16	.6	5½	170	511000	228
12¼	.48	4.79	125	377000	168
10½	.47	5	135	360000	161
10½	⅜	4½	105	286000	128
9	.58	4½	125	268000	120
9	⅜	4	85	189000	84.4
9	.3	3½	70	152000	67.9
8	⅜	4½	80	168000	75.0
8	.3	4	65	135000	60.3
7	⅜	3½	60	102000	45.5
6	.3	3½	50	76800	34.3
6	¼	3	40	62600	28.0
5	5/16	3	40	49100	21.9
5	¼	2¾	30	38700	17.3
4	5/16	3	37	36800	16.4
4	¼	2¾	30	30100	13.4
Channel iron, upright, thus].					
12¼	.68	4	140	381000	170
12¼	.42	3	85	238000	106
9	.43	3½	70	146000	65.2
9	.33	2½	50	104000	46.4
6	.40	2½	45	58300	26.0
6	.28	2¼	33	45700	20.4

Co also are extensive makers of other beams, rails, channel, angle, T, &c, &c. ble is one issued by them for finding the safe equally distributed loads for eams of the shape of Figs 17, p 210; said load being assumed to be ⅓ **of the nate or breaking one,** nearly.

give the following rules:

find the safe distributed load in lbs of any beam in the table: ut the corresponding number in the column of coefficients. Divide it by the pan in feet. Deduct the wt of the clear span of beam.

find the deflection in inches which this safe load will produce. Square an in ft. Divide this square by 70 times the depth of the beam in ins.

ample. What is the safe distributed load for the 4th beam, of 15 ft clear span; w much will the beam bend under this load?

e first, for the load, we have the coef for said beam, 377000. And $\frac{377000}{15\ (\text{span})} =$ lbs load. But we must deduct $125 \times 5 = 625$ lbs, the wt of 5 yds length of am itself; leaving 24508 lbs neat load.

t, for the def, we have the square of the span, or $15 \times 15 = 225$; and 70 times pth of said beam is $12\frac{1}{4} \times 70 = 857.5$; hence, $\frac{225}{857.5} = .262$ of an inch deflection.

262 of an inch is not as much as $\frac{1}{480}$ of the span; and is therefore not too much mit in a floor. See near top of page 197, "The extent to which," &c.

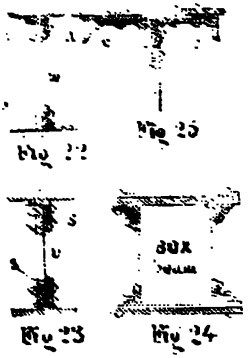
mark. Beams of no kind should be trusted with more *½ their breaking loads. If they are liable to receive heavy shocks, their loads not exceed from ⅓ to ¼ of the breaking ones, according to circumstances ections will diminish in the same proportion as the safe loads are reduce e may first find the load and deflection given by the table; and then mod uit any smaller assumed safe load.*

he most extensive and favorably known iron houses in Philadelphia.
in Philada.—1873. 15 inch beams to 20 ft long, 6½ cts per lb; 12¼ inch, to 25 long, 6 cts.

Art. 39. Cooper & Hewitt's box-beams. Fig [illegible] These beams consist of two [illegible] one at top, and one at bottom [illegible] channel-iron, 15 ins deep. They [illegible] heavier and variously [illegible] with a beam like the [illegible] span. The chain unconfined further [illegible] ways.

[illegible] Lbs	Deflection. Inches.	[illegible] Inches	Deflection. Inches.
[illegible]	[illegible]	[illegible]	$1\frac{1}{2}$
[illegible]	[illegible]	Interval of 25 days.	
[illegible]	[illegible]	[illegible]	$1\frac{11}{16}$
[illegible]	[illegible]	[illegible] as [illegible] nce a	$1\frac{13}{16}$
[illegible]	[illegible]	[illegible] some [illegible] commenced. In.	
[illegible]	[illegible]	[illegible] min.	$\frac{25}{16}$
[illegible]	[illegible]	[illegible] hour.	$\frac{27}{16}$
[illegible]	[illegible]	[illegible]	$3\frac{7}{8}$
[illegible]	[illegible]	With a [illegible] depth of [illegible]	$1\frac{1}{8}$
[illegible]	[illegible]	This [illegible] increased [illegible] the side plates gave way [illegible] tear bottom edges, in an [illegible] hour	
[illegible]	[illegible]		

[illegible] during the last day [illegible] With [illegible] lbs, def. = $\frac{1}{480}$ span.



Art. 41. Fairbairn plate, [illegible] beams, or girders. These ar[illegible] plates, and angle iron, rivete[illegible] The plates are usually from $\frac{1}{4}$ to $\frac{3}{8}$ [illegible] The angles vary from $2\frac{1}{2}$ by $2\frac{1}{2}$ by $\frac{3}{8}$ [illegible] inch. The rivets from $\frac{1}{2}$ to $1\frac{1}{8}$ [illegible] and driven from 3 to 6 ins apart, from [illegible] other shapes of rolled iron a[illegible] lately introduced, as the channel-i[illegible] &c. Figs 22 and 23 are com[illegible] constructing the plate beam for [illegible] and for bridges of moderate spans. F[illegible] other form. Frequently several thic[illegible] plates are riveted together, to fo[illegible] and bottom flanges, to obtain the re[illegible] section. The vertical web, v, or w, [illegible] frequently composed of two thickness[illegible] 24 of the box-beam, each of the two f[illegible] [illegible] two thicknesses. This form [illegible] when on a large scale, constitutes t[illegible] bridge.

Mr. Fairbairn directs to make the [illegible] top flange equal to $1\frac{1}{8}$ that of the l[illegible] He states that with his proportion no separate allowance need be made fo[illegible] but large experience sanctions it, and that no gain of streng[illegible] making either one of the flanges to exceed this limit. There is room [illegible] whether his proportion of the flanges is the best. More experiments are [illegible] He gives no rules for the thickness of the vert members. [illegible] channel-irons &c. are to be computed as part of the fla[illegible] as also any others used in the bottom flanges, are subject to [illegible] be strongly spliced together at their joints.

Many engineers, however, make both flanges of equal area; conseq[illegible] [illegible] safer in consequence of the weakness of the lower joi[illegible] flange being compressed, are not appreciably weakened by their joi[illegible]

e strength of the single-plate beam, Figs 22 and 23, Fair-
as follows:

$$\left.\begin{array}{l}\text{load in tons,}\\ \text{the weight}\\ \text{am itself,}\end{array}\right\} = \frac{\text{Area of bottom in sq ins} \times \text{Total depth of beam in ins} \times \text{Constant } 75}{\text{Clear span in ins.}}$$

box-beam, Fig 24, precisely the same, except use a constant 80 in-
Some very competent authorities, however, regard this as too great;
o that the constant should not exceed 75 in either case.
rt of the beam must be deducted, to obtain the neat center breakg load;
e of it for a uniform load. In beams for buildings, however, or in
r 40 to 50 ft span, this deduction will rarely be necessary.
ral approximation, when these beams exceed a very moderate span as
nt $\frac{1}{2}$ to $\frac{1}{3}$ part must be added to the wt of wrought iron as deduced from
sverse section, such as those in the above Figs, to allow for splicing-
ners, chucks, rivets, &c.

lowing girders have for many years sustained locomotive traffic:
23; single track; 2 girders, 15 ft apart from center to center; length
ft; clear span, 24 ft; total depth of girders, 28 ins; top and bottom T-
izontally $7\frac{1}{2} \times \frac{1}{2}$; vert flange, $3 \times \frac{1}{4}$; the four longitudinal splicing-
ach $6 \times \frac{1}{4}$; vert web v, $\frac{1}{4}$ thick. 2d, like Fig 22; single track; 2 gird-
rt from center to center; length of each 41 ft: clear span 37; total depth
0 ins; top plate, dotted, $8\frac{1}{2} \times \frac{1}{2}$; bottom plate, dotted, $16 \times \frac{5}{8}$; top
a, $2\frac{3}{4} \times 2\frac{3}{4} \times \frac{3}{8}$; bottom angle-irons, b, $3\frac{3}{8} \times 3\frac{3}{8} \times \frac{3}{8}$; vert web, w,
k. This is one of the first bridges of this kind built in the United States,
the proper proportions were known. 3d, like Fig 23; single track; 2
gth of each 68 ft; clear span, 60 ft; total depth of girders, 4 ft 3 ins; top
T-irons, t, $6 \times \frac{7}{16}$; vert flange, $3 \times \frac{1}{2}$; the 4 longitudinal splicing-plates
$\times \frac{1}{4}$; vert web, v, $\frac{3}{8}$ inch thick. Riveted to the top and bottom are
the dotted ones in Fig 22; each of these is 2 ft wide, $\frac{3}{4}$ inch thick at the
e girders, and $\frac{3}{8}$ at the ends. All have vertical stiffeners.

wing are in use on the **Charing Cross Railway,** England; all like
, for a single track, 41 ft clear span, total length 46 ft; total depth $2\frac{1}{2}$ ft;
otted, $16 \times \frac{3}{4}$; bottom plate $15 \times \frac{3}{4}$; vert web, w, $\frac{1}{4}$ thick; top angle-
$\times \frac{3}{4}$; bottom ones $4 \times 4 \times \frac{5}{8}$; rivets $\frac{5}{8}$ diam, 3 ins apart from center to
ght of 1 girder, $4\frac{1}{3}$ tons. 2d, for a single track, 51 ft clear span, total length
epth $3\frac{1}{2}$ ft; top plate, dotted, $18 \times \frac{7}{8}$; bottom plate $16 \times \frac{3}{4}$; vert web,
; top and bottom angle-irons, a, b, $4 \times 4 \times \frac{5}{8}$; rivets $\frac{3}{4}$ diam, 4 inches
center to center; weight of each girder, $5\frac{1}{2}$ tons. 3d. For three railway
rders only, 39 ft apart from cen to cen; clear span 73; total length 82;
, $7\frac{1}{2}$ ft at center of span; 6 ft at ends: top plate, dotted, 2 ft 2 ins wide;
f 4 thicknesses of plates, each $\frac{5}{8}$ thick, at center of span; and 3 such
e ends; bottom plate 2 ft wide. one thickness of $\frac{3}{4}$; and 3 of $\frac{5}{8}$ each, at
of the span; at the ends one of the $\frac{5}{8}$ is omitted. Vert web, w, $\frac{1}{4}$ thick
f span; $\frac{5}{8}$ for 11 ft at each end; top and bottom angle-irons, $6 \times 6 \times \frac{3}{4}$;
each girder, 25 tons; which would suffice for a single-track girder 100 ft
b stiffened every 3 ft; as at $t\,t$, Fig 28; by Ts of $6 \times 3 \times \frac{3}{8}$.
sverse iron floor-girders, when the main beams are but about 15 ft apart,
track, and the girders about 5 to 6 ft apart, are like the shaded parts of
at is, without top and bottom plates. Depth about 10 ins; vert web $\frac{1}{4}$
; 4 angle-irons, each $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$; weight about 800 lbs. In such of
g Cross railway bridges as have their main beams 39 ft apart, and where
rse floor-beams, also 39 ft long, support 3 tracks, the last, 3 ft apart, have
ng dimensions: Total depth, 16 ins; top and bottom plates, dotted, $15 \times$
ert web $\frac{1}{4}$ thick; four angle-irons of $5 \times 3 \times \frac{5}{8}$. Transverse floor-beams
pon the top of the main beams; or upon the inside portion of the bottom
e last is not as favorable to the strength of the main beams as the former;
the floor-beams are placed beneath the bottom of the main ones. In all
wo beams are riveted together.

2. The construction of plate and tubular girders is
ple as might be supposed from the foregoing Figs. They are composed of
eets of iron, not exceeding about 3×12 ft in breadth and length in the
dges; while in small ones, much smaller dimensions must be employed.
two plates come into contact, the joint, whether vert or hor, must be
ed by *riveting upon both* sides of it at least narrow strips of plate iron
ide, called covering-plates, or splicing plates. Or, since the girders re-
iffeners, the vert joints are frequently covered on both sides of the plates

by [illegible] rons, as *w w*, Fig 26; in which *a a* represent two plates wh is to be thus strengthened. When the middle web *w*, of the T-irons, d sufficiently t reqd stiffnes web, *a* or *g*, broad strips plate iron, r duced instea nection with as *b b*. Thes eted togethe vert plates *g* beam; and th

Fig 26.

Fig 27

eners *a a*; thus protecting the joint of the former, and holding the l place. These stiffening plates are frequently made to project more at top; and are at times [illegible] hened by angle-iron riveted along their at their base; thus m[illegible] them very effective as braces also.

Fig 28.

The transverse floor-bean ers, extending from one main beam are so spaced as to meet the stiffeners, ing upon the weaker intermediate par girders. These transverse beams thems requires to be stiffened in a similar ma vals of usually from 3 to 8 ft, as in th The stiffeners near the ends of the m those resting on the piers and abuts, in large spans) placed nearer togetl stouter than the others, because upon wt of bridge and load. The T and L to stiffen the sides of the Britannia tul weigh half as much as the plates whic sides.

Fig 29.

The vert T-irons *w*, or angle-irons *b*, generally h both at top and bottom, as shown at *i i*, Fig 28, in or hor angle-irons of the upper and lower flanges. Th long narrow strips *a*, Fig 28, which compose the t plates of the flanges, must also be connected toget splicing-plates. Those at the lower flanges require e this respect, inasmuch as they undergo tension. Th to the hor angle-irons *h h*, Fig 29, which also must nected at their ends. This is done by pieces *c c* o lengths of 18 to 24 inches.

Art. 43. Mr. William Fairbairn, of England, giv lowing table, calculated for a double-track railway, supporte beams, or by two girders He adopts 6 tons per foot run of the span breakg load of the bridge; or 12 tons per foot run, uniformly distribu the wt of the bridge itself. This 12 tons, he says, is equal to abou maximum load that can practically be brought upon the bridge. For s ft, he takes 1 ton per running foot as the wt of the bridge itself, and of the rolling load, or two trains; and from 100 to 300 ft spans, $1\frac{1}{2}$ ton the bridge; and $1\frac{1}{2}$ tons as that of the load; the total being 3 tons p cases.

The depths in the table are equal to $\frac{1}{12}$ of the span, for spans less t those exceeding 150 ft, they are $\frac{1}{13}$.* The breakg loads include the w itself. The depths are those at the center of the span. Except in quit some saving of material is effected by diminishing the depth from the ends. Single-track bridges will require but about $\frac{6}{10}$ the quantit given in the table; the depth of girder remaining the same. If the d ished, the areas of the flanges must of course be increased; or if increased, the flanges may be reduced; for, as is seen by the foregoing 44, the strength varies as the depth, when the flanges remain the same tions of the table will serve for plate beams, as well as box; the being but as 75 to 80.

* These depths are much smaller than is usual in truss bridges in the Unit $\frac{1}{10}$ to $\frac{1}{8}$ are the common ones; and are certainly to be preferred for practica

by Mr. Fairbairn, of the Proportions of Tubular or Bridges, consisting of two girders, of from 30 to t span; and intended to sustain two railway tracks. ufficiency of these dimensions has been questioned by high authority.)

an.	Center Breaking Load of Bridge, or of the two Girders.	Sectional Area of bottom of one Girder.	Sectional Area of top of one Girder.	Depth of a Girder, at the center of the Span.	
	Tons.	Inches.	Inches.	Feet.	In.
	180	14.63	17.06	2	4
	210	17.06	19.91	2	8
	240	19.50	22.75	3	1
	270	21.94	25	3	6
	300	24.38	[illegible]	3	10
	330	26.81	[illegible]	4	3
	360	29.25	34.13	4	7
	390	31.69	36.97	5	0
	420	34.13	39.81	5	5
	450	36.56	42.67	5	9
	480	39.00	45.50	6	2
	510	41.44	48.34	6	7
	540	43.88	51.19	6	11
	570	46.31	54.03	7	4
	600	48.75	56.88	7	8
	660	53.63	62 56	8	6
	720	58.50	68.25	9	3
	780	63.36	73.94	10	0
	840	68.25	79.63	10	9
	900	73.13	85.31	11	6
	960	90.90	105.00	10	8
	1020	95.63	111.56	11	4
	1080	101.25	118.13	12	0
	1140	106.88	124.69	12	8
	1200	112.50	131.25	13	4
	1260	118.13	137.81	14	0
	1320	123.75	144 38	14	8
	1380	129.38	150.94	15	4
	1440	135.00	157.50	16	0
	1500	140.63	164.06	16	8
	1560	146.25	170.63	17	4
	1620	151.88	177.19	18	0
	1680	157.50	183.75	18	8
	1740	163.13	190.31	19	4
	1800	168.75	196.88	20	0

girders are sometimes constructed with upper flanges of cast iron, or even .

44. Riveted joints. r steam boilers and water-terns, are usually propor-bout as per the following Fairbairn; and are made either by Fig 30, or Fig 30 is called a **single-d**, and Fig 31 a **dou-veted lap-joint.** The or c c, is the lap. Mr Fair-ates that the friction gen-etween the two plates by

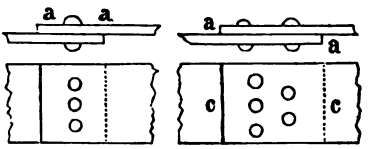
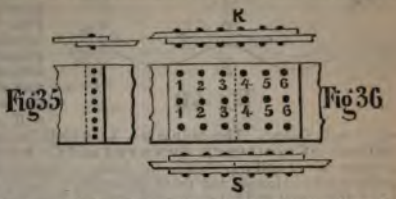


Fig 30 Fig 31

The tubular bridge will never be adopted by engineers. Trussed ones can be made much stronger, with much less material and expense. as Britannia tubular, each line of tubes carries inside of it but one track; and con. r cost more iron than is necessary for an equally strong Pink truss single-track bridge at the Britannia, and other tubes, shows that rust acts with fearful rapidity on the rivets of this iron. Moreover, the heads of the rivets become detached, and fall off m, owing to the combined effects of vibration, contraction, and expansion. It is s ours the rust has been arrested by careful painting.

ed; and when two are used, each is fully half as thick as one of the ence, however, proves that it is better in practice to make each of the r joints exposed to tension, nearly if not quite as thick as one of the

important feature in bridge joints, and which chiefly em from boiler joints, is that inasmuch as they are not required to ter tight, it is to place the together, and the plates by many rivet- two lines par- edges. There- ing the lap, as may diminish rivets which it e be necessary *e line or row lates*, as shown



may dispose of them, or of a greater number, by placing them in rows, 11, 22, 33, &c, **lengthwise** *of the plates*, as in Fig 36. The e use butt-joints and covering-plates, as at S. It is plain that in Fig h of the plate is diminished by the loss of the iron taken out of it for whereas in Fig 36 it is weakened by the loss of but the three holes in rows 11, 22, &c; for since the holes in these several rows are placed *ther*, it follows that a great number of such rows will reduce the and tensile strength of the plate no more than a single row; on the that a chain with one weak link will sustain the same tension or load ny such weak links, *any one* of which evidently limits the load. This Fairbairn's **chain riveting.** Since in this system the plate is in boiler riveting, (being less reduced in its cross-section,) it is only ke care that there shall be such a number of rows 11, 22, 33, &c, Fig ets also shall have at least an equal degree of strength with the plate: at may be no more liable to give way by the shearing-in-two of the the tearing-in-two of a plate.

rmination of the size and number of rivets, depends that the shearing and the tensile strengths of wrought iron are about ing say on an average 50000 lbs, or 22⅓ tons per sq inch of transverse re, in order that the united strengths of the rivets may be equal to ate where it is reduced by the united transverse areas *each* side of the joint, must e area of the reduced plate. d B, Fig 37, be two plates, a butt-joint *s*, and a cover- Then the reduced transverse te will be *equal to* the area us so much of it as is shown rk spaces, supposed to be e rivets. And the united transverse areas of the rivets on *each* side is equal to the areas of the *six circles* on each side of it. The two dark to be considered as semi-cylindrical, but each as having the thickness one dimension, and the diam of a rivet for the other.

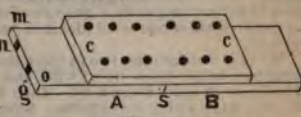


Fig 37

of a simple lap-joint, as Fig 38, the ts being of the same sizes as in the preceding vets would be reqd; because here each rivet *both* plates.

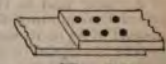


Fig 38.

t-joint with two covering-plates, Fig 36, the circular area of each rivet may be eat as in Fig 38, because in that case a rivet ithout being sheared through **in two places;** being thereby di- ee parts. The resistance to shearing is in proportion to the *area* herefore a rivet which exposes two such areas may the circular section of one which presents but one

Fig 39

pieces, *frequently reqd* in plate-iron bridges, *serted between two others* to prevent their being *gether by the rivets*; as the shaded piece in Fig

[illegible]

STRENGTH OF IRON PILLARS.

actical knowledge of the strength of pillars or columns, of different materials, ifferent forms of cross-section, is to this time very imperfect. The best es on iron pillars are Eaton Hodgkinson and Lewis Gordon, both of England. ults, however, differ considerably, although the rules of both are based in a asure upon the same experiments tried by Mr. Hodgkinson. Mr. Gordon's wever, are much the most simple, as well as more safe; and inasmuch as iple upon which they are founded seems to be equally as reliable as that y Hodgkinson, we shall confine ourselves to them. We do this with the fidence because a few experiments on a large scale with wooden pillars show gkinson's results are far too large.*

LLOW CYLINDRICAL IRON PILLARS.

on's rule for the breaking load in lbs. of hollow cylin- iron pillars, with flat ends, firmly fixed, and with the ressing upon them equally throughout. The thickness of or single thickness of metal, not to exceed about $\frac{1}{8}$ of the outer diam.

e area in square ins of solid metal contained in the ring, or transverse sec- e pillar. Square the length of the pillar in ins. Also square the outer ns. Then,

. **For Hollow Cast Iron Cylindrical Pillars,**

$$\text{Breakg load in lbs} = \frac{\text{Metal area in sq ins} \times 80000\,\dagger}{1+\left(\dfrac{\text{sq of length in ins}}{\text{sq of outer diam in ins}} \div 400\right)}$$

rds, mult the transverse metal area in sq ins, by 80000, and call the product the square of the length in ins, by the square of the diam in ins. Div the 400. To this last quot add 1. Div P by the sum. The result will be the l in lbs.

the breakg load in tons use 35.71 instead of 80000.

hat is the breakg load in lbs, of a round hollow cast iron pillar, 6 ins outer ft, or 120 ins long; and having a transverse metal area of 11 sq ins?

he sq of the length is $120^2 = 14400$ ins; and the $\text{diam}^2 = 6^2 = 36$ ins. Hence

$$\text{oad} = \frac{11 \times 80000}{1+\left(\dfrac{14400}{36} \div 400\right)} = \frac{880000}{1+1} = \frac{880000}{2} = 440000 \text{ lbs; or } 196.42 \text{ tons.}$$

equal to $\dfrac{196.42(\text{tons})}{11(\text{area})} = 17.86$ tons per sq inch of metal area.

ble 1, pages 224, 225.

. **For Hollow Wrought Iron Cylindrical Pillars.**

$$\text{Breakg load in lbs} = \frac{\text{Metal area in sq ins} \times 36000\,\dagger}{1+\left(\dfrac{\text{sq of length in ins}}{\text{sq of outer diam in ins}} \div 3000\right)}$$

the breakg load in tons, use 16.07 instead of 36000.

A wrought pillar of the same dimensions as the foregoing. Here,

ought iron columns shorten at the average rate of about $\frac{1}{8}$ feet, under loads of 4 tons per sq inch of metal cross-section; and cast-iron ones average e as much.

000 is Gordon's assumed crushing load in lbs (35.71 tons) per sq inch (a rather low and safe verage cast iron, in pieces about two diams high; the 36000 lbs (16.07 tons) is the corre- rushing load for wrought iron; both as assumed by Gordon. Therefore, to find the load tons, we may use 35.71 instead of 80000 in Rule 1; and 16.07 instead of the 36000 in Rule ing this, and at the same time omitting the metal area, we at once get the breakg load in

g inch. Thus, for the example in Rule 1, we should have $\dfrac{35.71}{1+\left(\dfrac{14400}{36} \div 400\right)} = \dfrac{35.71}{2} = 17.86$

inch; and for the example in Rule 2 we have $\dfrac{16.07}{1+\left(\dfrac{14400}{36} \div 3000\right)} = \dfrac{16.07}{1.133} = 14.18$ tons per

$$\text{Breakg load in lbs} = \frac{11 \times 36000}{1 + \left(\frac{14400}{36} \div 3000\right)} = \frac{396000}{1 + .1333} = \frac{396000}{1.1333} = 349411 \text{ lbs, or } 156 \text{ tons}$$

$$\frac{156 \text{ (tons)}}{11 \text{ (area)}} = 14.18 \text{ tons per sq inch of metal area.}$$

See Table 4, pages 228, 229.

HOLLOW SQUARE IRON PILLARS.

Breaking loads of hollow square iron pillars, with f ends, firmly fixed, and with the load pressing upon th equally throughout. The single thickness of the metal not to exceed a ⅛ of the width of a side.

Find the area of solid metal, in sq ins, contained in the square ring, or transv section of the pillar. Square the length of the pillar in ins; also square one o side in ins. Then,

RULE 3. **For hollow square cast iron pillars,**

$$\text{Breakg load in lbs} = \frac{\text{Metal area in sq ins} \times 80000}{1 + \left(\frac{\text{sq of length in ins}}{533.3 \times \text{sq of one side in ins}}\right)}$$

Or in words, mult the metal area in sq ins by 80000, and call the prod P. Mult square of an outer side by 533.3. Div the square of the length in ins by the p To the quot add 1. Div P by the sum.

For the breakg load in tons use 35.71 instead of 80000.

EX. A cast-iron hollow square pillar, ½ inch thick, 6 ins on each outer side; ft, or 120 ins, or 20 sides long; and having a transverse metal area of 11 sq ins.

Here the square of the length, or $120^2 = 14400$; and the sq of an outer side, o $= 36$. Hence

$$\text{Breakg load in lbs} = \frac{11 \times 80000}{1 + \left(\frac{14400}{533.3 \times 36}\right)} = \frac{880000}{1 + .75} = \frac{880000}{1.75} = 502857 \text{ lbs, or } 224.49 \text{ tons}$$

$$\frac{224.49 \text{ (tons)}}{11 \text{ (ins area)}} = 20.4 \text{ tons per sq inch; as per Table 8, page 232, for a square hol}$$

wrought pillar 20 sides high, Table 8 was drawn up from this rule and the a A round pillar of the same area, and 20 diams high, was found by Rule 5 to hav breakg load of 196.42 tons. Therefore, when both are 20 diams or sides high, square one is $\frac{224.5}{196.4} = 1.143$ times, or $1\frac{1}{7}$, as strong as the round one. The a proportion, however, does not hold good at other equal heights, as may be found comparing the round and square ones of Tables 7 and 8, p. 232.

See Table 8 and Rule, p. 232.

RULE 4. **For hollow square wrought iron pillars,** the same as cast iron, except that 36000 is to be used instead of 80000, and 6000 instead of 53

$$\text{that is, } \text{Breakg load in lbs} = \frac{\text{Metal area in sq ins} \times 36000}{1 + \left(\frac{\text{sq of length in ins}}{6000 \times \text{sq of one side in ins}}\right)}$$

For the breakg load in tons use 16.07 instead of 36000.

EX. A square hollow pillar of wrought iron, like the preceding.

$$\text{Breakg load in lbs} = \frac{11 \times 36000}{1 + \left(\frac{14400}{6000 \times 36}\right)} = \frac{396000}{1 + .0667} = \frac{396000}{1.0667} = 371238 \text{ lbs, or } 165.73 \text{ ton}$$

$$\text{hence, } \frac{165.73 \text{ (tons)}}{11 \text{ (ins area)}} = 15.07 \text{ tons per sq inch, as per Table 8, p 232.}$$

The breakg load of a given hollow square wrought i pillar may be found by Table 8, p 232, and rule at foot of said page.

SOLID CYLINDRICAL IRON PILLARS.

Breakg loads of solid cylindrical iron pillars, with f ends, firmly fixed, and with the loads pressing upon th equally throughout.

Find the area of metal in sq ins contained in the transverse section of the Square the length of the pillar in ins; also square its diam in ins. Then,

For solid cylindrical cast iron pillars.

$$\text{Breakg load in lbs} = \frac{\text{Metal area in sq ins} \times 80000}{1 + \left(\dfrac{\text{sq of length in ins}}{200 \times \text{sq of diam in ins}}\right)}$$

he breakg load in tons use 35.71 instead of 80000.

solid cylindrical cast-iron pillar 6 ins diam, and 10 ft, or 120 ins long.
e metal area in sq ins of the transverse section, by Table of Circles, p 18, is
he sq of the length in ins, or 120^2, is 14400; and the sq of the diam 6 is 36.

$$\text{reakg load in lbs} = \frac{28.274 \times 80000}{1 + \left(\dfrac{14400}{200 \times 36}\right)} = \frac{2261920}{1 + \left(\dfrac{14400}{7200}\right)} = \frac{2261920}{3} = 753973 \text{ lbs.}$$

$$\frac{3973}{240} = 336.6 \text{ tons.}$$

le 2, page 226, of such breaking loads.

For solid cylindrical wrought iron pillars.

$$\text{Breakg load in lbs} = \frac{\text{Metal area in sq ins} \times 36000}{1 + \left(\dfrac{\text{sq of length in ins}}{2250 \times \text{sq of diam in ins}}\right)}$$

he breakg load in tons use 16.07 instead of 36000.

olid cylindrical wrought iron pillar 6 ins diam; and 10 ft, or 120 ins long.
n the preceding case, the metal area is 28.274 sq ins; the sq of the length
14400; and the sq of the diam = 36 ins. Hence,

$$\text{treakg load in lbs} = \frac{28.274 \times 36000}{1 + \left(\dfrac{14400}{2250 \times 36}\right)} = \frac{1017864}{1 + \left(\dfrac{14400}{81000}\right)} = \frac{1017864}{1.18} = 862596 \text{ lbs.}$$

pillar and the foregoing wrought iron one are of the imensions; and each of them is 20 diams long. Hence at 20 diams
id wrought iron cylindrical pillar is $\frac{862596}{753973} = 1.144$ times as strong as a
whereas when both are *hollow*, and 20 diams long, the cast iron one is the
. See Table 5, page 230, of such breaking loads.

SOLID SQUARE IRON PILLARS.

kg loads of solid square iron pillars, with flat ends, fixed, and with the loads pressing upon them equally hout.

e area of metal in sq ins contained in the transverse section of the pillar.
ngth of the pillar in ins; also sq one side of the pillar in ins. Then,

. For solid square cast iron pillars.

$$\text{Breakg load in lbs} = \frac{\text{Metal area in sq ins} \times 80000}{1 + \left(\dfrac{\text{sq of length in ins}}{266.7 \times \text{sq of one side in ins}}\right)}$$

he breakg load in tons use 35.71 instead of 80000.

olid square cast iron pillar, 6 ins square, and 10 ft or 120 ins long. Here the metal area of
section is 36 sq ins. The sq of the length in ins, or $120^2 = 14400$; and the sq of one side in
= 36. Hence,

$$\text{Breakg load in lbs} = \frac{36 \times 80000}{1 + \left(\dfrac{14400}{266.7 \times 36}\right)} = \frac{2880000}{1 + \dfrac{14400}{9601}} = \frac{2880000}{2.5} = 1152000 \text{ lbs.}$$

le 3, page 227, of such breaking loads.

. For solid square wrought iron pillars.

$$\text{Breakg load in lbs} = \frac{\text{Metal area in sq ins} \times 36000}{1 + \left(\dfrac{\text{sq of length in ins}}{3000 \times \text{sq of one side in ins}}\right)}$$

the breakg load in tons use 16.07 instead of 36000.

olid square wrought iron pillar, 6 ins square, and 10 ft, or 120 ins long. Here the metal
ansverse section is 36 sq ins. The sq of the length in ins, or $120^2 = 14400$; and the sq of
ins, or $6^2 = 36$. Hence,

$$\textit{Breakg load in lbs} = \frac{36 \times 36000}{1 + \left(\dfrac{14400}{3000 \times 36}\right)} = \frac{1296000}{1 + \dfrac{14400}{108000}} = \frac{1296000}{1.133} = 1142100 \text{ lbs.}$$

6, page 231, of such breaking loads.

Table 1. HOLLOW CYLIND CAST IRON PILLARS.

Breaking loads, flat ends, perfectly true, and firmly fixed; and the loads pressing equally on every part of the top. By Gordon's formula.

For diams or lengths intermediate of those in the table, the loads may be found near enough by simple proportion.

For thicknesses less than those in the table, the breaking loads may safely be *assumed* to diminish in the same proportion as the thickness, while the outer diam remains the same. But for *greater* thicknesses than those in the table, the loads do not increase as rapidly as the new thickness. Still, in practice, they may be assumed to do so approximately **if the new thickness does not exceed about ⅛ part of the outer diam.**

Length in feet.	CAST IRON. THICKNESS ¼ INCH. (Original.) Outer Diameter in inches.											Length in feet.
	2	2¼	2½	2¾	3	3½	4	4½	5	5½	6	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	45.2	52.5	59.8	66.8	74.1	88.5	103.0	117.1	131.3	145.5	159.8	1
2	36.2	43.4	51.0	58.6	66.5	81.5	96.7	111.3	125.9	140.5	155.2	2
3	27.2	34.1	41.4	48.8	56.7	71.9	87.6	102.7	117.8	132.9	148.1	3
4	20.2	26.3	32.9	39.7	47.0	61.9	77.5	92.9	108.3	123.7	139.1	4
5	15.1	20.2	25.9	32.0	38.6	52.6	67.4	82.6	97.9	113.4	129.1	5
6	11.6	15.8	20.7	25.9	31.6	44.3	58.2	72.7	87.7	103.0	118.7	6
7	9.1	12.5	16.6	21.0	26.1	37.4	50.1	63.7	78.1	92.9	108.3	7
8	7.3	10.1	13.5	17.3	21.7	31.6	43.2	55.8	69.3	83.4	98.4	8
9	5.9	8.2	11.1	14.3	18.2	26.9	37.5	48.8	61.5	74.9	89.2	9
10	4.9	6.9	9.3	12.0	15.4	23.1	32.4	42.9	54.6	67.1	80.7	10
11	4.1	5.8	7.9	10.3	13.2	20.0	28.3	37.8	48.6	60.3	73.0	11
12	3.5	5.0	6.8	8.9	11.4	17.4	24.9	33.5	43.3	54.2	66.1	12
13	3.0	4.2	5.8	7.6	9.9	15.2	21.9	29.7	38.8	48.8	60.0	13
14	2.6	3.6	5.1	6.7	8.7	13.5	19.5	26.6	34.9	44.1	54.5	14
15	2.3	3.2	4.5	6.0	7.7	11.9	17.4	23.9	31.4	39.9	49.7	15
16	2.0	2.8	4.0	5.3	6.9	10.7	15.6	21.5	28.4	36.4	45.6	16
18	1.6	2.3	3.2	4.2	5.5	8.6	12.7	17.5	23.4	30.2	38.0	18
20	1.3	1.8	2.6	3.4	4.5	7.1	10.5	14.7	19.7	25.6	32.3	20
25	.....	.....	1.7	2.3	3.0	4.7	7.0	9.8	13.3	17.6	22.5	25
30	.....	.....	1.2	1.6	2.1	3.2	5.0	7.0	9.4	12.6	16.1	30
35	.....	.....	.....	.....	1.5	2.4	3.7	5.2	7.1	9.4	12.2	35
40	.....	.....	.....	.....	1.2	1.9	2.8	4.0	5.4	7.2	9.5	40
Weight of 1 foot of length of pillar in pounds.	4.31	4.91	5.53	6.13	6.75	7.97	9.22	10.4	11.7	12.9	14.1	
Area of ring of solid metal in square inches.	1.38	1.57	1.77	1.96	2.16	2.55	2.95	3.34	3.73	4.12	4.52	

Length in feet.	CAST IRON. THICKNESS ½ INCH. (Original.) Outer Diameter in inches.											Length in feet.
	5	5½	6	6½	7	7½	8	8½	9	10	11	12
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
2	239	268	297	325	354	383	412	440	469	526	583	640	2
4	203	235	266	296	326	356	387	416	445	504	563	622	4
6	166	196	227	257	288	319	350	380	410	471	532	593	6
8	131	159	188	218	248	279	310	340	371	432	494	557	8
10	103	127	154	182	210	240	270	300	330	391	454	517	10
12	82	103	126	151	177	205	233	262	292	351	413	475	12
14	66	84	104	126	149	174	200	227	255	313	372	434	14
16	54	69	87	106	126	149	173	198	224	277	334	394	16
18	45	58	73	90	108	128	149	172	196	246	300	357	18
20	37	48	62	77	93	111	130	151	173	219	270	323	20
22	32	41	53	66	80	96	113	132	151	194	241	293	22
25	25	34	43	54	65	79	93	109	126	164	206	252	25
30	18	24	31	39	48	59	70	83	96	126	160	199	30
35	13	18	24	30	36	44	53	63	73	98	126	159	35
40	10	14	18	23	28	35	42	50	59	78	102	129	40
45	8	11	15	19	23	28	34	41	48	64	84	107	45
50	6	9	12	15	19	23	28	33	39	53	70	89	50
60	4	6	9	11	14	17	20	24	28	38	50	65	60
70	3	4	6	8	10	12	15	18	21	29	38	50	70
80	3	4	5	6	8	9	11	13	16	22	30	38	80
Weight of 1 foot of length of pillar, in pounds.	22.1	24.5	27.0	29.4	31.9	34.4	36.9	39.4	41.9	46.9	51.8	56.8	
Area of ring of solid metal, in square inches.	7.07	7.85	8.64	9.43	10.2	11.0	11.8	12.6	13.4	14.9	16.5	18.1	

HOLLOW CYLIND CAST IRON PILLARS.

BREAKING LOADS.—(Continued.)

BY GORDON'S RULE.

CAST IRON. THICKNESS 1 INCH. (Original.)										Length in feet.
Outer Diameter in Inches.										
	14	15	16	18	20	22	24	27	30	36	
s.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
.	1415	1530	1645	1874	2103	2330	2557	2896	3236	3915	4
;	1368	1484	1601	1833	2066	2295	2525	2866	3208	3890	6
,	1303	1423	1543	1779	2015	2247	2479	2824	3170	3860	8
)	1231	1355	1475	1716	1957	2193	2430	2780	3128	3823	10
)	1152	1275	1399	1644	1889	2129	2369	2723	3076	3778	12
)	1071	1195	1320	1566	1813	2056	2300	2659	3016	3726	14
;	992	1114	1237	1484	1733	1979	2226	2589	2951	3668	16
;	913	1034	1155	1401	1651	1899	2147	2515	2879	3604	18
;	840	958	1077	1320	1568	1817	2066	2437	2805	3536	20
;	772	887	1002	1241	1486	1734	1982	2356	2726	3464	22
;	709	818	929	1163	1404	1650	1899	2272	2644	3387	24
;	651	756	863	1090	1326	1570	1816	2188	2560	3308	26
;	598	697	800	1020	1250	1489	1733	2103	2475	3226	28
;	549	644	743	954	1178	1412	1653	2020	2392	3143	30
,	506	595	689	893	1110	1338	1575	1938	2308	3059	32
)	467	552	641	836	1046	1268	1500	1857	2225	2974	34
)	432	511	596	783	984	1199	1427	1779	2143	2889	36
,	400	475	556	734	928	1136	1361	1704	2063	2804	38
,	370	441	518	687	874	1076	1296	1630	1984	2726	40
,	344	411	484	645	825	1019	1232	1560	1909	2636	42
,	321	383	452	605	776	963	1169	1491	1834	2553	44
,	299	358	423	569	734	915	1114	1428	1765	2474	46
,	280	335	397	536	694	869	1060	1367	1696	2396	48
,	261	314	373	505	656	824	1008	1306	1627	2319	50
-	222	269	320	438	573	725	893	1172	1474	2135	55
,	192	233	278	383	503	641	795	1054	1333	1964	60
,	165	202	242	335	443	568	709	951	1211	1808	65
,	146	178	213	297	394	507	636	853	1099	1664	70
;	113	139	168	235	315	408	516	705	914	1414	80
;	91	112	136	191	257	335	426	586	768	1209	90
)	75	92	112	157	213	279	356	491	651	1040	100
Weight of one foot of length of pillar, in pounds.											
	128	138	147	167	187	206	226	255	285	344	

CAST IRON. THICKNESS 3 INCHES. (Original.)										Length in feet.
Outer Diameter in Feet.										
;	4	4½	5	5½	6	7	8	10	12	
s.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
78	14908	16967	18986	21038	23052	27143	31196	39254	47375	10
54	14628	16712	18754	20825	22855	26972	31045	39133	47273	15
50	14250	16369	18440	20533	22585	26737	30837	38962	47131	20
02	13275	15459	17593	19743	21847	26084	30257	38486	46729	30
07	12113	14344	16531	18735	20891	25224	29416	37838	46175	40
02	10888	13126	15341	17580	19780	24107	28532	37037	45484	50
75	9690	11892	14100	16349	18570	23049	27460	36103	44666	60
70	8576	10702	12870	15097	17319	21824	26287	35058	43737	70
99	7570	9595	11692	13873	16070	20566	25048	33924	42712	80
53	6683	8588	10594	12706	14856	19304	23791	32725	41670	90
21	5909	7665	9588	11614	13700	18065	22521	31481	40438	100
88	5238	6888	8677	10606	12613	16869	21270	30213	39220	110
54	4400	5864	7483	9257	11132	14650	19448	27664	36692	125
50	3353	4547	5900	7417	9058	12701	16668	25182	34127	150
Weight of one foot of length of pillar, in pounds.										
0	1325	1503	1678	1856	2031	2386	2740	3444	4153	

thicknesses the breaking loads may safely be assumed to diminish at the thicknesses.

Table 2, of breaking loads, in tons, of SOLID CYLINDRICAL CAST IRON pillars, with flat ends, firmly fixed, and equally loaded. By Gordon's rule.

No pains have been taken to have the last figure of the loads perfectly correct in every case.

(Original.)

Length in feet.	DIAMETER OF CYLINDRICAL PILLAR IN INCHES.																							Length in feet.
	½	¾	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	1.81	6.92	16.31	30.0	47.8	69.5	95.1	124	157	194	234	278	324	375	429	487	548	614	683	754	829	908	990	1
2	.56	2.58	7.23	15.4	27.7	44.6	65.2	90.7	120	154	191	233	278	328	380	437	497	562	629	701	775	854	935	2
3	.26	1.26	3.75	8.5	16.3	28.8	44.7	63.6	86.1	115	146	184	224	270	319	373	430	492	557	626	698	776	856	3
4	.15	.73	2.24	5.2	10.3	18.0	28.9	43.4	61.6	85	111	143	177	217	260	310	362	421	480	546	614	685	765	4
5	.10	.48	1.48	3.5	7.0	12.5	20.4	31.2	45.2	63	84	110	139	174	211	255	301	353	408	469	532	601	673	5
6	.07	.33	1.04	2.5	5.0	9.1	15.0	23.1	34.1	48	65	87	111	140	171	209	250	296	344	399	457	521	587	6
7	.05	.25	.77	1.8	3.8	6.9	11.4	17.8	26.4	37	51	69	89	113	140	173	208	248	291	341	393	450	510	7
8	.04	.19	.60	1.4	2.9	5.3	9.0	13.9	20.9	30	41	56	73	93	116	144	174	209	247	291	338	389	443	8
9	.03	.15	.47	1.1	2.3	4.2	7.2	11.3	17.0	24	34	46	61	78	97	121	147	177	210	249	290	336	385	9
10	.02	.12	.38	.93	1.9	3.5	5.9	9.3	14.0	20	28	38	50	64	81	102	125	152	181	215	251	292	336	10
12	.....	.08	.27	.65	1.3	2.4	4.2	6.6	10.0	14	20	27	36	47	60	75	93	113	136	163	192	225	260	12
14	.....	.06	.20	.48	.99	1.8	3.1	4.8	7.4	11	15	20	27	35	46	58	72	88	106	127	150	176	205	14
16	.....	.05	.15	.37	.76	1.4	2.4	3.7	5.7	8.4	12	15	21	27	36	45	56	69	84	101	120	141	165	16
18	.....	.04	.12	.29	.63	1.1	1.9	2.9	4.6	6.7	9.4	12	17	22	29	36	45	55	68	81	97	115	135	18
20	.....	.03	.10	.24	.49	.90	1.5	2.4	3.7	5.4	7.6	10	14	18	24	30	38	46	56	67	81	96	112	20
25	.....	.....	.06	.15	.31	.58	1.0	1.6	2.4	3.5	4.9	6.8	9.1	11	15	19	24	29	37	44	54	63	75	25
30	.....	.....	.04	.10	.22	.40	.7	1.1	1.7	2.5	3.5	4.8	6.4	8.4	11	14	17	21	26	31	37	45	53	30
35	.....	.....	.03	.08	.16	.30	.5	.8	1.2	1.8	2.6	3.6	4.7	6.2	8	11	13	16	20	24	29	34	40	35
40	.....	.....	.02	.06	.12	.23	.4	.6	1.0	1.4	1.9	2.7	3.6	4.7	6	8	10	12	15	18	22	26	31	40
50	.....	.....	.01	.04	.08	.16	.3	.4	.6	0.9	1.3	1.8	2.4	3.1	4	5	7	8	10	12	15	17	20	50
	Weight of one foot of length of pillar, in pounds.																							
	.610	1.58	2.46	3.84	5.52	7.52	9.82	12.4	15.3	18.6	22.1	25.9	30.1	34.5	39.3	44.3	49.7	55.4	61.4	67.7	74.3	81.2	88.4	
	Area of solid metal, in square inches.																							

(Original.)

Length in feet.	ONE SIDE OF SQUARE PILLAR, IN INCHES.																							Length in feet.
	½	¾	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6	
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	2.82	10.25	23.2	41.5	64.8	93.0	126.0	163	205	252	303	359	419	483	552	626	704	787	874	965	1061	1162	1266	1
2	.93	4.15	11.3	23.4	41.0	64.7	92.7	127	166	210	259	313	372	435	503	576	653	735	822	913	1008	1108	1213	2
3	.44	2.14	6.1	13.6	25.4	42.2	64.5	92.3	125	164	209	259	313	373	438	509	584	664	748	837	931	1029	1133	3
4	.25	1.23	3.7	8.5	16.6	28.9	45.2	66.8	93.7	126	164	208	257	312	371	437	508	584	665	751	841	937	1037	4
5	.16	.80	2.5	5.8	11.5	21.1	34.9	49 7	70.6	97.0	129	166	208	257	310	370	434	505	581	662	747	839	935	5
6	.11	.56	1.7	4.1	8.3	14.9	26.1	37.3	54.3	75.6	102	134	170	212	258	311	369	433	502	577	657	744	835	6
7	.08	.42	1.3	3.1	6.3	11.3	18.8	29.0	42.6	59.3	81	108	138	174	215	262	314	371	435	503	576	656	741	7
8	.06	.32	1.0	2.4	4.9	8.9	14.8	23.0	34.0	47.9	66	89	115	146	181	222	268	320	376	438	505	578	656	8
9	.05	.25	.80	1.9	3.9	7.1	12.0	18.5	27.4	39.0	54	74	96	123	153	190	230	275	325	381	442	509	580	9
10	.....	.21	.65	1.6	3.2	5.9	9.9	15.5	23.1	33.2	46	62	81	104	130	162	197	238	282	333	388	448	514	10
12	.....	.14	.45	1.1	2.2	4.2	7.0	11.0	16.6	23.8	33	45	59	77	97	122	149	182	217	258	303	352	407	12
14	.....	.11	.34	.81	1.7	3.1	5.2	8.2	12.5	17.9	25	34	45	59	75	94	115	141	170	203	240	281	326	14
16	.....	.08	.26	.63	1.3	2.3	4.0	6.3	9.6	14.0	19	27	35	46	59	74	92	113	137	164	194	228	266	16
18	.....	.06	.20	.49	1.0	1.9	3.2	5.0	7.7	10.7	15	21	28	37	48	60	75	92	112	134	159	187	219	18
20	.....	.....	.17	.40	.83	1.5	2.6	4.1	6.3	9.1	13	18	23	31	39	49	62	76	93	112	133	157	184	20
25	.....	.....	.11	.26	.53	.96	1.7	2.7	4.1	6.2	9	12	15	20	26	33	41	50	62	74	89	105	124	25
30	.....	.....	.07	.18	.37	.68	1.2	1.9	2.8	4.2	6	8	11	14	18	23	29	36	44	52	63	75	89	30
35	.....	.....	.05	.13	.27	.50	.9	1.4	2.1	3.0	4	6	9	11	13	16	20	26	32	39	46	55	66	35
40	.....	.....	.04	.10	.21	.38	.7	1.1	1.6	2.3	3	4	6	8	10	12	15	20	25	30	35	43	51	40
50	.....	.....	.03	.06	.13	.25	.4	.7	1.1	1.5	2	3	4	5	7	9	11	13	16	20	24	28	33	50
	Weight of one foot of length of pillar, in pounds.																							
.781	1.76	3.13	4.88	7.03	9.57	12.5	15.8	19.5	23.6	28.1	33.0	38.3	44.0	50.0	56.5	63.3	70.5	78.1	86.1	94.5	103	113		
	Area of solid metal, in square inches.																							
.250	.563	1	1.56	2.25	3.06	4	5.06	6.25	7.56	9	10.6	12.3	14.1	16	18.1	20.3	22.6	25	27.6	30.3	33.1	36		

HOLLOW CYLINDRICAL WROUGHT IRON PILLARS.

Table 4, of breaking loads in tons of hollow cylindric wrought iron pillars, with flat ends, perfectly true, an firmly fixed, and the loads pressing equally on every pa of the top. Calculated by Gordon's formula. No pains have been taken to the last figure of the loads perfectly correct in every case.

(Original.)

Length in feet.	WROUGHT IRON. THICKNESS ¼ INCH. Outer diameter in inches.									
	¾	1	1¼	1½	1¾	2	2¼	2½	2¾	3
	BREAKING LOAD.									
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
1	3.64	5.27	6.88	8.50	10.1	11.7	13.2	14.8	16.4	18.0
2	2.94	4.64	6.32	8.00	9.6	11.2	12.8	14.5	16.1	17.8
3	2.30	3.86	5.57	7.28	8.9	10.6	12.2	13.9	15.6	17.3
4	1.77	3.13	4.74	6.36	8.1	9.9	11.6	13.3	15.0	16.7
5	1.36	2.51	4.07	5.66	7.3	9.1	10.8	12.5	14.2	16.0
6	1.04	2.03	3.46	4.91	6.6	8.3	9.9	11.6	13.4	15.2
7	.81	1.65	2.91	4.24	5.7	7.4	9.1	10.8	12.6	14.4
8	.61	1.36	2.46	3.67	5.1	6.7	8.3	9.9	11.7	13.5
9	.50	1.05	2.03	3.18	4.5	6.0	7.5	9.1	10.8	12.6
10	.41	.95	1.75	2.77	4.0	5.4	6.9	8.4	10.1	11.8
11	.34	.81	1.52	2.41	3.6	4.8	6.2	7.7	9.3	11.0
12	.29	.70	1.34	2.14	3.2	4.3	5.6	7.0	8.6	10.2
13	.24	.60	1.16	1.88	2.8	3.9	5.2	6.5	8.0	9.5
14	.21	.53	1.03	1.69	2.5	3.5	4.7	6.0	7.4	8.9
15	.19	.47	.91	1.50	2.3	3.2	4.3	5.5	6.9	8.2
16	.18	.42	.84	1.38	2.1	2.9	4.0	5.1	6.4	7.7
18	.14	.33	.67	1.11	1.7	2.4	3.4	4.4	5.6	6.8
20	.....	.27	.55	.91	1.4	2.0	2.8	3.7	4.7	5.8
25	.....	.....	.....	.....	.9	1.4	2.0	2.6	3.4	4.2
	Weight of one foot of length of pillar, in pounds.									
	.820	1.15	1.47	1.80	2.13	2.45	2.78	3.11	3.43	3.77
	Area of ring of solid metal, in square inches.									
	.246	.344	.442	.540	.638	.736	.835	.933	1.03	1.13

Length in feet.	WROUGHT IRON. THICKNESS ¼ INCH. Outer diameter in inches.									
	2	2¼	2½	2¾	3	3½	4	4½	5	5½	6
	BREAKING LOAD.										
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
1	21.9	25.4	28.3	31.4	34.5	40	47	53	60	66	72
2	21.1	24.3	27.6	30.7	33.9	40	47	53	60	66	72
3	19.9	23.1	26.4	29.7	33.0	39	46	52	59	65	71
4	18.6	21.8	25.3	28.5	31.9	38	45	51	58	64	71
5	17.0	20.4	23.5	27.3	30.7	37	44	50	57	63	70
6	15.4	18.8	22.1	25.7	29.2	36	43	49	56	62	69
7	13.9	17.3	20.5	23.8	27.8	34	41	47	54	61	68
8	12.5	15.6	19.1	22.3	25.9	32	40	46	53	60	67
9	11.2	14.2	17.5	20.6	24.3	30	38	44	51	58	65
10	10.0	13.0	16.1	19.1	22.7	29	37	43	50	57	64
11	9.0	10.7	15.7	17.6	21.1	27	35	41	48	55	62
12	8.1	10.6	13.5	16.4	19.6	26	33	40	46	54	61
13	7.3	9.6	12.4	15.1	18.2	24	31	38	44	52	59
14	6.6	8.8	11.3	14.0	17.0	23	30	36	43	51	57
15	6.0	8.0	10.4	12.9	15.8	21	28	34	41	49	55
16	5.5	7.3	9.5	12.0	14.6	20	27	33	40	47	54
18	4.5	6.0	8.0	10.3	12.7	18	24	30	37	43	50
20	3.8	5.1	6.8	8.7	11.0	16	21	27	34	40	47
25	.....	.....	.....	.....	7.9	12	16	21	27	33	39
30	.....	.....	.....	.....	.....	.....	13	17	22	27	32
35	.....	.....	.....	.....	.....	.....	10	14	18	22	27
40	.....	.....	.....	.....	.....	.....	.....	.....	14	18	23
45	.....	.....	.....	.....	.....	.....	.....	.....	11	15	19
50	.....	.....	.....	.....	.....	.....	.....	.....	8	12	16
	Weight of one foot of length of pillar, in pounds.										
	4.60	5.23	5.90	6.53	7.20	8.50	9.83	11.1	12.4	13.7	15.0
	Area of ring of solid metal, in square inches.										
	1.38	1.57	1.77	1.96	2.16	2.55	2.95	3.34	3.73	4.12	4.52

CYLINDRICAL WROUGHT IRON PILLARS.

Table 4, (Continued.) (Original.)

WROUGHT IRON. THICKNESS ½ INCH.										Length in feet.
Outer diameter in inches.										
5	6	6½	7	7½	8	8½	9	10	11	12	
BREAKING LOAD.											
ns.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
	139	152	166	177	189	201	214	238	263	290	2
	136	149	163	174	186	199	212	237	262	289	4
	132	145	158	171	184	197	210	235	261	288	6
	127	140	154	167	181	194	207	232	258	284	8
	123	136	149	162	176	189	203	228	254	280	10
	116	129	143	157	171	185	199	224	250	276	12
	108	122	137	151	165	179	194	219	245	272	14
	103	117	131	145	160	173	187	213	240	268	16
	97	110	124	138	153	166	180	207	235	263	18
	91	104	117	131	145	159	173	201	227	257	20
	83	96	109	123	138	151	165	192	220	250	22
	76	89	102	115	129	143	157	183	212	241	25
	63	74	87	100	113	127	141	167	195	224	30
	53	64	75	87	99	112	125	151	178	207	35
	44	53	64	75	86	98	110	135	163	190	40
	38	46	55	65	76	87	98	123	148	174	45
	32	38	47	56	66	76	87	109	133	158	50
	24	29	36	43	51	60	69	88	109	132	60
	18	23	28	34	40	48	56	73	91	111	70
	14	18	22	27	32	37	44	57	74	93	80
	11	14	18	22	26	31	36	49	63	78	90
	9	12	15	18	22	26	30	41	53	66	100
Weight of one foot of length of pillar, in pounds.											
3.2	28.8	31.4	34.0	36.6	39.3	42.0	44.7	49.7	55.0	60.3	
Area of ring of solid metal, in square inches.											
85	8.64	9.43	10.2	11.0	11.8	12.6	13.4	14.9	16.5	18.1	

Table 4, (Continued.) (Original.)

WROUGHT IRON. THICKNESS 1 INCH.											Length in feet.
Outer diameter in inches.											
14	15	16	17	18	20	22	24	26	28	30	
BREAKING LOAD.											
ns.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
53	704	753	805	854	955	1056	1157	1257	1357	1458	1
38	691	742	795	846	949	1049	1149	1248	1354	1457	10
35	651	702	759	810	913	1016	1120	1223	1327	1430	20
38	594	645	699	758	866	973	1077	1186	1289	1394	30
70	528	584	636	691	806	912	1027	1130	1237	1348	40
35	462	516	570	627	740	848	961	1067	1179	1294	50
48	400	452	505	559	669	781	891	1005	1115	1228	60
00	348	398	448	499	606	715	824	936	1046	1160	70
55	298	344	392	440	543	649	757	868	978	1092	80
22	261	303	347	392	489	590	694	800	910	1023	90
90	225	262	303	345	436	532	631	735	843	955	100
62	193	227	264	302	386	474	568	666	770	877	110
35	162	192	225	259	336	416	505	598	697	799	125
01	122	145	171	198	262	328	405	485	574	666	150
78	95	112	133	155	208	266	331	400	478	560	175
60	74	89	106	124	168	216	269	328	395	467	200
Weight of one foot of length of pillar, in pounds.											
136	147	157	168	178	199	220	241	262	283	304	
Area of ring of solid metal, in square inches.											
40.8	44.0	47.1	50.3	53.4	59.7	66.0	72.3	78.5	84.8	91.1	

king loads for less thicknesses **may safely be assume**
me rate as the thickness.

Table 5, of breaking loads, in tons, of SOLID CYLINDRICAL WROUGHT IRON pillars, with flat ends, firmly fixed, and equally loaded. By Gordon's rule.

No pains have been taken to have the last figure of the loads perfectly correct in every case.

(Original.)

Length in feet.	DIAMETER OF CYLINDRICAL PILLAR, IN INCHES.																									Length in feet.
	½	¾	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6			
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.			
1	2.51	6.37	11.9	18.9	27.6	37.9	49.7	63.1	78.1	94.6	112.8	132	154	177	201	227	255	284	315	347	381	416	454			1
2	1.56	4.95	10.0	16.8	25.5	35.7	47.4	60.7	75.7	92.3	110.5	129	151	175	199	225	253	282	313	345	379	414	451			2
3	.95	3.61	8.0	14.4	22.6	32.5	44.1	57.3	72.1	88.5	106.7	126	148	171	195	220	248	277	308	340	374	409	447			3
4	.62	2.52	6.2	11.9	19.5	29.0	40.2	53.0	67.7	83.9	102.0	121	143	166	190	215	243	272	303	335	369	404	442			4
5	.43	1.84	4.8	9.8	16.6	25.4	36.0	48.4	62.7	78.7	96.5	115	136	158	183	208	236	265	296	328	362	397	435			5
6	.31	1.40	3.8	8.0	14.0	22.1	32.0	43.9	57.6	73.1	90.4	109	130	152	177	202	230	258	289	320	353	389	427			6
7	.23	1.08	3.0	6.6	11.9	19.2	28.3	39.5	52.7	67.6	84.3	102	123	145	169	194	221	249	280	312	346	381	418			7
8	.18	.86	2.5	5.4	10.0	16.6	25.0	35.4	47.8	61.9	78.1	96	116	137	160	185	212	240	271	302	336	371	408			8
9	.15	.70	2.0	4.6	8.6	14.4	22.0	31.6	43.2	56.6	72.1	89	109	129	152	176	203	231	261	292	326	360	397			9
10	.12	.57	1.7	3.9	7.4	12.5	19.4	28.2	39.0	51.7	66.4	83	102	122	144	169	193	221	251	282	315	350	385			10
12	.08	.41	1.2	2.8	5.6	9.7	15.3	22.7	32.0	43.0	56.1	71	88	107	128	151	176	203	232	262	294	327	362			12
14	.06	.30	.93	2.2	4.3	7.6	12.2	18.3	26.3	35.8	47.4	61	76	93	113	134	158	183	211	240	271	303	337			14
16	.....	.23	.72	1.8	3.4	6.1	9.9	15.0	21.8	30.1	40.3	53	66	82	100	120	142	166	192	219	249	279	312			16
18	.....	.19	.58	1.4	2.8	5.1	8.2	12.5	18.3	25.5	34.4	45	57	71	88	106	126	148	173	199	227	256	288			18
20	.....	.15	.47	1.1	2.3	4.1	6.8	10.5	15.5	21.8	29.5	39	50	63	78	94	113	133	156	180	207	235	265			20
25	.....	.....	.31	.7	1.5	2.7	4.6	7.1	10.7	15.2	20.9	28	36	46	58	71	87	104	123	143	166	189	215			25
30	.....	.....	.22	.5	1.1	2.0	3.3	5.1	7.7	11.0	15.3	20	26	35	44	54	66	79	95	112	132	152	175			30
35	.....	.....	.16	.4	.8	1.4	2.4	3.7	5.7	8.3	11.7	15	20	27	34	42	53	63	76	90	106	123	143			35
40	.....	.....	.12	.3	.6	1.1	1.9	3.0	4.5	6.5	9.2	12	16	21	27	34	42	52	62	74	87	102	118			40
50	.....	.....	.08	.2	.4	.7	1.2	2.0	3.0	4.3	6.0	8	11	14	18	23	29	36	43	52	61	72	83			50
Weight of one foot of length of pillar, in pounds.	.655	1.48	2.62	4.09	5.90	8.02	10.5	13.3	16.4	19.8	23.6	27.7	32.1	36.9	41.9	47.3	53.1	59.0	65.5	72.2	79.2	86.6	94.3			

(Original.)

ONE SIDE OF SQUARE PILLAR, IN INCHES.

Length in feet.	½	¾	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6	Length in feet.
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
1	3.37	8.32	15.3	24.4	35.4	48.4	63.5	80.6	100	121	144	169	196	225	256	289	324	361	401	442	486	531	578	1
2	2.28	6.74	13.5	22.3	33.3	46.3	61.3	78.4	97	118	142	167	194	223	254	287	322	359	398	439	482	528	575	2
3	1.47	5.13	11.2	19.7	30.3	43.1	58.0	74.9	94	115	138	163	190	219	250	283	318	355	395	436	479	524	572	3
4	.99	3.82	9.05	16.8	27.0	39.3	53.9	70.7	89	110	133	158	185	214	245	278	313	350	390	431	474	519	567	4
5	.69	2.88	7.30	14.2	23.6	35.3	49.4	65.8	84	105	128	153	180	208	239	272	307	344	383	424	468	513	560	5
6	.51	2.22	5.89	11.9	20.4	31.5	44.9	60.7	79	99	121	145	172	201	232	265	300	337	376	417	460	505	552	6
7	.39	1.74	4.79	10.0	17.7	27.1	40.5	55.5	73	93	115	139	165	193	224	256	291	328	367	408	451	496	543	7
8	.30	1.40	3.94	8.4	15.3	24.6	36.4	50.6	67	86	108	132	158	186	216	248	283	319	358	398	441	486	533	8
9	.24	1.14	3.29	7.2	13.2	21.7	32.6	46.0	62	80	101	124	150	177	207	239	273	309	348	388	431	475	522	9
10	.20	.95	2.77	6.2	11.5	19.2	29.2	41.8	57	75	94	116	141	168	198	230	264	300	338	378	420	464	511	10
12	.14	.68	2.03	4.6	8.9	15.1	23.6	34.4	48	64	82	103	127	152	180	210	243	278	315	354	396	439	485	12
14	.10	.51	1.54	3.6	6.9	12.1	19.2	28.5	40	54	71	90	112	136	162	192	223	256	292	330	371	414	459	14
16	.08	.40	1.21	2.8	5.6	9.8	15.8	23.7	34	46	61	79	99	121	145	173	203	235	269	306	346	387	431	16
18	.06	.31	.98	2.3	4.6	8.1	13.1	20.0	29	40	53	69	87	107	130	156	185	215	248	284	322	362	404	18
20	.....	.26	.80	1.9	3.8	6.8	11.1	17.0	25	34	46	60	77	96	117	141	168	196	227	261	297	336	377	20
25	.....	.17	.52	1.2	2.5	4.6	7.6	11.7	17	24	33	44	57	72	89	109	132	156	183	213	245	279	316	25
30	.....	.12	.36	.9	1.8	3.3	5.4	8.5	13	18	25	33	43	55	69	85	104	124	147	173	201	231	263	30
35	.....	.09	.27	.6	1.3	2.4	4.1	6.5	10	13	19	25	33	43	55	68	83	100	120	142	166	192	220	35
40	.....	.06	.21	.5	1.0	1.9	3.2	5.0	8	11	15	20	27	35	44	55	67	82	99	117	137	160	185	40
50	.....	.04	.13	.3	.7	1.2	2.1	3.3	5	7	10	14	18	24	30	38	46	57	69	83	98	115	133	50

Weight of one foot of length of pillar, in pounds.

.83	1.88	3.33	5.21	7.51	10.2	13.4	16.9	20.9	25.2	30.0	35.3	40.9	46.9	53.4	60.2	67.5	75.3	83.4	91.9	101	110	120

Area of solid metal, in square inches.

.250	.563	1.00	1.56	2.25	3.06	4.00	5.06	6.25	7.56	9.00	10.6	12.3	14.1	16.0	18.1	20.3	22.6	25.0	27.6	30.3	33.1	36.0

Table 7. Deduced from Gordon.* (Original.)

HOLLOW ROUND PILLARS.

Length measd in diams.	Breakg loads per sq inch of metal area of transverse section.		Comparative strengths of cast and wrought hollow round pillars.	
	Cast.	Wrt.	Cast.	Wrt.
	Tons.			
1	35.6	16.0	1	.45
2	35.4	16.0	1	.45
3	34.8	16.0	1	.46
4	34.3	16.0	1	.47
5	33.5	15.9	1	.47
6	32.8	15.9	1	.48
7	31.7	15.8	1	.50
8	30.8	15.7	1	.51
9	29.6	15.6	1	.53
10	28.6	15.5	1	.55
11	27.4	15.4	1	.56
12	26.3	15.3	1	.58
13	25.1	15.2	1	.60
14	24.0	15.1	1	.63
15	22.9	14.9	1	.65
16	21.8	14.8	1	.68
17	20.7	14.6	1	.71
18	19.7	14.5	1	.74
19	18.8	14.3	1	.76
20	17.9	14.2	1	.79
21	17.0	14.0	1	.82
22	16.2	13.8	1	.86
23	15.4	13.6	1	.89
24	14.6	13.5	1	.92
25	13.9	13.3	1	.95
26	13.3	13.1	1	.98
26.4	13.0	13.0	1	1.00
27	12.7	12.9	1	1.01
28	12.1	12.7	1	1.05
29	11.5	12.5	1	1.09
30	11.0	12.4	1	1.13
31	10.5	12.2	1	1.16
32	10.0	12.0	1	1.20
33	9.60	11.8	1	1.23
34	9.18	11.6	1	1.26
35	8.80	11.4	1	1.29
36	8.42	11.2	1	1.33
37	8.08	11.0	1	1.37
38	7.75	10.9	1	1.41
39	7.44	10.7	1	1.44
40	7.14	10.5	1	1.47
41	6.87	10.3	1	1.50
42	6.60	10.1	1	1.53
43	6.36	9.93	1	1.56
44	6.12	9.77	1	1.60
45	5.90	9.59	1	1.63
46	5.68	9.42	1	1.66
47	5.48	9.25	1	1.69
48	5.28	9.09	1	1.72
49	5.10	8.93	1	1.75
50	4.93	8.77	1	1.78
55	4.17	8.00	1	1.92
60	3.57	7.30	1	2.05
65	3.09	6.67	1	2.16
70	2.70	6.10	1	2.26
75	2.37	5.59	1	2.36
80	2.10	5.13	1	2.45
90	1.68	4.34	1	2.58
100	1.37	3.65	1	2.66

Table 8. Deduced fr Gordon.* (Original.)

HOLLOW SQUARE PILLA

Length measd in sides.	Breakg loads per sq inch of metal area of transverse section.		Compar strengt cast and hollow sq.
	Cast.	Wrt.	Cast.
	Tons.		
1	35.6	16.0	1
2	35.4	16.0	1
3	35.1	16.0	1
4	34.6	16 0	1
5	34.1	16.0	1
6	33.5	16.0	1
7	32.7	16 0	1
8	31.8	15.9	1
9	31.0	15.9	1
10	30.1	15.8	1
11	29.2	15.8	1
12	28.2	15.7	1
13	27.3	15.7	1
14	26.2	15.6	1
15	25.3	15.5	1
16	24.2	15.5	1
17	23.3	15.4	1
18	22.3	15.3	1
19	21.4	15.2	1
20	20.4	15.1	1
21	19.6	15.0	1
22	18.8	14.9	1
23	18.0	14.8	1
24	17.2	14.7	1
25	16.5	14.6	1
26	15.8	14.5	1
27	15.1	14.4	1
28	14.5	14.3	1
29	13.9	14.2	1
30	13.3	14 0	1
31	12.7	13.9	1
32	12.2	13.8	1
33	11.8	13.6	1
34	11.3	13.5	1
35	10.8	13.3	1
36	10.4	13.2	1
37	10.1	13.1	1
38	9.70	13.0	1
39	9.30	12.9	1
40	8.93	12.7	1
41	8.60	12.6	1
42	8.28	12.4	1
43	8.00	12.3	1
44	7.71	12.2	1
45	7.45	12 0	1
46	7.19	11.9	1
47	6.95	11.7	1
48	6.72	11.6	1
49	6.50	11.4	1
50	6.28	11.3	1
55	5.35	10.7	1
60	4.61	10.0	1
65	4.00	9.43	1
70	3 50	8.84	1
75	3.08	8 30	1
80	2.75	7.77	1
90	2.21	6.88	1
100	1.81	6.02	1

*** To find the breaking load of a hollow pillar by this**

RULE. Mult the transverse area of solid metal in sq ins, by the corresponding number the table. The tabular quantities will be too great, if the single thickness of metal exceed *of the outer diam*, which is the one used in the tables.

For Phœnix pillars take the diam from out to out of the flanges.

s with rounded ends, as in Fig 1, are usually assumed to have
strength of those with flat ends, firmly fixed; or ⅔, if only one end
or bottom) is rounded. In iron bridges and roofs, the ends of pillars and
struts frequently have not full fair bearings for their ends;
tained by means of pins or bolts driven through (across)
ther one or both ends, as at *p*. These are about interme-
rength between those with flat and those with rounded
e writer is informed by Mr. C. Shaler Smith, as the results
ents by him. These remarks apply equally to hollow
d to both wrought and cast iron, and wood; but there is
l of uncertainty about this and all such matters. The
ged, jointed, or **pinned** are not applied to pillars
are merely rounded, as in Fig 1, but only to those, whether
aded, which have pins. See p 238.

Fig 1

kinson says that if each side of a long hollow square pillar of cast
ual to the diam of a round one; and if both have the
l area of transverse section, the square one will have
he strength of the round one. He also states that if,
fig, a cylindrical, a square, and an equilateral triangu-
ll solid, contain equal transverse sections, and of
al quantities of metal, the square one will be about
eaker; and the triangular one about $\frac{1}{12}$ stronger than
one. It would appear, however, that no such con-
ortion (ratio) exists between the strengths of pillars
engths, but of different forms of cross-sections. Thus
uare cast-iron one *a a*, is stronger than a round one
same area of metal, in these varying proportions:

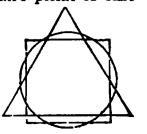


Fig 3.

are 10 diams (or sides) high, the sq one is 1.05 as strong.
20 ..... 1.14
30 ..... 1.21
40 ..... 1.25
60 ..... 1.29

ought iron, the ratios are different from cast.

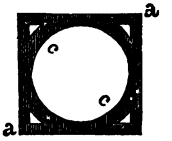


Fig 4

olled segment-column of the Phœnix Iron Co, of
, is coming into general use in iron bridges,
amon building purposes; for both of which it
be better adapted than any other device in
troduced. Fig 4½ shows a transverse section of
of only 4 segments, S S S S. The segments are
gether by rivets about six inches apart; by
langes along their sides, as shown in the Fig.
very two segments is (frequently, but not ne-
inserted an iron bar *a a*, called a *flat*, through
the rivets pass. These flats contribute much
ngth of the pillar. Cast-iron caps and bases are
e ends. An 8-segment column, 20¼ ft long, 16½
liam *s g s*; and $\frac{7}{16}$ thick, tested by hydraulic
howed no indication of weakness under a pres
, or 3.2 tons to each square inch (55.6) of its *en-*
erse area, including the flats.† See p 238.

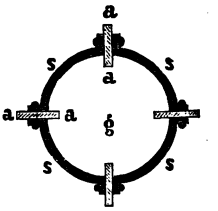
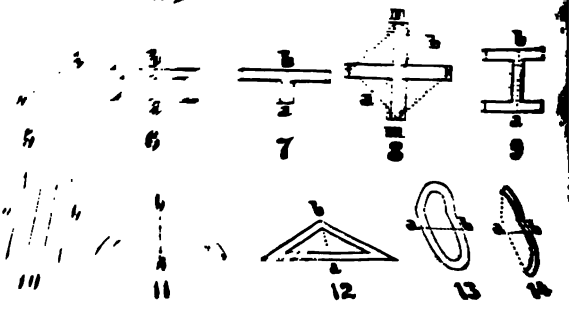


Fig 4½

D, Fig 42 of Trusses, is shown a combination of this column, with the Phœnix beams
ords of bridges &c, p 294.
hœnix pillars, Philada, 1873, 7¾ to 10 cts per lb.


5
6
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14

n such forms of cross-section as Figs 5 to 14, &c, in the greatest accuracy according to present theory, we must substitute s **the square of the least radius of gyration** (see Gloss-ss-section: in place of the diam or breadth, as the case may be. Thus, struts of cast or wrought iron, Gordon's Rules 1 and 2, on page 221, the following shapes:

Rule 1.

$$\frac{\text{of metal in sq ins} \times 80000}{+\left(\dfrac{\text{Length}^2 \text{ in ins}}{3200 \times \text{Rad gyr.}^2}\right)}$$

Rule 2.

$$\text{Breakg load in lbs} = \frac{\text{Area of metal in sq ins} \times 36000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{36000 \times \text{Rad gyr.}^2}\right)}$$

the *forms* of our rules 1 to 8 would not change the *results.* fied by inserting the actual square of the radius of gyration, the veral shapes would read thus, all the dimensions being in inches:

	CAST IRON.	WROUGHT IRON.
■	$\dfrac{\text{Metal area} \times 80000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{3200 \times \dfrac{\text{least side}^2 \text{ in ins}}{12.}}\right)}$	$\dfrac{\text{Metal area} \times 36000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{36000 \times \dfrac{\text{least side}^2 \text{ in ins}}{12.}}\right)}$
ow □	$\dfrac{\text{Metal area} \times 80000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{3200 \times \dfrac{\text{one side}^2 \text{ in ins}}{6.}}\right)}$	$\dfrac{\text{Metal area} \times 36000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{36000 \times \dfrac{\text{one side}^2 \text{ in ins}}{6.}}\right)}$
ow]a	$\dfrac{\text{Metal area} \times 80000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{3200\left(\dfrac{c^2}{12} \times \dfrac{c + 3a}{c + a}\right)}\right)}$	$\dfrac{\text{Metal area} \times 36000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{36000\left(\dfrac{c^2}{12} \times \dfrac{c + 3a}{c + a}\right)}\right)}$
●	$\dfrac{\text{Metal area} \times 80000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{3200 \times \dfrac{\text{Diam}^2 \text{ in ins}}{16.}}\right)}$	$\dfrac{\text{Metal area} \times 36000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{36000 \times \dfrac{\text{Diam}^2 \text{ in ins}}{16.}}\right)}$
ow ○	$\dfrac{\text{Metal area} \times 80000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{3200 \times \dfrac{\text{Diam}^2 \text{ in ins}}{8.}}\right)}$	$\dfrac{\text{Metal area} \times 36000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{36000 \times \dfrac{\text{Diam}^2 \text{ in ins}}{8.}}\right)}$
a ✚ o	$\dfrac{\text{Metal area} \times 80000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{3200 \times \dfrac{a\ o^2 \text{ in ins}}{24.}}\right)}$	$\dfrac{\text{Metal area} \times 36000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{36000 \times \dfrac{a\ o^2 \text{ in ins}}{24.}}\right)}$
, of ▬ o	$\dfrac{\text{Metal area} \times 80000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{3200 \times \dfrac{n\ o^2 \text{ in ins}}{24.}}\right)}$	$\dfrac{\text{Metal area} \times 36000}{1 + \left(\dfrac{\text{Length}^2 \text{ in ins}}{36000 \times \dfrac{n\ o^2 \text{ in ins}}{24.}}\right)}$

re believe, is meant that the single thickness of metal shall not *exceed* about ¼ part de. When the thickness is greater than ⅙ diam, or side, the loads given by the appreciably *too great for practical use.*

	CAST IRON.	WROUGHT IR
Angle iron of ribs of unequal lengths, but equal thicknesses.	$\dfrac{\text{Metal area} \times 80000}{1+\left(\dfrac{\text{Length}^2 \text{ in ins}}{3200 \times \left(\dfrac{o^2 \times c^2}{12(o^2+c^2)}\right)}\right)}$	$\dfrac{\text{Metal area} \times 36000}{1+\left(\dfrac{\text{Length}^2 \text{ in in}}{36000 \times \left(\dfrac{o^2 \times}{12(o^2+}\right.}\right.}$
H iron. Call the area of the web W, and that of the *two* flanges *a o*, *c n*, F.*	$\dfrac{\text{Metal area} \times 80000}{1+\left(\dfrac{\text{Length}^2 \text{ in ins}}{3200 \times \left(\dfrac{m^2}{12} \times \dfrac{\text{area W}}{\text{area F}}\right)}\right)}$	$\dfrac{\text{Metal area} \times 36000}{1+\left(\dfrac{\text{Length}^2 \text{ in in}}{36000 \times \left(\dfrac{m^2}{12} \times \dfrac{\text{are}}{\text{are}}\right.}\right.}$

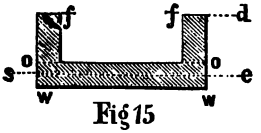


Fig 15

Channel iron. Let *d e* be the de the outer end *d*, of a flange *f o*, to the mi *e s* of the thickness of the web *w w o o* area of the web *w w o o*, in sq ins; a united areas of the two flanges *f o*, *f o*. **for cast iron,**

$$\text{Brkg load in lbs} = \frac{\text{Metal area} \times 80000}{1+\left(3200 \times de^2 \times \left(\dfrac{\text{area F}}{12 \times (\text{area F} + \text{area W})}\right) \times \dfrac{\text{area F} \times \text{are}}{4 \times (\text{area F} + \text{are}}\right.\right.}$$

For wrought iron, instead of 80000, and 3200, use 36000 in both e

The formulas give equal strengths to Figs 16, 17 a


Fig 16


Fig 17


Fig 18

The following table shows parative breaking strengths of holl drical pillars, and those of L, +, H se equal arms. The heights are measure or arms. The areas of metal cross se well as the outer diams and arms, are ail, as in Figs 16, 17 and 18. Deduced foregoing formulas.

(Original.)

Ht. in diams or arms.	Cast O	Wrt. O	Cast L, + or H	Wrt. L, + or H
10	1	.55	.71	.53
15	1	.65	.58	.61
20	1	.79	.50	.71
25	1	.95	.45	.81
30	1	1.13	.42	.91
35	1	1.29	.40	1.01
40	1	1.47	.38	1.09
50	1	1.78	.37	1.22
60	1	2.05	.36	1.32
70	1	2.26	.35	1.40
80	1	2.45	.35	1.45
100	1	2.66	.34	1.53

Phœnix beams as pillars.

Some rude and inconclusive experiments with the various patterns of **Phœnix beams**, Fig 17, p 210, employed **as pillars**, seemed to s with lengths equal to 30 times the width of top or bottom flange, their br *need not* be assumed at less than about 10 tons per sq inch, or say ¾ of t *for wrought-iron* cylinders of 30 diams, in Table 7; or $\frac{7}{10}$ of that for hollo *of 30 sides in length*, in Table 8, p 239.

* *In the case mentioned* immediately after these formulas, the web is supposed to ta *tirely across the H*; but we are not certain that it should not be taken only in the dea flanges. *If*, however, this is done, the result makes the H weaker than either the L o

es of cast iron for bridges, &c, it is usual among English : to allow more than 2½ tons (5600 lbs) of compression, or thrust, per sq l never subjected cast-iron pillars to more than 1½ tons (3360 lbs) per Shaler Smith * employs as maximum working strains, $\frac{1}{5}$ of the calcu- g strain for such hollow chords and posts of bridges as are 1 inch or ness, and not more than 15 diams long. For posts, only $\frac{1}{7}$; when not inch thick, nor more than 25 diams long; or from $\frac{1}{10}$ to $\frac{1}{20}$, when ⅝ and more than 25 diams long.

ing engineer must bear in mind that the breakg and the r sq inch, of pillars of any given material, are not constant quantities; as the piece becomes longer in proportion to its diam. If a very long ar be so braced at intervals as to prevent its bending at those points, th becomes virtually diminished, and its strength increased. Thus, if a long be sufficiently braced at intervals of 20 ft, then the load sustained due to a pillar only 20 ft long. Therefore, very long pillars used for &c, are thus braced; as are also long horizontal or inclined pieces, ompression in the form of upper chords of bridges; or as struts of any ges, roofs, or other structures.

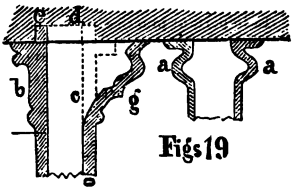
re sometimes made by assuming, say 5 or 6 tons per sq inch, as the safe load for cast iron; 4 tons for wrought; 1000 pounds for timber; without o the length of the piece.

gh the final crushing loads, as given in tables of strengths of materials, hose for pieces not more than about 2 diams high, they will not be much s not exceeding 4 or 5 diams.

remembered that a heavily loaded cast-iron pillar is easily broken by a Cast-iron ones are subject to hidden voids. All are subject to jars and om moving loads. It very rarely happens that the pres is equally dis- the whole area of the pillar; or that the top and bottom ends have per- at every part, as they had in the experimental pillars.† Cast pillars are ctly straight, and hence are weakened.

pillars intended to bear heavy loads should not be cast ouldings as *a a*; or with very ases or caps such as *g*, Fig 19. at these are weak, and would ider a much less load than the shaft of the pillar. When ing ornaments are required, be cast separately, and be at- prolongation of the shaft, as ins or rivets.

, it is better to adopt a more of base and cap, which, as at t in one piece with the pillar, kening it.



Figs 19

Strength of wooden pillars.

states that *while the quantity of material is the same s*, no strength is gained in *hollow* ones by making **s greater at the middle than at the** that in *solid* ones, *with rounded ends*, there is a gain part; and in those *with flat ends*, of about $\frac{1}{8}$th or making the diam at the middle about 1½ or 2 times ads. Also that a uniform round pillar has the same moderately tapering one whose diam at half-way up e uniform diam of the cylindrical one.

Fig 2

when a flat-ended pillar, Fig. 2, is so irregularly e pressure upon it passes along its diagonal *a a*, it rds of its strength. Hence the necessity for equalizing, as far as possi- ure over every part of the top and bottom of the pillar; a point very cure in practice.

skilful, experienced, and intelligent firm of Smith & Latrobe, civ engs, and bridge- sore, Md., The Baltimore Bridge Co.

t cases, both ends of iron pillars should be turned or planed perfectly true; as in &c.

Experiments, May, 1873, with Phœnix columns or pillars of lengths varying from 1 to 54 diams measured across the barrel, or circular part, showed that flat ended ones do not *cripple* until their loads are about 1.5 or 1.6 times as great as the ultimate ones deduced from Gordon's rule (p 221) for hollow circular columns; and that when applying his rule to the Phœnix, the diam may safely be taken *from out to out of the flanges.** This does not imply any defect in Gordon's rule. On the contrary, all the experiments showed a satisfactory coincidence in the comparative strengths of the cylindrical and of the Phœnix; thus strongly confirming the correctness of Gordon's views. They merely proved that the projecting flanges of the Phœnix are a stronger disposition of the metal than to use it in increasing the *thickness* of the barrel. The flanges beyond the outer circumf of the circle contain about .6 as much iron as the circle itself; and if they were used to give an increase of .6 to the *circumf* (and of course the *diam*) of the circle, without altering its thickness, the resulting cylindrical column would be at least as strong as the Phœnix with the same quantity of metal. The writer indeed is of the opinion that it would be stronger, inasmuch as Gordon's coefficient of 36000 lbs relates, very properly, to the *limit of elasticity* of average wrought iron; whereas the experimental Phœnix columns were loaded until they became slightly *crippled*. The Phœnix however possesses great advantages over a circular column in its facilities for attaching braces, ties, &c; and moreover manufacturers have not yet succeeded in making large cylindrical columns of wrought iron. **A Phœnix with both ends rounded** showed .54 of the strength of the flat ended one. Hodgkinson gives but about .3 for round ended *solid cast iron* cylindrical ones. This part of the subject, as well as that of pinned pillars, is but imperfectly understood.

WOODEN PILLARS.

The strengths of pillars, as well as of beams of timber, depend much on their degree of seasoning. Hodgkinson found that perfectly seasoned blocks, 2 diams long, required, in many cases, twice as great a load to crush them as when only moderately dry. This should be borne in mind when building with green timber.

In important practice, timber should not be trusted with more than $\frac{1}{8}$ to $\frac{1}{6}$ of its calculated crushing load; and for temporary purposes, not more than $\frac{1}{6}$ to $\frac{1}{4}$.

The strengths of wooden pillars, Hodgkinson's rules, are entirely too great.

Mr. Charles Shaler Smith, C.E., of Baltimore, prepared the following formula for the breaking loads of either square or rectangular pillars or posts, of moderately seasoned white, and common yellow pine, with flat ends, firmly fixed, and equally loaded, based upon experiments by himself.

It is Gordon's formula adapted to those woods; and gives results but about half as great as Hodgkinson's; although these pines are probably of nearly the same strength as his red deal.

Call either side of the square, or the least side of the rectangle, the *breadth*. Then

Rule. $$\left.\begin{array}{r}\text{Breakg load in lbs, per}\\ \text{sq inch of area, of a}\\ \text{pillar of W or Y pine}\end{array}\right\} = \frac{5000\dagger}{1 + \left(\begin{array}{c}\text{sq of length in ins}\\ \text{sq of breadth in ins}\end{array} \times .004\right)}$$

Or in words, square the length in ins; square the breadth in ins; div the first square by the second one; mult the quot by .004; to the prod add 1; div 5000 by the sum.

Ex. Breakg load per sq inch, of a white pine pillar 12 ins square, and 30 ft, or 360 ins long. Here the sq of length in ins is $360^2 = 129600$. The square of the breadth is $12^2 = 144$; and $\frac{129600}{144} = 900$; and $900 \times .004 = 3.6$; and $3.6 + 1 = 4.6$. Finally, $\frac{5000}{4.6}$ $= 1087$ lbs, the reqd breakg load per sq in. As the area of the pillar is 144 sq ins, the entire breakg load is $1087 \times 144 = 156528$ lbs, or 69.9 tons.

Recent experiments on wooden pillars 20 ft long, and 13 ins square, by Mr. Kirkaldy, of England, confirm the far greater reliability of Mr. Smith's formula. Hence we present the following new set of original tables based upon it.

For solid pillars of cast iron and of pine, whose heights range *from 5 to 60 times* their side or diam, we may say, near enough for practice, that a *cast iron one is about* $16\frac{1}{2}$ times as strong as a pine one; but no such approximate *ratio holds good* between wrought iron and pine, or between cast and wrought iron.

* *Each projects* 1 inch in a 4 inch column; $1\frac{3}{4}$ in a 6 inch; $1\frac{3}{4}$ in all larger.
† *The breaking load* in lbs per sq inch in short blocks, by Mr. Smith.

breaking loads in tons of square pillars of half white or common yellow pine firmly fixed and ded. By C. Shaler Smith's formula. (Original.)

Side of square pine pillar, in inches.

1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	Height in feet.
BREAKING LOAD.											
Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
3.99	5.73	7.80	10.1	12.8	15.7	18.9	22.3	26.1	30.1	34.5	1
3.59	5.26	7.25	9.6	12.2	15.1	18.3	21.7	25.4	29.2	33.7	½
3.19	4.80	6.74	9.0	11.6	14.5	17.6	21.0	24.7	28.6	33.0	½
2.81	4.35	6.19	8.4	10.9	13.7	16.8	20.2	23.9	27.8	32.1	¾
2.48	3.92	5.66	7.8	10.2	12.9	15.9	19.3	23.0	26.9	31.2	2
2.19	3.53	5.17	7.2	9.6	12.3	15.2	18.5	22.0	25.8	30.1	½
1.93	3.16	4.70	6.7	8.9	11.5	14.3	17.6	21.1	24.9	29.1	½
1.71	2.85	4.29	6.2	8.3	10.8	13.5	16.7	20.1	23.8	28.0	¾
1.52	2.55	3.89	5.6	7.6	10.0	12.7	15.8	19.2	22.9	27.0	3
1.21	2.07	3.23	4.8	6.6	8.8	11.3	14.2	17.4	20.9	24.8	½
.98	1.70	2.70	4.0	5.7	7.7	9.9	12.7	15.7	19.0	22.7	4
.81	1.42	2.28	3.4	4.9	6.7	8.8	11.4	14.1	17.2	20.7	½
.68	1.19	1.94	3.0	4.2	5.8	7.7	10.0	12.6	15.5	18.8	5
.57	1.02	1.67	2.6	3.7	5.1	6.8	8.9	11.3	14.0	17.1	½
.49	.86	1.44	2.3	3.3	4.6	6.1	8.0	10.2	12.7	15.6	6
.43	.74	1.26	2.0	2.9	4.1	5.4	7.2	9.2	11.6	14.2	½
.37	.66	1.11	1.8	2.6	3.6	4.9	6.5	8.3	10.5	12.9	7
.33	.59	.98	1.6	2.3	3.3	4.4	5.9	7.6	9.6	11.8	½
.29	.52	.87	1.4	2.0	2.9	3.9	5.2	6.8	8.7	10.8	8
.26	.47	.78	1.2	1.8	2.6	3.5	4.8	6.2	7.9	9.9	½
.23	.42	.71	1.1	1.6	2.3	3.2	4.3	5.6	7.2	9.1	9
.21	.37	.64	1.0	1.5	2.1	2.9	3.9	5.1	6.6	8.4	½
.19	.34	.58	.93	1.4	2.0	2.7	3.6	4.7	6.1	7.8	10
.17	.31	.53	.86	1.3	1.8	2.5	3.4	4.4	5.7	7.2	½
.16	.28	.48	.79	1.2	1.7	2.3	3.1	4.1	5.3	6.7	11
.14	.26	.44	.72	1.1	1.5	2.1	2.9	3.8	4.9	6.2	½
.13	.24	.41	.65	1.0	1.4	2.0	2.7	3.4	4.5	5.8	12
.....	.21	.35	.55	.84	1.2	1.7	2.3	3.1	4.0	5.0	13
.....	.18	.31	.46	.70	1.0	1.4	2.0	2.7	3.5	4.4	14
.....	.....	.27	.41	.63	.91	1.2	1.7	2.4	3.1	3.9	15
.....	.....	.24	.37	.57	.78	1.1	1.5	2.1	2.7	3.5	16
.....	.....	.....	.....	.50	.70	1.0	1.4	1.9	2.4	3.1	17
.....	.....	.....	.....	.45	.66	.92	1.3	1.7	2.2	2.8	18
.....	.....	.....	.....	.....	.....	.76	1.0	1.4	1.8	2.3	20

Side of square pine pillar, in inches.

4¾	5	5¼	5½	5¾	6	6¼	6½	6¾	7	7¼	Height in feet.
BREAKING LOAD.											
Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
45.7	51.1	56.8	62.8	69.0	75.5	82.3	89.4	96.8	104.5	112.4	2
41.1	46.2	51.8	57.7	63.8	70.2	76.9	84.0	91.3	98.9	106.7	3
35.9	40.8	46.1	51.7	57.6	63.8	70.4	77.3	84.5	92.1	99.9	4
30.8	35.4	40.5	45.8	51.5	57.4	63.7	70.3	77.2	84.5	92.1	5
26.4	30.5	35.2	40.1	45.4	51.0	57.0	63.3	69.9	76.8	84.1	6
22.6	26.2	30.5	35.0	39.9	45.0	50.5	56.5	62.8	69.4	76.3	7
19.2	22.5	26.3	30.4	34.9	39.7	45.9	50.4	56.2	62.4	69.0	8
16.5	19.5	22.9	26.6	30.7	35.0	39.9	44.8	50.2	56.0	62.1	9
14.2	16.9	19.9	23.2	26.9	30.9	35.2	39.9	44.9	50.3	56.0	10
12.4	14.8	17.5	20.4	23.8	27.4	31.3	35.6	40.2	45.1	50.4	11
10.7	12.9	15.4	18.0	21.1	24.3	27.9	31.8	36.0	40.6	45.5	12
9.4	11.4	13.6	16.0	18.8	21.7	24.9	28.5	32.4	36.6	41.1	13
8.3	10.1	12.1	14.2	16.7	19.4	22.4	25.7	29.2	33.1	37.3	14
7.4	9.0	10.8	12.7	15.0	17.5	20.2	23.2	26.4	30.0	33.9	15
6.7	8.1	9.8	11.5	13.6	15.8	18.3	21.0	24.0	27.3	30.8	16
6.1	7.3	8.8	10.4	12.3	14.3	16.6	19.1	21.9	24.9	28.3	17
5.5	6.6	8.0	9.4	11.2	13.0	15.1	17.4	19.9	22.7	25.8	18
5.0	6.0	7.3	8.6	10.2	11.9	13.8	16.0	18.3	20.9	23.7	19
4.6	5.5	6.6	7.8	9.3	10.9	12.6	14.6	16.8	19.2	21.8	20
3.8	4.6	5.6	6.6	7.9	9.2	10.7	12.4	14.3	16.3	18.6	22
3.2	3.9	4.7	5.6	6.7	7.9	9.1	10.6	12.2	14.1	16.0	24
2.8	3.4	4.1	4.9	5.8	6.8	7.9	9.2	10.6	12.2	13.9	26
2.4	2.9	3.5	4.2	5.1	5.9	6.9	8.0	9.3	10.7	12.2	28
2.1	2.6	3.1	3.7	4.4	5.2	6.1	7.1	8.2	9.4	10.8	30
1.9	2.3	2.7	3.2	3.9	4.6	5.4	6.3	7.3	8.4	9.6	32
1.7	2.0	2.4	2.9	3.5	4.1	4.8	5.6	6.5	7.5	8.6	34
1.5	1.8	2.2	2.6	3.1	3.7	4.3	5.0	5.8	6.7	7.7	36
1.3	1.6	2.0	2.4	2.8	3.3	3.9	4.5	5.3	6.1	7.0	38
1.2	1.5	1.8	2.1	2.6	3.0	3.5	4.1	4.8	5.5	6.3	40

Continued on next page.

Table of breaking loads in tons of square pine pillars, wit flat ends firmly fixed, and equally loaded. (Continued.)

Original.

Height in feet.	Side of square pine pillar, in inches.													Height in feet.
	7½	7¾	8	8¼	8½	8¾	9	9¼	9½	9¾	10	10¼	10½	
	BREAKING LOAD.													
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
2	120.6	129.1	137.9	147.0	156.3	165.9	175.8	186·0	196.4	207.2	218.2	229.5	241.0	2
4	107.9	116·3	125.0	133.9	143.0	152.6	162.4	172.5	182.8	193.4	204.2	215.3	226.6	4
6	91.7	99.7	108.0	116.5	125.3	134.5	143.5	153.0	162.8	173.5	184.4	195.6	207.0	6
8	75.9	83.2	90.8	98.7	106.8	115.5	124.4	133.6	143.0	153.0	163.2	173.7	184.4	8
10	62.0	68.5	75.3	82.4	89.7	97.7	105.8	114.3	123.0	132.3	141.8	151.6	161.6	10
12	50.7	56.5	62.5	68.7	75.1	82.2	89.6	97.2	105.0	113 5	122.2	131.2	140.5	12
14	41.8	46.7	51.9	57.3	62.9	69.2	75.7	82.4	89.4	97.1	105.0	113.2	121.6	14
16	34.7	38.9	43.4	48.1	53.0	58.5	64.3	70.3	76.5	83.3	90.4	97.7	105.3	16
18	29.1	32.7	36.6	40.7	45.0	49.8	54.9	60.1	65.6	71.7	78.0	84.6	91.4	18
20	24.6	27.7	31.1	34.7	38.5	42.7	47.2	51.8	56.7	62.0	67.6	73.5	79.6	20
23	19.6	22.1	24.8	27.7	30.9	34.4	38.0	41.9	46.0	50.5	55.2	60.2	65.4	23
26	15.8	17.8	20.1	22.5	25.2	28.1	31.1	34.4	37 9	41.6	45.6	49.8	54.3	26
29	13.1	14.8	16.7	18.7	20.9	23.4	25.9	28.6	31.6	34.9	38.2	41.9	45.6	29
32	10.9	12.3	13.9	15.7	17.6	19.7	21.8	24.2	26.7	29.5	32.3	35.5	38.7	32
35	9.3	10.6	11.9	13.4	15.0	16.8	18.7	20.7	22.8	25.2	27.7	30.5	33.3	35
38	8.0	9.1	10.2	11.5	12.9	14.6	16.3	18.0	19.7	21.8	23.9	26.3	28 8	38
41	6.9	7.9	8.9	10.0	11.2	12.6	14.1	15.6	17.2	19.1	21.0	23.1	25.2	41
44	6.0	6.9	7.8	8.8	9.8	11.0	12.3	13.6	15.0	16.7	18.4	20.2	22.1	44
50	4.7	5.4	6.1	6.9	7.7	8.8	10.0	11.3	12.6	13.7	14.9	16.2	17.5	50

Height in feet.	Side of square pine pillar, in inches.												Height in feet.
	10¾	11	11¼	11½	11¾	12	10¾	11	11¼	11½	11¾	12	
	BREAKING LOAD.						BREAKING LOAD.						
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
4	238.9	251.0	263.2	275.9	288.8	302.1	26.5	28.6	31.1	33.8	36.7	39.9	4
6	218.8	230.6	242.7	255.2	267.9	281.0	24.2	26.4	28.7	31.2	33.9	36.8	6
8	195.5	207.0	218.5	230.4	242.5	255.1	22.4	24.3	26.4	28.7	31.2	33.9	8
10	172.2	183.0	194.1	205.6	217.3	229.5	20.7	22.6	24.5	26.7	28.9	31.4	10
12	150.3	160.3	170.7	181.5	192.5	204.0	19.2	20.9	22 7	24.7	26.8	29.2	12
14	130.5	139.8	149 4	159.4	169.6	180.2	17.8	19.5	21.1	23.0	25.0	27.3	14
16	113.2	121.7	130 4	139.6	149.0	158.8	16.6	18.2	19.7	21.5	23.3	25.4	16
18	98.7	106.2	114.1	122.5	131.0	140.0	15.5	17.0	18.4	20.1	21.8	23.7	18
20	86.2	92.9	100.0	107.6	115.4	123.6	14.5	15.9	17.2	18.8	20.4	22.2	20
22	75.6	81.7	88.1	95.0	102.0	109.5	13.6	14.9	16.2	17.7	19.2	20.9	22
24	66.7	72.2	77 9	84.1	90.5	97.3	11.8	12.9	13.9	15.2	16 5	18.0	24
26	59.1	64.0	69.2	74.9	80 6	86.8	10.1	11.1	12.0	13.2	14.3	15.6	26
28	52.6	57.1	61.8	66.9	72.1	77.7	8.9	9.8	10.6	11.6	12.5	13.7	28
30	47.0	51.1	55.3	60.0	64.7	69.9	7 8	8.6	9.4	10.2	11.1	12.1	30
32	42.1	46.0	49.9	54.0	58.4	63.0	7.0	7.7	8.4	9.1	9.9	10.7	32
34	38.2	41.5	45.0	48.8	52.8	57.1	6.2	6.8	7.4	8.1	8.8	9.6	34
36	34.6	37.7	40.9	44.3	48.0	51.8	5.6	6.1	6.7	7.3	8.0	8.7	36
38	31.5	34.2	37.2	40.4	43.9	47.4	5.1	5.6	6.1	6.6	7.2	7.8	38
40	28.8	31.3	34.0	37.0	40.1	43.5	3.5	3.9	4.2	4.6	5.0	5.5	40

Continued on next page.

le of breaking loads in tons of square pillars of half-med white or common yellow pine, with flat ends y fixed, and equally loaded. By C. Shaler Smith's formula. ued.)

is table was partly made by interpolation, the last figure is not always pre-orrect.

Original.

Side of square pine pillar in inches.												Height in feet.
13	14	15	16	17	18	19	20	21	22	23	24	
BREAKING LOAD.												
ns.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	
8	418	482	552	625	703	786	872	964	1060	1161	1265	4
5	394	456	526	599	676	760	847	938	1033	1134	1236	6
8	367	429	500	572	649	732	818	910	1005	1106	1208	8
1	339	400	466	537	612	694	780	870	964	1064	1166	10
2	307	365	432	502	576	656	740	829	922	1022	1124	12
5	277	333	397	464	536	614	696	784	876	973	1074	14
	250	303	363	428	497	573	652	739	829	925	1024	16
	224	274	331	392	458	531	608	692	780	873	972	18
	201	248	301	359	422	492	566	647	732	822	919	20
	182	224	274	329	388	455	526	604	686	773	866	22
	163	203	249	301	357	421	488	563	642	726	816	24
	148	184	226	275	328	389	453	523	599	680	767	26
	133	167	206	252	302	359	420	490	560	638	721	28
	121	152	189	231	278	332	389	453	522	597	677	30
	109	138	173	212	256	307	361	421	487	558	635	32
	99	126	159	196	237	284	335	392	455	523	597	34
	91	116	146	180	219	264	312	366	426	490	560	36
	84	107	134	166	203	245	290	341	397	458	525	38
	77	99	124	154	188	227	270	318	372	429	494	40
	71	91	115	143	175	212	253	298	349	403	465	42
	66	84	107	133	163	198	236	280	328	380	438	44
	61	78	99	123	152	185	221	263	308	358	413	46
	57	73	92	115	142	173	207	247	290	337	389	48
	53	68	86	107	133	162	194	231	272	317	367	50
	50	64	81	101	124	152	182	217	256	300	347	52
	47	60	76	95	117	144	172	205	242	283	328	54
	44	56	71	89	110	135	162	193	228	267	310	56
	41	52	67	84	103	127	153	182	215	253	294	58
	38	49	63	79	98	120	144	172	204	240	280	60
	33	43	55	69	86	105	126	151	179	211	246	65
	29	37	48	60	74	92	111	134	159	187	218	70
	25	33	42	53	66	82	98	118	141	166	195	75
	22	29	37	46	58	72	87	105	125	148	174	80
	19	26	33	41	52	65	78	94	112	132	156	85
	17	23	30	37	46	58	70	85	102	120	141	90
	16	21	27	33	42	53	64	77	93	108	127	95
	14	19	24	30	38	48	58	70	84	99	117	100
	12	16	20	26	33	40	48	58	70	82	97	110
	11	14	17	22	28	34	41	49	60	71	83	120
	9	12	14	18	23	29	36	43	52	61	72	130
	8	10	12	16	20	25	31	37	44	53	62	140
	7	9	11	14	18	22	27	32	38	46	54	150
	6	8	10	13	16	20	24	29	34	41	48	160
	5	7	9	11	14	17	21	25	30	36	43	170
	5	6	8	10	12	15	19	22	27	32	38	180
	4	5	7	9	11	14	17	20	24	29	34	190
	4	5	6	8	10	12	15	18	22	26	31	200

Table of breaking loads in tons, or in lbs per sq inch, cross section of half seasoned square pine pillars, wh heights are measured by one of their sides. By C. Shaler Sm formula.

Original.

Height in sides.	BR LOAD PER SQ IN.		Height in sides.	BR LOAD PER SQ IN.		Height in sides.	BR LD PER SQ IN.		Height in sides.	BR LD PER S
	Tons.	Lbs.		Tons.	Lbs.		Tons.	Lbs.		Tons.
1	2.2232	4980	26	.6027	1350	51	.1960	439	76	.0924
2	2.1969	4921	27	.5697	1276	52	.1888	423	77	.0902
3	2.1544	4826	28	.5398	1209	53	.1826	409	78	.0879
4	2.0978	4699	29	.5116	1146	54	.1763	395	79	.0862
5	2.0290	4545	30	.4853	1087	55	.1705	382	80	.0839
6	1.9513	4371	31	.4607	1032	56	.1647	369	81	.0821
7	1.8665	4181	32	.4379	981	57	.1598	358	82	.0799
8	1.7772	3981	33	.4165	933	58	.1545	346	83	.0781
9	1.6857	3776	34	.3969	889	59	.1496	335	84	.0763
10	1.5942	3571	35	.3781	847	60	.1451	325	85	.0746
11	1.5040	3369	36	.3599	809	61	.1406	315	86	.0728
12	1.4165	3173	37	.3447	772	62	.1362	305	87	.0714
13	1.3317	2983	38	.3295	738	63	.1321	296	88	.0696
14	1.2513	2803	39	.3152	706	64	.1286	288	89	.0683
15	1.1745	2631	40	.3018	676	65	.1250	280	90	.0670
16	1.1027	2470	41	.2889	647	66	.1210	271	91	.0656
17	1.0353	2319	42	.2772	621	67	.1179	264	92	.0638
18	.9723	2178	43	.2661	596	68	.1147	257	93	.0625
19	.9134	2046	44	.2554	572	69	.1112	249	94	.0616
20	.8585	923	45	.2455	550	70	.1085	243	95	.0603
21	.8076	1809	46	.2357	528	71	.1054	236	96	.0589
22	.7603	1703	47	.2268	508	72	.1027	230	97	.0576
23	.7165	1605	48	.2183	489	73	.1000	224	98	.0567
24	.6755	1513	49	.2100	472	74	.0973	218	99	.0554
25	.6380	1429	50	.2031	455	75	.0951	213	100	.0545

Steel pillars. Mr. Kirkaldy experimented with a small steel pillar or t of Shortridge, Howell & Co's homogeneous metal. Its length was 4 ft, or 25.6 dia outer diam 1⅞ inch: inner diam 1⅛; thickness ⅜ inch. Area of cross-section sq ins. Flat ends. Under 67300 lbs, or 30 tons total pressure, or 17.14 tons per inch of solid metal section, it bent very slightly. On increasing the pressure c siderably, the pillar sprang out from under the load. Our preceding table gives 5 tons total, or 13 tons per sq inch, as the ultimate load for a wrought-iron tube of same size.

Mr. M. G. Love, Paris, as the result of a trial with **small steel rods**, abou inch diam, and which had a tensile strength of 48 tons per sq inch, suggests that comparative strength of pillars of wrought iron, cast-iron, and steel, are **probab** about as follows: At from 1½ to 5 diams in length, steel and cast-iron ones h equal strengths; and either of them is about twice as strong as wrought iron. At diams, steel is 1½ times as strong as cast; and 2.2 times as strong as wrought ir At 40 diams, steel is 4 times as strong as cast; and 2.7 as strong as wrought. I this needs confirmation. Now that powerful and accurate testing machines coming more into use, we may hope that the doubts at present existing on su subjects will be set at rest.

TRUSSES.

hen the span of a bridge, roof, &c, becomes so great that single solid
:ed at their ends, cannot be employed, we resort to compound beams,
s. composed of several pieces so arranged and united as to furnish
gth. The designing, construction, and erection of trusses of great
y when of iron, involve such a multiplicity of important detail, that,
ing of locomotives, cars, &c, they have become a specialty, or a dis-
f business, to which persons confine themselves to the exclusion more
r departments; and thus attain a degree of skill beyond the reach of
gineer.* The latter, however, should possess a knowledge of the sub-
at least to enable him to form a well-grounded opinion of the general
sign; and to guard him against the adoption of one involving serious
In a volume like this we can aim at nothing more than an attempt
me few general principles. We shall confine ourselves to such trusses
non use; showing first the effects of uniform stationary loads, as in
fs; and then those of moving loads, such as an engine and train on a

Iost of the bridge trusses in common use have two long, straight, par-
ıd lower members *l t*, *a p*; and *l u*, *a p*, Figs 10, 11, p. 254, called the
in England, the **booms.** Vertical pieces placed between, and con-
per and lower chords, are called **posts,** when they sustain compres-
tical ties, or **suspension rods,** &c, when they sustain tension
oblique pieces seen in these figs are called **braces, strut-braces,**
es, &c, when resisting pres or thrust; or **tie-braces, tension-**
ain oblique ties, oblique suspension-rods, &c, when
. Sometimes the same piece is adapted to bear both tension and com-
nately; and may then be called a **tie-strut** or a **strut-tie.** The
ers alluded to are sometimes called main-braces, whether they are
to distinguish them from **counter-braces,** or **counters.** These
hown in Figs 10 and 11, but are seen in Figs 28 and 31, crossing the
liagonally. These posts, braces, counters, ties, &c, serve not only to
chords asunder, and to prevent them from bending; but to transform
strains produced by the wt of the truss and its load, into other strains,
idinally, or *lengthwise,* along the diff members; and to conduct said
the truss, to the firm supports of the abuts. A load placed at any one
ers is, of course, partly supported by each abut; one part of it travels
alternately between the chords, and along the successive members,
s one abut; and the other part, in like manner, goes to the other abut.
rs, therefore, perform the duty of the vert web of the Hodgkinson
e I rolled beams, or of the tubular girder; and on this account are col-
d the **web members,** in contradistinction from the chords. Each
load, while being transferred by the web members, from the spot at
ced on the truss, to its final point of support on the abut, produces a
itself upon every *vert* web member along which it travels between the
s; while upon each *oblique* member encountered on its way, it produces
r than itself, in the same proportion that the oblique member is longer
e.

e web members are strained by compression, or by tension; or, in
whether they act as struts, or as ties, the *amount* of strain will be the
ıer case the straining agent is the same identical force, namely the wt,
f gravity of the truss itself, and of its load; and (Art 18, of Force in
whether this force exhibits itself as a *push,* or as a *pull,* neither its
s *direction* undergoes any change. So far, therefore, as regards the
e involved in the duty performed by the web members, they might be
y into **verticals,** and **obliques.** We shall frequently so desig-

mount of strain the upper end of an oblique produces in one direction
per chord, that same amount will its lower end produce against the
chord; but in the opposite direction. That is, if the top or head of
ushes the upper chord toward the right hand; its foot will pull the
to the same extent toward the left hand. This, however, is not *pre-*

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ter to whom we are indebted for a knowledge of correct principles on this subject is
E., *the first edition of whose* book (beyond all doubt the pioneer one) bears date,
847. He was followed by Bow, of England, and Houpt, of this country, both in 1851.
ipple bridge (of which Mr. John W. Murphy, C. E., has built several of the best)
hese two gentlemen.

cisely correct, inasmuch as when the oblique is a strut, the pres at its f what greater than at its head, because the foot supports also the wt itself; or if the oblique is a tie, with its head attached to the upper chor strain is a little greater at the head than at the foot; because then the h the wt of the oblique, and the foot sustains none of it. This remark course, to verts also. Another exception is, when the ends of two ob each other; as those at the center of the trusses, in Figs 10 and 11. If, in the ends of the obliques abut *against each other*, instead of being separat to the chord, they will at that point exert their strains against each othe against the chord. This, however, does not cause the chord to be a strained at that point, as will be seen hereafter.

In any oblique, as *c d*, Fig 1, the vert dist *a c* between its ends; and t *a d* between the same, are called its vert and hor **spreads,** or **stre reaches.**

Art. 3. There is a great diff in principle between two classes o common use. In some of them, two chords are absolutely essentia Howe truss, p 283; the Pratt, p 284; the Lattice, p 285; the Warren, their various modifications, known as the Murphy-Whipple, the Linville, &c, &c, which differ only in certain unessential details. In the Howe trusses there is no diff whatever of broad principle. The distinction bet consisting chiefly in the fact that in Howe's the verts are ties, or suspe and the obliques, struts; while in Pratt's, the verts are posts; and the ol In all these the strains on the verts and *main* obliques (not on the counte at the center of the truss; and increase gradually toward the end of it; on the chords (as in an ordinary wooden beam) are greatest at the cente at the ends. Hence, also, such are called **beam trusses.** The str counters are also greatest at the center.

But there is another class, called **suspension trusses,** of whic Figs 26 and 27; and the Bollman, Figs 22, 24, 25, are the principal repr In these but one chord is essential for a perfect truss. See Figs 25, 27. chord the web members are suspended; and to it alone do they all tr strains; and the strain upon this chord is uniform from end to end. show perfect bridges, with but one chord each. In Figs 24, 26, *n n*, *n* be chords; but strictly speaking they are not; they are merely longitu for upholding the cross-beams of the flooring, when the roadway is pl bottom of the truss. They have not to resist tension, as in beam trusse

In all the forementioned trusses the roadway may be placed on either the top or the constituting in the first case a TOP-ROAD, or a DECK bridge; and in the second, a BOTT TRUSSED bridge. When a given height of headway is required beneath the bridge, ther trusses whose end web members are *obliques*, be a saving of material in the end support (whether these be of masonry, or of iron posts) by placing the roadway on the lower the Bollman and in the Fink *suspension* trusses there need be no difference in *this* res changes in the mode of action, (whether as tension or compression,) and also in th material in the trusses themselves, are dependent upon whether the roadway is at bottom of the truss; but these will be more fully alluded to in their proper places.

Art. 4. That part of a truss, such as Figs 10, 23, 23 *a*, &c, that is co tween two adjacent verts, is called a **panel**; thus, in Fig 10, *e i j d*, *d j* in Fig 31, of Pratt, *t y n w*, is a panel. The Triangular or Warren trus Figs 23 *e*, has no verts, as essential parts of it; and its subdivisions are c triangles; and a panel is a length of truss equal to the width of a triar are sometimes added to it when the spaces *a b*, *b c*, *c d*, Fig 11, become t safely supporting the roadway without them; thus dividing the truss in els. It is not a matter of practical importance as regards strength, number of panels in a truss be odd or even; but it is usually even, with center of the truss. In Fig 1, *a b c o*, *b d o n*, are *half panels.*

The most economical inclination of obliques, as regards the quantity in the web, is when their least angle (*t p o*, or *s o n*, &c, Fig 10) with the This applies also to the admirable Warren truss; in which the triangles a usually made equilateral. When great, and the height of truss corresp if the panels be made square, or nea a view to secure this inclination of the obliques, the verts (as *t o*, *s n*, will become so far apart, that the dists *p o*, *o n*, &c, become too long t upholding their loads of engines, ca out additional precautions. When the expense or inconvenience resulti would be too great, the verts may placed so near together as to ma

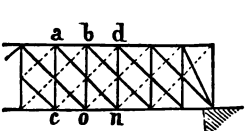


Fig 1

n they are long;* and the obliques (both the main ones, and the
then run across one vert, as in the Fig; or across two, if neces-
rren girder, the expedient is to introduce verts; or else a second set
ı Fig 1, omitting the verts. From 8 to 12 or 15 ft apart are ordinary
bridges of moderately large spans. Frequently panels are made
her than long; disregarding the economical angle of 45°. In large
ı obliques, instead of being each in one piece, are usually made of
allel pieces, disposed in such a manner as to let the counters pass
agonally, without mutual interference. Each lower chord (and fre-
er one also) in large spans, is usually made up of several parallel
or bars of iron, side by side.

ke Bridge, England, a transverse section of *each* lower chord shows 14 iron bars.
mploy smaller beams and bars; and, moreover, secures greater *width* of truss;
the tendency to lateral or sideways motion. It in no way affects the amount of
aode of calculating them; but is a mere matter of mechanical expediency, or of

s is very high, the posts are sometimes made as in
ıd c are the upper and lower chords; and *pp* a post
o hollow cast-iron pillars, bolted together by their
s s. At *s s* is also placed a hor + shaped casting, of
er extremities of which serve to keep in place the
stiffening the post sideways.

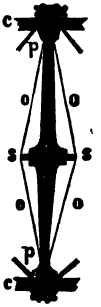


Fig 2

rs are in use for building or composing large posts, as well as
eans of T, +, U, H, O, or other shaped irons, riveted together.
low column of the Phœnix Iron Co. of Philadelphia, (p 293,
illars,") is coming into very common use for such purposes.
n may be made of round or square bars; or of flat ones joined
ans of splicing-plates, as at *s*, Fig 37, of "Strength of Mate-
or pins passing through eyes at the ends of the bars: see Fig
ow modes of joining the beams for lower chords of wood so as
pper chords sustain compression only; and need but a simple

ree expansion and contraction from temperature, when the span of an iron bridge
out 75 or 100 ft, one end of it should rest upon roll-
her device (see p 295) be substituted for the same
be neglected, the abuts will be exposed to displace-
se be sufficient to resist the expansive force, the
e likely to buckle, if of wrought iron; or to be frac-
iron, or having a cast-iron upper chord.

imagine lines crossing the panels diag, as the main obliques shown
in the opposite direction, as shown in Figs 28 and 31, they will rep-
r-braces, or **counters.** These, like the main obliques, are in
, and in others ties. Although important members, they are less so
bliques. They are unnecessary when the load is uniform and station-
assumed to be the case in roofs: and are required only when the
or a moving one, as in a train crossing a bridge. In this last case they
the span is but partially loaded. If the train at any moment covers
and is of uniform wt, their action ceases for that time. Their office
eract the deranging tendency of the unequal loading of diff parts
hown in Figs 9½, p 253. In Fig 9 *b*, an excess of load along *a o*
range the main braces *h o* and *t a*; and this would be counteracted
s *c o* and *t s*. The same thing may be effected by arranging the main
as to bear tension as well as compression. The bad effects of une-
plainly become greater in proportion as the load is heavier than the
when the bridge becomes very heavy, so that the load must extend
els before its effects become serious, but little counterbracing is
t at and near the center only; whereas, in a very light bridge, the
extend from the center, where they are most strained; to near the
strain upon them is least. Inasmuch as we shall first speak of *uni-*
usses, we shall not here say more respecting counters. See Remark,

at sight appear that the several parts of a bridge truss must be most
vered from end to end with its maximum load; but this is true only
nd of the main obliques and verts, as *l a*, *t p*, Fig 10, at the *ends* of
her web members are more strained by a part of the load as it passes
so that if *they be correctly* proportioned for a full load, they will

anel is its horizontal measurement; although this may be, and usually is, con
vert one, or height; thus, in Fig 31, *t y* or *a w* is the length of the panel *t y a*,

[illegible] made as strong as the end [illegible] require an expen[illegible] moderate spans, especially [illegible] expense of getting out and fitting t[illegible] sizes, may more than counterbalance the saving in mat[illegible]

Art. 5. [illegible] requires a process of cal[illegible] mathematics. We shall first [illegible] the latter only; and [illegible] methods. [illegible] will render the gener[illegible] with [illegible] of the composition [illegible] forces. [illegible] with all the neces[illegible] drawn to a scale. The same [illegible] that answers [illegible] may also represent a ton. It [illegible] convenient [illegible] may also be used for measurin[illegible] such.

The [illegible] and [illegible] before mentioned, constitut[illegible] **truss**; [illegible] other pieces are necessa[illegible] **bridge**; such as [illegible] or beams; transverse bracing for co[illegible] trusses with one [illegible] is better to resist lateral or side[illegible] arrangements of braces for giving the truss to the piers [illegible] posts, &c. The same may be said of the extension frequently [illegible] either an upper or a lower chord of a bridge, as shown at *n n* in [illegible] Here the [illegible] are needed with the extensio[illegible] allow the load to reach and to leave it. They m[illegible] same purpose in an upper chord of a lower-road bridge; or for [illegible] over an open span, &c. The [illegible] posts *e e*, of the same Figs. [illegible] the truss, and supports for uprooting it, also, the posts *p* and *d*, [illegible] to the truss.

Thus, besides the forms of truss already mentioned, there are many othe[illegible] which are introduced either as structural members, or merely as authorities; suc[illegible] Fig. 3, the Howe and Strouse, Fig. 3, and the Burr, Fig. 36. All these posse[illegible] and no doubt have each been introduced in practice, from the fact [illegible] were extensively introduced into the United States in this country, the [illegible] heavy engines and trains of the present day, and consequently failed.

Art. 6. Chords. When a beam [illegible] 5, Fig 3, supported at either under its own [illegible] action of a load plac[illegible] suspended from it b[illegible] cause the lower fibr[illegible] are *pulled asunder* [illegible] ones at *u*, *crushed to* [illegible] extent as to offer n[illegible] ance. The fig show[illegible] what exaggerated [illegible] treme upper particle[illegible] treme lower ones at

Fig 3.

strained, give way first; and the strength of the beam being there[illegible] adjacent ones give way in rapid succession. The compressed part[illegible] are [illegible] a certain point *n*; while the extended ones are *below* i[illegible] an infinitely fine needle to be held perp to this page, and in that p[illegible] the plane the point *n*, passing *entirely through* the beam, or page, th[illegible] fine hole thus made will pass along what is called the **neutral a[illegible]** It is so named because the fibres situated in that line, and which w[illegible] the needle, are neither compressed nor extended, until the strain [illegible] that on its removal the beam will not entirely recover itself; o[illegible] until the strain exceeds the *elastic limit* of the beam. Within the li[illegible] the neutral axis may be assumed to pass through the cen of grav o[illegible] of the beam. Thus, if the c[illegible] any of the forms shown in [illegible] as the beam is safe, or th[illegible] elastic limits, the line *n a* [illegible] cen of grav; which is at [illegible] neutral axis.

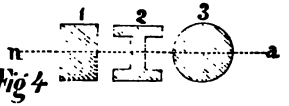
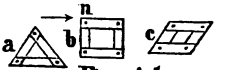


Fig 4

Now the parallel upper [illegible] of a truss of the beam sys[illegible] *siderable extent*, in the same condition as the upper and lower fib[illegible] *Fig 3; for the truss* also has its neutral axis; so that all the [illegible] *chord are compressed*; and all those of the lower one are extend[illegible] *lower chords are* of the same size and shape, this axis will be hal[illegible] *at the center of* the height of the truss. If the chords be

en the centers of grav of their cross-sections. The same quantity of ver, that composes the beam *a b*, Fig 3, will present far more resist-g or breaking, if it be cut in two lengthwise, and converted into top ords of a truss; for the reason that both of the chords are then so far the neutral axis, that *all* their particles are strained *to nearly the same* me time; so that all the fibres in the upper one must be crushed, and lower one be pulled apart, *at the same instant*, before the truss can reas, in the single beam *a b*, the extreme upper and lower fibres break se next to them, and so on, one after the other. They do not all act y do in the chords. It is this principle that gives so much strength e Hodgkinson beam, &c.

n the designing of trusses, especially such as may have to oads at different parts, as in a bridge, the to be aimed at is to dispose its various orm a series of properly connected **tri-**use in that shape they present more derangement of form, than in figs of a r of sides. Thus, in the three beams at th a bolt at each junction or joint, the n evidently cannot be changed by any but a force sufficient to either either the beams or the bolts. But in the 4-sided fig *b*, the form may nged to that at *c*, by a force at *n* entirely too small to injure either he bolts. In *a* the bolts assist to prevent change of form; but in *b* y pivots, around which great changes may easily take place.



Figs 4 1/2

the strains on any truss can be correctly calculated, and its parts properly proportrains, ITS WEIGHT MUST BE KNOWN; for this tends to break it, as well as the extrane the other hand, we cannot learn its *wt* until we know the size of its diff members. we must *assume* for it an *approximate* wt, based upon our knowledge of somewhat ready built. This becomes the more necessary as the truss increases in size, so that es greater in proportion to that of the load. The table, p 296, gives *safe* assumed tusses; and p 298 will aid in the case of roofs. In very small spans, especially of is generally so much greater than the wt of the truss, that the latter might almost rely.

he most simple form of a roof truss.* It consists of two *a, o b*; m *a b*. 's gen- weight d of its overing, may be iformly ns the oof con- trusses, om 6 to t some-, and at e. The agitati-, called etching e wall; tribute to ns and greater

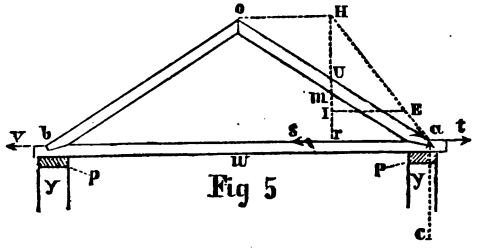


Fig 5

fters, and at intervals of a few ft, are fixed long pieces of timber called *purlins*, of running across from truss to truss; to which the laths or boards are nailed which gles, tin, or slate, &c, which forms the roof-covering.

supports all the purlins, roof-covering, snow, &c, &c, which occupy the space half of it to the next truss. Thus, suppose a span of 20 ft; and each rafter to be 14.8 ft trusses are say 12 ft apart from center to center, and if we assume (as it is generally the wt of the truss, covering, snow, &c, may amount to 40 lbs for every sq ft of area as has to sustain 33.6 × 12 × 40 = 16128 lbs, including its own weight. Strictly the m should be omitted; because in Fig 5 no part of it is upheld by the rafters. It is rever in comparison with the load.

the strains upon the different parts of a truss, rst calculate in the manner just shown, the entire wt in lbs of a truss

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pages 292, 294, &c, exhibit details of *modes of* uniting the various pieces in a truss. , will be found remarks on the *wt*, &c, of roofs and trusses. given by Prof Rankine, (see his Civ Engineering, edition of 1862, p 470;) by Fenwick, ples of Construction, ed 1861, p 169;) and by other authorities, for calculating the *truss, is erroneous, and unsafe.* See last foot-note, p 252.

and its load. Through the center U of either rafter draw a vert line H *r*. From *a* o H. Join H *a*. New on the vert line H *r*, lay off H I by any convenient scale to rep uniformly distributed wt of *one rafter* and its load; and draw the hor line I E. Th by the same scale the hor force at the head of the rafter; and H E the amount and oblique force * which presses *the foot* of the rafter; but which diminishes gradually s head.¶ The *hor* force at the foot of the rafter will be equal to that at its head;† a the hor pull along the whole length of the tie-beam.‡

There is no force acting in the direction *o a* of the actual length of the rafter *in such*

But the rafters have to be considered in another point of view no less important t to sustain this pres at their feet. Each rafter is an inclined beam supported at bearing a heavy load equally distributed along it; and we must find the dimensio purpose also; and add them to those reqd for other purposes.§

These dimensions may be found by the rules, p 189, or by tables, pages 191, or 1 the case.

The strains in Fig 5 may be found in the followin

Take, for example, a truss of white pine, of 30 ft span, and 7½ ft rise. The wt o snow, &c, &c, 40 lbs per sq ft of roof area. Trusses 12 ft apart from center to cen truss will have to sustain a total load (including its own wt,) of $33.6 \times 40 \times 12 = 16$ may call 16000, or each rafter 8000 lbs. We will calculate each part with a safety think is abundantly sufficient, with the assumption of 40 lbs per sq ft. First pr of the truss, on a scale of say ½ inch to a ft. This diagram will consist of but thre use the same scale of ½ inch to represent 1000 lbs of either wt or strain. Make H I 1 inch; that is to the 8000 lbs uniformly distributed wt of one rafter and its load. A measure it. It will be equal in this case (accidentally) to H I, or 8000 lbs; and this pull along the tie-beam. Now we see by table, page 177, that average white pine bre

of 10000 lbs per sq inch; so that for a safety of 3, we must not subject it to more t

lbs pull per sq inch. The weakest part of the tie-beam is where it is cut into, near t ing the rafters; and even what is there left by the cut, is usually still farther red of the bolts or spikes driven into it through the feet of the rafters. Therefore, a things, we must give to the tie-beam *at that point* a transverse section of solid wood, $\frac{8000}{3333} = 2.4$ sq ins. This would no doubt be sufficient to resist the pull; but there are tions, such as danger of sagging or breaking down if persons should get on it; or if should chance to be laid upon it, &c, which cause the tie-beam (even when unloaded ev plastered ceiling below,‖ as is here supposed to be the case,) to be made about as

If, instead of a beam, we had used an iron rod to resist the 8000 lbs pull, we shou with a breaking strength of $8000 \times 3 = 24000$ lbs; and by the table of bolts, page 3 diam of full $\frac{13}{16}$ inch would suffice if upset; or of full 1.04 inch if not upset.

Now, as to the rafters, each of them is an inclined beam, sup

---

* As this strain H E arises from the wt of the rafter and load, it is greatest a rafter which bears *all* said wt. At U, half-way up, it is only half as great; and a rafter, which has no wt of truss above it, it is nothing; and therefore, so far as sai concerned, the rafter might terminate in a point at top. But it requires some thicke vent being snapped off by the *hor* pres caused by the rafters leaning against each more to enable it to bear the *transverse* strain arising from its own wt, and from spread uniformly over it; as will be seen. The increase reqd for this last purpose, is to provide in itself against the top hor pres; which, therefore, requires no special p

† The foot of each rafter tends to slide or push outward horizontally in the directi *t* and *v*; each with a force equal to I E. But the tie-beam prevents them from so doin verts their *pushes* into *pulls* against each other; and thereby into a pulling strain tie-beam itself, to an amount equal to one of the forces; as two men pulling against two ends of a rope, each with a force of 10 lbs, only strain the rope 10 lbs. In other *two* equal opposing *forces* of 10 lbs each, to produce *one strain* of 10 lbs. See Arts 13 in Rigid Bodies.

‡ And there is a hor strain to the same degree generated at every point along th rafter; for two equal forces in opposite directions against the two ends of a stationa a strain equal to one of them, and parallel to their own directions, throughout the the body. Thus, in an arch, the hor pres at its center and foot exists equally throu curve. See page 493, foot-note.

§ Morin, a celebrated French authority in these matters, asserts that if we provid tudinal strains in a rafter, we may neglect the transverse ones; but this is a great last may be much more important than the first.

‖ **The weight of an ordinary lathed and plastered** about 10 lbs per sq ft; and **that of an ordinary floor** of 1¼ in gether with the usual 3 by 12 inch joists, 15 ins apart from center to center, is from 1 ft. In preliminary calculations it is well to take the two together at 25 lbs per sq ft.

¶ **The principle** upon which the lines I E, H E, become the me strains at *a* upon the tie-beam and upon the foot of the rafter, is explained at Rem 2,

loaded with 8000 lbs; which is equal to a center load of 4000 lbs. But for a safety e will find its dimensions for a *breaking* center load of 12000. Being *inclined*, be taken as if measd *horizontally* between its end supports, or 15 ft. See Art 16. ıme for it some probable approx depth; say 9 ins. Then by Art 21, p 189, we find center load of 12000 lbs, its breadth will be 4.94, say 5 ins. Therefore, to be safe r load, each rafter must *at its head* be 5 ins broad, by 9 ins deep.* But it must because we have not yet provided for the crushing pres at its foot, indicated by by scale measures, say 10500 lbs. Now, we find by table, page 174, that average e absolutely crushes under a pres of say 6000 lbs per sq inch; therefore it will have 2000 lbs per sq inch; so that we must provide for the foot of each rafter $\frac{10500}{2000} = 5\frac{1}{4}$ sisting *this* pressure alone. These $5\frac{1}{4}$ sq ins may be added either in the *breadth* thus making it say 5.6 by 9; or to its *depth*, making it full 5 by 10 at foot. We rafter regularly from the bottom, until it becomes equal to the top: but in practice le to do this, unless timbers with the proper taper happen to be at hand. Make the same size as a rafter.

:t three trusses we shall not enter into this detail on; as we conceive that this example suffices to s principle.

ct to Fig 5, in point of simplicity, is Fig 6; which represents a truss r a roof of mode- wo equal rafters, n *a b* as before; ıtion of a king- suspension-rod ie-beam, or the ıay be uniformly ıaterial whether rmer be equal to r not. We shall russ only as that be points *half-* ting-rod and the will the king-rod ht of the portion m and its load. tie-beam and its , and the walls directly by the

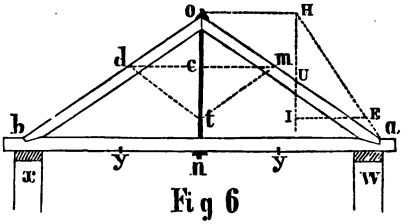

Fig 6

wt of the truss and its load, it is plain, is sustained ultimately by the abuts. or wt of *y y* and its load does not reach the walls until after having, as it were, first ing-rod to *o*, and from there down the rafters to *a* and *b*; or, *indirectly*, by a cir- at the king-rod sustains all between *y* and *y*, will be evident when we reflect that firmly suspended at its center *n*, may be regarded as two separate beams *n b*, *n a*. eam *n b*, and its load would, in that case, manifestly be borne by the wall *x*, and s; and so with *n a*. Therefore, *n* upholds one-half of the beam *a b* and its load; all between *y* and *y*. The king-rod transfers the wt of and on *y y*, to the heads

This wt may, therefore, be considered precisely in the light of one resting upon ceed to find the strains which it produces upon the rafters and tie-beam, by Art ce in Rigid Bodies. Namely, on *o n* make *o t*, by scale, equal to said total wt of nd the wt of the king-rod itself. Complete the parallelogram of forces *o m t d*; lling *m d*. Then will *o m*, *o d* measure the strains produced *by said total weight* espective rafters; and *c m*, *c d* the pulling forces produced by the same wt only *a b*; causing strain all along it equal to one of them. Art 13, p 449, of Force in

d that in this truss, therefore, unlike Fig 5, there *is* a strain *o m*, or *o d*, running the rafters from head to foot. **To resist this strain, the rafter arded as a pillar,** with a load on its top equal to the strain; and the pholding it may be found by means of the table of strength of wooden pillars, on s area when found, *a*. Next, consider a rafter as a *beam* supported at both ends, enter load equal to *half* its own wt and actual load; and in that respect proceed 5, to find its safe breadth and depth by rules, page 189. To the area of section area *a*; and the sum will be the final area reqd for the *head o* of the rafter. Call e H E, as in Fig 5, for the pres caused at the *foot only* of the rafter by the weight s load. Divide this by the safe amount of 2000 lbs, as was done in Fig 5; or by safety greater than 3 is reqd. The quot will be the area needed for that purpose rea *b*: and the sum will be the *total* area reqd at the *foot*.† This area may now ished upward, by reducing either the breadth or the depth, until at the head it is t this is rarely done even in iron roofs: the area at foot being continued to the head. minish both the breadth and the depth, because we should thereby reduce the pro- ired at every point of the length, to resist this pressure, which, although greatest t disappear entirely until at the very head *o*.

on the tie-beam will be I E added to *c m* or *c d*. Find the

---

ad proportion of breadth to depth. If we had assumed say 15 ins for the depth, a rafter so thin as to be laterally weak. Frequently, two or three assumptions ay have to be made before we hit upon a satisfactory proportion.

91.

safe area by dividing their sum by 3333, which is the number of lbs per sq inch, giv
Then regarding *half* the length of the tie-beam supported at both ends, and loaded a
only *one-fourth* of the wt of and on the entire tie-beam, find its safe dimensions by
or by table, page 191. The resulting area, added to the safe area for the pull just fo
entire section of the tie-beam, unless some addition be made to the depth, to allow
away for the feet of the rafters.*

As to the vertical king-rod, *n o*, it must be strong enough
a pull equal to its own weight, added to the weight of and upon *y y*. If the rod is o
it should have one square inch for a safety of 3, of cross-section for each 20000 lbs
or see table, page 376. If of wood, it must, for a safety of 3, have at least one sq inch
3333 lbs of said weight. A safety of 3 will be enough if the bar is not liable to vibra
When the king-rod is of wood, it is improperly termed a king-*post*. Since a post is
tain a load on its top, the term might lead to the inference that the upper ends of t
upon, or were upheld by the king-post; whereas, as we have seen, *they* actually up

We add the calculated approximate dimensions f
Fig 6, of 30 ft span; and 7½ ft rise. Wt of roof and its maximum lo
sq ft of area of roof. Wt of and on the tie-beam, including floor, ceiling, load, and
lbs per sq ft. Timber white pine. Safety of each piece 3. Rafters 8¼ ins broad, by
and 7¾ by 11 at head. Tie-beam 8¼ broad, by 11 deep, without any allowance fo
of rafters. King-rod 1 $\frac{5}{16}$ inch diam if upset; or full 1⅜ if not upset.† See Note, p

With no floor or loading on the tie-beam, except its
1000 lbs, we have, approximately enough, rafters 6½ ins broad by 9 ins deep, at foot;
Tie-beam, say same as rafter, or 6½ × 9. King-rod, ¾ inch diam if upset; ½ inch if
it would be expedient to make it rather more. Trusses 12 ft apart center to center.

Art. 9. In Fig 7 we have a truss consisting of two rafters, *a b*, *a d*
h d; a king-rod, *a c*; and two struts or braces, *e c*, *h c*. Either the rafters or the tie
may be supposed to be uniformly loaded.

Here, as in Fig 6, **the king-rod *a c*, upholds** the weight of th
of the tie-beam, and
floor, ceiling, people,
placed upon that pe
with its own weight.
tains, in addition to i
of the two struts *e c*,
weight of the portion
of the rafters; and pa
of the roof-covering,
may rest on said porti
holds itself, *y y*, and t
most self-evident; be
part of *x r*, and *x w*, ar
not at first sight so a
struts are in
into trusses
become so long as to
bending too much, or
der their loads; or el

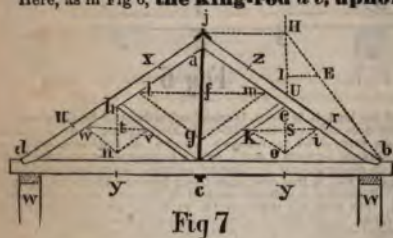

Fig 7

use of inconveniently large timbers to make them of. They act like posts in affording
to the rafters. They carry a part of the strain upon the rafters down to the foot *c*, a
and the king-rod carries it from there up to the tops, *a*, of the rafters. From *a* it passe
the entire length of the rafters to their feet. Thus, it is seen that the action of the st
relieving the rafters from a *transverse*, or cross-strain which would endanger their
converting it into a *longitudinal* strain in the direction of their length, in which th
with less danger. As we proceed with the subject of trusses for bridges as well as
seen that this is the grand duty of such struts and obliques generally. In roofs they
rafters; and in bridges, the chords.

To find the actual amount of strain in lbs which t
thus convey from the rafters to the foot of the king-rod, draw *c o* s
cally; and make each of them, by any convenient scale, equal to the weight in lbs of e
and its load. From *o* and *n* draw the dotted lines, *o i*, *n w*, parallel to the struts; a
allel to the rafters; thus completing the parallelograms of forces, *c k o i*, and *h w n v*.
zontal diagonals *i k*, and *v w*.

Then by Composition and Resolution of Forces, either *e k* or *h v*, measured by the sa
fore, will give the longitudinal strain in lbs upon each one of the struts. This strain pr
lengthwise from head to foot. Their feet are pressed in addition by the weight of t
selves; and *this* pressure diminishes regularly toward their heads or tops, where it is
the case of the rafters in Fig 5. In practice, the wt of the struts is so trifling in compa
of the roof portions which they sustain, that it may be neglected, and its safe dimension
by pages 238, 239. **Therefore, each strut may be regarded a**
pillar, bearing a load equal to *e k* or *h v*. Now, the strain *e k*,
e c, is compounded or composed of the vert strain *c s*, (which is equal to *half of e n*, or
wt of and on *x r*;) and of the hor strain *s k*. And the strain *h v* along the strut *h c*,

* *In cases where no appearance of sagging would be admissible, it is not always e rafters and tie-beam be safe;* for they may be perfectly safe, and yet sag too much for *When such is the case*, refer to table, page 204.

† *We have known* country road bridges, Fig 6, of 30 ft span, and 7½ ft rise, coun *trusses 18 ft apart*, in which the timbers were no larger than in this example.

which is equal to *half* of *h n*, or one-half of the wt of and on *x u*;* and of the
;wo hor strains *s k* and *t v* neutralize or counteract each other, by pressing
he feet of the struts; and therefore only the vert ones *e s* and *h t* pull upon
pull it to an extent equal to half the weights of and on *z r* and *x u*.*

l, therefore, upholds in all, 1st, the weight of the two
nd on *y y*; 3d, *half* the wt of and on *z r* and *x u*;* and 4th, its own wt. It
ufficient sectional area to safely sustain a pull equal to the sum of these four.
l by means of the table of bolts on p 376.
ual to the sum of these four wts. Draw *g m*, *g l* parallel to the rafters; and

ensions of the rafters, *a b*, *a d*, commencing with what
supported at the ends, bear in mind that the introduction of the struts *e c*, *h c*
s *a b*, into two shorter ones, *a e*, *e b*; each of which sustains, in the present
load of and on the whole rafter; or only ¼ of it as a *center* load. Find the safe
t beam, with its smaller center load, by rules, p 189, or by table, p 191.
er as if a vert pillar, which has to support on its top the pressure indicated by
in doing this, remember that the struts virtually reduce each rafter, as *a b*, to
e b; which require much less area than the whole rafter would. This area
rule on page 238; or by the table, page 239; and added to that required as a
e the total area at the head of the rafter. See footnote p 191.
of a line, H E, drawn on the same principle as in Figs 5, 6, and 9, (H I being
r and load,) the amount of pres which the *foot* of the rafter sustains from its
g on it. Divide it by 2000, (or by whatever other number of lbs may be con-
ng strength of timber.) The quot will be the safe area in sq ins reqd at the
and added to the areas previously found for a beam, and for a pillar, it gives
foot.
ther the breadth or the depth, (not both,) of the rafters regularly from foot to
e total areas found for those points respectively; or, which is generally better,
ghout the same area they have at foot.

n. Here I E added to *f m*, or to *f l*, will give by scale all the pull
de this pull by 3333, the safe pull in lbs per sq inch. The quot will be the safe
n. Then consider *one-half* of the tie-beam to be a uniformly loaded beam sup-
d find the safe dimensions by rules, p 189, or by table, p 191. To these dimen-
found for the hor pull; the sum is the entire area reqd for the tie-beam, un-
nade to compensate for the cutting away at the feet of the rafters.† See foot-

ne calculated dimensions for two trusses, Fig 7,
10 ft rise: and 12 ft apart from center to center. In the first of
h its floor, ceiling, and other load, are assumed at the rate of 100 lbs per sq ft
second, no specific load is assumed for that member, for reasons before given.
rafters, with their roof-covering and load of snow, and wind, is taken at 40 lbs
e between the centers of two trusses. The safety of each separate part is taken
uloaded tie-beam is fixed by rule of thumb. Timbers white pine. The great-
case is the depth. Dimensions in inches. 1st. Rafters 8 × 10 at foot; and
am 8 × 15. Each strut 4½ × 4½. King-rod 1⅝ diam if upset; or 2 ins if not
is better to make the struts as broad as the rafters. 2d. Rafters 6 × 8 at
ad. The tie-beam requires, theoretically, only 16 sq ins area; we will make
rs. Each strut 4½ × 4½; (the same as in the other.) King-rod ⅝ diam if
if not upset. See Note, p 260.
d instead of a tie-beam, its diam would be 1¼ inch if upset; or 1.6 if not.

9 is a truss with a tie-beam *a b*; two rafters *w a*, *z b*; two queen-
z t, and a hor
t may repre-
loaded along
ng beam; and
along the tie-
of these loads
exist, as in a
ong *a b*; or a
g *a w z b*. The
esides its own
of and on the
eam; and the
and on *u y*: *z*
lfway between
These are the
ueens; so that
n be found by
The parts of

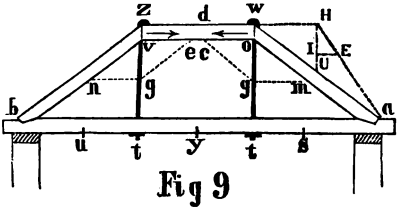

Fig 9

ns bear *half* of the wt of and on *z r*, or *h u*, only when, as in Fig 7, the incli-
the same as that of the rafter. If the strut is steeper than the rafter, it will
but if it is less steep than the rafter, it will bear less than half; the remainder
orne by the rafter. The parallelogram of forces will of course show all this.
s of a rafter and strut are not equal, we cannot draw hor diags *t k*, *v w*; but
s, *w*, we must draw hor lines to the vert diags *e o*, and *h n*.
of the struts in Fig 7, renders our process for that form of truss in some meas
however safe.
* SO LONG THAT IT MUST BE SPLICED, allowance must be made for the weakening
or Splices, see p 292; and for other joints, p 294.
uently made of wood.

the tie-beam from *s* and *u* to the abuts, or walls, as well as whatever loads those parts may bear, sustained directly by the abuts.

The queens transfer, as it were, the weights of themselves and of *s y* and *u y*, with their loads, rectly to *w* and *x*. To find the strains on the various parts of the truss, first from the centre U rafter *a w*, draw a vert line U H; and from *w* draw a hor line *w* H to meet it. Join H *a*. Mak by scale equal to the wt of only *one rafter* and its uniformly distributed load. Also draw *o g* and equal, by the same scale, to the wt upheld by the queen-rod *w t*, added to *one-half* the wt of straining-beam *d*, and its load; for it also presses vert at o. Draw *g m* hor, or parallel to the stra ing-beam; and *g c* parallel to the rafter; thus completing the parallelogram *o c g m* of forces.

The strains on the straining-beam *d*. The hor line I E and o gether, give all the hor pres against the end *w* of the straining-beam *d*; and it is plain that a cor process on the other side of the truss, would give an equal pres against the end *x*. These two c pressures reacting against each other, produce a strain, equal to one of them, throughout the e length of the straining-beam; and therefore, the beam must be regarded as a pillar with a load c to this strain, on its top; and the dimensions and area of section, for safely supporting it, may found by the rule, p 238; or table, p 239.

But beside this, the straining-beam, if loaded, must be regarded also as a beam supported at ends; and the area necessary for this, as found by tables, page 191, or 204, must be added to already found. See footnote p 191.

The strains on the rafters. *First*, consider a rafter *w a* as an inclin loaded beam supported at both ends; and find the proper dimensions and area, by the rules on 189; or by the tables, p 191, or 204. *Second*, consider it as a pillar supporting a load equal to and by p 238 or 239, find its safe area. Add this area to the one already found; and their sum wi the area of the rafter at its *head*. *Third*, measure by scale the number of lbs pres indicated by Divide it by 2000, (the crushing pressure in lbs which ordinary average building timber can be *short blocks* with a safety of 3.) The quot will be the area in sq ins reqd at the *foot* of the rafter, pressly for resisting the crushing effect at that point. Add this area to the two preceding ones the sum will be the total area for the foot. We may diminish to the top; or may make it of uni section throughout its length; the last is generally most convenient.

The tie-beam. The hor strain, or pull on the tie-beam, will be equal to push on the straining-beam; and is represented by I E and o c together. Find the safe area by t page 177; or by dividing the hor strain by 3333, which is the pull in lbs per sq inch that ordi building timber will bear with a safety of 3.

Then, since in this truss the queens divide the tie-beam into *three* lengths, each of these must be sidered as a separate beam, (loaded or unloaded, as the case may be,) supported at each end. Its dimensions being found by rules, p 189; or tables, pages 191, or 204, as may be reqd, add the area found for resisting the pull. Add, if reqd, an allowance for the cutting away at the feet of the ra

Below are the calculated approximate dimensions for t trusses, Fig 9, of sixty ft span; 15 ft rise; and 12 ft apart from cent center. All the conditions the same as for the preceding example of Fig 7. **1st.** Rafters 12 ins br by 14 ins deep at the head; and 12 × 14½ at foot. Straining-beam 12 broad, by 12 deep. Tie-b 12 broad, by 12 deep. Each queen-rod 1½ ins diam if upset; 1⅛ if not.†

2d. Rafters 10 × 11¾ at the head; and 10 × 12½ at foot. Straining-beam 10 × 11. Tie-beam, 10 × 12. Each queen-rod $\frac{5}{16}$ inch diam if upset; ½ if not. Unloaded tie-rod, 1 $\frac{3}{16}$ or 1½.

The proper size for each piece, ***so that they shall all be suita for the truss,*** cannot be determined at once. We must find any dimensions will answer for each piece by itself; and afterwards adjust them by recalculation, perhaps 3 times. Great accuracy is not necessary in doing this.‡ See Note, p 260.

For more on roof trusses, see pp 257 to 265; 294; and 298 to 302.

Rem. The truss in Figs 9 and 9¼ affords a good opportunity for alluding, in a general way the **principle of counterbracing,** and to the necessity (as stated in Art 1) of bering to a triangular arrangement of the parts of a truss. So long as this truss is uniformly loa throughout its length, it is well arranged for sustaining the resulting strains; because the strain each side of the center are equal, and balance each other. But if a heavy load be placed along only, its tendency to depress that portion of the truss will produce the derangement shown in Fig because the hor pressure from *s* toward *t*, Fig 9½, will then become greater than that from *t* towar The two triangular portions will still retain their original *fig;* but owing to the case with which 4-sided portion, *n m c e*, has been deranged, and changed to *s t c e*, their *position* becomes alter

† We have seen country bridges which have withstood common travel for many years, consisting but two trusses like Fig 9; and of no larger timbers and rods than in this example. Trusses 1 apart. They, however, trembled considerably. They should have had diagonal braces in each bay tion across the central division. They were, however, about 13 ft high, so as to be braced togeth overhead.

‡ The writer believes that the mode of treating the four preceding trusses, although somewhat em pirical, is more correct than that given by the authorities. He is indebted to Prof De Volson Wo for the correction of some erroneous views expressed in the first edition. See Prof Wood's paper the Jour Franklin Institute, Feb. 1872.

Fig 9¾ Fig 9¼ Fig 9½

s in Fig 9¼. The diag c m
ned to c t; while the diag
tened to e s. Now, if there
r bar of iron reaching from
ids firmly attached to those
ave divided the whole truss
nd then the diag c m could
lengthened to c t by any
one sufficient to break this
ag it apart; therefore the truss would have remained safe, and unchanged in figure;
s preventing c m from lengthening to c t, would, as a consequence, prevent e n from
. Or, omitting the iron bar at c m, suppose a stiff, unbending inclined post to
een e and n.- This also will divide the whole truss into triangles; and it is then
ld not be shortened to e q by any strain less than one sufficient to break the post by
erefore, in this case also, the truss would have remained safe, and unchanged in
st, while preventing e n from shortening to e s, would, as a consequence, prevent c m
to c t. Either the bar or the post would be a counterbrace against the effect of un-
With a uniform load it is not needed. Neither are additional counterbracing pieces
trusses of the forms Figs 10, 11, 12, 13, provided each web member is so constructed
ately compression and extension. See Rem 1, foot of p 308.

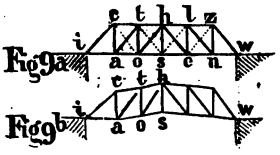

Fig 9a Fig 9b

b, shows the bad effect produced in a
a Fig 9¼, when the web members are
d. In the Burr bridge, Fig 36, and in
ough the truss is divided into trian-
lined braces, i c, &c, are often impro-
bear compression only; their ends not
ached to the chords. Consequently,
d at a, the derangement shown in Fig
that in Fig 9½) takes place. To pre-
racing must be resorted to, either by
r ties along the dotted diagonals; or
rces capable of resisting tension as
ion. The last method shows that counterbracing can be performed without the ad-
specially called counterbraces, and denoted by the dotted diagonals. All that is re-
nciple of counterbracing, is to so arrange and connect the several web members,
roduced by *unequal* loading at any point, as a, between the abuts; or along any
istance, shall be properly transferred by them to *both abutments*.

The weight of the truss and its load being uniform, the strains along
ers of all the following trusses may be found by three simple processes,
tch drawn to a scale, and representing the members by single straight
gs 10 and 11. Having prepared the sketch, (for which a scale of from
o a foot will generally suffice,) the **first process** is the very easy
ining how much of the total uniform weight is to be considered as
ach point of support along either the top or the bottom of the truss,
may be; remembering that one *half* of each end panel is sustained
e abut nearest to it, as in the preceding cases.*
re fully to illustrate the following Articles, we shall assume each of
Fig. 22 inclusive, to be 64 ft long, and 16 ft high. The height (called
y the depth) is understood to be measured *vert between the centers of*
ss-sections of the upper and lower chords. Each truss is assumed to be
equal panels. Total uniform wt of one truss and its load, 32 tons; or 4
l. Consequently there will be 9 points of support to each truss. Thus,
and 16, in which the load is supposed to rest on top of the truss, and
11, in which it rests upon the bottom, the points of support are at *c*,
, *o*, *p*. Some of these are not shown in the last three Figs. Now, in
, *z*, *h*, being midway between the several points of support, it is plain
g *all* the weight to be on the lower chord) the central point, *g*, must
portion of it comprised between *z* and *h*; *d* all between *y* and *z*; *c* all
l *y*; *b* all between *w* and *x*; while the abut *a* sustains *directly* the por-
a; and the abut *p* that from *g* to *p*. The same principle applies to all
ses; and equally so whether the panels be of the same width or not;
support is assumed to sustain all the uniform wt of truss and load be-
nd the two points midway to the adjacent points of support. It might
at first sight that the *lower* parts of the truss in Figs 14, 15, 16, (or
11, if *inverted*,) are sustained by these *upper* points of support; but
plain when we reflect that these lower parts are *suspended from* the
y the web ties, and are therefore upheld by it as fully as if they rested
ur Figs 10 to 16, the strong dotted lines of the web members represent

* Considering all the weight to be concentrated on one chord; although usually suf-
for roofs, is not so for *large* bridges. See Arts 18, 19, and 20.

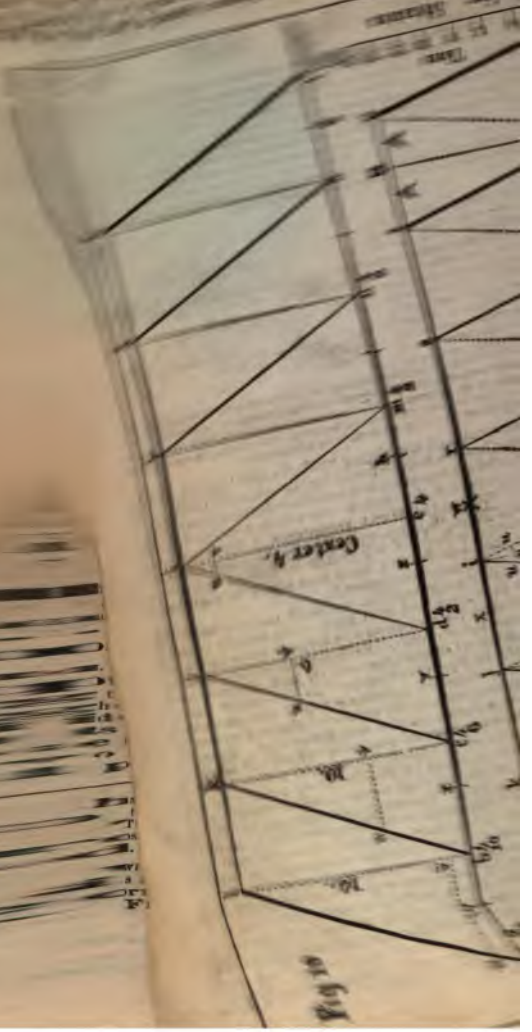

Fig. 10

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each of the others. Therefore, to find the amount of wt sustained at
nine points of support, we have only to div the total wt (32 tons) by a
by 1 than the number of points. The quot $\frac{32}{8} = 4$ tons, will be a *full*
to be at each point, except the two end ones, *a* and *p*, at the abuts; at
ch it will be but half of one of the full panel-loads, or two tons. The
these panel and half-panel loads should at once be figured on the sketch
per points, as is done in our Figs; a 2 being placed at each end of the
a 4 at the other points. The other numbers at the same points are the
other vert strains, which will be explained as we proceed. As the strains
of the truss are the same as those on the other half, the numbers need
tten on one of them; indeed, the sketch, as a general rule, need show
f of the truss.

cond process. All the panel-loads are of course eventually trans-
ugh the truss to the abuts; as is manifest from the fact that each abut
lf the total load. But each panel-load, while travelling, as it were, from
point of support, to the nearest abut, produces an additional strain upon
point of support encountered on its way. Our second process consists
he *vert* component of this additional *strain* at each point of support.
0 and 11. the routes taken by the panel-load strains lead at once toward
abut; but in Figs 14, 15, and 16, they first go forward to the center *c* of
and from thence are transmitted backward to the abuts, along the in-
r member *ca*, Figs 14 and 16; or the inclined lower one *ia*, of Fig 15. In
culiar force is generated by the raising of the center of the tie-bar *a i u*;
ill be considered after the others. It is very easy to determine, at a
ther the panel-load strains travel at once toward the abuts, or toward
f the truss; and it is essential that this be first done. We have only to
uppose, for the moment, that a tie can, like a chain, sustain no pressure;
no pull; then, by looking at any point of support, as at *c*, Fig 10, we see at
s strain, being a pull, cannot travel toward the center, along the strut *cj*,
toward the abut *a*, through the tie *ck*. Whereas, in Fig 14, the pressing
annot go toward the abut through the tie *ck*, but only toward the cen-
the strut *cj*. When, as in Figs 10 and 11, the strains travel at once
nearest abut, the web members are slightest at the center of the truss,
t near the abuts; while the chords are stoutest at the center, and slight-
uts. But when the strains first move toward the center of the truss, all
sed; as indicated by the diff thicknesses of the lines in the Figs.
) and 11, with *parallel horizontal upper and lower chords*, the vert strains
sily found, thus: Remembering that only *half* of the *center* panel-load
goes to *each* abut, begin with 2 of the 4 tons at *e*. Add those 2 tons at *d*,
m down as in the Figs; thus making 6 tons of vert strain at that point.
From the *whole* of these 6 tons goes to the abut at *a*; therefore, add 6 tons
aking 10 tons of vert strain at that point. The *whole* of these 10 tons go
fore, add 10 tons at *b*, thus making 14 tons of vert strain at that point.
f these 14 tons go to *a*; therefore, add 14 tons to *a*, thus making 16 tons
in at that point. Now, 16 tons is half the total wt of our truss and its
nce this must always be the case, it proves our work to be correct thus far.
e numbers 4, 6, 10, 14, written on the verts of Fig 10, denote the tons of
those members; but are placed there to be referred to in Art 18.

ird and last process consists in completing our sketch, or diagram,
anner as to enable us to measure by scale the strains produced along
ber of the truss, by these vert strains thus accumulated at the diff points
a, *b*, *c*, *d*, *e*, &c. In Figs 10 and 11, from the points *i*, *j*, *k*, *l*, (and observ-
Fig 11, *i* is not at the center,) lay off by any convenient scale, upward or
as the case may be, (upward if the Figs, or, rather, the trusses, are in-
vert dists *i v*, equal to 2 tons, or *half* the vert strain at the center point *e*;
', *l v'''*, respectively equal to the *entire* vert strains, 6, 10, 14 tons, at *j*, *k*,
16-ton strain at *a* is not sustained by the truss, but by the abut.
'', *v'''*, draw the hor lines as shown in the Figs. Then, the triangles formed
r lines, and by the portions which they cut off from the oblique and
embers, if measd by the scale used in laying off the vert dists, will (with
on below) give all the strains along the several members of the truss.
of the triangles, as *l u*, *k u*, *j u*, *i u*, &c, which are formed by portions of
mbers, give the strains along said members; while the hor dotted sides,
0, and *u u in Fig 11, give those* along the hor upper and lower chords.*

* *along the obliques may be found correctly by calculation, thus:* Mult the vert strai

The exception just alluded to is in the *center vert e i*, of Fig 10, in which the st must be mead by *twice i v*, because this member itself sustains *all* of the 4 tons load from *z* to *h*; although at its upper end it transfers only *half* of the rend strain to *each* of the two obliques, *i d*, *i m*.

Below we add the amounts of all the strains, to enable the young student to test his own trial the same Figs. If the Figs be inverted, the process remains the same; as do also the amounts of strains; but what was before a pull, will now become a pressure; and the pressures will be pulls; so that the ties become struts; and the struts, ties. These remarks apply to the chords, as as to the web members.

Strains along the web members.

Fig 10.		Fig 11.		Fig 11.	
Along $e i = i v \times 2 =$	4 tons.	Along $i d =$	2.24 tons.	Along $e i$ and $i d$, each =	2.24
$d j = j v$	$= 6$	$j c =$	6.71	$d j$ and $j c$, " =	6.71
$c k = k v''$	$= 10$	$k b =$	11.8	$c k$ and $k b$, " =	11.8
$b l = l v'''$	$= 14$	$l a =$	15.7	$b l$ and $l a$, " =	14.4

Strains along the horizontal chords.

Along the portion $a b$, or $l k$, each = the hor line $v''' = 7$ tons.
$b c$, or $k j$, " $= v''' + v'' = 7 + 5 = 12$.
$c d$, or $j i$, " $= v''' + v'' + v' = 7 + 5 + 3 = 15$.
$d e$, or from i to center; or at i at the center, $= v''' + v'' + v' + v = 7 + 5 + 3 + 1 = 16$

The hor strain at the center of either hor chord, must th equal to

$$\frac{\text{½ of the entire uniform weight of the truss and its load} \times \text{¼ of the span, } a p}{\text{Height or depth of truss,}}$$; which, in Figs 10 and 11, is $\frac{16 \text{ tons} \times 16}{16 \text{ feet}}$

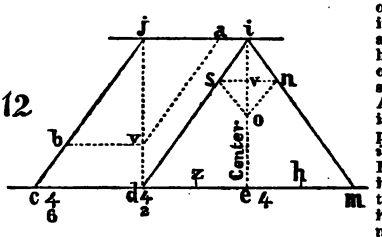
16 tons, as above. This serves as an easy proof of the work.

As to the *transverse* strains on the several parts *a b*, *b c*, *c d*, &c, of the lower ch considered as beams, see foot-note p 191.

Art. 12. We will briefly explain the *principle of* the foregoing process, wh indeed, is almost identical with that involved in the following trusses; and wh seen to require no more knowledge of the laws of force than might be expect any intelligent school-lad.

Referring to Fig 10, we shall employ Fig 12. Bearing in mind that as the wt (4 of one panel of the truss its uniform load, is sp along *z h*, it is plainly al held by the center vert ti or, in other words, sai sustains a vert pull of 4 t At its upper end *i*, this is transmitted in shape push to the two obliques *i d*, *i m*; and from Comp Res of Forces we know if we make *i e* by scale a to these 4 tons, and if w it as a diag we complete parallelogram of forces fa

Fig 12



then *i s* and *i n* will, by the same scale, give the oblique strains produced by the tons along the struts *i d*, *i m*. Now it is plain that this amounts to the same th as was done in Fig 10, in which we merely drew *i v* and *v s*; nothing more ha actually necessary.

Then as to the 6 tons at *d*, it is evidently all borne by the tie *d j*. In other wo from *d* there is a vert pull of 6 tons against *j*; and this pull must plainly be resi or sustained by the chord *i j*, and the strut *j c*. Hence, if, as before, we take the dist *j v*, by scale, to represent this vert pull of 6 tons; and if on *j v*, as a diag complete the parallelogram of forces *j a v b*, then will *j a*, or *its equal*, *b v*, give hor push along the chord; and *j b* the oblique one along the strut. And so with other strains. This also amounts to the same as was done in Fig 10.

We shall illustrate Fig 11 by Fig 13. The principle is precisely the same as bef The center load of 4 tons, at *e*, produces pulls along the two ties *e i*, *e v*; and if make *e t* equal to 4 tons; and on it as a diag, draw the parallelogram *e n t s*, th will *e s* and *e n* give the amounts of the two oblique pulls. But as this is not *center of the* truss, and consequently only one of these pulls goes to *each* chord,

(except the center one, of which mult *one-half*,) by the nat secant of the angle t e s, Fig 13, or t *Fig 11, which the oblique* forms with a vert. To find this nat secant, divide the length of an *by its height, or* vert stretch. Or, for strain along an oblique, mult the vert strain by the *the oblique*, and divide by its vert stretch.

to examine but one of them, say *e s*. This acts to pull *i* toward *e*; and at *i* ill is resisted by the chord *a i*, and by the strut *i d*. Hence, if we make *i o*, *o*, equal to *e s*; and upon it as a diag draw the parallelogram *i a o b*, then will *its equal o b*, give the pres along the chord; and *i b* that along *i d*. But since e made *i o* equal to *e s*; and have drawn *o b* hor, or parallel with the hor chords, ain that the vert *i v* must al to the vert *e v*; that is, *half of the vert force* at *e*, cted in Fig 11. The same ing applies to the pull on the 6 tons at *d*; except nasmuch as this is *all* by the *one* tie *d j*, we must *j v* equal to the *whole* 6 s directed in Figs 10 and

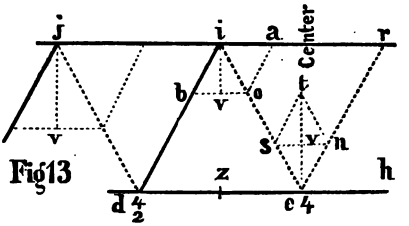


Fig 13

20 gives a much more ilent mode of finding the along such trusses as regoing, when we do not trace their progress.

t. 13. In Figs 14, 15, and 16, (in which, as before, the span is 64 ft, the rise and the load to a full panel 4 tons,) the **second process** (or that of finding

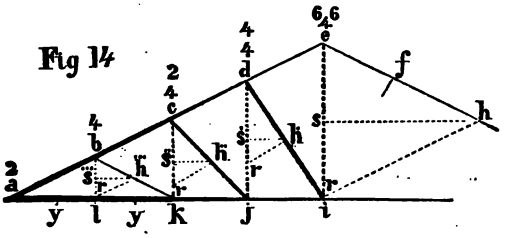


Fig 14

uch additional vert strain is produced at the several points of support by each Load, on its way to the nearest abut) differs somewhat from that pursued with 10 and 11, which have *parallel* upper and lower members. The principle in-d, however, is precisely the same.* In Fig 14, *beginning at the panel-load b, at the abut*, lay off by scale the vert *b r* equal to that panel-load. 4 tons. Draw parallel to the rafter; and *h''' s'''* hor. Measure *b s'''*, (2 tons,) and carry forward amount to *c*, writing it down over the panel-load 4; thus making the total load 6 tons (See Remark, Art 15.) Make *c r* equal to this total load: draw *r h''* lel to the rafter; and *h'' s''* hor, as before. Measure *c s''*, (4 tons,) and carry it ard to *d*, writing it down, and thus making the total load at *d* 8 tons. Make *d r* l to this total load; draw *r h'* parallel to the rafter; and *h' s'* hor, as before. ure *d s'*, (6 tons,) and carry it forward to the center *e*, writing it down. Now it is that the same process, on the other half of the truss, would bring another 6 to *e*. Write this down also, as in Fig 14. Thus we get for the vert strain at *e*, 4 + 6 = 16 tons, or one-half of the wt of the entire truss and its load. As this always be the case in *such* trusses, it proves our work to be correct so far. See 18.

hird process. From *e* draw a vert line *e r*, by the same scale as before,

---

hen wishing merely to know the *amount*, without caring to *trace* the *progress of* these vert ns at the points of support, in order to calculate the other strains, we may omit part of the folg, and proceed thus: At the peak *e* of the truss, the vert strain will always in such trusses be l to *half* the entire weight of the truss and its load, (for which, *per sq ft*, see Table 4, p 301,) may therefore be written down at once at *e*. Next count the entire number of points of support *a, d, e, f, &c*, of the truss. From this number, whatever it may be, subtract 1. Div the weight of the truss and its load by the rem. One *half of* the quot will be the vert strain at the end of the truss; which write down at *a*. At *b* the strain will be *twice* that at *a*; at *c* *three* times; 4 times; and so with any number of points of support along a rafter, except at the center one *e*, re the strain will be *twice as great* as at the preceding one *d*. But it has been found already, *w this has been done, the vert lines b r, c r, d r, e r, &c*, may at once be drawn equal by scale *several vert strains*: then *r h, r h, r h, &c*, parallel to the rafter; and *s h, s h, &c*, parallel t *Then begin as "Third Process."*

equal to the 16 tons of total load at e; draw $r\,h$ parallel to the rafter; and h
The fig is now ready for giving by scale the strains along all the diff members
truss. It is by mere chance that the two vert lines $b\,l$ and $e\,r$, representing th
at b and e, happen just to extend to the hor tie $a\,i$ in our fig. As with the pa
figs, we add the strains in this case also. When as usual, the points of suppo
c, &c, are equidistant, the continuous additions for the strains along the w
along the several divisions of a rafter or main tie, will be equal to each other
roofs; but those along the obliques will be unequal, on account of their
obliquities.

Strains along the verticals.

Along $b\,l$ = nothing, except weight of tie-bar from y to y.*

" $c\,k$, $= b\,\dddot{s} = 2$ tons.

" $d\,j$, $= c\,\ddot{s} = 4$.

Strains along the obliq

Along $b\,k = b\,\dddot{h} = 4.47$ tons.

" $c\,j = c\,\ddot{h} = 5.66$.

" $d\,i = d\,\dot{h} = 7.21$.

$e\,i = 2\,d\,\dot{s} = 2 \times 6 = 12$; for while each other vertical tie bears only the vert strain brought
the oblique strut next nearer the abut; the *center* tie $e\,i$, of course bears those from
one on each side of it.

Strains along the horizontal tie-bar $a\,i$.

$$\text{At } i = s\,h = 16 \text{ tons; also} = \frac{\tfrac{1}{2} \text{ wt of truss and load} \times \tfrac{1}{2} \text{ span}}{\text{height of truss.}}$$

From j to $i = s\,h + \dot{s}\,h = 16 + 4 = 20$.

" k to $j = s\,h + \dot{s}\,h + \ddot{s}\,h = 16 + 4 + 4 = 24$.

" a to $k = s\,h + \dot{s}\,h + \ddot{s}\,h + \dddot{s}\,h = 16 + 4 + 4 + 4 = 28$.

Strains along the rafter $e\,a$.

From e to $d = h\,r = 17.9$ tons.

" d to $c = h\,r + \dot{h}\,r = 17.9 + 4.47 = 22.4$.

" c to $b = h\,r + \dot{h}\,r + \ddot{h}\,r = 17.9 + 4.47 + 4.47 = 26.8$.

" b to $a = h\,r + \dot{h}\,r + \ddot{h}\,r + \dddot{h}\,r = 17.9 + 4.47 + 4.47 + 4.47 = 31.3$.

It will be observed that the hor components $h\,s$, except the center one, hav
lengths; also those marked $h\,r$, parallel to the rafter; while the oblique on
not.

For transverse strains, see foot-note p 191; also Art 29, p 298.

For a span of 100 feet, rise 20 ft, or $\frac{1}{5}$ of the span; trusses 10 fe
from center to center; loaded on top only; the following dimensions will be
sufficient for a covering of slate. Rafters and tie-beam, each 10″ × 12″ deep
rafters may, if preferred, be reduced to 9 × 12 at top. The verts of round b
of good quality, if not upset at the screw ends, ½, ¾, 1, and 1¾ ins diam
obliques or braces, 5 × 10, 6 × 10, 8 × 10; thus making the truss-thickness uni
10″. See Table 2, p 299; also Note, p 260.

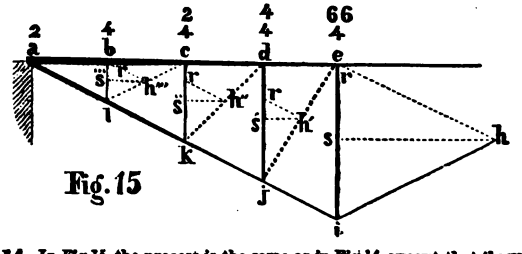


Fig. 15

Art. 14. In Fig 15, the process is the same as in Fig 14, except that the ver
representing the strains at the points of support a, b, c, d, e, are to be drawn s
from i, k, j, i; and the strains $i\,s'''$, $k\,s''$, $j\,s'$, are to be carried forward to th

* On this account a vertical tie at $b\,l$ is rarely required, except when the main tie $a\,i$ is w
which case, see Art 18.

l. The strain on $b\,l$ will be the 4 tons at b. All the other strains will be those in Fig 14; the hor lines $h\,s$, &c, giving those along the hor upper ; and the oblique ones $h\,r$, &c, those along the oblique tie $a\,i$. It is easy to ee 4 tons at b must strain the vert post $b\,l$ to that same amount, since they l by the post to l, where they are transferred to $l\,a$ and $l\,c$. But in Fig 14, at b, rest directly upon $b\,a$ and $b\,k$, and produce no strain whatever upon le $b\,l$.

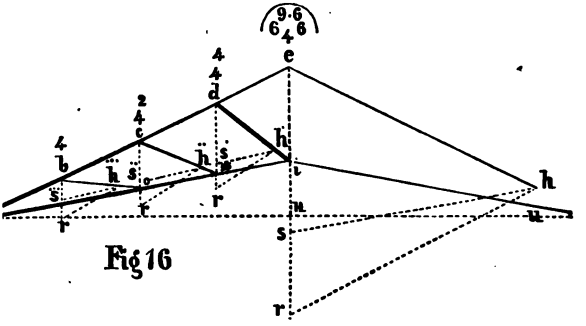


Fig 16

15. In Fig 16, the process is the same as in Fig 14, except that the lines $h\,s$, be drawn and measured parallel to the inclined tie $a\,i$; instead of being hor. ' must be carried forward to c; $c\,s''$ to d; $d\,s'$ to e. In this manner, we find, , the center load of $6 + 4 + 6 = 16$ tons at e. But we must now add to tons, another strain generated by the obliquity of the tie-rod $a\,i$. This strain by mult the load at e, (16 tons, or half the wt of the entire truss and its the vert dist $n\,i$, (6 ft,) which the center i of the tie-rod is raised above ontal $a\,u$, (see Remark 1, Art 16;) and div the prod by the dist $i\,e$, (10 ft.) $\frac{16 \times 6}{10} = 9.6$ tons; which also write down as in the Fig; making the total in at e 25.6 tons, instead of the 16 tons of Fig 14.*

nake $e\,r$ by scale, $= 25.6$ tons; draw $r\,h$ parallel to the rafter $e\,a$, and meet- ther rafter; also draw $h\,s$, parallel to the raised tie-bar $i\,a$. Then the strains members of the truss will be as follows, taken from a Fig on a larger scale.

s along the verticals.

one at b = nothing, except weight of tie-bar for the width of one panel (8 ft.)
Along $c\,o = b\,s'' = 2$ tons.
" $d\,x = c\,s'' = 4$ "
" $e\,i = 2\,d\,s' + 9.6 = 21.6$.

Strains along the obliques.

Along $b\,o = b\,h''' = 6.43$.
" $c\,x = c\,h'' = 6.96$.
" $d\,i = d\,h' = 8.04$.

Strains along the raised tie-bar $a\,i$.

At $i = h\,s = 26$ tons.
From x to $i = h\,s + h'\,s' = 26 + 6.5 = 32.5$.
" o to $x = h\,s + h'\,s' + h''\,s'' = 26 + 6.5 + 6.5 = 39$.
" a to $o = h\,s + h'\,s' + h''\,s'' + h'''\,s''' = 26 + 6.5 + 6.5 + 6.5 = 45.5$.

Strains along the rafter $e\,a$.

From d to $e = h\,r = 28.5$ tons.
" c to $d = h\,r + h'\,r = 28.5 + 7.13 = 35.63$.
" b to $c = h\,r + h'\,r + h''\,r = 28.5 + 7.13 + 7.13 = 42.76$.
" a to $b = h\,r + h'\,r + h''\,r + h'''\,r = 28.5 + 7.13 + 7.13 + 7.13 = 49.9$.

nverse strains on the rafters, see foot-note p 191.

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obable that the tie-rod is sometimes raised in this manner by persons ignorant of the fact hereby greatly increase the strains on the rafters, &c.

Rem. The reason for measuring only *parts* of the vert lines which, in Fi 16, represent the whole panel-loads, is that the rafter *a e*, Figs 14 and 16; *a i* of Fig 15, being *inclined*, also bear a part of each panel-load; and since does not go forward to the next point of support, but goes backward, alon clined member, to th *a*, it must be omitt *second process*. Thu 17, if *b a* be an *inclin* resting on an abut strut; and *b r* a vert l senting the load sus *b* by *b a* and *b g*: if plete the parallelogr of forces, then will by scale the strain strut; and *b n* that rafter. The strain strut is made up or of the portion *b s* of and the hor force

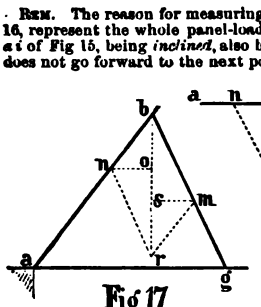


Fig 17

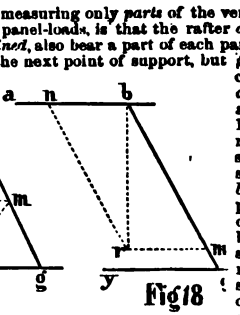


Fig 18

vert portion *b s* alone goes to the next point of support; while *s m* strains hor. So also the strain *b n* is made up of the other portion (*b o* or *s r*) o force *b r*; and of the hor force *o n*; which, when the strain *b n* reaches again resolved into two; one of which, *b o*, presses vert upon the abut: o words, transfers to the abut the portion *b o* of the load resting on *b*; whil tion *o n*, which is equal to *s m*, strains the tie *a g* hor.

But in Fig 18, where *b r* also represents a load resting on *b*, and suppo strut *b g*, and by a *hor* chord *b a*, if we complete the parallelogram *b m r n* the strain *b m* along the strut, composed of *all* the vert force *b r*, and the

NOTE.

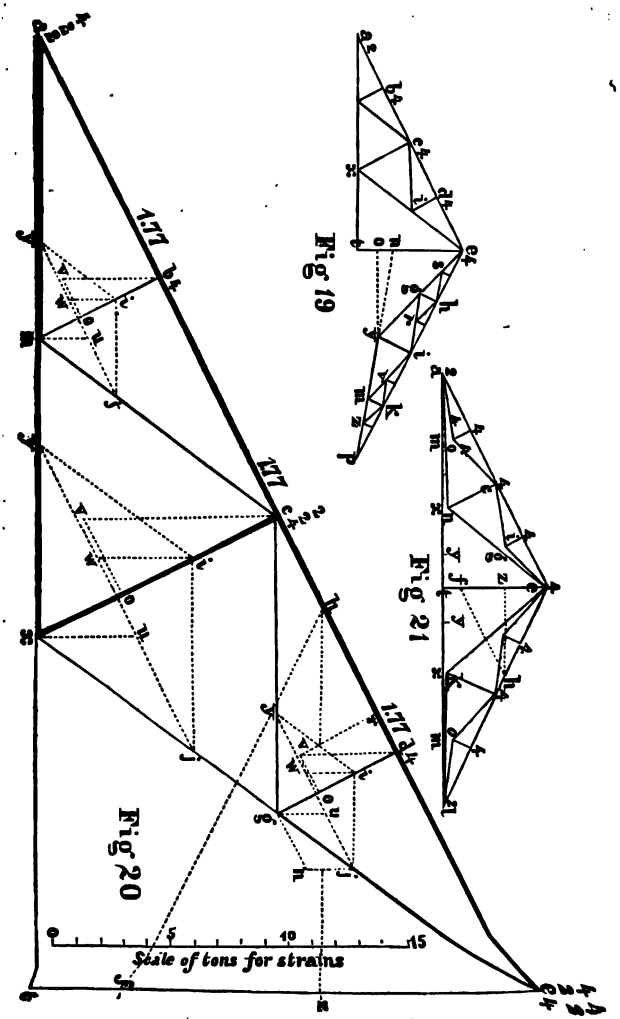
The following may at times save trouble in designi trusses. After the dimensions of all the members of a roof truss o have been calculated, then those of any *smaller* span arranged in the same manner, an same rise in proportion to its span: but with the trusses at the same distance apart a one; may be found *safely*; and often near enough for practice, thus:

Find the area of cross section of each member of the large truss, in sq ins. Then m of cross section of the corresponding members of the small truss in the same proporti is smaller than that of the large one. The small truss thus obtained will in fact be at span than the large one. If the total loads sustained by trusses of different spans, *includin of the trusses themselves*, were in proportion to the spans, then this method would be o with the trusses at the same distance apart in both cases, only the *extraneous* total load is is in proportion to the span; while the total weight of the trusses themselves is as the s spans. In our table of wts of iron roofs on p 300, it will be seen that it is based upon 40 lbs per sq ft of ground covered, *including the wts of the trusses themselves*. Also th the truss itself of 175 ft span is 8.05 lbs per sq ft of ground covered; so that the greates load of purlins, slate, snow, &c, for this span is $40 - 8.05 = 31.95$ lbs. If now we prep of 35 ft span by the above mode, or by the table (which is based on that mode), we find th as 35 is but $\frac{1}{5}$ of 175, the short truss will weigh but 396 lbs, or $\frac{1}{23}$ part of 9379 lbs, the t the 175 ft span. We also see that the 35 ft truss will weigh but 1.61 lbs per sq ft of gro while the 175 ft one weighs 8.05 lbs or 6.44 lbs more. Therefore the small truss will be an extraneous load of $40 - 1.61 = 38.39$ lbs per sq ft, as the large one is under $40 - 8.05 =$ in other words, if the large one is strong enough, the small one will be about $\frac{1}{6}$ part as necessary. Reductions however will rarely be made to as small as $\frac{1}{8}$ of the original; an do not exceed $\frac{1}{4}$, the method will answer very well in practice.

For examples of reducing, see p 301.

The whole of $b\,r$ is transferred to the next point of support; while $r\,m$ and $b\,n$ use only *hor* strains along $b\,a$ and $g\,y$.

Art. 16. The roof truss shown by Figs 19, 20, and 21, consists of two complete truss, $a\,e\,x$ and $e\,y\,p$, Fig 19. It is supposed to be of the same span and height to others; and to have the same number (9) of points of support for the weight



(32 tons,) supposed to be uniformly distributed along its top. Consequently th
sults from our first process, that there will, as before, be 2 tons of panel-load at
of the end supports; and 4 tons at each of the others. Write these down as in
20. Now, at either strut, as $d g$, draw a vertical line $d v$ in pencil, by any convenie
scale, to represent a whole panel-load, (4 tons;) and draw $v o$ parallel to the rafter
and $v r$ parallel to the strut. Measure $d r$ or $v o$ by the scale, and write down
result (1.77 tons) near every strut, as in the Fig. The object of this will be sh
hereafter.* The lines $d v$, $v o$, $v r$, may then be rubbed out. Now the strains f
the several panel-loads, in passing to their final points of support at the abuts, ta
by a route diff from that in either of the preceding cases. The part truss $e x a$
be regarded, to some extent, as being composed of three separate trusses; nam
$e x a$, $e g c$, $a c m$; as will appear more plainly from $e n a$, $e i c$, and $a c o$, in Fig
These may be called *first* and *second secondary trusses*. In Fig 19, the half $c p g$
hibits a truss on the same principle, but having a greater number of points of
port for the uniform wt. That half truss consists of first, second, and *third* sec
aries, as shown by $e y p$, $e g i$, and $e s h$. However far this subdivision may be ca
if the struts occur at equal dists, and if the wt or panel-load supported by each
is the same, then each panel-load resting on a short strut will travel (one-half
each way) either to the panel-load of the next longer strut; or else to one end o
rafter. Thus, beginning our second process with one of the shortest struts, as
Fig 20, 2 of its 4 tons go to c at the next longer strut $c x$; and 2 of them to the e
of the rafter, as written on the Fig. Then, at the other of the shortest struts, 2
tons go to c: and 2 to the end e of the rafter. We thus have 4 tons at b; 4 at d; a
at c. But of these 8 tons at c, 4 travel down the strut $c x$, and along the tie x
the end a of the rafter; and 4 along $c x$ and $x e$ to its other end e; both of th
tons are therefore set down as at a and e. When there are more points of sup
as along the rafter $e p$, Fig 19, the process is precisely the same: we first adjus
strains of the four third secondaries, $e s h$, $h r i$, $i v k$, $k z p$; placing them at e, h,
and p; then we transfer those thus accumulated at h and k, to e, i, and p; and th
transfer them from i to e and p, at the ends of the rafter. Now, returning to Fi
we see that in addition to the original panel-load of 4 tons at e, we have accumu
6 tons of vert strain from the other panel-loads; and it is plain that the same
cess, performed along the other half of the truss, would bring $2 + 4 = 6$ tons
to e, as written in the Fig. Thus it appears that we have 16 tons in all at e; and
is precisely half the weight (32 tons) of the entire truss and its load; and as
will always be the case in trusses on this principle, it proves our work to be co
thus far. In like manner, the total strains accumulated at the other end a a
rafter; as well as those at its center c, must always each be equal to one quart
the weight of the entire truss and load. Also the total strains thus accumu
at any longer strut, will be twice as great as that at any *next shorter* one. See A

Having thus finished our second process of finding the additional strains a
several points of support produced by the panel-loads on their way to their
points of support, nothing remains but the third process; that is, to complet
drawing, so that we may measure by scale the strains along all the members o
truss. To do this, from the tops of the struts draw vert lines $b r$, $c r$, $d v$ to r
sent the *total* vert strains accumulated at those respective points; namely, 4 ton
b, 4 at d, 8 at c. Draw $r o$, $v o$, $v o$ parallel to the rafter $a e$. Then $b o$, $c o$, $d o$
give the strains along the struts; 3.6 tons on $b m$ or $d y$; and 7.2 tons on $c x$. La
$m i$, $x i$, and $g i$ respectively equal to $b o$, $c o$, and $d o$; draw $i j$, $i j$, $i j$; and $i j$,
$i y$, parallel to the ties; thus completing the parallelograms of forces $i j m y$, $i j$
and $i j g y$. Also draw the diags $y j$, $y j$, $y j$; and the vert lines $i w$, $i w$, $i w$,
$x u$, and $g u$. Also lay off the vert dist $e f$ equal to the total vert strain (16 t
at e; draw $f h$ Fig 21) parallel to the rafter $a e$, and terminating at h in the other m
$e l$; and $h z$ hor or parallel to the tie $a l$. Or, which amounts to the same thing
Fig 20 make $e f$ equal to the strain (16 tons) at e; make $e z$ = to half of $e f$; d
$z h$ hor; and $h f$. This saves the necessity of drawing more than half the truss.

Now $m y$ and $m j$ give the strains (4 tons each) along the ties $m a$, $m c$, caused
the 4 tons at b; which strains extend from m to a and c.† In like manner, $g y$

* When merely wishing to ascertain the strains along the members of such a truss, without ca
to trace their progress, we may omit part of the following; and, after having made a correct diag
of the truss; and found and noted down the force $d r$ or $v o$ mentioned above, we may at once w
down the vert strains at the points of support, thus: At e (the apex or peak of the truss) write
half of the entire wt of a truss and its load, (for which, *per sq ft*, see Table 4, p 301,) at the en
strut c, one fourth; at the foot a of the rafter, also one *fourth;* at b and d, one *eighth* at each;
when there are four intermediate subdivisions of the same kind, as along the rafter $e p$, Fig
one sixteenth of said entire weight at each of such additional points, &c. Then begin at "Hav
thus finished our second process."

† *These strains* along the ties will be equal to those at the points of support, only where the
of the truss is equal to ¼ of its span; as in the case before us. When the height is less than
strains on the ties will be greater than those at the points of support; and vice versa.

$g\,j$ give the strains (4 tons each) extending from g to c and e. In Fig 21, the short lines, $o\,a$, $o\,c$, $i\,c$, $i\,r$, show this more distinctly.

Next, $x\,y$ and $x\,j$, Fig 20, give the strains (8 tons each*) produced along $x\,a$ and $x\,e$ by the 8 tons at c.* This also is shown more plainly in Fig 2 , by the ties $n\,a$ and $n\,e$. Again, the hor line $h\,x$ will give the strain (16 tons) produced along the entire hor tie $a\,l$, Fig 21, by the 16 tons at e. Fig 20 may be considered one half of Fig 21.

In practice, the ties $a\,o$, $a\,n$, &c, Fig 21, of the secondaries, are not always made distinct from that ($a\,l$) of the primary truss $a\,e\,l$; but they are so represented in Fig 21, merely to show more plainly that the central portion $x\,x$ of the primary tie $a\,l$ needs only such dimensions as will enable it to sustain the thrust produced by the 16 ton strain at e: whereas, along its portions $x\,m$, $x\,m$ it must be stout enough to bear, *in addition*, the thrust along the first secondary ties $n\,a$, $k\,l$; while at its ends $n\,a$, $m\,l$ it must resist not only the two preceding thrusts, but also those along the second secondary ties $o\,a$, $o\,l$. Likewise, it is plain that the portion $g\,e$ of the first secondary tie $n\,e$, must be stouter than the portion $n\,g$; because when $n\,e$ is formed of one bar, its portion $g\,e$ has to bear also the thrust along the second secondary tie $i\,e$. In Fig 20, those portions of the ties which are most strained are shown by stouter lines.

We have for the strains on the ties, as follows:

Along $c\,m$ and $c\,g$, strain $= m\,j$ or $g\,y = 4$ tons.
Along $x\,e$, from x to g, strain $= x\,j = 8$ tons.
Along $x\,e$, from g to e, strain $= x\,j + g\,j = 8 + 4 = 12$ tons.
Along $t\,a$, from t to x, strain $= h\,x = 16$ tons.
Along $t\,a$, from x to m, strain $= h\,x + x\,y = 16 + 8 = 24$ tons.
Along $t\,a$, from m to a, strain $= h\,x + x\,y + m\,y = 16 + 8 + 4 = 28$ tons.

The strain along a rafter $a\,e$ would be equal throughout, were it not for the small strains, of 1.77 tons each, at b, c, and d first found.† This will be seen thus: The 8 tons at c, Fig 20, produce *forces* at a and e of 5.4 tons each; which are found by measuring $u\,j$ and $w\,y$, of the middle parallelogram of forces $i\,j\,x\,y$. Consequently these two forces acting against each other at the opposite ends of the rafter, produce a uniform pressing *strain* of 5.4 tons throughout its entire length. Again, the 4 tons at b produce *forces* at a and e of 2.7 tons each, found by measuring $u\,j$ and $w\,y$ of the parallelogram $u\,j\,m\,y$. Consequently these two forces acting at the opposite ends of the half rafter $a\,e$, produce a strain of 2.7 tons along said half. But the 4 tons at d produce in like manner the same amount of strain along the other half, $e\,e$. Finally, the 16 tons at t, produce two forces, each equal to $e\,h$ (17.9 tons;) one of which presses the entire length of each rafter. Consequently we have, as pressing its entire length, the forces $e\,h$, $u\,j$ of the middle parallelogram; and $u\,j$ of the parallelogram $i\,j\,g\,y$; or $17.9 + 5.4 + 2.7 = 26$ tons. Nothing but these 26 tons press it from e to d; but from d to c we must add 1.77 tons; from c to b 1.77 more; and from b to a 1.77 more; so that we have at last, for the total strain along a rafter,

From e to d, strain $= e\,h + u\,j$ (of the parallelogram $i\,j\,g\,y$), and
$u\,j$ (of the parallelogram $i\,j\,x\,y$) $= 17.9 + 5.4 + 2.7 = 26$ tons.
" d to c, " $= 26 + 1.77 = 27.77$.
" c to b, " $= 26 + 1.77 + 1.77 = 29.54$.
" b to a, " $= 26 + 1.77 + 1.77 + 1.77 = 31.31$.

The center vert tie $e\,t$ is sometimes omitted in short spans; since it sustains nothing but the wt of the half ($j\,y$) of the central spread $x\,x$ of the hor tie $a\,l$. Thus we have the strains along every member of the truss. See Art 29, p 298.

Rem 1. If the main or primary tie, instead of being hor like $a\,l$ in Fig 21, is raised at its center, as $p\,n$, Fig 19; or if it is raised only as far as y, and is then continued hor, as $y\,o$, (as is frequently done,) *in either case* we must proceed as at Fig 16, p 259; and, after having found the vert strains at all the points of support, as before, we must add to that (16 tons) at e, the amount arising from mult said 16 tons by the vert height $t\,n$ (Fig 19) to which the tie $z\,n$ is actually raised above the hor $a\,t\,p$, (or to which it *would be* raised if the inclination of the tie $p\,y$ were continued to n, instead of being hor like $y\,o$:) and from div the prod by the remaining height rise of the truss. Then, as in Art 15, we must lay off the vert $e\,f$, Fig 20, equal to the total vert strain at e, thus found; and, after drawing $f\,h$ parallel to the rafter, must draw $h\,x$ parallel to the inclined tie, instead of hor.

Rem 2. We will explain the reason for using the portions $u\,j$ and $w\,y$, of the diags , Fig 20, for measuring strains along the rafter $a\,e$. Take, for instance, the strut $d\,g$. re it is evident, that since $i\,g$ represents by scale the strain along the strut, the sides $g\,y$ and $g\,j$ of the parallelogram, give the resulting strains along the two

* See second note at foot of preceding page.

† These small strains become proportionally greater as the rise of the truss increases; so that when the rise is as great as $\frac{1}{5}$ of the span, they cause the pressure at the foot of the rafter to be about twice that at its head; while at $\frac{1}{10}$ rise, it is but about 1.04 times as great.

ties. Now if we take one of these strains, say *g j*, and on it as a dia allelogram of forces *u j n g*, with two of its sides vert, and two parall then *j n* will give the vert strain which *g j* produces at *e*; and *j u*, th it produces at *e*, in the direction of the rafter. Then, if we measure *j n*, or *g u*, we shall find it to consist of the 2 tons which we *origin* from *d* to *e*, by our *second process;* and, in like manner, *i w* repre transferred from *d* to *c*. Since, then, the strain *g j* is made up of since *j n* was transferred to *e* by *calculation* during our second proc mained to be determined by measurement in the third process.

REM. 3. It is not necessary actually to draw all three of the parallelograms, as it or center one alone will suffice; for we need only div the several strains measured Fig 21; and along the ties *n a*, *n e*, by 2, to get those along the struts 4 *o* and 4 *i*; *a o*, *c o*, *c i*, *e i*. And these, in turn, div by 2, will give those along the smaller between *e* and *p*, Fig 19, if there are such; and so on with any number of still sm

REM. 4. The student need now have no difficulty in finding the st by an *equally distributed* load, on a hor Fink truss, Figs 26, 27; the simple than in the roof: for the chord being hor, and the struts vert, forces like the 1.77 tons at *b*, *c*, and *d*; and the strain along the chor be uniform from end to end. In Fig 26, *n n* is not, properly speaking but merely a beam added only for supporting the roadway. If the tended for a hor *roof*, *n n* would be omitted. The vert strains at t strut may be at once written down, without tracing their progress Thus, at the *half-way*, or center strut, *d c*, write one *half* the entire and its load: at each *quarter-way* strut *m m*, write one-quarter of t weight; at each *eighth-way* strut, *h h h h*, one-eighth of it, &c. Ther of the struts, set up vert dists along the struts, by scale, to represent just written at the top of each. From the upper ends of these d downward parallel to the two ties at the foot of the strut, and endin thus completing a parallelogram of forces at the foot of each str Draw the hor diag of each of these parallelograms. Measure and ad half of the half-way or central diag; one-half of *one* quarter-way or *one* eighth-way one, &c. The sum will be the uniform hor strain al chord.* The strain along each inclined tie will be found by measuri the parallelogram which is on said tie. For Fink bridge trusses, see

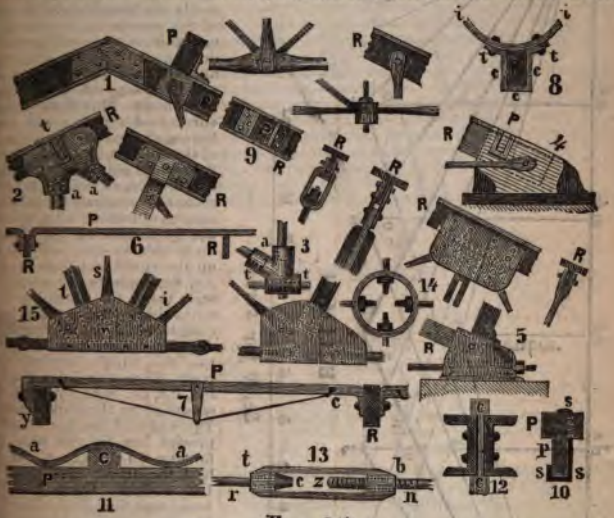
REM. 5. **Figs 21½ show a few of the many forms of of iron roofs.** Every maker has his own modifications of ther figs explain themselves. They will serve as hints. For more on iro R and P stand for rafter and purlin. In small roofs, with the tru ft apart, the purlins may, as at 6, be simple ½ inch or ⅝ round r apart; and the slates may rest immediately on them, being tied t wire. They may be bent down at their ends, and riveted to the rafte between the trusses increases, these purlins may be made of flat iron, deep, and ½ inch thick: or of light T iron, &c; and may be trussed, admit of being placed several feet apart. When, however, they hav weight, the mode at *c*, Fig 7, of confining their ends to the rafters, w Sometimes they may be arranged as at *y*. Or the purlins, of either may rest on top of the rafters, as at 1 and 5; or their ends may re stirrup, as at *t*, Fig 2; and at P, Fig 4: in castings placed at the " port" of the truss; or they may be confined to the sides of the rafter irons, as at P, Fig 9. **Purlins should, when practicab ported only at or near the "points of support"** of as a general rule, it will be expedient to arrange the number of th reference to this particular. The rafters are then relieved from tran and may be proportioned with regard only to the compressive strain of their length. Too little attention is sometimes given to this point verse strain is overlooked, to the serious injury of the roof. It is w bear in mind that thin deep rafters are liable to yield by buckling that this tendency is diminished by purlins well secured to them *betw* of support." Sometimes castings similar to 2, are used at the head feet, of the struts and vert ties; which last have their ends cut into hand screws, for insertion into corresponding female screws cut i At 3, *t t* is the main tie passing loosely through the lower opening th ing. Below it, is seen the head of a small set-screw, for tightenin casting and the tie; to prevent the former from slipping out of plac be different patterns of these castings, to suit the obliquities of the s *or, in small roofs*, the parts *a a* may be made with hinges, for the san

* *If the struts are equidistant, and of equal length, each succeeding one of these ¾ part as long as the one that precedes it. The strains along the oblique ties same proportion.*

At 4 and 5 are cast-iron shoes for supporting the ends of the trusses upon the walls. With the exception, perhaps, of these shoes, it is better that the details generally should be of wrought iron.

At 8 is a mode of confining thin metal roof-covering *i*, **to the purlins** P, by means of short (about an inch) U-shaped pieces (*c c t t* is one of them) of the same metal; to which *i* is riveted by an ⅛ inch rivet through each flange *t t*. This may be adopted with corrugated iron covering, which, by its strength, allows the purlins to be placed several feet apart. See Corrugated Iron. Flat sheets require boards beneath them.

At 10 is a mode of confining a wooden purlin P on top of an iron one *p*, by means of a crooked spike *s s s*; which, after being driven from below, is clinched or bent on top. Wooden purlins are sometimes thus required, for nailing slates or plain sheet metal. At 11, *c*, is a stick of timber inserted between an iron



Figs 21½

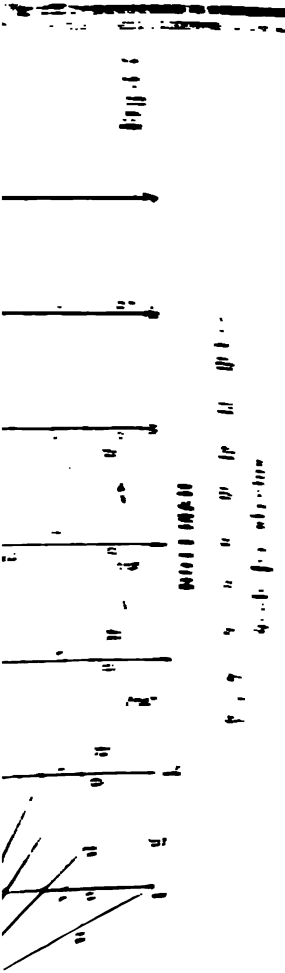
purlin P and the corrugated roof-covering *a a*. To such sticks plastering-laths may be nailed, when the roof is to be plastered beneath, to avoid condensed moisture. There is room for much ingenuity in all these details. Fig 12 is a rafter made of two channel-bars riveted together; with a web member *c c* between them. Two angle-bars are often thus riveted together for a rafter. For more on iron roofs, p 298.

Fig 13 shows a mode of tightening two lengths of a tie-bar *t t* **a swivel** or **turnbuckle**, *t b*. The end of one length, *n*, is cut into a screw; and the corresponding end, *b*, of the turnbuckle is a female screw, into which *n* fits. The end *r* of the other length may either be made in the same manner, or, as in the fig, may be plain, and be furnished with a head *c*. The turnbuckle revolves around both rods, and of course can thus tighten them. Sometimes *t b* is made of a solid round bar, called a **double nut**; with a female screw tapped a few inches into each end, right and left. The end of each rod, *c* and *z*, is then cut to a screw.

Fig 14 is a mode of tightening four lengths of tie-bars crossing each other, by means of a ring. The ends inside of the ring are cut into screws, and provided with tightening nuts, as in the fig. The rings are usually ¾ to 1¼ inch thick; 3 to 5 deep; and 7 to 10 diam.

In Art 29 are remarks on the comparative merits of the foregoing plans of roofs, inasmuch as they are the *kinds usually* employed.

In *proportioning the sizes of struts* for roofs or bridges, bear in mind the Remark on *Rule 1, Strength of Iron Pillars.*

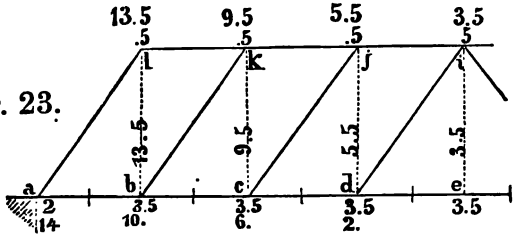


The strain $l\ u = 3.91$ tons.	The strain $l\ g = 1.82$ tons.
" $k\ u = 4.25$ "	" $k\ g = 3.17$ "
" $j\ u = 4.52$ "	" $j\ g = 4.05$ "
" $i\ u = 4.47$ "	" $i\ g = 4.47$ "

post or vert is of course strained to the amount of a full panel-load, when ole wt is supposed to be on top of the truss. If the load is at the *bottom* of ss, see "**In the Bollman truss,**" near the end of Art 20¾.

. 18. More correct views. We have already hinted, in Art 11, that f the results given by the method pursued in the foregoing cases, of supposing ire wt of the truss and its load to be concentrated at points of support along the top alone, or the bottom alone, of the truss, will be correct only in pro- as the truss alone weighs but little in comparison with its uniform load. hough the entire weight may with propriety be considered as so concentrated, wt of the *truss itself* is actually distributed along both the top and the bot- nd before we can correctly assume these separate portions of its weight to be at points on one chord only, we must take into consideration the strains which per portion produces along those web members which carry it to the lower r which the lower portion produces along those which carry it to the upper s the case may be. An example will best illustrate our meaning; and will at ne time exhibit the mode of proceeding to be adopted, when the truss has uni- oads along both chords. It is immaterial in that case whether the loads be o each o her or not; the process is the same. In roofs, the weight of the truss lly so small in comparison with the load, that the preceding methods are suf- y accurate for all probable cases; but a great bridge-truss, may not only bear derable proportion to its load; but may even exceed it. After explaining prin- n this Art, we will in Arts 19 and 20, give short methods of calculation, adapted ctice.

g. 23.



23 may be considered as similar to Fig 10; but in Fig 10 the entire wt of both uss and its load, or 4 tons per panel, was assumed to be placed along the lower ; whereas in Fig 23, we shall call the wt of the *load*, 3 tons per panel; and that *truss*, 1 ton per panel; and shall div the latter among points of support along hords. We thus get .5 of a ton at each upper support; and 3.5 tons at each lower s written in the Fig. Now, beginning at *e*, with our *second process*, the 3.5 tons 4 point, pass up the tie *e i*, adding a strain of 3.5 tons to the .5 ton already there. we have 4 tons vert strain at *i*; but since this is at the *center* of the truss, only alf of it, or 2 tons, passes to each abut. Therefore, 2 tons will pass to the next of support, at *d*, on the lower chord; and here they will meet with the 3.5 tons ly there; making 5.5 tons in all at *d*. These 5.5 tons of vert strain, pass up the *i* to *j*, the next point of support on the upper chord; where they meet the .5 ton ly there; thus making 6 tons of vert strain at *j*. These 6 tons pass down the *j c*, making 9.5 tons at *c*; and thus on throughout; until the final vert strain tons at *l* is transmitted along the strut *l a*, to the abut *a*; where it meets the 2 already there; thus making up 16 tons, or one-half the entire wt of both truss oad, as in Fig 10; and which, as already remarked, must always be the case in ss of this kind.

w, in Figs 10 and 23, we have written along each vert, the strain in tons which rains, according to calculation; and it will be seen that when, as in Fig 10, we ose *all* the uniform wt of truss and load to be upon the *lower* chord, then the lated strain along each vert will be *greater* than when we adopt the more cor- assumption that *one-half of the wt* of the truss itself is borne by the upper 1. *The error is the same in amount*, along each vert; and at each of them it is *to half the wt of a panel of truss* alone; or .5 of a ton, in our examples; whic *be subtracted from the strain* on each vert when greater accuracy is require *be observed that in Fig 23*, the half weight of the truss along the up

chord, is actually transferred to the lower one, by the correct method also portion of it at *i* is not transferred to *e*, as is assumed in Fig 10, but to *d*. is transferred, not to *d*, but to *c*, &c. Hence the diff between the results o modes of calculation. This diff does not affect the strains along the other of the truss: but they remain the same by either process. In the Warren g 11, the obliques are affected. Now, if in all cases, the assumption that al form wt is on one chord, gave the results too great, the error would be o of safety, and therefore not very objectionable; but it does not. For, if in assume the entire wt to be on the *top* of the truss: and that in Fig 23, 3½ t each upper point of support, and .5 of a ton at each lower one, (except th abuts,) then the result will be reversed; and the calculated strain along ea Fig 10 will be too *small*, by the wt of half a panel of truss alone, or .5 of s cannot spare the space to illustrate this, and other cases, by a fig, as we di ceding one. The student can readily do this for himself, by drawing a ske truss, and performing the process by its aid; and the same remark app next Art. The method of the present Art is extremely useful, as it enables the true strains, be the load where it may.

Art 19. We, however, now give the corrections to be made in the ap strains along each of the verts of several trusses calculated on the erroneous as of all the weight being along one chord only. These corrections are based o position that the upper and lower members themselves are of equal weigh they generally are, approximately enough.

In Fig 10, or in Figs 23 *a*, 23 *b*, from the strain on each vert, deduct the w panel of the truss alone. In Figs 23 *c*, 23 *d*, add this ½ panel wt of truss each vert.

In Fig 14, if all the wt has been assumed to be on the rafters, add the panel of truss alone; or if on the chord, subtract it, from each vert.

Also, in Fig 15, weight along the hor top, subtract ½ panel wt of truss

In Fig 16, wt along rafters, add; wt along chord, subtract this ½ panel

In Fig 11, (the Warren truss,) of two hor chords, and a series of equal whose sides are equally inclined to their bases, if we assume the entire w and load to be equally distributed along *either the top, or the bottom chord* al the two Warrens at Figs 23 *e*,) then the calculated strains along the tw *obliques e r, e i*, one on each side of the center, will be too great; and g them each way, toward the abuts, the strain on the next web member, as too small; that on the one, *d j*, too great; and so on alternately with the

The amount to be alternately subtracted and added, is the same for all th of the Warren: and is equal to

$$\frac{\frac{1}{4}\ \textit{weight of one panel of truss alone} \times \textit{length of an oblique}}{\textit{vert spread, or height of oblique.}}$$

Or, (which amounts to the same thing,) to ¼ of the wt of a panel of truss a by the nat secant of the angle *t o* B, Fig 11, which the oblique *t o* forms w line *o* B. To find this nat sec, div the length *t o*, of an oblique, by its vert

REM. Observe that in the Warren, only one-*fourth* of the wt of a panel made use of, instead of one-*half*, as in the preceding cases.

Verts are sometimes added to a Warren truss, either ties, when the stretches of chord between the obliques become so long as additional support. These in no way affect the strains on the obliques of a truss; or of one equally loaded along both chords: except that they increas the very end ones, when there is a vert in the end triangle. This excep ever, is generally too small to require attention in ordinary practice. Eac tains the weight of half the stretch of chord to which it is applied; together of any load that may be on said stretch. Thus, in Fig 11, the vert post B, the stretch *t u* of the upper chord, sustains the half stretch A A, and wha it may have on it. So, if there were a vert tie from *t* to V, it would sust of, and on, J J. The strains on the obliques *u o. o t, t n*, would remain t if there were no verts; for the latter are in a great measure independ former. Strains travelling along the obliques, from the center of the tr abuts, do not pass through the verts of a Warren truss. If, however, t loaded along one chord only, then the verts change the strains on all the o the extent of ¼ panel of load mult by nat sec of brace angle. On a cent the strain will be *reduced* that amount; on the next one it will be *incr much; and so* all through the truss, alternately reducing and increasing.

In Fig 22 (the Bollman truss) the vert posts do not appear to sustain the *below them*; but in practice they generally do so in a great measure; *prevent these* rods from sagging, they are made to rest upon hooks (or *arrangement)* fixed to the sides of the posts.* Therefore, when this is

* *The same with long rods in the Fink.* It lessens their vibrations under

all the wt of truss and load is assumed to be along the top, we may, near enough for practice, deduct nothing from the calculated strain on each post: but give to each one a strain equal to the full wt of one panel of truss and load. If the bars are not thus attached to, and upheld by the posts, we may deduct ½ the wt of a panel of truss only, from the strain on each post. If all the wt is assumed to be along the bottom chord, Fig 24, calculation would of course give no strain on the posts, except the wt of 1 panel length of upper chord; but in practice we had better give to each one a strain equal to the wt of a whole panel of truss, if the posts support the bars; or half as much, if they do not.

In Fig 20, of a Fink roof; or Figs 26, 27, of bridges; owing to the fact that when there are several panels, the obliques transfer nearly all the wt of the lower half of the truss to the tops of the posts, in such a manner as to cause the last to sustain almost the whole wt of the truss, whether the load be on the top or the bottom, it is best to assign to each post, in both cases, the full strain due to a top load. Some reduction might, it is true, be made in that series of posts which bear the least strains; but it hardly merits notice in ordinary practice.

Rem. We repeat that in this Art the wts of the upper and lower members (chords and rafters) are supposed to be equal. Generally they are so approximately equal that our rules are applicable; but if there should be a diff sufficient to require attention, the process in Art 18 will always give correct results; first assigning to each upper and lower point of support, its due portion of the upper and lower wts.

Art 20. Shorter processes for finding strains on trusses. After they are found approximately by this Art, they may readily be corrected by Art 19. When, as is usual in proportioning *roof* trusses, the inaccurate, but sufficiently approximate (for roofs) assumption is made, that all the weight of both truss and load is uniformly distributed along one chord only, then in Figs 23 *a*, 23 *b*, 23 *c*,* and 23 *d*, (in which the dotted web-members sustain tension, and the full-lined ones compression,) the only members which an assumed load along a top chord, will strain

Fig 23.a

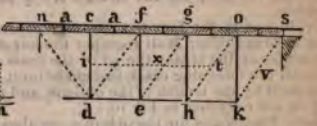


Fig 23.b

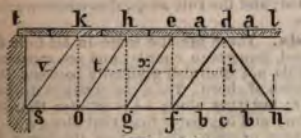


Fig 23.c

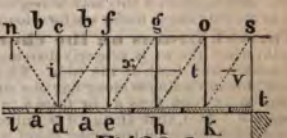


Fig 23.d

differently from one on a bottom chord, are the verticals. The strains on the chords and obliques are the same in all four figs. Thus, in Figs 23 *a* and 23 *b*, it is seen that, supposing all the wt to be on one chord, the center vert *c d* sustains the wt along a whole panel *a a*. Said vert transfers said wt to the ends of the two central obliques *d f*, *d n*, at *d*; and is therefore strained throughout its length to an amount equal to said wt of one whole panel of truss and load. But in Figs 23 *c*, and 23 *d*, (all the wt supposed to be on one chord,) the wt of the center panel *a a* is already at the ends *d*, of the center obliques; without passing to them through the center vert *c d*. Said vert really sustains nothing but the wt of the one panel length *b b* of the unloaded chord itself: which it of course transfers to *d*. Thus, in all four figs, the entire wt of the central panel comes to the ends *d* of the central obliques *d f*, *d n*; which consequently are equally strained in all four; and the same is the case with all the corresponding braces of the four figs.

Approximate strains on the verticals. In Figs 23 *a* and 23 *b*, (all the wt supposed to be on one chord,) the center vert *c d* sustains, and is strained to

* Or any *Howe or Pratt truss; because the counters in them are not affected by a uniformly distributed load; and when their counters are omitted*, they become the above figs.

the amount of the entire wt of truss and load along one panel; the next vert $e f$, 1½ panel; the next, $h g$, 2½ panels; and so on, through any number of verts; adding the wt along one panel, to each succeeding vert.

In Figs 23 c, and 23 d, on the same supposition, the centre vert $c d$ sustains nothing but the trifling wt of the panel length $b b$ of the unloaded chord itself. The next vert, $e f$, sustains ½ panel wt of truss and load; the next one, $h g$, 1½ panel; the next, $k o$, 2½ panels; and so on, with any greater number. These may then be corrected by Art 19. In any of the four figs, in a simple unloaded truss, with uniform and lower halves of the same wt, the strain on the center vert is equal to half the wt of a panel; on the next, a whole panel; on the next, 2 panels; then 3, 4, panels; and the same if the truss be equally and uniformly loaded along both chords. The weights in the last case of course include both truss and load.

Correct strains on the obliques. These are the same in all the four figs; not only when the entire wt is incorrectly assumed to be on one chord only, but also when it is properly proportioned along both chords. If we div the length $e g$, of any oblique, by its vert spread or stretch, $f e$, or $g h$, the quot will be the nat sec of the angle $g e f$, or $e g h$, which the oblique forms with a vert line $f e$, or $g h$; and it is evidently immaterial, therefore, whether we first mult any given wt by the length of an oblique, and then div the prod by the vert stretch; or whether we at once mult the wt by the nat sec. But when there are many obliques of the same length and inclination, the nat sec becomes the most convenient. If the length of an oblique be, say 21.4 ft, and its vert spread 14.8 ft, then the nat sec will be $\frac{21.4}{14.6} = 1.47$; near enough for practical purposes.

For the correct strain on each center oblique $d f$, $d n$, no matter where the uniform load may be, mult the wt of ½ a panel of truss and load by nat sec. For the next one, $e g$, mult 1½ panels by nat sec; for the next one, $h e$, 2½ panels × nat sec; for the next, $k s$, 3½ panels × nat sec; and so on, with any number.

Or, in any of the four figs, the correct strain on any oblique, as, for instance, $h e$, may be found thus: Mult together the hor dist $t i$, from the center of the oblique to the center of the truss; the wt of the truss and load *per foot run;* and nat sec.

In all the foregoing cases, instead of mult by nat sec, we may, if more convenient, first mult by the length of the oblique, and then div the prod by the vert spread. In Art 19, see reference to Fig 10.

Rem. These rules for the obliques are always safe; and are perfectly correct for all, except that in cases like Figs 23 a, 23 b, where one of the chords is not extended to the length of the other, the rules give the strain on the very end obliques rather too great. But in practice this may be neglected; and the rule adopted as a safe working one in all four cases, under *uniform* loads.

Correct strains on the chords. These are the same in all four figs, as also when the wt is properly distributed along both chords; and are greatest at the center, and least at the ends. They do not diminish gradually between those limits; but are uniform along any one stretch of chord. Thus, the strain is uniform, or the same, from k to h; or from h to e; but that along $h e$ is greater than that along $k h$. Also, that along $e d$ is uniform; and is greater than that along $h e$. In all cases the strain on the upper chord is pres, or push; and that on the lower, tension, or pull: all of which has been shown in previous Arts. The strain between the upper ends of two adjacent obliques, is equal to that between their lower ones; thus, that along $h e$ is equal to that along $o g$; that at $e d$, to that at $g f$; that at $f n$, to that between the ends which meet at d. See foot-note, page 276.

For the correct strain at the center d or c, of either chord, mult ½ the wt of the truss and load, by ¼ of the span; and div the prod by the height or depth of the truss, measured from center to center of chords, (see Art 16;) or, which amounts to the same thing,

$$\text{Strain at center of either chord} = \frac{\text{entire weight of truss and load} \times \text{span}}{8 \text{ times height of truss.}}$$

These two rules apply to the Warren (Figs 23 e) also; as do likewise the following, using the four letters i, x, t, v, of those figs; and remembering that the hor spread *of an oblique* in a Warren, is *half* the base of a triangle.

***For the* correct strain at each end k or s, of each chord,** *mult all the wt of truss* and load between the center i, of the truss: and the center n, *of the end oblique* $s k$; by the nat tang* of the angle $s k o$, formed by such oblique,

* *For this* nat tang, div the hor spread $o s$, of the oblique $s k$, by its vert spread $o k$.

Or, which amounts to the same thing, mult the same wt by the hor ıe end oblique *s k*, and div the prod by its vert spread *o k*.

rain at any point between the center, and the end ıord; as at the point *g* or *e*; and from there toward *f* and *d*. If all alike, add together the dist *i x* in ft, from the center of the truss to he oblique at whose end, *g* or *e*, the strain on the chord occurs; and similar dists *i t*, *i v*, to the center of each oblique; to the end one, *s k*, together the sum of these dists; the wt of truss and load *per foot run*; of a panel; and div by the height of a panel. Or mult together the ft, and nat tang.

are not alike, first find the end strain at *k* or *s*, as previously directed d; then find separately the strains at the intermediate points *h*, *e*, or e process as for finding the end strain at *k* or *s*; assuming *h o* and *e g* end obliques. Add all these strains together.

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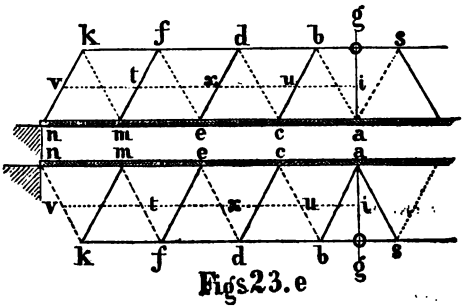


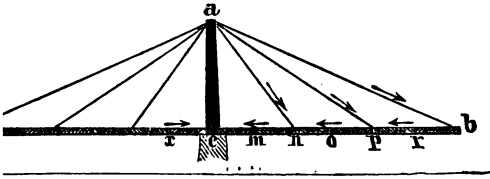
Figs 23. e

and b c; *each* = *wt on* $\frac{1}{2}$ *panel* × *nat sec of angle* g a b.*
and d e; *each* = *wt on* $1\frac{1}{2}$ " × " " " "
and f m; *each* = *wt on* $2\frac{1}{2}$ " × " " " "
and k n; *each* = *wt on* $3\frac{1}{2}$ " × " " " "

ı any more pairs on the same side of the truss. The strains thus ıecessary, be readily corrected by Art 19.

ect strains on the chords, whether at the center, ends, or ɔints, no matter where the uniform load may be placed, may be found mediately preceding, for the four Figs 23 *a*, &c; using the four letters in all six figs; remembering that the hor stretch *b o*, or *o s*, of any *s a* of a Warren truss, is equal to *half* the base of one of the triangles. ren, the only obliques used for finding the strains along the chords, *i e*, *f m*, *k n*,) which are parallel with the end one *k n*. Use the nat gle *b a o*, or *s a o*, which an oblique *b a* or *s a* forms with the vert *a o*. n a Warren is found by div *half* the base *b o*, or *s o*, of one of the tri- vert spread *a o* of an oblique *b a* or *s a*. See foot-note, page 276.

In cases like the following fig, which is **an opened swing-** ɔorted on rollers at *c*; and by ties *a b*, *a p*, *a n*, &c, we may consider the



* *For nat sec, divide the length of an oblique by its vert spread.*

tie $a\,b$ as sustaining all the wt along $r\,b$; $a\,p$, that along $o\,r$, $a\,n$, that along m the wt of $x\,m$ being directly sustained by the rollers on top of the pier. Then

strain on $a\,b = \dfrac{\text{wt of } r\,b \times a\,b}{a\,c}$. That on $a\,p = \dfrac{\text{wt of } o\,r \times a\,p}{a\,c}$. That on

$\dfrac{\text{wt of } m\,o \times a\,n}{a\,c}$. The strains on the obliques on the opposite side will of cour

the same. The wts of $r\,b$, $o\,r$, $m\,o$, &c, produce, moreover, hor *pressing* strains fr b to c. These are least at the end, from b to r; and greatest toward the cen from o to c. In such bridges the chord must be so designed as to resist these strai as well of those of tension, to which it is subject under passing loads when shut.

These strains are as follows: That from the wt

$$\textit{of } \mathrm{r\,b} = \frac{\textit{wt of } \mathrm{r\,b} \times \mathrm{b\,c}}{\mathrm{a\,c}}; \textit{ and extends from } \mathrm{b} \textit{ to } \mathrm{c}.$$

$$\text{“ } \mathrm{o\,r} = \frac{\textit{wt of } \mathrm{o\,r} \times \mathrm{p\,c}}{\mathrm{a\,c}}; \textit{ and extends from } \mathrm{r} \textit{ to } \mathrm{c}.$$

$$\text{“ } \mathrm{m\,o} = \frac{\textit{wt of } \mathrm{m\,o} \times \mathrm{n\,c}}{\mathrm{a\,c}}; \textit{ and extends from } \mathrm{o} \textit{ to } \mathrm{c}.$$

Hence, the length $o\,c$ bears the strains from all the wts $m\,o$, $o\,r$, and $r\,b$; the le $o\,r$, bears those from the wts $o\,r$ and $r\,b$; and the length $r\,b$, only that from the of $r\,b$. The other half, or $d\,c$, will be strained the same.

If the bridge, instead of resting on rollers on top of the pier, is supported eath by means of the obliques, and of the center post $a\,c$, then we must consider the $a\,n$ as supporting all the wt along $c\,o$.

If the fig be inverted. the strains will be the same in amount; but obliques will then become struts; while $b\,c$, and $d\,c$ will become ties. If one $a\,b\,c$, of the fig, by itself, be a projecting platform, resting on a wall at the en and otherwise upheld by the oblique ties; the calculation will be the same; so if it be inverted.

Art. 20½. Moving loads, and counterbracing. We shall no glance at this subject in a superficial manner; without pretending to discuss comparative merits of diff forms of bridge truss. As already stated in Remark, 10, counters are not needed in a truss uniformly loaded from end to end. The m which we propose for finding the strains produced in beam trusses by moving lo differs from that commonly used. It renders the principle more apparent; an equally reliable and safe.

To assist in illustrating the calculations for strains produced by moving loads, shall suppose the following Fig 23*f*, to represent the two trusses, combined into of a span $m\,x$ of 120 feet; divided into 6 panels, each 20 ft long, and 30 ft high. in other words, we (for convenience of calculating) suppose all to be borne by truss only. The weight of this double truss is 48 tons; or 8 tons per panel; or .4 per ft run. The floor, and its several timbers, such as cross girders, hor bracing, which, although not portions of the *truss proper*, are essential to the *bridge*: m be considered as so much permanent load, equally distributed along the truss, assumed to weigh an additional 24 tons; or 4 tons per panel; or .2 ton per ft ru the double truss. Therefore, the truss and its accompaniments together, or in o words, the bridge superstructure, weighs 72 tons; or 12 tons per panel; or .6 of a per ft run. Since each panel is 20 ft long, and 30 ft high, each oblique is $\sqrt{20^2 + 30^2}$ = 36 ft long: and the nat sec of the angle which it forms with a vert, (or of the ang *angle*,) is $\dfrac{\text{length of oblique}}{\text{its vert spread}} = \dfrac{36}{30} = 1.2$; and the nat tang of the same angle $\dfrac{\text{hor spread}}{\text{vert spread}} = \dfrac{20}{30} = .6667$.

The moving load is assumed to consist of an engine, $y\,z$, weighing 36 tons; all which is supposed to be so concentrated on its drivers, as to stand upon the leng of one panel; and be upheld by each vert in turn. The engine is supposed to be fo lowed by a tender and cars, weighing 21 tons per panel; or 1.05 ton per ft run. Th last (or rather, 1 ton) is an ordinary allowance, and a safe one; inasmuch as tru *of loaded cars* never, perhaps, weigh 1 ton per foot; and comparatively few engin *even, exceed* that limit.

The greater danger from engines, arises from the fact that in ma *of them, especially* in heavy freight engines, all the wt is concentrated upon the *driving wheels*; which are so close together that a 30 or 40 ton engine on 4 whe

ten have its entire wt sustained upon only 12 to 15 feet length of the truss; ringing a great strain upon each individual web-member, as it crosses the . This point is one of the greatest importance in arranging the preliminary which is to form the basis for calculating the strains on a bridge truss; and we l allow liberally for the greatest load that can possibly be brought upon one at a time. When the panels are quite short, each one will have of course tain only a smaller portion of the wt of engine; it may, indeed, have to be di- among 2, 3, or 4 of them. But few engines weigh over 42 tons; but with the steel rails, it is probable that both their total wt, and the wt on each driver, y degrees be increased. This should be considered in designing bridges. We ; think they should be proportioned for *less* than 5 tons on a driver.

fact that the engine produces upon each panel in succession, a greater strain an equal length of loaded cars can do, causes a modification of the calcula- as will be seen as we proceed.

h data must be prepared before beginning the calculations. Our assumptions case before us have been made entirely with reference to ease of calculating :ample with but few figures. Our truss, together with a load of cars extending nd to end, will weigh 198 tons; or 1.65 tons per ft run; or 33 tons per panel. ing prepared a diagram, begin by finding the strains caused in only the verts bliques, by the bridge itself. By Art 20, *Fig 23 a*, the *approximate* strains on rts, when there is a uniform load (in this case the floor) on the lower chord, und to be as follows:

Strain on e p = *weight of* 1 *panel of truss and floor*, = 12 *tons.*
" *on* h o *and* c q, *each* 1½ " " " = 18 "
" *on* i n *and* a r, *each* 2½ " " " = 30 "

we must correct these by Art 19, by deducting from each the wt of ½ a panel s alone, or 4 tons; which leaves for the true strains as follows: On *e p*, 8 tons; and *c q*, each 14 tons: on *i n* and *a r*, each 26 tons. All of which write on the m, as shown in our Fig 23 *f*, near the feet of the verts. We shall write all our or bridge strains, near the *feet* of the web members; the strains from the g load, near their *tops;* and the total strains, near their *centers.*

t find, by Art 20, the strains on the obliques, resulting from the bridge alone, ows:

e o *or* e q, = ½ *pan truss and floor*, × *nat sec*, = 6 × 1.2 = 7.2 tons.
h n *or* c r, = 1½ *pan* " " × *nat sec*, = 18 × 1.2 = 21.6 "
i m *or* a x, = 2½ *pan* " " × *nat sec*, = 30 × 1.2 = 36.0 "

also place on the diagram near the feet of the obliques, as in our fig.

ow have the strains on the web members, resulting from the truss itself, and from formly distributed permanent load of floor, &c: and are prepared to begin with ditional strains resulting from the moving load. We find those on the coun- n one-half of the diagram; and those on the main obliques and verticals, on the half. First suppose the engine to be in the position *y z*, on the half diagram rith the train reaching to the farthest end *m* of the truss. In this position of oving load, the vert *a r*, and the end oblique *a x*, will be more strained by it in any other; and their strains will be produced by that portion of the load ay be regarded as being first concentrated at *r*; and as passing thence up *r a* and from *a*, down *a x* to the abut *x*; together with another portion which, after ling from *r* to *a*, passes off to the other abut *m*, without descending along *a x*. d these portions, the truss may be considered as a simple beam, or a lever, d off into six equal divisions at *r*, *q*, *p*, *o*, *n*. In that case we know (from the ple of the lever) that if any load, as for instance our engine of 36 tons, be l uniformly along *y z*, its whole wt may be assumed to be concentrated at the e point *r*; and that $\frac{5}{6}$ of it will be borne by the nearest abut *x*; and only $\frac{1}{6}$ by rthest abut *m*. So also with the panel length *y w* of cars; its wt being supposed ntrated at *q*, $\frac{4}{6}$ of it must be borne by *x*; and $\frac{2}{6}$ of it by *m*. Of the cars *w v*, $\frac{3}{6}$ rne by *x*; and $\frac{3}{6}$ by *m*. Of those along *v u*, $\frac{2}{6}$ are borne by *x*; and $\frac{4}{6}$ by *m*; of those u *s*, $\frac{1}{6}$ by *x*; and $\frac{5}{6}$ by *m*. Of the half panel of cars, *s m*, $\frac{1}{24}$ part goes to *x*, be- *t m* is $\frac{1}{24}$ part of the span; *t* being the cen of grav of the half panel of cars. present, however, we have nothing to do with any of the portions that go to cept to remark that each *vert* in turn really sustains the *entire* wt of the *engine*,* s strained by it all; but when this entire wt reaches the top of the vert, only ortion $\frac{5}{6}$, as above *stated, go down to a x* to the abut *x*. The remaining ⅙ goes

strictly correct; but a safe and advisable assumption in practice, when the entire wt of th m stand on one panel.

to the other abut m. By this process, then, we find that $\frac{2}{3}$ of the en[illegible] 30 tons; $\frac{1}{3}$ of t[illegible] cars $y\ w$, or 14 tons; $\frac{2}{3}$ of the cars $w\ v$, or 10.5 tons; $\frac{2}{3}$ of $v\ u$, or [illegible] tons; $\frac{1}{6}$ of $u\ s$, or [illegible] tons; and $\frac{1}{24}$ of $s\ m$, or .44 of a ton; making 65.44 tons in all, are borne by the ab[illegible] x, when the moving load is in the position shown in our fig. This load of 65 to[illegible] passes from r to a; and from a to x. The calculation is made, as shown under [illegible] fig; below the heading "Engine at r."

We next suppose the engine to back into the position $y\ w$; with the train reach[illegible] to m; and with no load between y and x. In this position the strains on $q\ c$, and [illegible] are greater than in any other; and by a process precisely as before, and as sho[illegible] under our fig. below the heading "Engine at q," we find that 45.44 tons of the mov[illegible] load go to the abut x; passing from q to c; and from c to r, on their way to it. T[illegible] placing the engine at $w\ v$, the cars reaching to m, and no load between w and x, [illegible] strains on $p\ e$ and $e\ q$, are the maximum that they can be subjected to by an eng[illegible] and train; and are found like the others, as shown under "Engine at p." We h[illegible] now reached the center of the truss, and have obtained data for the greatest str[illegible] that can occur on the verts and main obliques on one-half of it. So far as reg[illegible] the corresponding members on the other half, we stop here; for it is manifest [illegible] an engine and train crossing the bridge in the opposite direction, must produce [illegible] same maximum strains upon them as we have already found for the others. Th[illegible] $i\ m$ will be strained the same as $a\ x$; $n\ i$ the same as $r\ a$; $e\ o$ the same as $e\ q$, as [illegible] on, by a returning train.

But as yet, we have not sufficient data for determining where *counters* wil[illegible] required. To obtain it, we draw the two lines $h\ p$, and $i\ o$, parallel to the obli[illegible] on the opposite side of the truss; and taking it for granted for the present, that t[illegible] two lines are counters, we back the engine to $v\ u$, and then to $u\ s$; performing [illegible] time, calculations precisely similar to the former ones: and as shown under the [illegible] ings "Engine at o," &c. We have stated in a former Art. that a counter is [illegible] strained when the moving load extends from it to the *nearest* abut. Therefo[illegible] counter $h\ p$ would be most strained with the engine on $v\ u$, and the train reac[illegible] to m; and $i\ o$, by the engine on $u\ s$, with a train to m.

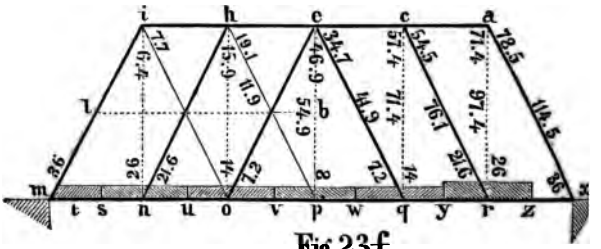


Fig 23f

MAIN BRACES.

With Engine at p, *there go to* x.	With Engine at q, *there go to* x.	With Engine at [illegible] *there go to* x.
Tons.	Tons.	Tons.
18 = 3-6 *engine.*	24 = 4-6 *engine.*	30 = 5-6 *engine.*
7 = 2-6 v u.	10.5 = 3-6 w v.	14 = 4-6 y w.
3.5 = 1-6 u s.	7 = 2-6 v u.	10.5 = 3-6 w v.
.44 = 1-24 s m.	3.5 = 1-6 u s.	7 = 2-6 v u.
	.44 = 1-24 s m.	3.5 = 1-6 u s.
		.44 = 1-24 s m.
28.94 *go to* x.	45.44 *go to* x.	65.44 *go to* x.
1.2 *nat sec.*	1.2 *nat sec.*	1.2 *nat sec.*
34.728 *on* e q *from load only.*	54.528 on c r *from load only.*	78.528 on a x *from load only*

VERTICALS.

Vertical e p.

28.94 *go to* x.
18 = *remaining*
3-6 *engine.*

46.94 *on* e p
from load only.

Vertical c q.

45.44 *go to* x.
12 = *remaining*
2-6 *engine.*

57.44 *on* c q
from load only.

Vertical a r.

65.44 *go to* x.
6 = *remaining*
1-6 *engine.*

71.44 on a r
from load only.

COUNTERS.

Counter i o.

Engine at *n*.

6 = 1-6 *engine.*
.44 = 1-24 s m.

6.44 *go to* x.
1.2 *nat sec.*

7.728 *on* i o
from load only.

Counter h p.

Engine at *o*.

12 = 2-6 *engine.*
3.5 = 1-6 u s.
.44 = 1-24 s m.

15.94 *go to* x.
1.2 *nat sec.*

19.128 *on* h p
*from load only.**

Supposing the calculations to have been made as above, we have thereby found the several loads, say 65.4; 45.4; 28.9; 15.9; and 6.4; which go to the abut x, from the several points r, q, p, o, n, when the engine is at each of those points in succession; with a train reaching in each case to m. Each of these loads, in travelling, as it were, from its particular point, to the abut x, first ascends to the top of its own vert; and then descends along the adjacent oblique; producing in said oblique a strain as much greater than the load itself, as the length of the oblique is greater than its vert spread. Now, each of our obliques, as before stated, is 36 ft long, or 1.2 times as long as its vert spread 30 ft, (or the height of truss;) that is, $\frac{36}{30} = 1.2$; which 1.2 is therefore the nat sec of the angle which any of our obliques forms with a vert line. Therefore, to find the strain which each of our moving loads produces on the oblique next to it on its way to the abut x, mult each said load by 1.2, as in the above calculations. We thus obtain for these strains, 78.5 tons on $a\,x$; 54.5 on $c\,r$; and 34.7 on $e\,q$: all which write near the tops of the obliques, as in our fig, in which we omit some of the decimals, to avoid crowding it.

Next, as regards the strains from the moving load alone, upon the verts, we must bear in mind that these last have to sustain not only the loads just considered, which pass up them on their way to the abut x; but also the remaining portion of the wt of the engine; which also passes up them, on its way to the *farthest* abut m. Thus, the end vert $a\,r$ sustains the entire wt of the engine; whereas our load of 65.4 tons before found, at r, includes only $\frac{5}{6}$ of it. The remaining $\frac{1}{6}$, or 6 tons, must therefore be added to it, as is done under the head, "Vertical $a\,r$," making 71.4 tons strain on $a\,r$, from the moving load only. Write this near the top of $a\,r$. For the same reason, at the next vert $c\,q$, we add to the load 45.4 before found, that goes from q to x, the remaining $\frac{2}{6}$, or 12 tons, of the engine, that go to the abut m, after ascending to c. This we have done under the head "Vertical $c\,q$," thus getting 57.4 tons for the total moving load-strain on $c\,q$. And so at the center vert $e\,p$, by adding 18 tons, or the remaining $\frac{3}{6}$ of the engine, that go to m, we get 46.9 tons total load-strain on that member; to be written near its top. The strains on the verts of the other half, $p\,m$, of the truss, are not to be calculated. They will be the same as those already found. Having now the load-strains written near the tops of the verts; and

* De Volson Wood, the accomplished professor of civil engineering in the University of Michigan, objects to the writer's calculations as given above; stating that they involve two errors, each of which increases the strains on the web members beyond the truth; or in other words, that the writer makes his web members stronger than is necessary. Mr. Wood says, first, that in our calculations for the main braces and counters, we should omit the .44, or $\frac{1}{24}$, $s\,m$. He remarks, that *all* of the load $s\,m$ is borne directly by the abut m. This we do not comprehend; for the whole truss is a lever, or a beam supported at both ends; hence, a load placed at any part of it, must be partially sustained by each abut. It will be observed there is no load at the other end of the truss, to counteract in any way the load $s\,m$. He says, next, that in the calculations for the verticals, we should omit the "Remaining parts of the Engine." He had the kindness to send us his reasonings upon this point also; but from our want of ability to *understand them*, we have been compelled to allow our calculations to remain as *they are*. *The only injury that* can result is, that some parts of the truss will be slightly *stronger than theory requires.*

the truss strains near their feet; we have only to add these together to find the strain on each; namely, 97.4 tons on $a\,r$; 71.4 on $c\,q$; and 54.9 on $e\,p$. We have done with the main obliques, and the verts.

For the counters, we go to the other side, $p\,m$, of the truss. Begin with the panel $e\,h\,o\,p$, we examine its two diags $h\,p$, $e\,o$, and see that with the s at $v\,u$, and the cars reaching to m, there is produced in the counter $h\,p$ its max strain of 19.1 tons; which tends to cause in the truss the kind of derangement in Figs 9½, 9¾, and 9 b. Now, to resist this derangement, there is nothing b 7.2 tons produced by the truss, floor, &c, upon the opposite diag $e\,o$ of the same Since, therefore, the deranging effect of the load is greater than the preventiv of the wt of the truss, there must be a counter at $h\,p$, able to bear a strain e least to the diff between the two, or to 19.1 — 7.2 = 11.9 tons.

We now go to the next panel, $h\,i\,n\,o$. But here we find that the deranging of the load on the counter $i\,o$, with the engine at $u\,s$, is but 7.7 tons; while t ventive effect of the wt of the truss, exerted through the opposite diag $h\,n$, tons. Hence, the moving load can produce no derangement of the truss; a sequently the counter $i\,o$ may be omitted. On this same principle each panel side of the truss must be examined, when there are many of them; and the in or omission of counterbraces be determined upon. When we thus arrive at at which no counter is reqd, none will be needed between it and the nearest the bridge. Similar counters will, of course, be needed on the other side of th In practice it is better to retain the first apparently unnecessary counte counting from the center of the truss. Thus, although calculation shows t unnecessary, it is well to retain it. The lighter a bridge is, in proportion moving load, the greater will be the number of panels requiring counters.*

The strains on the chords are greatest when the truss is load end to end; and may be found by Art 20.† Thus, the entire wt of the bridge, uniform load of 21 tons of cars per panel, being 198 tons, we have for the s the center of either chord,

$$\frac{198 \times \text{span,}}{\text{8 times height of truss}} = \frac{198 \times 120}{8 \times 30} = \frac{23760}{240} = 99 \text{ tons,}$$

both at e of the upper chord; and along the stretch $o\,q$ of the lower one. A end of either chord it will be equal to

$$\text{the dist } l\,b, \text{ in feet} \times \text{total wt per ft run} \times \text{nat tang} = 50 \times 1.65 \times .666 = 55 \text{ tons.}$$

For intermediate points see Art 20. The strains on the chords might in a trifle greater; because we have calculated them for a full load of *cars* only; w a very heavy *engine* at the head of the train would somewhat increase the to but on the other hand, loaded cars do not weigh as much as our assumed 1 ton

Finally, each of all the strains in our double truss must be div by 2: for p tioning them among the two actual trusses, which we have all along suppos convenience) to be combined into one.

Rem. If our diagram were inverted, together with its load, so that the load *then* be on its top; all the strains in every part would remain the same in an but the obliques would become ties; and the verticals, posts. Also, if the load *diagram as it stands*, instead of being on the bottom chord, had been on the to the strains on all the obliques, and on the chords, would have remained unchan and be found in precisely the same way. Also, all the strains produced in the by the bridge itself would be the same: so that the working diagram would a for either case; but the strains which the *moving load* brings upon the *verts*, w that case be less with a top load. And in all cases when, as in our diagram Figs 23 a, 23 b, the load $w\,v$, or $a\,a$, at the center panel of the truss, is sustain rectly by the center vert, the strain on that member, and on the other verts, is g than when, as in Figs 23 c, 23 d, the center load $a\,a$ is sustained directly by th center obliques $d\,n$, $d\,f$. In Fig 23 c, this center load rests upon the tops d obliques; producing compression along them; and in Fig 23 d, it is suspended their feet d; producing extension or pull.

When the load at the center of the truss (as the load $a\,a$, Figs 23 c, and 23 sustained directly by the center obliques $d\,n$, $d\,f$, first find, and write upon th gram, precisely as in our foregoing case, all the strains produced by the bridge i *all* the portions of load that go to the abut s, Figs 23 c, 23 d; the total strains o *main obliques* of the side $c\,s$ of the truss; and those on the counters of the *side*: *in short*, all but the strains on the verts. Finally, for the total strain o *end vert* $k\,o$, ($s\,t$, $s\,t$, are not truss-verts, but uprights for upholding the trus

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* See Rem 1, Art 31.

† For resisting their hor strains the chords must be regarded as pillars with a length e f a panel; unless they also bear transverse loads; in which case, see foot-note, page 1

er the strain produced on it by the truss, and the load *that goes to the abut s*, (not ain on the oblique,) from the next point of support *h* toward the center of the For the total strain on the next vert *h g*, add together its strain from the truss; at of the load that goes to *s* from the next point *e* toward the center. And so he total strain on *e f*; and last, for that on the center vert *c d*, add together its train, and the load that goes to *s* from the point *l* on the opposite side of the

will now proceed to the Warren truss.

20¾. The Warren or triangular truss. In this truss the pro- r finding the strains from moving loads, is not *precisely* the same as in the ing one. The following Fig 23*g*, represents such a truss; in which the span, weight, and number of panels will be taken to be the same as in the foregoing *f*. Here the dotted web-members which supplant the ties in Fig 23*f*, are not ut inclined to the same extent as the struts or braces. Hence the hor stretch oblique will be but *half* the length of a panel, or 10 ft; or only half as great as in *f*. Consequently, the length of each oblique will be $\sqrt{10^2 + 30^2} = 31.6$ ft; e nat sec of the brace angle will be $\frac{31.6}{30} = 1.05$; and the nat tang of the same will be $\frac{10}{30} = .333$. All the other data being the same as in the foregoing ex- , first find by Remark at end of Art 20, the *merely approximate* strains on all liques, produced by the weight of only the truss itself and its floor; supposing o be on the lower chord.

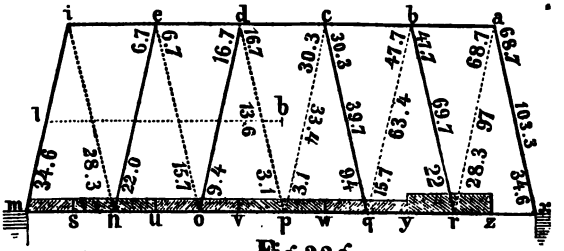


Fig. 23g

se will be as follows:

rox strains on	*Wt of*		*Tons.*
and d o, *each*, =	½ *panel of truss and floor*	× *nat sec* = 6 × 1.05 =	6.30
and e n, *each*, =	1½ *panels* " "	× *nat sec* = 18 × 1.05 =	18.90
and i m, *each*, =	2½ *panels* " "	× *nat sec* = 30 × 1.05 =	31.50

ose on the other half of the truss will be the same.

se strains must be corrected by Art 19: subtracting from that at the center of uss; and then alternately adding to, and subtracting from, each successive one, ount equal to the weight of

$$\frac{1}{4} \text{ panel of } \textit{truss} \text{ only} \times \text{nat sec} = 3 \times 1.05 = 3.15 \text{ tons.}$$

s, the strains become, on Tons.

$$\begin{aligned} p\,d &= 6.30 - 3.15 = 3.15 \\ d\,o &= 6.30 + 3.15 = 9.45 \\ o\,e &= 18.90 - 3.15 = 15.75 \\ e\,n &= 18.90 + 3.15 = 22.05 \\ n\,i &= 31.50 - 3.15 = 28.35 \\ i\,m &= 31.50 + 3.15 = 34.65 \end{aligned}$$

hose of the other half of the truss will be the same. All of which write on iagram, near the *feet* of the obliques. Having now the correct strains arising

he student understands the principles laid down in the preceding Arts on Trusses, he will have ously whatever in seeing the cause of this. We cannot afford space to explain such simply n full more than once. In Figs 23 *c*, 23 *d*, the obliques transfer the load-strains to the vert In Figs 23 *a*, 23 *b*, and in our preceding diagram, the verts transfer them to the obliques; vident from examining the center loads in the figs. These matters must be not merely r tudied, to be of any use.

from the weight of the truss and floor; next find, precisely as in the prec ample, how much of the moving load will go to x when the engine is at r, fig, with the train reaching to m; and afterward with the engine at q, p, in succession. These loads will of course be the same as in the former namely:

Engine at n.	*Engine at* o.	*Engine at* p.	*Engine at* q.	*Engi*
6.44 *go to* x.	15.94 *go to* x.	28.94 *go to* x.	45.44 *go to* x.	65.44

Multiplying each of these by the nat sec 1.05, we get the compressi which they produce on the end oblique strut $a\,x$; and on the other obl *are parallel to it;* namely:

On e o.	*On* d p.	*On* c q.	*On* b r.	*On* a x.
6.76	16.73	30.39	47.71	68.71

which write near the *tops* of said obliques, as done in our fig. But produce precisely the same amount of strains, in the shape of tensions o the respective dotted ties which carry them to the struts between x and p; struts which carry them to the dotted ties between m and p. Write the the heads of said obliques also, as is done in our fig; except that we have t but one decimal, to avoid crowding.

Now, if on the half truss $x\,p$, we add together the strains written at the foot of each oblique separately, the sums will be the total or max strain sive on the struts, and tensile on the ties.) which said obliques will be subje the passage of the train: approximate enough for practice: but a little in some of them, and therefore safe.* These max strains are written at the each oblique between x and p. They will of course be the same on the of the truss, when the train crosses in the opposite direction.

Finally, as to **counterbracing**, (which, in a Warren truss, consists finding which obliques may have to act as both ties and struts,) we go to half, $m\,p$, of the truss. Beginning with the oblique $d\,p$, we see that with t at o, and the train reaching to m, the deranging compressive strain, 16.73 t moving load, is greater than the preservative tensile strain, 3.15 tons, of and floor, acting on it at the same time. Therefore, $d\,p$, although a tie, th $c\,p$, is liable *at times* to be compressed rather than pulled. Therefore, it a arranged as to act also as a strut; at least so far as to bear a pres equal t between the 16.73 tons of pres from the load, and the 3.15 tons of tension wt of the truss and floor; or to 13.58 tons. See Rem 1, Art 31, p 306.

On the next oblique $o\,d$, which is a strut, the same as $c\,q$, the moving lo produce a pull of 16.73; while the truss and floor produce on it a pres of at the same time. Therefore, although it is a strut, it is liable *at times* to rather than compressed; and consequently it must be made able to bear a equal at least to $16.73 - 9.45 = 7.28$ tons. On the tie $e\,o$, the deranging co load strain 6.76, is less than the preservative tensile strain 15.75 of the floor acting upon it at the same time. Therefore, it may remain as a tie o other words, it requires no counterbracing. When this is the case, no othe between it and the nearest abutment m needs counterbracing. It is almost to remark, that the half $x\,p$ of the truss requires the same as the half $m\,p$, engine crosses in the opposite direction.

The strains on the chords are found as in the last example.

Rem. In a Warren truss, the strains on all its members are the same i whether the load be on the top, or on the bottom chord; but those oblique are ties in one case, become struts in the other.

Rem. When the length $x\,q$, $a\,c$, $q\,o$, &c, Fig 23 h, of the panels of a War becomes so great that the stretches $x\,q$, $a\,c$, &c, of chords, need support, t $a\,r$, $b\,q$, &c, are introduced for that purpose; dividing the truss into hal Thus, the vert strut $b\,q$ prevents the long stretch $a\,c$ of the upper unload-d ch

* This arises from our here neglecting what in the foregoing example, of Art 29¼, we c remaining ⅙, ⅔, &c, of the *Engine*," which go to the abut m. Thus, when the engine the train reaching to m, ⅙ of the eng alone goes to m. It first passes up from r to b; as oblique $r\,b$ is a *strut*, and since the ⅙ eng acts on it as a *pull*, it follows that $r\,b$ is there from pres to the amount of ⅙ eng. From b this ⅙ eng passes down $b\,q$ as a *pres;* but sin *tie*, it thus becomes relieved of that much *pull*. From q, this ⅙ passes up the strut q thus relieving it from that much pres; and so on alternately, until the ⅙ eng reaches m. I *way, when the eng is at* q ⅔ of it alone pass up $q\,c$, down $c\,p$, and so on to m, relieving e *as before; but to twice the extent.* Arriving at q, (or in any case, at the panel next before *one p.) and making* the allowance for the remainder of the engine, we need go no farth *same result will take place on the other half of the truss when the train crosses in* the *direction. In practice we should recommend the omission of small considerations* like *here, it conduces to safety to do so.* This note is added merely to show the principle.

. If the load were on the upper chord, vert ties may in the same way become
y merely to prevent the unloaded lower chord from sagging. But in either
ts for thus supporting the *unloaded* chord may often be omitted even when
r supporting the loaded one may be necessary. Thus, in Fig 23 *h*, the verti-

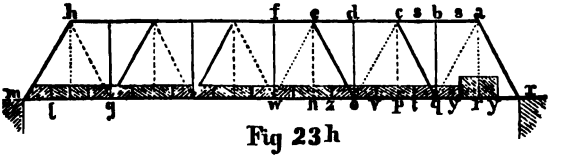


Fig 23 h

z *r*, *c p*, &c, may be absolutely required, even when the stretches *a c*, *c e*, &c, unloaded chord, are not long enough to demand the vert struts or posts *b q*,
Such verts (whether struts or ties) as assist the unloaded chord, (whether er or lower one,) as for instance *b q*, support nothing more than the wt of a etch *s z*, of chord alone.
ach vert, whether tie or post, which, like *a r*, assists a *loaded* chord, should ice be considered to sustain at least the wt of the part *y y* (a half panel) of d floor, in addition to the greatest moving load that can come upon that s a train passes, each such vert in succession has to sustain these united
In other words, they are all strained alike, as are the other verts; but the are strained to diff extents.
g first prepared a skeleton diagram to a scale, calculate, correct, and write near the feet of the obliques, the *truss strains* on said obliques. These may l precisely as in the preceding case; for the verts produce no effect upon cept to increase those on the extreme end ones *a x*, and *h m*, each to the ex- the wt of $\frac{1}{8}$ part of a panel of truss and floor. mult by nat sec. Each vert ps sustain the loaded chord, must have written near its foot, a strain equal to f $\frac{1}{2}$ pan length of chord and floor. And each of the other verts, the wt of length of unloaded chord alone.
for the moving load, and its resulting strains, divide it into half panel lengths, *t v*, &c; and then, as in the foregoing case, find the loads that go to *x* from ts *r*, *q*, *p*, *o*, *n*, &c. Thus, (the truss, considered as a lever, being divided into parts,) from *r* there would go to *x*, $\frac{11}{12}$ of the engine; $\frac{10}{12}$ of *y t*; $\frac{9}{12}$ of *t v*; z, &c, &c, and $\frac{7}{48}$ of *l m*. From *q* there would go to *x*, $\frac{10}{12}$ eng; $\frac{9}{12}$ *t v*; $\frac{8}{12}$
Then, for the strains produced on the obliques by these moving loads, and ritten near their tops, proceed thus: Mult the load at *r* by nat sec of brace the prod will be the load strain on the oblique strut *a x*; and it will be a pres. ult the load at *q* by nat sec, for the load strain (a pull) on the oblique tie *a q*. d at *p*, mult by nat sec, will be the load strain (a pres) on *c q*. And so on to er load at *w*. From *w* to *m*, the loads which go to *x*, will, as in the preceding oduce *pulls* upon the oblique *struts*; and *pressures* upon the oblique *ties*; and ressures and pulls on that half of the truss, must be used, as in said case, for which obliques must act as both ties and struts; or in other words, as coun- s. For the load strains on the verticals which assist the loaded chord, write e top of each, the greatest moving load, as *y y*, that can come upon a half This added to the truss strain of $\frac{1}{2}$ panel of chord and floor, already written foot, will be the total strain on each such vert. The verts which assist the d chord may be practically considered as unaffected by the moving load; and e each of them is strained only to the amount of the wt of a half panel of rd.
trains on the chords are found precisely as in the foregoing case.
1. Respecting the dotted verts *a r*, *c p*, *e n*, &c, which assist the loaded chord, t first sight appear that each would be strained by all the moving load which osed to be concentrated at its foot, and to pass from there to the abut *x*; s we have stated that the greatest moving load strain on each of them in turn, aximum moving load *y y* that can come upon the width of half a panel. Thus, ance, at the foot of the vert *a r* we have assumed to be concentrated, $\frac{11}{12}$ *y t*, $\frac{9}{12}$ *t v*, &c; whereas we also say that it is strained or pulled by *y y* only, a correct. *The reason of this is* that at the feet of these verticals, the several *of the moving load are* (for convenience of calculation) only assumed to be *ated as they go to the* abut *x*. For since these portions travel along th *ly, they cannot reach the feet of these verts*; and nothing can reach them

that portion of the load which *actually* comes to them. But it will n that so far as regards the calculations for the strains on the *obliques*, th is justifiable. Take, for instance, *a r*. Here we *assume* to be concentr way to *x*, along the oblique *a x*, $\frac{11}{12}$ eng, $\frac{10}{12}$ *y t*, $\frac{9}{12}$ *t v*, &c; and we that all these added together, and mult by nat sec, give the strain on all these portions of the moving load, the $\frac{11}{12}$ eng, or *y y*, is all that ge of *a x* by ascending the tie *a r*; the other portions really accumulate there go to the head of *a x* by way of the oblique *q a*. Therefore, *all* *a*; and from that point go down *a x* to the abut. All the $\frac{12}{12}$ of *y y* res but after reaching *a*, $\frac{1}{12}$ (by the principle of the lever) goes to th *m*, along *a q*, *q c*, *c o*, &c. The same reasoning applies to each other ve the loaded chord. With the roadway on the upper chord, the same vert posts which then assist that chord.

REM. 2. If the moving load were uniform in practice; that is, if every of it were of the same wt, we might readily calculate the strains on s Fig 33, as if it were a simple Warren, (Fig 23 *g*.) Thus, in Fig 33 we m the shaded lattices *c v*, *u y*, *s h*, &c, as the obliques of a WARREN; and strains for such. These strains might then be divided by the numl which take the place of a Warren oblique; and the quot would be a suf approximation to the strain on each lattice. Thus, the strain on the W *u y* would be divided among the five lattices *g i*, *w v*, *u y*, *t x*, *l e*, parall extending from center to center of the panels adjacent to *u y*; and so w oblique in either direction. And this method may be used when the n actually uniform; or for a permanent uniform load *from end to end*. erally the greater strain produced at *every point* along a bridge by th trated on the driving-wheels of engines, appears to be lost sight of; an is stated to be sufficiently correct under any circumstances. But in rai it must make the strains on the lattices too small; that is, each of the lattices will be strained in turn by the passing load, to a greater exte the strain on the Warren oblique *u y*.

In the Fink truss, Figs 26, 27, the effects of a mov may be calculated as for a full uniform max load from end to end. T at first, as in the preceding cases, that everything is borne by one trus when the load is upon the *top* chord of the truss, each vert post may i regarded as upholding one-half of that portion of the entire wt of br tributed load which is between the two extreme ends of the two ob uphold said post. Thus, in Fig 26, the half-way post *d c*, bears half o *a* and *b*. The post *m g*, half of all between *a* and *d*; the post *h o*, half o *m* and *d*.* This is equivalent to saying that the half-way post bears h wt of the bridge and load; each quarter-way post, one-quarter; eac post, one-eighth, &c, &c, of this same entire wt of bridge and load; and tute theoretically the strains on the several posts. But after having go is necessary to examine whether some of the smaller ones may not b creased, for the following plain reason: Suppose we have assumed our m a string of heavy engines, weighing 1 ton per foot run; or, including bridge itself, say 1.4 ton per ft; and suppose our posts to be as close it; then the least loaded posts would each bear $5 \times 1.4 = 7$ tons. But from 16 to be concentra length of 5 drivers of an half of it wil supported by succession as When we thu which posts strained by ar by our assum run of the wl

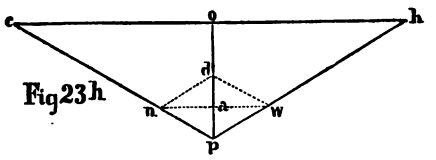


Fig 23 h

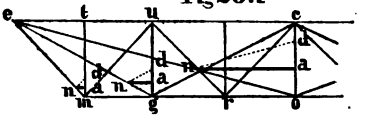
must increase the load first found, correspondingly. In the Fink, trusses, the verts are always struts or posts. Having fixed upon the l *post*, as *p o*, Fig 23 *h*, then for the strain which said load will produce *the obliques*, or *ties*, *p c*, *p h*, upholding said post, take any dist *p d* o

* *In Fig 26, the load is really on the bottom chord. But in this case also, the carried to the tops of the posts by the obliques: except at the four posts h e, h a, h obliques at their heads. With a bottom load, these may in practice be considere only one half-panel wt of truss alone; but their obliques sustain the same load w n the top chord.*

the load by scale; and draw $d\,w$, $d\,n$, parallel to the ties; then $p\,w$, $p\,n$, by the same scale, will respectively give the strains on each; whether they inclined as usual, or not. The two hor lines $n\,a$, $w\,a$, by the same scale, wo hor forces which the load at the top o of the post, acting through the ces upon the chord at c and h; which two equal and opposing *forces* pro- ; the intermediate stretch $c\,h$ of chord, a *strain* equal to one of them. In is, either $n\,a$, or $w\,a$, gives the hor strain produced along $c\,h$, by the load See "Strain," Art 2, of Force in Rigid Bodies, p 444.

on the chord. This, from a uniformly distributed load, is the same t the entire length of a Fink chord.* To find it, observe which obliques, , $o\,e$, Fig 23 i,) of one-half of the truss, *terminate at one end*, e, of the chord. ng previously found the loads on the posts, $c\,o$, $u\,g$, $t\,m$, which pertain to ues, ascertain by the pro- g 23 h, the hor force $n\,a$, gs,) which each of those nces on one oblique. Add hese forces $n\,d$, (there three of them in Fig 23 i, by the dark lines;) their e the strain along the en- . The obliques $m\,u$, $u\,r$, not terminate at e; and fore, omitted in finding strain. The process is the same whether the verts are all of the same not.

Fig23.i



nor chord-strain produced by each of the loads on the posts $c\,o$, $u\,g$, $t\,m$, culated thus, and added together.

$$\textit{Horizontal strain} = \frac{\textit{entire load on post} \times \textit{hor dist from post to end e of chord}}{\textit{twice the length of the post.}}$$

Bollman truss, Figs 22, 24, 25, for a *moving* load, having first pre- working diagram, determine the max weight that can come upon a post. e the same for each post. If the moving load is on top of the truss, this ch post will consist of the greatest wt of engine that can stand upon one th of truss; together with (approximately enough for practice) the wt of length of truss, floor, &c, if the posts uphold the oblique ties; (see Boll- , in Art 19;) or one panel-length of floor; and the half of a panel-length the posts do not uphold the ties. If the load is at the bottom of the truss, bear no part of either the moving load, or of the floor; but each of them ained to the amount of the wt of say one panel of truss, if the posts up- lds; or of half a panel of truss, if they do not.

ls on the posts may then be written upon the diagram.

bliques or ties, however, when the load is at the bottom, bear (as in the same amount of strain from the moving load and floor, as when it is on top. when it is at the bottom, each pair of ties sustains not only the load rest- post which they uphold; but the wt of one panel-length of floor, and the -weight of engine. In other words, *whether the load be on top, or at bottom*, es at the foot of each post, sustain a wt equal to a full panel-length of truss together with the max panel-wt of engine. Having added these wts to- off their sum by scale at each post, as shown at $l\,r$, $k\,r$, $j\,r$, $i\,r$, Fig 22; com- u, $k\,g\,v\,u$, &c; and measure the strains $l\,u$, $l\,g$, $k\,u$, $k\,g$, &c, along the ties.

trains on any pair of ties, may also be calculated thus; having ley sustain.

$$\textit{Strain on short tie} = \frac{\textit{load} \times \textit{hor dist from post to farthest end of chord}}{\textit{total length of truss}} \times \frac{\textit{length of short tie}}{\textit{depth of truss.}}$$

$$\textit{Strain on long tie} = \frac{\textit{load} \times \textit{hor dist from post to nearest end of chord}}{\textit{total length of truss}} \times \frac{\textit{length of long tie}}{\textit{depth of truss.}}$$

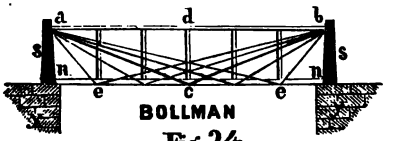
or strain on the chord will be uniform throughout, as in the Fink*

compressed chord of any form of truss, to resist the hor strain, regard the chord as ngth is equal to the dist between two adjacent points of support.

truss; and will depend upon the max uniform load that can cover the whe and not, as in the case of the ties, upon the greatest load which each pair o have to sustain *in succession;* unless we assume our max uniform load to l of engines which may bring a max panel-wt of engine upon *every pair once.* In that case we have only to measure upon one-half of our working the several hor lines corresponding to *u o*, &c, in Fig 22; and their sum v reqd hor strain on the chord. But if we take our max uniform load on truss, to be a string of *cars*, we must diminish the chord-strain thus fou manner: Add together a full panel-weight of truss, floor, and *cars;* then, panel-wt of truss, floor, and *engine*, (which we before assumed as the stra of each pair of ties,) is to the panel-wt of truss, floor, and cars, just found hor chord-strain before found, to the one reqd.

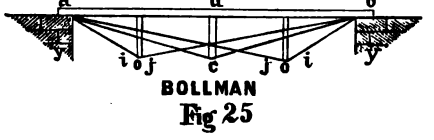
We will repeat, that chords must be strong enough to bear not only th or push to which they are exposed; but also to sustain safely, *as beams,* verse strains from the floor, and from the moving load, when these rest u See foot-note p 191.*

Art. 21. If a long beam *a b*, **Figs 25 and 27, require strengthened,** this may be done by the addition of a vert post *d c*; a clined tie-rods *c a*, *c b*. And if after this, the two halves *d a*, *d b* of the are found to be too weak, additional intermediate posts *o*, *o*, may be introdu other ties *i*, *j*, to sustain them.

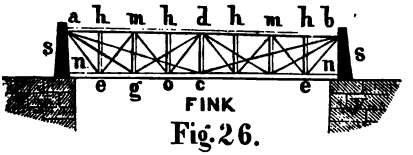


BOLLMAN
Fig 24.

In Figs 25 and 27, th is at the chord *a b*; r lower chord being ne may be omitted. Th ties act as substitutes f if the bridge is so near as not to allow the pos to be placed *beneath* th *a b*, we may raise the e upon two posts or pie 24, 26; and place th *n n*, at the lower ends o and ties: instead of letting it rest on top of the chord, as in Figs 25 and 27 24 and 26, the truss and its load do not then rest *directly* upon the abut upon the tops of *s*, *s*; and the only does rest direct abuts, is *one-hal* small portion of way comprised a between *s* and *n*; words, only one h of the roadway c panels; the other sustained by th ties which meet s When the tie-rod from the feet of the p ends of the chords, 24 and 25, we have t **man truss.** And in Figs 26 and 27, c which sustain the c *d c*, *both* pass to th the chord, while t are disposed as in sa

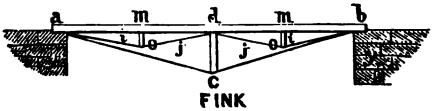


BOLLMAN
Fig 25



FINK
Fig. 26.

Fink truss is the result.



FINK
Fig 27.

* *As respects* bearing their compressing strains only, chords and roof rafters are w *tilars* having the length of one panel.

2. **Fig 28, shows the general arrangement of a small Howe bridge-truss;** Fig 29, some of its details; and Fig 30, those truss. High trusses are sometimes made as in Fig 1. The top and bottom the wooden one are up of three or more umbers c c c, placed a apart, to let the vert r pass between them. braces, o o, are in pairs ces. The pieces com- m, about at top and bot- ast triangular **angle** s; which if of hard solid; and if of cast- ow; as shown at T, strengthened by inner ribs. tend entirely across the three chord-pieces. Against their but also the counterbraces e. single pieces in small bridges; s, in large ones; and pass be- pieces which compose a main here the wooden braces and cross each other, they are gether. For wooden chords, blocks are cast,* as at T. The nes show the strengthening x serves to keep the block in place. The vert tie-rods r r, of iron, are in ees, or fours, &c, according to size of bridge; with a screw and nut at each heads and feet of the braces and counters, butt square against the angle- and are kept in place only by the tightening of the screws of the vert ties. floor is below, as in Fig 28, the end posts p d; and the ends g i and w b, of chord, *may* be omitted; also i c and b y; but it is seldom done.

Fig 28

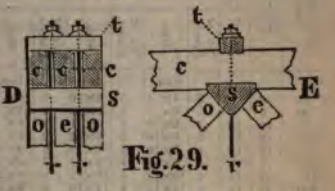


Fig. 29.

Figs 30

30, of an iron Howe truss, the top chord P, M, and W, is cast in one piece ly, as at P. Its separate lengths are connected together by flanges and ewhat as shown at W; where, a a, are cast longitudinal flanges for strength- transverse bolting-flanges g. Instead of separate angle-blocks at the upper ld ones may be cast in the same piece with the chord itself, as shown at M. r chord usually consists, as in other iron bridges, of four or more flat bars of n, c, placed on edge: and some dist apart, as at R. On top of them rest the gle-blocks s, which have shallow channels below, for receiving the chord nd thus securing them from lateral motion. A cast washer, a, below the provided with similar channels on top, for the same purpose. The braces

spans, in order to prevent the pressure of the heads and feet of the obliques from crushing the angle-blocks are *cast with deep projecting flanges* under their bases; and which, pass- *the pieces which compose a chord*, extend to the opposite face of the chord. There the *upon broad washers at the ends of the vert rods.* By this means the strains along the *transferred directly to the verts, without* at all affecting the chords. Angle blocks of *openings for the passage of the vert rods.*

and counters, *o*, *e*, in moderate spans are usually cast in a star-shape, following table gives dimensions sufficient for a strong Howe bridge; before remarked, it is advisable to add arches when the span exceeds a

Dimensions for each of two trusses of a Howe br single-track railway. Timber not to be strained more than inch; nor iron more than 5 tons per sq inch. Iron supposed to be of r quality, requiring from 25 to 27 tons (60480 lbs) per sq inch to break to be *upset* at their screw-ends. To each of the two sides of each lower posed to be added, and firmly connected, a piece at least half as thick chord-pieces; and as long as three panels; at the center of the span.

Clear Span. Feet.	Rise. Feet.	No. of Panels.	An upper Chord.		A lower Chord.		An End Brace.		A Center Brace.		A Counter.		End R	
			No. of Pieces.	Size.	No. of Pieces.	Size.	No. of Pieces.	Size.	No. of Pieces.	Size.	No. of Pieces.	Size.	No. of Rods.	
				Ins.		Ins.		Ins.		Ins.		Ins.		In
25	6	8	3	4× 5	3	4×10	2	4× 6	2	4× 5	1	4× 5	2	1
50	9	9	3	6× 7	3	6×10	2	6× 7	2	5× 6	1	5× 6	2	1¼
75	12	10	3	6× 9	3	6×11	2	6× 8	2	6× 6	1	6× 6	2	1½
100	15	11	3	6×10	3	6×12	2	8× 9	2	6× 8	1	6× 8	2	2
125	18	12	4	6×10	4	6×13	2	9×10	2	6× 9	1	6× 9	2	2¼
150	21	13	4	8×10	4	8×14	3	8×10	3	6× 8	2	6× 8	3	2½
175	24	14	4	10×12	4	10×15	3	8×11	3	8× 8	2	8× 8	3	2¾
200	27	15	4	12×12	4	12×16	3	9×12	3	8×10	2	8×10	3	2¾

The same dimensions will serve for a *double* road for common travel.

For bridges of iron, assuming the safe strain for iron to be 5 tons per times as great as the 800 lbs assumed for wood; * the areas of the cross-s individual members will as a general rude approximation, be about $\frac{1}{1}$ as those of wooden ones. Equally strong wooden, and iron bridges of t will not differ very materially in weight. The rods in our table are those usually employed; but we consider them none too large.

Fig 31

Figs. 32.

Art. 2 shows in a **wood truss:** an details of one. After they do no explanatio angle-block sist tension thrust, the above the below the The main in pairs; as single con tween the Howe. In they are in &c. The which, whe hollow, are their positi the strai obliques, nate above their ends; let into the large spans generally

less from those in the figs. In Fig 31, c c c are the main braces; and o o

* See page 174, "Mistakes are sometimes made, &c.

his roadway is below, as in Fig 31, the ends, *r b*, *y x*, of the upper chord; the als *p* and *u*; and the two tension obliques in each end panel, may be omit- two diagonal *struts* from *b* and *y* must then be substituted, extending to ments, for upholding the upper chord, &c. In the Pratt truss, the chords f the same dimensions as in the foregoing table for Howe's. The posts may at $\frac{1}{5}$th less area than the main braces of the Howe. The main brace rods, ra, (of the same number as the main brace pieces of the Howe,) may have ving diams in ins; allowing the safe strain to be five tons per sq inch.

For each Truss of a Pratt Bridge.

Spans.	50 Ft.	75 Ft.	100 Ft.	125 Ft.	150 Ft.	175 Ft.	200 Ft.
End Main-brace Rods.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
	2 of 1⅙	2 of 2⅙	2 of 2⅓	2 of 2⅚	3 of 2⅓	3 of 2⅚	3 of 3⅙
Center Main-brace Rods.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
	2 of $1\frac{3}{16}$	2 of $1\frac{5}{16}$	2 of $1\frac{7}{16}$	2 of 1⅙	3 of $1\frac{5}{16}$	3 of 1⅚	3 of 1⅚
Counter Rods at Center.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
	1 of 1⅚	1 of 1⅚	1 of 2	1 of 2⅙	2 of 1⅚	2 of $1\frac{11}{16}$	2 of $1\frac{11}{16}$

tt's truss the directions of the main braces and counters are respectively ne of the Howe. Many of the remarks in the preceding Art, apply equally Neither the Howe nor the Pratt possesses any special advantage over the regards ease of adjustment, &c. In both trusses, arches are frequently added a railroad bridges when the span exceeds about 150 ft.*

24. Town's lattice truss, Fig 33, as originally introduced, and very ely employed, was of extremely simple construction; being composed en- planks from 2 to 3 ins d from 9 to 12 wide; de- on the span. Two sets were placed crossing ier at angles of about d were connected to- t their intersections by or 4 treenails of locust, hard wood, about 1¾ n. At the top and bot- nilar planks, *a a*, *c c*, enailed hor, to form the or in large spans, (many f 150 ft,) there were two nd two lower chords, (sometimes of timber 6 ins thick,) as shown in the fig, *o c*. The transverse section A shows the two upper chords on a larger scale; ord consisting of two planks; one on each side of the lattices. Two trusses tind, with a depth equal to ⅙ or $\frac{1}{5}$ of any clear span not exceeding about 175 ks of 3 × 12 white pine; the open squares 2½ ft on a side, in the clear, were con- sufficient for a common-road bridge 20 ft wide. Many of these bridges warped n very badly; and when applied to railroad purposes, failed entirely. In ses the better mode of three lattices was employed; two of them running in ction; and the third in the other direction, passing between them. A funda- defect was that the parts were of equal size throughout the span; whereas the should be stoutest at the center; and the lattices, near the ends. These de- sed the lattice *to fall into unmerited* neglect, in the United States; whereas

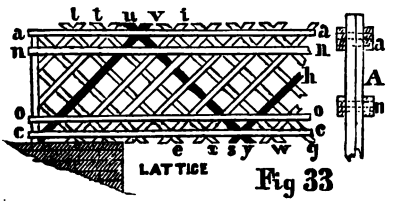


Fig 33

Baltimore Bridge Co (Smith, Latrobe, &c) have built a wooden Howe of two trusses, of 3 *high, 28 wide, from center to center of chords, without any arch.* It has a wrought-ir *ft, and is proportioned for a moving load of but 1000 lbs per ft run.*

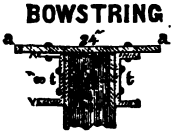
in Europe it is, when properly proportioned, highly esteemed; especially f bridges; some of which, on this principle, ha constructed of more than 500 ft span. Anoth was lateral weakness; or a tendency to warp si owing to the thinness of the trusses. This is e in the large bridges referred to, by placing a truss, Fig 34, (instead of a single one,) at each the bridge. The trusses T, D, composing a dou are placed a foot or more apart; and are conne gether at proper intervals, by short pieces ri each one, for stiffening them. At T and D are three lattices or lattice-bars, of each truss;

Fig 34

which, on the outside, constitute the main braces; while the center one is t terbrace. Such a double truss bears some resemblance to the Fairbairn box the diff being chiefly that in the former the sides are composed of lattice-ba in the latter of solid plates.

The Bowstring truss, Fig 35, is an excellent one as regards streng economy of construction. It has, however, the disadvantage, in large spans, c culty in connecting together overhead the two trusses of a span, so as to b from lateral vibration as with parallel upper an chords. In the latter, t nection can be made fro end of the span; but in t string it can be done some distance each way f center; from want of l near the ends. In sho with low trusses, this not felt. The verts and shown in the fig, may struts, or all ties; or Howe, a Pratt, or a Wa rangement.

Fig 35
BOWSTRING



The pull along chord *y y* in a Bo truss, is uniform throughout its length; and is the same in amount as the l at the center or crown, 4, of the arch. Either may be found thus; remem that in our calculations of bridge trusses, we assume at first, as in previou ples, that all the weight of the bridge and its load is sustained by one truss c that the resulting strains must finally be divided by 2, to distribute them am two actual trusses: *

$$\textit{Hor pres at crown; or pull on chord} = \frac{\textit{half the united wt of bridge and max load} \times \tfrac{1}{4}\textit{ span}}{\textit{rise measured between the half-depths of the arch itself, at its foot and crown.}}$$

Which is the same as for the Howe, and other beam-trusses; but not for Fis Bollman's; in both of which it is greater. If the chord has also to act as a b supporting cross-floor girders, it must be made stronger. See footnote, p This may be avoided by placing such girders only close to the verts.

For the pres along the direction of the arch, at its f square half the united wt of the bridge and its max load. Also square the h at crown just found. Add these two squares together. Take the sq rt of the This is the same process as for a stone arch. The pres does not increase m from the crown to the foot; but in bridge practice it may safely be assumed t

Strains on the obliques. Whether these (Fig 35) are all strut ties, the strains upon them, according to Prof. Rankine, remain unaltered; a along any one of them is found as follows, using the numbers of the two verts b which it is situated, after having first numbered the verts along *one-half* of th as at 1, 2, 3, 4, in Fig 35. The moving load is supposed to be uniform; and the Professor's formulas do not allow for the extra load concentrated upon th *ing-wheels* of an engine.

$$\textit{Strain along any oblique} = \left(\frac{\textit{weight of one panel-length of moving load}}{\textit{vert stretch of the oblique}} \times \textit{length of the oblique}\right) \times \left(\frac{\textit{No. of one vert}}{\textit{twice the number of panels in one h}} \times \textit{No. of a vert}\right)$$

That is the strain along the oblique *e c*, Fig 35, situated between verts No. 2 ; the weight of a panel-length of the moving load alone being 10 tons; the , $12\frac{1}{2}$ ft; and its vert stretch *c n*, 7.5 ft.? There are 8 panels in one truss.

$$\frac{0 \times 12.5}{7.5}\Big) \times \left(\frac{2 \times 3}{16}\right) = 16.7 \times .375 = 6.26 \text{ tons, Ans; or } \frac{6.26}{2} = 3.13$$

the oblique *e c* of each of the two *actual* trusses of a span.

ns on the verticals. When the arrangement of verts and obliques Fig 35, then the strains on the verts will depend considerably upon whether ques are all struts, or all ties. In either case, however, they will be greatest center of the truss, and least near its ends; resembling in this respect the s 14, 15, and 16; but the reverse of the *beam* truss, such as a Howe, Pratt, n. In the Bowstring, neither the verts nor the obliques sustain any portion ow or arch itself; but on the contrary, they are all upheld by it.

n the obliques are all struts, first find the wt of one panel-length ridge and its max load; and from this take the weight of one panel-length rch or bow itself. We will call the remainder, the "*reduced total panel-*" then the

$$\text{strain on any vert} = \left(\begin{matrix}\textit{reduced total}\\ \textit{panel-weight}\end{matrix}\right) \times \left(1 + \frac{\textit{No. of vert} \times \textit{next less No.}}{\textit{twice the number of panels in one truss.}}\right)$$

What is the greatest strain that can come upon vert No. 3, Fig 35, the weight uced total panel-weight being 13 tons? Here

$$13 \times \left(1 + \frac{3 \times 2}{16}\right) = 13 \times \left(1 + \frac{6}{16}\right) = 13 \times 1.375 = 17.875 \text{ tons.}$$

$\frac{75}{}$ = 8.9375 tons on the vert No. 3 of each of the two actual trusses.

If the obliques in Fig 35 are all ties, the greatest strain upon of the verts, is simply equal to one reduced total panel-weight.
Rankine also says that in *some cases*, under a moving load, if the obliques in re *ties*, some of the verts may have to act at times as struts, as well as ties. tain if this is the case with any given vert, use the following:

$$\text{load vert} \times \left(\frac{\textit{No. of vert} \times \textit{next greater No.}}{\textit{twice the number of panels in one truss}}\right) - \left(\begin{matrix}\textit{wt of one panel of}\\ \textit{bridge only,}\\ \textit{without the arch itself.}\end{matrix}\right)$$

sult is minus, the vert will act as a tie only; but if the result is either 0 it will show to what extent the vert may have to act as a strut also.

m a Bowstring truss has no obliques, but only verts, the arch upposed to be sufficiently stiff in itself to resist change of form by a moving e greatest strain on each vert from a uniform moving load, is equal to a re- tal panel-weight; but in practice we must allow for the greatest load that e upon a panel. The proper shape for the arch, under a uniform *quiescent* a parabola; but when the rise does not exceed about $\frac{1}{8}$ of the span, (which d not,) a circular segment is sufficiently approximate for common practice. uniform quiescent load, obliques are not needed.

an arch trussed as in Fig 35½, with vertical struts, and with ob- ll struts, or all ties; the strains arch and obliques may, according . Rankine, be found by the rules ren for the Bowstring. The upper w, need only be strong enough the load upon it.

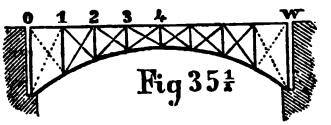


Fig 35½

en the obliques are all the verts will always be struts or and the strains on any one of will be given by the foregoing formula for any vert of a Bowstring.

en the obliques are all struts, the strain on each vert is equal to duced total panel-weight; and in this case it is possible that some of the verts t times during the passage of a moving load, have to act as ties, as well as To ascertain if this *is the case* with any given vert, use the last preceding h. *If the result is plus, it gives the* pull or tension which the vert may have *t as a tie; but if it is either 0* or minus, the vert acts as a post only. Prof *says that the dotted obliques in* the end panels are not absolutely necessar

as the formula will show; but that it is still well to insert them, f greater stiffness.

The Lock Ken viaduct, England, of 130 ft clear span, s has two trusses, 13 ft 8 ins apart clear, for single-track railroad, on principle, Fig 35, omitting only the verts; which, however, affects the obliques. For convenience of construction, it was built chiefly of r iron, (see *t t*,) of 8 ins by 4, by 4, by ½ inch. At Fig 35, *d* shows a tra of the arch or bow; which is uniform throughout. That of the uni string is of the same fig and size as the arch. Each has an area of 33 truss. The top strip, *a*, *a*, of rolled iron, is 24 ins by ¾; and is rivete flanges of the two channel-irons *t t*; which are 8 ins apart, so as just to all between them of the obliques *d*; which also are of the same channel-iro are about 12 ft long near the center of a truss; and 8 ft at its ends. two trusses alone, without the roadway, is very nearly 50 tons. The wi beyond the theoretical requirements, to save the trouble and expens and fitting together many pieces of diff dimensions; yet, although strong one, the trusses alone, weigh together but .37 of a ton per foo two obliques cross, one is in two pieces, riveted to the other by straps.

In the State of New York are many much-used **bridges fo travel, by Mr. Whipple,** of 100 feet span, and 12½ ft rise; with ft apart from center to center; for two roadways; and having two out each 6 ft wide. Each truss has 9 panels, braced altogether by vert a rods (no struts) arranged as in Fig 35. The verts next the center of *eac* each of two rods of 1⅝ ins diam, welded together at top; and straddling The other verts are single, and 2 ins diam. The obliques are all single, an The arches are of cast-iron. The transverse area of metal of each arch is crown; and 21 sq ins at the spring. The shape of arch transversely resem iron with its back upward; the total depth of flange 7 ins; the width of ins at center of span; but increasing uniformly by means of wide oper to 3 ft at springs. Each consists of 9 straight segments, held togethe ting flanges, by the verts themselves; which pass through them, and h nuts at their ends. The screw-ends are not upset. The thickness o arches nowhere varies much from ¾ inch. Under the floor, and betwe are horizontal diagonal braces of rods ¾ inch diam; two of them to eac of them with a tightening swivel. The chord of each arch consists of diam. In the same State, are also many similar common road bridges Rise 9 ft; two trusses, 19 ft apart from center to center; * and two on of 6 ft each in addition. Each truss has 7 panels, with vert and oblique ti Each cast-iron arch is in 7 straight segments, of the same shape as with a cross-area of metal of about 12 and 15 sq ins. Its width at cen ins; at springs, 30 ins. The two verts next the center of each truss, co rods of 1½ diam; the other verts are single, each 1¾ diam. The o single, 1 inch diam. The chord or string of each arch, is 4 rods of 1 Horizontal diag bracing of ¾ inch rods under the floor, as in the fore

Some cast-iron bridges of the Severn Valley England, of 200 ft clear span, consist of arches rising 20 ft, and s railroad *on a level* with the tops of the arches, instead of *above* them a There are no diags between the arches and the roadway, as in that iron verts only, placed 4 ft apart. The railroad is double track; and arches, one under each line of rails. The transverse section of an ar flange is 15½ ins wide, by 2 ins deep; the web is 2 ins thick. Total de of span, 4 ft; and at the skewbacks, 4 ft 9 ins Transverse area of each 150 sq ins. Each arch is cast in 9 segments of equal length.

The cast-iron bridge across the Schuylkill at Ch Phila. Strickland Kneass, Esq, Engineer, roadway on top, has two a clear span each, and 20 ft rise. Clear width, 42 ft. Each arch has 6 ri apart in the clear; and of the uniform depth of 4 feet, including a hor wide; and a similar one at the bottom. Thickness everywhere 2½ ins; each rib a transverse area of 147½ sq ins. The standards are vert, wit tion. It is a city street bridge. The roadway consists of cast-iron plat port a pavement of cubical blocks of granite, laid in gravel. The arch segments 12 ft 10 ins long; each with end flanges 12 ins wide, for bolt gether with four 1¾ inch diam screw-bolts at each end. For a chang *ture from* 12° *to* 99° Fah. the crowns of the arches rise $2\frac{5}{16}$ ins. Unc *extraneous* load of 100 lbs per sq foot, the greatest pres on the arches

* *With only two trusses*, the width between them, in the clear, should not be *allow two ordinary* vehicles to pass each other readily; but 18 or 20 ft is still bet *unnecessary* when there are outside footways. The headway should not be less

' their cross-section; or not more than $\frac{1}{28}$ of the ultimate crushing ›rage cast iron, in short blocks.

›ley Bridge, Figs 35¾, by Thos. W. H. Moseley, of Kentucky, is rought-iron Bowstring, with a hollow plate-iron arch of triangular ,pex up; and formed of three plates riveted together; the two side- having their top and bottom form flanges for this purpose. the verts; and the two counter- also of iron. These counter- *nded* as a substitute for the ob- ›. Each of them consists of two ıck to back, riveted together, icals, which pass between them. has a sectional area equal to e main arch. The verticals are ! ft apart. They are flat (not r convenience of riveting. They ıoles in the bottom-plate of the › dotted line of top Fig,) and are ; the same rivets which connect

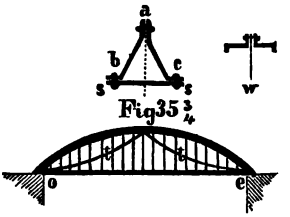


Fig 35 ¾

ges of the two side-plates, *a b*, *a c*. The chords, *o e*, are also flat bars; nsverse area half as great as that of the main arches. At their ends ıed to strong wrought-iron shoes upon which the feet of the main d abut. The rise of the main arches, measured to the bottom of the $\frac{1}{6}$ of the clear span.

ıg are the principal dimensions of a single-track bridge of 93 ft clear m out to out of arch,) carrying the "Iron Railroad" at Ironton, Ohio. › 1860, and is traversed by heavy engines with trains of pig iron, coal, ttom of arch 10⅓ ft; to middle of arch 11 ft. Bottom-plate *s s* of arch, ‹ inch. Side-plates *a b*, *a c*, in clear of flanges, 14½ ins, by .29 inch. *a*, each 3 ins. Bottom flanges *b* and *c*, each 1.88 ins. Vert rods, 3 ins, d 2 ft apart from center to center. The chord of each arch consists of each of 4½ sq ins of cross-section. The bridge was tested for three ıd load of ½ a ton; and a rolling load of 1 ton at the same time, per deflected only ⅝ inch. With a load of 1 ton per foot, the pressure at he arches would be about full 4 tons per sq inch of metal; and a trifle ›t. The sectional area of metal in each main arch is 18½ sq ins. These ' easy construction, and consequently cheap. For long spans, vert ng (see Fig 35) would probably be essential for preserving the form ınder heavy moving loads.*

arch roof in Philadelphia, clear span 80 ft, rise 16 ft, con- :m arch of single 7-inch Phœnix beam of 6 sq ins sectional area; weigh- foot. This rests on cast-iron shoes on the walls. The hor chord or tie ›f two rods of 1⅜ diam. At the center of this tie is an arrangement 15, of Figs 21½, from which diverge upward, to the arch, a central vert a; two struts of 6-inch Phœnix beam, 20 ft apart at the arch; and two iam, reaching the arch at half-way between the struts and the feet of ›re are 10 such trusses, each of which by itself weighs about 3900 lbs; l 16 ft apart; and by means of purlins resting upon them, support the ›f the roof, which is of inch boards, covered by thin sheet-iron.

roof of a rolling-mill near Boston, Mass, of about 80 ft rise, has arches of the Moseley section *a*, *b*, *c*, Fig 35¾; but without . The trusses are 12 ft apart. Sides of the arches, clear of the flanges, nges, 2 ins; lower ones, 1 inch; total of each side, 10 ins, by .19 inch thick. ½ inch by ¼ inch thick. Total sectional area of an arch, 5.925 sq ins. ies, a chord; and 24 vert suspending rods; but no obliques. The roof ı corrugated iron, on purlins. When required, the heavy iron rolls of 'ted by tackle supported by a roof-truss.

represent the Burr truss; which was formerly more used

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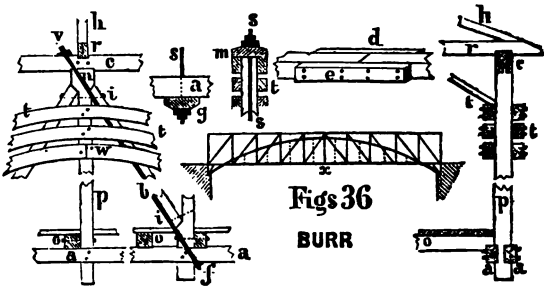
s bridge seems to have stood very well for several years, the writer would prefer not ıs of compressive strain per sq inch *on plate-iron* in such structures. In several Moseley has used a *continuous web* of ⅛ inch iron, instead of the vert suspenders. ıs the triangular tube is not applicable for the arch; and he substitutes two T bars, and to *the web, w, which passes* between them, as in the foregoing fig. The counter › *unnecessary, are omitted.* This web would be objectionable in large spans, espe- *idges, on account of the wind.* More recently still, he has also introduced *vert bars, in some of his* bridges; together with many innovations on the arrang *t. At 1 ton per ft run,* the pull on the chord above, = 8 tons per sq in.

than any other in the United States. It is at present regarded with disfavor by m
because many early ones failed under railroad traffic, in consequence of bad pro
tions, and the absence (as in our Fig 36) of counterbracing. When properly
structed it makes an excellent bridge. The common objection to it, and not wit
reason, is that a truss and an arch cannot be so combined as to act entirely in
cert; yet, as soon as any ordinary truss begins to fail, the almost invariable re
is to add an arch. When, however, the two are to be united, it is better to so
portion the arch as to be capable by itself of safely sustaining the max load at



Figs 36
BURR

and to confine the duty of the truss to preventing the arch from changing its
under a *moving* load. Counterbracing may be effected by strapping the heads
feet of the braces to the chords; or by iron rods parallel to the braces; two
brace; with screws and nuts, as at *v f*. Or by similar rods across the other diag
the panels. The following dimensions will answer for a substantial single-t
railroad bridge of about 150 feet span. Rise from out to out of chords, 1/8 of
span: about fourteen or sixteen panels. Width in clear of arches, 13 ft. Six
pieces *t*, of 8″ × 12″ each, to each truss. Upper chord *c*, 12″ × 14″. Lower chord
two pieces each 8″ × 14″. Posts *p*, 12″ transversely of the bridge, as in the r
hand fig; by 8″; except at the heads and feet, where enlarged to receive the end
the braces. Braces 8″ deep, by 12″ wide. Floor girders *o*, 8″ × 15″; and 2½ to
apart from center to center. Suspending rods (shown at *s*; and dotted in *s*)
diam. Counterbrace rods in pairs parallel to braces, about 1¼″ diam. Bolts
arches, lower chords, &c, 1¼″ diam. Theoretically, the posts, braces, and are
should gradually diminish from the ends, toward the center of the truss; while
chords should increase; but in practice, the additional labor of getting out and fit
pieces of diff sizes, frequently makes it more economical to use uniform sizes.*
same amount of arch would answer also, if trussed, as in Fig 35; and the arch
itself with a full max load, would be strained less than 800 lbs per sq inch, at its
For a span of 200 ft, with the same proportion of rise, the transverse areas of
several arch and truss pieces should be increased 33 per cent; and for one of 100
they may be diminished the same. None of these dimensions are the result of
calculation. The dimensions just given will answer for *common travel*, for a span
200 ft; with a depth from out to out of chords, of 1/8 the span; panels 10 to 12 ft
Many such spans have been built with timbers of about ⅕ less transverse sect
and without counterbracing. The heads of the posts are notched about 2″ to 3″
the bottom of the upper chords; and are moreover tenoned into it some ins furth
with two wooden pins through the tenon; see *n*, Figs 36. Their feet are notch
both *into* and *upon* the lower chords, so as to leave the two chord-pieces *a a* a
about 2″ apart. Through these and the post pass two bolts of about 1″ to 1⅛ dia

Since the upper chord resists compression only, its pieces may come together w
a plain butt joint, *d*. To this may be added fishes *e d*, of stout plank, on the
of the chord, bolted through by 4 or 8 bolts.

The lower chords resist pull: and the pieces composing each lower chord
therefore be joined together somewhat as in Fig 37, &c. These pieces should be

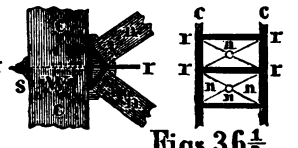
* *It will be borne in mind that our examples are not intended to illustrate perfectly proportio structures. None of them would endure strict criticism. There is more waste of timber in an a built with a uniform transverse section throughout, than in the straight upper chord of a Howe Pratt similarly built; for both must be proportioned to the greatest strain. This is at the mid the two last; but while that at the center of the arch is as great as in these, that at its feet greater.* See Example 2, Art 33, of Forces in Rigid Bodies, p 403.

ıd should never be jointed opposite to each other, but one opposite other; see Fig 39.
nerely cut to fit to the heads and feet of the posts, after these last their places; and usually have no other connection to them than at each end, for small bridges; or screw-bolts for large ones.
timbers composing the arches, butt full square against each other; a wooden dowel. The joints should occur at the posts, as shown are screw-bolted to the posts, as shown in the figs, by bolts of 1 to ıere the arches pass the lower chord, both are notched, and well The feet of the arches abut against cast-iron plates.
n rods (dotted in Fig 36) are used for assisting to support the road-ed as shown at *s s*; *m* being a strong block of wood slightly notched r arch-pieces. The rods are suspended by a washer and nut on top after passing down between the arches and chord, have a similar inverted, below the last; as shown at *g*.

rs not exceeding 14 ft clear span, may be 8 ins, by 15 ins deep; ore than about 2½ ft apart from center to center. Upon them and spiked stout string-pieces, say 12″ wide, by 9″ deep, to carry istribute the pressure of the load.

racing for diminishing lateral motion, can be used only under bridges; but in high ones it is introduced also at the top of the timber, these braces are about 4 to 6 inches thick; by 6 to 9 deep; ross between each two opposite panels of the two trusses. If the ıd has girders *r*, Figs 36, 36½, upon and well secured to the upper per lateral bracing may consist simply of 4 iron rods *n n*, passing s about midway of their depth; and having heads and washers on At the center of the cross the rods terminate in an adjusting-ring; 21½. In a bridge of 150 ft span, these rods need not exceed ¾ center panel, and 1½ at the end ones. If the bridge is high, and en at top, then cross-struts *r r*, Figs 36½, must be inserted pur-rod-bracing is used. If it is also used at the lower chords, the floor-e duty of these struts. Iron-bracing is not liable to catch fire from

node of lateral bracing. W, Figs 36½, resembles a Howe ts side. In it the diags of the cross are struts of timber; and the d rods. One of the struts is whole, with the exception of a slight ert side, at its center, for receiving tenons cut on the inner ends of ich compose the other diag. At the sides of the chords, the ends ıpon a ledge, (shown by) about 1½ ins wide, cast he cast-iron angle-block. sing through the chords ing tightened by means the diags firmly in place; eir shrinking a little in tightened up by the same



Figs 36½

ations of these methods cannot afford them space ıgle-block is as deep as a ss need not exceed ½ inch, in a large bridge. The dark triangle is It has holes for the passage of the rod *r r*.
d somewhat into the details of the Burr truss, because many of less applicable to others. A good light water-and-fire-proof ma-oofs; and one not corroded by the smoke from the engines, is yet a

ıgthening-scarfs, splices, or joints. The lower chords exposed to great pulling strains, require much care in connecting of the several pieces of which they are composed. There is much ling the strength of the joint-fastenings in common use for this nents on the subject are much needed. When only two pieces, as 38, are joined by any of the ordinary methods, it is probably not ı their possessing more than ⅓ of the tensile strength of a single ıl cross-section. When the chord, as in Fig 39, is composed of two *n n, made up of long* pieces, breaking joint with each other, as at *two parts may be* made somewhat stronger than either one of *itself. This is owing to* the opportunity afforded of connectin

them also by bolts *b b*, and packing-blocks, *c c*, of wood or iron, intermed joints *j j*, &c. By this means the strength of the entire chord may probal tically rendered equal to one-half of what it would be if solid. If the sists of 3 or 4 parallel parts, of long pieces, breaking joint, and connec same way, it will probably have about ⅔ of the strength of the solid. of course be taken that the serviceable area of the pieces shall not be red intermediate point, to less than it is at the joints.

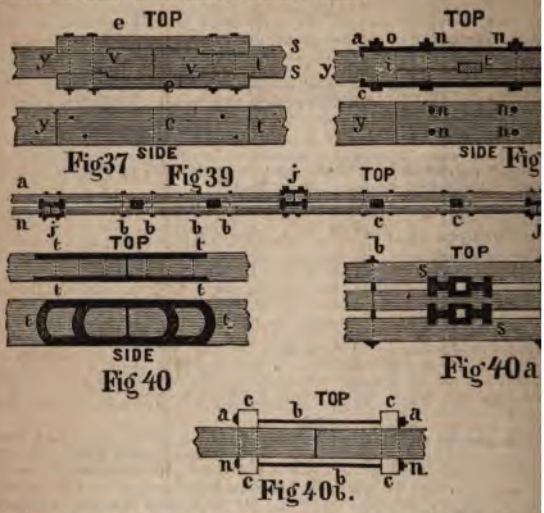


Fig 38 is a simple and efficient form of scarf. Its length *i i* may be a times the greatest transverse dimension of the beam. At the center is a hard wood, with a thickness equal to ⅓ that of the beam; a width of 2 or thickness; and a length just sufficient to reach entirely through the b beams are connected by 4 screw-bolts *n n*; or by 8 of them, if the length Plates of stout rolled iron, *a a*, *c c*, with their ends bent down into the occasionally added. These require bolts *o o*, beyond the ends *i i* of the sca bolts are not shown in the side view.

Fig 37 is another excellent joint with *splicing-blocks e e*, instead of the Fig 38. The indentations *v v*, may each be about ⅙ as deep as the beam is t length of each splice-block, about 6 times *s s*. From 4 to 8 screw-bolts, may require. Length of each indent about ¼ that of the block itself.

Fig 40 is a joint formed by two flat iron links or rings *t t*, let flush in bers, and retained in place by spikes. The iron may vary from ¼ to 1 inc ness; from 1 to 4 or 5 ins in width; and 2 to 6 ft in length; as occasion m

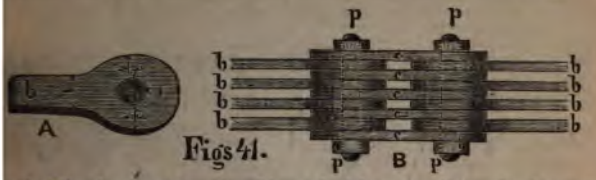
Fig 40 *b*, is a joint formed by two blocks *c c*, of hard wood, passing th timbers; and connected by bolts, *a a*, *n n*.

In Fig 40 *a*, *s s* are cast-iron packing-blocks, sometimes used instead of pla ones, at points *b b*, Fig 39, intermediate of the joints *j j*. The openings in of the blocks are needed only when vertical truss-rods have to pass thr points. At *e e*, of the same fig, is shown another form, much used in chord of two or more parallel strings. Both these are as deep as the chord; cross-sections, or end views shown in the fig, may be from 4 to 10 ins long wide; and from ½ to 1½ ins thick; according to size of bridge, &c.

Rem. In selecting hard wood for splicing-blocks, treenails, or for any *bridge, it is well to* remember that the oaks when in contact with the pine *the decay of the latter*; therefore, it is generally better to employ the bes *yellow pine heart* wood for such blocks, &c, or interpose sheet iron.*

* *The tendency of* some kinds of timber to produce rapid decay when brought in

bolting together pieces of timber to resist tension, it is very important to bear ind the principle shown in Figs 35, 36, (of iron pieces,) in Strength of Materials. the strain is greatest at the center of the chord, it is well in practice in large es, to bolt a stout plank of 3 to 6 ins thick, and as long as can conveniently be ned, on both sides of the chord; and reaching from the center each way.



Figs 41.

1. The lower chords of iron bridges usually consist of a number at links, or bars *b*, Figs 41; seen also in Figs 30 and 32; the broad sides being for the sake of stiffness. The consecutive sets of links are connected together own in the top view B; where *c c c* show the *connecting links;* through which the main links, pass the pins *p p*. At A is a side view of the end of a main link, ed into an eye for the pin or bolt *p*. In order that these eye-ends shall be every- e as strong as the body of the link, their width on each *side* of the pin-hole must as great as the depth of the link; and *back* of the pin-hole, the width must be l to the entire depth of the link. These proportionate dimensions are figured .

2. To find the diam of the wrought-iron (it should be of steel) *p*, we must bear in mind that it will give way by *shearing;* (see Art 7; and near of Art 44, of Strength of Materials.) Moreover, this shearing would take place each pin at twice as many places as there are main links perforated by it; nuch as each link would shear the pins across at two places; therefore, in Fig ch pin would be sheared at eight places in its length; and its diam must therefore ficient to enable it to withstand safely a shearing force equal to ⅛ part of the on the chord. An ordinary iron railroad bridge of 160 ft span, single track, loaded by a train, would have a strain of about 250 tons at the center of its chords; or about 125 tons on the chord of one truss. Suppose this chord to st of 4 parallel links, as in Fig B. Then the pin must be strong enough to a shearing strain of $\frac{125}{8}$ = say 16 tons, or about 36000 lbs. To be safe, it should eak with less than say six times as much, or 216000 lbs. Therefore, assuming hearing strength of wrought iron to average 45000 lbs per sq inch, which is ably near the truth, the *transverse* area of the pin should not be less than

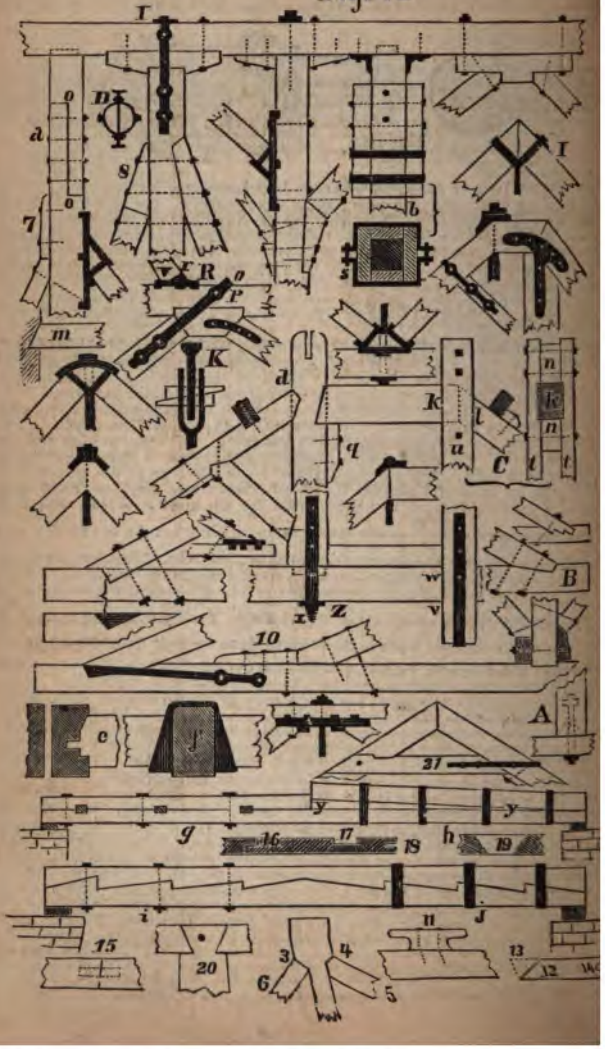
$\frac{\ldots}{\ldots}$ = nearly 5 sq ins; or a circle of 2½ ins diam. Such pins should be made of ; and a slight addition made to the diam to allow for wear from vibrations.

rt. 26. The following Figs 42, exhibit joints adapted to most of the cases that r in practice with wooden beams, &c. They need but little explanation. Fig *a* good mode of splicing a post; in doing which the line *o o* should never be in- d or sloped, but be made vert; otherwise, in case of shrinkage, or of great sure, the parts on each side of it tend to slide along each other, and thus bring eat strain upon the bolts. When greater strength is reqd, iron hoops may be , as at *b*, *h*, and *j*, instead of bolts. Fig *b*, a post spliced by 4 fishing pieces; ch may be fastened either by bolts, as in the upper part; or by hoops, as in the r. The hoops may be tightened by flanges and screws, as at *s*; or thin iron gs may be driven between them and the timbers, if necessary. Fig C shows a d strong arrangement for uniting a straining-beam *k*, a rafter *l*, and a queen-post by letting *k* and *l* abut against each other, and confining them between a double en-post *t t*; *u u* are two blocks through which the bolts pass. A similar arrange- t is equally good for uniting the tie-beam *w*, with the foot *v*, of the queens; with addition of a strap, as in the fig. Fig *e* is a method of framing one beam into ther, at right angles to it. **An iron stirrup,** as at *f*, may be used for the e purpose; and is stronger. Figs *g h*, *i j* are **built beams.** When a beam finder of great depth is required, if we obtain it by merely laying one beam flat

other blade, is a subject of great practical importance; but one which hitherto has received but attention. Black walnut and cypress are said to cause mutual rot within a year or two. Ob- tances of this kind should be reported to the leading professional journals.

upon another, we secure only as much strength as the two beams would have if separate. But if we prevent them from *sliding* on one another, by inserting transverse blocks or keys, as at *g*; or by indenting them into one another, as at *j*; and

Figs 42



) them firmly together to create friction; we obtain nearly the strength of the total depth; which strength is as the *square* of the depth.

built beam is increased by increasing its depth at its center, where it is most upper chords of a bridge. This may be done by adding the triangular strip *y y* ams.

hat the combined thicknesses of all the keys shall not be less than 1.4 times the girder; or when indents are used, as in *i j*, that their combined depths be at least r. If the girder *i j* be inverted, it will lose much of its strength.

ron may be placed at the joints of timbers when there is a great pressure; which red over the entire area of the joint; or cast iron may be used.

gle strap will not suffice, when it is necessary to draw the two timbers very n such cases, one end of each strap may, as at *x*, terminate as a screw; and after ross-bar Z, all may be tightened up by a nut at *x*. Or the principle of the pou- K, may be applied. Sometimes, as at A, the hole for the bolt is first bored; then side of the timber, and reaching to the bolt-hole, large enough to allow the screw This being done, the hole is refilled by a wooden plug, which holds the nut in rew-bolt is inserted, passing through the nut. By turning the screw the timbers ed together. Figs 21½, page 265, may be consulted also.

beams, joists, &c. are inserted into walls in the usual square manner, there is of being burnt in two, they may, in falling, overturn the wall. This may be the ends into the shape shown at *m*.

Fig R, has to bear a strain so great as to endanger its crushing the timber *p*, on sting like *v* may be used under it. The strap will pass around the back *r* of the l projections in the bottom being notched into the timber, will prevent the casting the oblique strain of the strap. The same may be used for oblique bolts, and ell as above it. When below, it may become necessary to bolt or spike the casting the timber. When the pull on a strap is at right angles to the timber, if there iece of plate-iron, instead of a casting, may be inserted between the strap and the the latter from being crushed or crippled; see I and I.

Expansion rollers, Fig 43, or planed iron **Slides;** or 44; or **suspension-links,** Figs 45, 46; must be provided when eeds about 50 ft; in order to allow the trusses to contract and expand anges of temperature, without undue strain upon some of its mem-

Fig 43

ows the general arrangement of roll- cylinders of cast iron or steel, from 3 d 1 to 4 ft long; planed smooth. From f these are connected together by a , *n n*; and one such frame is placed *e end* of the truss. The rollers rest laned cast bed-plate *o o*; bolted to the Under the end of the truss is a sim- which it rests on the rollers. Since 200 ft span will scarcely change its as 3 ins by extremes of temperature, ollers is but small. They are kept in cast along the side of the bed-plate. *s s*, so as completely to protect dust, rain, &c.

Flanges should also project

Fig 44

Fig 45

Fig 46

es a general idea of a ROCKER; and Fig 45, -LINK. U U in each fig is a side view of pper chord, through each end of which o, which sustains the entire weight of the ; and which is sustained by from 4 to 6 the case may be. In a railroad bridge of he Monongahela, the links are 3½ ft long; liam; and in others of the same size, over ivers, the rockers are a foot wide from *r* s wide transversely on the curved tread commodation of these several links and of the various bars *b b*, which constitute a Fink truss; the ends of the octagonal rds are widened out, as shown by Fig 46; of a longitudinal section of such an end. s, and bars, *b*, occupy the spaces *n n*, &c, partitions of the chord; and the pin *o o* em all, except when it is expedient to bars to the sides, or to the top of the se figs are intended merely to illustrate e, without regard to detail of construction. bridges of considerable size, such as the of 150 ft spans; and the Newark Dyke ; (both of them Warren girders,) and foregoing, by the ends of the upper precaution is taken with regard to expan- n, than merely to rest the ends of said *bly planed iron plates*, upon which they *everal American bridges.*

he weight of bridges of *signed by different* persons, va- *rom several* causes; such as the

form of truss; quality of iron; proportions of wrought and cast iron; adopted for safety, and for strength of materials; whether the roadway is chord, or the bottom one, &c, &c.

For a mere approx wt to be assumed as a preliminary in calculating and proportioning the parts of a bridge, or for forming some rude idea of the quantity we suggest the following purely empirical rule. It is for the wt per foot run of bridge, of *trusses* together, and their lateral bracing, for a single-track railway bridge. The wt of flooring, rails, &c., is not included.

Wt of only the two trusses together (and their lateral bracing) in tons per ft run of bridge. } If under 250 ft Span $= \frac{\text{Span}}{500} + \frac{\text{Sq rt of span}}{100}$. But if over 250 ft span $= \frac{\text{Sq}}{1}$

As a rude average, we may set down for spans not exceeding about 200 ft. cross flo about. 7'' × 15'', and about 2½ to 3 ft apart from center to center; clear bearing 14 ft; t substantial string-pieces of about 10'' × 12'', for supporting the rails; the rails thems plank pathway between the rails, all complete, at about .14 ton, or 314 lbs per ft of spa full floor of 3'' plank, 14 ft wide, about .2 ton, or 448 lbs. For greater spans, with the tr apart, increase this to .25 ton, up to 300 ft; .3 ton, to 400 ft; .35 ton, to 500 ft and .4 t as is done in the following table.

If trussed Phœnix beams, p 210; or Fairbairn beams, page 214; of Strength of Mate instead of wooden ones, the weight of roadway will not be seriously increased thereby.

When a bridge is to be roofed and weather-boarded, an addition must, of course, be weights.

Wooden bridges weigh about the same as iron ones of equal strength.

Table of approximate average weights per foot single track railroad bridges of iron, by above rule. (O

Clear span.	Wt of only the two trusses together; with their lateral bracing; per foot run.		Wt of trusses, roadway, &c, complete; per foot run.		Entire wt of bridge, and max load; per ft run.	Clear span.	Wt of only the two trusses together; with their lateral bracing; per foot run.		Wt of tru roadway complete foot ru
Feet.	Tons.	Lbs.	Tons.	Lbs.	Tons.	Feet.	Tons.	Lbs.	Tons.
5	.032	72	.232	520	4.23	120	.350	784	.550
7½	.042	94	.242	542	3.50	130	.374	838	.574
10	.052	116	.252	564	3.50	140	.398	891	.598
12½	.060	134	.260	582	3.50	150	.422	945	.622
15	.069	155	.269	603	2.94	160	.446	999	.646
17½	.077	172	.277	620	2.80	170	.470	1053	.670
20	.085	190	.285	638	2.79	180	.494	1107	.694
25	.100	224	.300	672	2.30	190	.518	1160	.718
30	.115	258	.315	706	2.00	200	.541	1212	.741
35	.129	289	.329	737	1.95	225	.600	1344	.850
40	·143	320	.343	768	1.90	250	.658	1474	.908
45	.157	352	.357	800	1.85	275	.775	1736	1.025
50	.171	383	.371	831	1.80	300	.904	2025	1.154
60	.197	441	.397	889	1.80	350	1.194	2674	1.494
70	.224	502	.424	950	1.75	400	1.529	3425	1.829
80	.249	558	.449	1006	1.70	450	1.908	4274	2.258
90	.275	616	.475	1064	1.70	500	2.332	5224	2.682
100	.300	672	.500	1120	1.70	550	2.801	6274	3.201
110	.325	728	.525	1170	1.70	600	3.314	7423	3.714

The Fink truss, when the roadway is on top, requir chord, and makes a lighter bridge than any of the beam-systems. The fol the weights per ft run, of some single-track Fink railroad bridges of that kin first-class builders, Smith & Latrobe, of Baltimore. These gentlemen a coefficients of safety; and use only iron of superior quality. The wts i roadway; which, however, is open, and consequently considerably lighte assumed .18 ton per ft run. See Art 31.

Clear span 50 ft. Wt per ft .28 ton. | Clear span 150 ft. Wt per ft .
" " 100 " " " .38 " | " " 210 " " " .

The contemplated Fink bridge across the Missouri at St. Charles, desi Shaler Smith, bottom road, single track, with spans of 325 ft, is estimate (the two trusses alone) 1700 lbs, or .76 ton per ft of span; the roadway 38 *ton; total* .93 ton per ft run. The estimate for 400 ft spans, is but 1.43 t *Two trusses of the* Newark Dyke bridge, England, Warren girder, 240½ ft s *1.02 tons. The single-track* tubes of the Victoria bridge at Montreal, 244 ft *tons: the 330 ft span*, 2 tons. The single-track Britannia tube, England, *3.43 tons. Two trusses* alone, of the Penn'a Central R R, at Philad'a,

m,) by Mr Linville, .52 ton. The fine 320 ft span across the Ohio at Steu-
tt,) also by Mr Linville, 1.6 tons per ft. All these are of iron; single track.
imensions for short bridge trusses, see Art 31, p 304.

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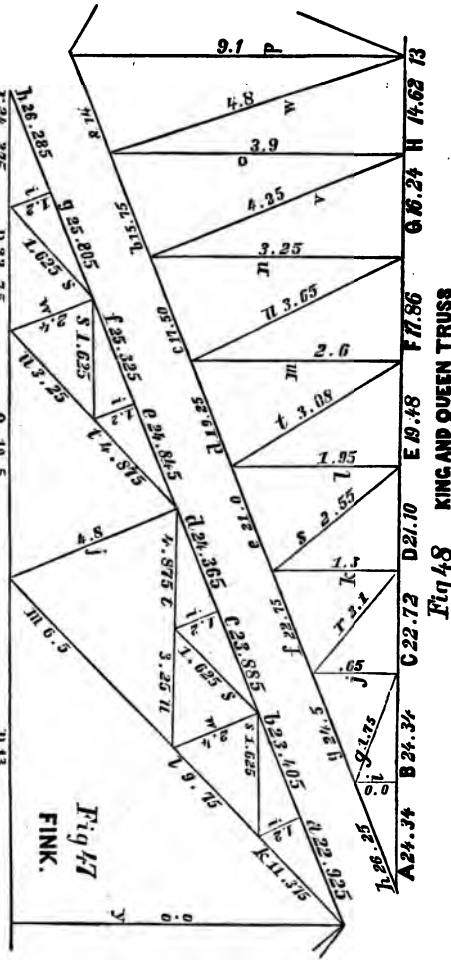


of such an engine is, however, concentrated within about 20 ft length of road; a
produce in a 20 ft span a load of 2¼ tons per ft, independently of the wt of the bri
allowance for momentum.
m of the Chelsea bridge, London, packed picked men upon the platform of a w
vault of 84 lbs per sq ft. Mr Nash, architect of Buckingham Palace, experim
fire-proof floors for that building, *wedged* men together as closely as they cou

60 ft span. But in a common bridge also, the greatest load per ft run, on a very short spa greater than in a long one; as in the case of two wheels of a truck hauling a large block of s and this must be taken into consideration in building such.

It must also be remembered that each transverse floor-girder must bear at the weight resting upon two wheels; no matter how close together the gird be placed. If they are farther apart than the dist between two axles of a they will have to bear more than the load on one pair of wheels.

The allowance for safety in a truss bridge.

As the result of a long-continued series of deflections applied to an experimental plate-b of 20 ft span, Mr Fairbairn concludes that a bridge subject to 100 deflections per day, ess that produced by ⅓ of its extraneous breaking load, would probably break down in about while, with 100 daily deflections equal to that arising from but ¼ of its breaking load, it fully 300 years. Mr Fairbairn's experimental bridge weighed but little in comparison to t ing strains to which it was subjected. Had the reverse been the case, we conceive that a might have ensued. We are of the opinion that a bridge should not have a safety of *less* its max extraneous load, and its own weight, combined: nor do we see any use in ex A well-proportioned bridge should not deflect *more* than $\frac{1}{1200}$ part of its span, under its It will not deflect more than $\frac{1}{1500}$ or $\frac{1}{2500}$ part, if due care be taken in fitting its j throughout. See on the Camber of bridge trusses, Art 30, p 302.

Art. 29. Remarks on king and queen; and on Fink tr for roofs. The following comparison is founded upon total spans, or of truss, of 154 ft. Rise 30.8 ft; or $\frac{1}{5}$ of the total span. Trusses 7 ft apart f ter to center. Each rafter 83 ft long. Total load, including the truss i lbs per sq ft of roof; or 20.8 tons to each truss. There are seventeen point port in each truss; consequently a full panel-load (Art 11) is $\frac{20.8}{16} = 1.3$ tons, as shown, one-half of each, in Figs 47 and 48. The strain in tons (c as if all the weight of truss and load were on the rafters) is marked member. The assumed coefficient of safety for ties is 3. Iron is supposed t that will not break with a less pull than 20 tons per sq inch; the assumed sa able pull being therefore here taken at $\frac{20}{3} = 6\frac{2}{3}$ tons per sq inch. The safe *along* the rafters is taken at $3\frac{1}{2}$ tons per sq inch. The struts are assumed to be cylindrical tubes, with an outer diam equal to $\frac{1}{40}$ of their length; and thickness as will give them a metal area of 1 sq inch for each 2 tons of stra rafters are in the present case supposed to be 9-inch rolled Phœnix beams; 7 transverse area; weighing 25 lbs per foot run. The ends of all ties are s to be enlarged, or *upset;* so that the cutting of the screw-threads shall not their effective area. The purlins are supposed to be at or near the "point port," so as to produce no cross-strains on the rafters.

Table I. Weight of the Fink truss, of which Fig 47 one-half. See also Figs, p 261.

Length 154 ft. Rise $\frac{1}{5}$ length. Trusses 7 ft apart. Load 40 lbs per sq ft of roof; includin See Note, p 260. (Original.)

Name of part.		Number of parts.	Area of each part. sq ins.	Lbs. per foot run of each part.	Total weight of all the parts. Lbs.
Rafter		2	7.50	25.	4150
Main tie	n	2	1.95	6.5	430
	o	2	2.93	9.77	450
	p	2	3.41	11.37	272
	r	2	3.66	12.22	294
Secondary ties	u	4	0.50	1.67	78
	t	4	0.73	2.44	118
	m	2	0.98	3.27	150
	l	2	1.47	4.89	118
	k	2	1.71	5.70	136
	s	8	0.25	0.83	76
Struts	j	2	2.40	8.0	272
	w	4	1.20	4.0	136
	i	8	0.60	2.0	68
Center vertical y		1	.....	.....	40
Joint and splicing-pieces, nuts, &c, &c... say		.....	.....	.....	400
Shoes at ends of rafters, say		.....	.....	.....	400
Wt of purlins not included. Total wt of truss		.....	.....	.....	=7588

thly stand upon an area of 20 ft diam; the last man being lowered down from abov *bers. Result, 120 lbs per sq ft.* See foot-note, p 595.

ncluding *the trusses*, the *area*, and wt *per foot run* of each part, as well as ı upon it, when the trusses are at the same distance apart, with a rise of ıry directly *as the spans;* but the *total* wts, as the *squares* of the spans. is easy to deduce from the table the areas reqd for smaller spans. The r small spans are frequently made of round iron rods from 1¼ to 1¾ ins of ordinary flat bars. Tubes with the same area of metal, would be better. es also of different spans, and rise of ⅕ the span, 7 ft apart, in which the ıd struts are of wood, with ties of iron, the strains may be deduced quite om those in Figs 47 and 48. They will, however, be somewhat greater, ooden struts, not being hollow like our assumed iron ones, must be heavier latter to prevent bending. The weight of the load, however, is generally so ıater than that of the truss, that this consideration of the strut is not very so that a roof partly of wood may be assumed in practice to weigh, *together ıad*, but little more than an iron one; and the strains on the several parts ırly the same in both cases.

2. Weight of the king and queen truss, of which Fig 48 shows one-half.

See also Fig 14, p 257.

ı ft. Rise ⅕ length. Trusses 7 ft apart. Load 40 lbs per sq ft of roof; including truss. p 260. (Original.)

Name of part.		Number of parts.	Area of each part. sq. ins.	Lbs. per foot run of each part.	Total weight of all the parts. Lbs.	
.....		2	7.5	25	4150	4150.00
.....	H	2	2.2	7.33	146 70	
	G	2	2.44	8.14	162.80	
	F	2	2.68	8.94	178.80	
	E	2	2.92	9.74	194.80	1606.30
	D	2	3.16	10.54	210.80	
	C	2	3.41	11.34	226.80	
	B	2	3.65	12.14	242.80	
	A	2	3.65	12.14	242.80	
.....	*i*	2	.0	.0	.0	
	j	2	.1	.33	5.34	
	k	2	.2	.67	16.00	
	l	2	.3	1.00	32.00	298.66
	m	2	.4	1.33	53.33	
	n	2	.5	1.67	80.00	
	o	2	.6	2.00	112.00	
tical....	*p*	1	1.4	4.67	150.00	150.00
.....	*q*	2	.88	2.92	64.20	
	r	2	1.05	3.50	91.00	
	s	2	1.28	4.25	136.00	
	t	2	1.55	5.17	196.37	1587.50
	u	2	1.82	6.08	267.63	
	v	2	2.12	7.08	368.30	
	w	2	2.40	8.00	464.00	
ıplicing-pieces, nuts, &c, &c .. say		.....	.....	.....	400.00	400.00
ıds of rafters..... say		.....	.....	.....	400.00	400.00
Total weight of truss = ins not included.		.....	.....	.....	8592.46	8592.46

, the wt of the king and queen truss in this instance is equal to $\frac{8592.46}{7588}$ times (say 1⅛ times) that of the equally strong Fink; or the Fink is about ghter than the K and Q. Theoretically the diff would be less, because the rafters and Q truss being so much less strained at top than at foot, may be diminished heir upper ends, instead of being proportioned throughout with reference to strain at their feet. If the theoretical diminution toward the tops of the were made in both cases, the wts of the two forms of truss would be nearly But in practice, on the score of inconvenience, this is rarely done in roofs rate span: say less than about 100 ft. No such diminution, or but very slight, e admissible even theoretically, when the purlins are not placed at the points nt only. *With the same total load per sq ft*, including the trusses, the writer s *that when similar trusses* of different spans are placed at the same di *may in practice consider* their *total* wts to vary directly as the squares *(lengths.) And their wts per foot of span*, (length.) as well as the tr

verse areas; the wts per *foot run* of each individual member; and the st
member; to vary *directly*, or *in the same proportion as the spans themselv*
when the dist apart of the trusses is 7 ft from center to center; the
span; assumed load, including the wt of the trusses themselves, 40 lbs
roof covering; and the various parts proportioned for the several strain
assumed in Tables 1 and 2; the wt of a properly constructed Fink t
approximately as follows: (See NOTE p 260.)

$$\textit{Total wt in lbs of a Fink roof-truss} = \frac{\textit{square of span in ft}}{3.1};$$

and the wt in lbs of the truss per foot of span $= \dfrac{\textit{span in feet}}{3.1}$.

And the total wt of a K and Q truss, will be about $\frac{1}{7}$ part more; or $\frac{\textit{square o}}{}$

Or per foot of span, $= \dfrac{\textit{span in feet}}{2.7}$.

These rules give $\dfrac{23716}{3.1} = 7650$ lbs, for the foregoing Fink; and $\dfrac{23716}{2.7}$ for the K and Q truss. From these rules we have drawn up the followi

Table 3. Approximate weights of roof-trusses of t system. (Original.)

Rise $\frac{1}{5}$ span. Trusses 7 ft apart. Load 40 lbs per sq ft of roof, including tr

Total Span.	Total wt of a Truss.	Wt per ft. of Span.	Wt per sq ft of ground covered.	Total Span.	Total wt of a Truss.	Wt per ft. of Span.
Feet.	Lbs.	Lbs.	Lbs.	Feet.	Lbs.	Lbs.
20	129	6.46	.92	100	3228	32.3
25	202	8.08	1.15	105	3557	33.9
30	290	9.67	1.38	110	3904	35.5
35	396	11.3	1.61	115	4267	37.1
40	516	12.9	1.84	120	4640	38.7
45	654	14.5	2.07	125	5041	40.4
50	807	16.1	2.30	130	5452	42.0
55	976	17.8	2.53	135	5880	43.6
60	1160	19.4	2.76	140	6336	45.2
65	1363	21.0	2.99	145	6782	46.8
70	1584	22.6	3.22	150	7260	48.4
75	1815	24.2	3.45	155	7750	50.0
80	2064	25.8	3.68	160	8256	51.6
85	2331	27.5	3.91	165	8782	53.3
90	2616	29.1	4.14	170	9324	54.9
95	2912	30.7	4.37	175	9879	56.5

For king and queen trusses add $\frac{1}{7}$ part to the tabular wts; when the
as usual of the same size throughout. See Note, p 260.

The wts in the 4th column will remain nearly the same, wh
be the dist apart. For if this be increased say to 14 ft, each truss will s
as many sq ft of roof; and must itself be at least twice as strong and hea
to do so. We say "*at least*," because if the dist apart is increased, the wt
lins will generally increase more rapidly than said dist. Thus, if the dist
the purlins will not only be doubled in length, which alone would doub
but they must also be deeper. In practice, however, long purlins are
vented from becoming very heavy, by trussing them, as at 7, Figs 21½, 1

The cost of trusses alone for iron roofs, generally varies bet
12 cts per lb put up; depending on the price of iron and labor. The putti
from ¾ to 1¼ cts per lb. With trusses 7 ft apart, iron purlins will weigh
per sq ft of ground covered by the roof. Therefore to any wt in the 4th c
Mult the sum by from 10 to 12 cts, for the cost of trusses and purlins alone
ground. Add for covering with tin or slate on boards, say 15 to 20 cts per
corrugated iron on the bare purlins say 35 cts; or if on boards, 38 to 40 c

Bridge trusses, at shop, wrought iron parts 8 to 11 cts per lb; cast irc

REM. 1. **As to the proper total weight, or load, p**
of roof, (including snow and wind.) that should be assumed to be sustained by the trus
differ considerably. The French appear to consider 30 lbs as sufficient; while the E
Since roofs are not subject to violent vibrations like bridges, they do not require as big
of safety; this should not, however, in our opinion, be taken at less than 3; and w
sufficient. The load is evidently influenced by the character of the roof-covering.
limits, for spans not exceeding about 75 ft, and with trusses 7 ft apart, the total load
ing the truss itself, purlins, &c, complete, may be safely taken as follows:

Table 4.

Span 75 ft or less.		Wind and Snow.*	Total.
Roof covered with corrugated iron, unboarded,†	8 lbs.	20 lbs.	28 lbs.
" " " If plastered below the rafters,	18 "	20 "	38 "
" " " corrugated iron, on boards,	11 "	20 "	31 "
If plastered below the rafters,	21 "	20 "	41 "
" " " slate, unboarded, on laths,	13 "	20 "	33 "
" " " " on boards, 1¼ ins thick,	16 "	20 "	35 "
" " " " if plastered below the rafters,	26 "	20 "	46 "
" " " shingles on laths,	10 "	20 "	30 "
If plastered below rafters or below tie beam,	20 "	20 "	40 "

For spans from 75 to 150 ft, it will suffice to add 4 lbs to each of these totals.

Example of use of foregoing tables. A Fink roof 60 feet span; rise $\frac{1}{5}$; trusses 14 ft apart; and to be covered with slate, on boards 1¼ inch thick. Here we see at once from Table 3 that at 7 ft apart, its wt would be about 1160 lbs; therefore, at 14 ft apart, it would be 2320 lbs. But our table is for 40 lbs per sq ft of roof: while, for slate on boards, 35 lbs, or ⅛ part less, is sufficient. Therefore, we may reduce the weight of the truss ⅛ part, making it only 2030 lbs.

Ex. 2. Roof as before, 60 span; *trusses only 7 ft apart.* Turn to Table 1, where the areas are given for a total length or span of 154 ft. But 60 ft is the $\frac{60}{154}$ = say the part of 154 ft; therefore, the areas, and the wts *per foot run* of each member of the 60 ft span, will be .4 of those of the 154 ft one. Thus, the area of a rafter will be 7.5 × .4 = 3. sq ins; which corresponds with a rolled T iron of 3 × 3½ ins, and ¼ inch average thickness. Its wt per foot run will be 25 × .4 = 10 lbs. The area of the part *n* of the main tie will be 1.95 × .4 = .78 sq inch, which we see at once from a table of circular areas, is equal to a round rod very nearly 1 inch diam. Its wt per ft run = 6.5 × .4 = 2.6 lbs; and so with all the other members. But the *total* wts will be as the *squares* of the span. The square of 154 is 23716; and that of 60 is 3600. And $\frac{3600}{23716}$ = .152; therefore, the total wts will be .152 of those in Table 1. Thus, the two rafters will weigh 4150 × .152 = 631 lbs. The main tie, 1446 × .152 = 219 lbs, &c. Lastly, if for 35 lbs per sq ft, reduce each area and wt ⅛ part.

Since the rafters are generally made of T or I iron, a pattern precisely adapted to the calculated values, will not always be procurable; and in that case we may either take the nearest one in excess; or change the dist apart of the truss to suit the pattern on hand. Owing to the variety of modes of arranging the details of the junctions, &c, an exact coincidence between the calculated and the actual wts is not to be expected; but we suspect that in properly proportioned roofs, the discrepancy will rarely be found to vary more than about 5 per ct from the results of our rules.

It might be supposed that with iron of a tensile quality considerably higher than our assumption of 20 tons per sq inch; as say of 25 to 30 tons, the truss might be made *much* lighter. But this is not the case; because the superiority would affect the ties only; inasmuch as the compressive strength of iron does not increase with its tensile strength; but to a certain extent rather the reverse. Now, by Table 1, it appears that the ties in a Fink roof-truss, constitute less than $\frac{3}{10}$ of its entire wt. Therefore, iron of even 30 tons, would reduce the weight of the truss less than ⅓ part of $\frac{3}{10}$ part; or $\frac{1}{10}$; and 25 ton iron, about $\frac{1}{20}$ part.

Short spans need not have as many subdivisions, or "points of support," as a large one; and this will lessen the number of parts of the truss: but inasmuch as the remaining parts will require to be proportionally stronger, this consideration will not materially affect the wts. While on this subject, we will remark that too few points of support are probably used at times; owing to either an under-valuation, or an ignorance of the effect of the transverse strains produced by the load on the parts of the rafter between said points. These parts must be regarded as so many separate *beams* supported at both ends: or rather, as *firmly fixed* at both ends, when the pieces composing a rafter are, as usual, strongly connected together; in which case the beam is about twice as strong as when merely supported. If the separate parts be trussed, like the purlin at 7, Figs 21½, to neutralize this transverse action, it must be remembered that additional *compression* will be thereby produced lengthwise along the rafter. The best practice is, as far as practicable, to increase the number of points of support, so that the purlins may rest upon them alone, or near them; and thus relieve the rafters entirely, or in part, from transverse strain.

Rem. 2. **As to the effect produced on the weight of a truss, by changing its rise,** no short correct rule can be laid down. Although as a roof becomes flatter, its area becomes less, so that each truss has less total wt of roof-covering, snow, and wind, to sustain, still the strains on most of its members become greater; requiring greater wt of truss. To find this increase with accuracy, it is

* See Snow and Wind, p 519, 520.

† The corrugated iron itself will weigh from 1½ to 2 lbs per sq ft on the roof. If not plastered underneath, the condensed moisture of the air, especially from crowded rooms, will fall from the iron into the rooms below. Mere boarding will not prevent this, even if tongued and grooved, unless the circulation of air against the under side of the iron is effectually cut off.

necessary to make a diagram, and perform all the calculations. The strains on a Fink rafter become more nearly uniform throughout its length, as the pitch of the roof becomes less; while, with a rise of $\frac{1}{8}$ span, the strain at its foot is about $1\frac{1}{3}$ times that at its head. On the contrary, the strains on its struts remain *nearly the same in amount* for all ordinary rises.

In the king and queen truss the strains at the heads and feet of the rafters retain the same *proportions* to each other, at all rises; the strains on the verticals become *less* as the roof becomes flatter; while those on the obliques vary according to their several obliquities. Under these irregularities, which affect the K and Q, much more than the Fink, we can do nothing more than say that when it is merely wished at the moment to form a rough idea of the effect of changing the rise, we may assume the weight of a *Fink* truss to increase about in the *same proportion* as we diminish the rise; or to diminish as we increase the rise. Thus, if we increase the rise of the roof in Table I, one-fourth part, so as to make it equal to .25 or $\frac{1}{4}$ of the span, instead of .2 or $\frac{1}{5}$ of the span, we may diminish its wt $\frac{1}{4}$ part; making it about 6000 lbs, instead of 8000. Or if we reduce the rise from $\frac{1}{5}$ to $\frac{1}{10}$, making it only half as great, we shall double its weight, making it 16000 lbs; as rude approximations.

Art. 30. On the camber of truss bridges. In practice, the upper and lower chords of bridges are not made perfectly straight, but are curved slightly upward; and this curve is called the **camber** of the truss or bridge. Its object is to prevent the truss from bending down below a hor line when heavily loaded. A cambered chord is of course longer than a straight line uniting its ends; but in practice the camber is so small that this diff is inappreciable, and may be entirely neglected. But when the chords are cambered, (see *y s* and *c d*, Fig 51,) they become concentric arcs of two large circles, of which the center is at *t*; and the upper one plainly becomes longer than the lower, to an extent which, although much exaggerated in our fig, cannot be overlooked in practice. The verticals, instead of remaining truly vert, become portions of radii of the aforesaid large circles; and although their lengths remain the same, yet their tops become a little farther apart than their feet; and this renders it necessary to lengthen the obliques or diags a trifle. Therefore, we must find how great is this increase of length of the upper chord beyond the lower one; and divide it equally among all the panels, along said chord; otherwise the several parts of the truss will not fit accurately together.

1st. To find the amount of camber of the lower chord. Divide the span in feet, (measured from center to center of the outer panel-points,)* by 50. The quot will be a sufficient camber, in inches; as shown in the following

Table of cambers for bridge trusses.

Span. Feet.	Camber. Ins.	Span. Feet.	Camber. Ins.	Span. Feet.	Camber. Ins.
25	0.5	100	2.0	250	5.0
50	1.0	150	3.0	300	6.0
75	1.5	200	4.0	350	7.0

It is by no means necessary to adhere strictly to this rule; and the camber by experienced builders is frequently less than the rule gives.

2d. To find the increase of length in the upper chord, beyond the lower one, having the span; the depth of truss; and the camber; (all in feet, or all in ins.)

With any camber not exceeding $\frac{1}{50}$ of the span; (which, however, is about 7 times as great as is usually given to trusses;) mult together the depth of truss, the camber, and the number 8; div the prod by the span. The quot will be the increase, in ft, or in ins, as the case may be. Or as a formula,

$$\text{Increase in ft, or in ins,} = \frac{\text{depth} \times \text{camber} \times 8}{\text{span,}} \quad \text{all in feet; or all in inches.}$$

This rule may be considered practically perfect with any camber not exceeding $\frac{1}{50}$ of the span. Based upon this principle, we have prepared the following table, which may be used instead of making the above calculations.

* A PANEL-POINT is any point *c*, Fig 49, in either chord, at which the axes (dotted) of a chord, of a vert, and of an oblique, meet. For making calculations of strains, or of dimensions, &c, of trusses, the measurements are taken at these points; and frequently from diagrams which show these dotted lines only, and hence called SKELETON DIAGRAMS, and skeleton measurements. Thus, the depth of a truss is measured from *c* on the lower chord, *to* a corresponding point, vertically above it, on the upper chord.

r finding increase of length of upper chord beyond lower one.

Mult Camber by	Depth of Truss.	Mult Camber by	Depth of Truss.	Mult Camber by	Depth of Truss.	Mult Camber by
2.00	⅛ span.	1.00	1-12 span.	.666	1-16 span.	.500
1.60	1-9 "	.888	1-13 "	.614	1 17 "	.470
1.33	1-10 "	.800	1-14 "	.571	1-18 "	.444
1.15	1-11 "	.727	1-15 "	.533	1-20 "	.400

y much longer is the upper chord than the lower one, when the depth of $\frac{1}{7}$ of the span; and the camber 5 ins? Here in the table, and opposite $\frac{1}{7}$ nd the multiplier 1.15. Therefore, 5 ins $\times$ 1.15 = 5.75 ins, Ans. If the ay 8 panels, then $\frac{5.75}{8} = .72$ inch of this increase must be given to each g the upper chord.

ngth of a diag. or oblique, *b c*, Fig 50, may readily be found *a s n c* in this fig represent a panel when there ber; then *o b n c* will represent a panel when camber; and *o a* and *s b* together are the the increased length of upper chord given to ; but to an exaggerated scale. Now, to find e the right-angled triangle *a b c*, in which we ide *a c*, (the depth of truss;) and the side *a b*, he panel width *c n* on the lower chord; added *half* the portion of the increased length of rd given to one panel.) Hence, we have only each of those two sides; add the two squares and take the sq rt of the sum. This sq rt is *b c*. Span 200 ft. Height (*a c*) of truss ⅛ of the ft, or 300 ins. Camber 5 ins; 10 panels each 0 ins, (*c n*,) measured on the lower chord or , the height being ⅛ of the span, the increase f upper chord will be equal to the camber, 5 ins; and this divided among

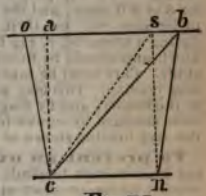


Fig 50

will be $\frac{5}{10} = .5$ inch to each panel; or *o b* will be .5 inch longer than *c n*; d *s b* will each be .25 inch. Hence, in the right-angled triangle *a b c*, we = 240.25 ins; and *a c* = 300 ins. Hence,

$$\sqrt{a\,b^2 + a\,c^2} = \sqrt{57720.0625 + 90000} = \sqrt{147720.0625} = 384.344 \text{ ins};$$

44 ins. Without any camber, *b c* would be in the position *s c*, which is 32 long; or .157 of an inch (about ⅙ inch) shorter than *b c*.

o this extent would prove seriously inconvenient if the oblique were a cast-iron strut ly planed ends, intended to fit closely between planed bearings at the chords; or a bar d hole at each end for fitting over pins whose position was fixed and unalterable. In as when the obliques are merely rods with screw-ends, it is only necessary to be sure *long enough;* because their exact length can then be adjusted when put into place, by nuts on their ends. So also when the obliques or other pieces are flat bars intended to be rted to the sides of the chords; for the final rivet-holes may be made when the pieces ually fitted in place. In the case of wooden obliques, &c, if too long, they can readily be he saw or chisel.

e panels are all of one size, as is generally the case, it is usual for builders e of them full size on a board platform or floor, to guide in fitting the ther.

ising a truss, or in other words, when putting its parts together in er position on the abutments and piers, a kind of scaffolding or *false-works*, be erected for sustaining the parts until they are joined together so as to complete self-sustaining truss. Upon the false works the bottom chords id as nearly level as may be; and the top chords are then raised upon tem- pports which foot upon the one that carries the lower chord. The upper at first placed a few inches higher than their final position, or than the t of the truss, in order that the obliques and verts may be readily slipped . After this is done, the top chords are gradually let down until all rests *lower chords. The* screws are then gradually tightened to bring all the *the joints into their* proper contact; and by this operation (the upper *supposed to have the* increased length given by the foregoing rule) th

camber, as it were, forms itself; and lifts the lower chords clear off from their fo works; leaving the truss resting only upon the abuts or piers, as the case may be.

Fig 51

In the case of a draw bridge, *g b c s*, Fig 51, of two leav *g b*, *c s*, revolving on a central pier the short upper chord *b c*, immediate over the pier, must be made so mu shorter than the lower one *o y*, as allow for *one-half* of the increa length of *each* cambered upper ch *a b*, and *c d*; or in other words, must be shorter than *o y*, by an amou equal to the *entire* increase of *one* cam bered upper chord. If this be neglecte the outer ends *g* and *s* of the leaves w droop or sag down when the draw opened; and it will be difficult to sh it again.*

Art. 31. For very short spans, the rolled Phœnix beam, pag 210, answers every purpose. A single 20 lb, 7-inch beam under each rail, will suffi for 3 or 4 ft span; and the 52 lb, 15-inch one, for 8 to 10 ft. *Two* 52 lb, 15-inch on under *each* rail, for 15 ft. By employing a greater number of beams, or by intro ducing a truss-rod, *r*, Fig 52, the spans may be increased; or lighter beams be use The beams should be fitted at their ends into wide cast-iron shoes, well bolted to th abuts. Care must be taken to insure lateral stability, by means of hor cross-ties an diag bracing. This may generally be secured by notching and bolting the cross-ties to the beams; and by diag rods passing through the chords; and having their inne ends confined by screws to such a ring as is shown at 14, Figs 21½, page 265; or by the diag bracing shown at W, Fig 36½, p 291.

To prevent an overturning tendency in a whole truss when it is not high enough to admit of being horizontally braced overhead, we may introduce wooden knees, or short straight struts or ties, of either wood or iron; which may foot upon the cross-girders of the floor; and head against either some of the wi members, or the upper chord. These braces or ties may be placed either between the two trusses of a span; or outside of them; or both. When outside, some of th floor-girders may be lengthened out a few feet beyond the lower chord, for receiving the feet of the braces or ties.

See Art 41, of Strength of Materials, for other iron beam bridges of still greater spans.

Or, **a single beam of wood under each rail,**† and firmly brace against lateral motion, will suffice. Assuming the weight of entire bridge and load at two tons per foot, the following dimensions may be used:

Span in Ft.	Size of Beam.	Span in Ft.	Size of Beam.
5	8 × 10 ins.	15	12 × 18
7½	9 × 12 "	17½	13 × 20
10	10 × 14 "	20	14 × 22
12½	11 × 16 "	22½	16 × 24

The greatest dimension to be the depth. The ends should be well bolted down t

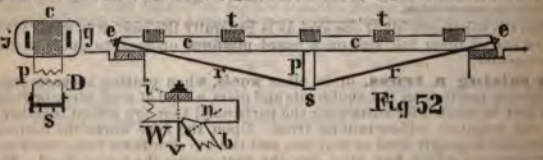


Fig 52

* The strains upon a drawbridge are very complicated; and the designing and construction of su *bridges, when of large spans*, should be intrusted to experts only. When such a bridge is opened, its ou *ends, being no longer supported by the abutments*, will sag a little, owing to the extension produc *by its own weight. This does not necessarily imply weakness*, because, as is well known, materia *lengthen or shorten under strains that are perfectly safe*, or far within their elastic limits.

† If *single beams of sufficient depth cannot be procured*, built-beams may be used; see g, and *Figs 42, p 294.*

. These are long stout sticks of timber, from 10 to 15 ins square, (accord- span.) laid across the abuts at the bridge-seat, for the chords to rest on. 31. Frequently two are used at each abut, even in small spans; and we ut one, under railroad spans of 150 feet. Large spans may require three They are not necessarily placed in contact with each other; but may be part, if required.

ans of about 15 to 30 ft, we may use somewhat lighter beams; and truss n as in Fig 52, by an iron bar *e s e*; and a center post *p*. In this case the imensions will answer; the total deflection of the rod being ⅛ of the The screw-ends of the bars are supposed to be upset; but the areas are e body of the rods.

For each beam.

a.	Beam. Ins.	Section of Rod. Sq Ins.	Section of Post. Sq Ins.
	12 × 15	3⅝	25
	13 × 17	4¾	33
	14 × 18	6	42
	15 × 20	7	50

r to have two rods instead of one under each beam; each rod being of half here given; and the two placed several ins apart. This affords a better the post. The ends of the beam should be at right angles to the direction and be provided with ample washers *e e*, of wood or iron, for distributing e from the rod, over the whole area of the ends. The ends of these wash- end a few ins each way beyond the sides of the beam, as shown on a larger This allows the rods to be outside of the beam; instead of requiring holes in the latter, for passing the rods through them. They may be nearer the foot of the post.

f the post may be tenoned into the bottom of the beam; and be further united to it by To prevent the foot from being worn by the rods, it should be shod with iron. A cast- at *s*, may be bolted to it; having ribs for keeping the bars in place. Or a stout wrought- y be well secured to it. In either case the rod at *s* should be so united to the shoe as to dency in the foot to slide toward *r* or *r*, under the vibration of passing loads. Perhaps ost conveniently done by making each rod *e s e*, in two separate lengths, *r*, *r*; and by lower ends to the shoe at *s* by hooks and eyes; or by eyes and bolts, &c. Various in use for the heads and feet of the posts of large spans; but we cannot here treat upon pertain more to the professional bridge-builder.

of trussing is also well adapted to long floor beams; and has been used in long oblique ; as well as in long stretches of chords from one point of support to another. Fig 2 is a trussed in more than one direction.

llowing dimensions for single-track Fink bridges.

s and posts of wood; and iron suspension bars; are on the assumption bars deflect ⅛ of the span; that the road is on top; that the bars shall ned more than 10000 lbs, or 4½ tons per sq inch, under a weight of bridge mounting in all to two tons per running foot. Assumed wt on each el of engine, 5 tons. Screw ends upset.

ons for one truss only, of a single-track Fink bridge.

are in feet; the other dimensions are square inches of cross-section of er.

Areas in sq ins.				Areas in sq ins.						
1st Rod.	2d Rod.	3d Rod.	1st Post.	Span. Ft.	Chord.	1st Rod.	2d Rod.	3d Rod.	4th Rod.	1st Post.
8¾	2½	...	56	90	470	21	5¾	1¾	...	146
9½	2½	...	64	100	505	23¾	6¼	2	...	169
10½	2⅞	...	72	125	580	29¼	8	2½	1¼	225
11¾	3¼	...	81	150	650	35	9½	3	1½	280
14	3¾	1¼	95	175	730	41	11	3½	1½	335
16½	4½	1¾	110	200	800	46¾	12½	4	1¾	390
18¾	5	1¾	126							

ven the area of the 1st post only. For that of the 2d, we may take ⅔ of the first; for he second; and for the 4th, ⅔ of the third; without pretending to any great accuracy. may be flat, square, or round, so that the proper area be maintained. It will usually *e them flat.*

***amed total* weight of 2 tons per foot of span**, for bridge *ther, up to spans of 200 ft*, is greater than is usually adopted. Still we

would recommend not to diminish the areas in the last three short tables, or in the following one, more than ⅓ part. They will then be about up to ordinary practice for railroads; and will also be sufficient for bridges for common travel, with a clear width of 18 ft between the two trusses; and two outside footpaths, each 5 ft wide, in addition. A width of 18 ft is necessary for allowing two ordinary vehicles to pass each other *readily*.* It should never be less than 16 ft; and nothing is gained by exceeding 20 ft.

It will of course be understood that each member, especially in large spans, will consist of two or more pieces, side by side. Thus, in a span of 200 ft, the 800 sq inches of each chord, will probably consist of four beams of about 10″ × 20″, or 12″ × 16″, placed side by side; but with sufficient intervals between them to allow the several oblique bars to pass. Or it may consist of six beams of smaller size. So also, the 1st, or main rod, of 46¾ sq ins, will probably be made up of from 4 to 8 bars, of 11.7 or 5.85 sq ins each, placed side by side; occasionally some inches apart. And so with the others. The feet of the opposite posts of the two trusses of a Fink span, are connected together by ties of wood or iron, to prevent lateral motion; and for the same purpose, diagonals are carried from each upper chord to the foot of the opposite post; as is usually done in top-road bridges of any kind. These had better be tie-struts.

A simple truss like Fig 6, of 30 ft span, and 10 ft high, may have a chord of 15″ by 18″; rafters 10″ by 10″; one rod of 2⅜″ diam; or two rods of 1⅝″ diam and several inches apart transversely of the bridge; which is far better than one.

The Pratt truss, when put together as in Fig 31; each member (except the *main* obliques *x w*, *y n*, &c,) being of one piece, is perhaps as easy of construction, and as suitable for small spans, as any other. With the same assumptions as before with regard to total load, upset rods, &c, the following dimensions may be used. They are, however, by no means the result of *close* calculation; but are purposely full.

Table of dimensions for small, single-track Pratt trusses.

Height from top of top chord, to bottom of bottom chord, $\frac{1}{7}$ of the clear span. Twelve panels to each truss. Each main oblique consists of two parallel rods or bars. Each counter oblique is a single rod passing between the mains.

For one truss only.

Clear Span.	Chords. Each.	Center Post.	End Posts.	Diam in ins of each of the two bars of a main oblique.		Center Counter. Diam.	End Counter. Diam.
Ft.	Ins.	Ins.	Ins.	At center of Truss.	At ends of Truss.	Ins.	Ins.
30	9 × 11	4 × 9	7 × 9	1	1⅛	1⅜	1
40	10 × 12	4 × 10	8 × 10	1⅛	1⅜	1⅜	1
50	10 × 14	5 × 10	9 × 10	1¼	2⅛	1¾	1
60	12 × 15	5 × 12	9 × 12	1⅜	2⅜	2	1
70	12 × 17	6 × 12	11 × 12	1½	2½	2⅛	1

The diams of the main rods may *increase;* and those of the counters *diminish*, regularly from the center each way to the abuts.† The chords and posts are supposed to have the same dimension *transversely* of the truss. When more convenient, all the posts may be of the same size as the end posts; their ends tenoned an inch or two into each chord. The angle-blocks above and below the posts should be of oak, or other hard wood; or of cast iron. In such small spans they need not be notched more than an inch into the chords; or at most, 1½ ins, near the ends of the larger ones in the table.

Fig 9 *b*, or Fig 23, without any special counterbraces, will answer very well for spans of 25 to 45 ft. The height of truss, number of panels, and size of chords, to be the same as in the preceding table for Pratt. W, at Fig 32, shows the arrangement at the head and foot of each brace *b*; except that the heads of the two center braces *abut against each other*.

The vert rods, *v*, are in pairs; like the main-brace rods of the Pratt above; but their areas of section may be $\frac{1}{5}$ less than those given for the Pratt braces. The areas of the wooden braces or obliques, *b*, should be ¼ greater than those of the vert posts of the Pratt. The notches of the heads and feet into the chords, may be 2½ to 3 ins deep. Two long spikes, *n*, may be used at each head and foot. At *i* is a wooden block washer, across the chord.

REM. 1. It may appear strange that we refer to Fig 9 *b*, as being appropriate; after having used it for illustrating the necessity for counterbraces under moving loads. But experience has shown that this necessity is not *always* so urgent as some suppose; and that in *short* spans at least, with *stout chords, well connected* with each other at *short* panel-lengths, it may be dispensed with.

For instance, in Fig 23 *f*, page 274, the counter *h p* sustains a strain of 11.9 tons. Now, if the counter be taken away, this strain is by no means left entirely unprovided for; but it becomes reversed

* *These remarks of course* apply to railroad bridges of any form of truss.

† *Not theoretically* correct under moving loads; but in all cases safe. The same remark applies to verticals.

hot push against the chord at *h*; and from there the chord carries it to the center of the truss. From *e*, it is taken up by the obliques *e g*, *c r*, &c; and by them carried safely to the abutment. If the chord is sufficient for its extra duty from *h* to *e*, as it usually is in short spans, the writer not see the need of counters. At any rate, he imagines that the two long spikes at each head and f the braces give sufficient counterbracing in the present case.

Newark Dyke Warren, England, span 240 ft, has no counterbracing; and so with some other r bridge trusses of various kinds. The writer is inclined to believe that many failures which been ascribed to want of counterbracing, have in fact been owing to the common mistake alluded r the bottom of p. 174; and to not giving the main braces a sufficient depth of insertion into the ; thus causing an undue yielding of the timber at these points; as he has frequently seen.

r wooden arches, with only a lower chord; or in other words, bowstring bridge, *Fig* 35, the following rule will give quite sufficient dimensions e arches alone, for a single-track railway. The rise of the lower arc of the arch posed to be $\frac{1}{9}$ of the clear span.

$$\text{Area of arch of one truss only, in sq ft.} = \frac{\text{clear span in ft} + 25}{37.5}.$$

gives as follows, for *each* truss of the two which constitute a span; each truss two arches, side by side; or within say about a foot of each other.

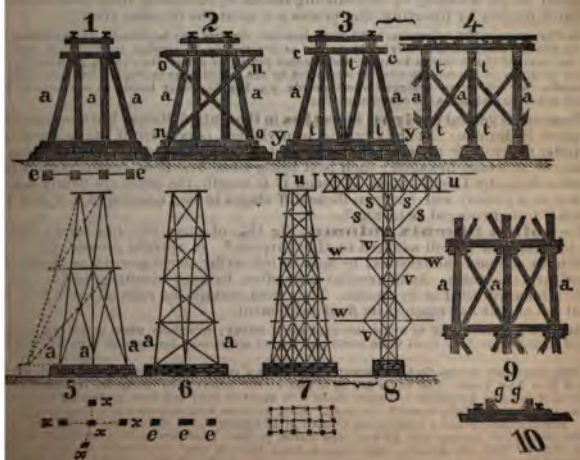
ar Span.	Inches.	Clear Span.	Inches.
50 ft	2 of 10 × 14	125 ft	2 of 11 × 26
75 ft	2 of 10 × 19	150 ft	2 of 12 × 28
100 ft	2 of 10 × 24	175 ft	2 of 12 × 32
110 ft	2 of 11 × 24	200 ft	2 of 12 × 36

span to be divided into about 16 panels: diminishing in horizontal length he center, toward the abuts. Either Howe or Pratt, or the Warren trussing arches will be suitable. The arches may be made of planks 3 or 4 ins thick; o form the curve; and well bolted together.

ilroad trusses for 4 ft 8½ inch gauge, should be at least apart in the clear; 14 ft is better. For lateral stability in long spans the from out to out should not be *less* than about $\frac{1}{14}$ of the span.

eadway of at least 16 ft, should be allowed for clearing smoke-stacks.

TRESTLES.



, 2, 3, 5, 6, 7, are **elevations of trestles**; taken across the track
We may consider *Fig* 1 as adapted to a height of about 10 to 20 ft.; Fig

and 3, from 20 to 30; Fig 5, from 30 to 40; Fig 6, from 40 to 60: as mere app
tions. These figs of course admit of many modifications. They are called
These bents are usually supported by bases of masonry, as in the figs; to
the lower timbers from contact with the earth, which would hasten decay.
well to make these bases high enough to prevent injury from cattle, or
vehicles, &c. Up to heights of about 40 or 50 ft, a single row of posts or
a, a, a, Figs 1 to 9, as shown at *e e* under Figs 1 and 6, will answer. But as th
becomes greater, more posts should be introduced, as shown at *x x* under
two entire rows of them; or three rows, as under Fig 7: and as also in Fig
is an end view of Fig 7. Figs 7 and 8 bear much resemblance to the trest
high, with masonry bases 30 ft high, (S. Seymour, C. E.,) which carry the
and Erie R R across the Genesee river at Portage. There each bent has 21
ins square at its base; and 15 posts of 12 × 12, at its top. The other tim
6 × 12; many of them are in pairs, embracing the posts. Frequently the
trestles are in pairs; and the other timbers pass between; all bolted togeth

In Fig 4, the posts *a, a, a*, are end views of three trestles or bents, such
and *t t* are diag braces extending from trestle to trestle; the two outer ones
in one direction; and the central one crossing them. These may be plac
intermediate of the posts, as in Fig 3; with the heads of the two outer ones
to the cap *c c* of one trestle; and their feet to the sill *y y* of the next one;
may all be spiked or bolted to the posts themselves, as in Fig 4. The last is
as it serves also directly to stiffen the posts: as do also the braces *o o, n*
Such bracing is too frequently omitted. During the passage of trains the
(so to speak) of the rails against the adhesion of the driving-wheels of the
produces a serious strain lengthwise of the road, and tending to upset the
and this becomes more dangerous as the height increases. Hence the nec
such braces.

The posts should not be less than about 12 ins square, except in quite low
and even then not less than about 10 × 10. The diag bracing may generally
as wide as the posts; and half as thick. The dist apart of the bents, when
way is supported by simple longitudinal beams, should not exceed 10 or
railroads. But if these beams receive support from braces beneath, like *x x*,
from iron truss rods, as at Fig 52, page 304; the dist may be extende
20 or more ft. But when the trestles become very high, and contain a gre
timber, it becomes cheaper to place them farther apart, say 30 to 60 feet; an
the railway upon regular framed trusses, as at *u u*, Figs 7 and 8; as in a br
stone piers. In the forementioned Genesee river viaduct, the trestles are 5
from center to center.

When such a trestle as Fig 8 becomes very narrow in proportion to its h
may add to its stability by introducing beams *w*, extending from trestle t
and still further by inserting diag braces *v v*, as at the Genesee river.

The Figs 42, page 294, "Trusses," will show how the timbers may be joined. In designi
(as in wooden bridges,) it is advisable, as far as practicable, to arrange the pieces so t
may be removed if it becomes decayed; and another put in its place. On curves, addition
should be given on the convex side; as suggested by the dotted lines in Fig 5. On very h
especially, (as well as on bridges,) wheel-guards, *g g*, Fig 10, either inside or outside o
should never be omitted, as is commonly done.

There are as yet but few **iron trestles** in the United States; but they ar
for piers in Europe. At the Crumlin iron viaduct, (spans 150 ft,) they are
ft high; 60 by 27 ft at base; 30 by 18 at top; each composed of 14 cast-i
arranged as a long hexagon; each post being formed of 17 ft lengths of iro
ft outer diam, by 1 inch thick. At each 17 ft length, the pipes are firmly
by hor iron pieces; and between these diff stages is diag bracing of 4 × 4
rolled T iron, arranged as in Fig 7.

The **rolled Phœnix column,** Fig 4½, of page 233, has lately
for trestles, and is well adapted to the purpose.* *In marshy ground*, pile
driven to support the trestles; or may be left so far above ground, as then
constitute the posts. Such trestles may often be used advantageously, e
to be afterward filled in by embkt. They then sustain the rails at their pr
until the embkt has reached its final settlement.

They are generally used to avoid the expense of embkt; especially when earth can
tained from a great dist. Even when earth and timber are equally convenient, they will

* The Baltimore Bridge Co have recently used it in a single-track trestle-work about
on a R R in New York. The bents are 30 ft apart; the tallest ones being 120 ft high.
consists of but two 4-segment columns, about 8 ft apart at top; and each spreading out
1¾ ins per foot of height, toward the base. Each of these long columns is in four leng
each. The top length has a transverse section of 10 square ins of metal; the 2d, 11 ins;
ins; and the 4th, or bottom one, 13 ins. See Table, p. 234. There are no ties between th
Beams trussed like Fig 52, of p 304, extend from bent to bent, to sustain the rails.
firmly braced in every direction, especially at all the joints of the columns. The
since built piers of Phœnix columns to a height of 252 ft.

Weight of powder in one foot depth of hole.

Diameter of hole in inches.

1	1¼	1½	2	2½	3	3½	4	4½	5	5½	6
Weight of powder in pounds and ounces avoir.											
0"5	0"8	0"11	1"4	2	2"13	3"14	5"0	6"6	7"14	9"8	11"5

lling holes for blasting. The holes are generally from 2½ to 4 ft and from 1½ to 2 ins diam. **Churn-drilling** is much more expeditious onomical than that by *jumping*, mentioned below. The churn-drill is merely l iron bar, usually about 1¼ ins diam, and 6 to 8 ft long; with a steel cutting r bit, (weighing about a lb, and a little wider than the diam of the bar,) welded lower end. A man lifts it a few inches; or rather catches it as it rebounds, t partially around; and lets it fall again. By this means he drills from 5 to of hole, nearly 2 ins diam, in a day of 10 working hours, depending on the ter of the rock. From 7 to 8 ft of holes 1¾ ins diam, is about a fair day's n hard gneiss, granite, or compact siliceous limestone; 5 to 7 ft in tough com- ornblende; 3 to 5 in solid quartz; 8 to 9 in ordinary marble or limestone; 9 to andstone; which, however, may vary within all these limits. When the hole than about 4 ft deep, two men are put to the drill. Artesian, and oil wells, , are bored on the principle of the churn-drill.

umper, as now used, is much shorter than the churn-drill. One man (the *holder*) sitting fts it slightly, and turns it partly around, during the intervals between the blows from about 5 hammers, wielded by two other laborers, the *strikers*. It can be used for holes of smaller s than can be made by the churn-drill; because the holder can more readily keep the cutting he exact spot required to be drilled. It is also better in conglomerate rock; the hard siliceous of which deflect the churn-drill from its vertical direction, so that the hole becomes crooked, tool becomes bound in it. The coal conglomerates are by no means hard to drill with a The jumper was formerly used for large deep holes also, before the superiority of the churn- ame established.

r tool requires resharpening at about each 6 to 18 inches depth of hole; and the wear of the ge requires a new one to be put on every 2 to 4 days. With iron jumpers, the top also be- attered away rapidly. As the hole becomes deeper, longer drills are frequently used than at ning. The smaller the diameter of the hole, the greater depth can be drilled in a given nd the depth will be greater in proportion than the decrease of diam. Under similar circum- , three laborers with a jumper will about average as much depth as one with a churn-drill.

and-drill, in which the same man uses both the hammer and the short drill, is chiefly used low holes of small diam. With it a fair workman will drill about as many feet of hole from ns deep, and about ¾ inch diam, as one with a churn-drill can do in holes about 3 ft deep, and im, in the same time. Only the jumper or the hand-drill can be used for boring holes which zontal, or much inclined.

e Burleigh Rock Drill, (Co at Fitchburg, Mass.,) is much more rapid and eal than the foregoing if the work is so great as to justify the preliminary outfit. It drills at le; and is worked either by steam directly; or by air compressed by steam into a tank, and ed to the drills through iron pipes. The air is best for tunnels and shafts, because after the drills it aids ventilation. The cost of each drill and its support is about $450 to $1200 at , depending on the size of the holes, (¾ to 6 ins diam.) Each compressor, with attached $1100 to $2800. Add boilers, air-tank, air-pipes, house, extra parts, &c. A $2000 compressor k two drills; and require 3 inch air-pipes; and a tank 15 ins diam, by 4 ft long. The boiler pressor together require 1 man; each drill 2 men, and will drill from one to two holes of ins diam, and 5 ft deep per hour, depending on the kind of rock; and about ⅕ more of 2 inch n limestone from 25 to 45 ft of hole with one sharpening. One blacksmith and helper can for 5 or 6 drills.

t of quarrying stone. After the preliminary expenses of purchasing e of a good quarry; cleaning off the surface earth and disintegrated top rock; roviding the necessary tools, trucks, cranes, &c; the total neat expenses for *out* the rough stone for masonry, per cub yard, ready for delivery, may be y approximated thus: Stones of such sizes as two men can readily lift, meas- *piles*, will cost about as much as from ¼ to ½ the daily wages of a quarry r. Large stones, ranging from ½ to 1 cub yd each, got out by blasting, from daily wages per cub yd. Large stones, ranging from 1 to 1½ cub yds each, in most of the work must be done by wedges, in order that the individual stones ome out in tolerably regular shape, and conform to stipulated dimensions; to 4 daily wages per cub yard. The smaller prices are low for sandstone, the higher ones are high for granite. Under ordinary circumstances, about b yds of good sandstone can be quarried at the same cost as 1 of granite; or, r words, calling the cost of granite 1, that of sandstone will be ¾: so that ans of the foregoing limits may be regarded as rather full prices for sandstone; scant ones for granite: and about fair for limestone or marble.

t of dressing stone. In the first place, a liberal allowance should be for ***waste.*** *Even when the stone* wedges out handsomely on all sides from arry, *in large blocks of nearly* the required shape and size, from ⅙ to ¼ of gh *block will generally* not more than cover waste when well dressed. In e-sized *blocks, (say averaging* about ½ a cub yard each,) and got out by

blasting, from ¼ to ⅓ will not be too much for stone of medium ch
straight splitting. About the last allowance should also be made for
rubble. The smaller the stones, the greater must be the allowance
dressing. In large operations, it becomes expedient to have the ston
far as possible, at the quarry; in order to diminish the cost of transport
when the distance is great, constitutes an important item—especially
and on common roads.

A stonecutter will first take out of wind; and then fairly patent-hamm
to 10 sq ft of plain face in hard granite, in a day of 8 working hours; or twice as mu
rior dressing as is usually bestowed on the beds and joints; and generally on the fac
masonry, &c, when a very fine finish is not required. In good sandstone, or marble,
¼ more than in granite. Of *finest* hammer finish, *granite*, 4 to 5 sq ft.

Cost of masonry. Every item composing the total cost is l
variation; therefore, we can merely give an example to show *the gen*
upon which an approximate estimate may be made; assuming the wa
mon laborer to be $2.00 per day of 8 working hours; and $3.50 for a r
monopoly of quarries affects prices very much.*

Cost of ashlar facing masonry. Average size of the s
long, 2 ft wide, and 1.4 thick; or two such stones to a cub yd. Then,
stone to be granite or gneiss, the cost per cub yd of masonry will be

Getting out the stone from the quarry by blasting, allowing ¼ for waste in
dressing; 1⅓ cub yds, at $3.00 per yard.....
Dressing 14 sq ft of face at 35 cts.....
" 52 " beds and joints, at 18 cts.....

Neat cost of the dressed stone at the quarry.....
Hauling, say 1 mile; loading and unloading.....
Mortar, say.....
Laying, including scaffold, hoisting machinery, &c.....

Neat cost.....
Profit to contractor, say 15 per ct.....

Total cost.....

Dressing will cost more if the faces are to be rounded, or moulded. If the stones
we have assumed, there will be more sq ft per cub yd to be dressed, &c.

If in the foregoing case, the stones be *perfectly* well dressed on all sides, includ
cost per cub yd would be increased about $10; and if some of the sides be curved, a
say $12 or $14; and if the blocks be carefully wedged out to given dimensions, i
making the neat cost of the dressed stone *at the quarry* say $36 to $38 per cub yd.

The item of laying will be much increased if the stone has to be raised to great hei
to be much handled; as when carried in scows, to be deposited in water-piers, &
large work presents certain modifying peculiarities, which must be left to the judg
neer and contractor. The percentage of contractors' profit will usually be less on l
on small ones. See App p 630.

Cost of large scabbled granite rubble, such as is gen
backing for the foregoing ashlar; stones averaging about ½ cub yd e

At above rates of wages.

Getting out the stone from the quarry by blasting, allowing ¼ for waste i
scabbling; 1¼ cub yds at $3.00.....
Hauling 1 mile, loading and unloading .....
Mortar; (2 cub ft, or 1.6 struck bushels quicklime, either in lump or ground
and 10 cub ft, or 8 struck bushels of sand, or gravel; and mixing).....
Scabbling; laying, including scaffold, hoisting machinery, &c.....

Neat cost.....
Profit to contractor, say 15 per ct.....

Total cost.....

Common rubble of small stones, the average size bei
men can handle, costs, to get it out of the quarry, about 80 cts per
or to allow for waste, say $1.00. Hauling 1 mile, $1.00. It can be rou
and laid, for $1.20 more; mortar as foregoing, $1.50. Total neat cost, $
15 per ct profit, $5.40, *at the above wages for labor.* See p 630.

* The large blocks of granite for the Bunker Hill Monument at Boston, averagin
were quarried by wedging, and delivered at the site of the monument, at a neat co
per cub yd; by the Monument Association; from a quarry opened by themselves for
Association received no profit; their services being voluntary. The average contra
same, were $24.30! The actual cost of getting out the rough blocks at the quarry w
ing upon trucks at quarry, about 15 cts. Transportation 6 miles by railway and com
Total, $5.40. In 1825 to 1845; common unskilled labor averaging $1 per day.

***In 1873* large blocks of granite** of about a cub yd each, wi
joints, but with only a 2 inch draft around the showing-face (which is left rou
wharf at Philada from Maine &c, at $27 per cub yd. And from Port Deposit at

ith smaller stones, such as one man can handle, we may say, stone 70 cts; hauling $1; : and scaffold, tools &c, $1; mortar $1.50. Making the neat cost $4.20; or with 15 per ct profit, $4.83. cobbled irregular range-work costs from $2 to $3 more per yd than rubble: according to the charac- the stone &c. The laying of thin walls costs more than that of thick ones, such as abutments &c.*

ne cost of plain 8 inch thick ashlar facings for dwellings &c in ta, in 1873, is about as follows per square foot showing, put up, including everything. Sand- $1.50 to $2.25. Pennsylvania marble, $2.50. New England marble, $2.75 to $3.25. Granite, to $2.75. If 6 ins thick, deduct one-eighth part. **First class artificial stone** be made and put up at one-third the price. See p 508. **North River blue stone** s, 3 ins thick, for footwalks, put down, including gravel &c, 70 cts per sq foot. **Belgian** et **pavement,** with gravel, complete, $3.50 per sq yard.

ien dressed ashlar facing is backed by rubble, the expense per cub yard of the e mass will of course vary according to the proportions of the two. Thus, if r at $12 per yd, is backed by an equal thickness of rubble at $5, the mean cost be $\frac{\$12 + \$5}{2} = \$8.50$; or, if the rubble be twice as thick as the ashlar, then $\frac{\$5 + \$5}{3} = \$7.33$; &c.

imes the contractor must be allowed his extra expense in opening new quarries; in forming roads to his work; in digging foundations; or for pumping or otherwise draining them, when ie are unexpectedly met with; for the centers for arches, &c; unless these items are expressly ed in the contract per cub yd.

FOUNDATIONS.

OLUME might be occupied by this important subject alone. We have space for a few general hints; leaving it to the student to determine how far they may plicable in any given case. In ordinary cases, as in culverts, retaining walls, excavations, or wells, &c, in the vicinity, have not already proved that the soil iable to a considerable depth, it will usually be a sufficient precaution, after ig dug and levelled off the foundation pits or trenches to a depth of 3 to 5 ft, to t by an iron rod, or a pump-augur; or to sink holes, in a few spots, to the depth to 8 ft farther; (depending upon the weight of the intended structure:) to ascer- if the soil continues firm to that distance. If it does, there will rarely be any in proceeding at once with the masonry; because a stratum of firm soil, from 4 ft thick, will be safe for almost any ordinary structure; even though it should nderlaid by a much softer stratum. If, however, the firm upper stratum is ex- d to running water, as in the case of a bridge-pier in a river, care must be taken reserve it from gradually washing away; or from becoming loosened and broken y violent freshets; especially if they bring down heavy masses of ice, trees, and r floating matter. These are sometimes arrested by piers, and accumulate so as rm dams extending to the bottom of the stream; thus creating an increase of city, and of scouring action, that is very dangerous to the stability both of the m and of the structure. When the testing has to be made to a considerable h, it may be necessary to drive down a tube of either wrought or cast iron, to ent the soil from falling into the unfinished hole. If necessary, this tube may short lengths, connected by screw joints, for convenience of driving; and the inside of it may be removed by a small scoop with a long handle.†

orings in common soils or clay may be made 100 ft deep in a day or two by a ne wood augur 1½ ins diam, turned by two to four men with 3 ft levers. This will bring up ks. The iron of the jointed rod should be of best quality.

starting the masonry, the largest stones should of course be placed at the bot- of the pit, so as to equalize the pressure as much as possible; and care should ken to bed them solidly in the soil, so as to have no rocking tendency. The few courses at least should be of large stones, so laid as to break joint thoroughly those below. The trenches should be refilled with earth as soon as the masonry permit; so as to exclude rain, which would injure the mortar, and soften the lation. It is well to ram or tread the earth to some extent as it is being deposited. the tests show that the soil (not exposed to running water) is too soft to support masonry, then the pits should be made considerably wider and deeper; and after- be filled to their entire width, and to a depth of from 3 to 6 or more ft, (de- ng on the weight to be sustained,) with rammed or rolled layers of sand, gravel, me broken to turnpike size; or with concrete in which there is a good propor- f cement. On this deposit the masonry may be started. The common practice ih cases, of laying planks or wooden platforms in the foundations, for building

Philada, in 1873, cellar and other walls of rough rubble, $5 to $6 per perch of 22 cub ft of wall. walls with a facing of broken range rock-work of sandstone, (as common in Gothic churches,) $ per 22 cub ft, including everything.

†Russian caverns in limestone regions are a frequent source of trouble, against which it v adopt precautions.

upon, is a very bad one. For if the planks are not constantly kept thoro they will decay in a few years; causing cracks and settlements in the ma

Some portions of the circular brick aqueduct for supplying Boston w gave a great deal of trouble where its trenches passed through running q and other treacherous soils. Concrete was tried, but the wet quicksand m with it, and *killed* it. Wooden cradles, &c, also failed; and the difficulty overcome by simply depositing in the trenches about two feet in depth gravel.* Sand or gravel, *when prevented from spreading sideways*, forms best of foundations. To prevent this spreading, the area to be built on n rounded by a wall; or by squared piles driven so close as to touch each o less important cases, by short sheet piles only. But generally it is suffici to give the trenches a good width; and to ram the sand or gravel (which better if wet) in layers; taking care to compact it well against the *sides* of also. Under heavy loads, some settlement will of course take place, as in all foundations except rock. If very heavy, adopt piling, &c.

When an unreliable soil overlies a firm one, but at s that the excavation of the trenches (which then must evidently be ma well as deeper,) becomes too troublesome, and expensive; especially whe ally happens in that case) water percolates rapidly into the trenches from t strata, we may resort to piles. See "Piling." When making deep foun in damp clay, it must be remembered that this material, being soft, has, degree, a tendency to press in every direction, like water. This causes inward at the sides; and upward at the bottom. The excavations for tur vertical shafts, often close in all around, and become much contracted th they can be lined; therefore they should be dug larger than would othe cessary. The bottoms of canal and railroad excavations in moist clay are pressed upward by the weight of the sides. Dry clay also rapidly absor from the air, and swells, producing effects similar to the foregoing. It is attended by great pressure; so that retaining-walls backed with dry r will be in danger of bulging if the clay should become wet. It is a treac terial to work in. **For concrete foundations,** See p 507.

Fig 1 shows an easy mode of obtaining a foundation in certain cases **"pierre perdue"** (los the French; in English, "**stone,**" or random work.

Fig 1

It is merely a deposit of rough a stone thrown into the water; th being at the outside, to resist dis freshets, ice, floating trees, &c. A terior may be of small quarry ch gravel, sand, clay, &c. When the regular rock, this process saves t levelling it off to receive the maso 3 feet below the surface of the wa may generally be disposed by hand so as to lie close and firmly. Small spawls packe larger ones will make the work smoother, and less liable to be displaced by violenc chains may at times be useful for connecting several of the large stones together for gr

If the bottom is so yielding as to be liable to wasl freshets, it may, in addition, be protected, as in Fig 2, by a covering of th of stones, as ing all aroun ture. Or th of stones ma ed as per dot so that if should wash dotted line stones *d* wi the cavity, a vent furth Sheet-piles,

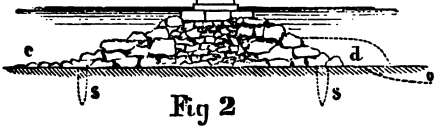


Fig 2

driven as an additional precaution. For greater security, the bed of th be dredged or scooped under the entire space to be covered by the mai per dotted lines in Fig 3, to as great a depth as any scouring would be s

* *Smeaton mentions* a stone bridge built upon a natural bed of gravel only about 2 *lying deep mud so soft* that an iron bar 40 ft long sank to the head by its own weig *piers, however, sank* while the arches were being turned; and was restored by Sme *a wretched precedent* for bridge-building, this example illustrates the bearing pow *of well-compacted gravel.*

also to be filled with stone. Such foundations are evidently best t water. The masonry should rest on a strong platform.

of stone, as in these increase the velocity, action of the stream cially in freshets; un- each side from the de- out to such an extent rea of water shall not bottom is treacherous, e before depositing the Fig 2. Judgment and essary in such matters, nnected with engineer- will not guard against Theory and practice her.

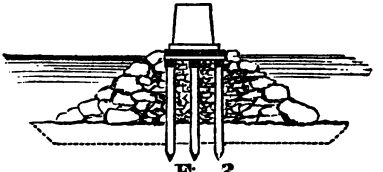


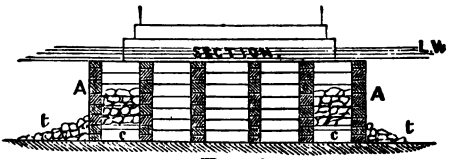
Fig 3

simple method; and reate too great an ob- rigation of the stream, or to the escape of its waters in time of high freshets, is a

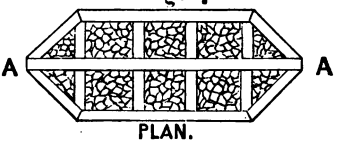
Here the piles are first driven into the river bottom, for the support of the pier; stone is thrown in, for the support and protection of the piles; preventing them r their loads; and shielding them from blows from floating bodies. The tops of off to a level, a strong platform of timber is laid on top of them, as a base for the of the platform should not be less than about 12 or 18 ins below ordinary low lecay. Mitchell's iron screw pile; or hollow piles of cast iron, may be used instead lee PILING, p. 320.

ent a convenient method of establishing a foundation in water, by **mber crib, A A, without a bottom.** It should be built of

, notch- t their own at h being of the means upport- s entire e below pulling s. Bolts at the at least he crib, portion ted off r. The ded into angular 4 or 5 ft ding to ts of the rtitions ells are in the s those e cribs; ly, like n solid



Figs 4



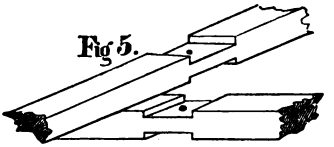


Fig 5.

framed afloat, at any convenient spot; and when finished, may be towed to its it is carefully moored in position, and then sunk by throwing stone into a few platforms, as at c c, for that purpose. These platforms should be placed a little ge of the cells, so as not to prevent the crib from settling slightly into the soil, and ll bearing upon the bottom. After it has been sunk, all the cells are filled with ut top platform may be added or not, as the case may be; also, a protection, t t, of revent undermining by the current. If the sides are exposed to abrasion from e covered in whole or in part with plank, or plate iron; and the angles strength- , &c. In deep water, a foundation may be made partly of random stone, as in n top of this may be sunk a crib, with its top about 2 ft under low water, as a base This is much safer than random stone alone.

ron bridge across the Schuylkill at Chestnut St., *lckland Kneass, Engineer,* affords a striking example of crib foundn *pier stands on a crib,* an oblong octagon in plan; 31 by 87 feet. *t top; and (with its platform)* 29 ft high. Its timbers are of yell

pine, hewn 12 ins square; and framed as at Fig 5. The lower timbers were cut or scribed to conform to the irregularities of the tolerably level rock up it rests. These were ascertained (after the 8 ft depth of gravel had been off) in the usual manner of mooring above the site a large floating wooden composed of timbers corresponding in position with all those of the lower the intended crib, both longitudinal and transverse. Soundings were th close together along all these lines of timber. Most of the cells are about on a side, in the clear. A few of them had platforms at the level of th course from the bottom, for receiving stone for sinking the crib; the other to the bottom.

The crib was built in the water; and was kept floating, during its construction, with its top continually just above water, by gradually loading it with more stone as new timbers The stone required for this purpose alone was 300 tons. When the crib was towed into p moored, 150 tons more were added for sinking it. All the cells were afterward filled with stone, and coarse gravel screenings; making a total of 1666 tons. A platform of 12 by 12 ft timber covered the whole: its top being 2½ ft below low water. The pier alone, which st crib, weighs 3255 tons; and during its construction it compressed the crib 6½ ins. Th superstructure resting on the pier, may be roughly taken at 1000 tons more.

An ordinary caisson is merely a strong scow, or a bo a lid; and with sides which may at pleasure be readily detached from its b is built on land, and then launched. The masonry may first be built in it, whole or in part, while afloat; and the whole being then towed into p moored, may be sunk to the bottom of the river, to rest upon a foundation prepared for it, either by piling, if necessary; or by merely levelling off th surface, &c. The bottom of the caisson constitutes a strong timber platfo which the masonry rests; an ranged, that after it is sunk, may be detached from it, and to be rebottomed for use a pier, if needed. This detachi effected by some such contr that shown in Fig 6, where P bottom of the caisson, to w firmly attached at intervals s eyes t; which are taken h hooks d, at the lower end of E n, reaching to the top tim the crib, where they are co screw nuts n. By loosening t the hooks d can be detached eyes t; and the sides can th moved from the bottom: th

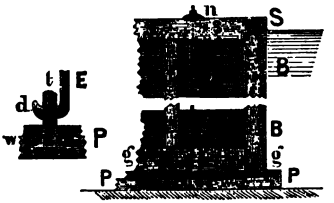


Fig 6.

no other connection between the two. These hooks and eyes are usually p side of the caisson; the screw nuts n being sustained by the projecting en pieces, as t t, Fig 9. The improper position given them in our Fig was convenience of illustrating the principle. It will sometimes be necessa one side detachable from the others, in order to float the caisson away clea finished pier; unless it be floated away before the masonry has been built to render the precaution useless. Fig 6 shows one of many ways of cons caisson; with sides consisting of upright corner-posts, T; cap pieces S, on sills g at bottom, resting on the bottom platform P P w; intermediate u framed into the caps and sills; the whole being covered outside by one or nesses of planking B, which, as well as the platform, should be well calke vent leaking. Tarpaulin also may be nailed outside to assist in this. Th trouble from leaking is where the sides join the platform. On top of the p firmly spiked a timber o o, extending all around it just inside of the in edge of the sides of the caisson. Its use is to prevent the sides from be inward by the pressure of the water outside. The details of constru of course vary with the requirements of the case. In deep caissons, ins braces or struts from side to side, as at c c, Fig 7, will be required to p sides from being forced inward by the pressure of the water, as the vessel sinks while the masonry is being built within it. As the masonry is carri struts are removed: and short ones, extending from the sides of the caiss masonry, are inserted in their place. When the caisson is shallow, only *course of braces* will be required; they also support a platform for the *and their materials.* In deep caissons, in order not to be in the way of th *the outer planking of* the sides may, in part, be gradually built up as the *progresses. It may* sometimes be expedient to build the masonry hol *with thin transverse* walls inside to stiffen it if necessary; and to compl *for after sinking* the caisson. Indeed, masonry or brickwork, in come

low at first, resting on the platform; the masonry itself forming the sides on. Or the sides may consist of a water-tight casing of iron, or wood, of f the intended pier, &c. This casing being confined to the platform, be- ict, a mould, in which the pier may be formed, and sunk at the same time with hydraulic concrete.

r rock bottom, the under timbers of the platform may be cut to suit the irregularities, ted under "Cribs." Or the bottom may be levelled up by first depositing large stones ea upon which the caisson is to rest; and then filling between these with smaller stones sting the depth by sounding. Or a level bed of cement concrete may, with care, be he water. If there are deep narrow crevices in the rock, through which the concrete hey may be first covered with tarpaulin. Diving bells may often be used to advantage, erations. But in the case of very irregular rock, it will often be better to resort to cof- ne draft of a caisson (the depth of water which it draws) whether empty or loaded, can art 19 of page 534. Valves for the admission of water for sinking the caisson are reduced. If, after sinking, it should be necessary to again raise the whole, it is only lose the valves, and pump out the water. Guide piles may be driven and braced along- sson. to insure its sinking vertically, and at the proper spot. Or it may be lowered by ted by strong temporary framework.

he uprights I, T, &c. Fig 6, to be sufficiently braced, as at *c c*, Fig 7, the following table thickness of planking necessary for different distances apart of the uprights, (in the re a safety of six against the pressure of the water at different depths; and at the ; to bend inward under said pressure, more than $\frac{1}{480}$ part of the distance to which rom upright to upright; or at the rate of ¼ inch in 10 ft stretch; ⅓ inch in 5 ft, &c. nay be of use in other matters.

thickness of white pine plank required not to bend than $\frac{1}{480}$ part of its clear horizontal stretch, under nt heads of water. (Original.)

Stretch in Ft.	HEADS IN FEET. 40	30	20	10	5
	Thickness in Inches.				
3	3½	3	2¾	2¼	1¾
4	4½	4	3½	2¾	2¼
6	6¾	6	5¼	4¼	3¼
8	9	8	7	5½	4½
10	11¼	10	8¾	7	5½
12	13½	12¼	10¾	8½	6¾
15	16¾	15	13	10½	8½
20	22¼	20	17½	14	11

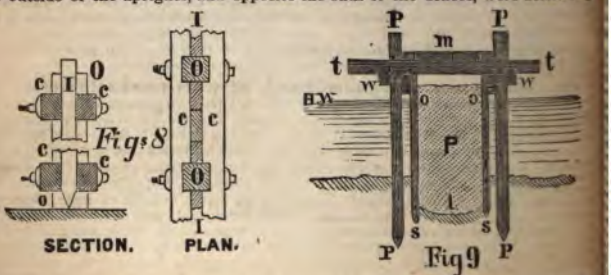
dams are enclosures from which the water may be pumped out, so as e work to be done in the open air. Their construction of course varies n still shallow water, a mere well-built bank of clay and gravel; or of filled with those materials when there is much current, will answer se; or (depending on the depth) a single or double row of sheet-piles; or of es of larger dimensions, driven touching each other; their lower ends a the soil; and their upper ones a little above high water, and protected heaps of gravelly soil or puddle, (as at P in Fig 7,) to prevent leaking. iles may be of wood; or of cast iron, of a strong form.

ifficiency of a mere bank of well-packed earth in still iown by the embankments or levees, thrown up in all countries, to pre- from overflowing adjacent low lands. The general average of the levees iiles of the Mississippi, is about 6 ft high; only 3 ft wide on top; side- o 1. In floods the river rises to within a foot or less of their tops; and ourst s through them, doing immense damage. They are entirely too slight. iod of a single row of 12 by 12 inch squared piles, driven in contact with (*close piles*,) and simply backed by an outer deposit of impervious soil, ctive; and with the addition of interior cross-braces or struts, like *c c*, Fig it crushing inward by the outside pressure of the water and puddle when t, has been successfully employed in from 20 to 25 ft depth of water, in e was not sufficient current to wash away the puddle. The cross-braces l successively, as the water is being pumped out; beginning, of course, pper ones. The ends of these braces may abut on longitudinal timbers, ie piles for the purpose. Another method is by means of a strong crib, *f uprights framed into caps* and sills; and covered outside with squared *plank, laid touching each* other, and well calked; as in the caisson, Fig *ut a bottom. Between the* opposite pairs of uprights are strong interic *Fig 7, reaching from* side to side, to prevent crushing inward. T

upper series of these usually supports a platform for the workmen, windlasses, &c. The crib having been built on land, is launched, taken to its final place, and sunk by piling stones on a temporary platform resting on the cross-struts; the bottom of the stream having been previously levelled off, if necessary, for its reception.

To prevent leaking under the bottom of the crib, sheet-piles may be driven around it, their heads extending a few feet above its bottom; or a small deposit of outside puddle may be placed around it, as shown at the stone deposits *t t*, Fig 4. Or a broad flap of tarpaulin may be closely nailed around and a little above the lower edge of the crib; so arranged that it may be spread out loosely on the river bottom, to a width of a few feet all around the outside of the crib; and the puddle may be placed upon it. Such a tarpaulin is also very useful in case the river bottom is somewhat irregular, and cannot be levelled off without too great expense; in which case the crib cannot come to a full bearing upon it; and consequently the water would leak or flow beneath freely. It is especially adapted to uneven rock; where sheet-piles cannot be driven. An artificial stratum of impervious soil may, however, be deposited on bare rock; in which case the sinking of the crib, and the subsequent operations will be the same as on a natural stratum. These expedients are evidently more or less applicable in other cases, where, to avoid repetition, they are not specially mentioned.

Fig 7 is another crib coffer-dam; in which the sides, instead of being planked longitudinally, as in the last instance, are sheathed with vertical sheet-piles *s*, driven after the crib is sunk. It is much inferior to the last, owing to its greater liability to leak. In one of this description, Fig 7, successfully used in 16 ft water, the dimensions of the crib were 34 ft by 80 ft. Along each long side were 7 uprights *t t*, 19 ft long, 12 ins square, 12¾ ft apart. Into each opposite pair of these were notched and held by dog-irons, 6 cross-braces *c c*, of 12 ins square. The distance between the two upper ones was 3 ft in the clear; gradually diminishing to 18 ins between the two lower ones, on account of the increased pressure of the water in descending. On the outside of the uprights, and opposite the ends of the braces, were bolted longi-



tudinal timbers to support the outside pressure against the 3-inch sheet-piling *ss*. Other longitudinal pieces *o o*, confine the heads of the sheet-piles to the top of the crib after they are driven. The feet of the sheet-piles were cut to an angle, as at *w*, to make them draw close to each other at bottom in driving.

The sheet-piles will drive in a far more regular and satisfactory manner, with the arrangement shown in Figs 8. Here *o o* are the uprights; *c c* are pairs of longitudinal

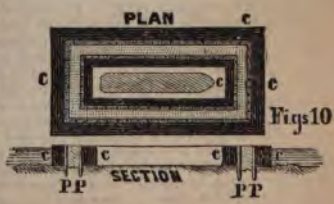
ed and bolted to the uprights, near both their tops and their feet; and intermediate points as may be desired. The sheet-piles I, are inserted e; and of course are guided during their descent, much more perfectly

current is too strong to permit the use of outside puddle, P, Fig 7, the offer-dam shown in Fig 9, is generally used; in which both sides of the otected from washing away. The space to be enclosed by the dam is sur- wo rows of firmly-driven main piles *p p*, on which the strength chiefly ey may be round. In deciding upon their number, it must be remem- ey may have to resist floating ice, or accidental blows from vessels, &c. ce to this, extra *fender*-piles may be driven. A little below the tops of s are bolted two outside longitudinal pieces *w w*, called *wales;* and oppo- two inner ones, as in the fig. The outer ones serve to support cross- which unite each pair of opposite piles, and steady them; and prevent ng apart by the pressure of the puddle P. The inner ones act as guides t-piles *s s*, while being driven; after which the heads of the sheet-piles them. In deep water these sheet-piles must be very stout, say 12 ins esist the pressure of the compacted puddle.

w, of plank, is often laid on top of the cross-pieces *t t*, for the use of the wheeling materials, &c. The puddle P is deposited in the water in the king, between the sheet-piles. It should be put in in layers, and com- ll as can be done without causing the sheet-piles to bulge, and thus open The bottom of the puddle-ditch should be deepened, as in the fig, in its, as it often does, of loose porous material which would allow water to ath it and the sheet-piles. This leaking *under* the dam is frequently a ch trouble and expense. Water will find its way readily through almost nd distance of clean coarse gravelly and pebbly bottom, unmixed with is also troublesome; and if a stratum of either should present itself ex- great depth, it will generally be expedient to resort to either simple or to caissons; with or without piles in either case, according to cir- But if such open gravel, or any other permeable or shifting material, quicksand, &c, is present in a stratum but a few feet in thickness, and stiff clay, or other safe material, leaking may be prevented, or at least d, by driving the sheeting-piles 2 or 3 ft into this last; and by deepening rench to the same extent. It may sometimes be better, and more con- lredge away the bad material entirely from all the space to be enclosed and for a short distance beyond, before commencing the construction of If the dam, Fig 9, is (as it should be) well provided with cross-braces, 7, extending across the enclosed area, the thickness or width *o o* of the not be more than 4 or 5 feet for shallow depths; or than 5 to 10 ft for great e its use is then merely to prevent *leaking*. But if there are no braces, ade wider, so as to resist *upsetting* bodily; and then, with good puddle, rule of thumb, be ¾ of the vertical depth *o l* below high water; except ives less than 4 ft: in which case make it 4 ft; unless more should be the use of the workmen, for depositing materials, &c. Or if the excavation nry is sunk deeper than the puddle, the dam must be wider; else it may the excavated pit.

vated inside of the dams may be ets by windlasses, or by hand, in es. The pumps may be worked steam, as the case may require; indlasses generally needed for r, stone, &c. More or less leak- be anticipated, notwithstanding n.

fer-dam is exposed to a violent eat danger from ice, &c, the ex- shown in Figs 10 may become two black rectangles *c c*, repre- of rough cribs filled with stone. sition: one row being enclosed ith a space several feet wide be- Sheet-piles *p p* are then driven posite faces of the two rows of puddle is deposited within the boxing thus provided for it, as shown in the fig.

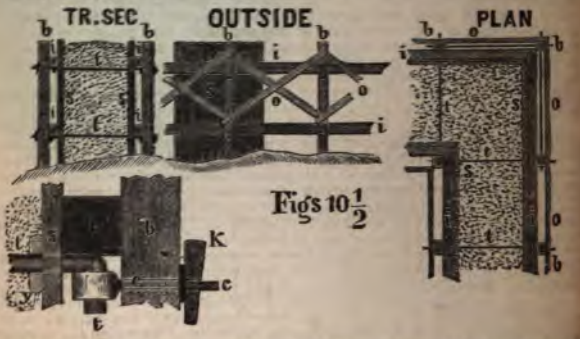


urrent is not strong enough to wash away gravel backing, we may, on rock especially, ce to be built on, by a single quadrangle of cribs sunk by stone; and after adopting prevent the gravel from being pressed in beneath the cribs, apply the backing.*

show the plan, outside view, and transverse section, to a scale of 20 ft to *offer-dam on rock*, in 8 to 9 ft water, used successfully on the Schuylkill

coarse gravel is entirely unfit for such purposes. A considerable proportion *or preventing leaks.*

The construction is very simple. Uprights *b*, about 1 ft square, and 10 ft apart from center to center along the sides of the dam; and 10 ft in the clear, transversely of the dam, support two lines of horizontal stringers, *i i*; inside of which are the two lines of sheeting-piles, *s s*, enclosing between them a width of 7 ft of gravel puddle. Two flat iron bars (*t t*, of the transverse section) tie together each pair of uprights *b b*. These bars are ⅝ inch thick, by 2½ ins deep, and 9 ft long. Their hooked ends fit into eye-bolts *c*, which pass through the uprights *b*; outside of which they are fastened by keys, *k* (see detail sketch.) Between the keys and *b*, were washers. At the corners of the dam (or point were additional tie-bars, as shown. A small band of straw, as seen at *y*, wrapped around the tie-bars just inside of the sheet-piles; and kept in place by the puddle; effectually prevented the leaking which generally proves so troublesome in such cases. The short oblique braces, *o o*, were merely spiked to the outside faces of the uprights *b*. They are not shown in the transverse section. This dam was built on shore; in sections 30 to 40 ft long. These were floated into place, and weighted down, sheet-piled, and puddled with gravel. The dam had sluices by which water was admitted when necessary for preventing the outside head from exceeding 9 ft. The lengths of the uprights *b b* were first found by careful soundings.



Valuable hints for coffer-dams may be taken from what is said under the head of "Dams," where Fig 1 affords useful suggestions for coffer-dams also, on rock in shallow water: p 583.

The proper mooring of large caissons, or cribs for coffer-dams, preparatory to sinking them, is sometimes troublesome, especially in strong currents. It may be necessary to drive clumps of piles; or to temporarily sink rough cribs filled with stones, to which to attach the long guide-ropes by which the manœuvring into position, &c, is done. Frequently dams are left standing after the work is done; if not in the way of navigation, or otherwise objectionable; inasmuch as the materials are rarely worth the expense of removal. But if removed, the piles should not be *drawn* out of the ground; but be *cut off* close to river bottom; for if drawn, the water entering their holes may soften the soil under the masonry. It is often expedient to drive two rows of piles from the dam to the shore, for supporting a gangway for the workmen, or even for horses and carts; or for a railway for the easy delivery of large stones, &c.

Coffer-dams to be sunk through soft bottom, to a firm soil, are successfully used in shape of a box of cribwork, either rectangular or circular, and without a bottom. This being strongly put together, and provided with proper temporary internal bracing, (to be gradually removed as the masonry is built up,) is floated into place, and after being loaded so as to rest on the soft bottom, is sunk by dredging out the soft material from inside. Additional loading will sometimes be required for overcoming the friction of the soil against the outside; or it may even become necessary to dredge away some of the outer material also. On rock bottom it may at times be expedient to drill holes in deep water, for receiving the ends of piles, or of iron rods, &c. This may be done by means of long drill-rods, working in an iron tube or pipe sunk as a guide to the rod; with its lower end over the spot to be bored. Or a diving bell may be used. Or a hollow cylinder, or box without top or bottom, long enough to reach above the surface, and having a broad tarpaulin flap or apron around its lower edge, to be covered with gravel to prevent leaking: may be sunk, and the water pumped out, to allow a workman to descend, and work in the open air.

Piles. When driven in close contact, as on Fig 11, for preventing leakage; for *confining puddle* in a coffer-dam; or for enclosing a piece of soft or sandy ground, to *prevent its spreading* when loaded; or if the outside soil should wash away from

Cost of piles delivered at wharf, Philada. 1873. Hemlock, 6 to 8 cts per lineal foot, Yellow pine, 10 to 15 cts, Southern yellow pine, 18 to 25 cts.

nd them, &c, they are called **sheet-piles.** Generally these are thinner than are wide; but fre- tly they are square; s large as bearing ; and are then called **e piles.** To make . drive tight to- r at foot, they nt obliquely as at occasionally, when n down to rock gh soft soil, their are in addition cut edge, as at *i*, so as become somewhat ed when they reach

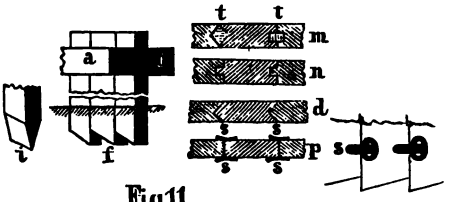


Fig 11

ock, and thus fit closer to its surface. Their heads are kept in line while driv- by means of either one or two longitudinal pieces *a* and *o*, called **wales** or *gers.* These wales are supported by *gauge-piles*, or *guide-piles*, previously driven e required line of the work, and several feet apart, for this purpose.

g-iron, d, Fig 12, of round iron, may also be used for keeping ges of the piles close at top to those previously driven, both g and after the driving. Its sharp ends, *c c*, being driven into s of the wales *w w*, (shown in plan,) it holds the descending firmly in place. At *n*, *d*, *p*, Fig 11, are other modes occasionally for keeping the piles in proper line. At *p*, the letters *s s* denote pieces of iron well screwed to the piles, a little above their feet, as guides; very rarely used. At *m* are shown wooden tongues metimes driven down between the piles after they themselves been driven; to assist in preventing leaks. In some cases -piles are employed without being driven. A trench is first e their full depth for receiving them; and the piles are simply d in these, which are then refilled. Closer joints can be secured in this manner than by driving.

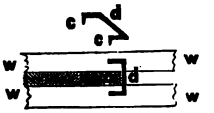


Fig 12

hen piles are intended to sustain loads on their tops, whether driven all their th into the ground, or only partly so, as in Fig 3, they are called **bearing es.** They are generally round; from 9 to 18 ins diam at top: and should be ght, and clear of bark and of projecting stubs. White pine, spruce, or even lock, answer very well in soft soils; good yellow pine for firmer ones; and hard , elm, beech, &c, for the more compact ones. They are usually driven from about to 4 ft apart each way, from center to center, depending on the character of the and the weight to be sustained. The tread-wheel is much more economical than winch for raising the hammer, when this is done by men. Morin found that work performed by men working 8 hours per day, was 3900 foot-pounds per man, minute by the tread-wheel; and only 2600 by a winch.

he gunpowder pile driver invented by Mr. Thomas Shaw,* the well-known nical engineer of Philada, is a very meritorious machine. The hammer is worked by small ridges of powder, placed one by one in a receptacle on top of the pile; and exploded by the ham- itself. It can readily make 30 to 40 blows of 5 to 10 ft. per minute; and, since the hammer does come into actual contact with the piles, it does not injure their heads at all; thus dispensing iron hoops, &c, for preserving them. When only a slight blow is required, a smaller cartridge sed. This machine, however, does not assist in raising the pile, and placing it in position, as is by ordinary steam pile-drivers; the latter, however, average but from 3 to 6 blows per minute.

he force in lbs with which a pile-hammer makes its blow upon the head of a pile, not be calculated. All rules for that purpose are founded in error. We must e depend upon experience alone. But, unfortunately the recorded facts are so , and so incompletely described, as not to furnish grounds for reliable rules. We never have rules by Maj Sanders, and others; and to these we venture to add one our own, which is also purely empirical. Molesworth's rule is Sanders.

hey differ very much. No rule can apply correctly to all conditions. The ground itself between piles, in most cases, supports a part of the load; although the whole of it is usually assigned to piles. Again, in very clayey soils, there is greater liability to sink somewhat with the lapse of , in consequence of the admission of water between the pile and the clay; thus diminishing the tion between them. The less firm the soil, the more will the piles be affected by tremors; which tend in time to cause sinking. In some cases this sinking will not be that of the piles settling per into the earth *around* them; but of that of the entire compacted mass of piles and earth into ch they were driven, settling down into the less dense mass *below* them. Squared piles, and ta- ing round ones, will not bear equal loads. Piles are sometimes blamed for settlements which are lly due to the crushing (flatways) of the timbers which rest immediately upon their heads.

In the fine *stone London bridge across* the Thames, each pile under some of the *rs sustains the very heavy load of* 80 tons. They are driven but 20 feet into the *blue London clay; and are placed* nearly 4 ft apart from center to center; whi

* *For cost and performance, see foot of page 322.*

is too much for such piers and arches. At 3 ft apart scant, they would have had to 45 tons to sustain. They are 1 foot in diameter at the middle of their length. Large settlements, some of them to the extent of about a foot, have occurred under these piers. Blackfriars bridge, in the same vicinity, exhibits the same defect. By some this is ascribed in both cases to the gradual admission of water between the clay and the piles, perhaps by capillary action of the piles themselves; or perhaps by direct leaking. It may, however, be owing in part to the crushing of the platforms on top of the piles; or to a bodily settlement of the entire mass of piled clay, into the unpiled clay beneath, under the immense load that rests upon it. This here amounts to 5½ tons per sq foot of area covered by a pier; and is probably too much to trust upon damp clay, when even the slightest sinking is prejudicial.

Maj J. Sanders, U. S. Engs, experimented largely at Fort Delaware; and in 1861 gave the following as reliable for ordinary pile driving. For the *safe* load for a common wooden pile, driven until it sinks through only small and nearly equal distances, under successive blows, divide the height of the fall in ins, by the small sinking at each blow in ins. Mult the quot by the weight of the hammer, ram, or monkey, in tons or pounds, as the case may be. Divide the prod by 8.

EXAMPLE. At the Chestnut St bridge, Philada, the greatest weight on any pile is 18 tons. Mr Kneass had the piles driven until they sank ¾, or .75 of an inch under each blow from a 1200 lb hammer, falling 20 ft. Was he safe in doing so? Here we have the fall in ins $= 20 \times 12 = 240$. And $\frac{240}{.75} = 320$; and $320 \times 1200 = 384000$ lbs; and $\frac{384000}{8} = 48000$ lbs, $= 21.4$ tons safe load by Maj Sanders' rule. Comparing this rule with some other instances, it would at first sight appear to be greatly in error. Thus the piles under some of the piers of the Royal Border bridge, England, support 70 tons each.* They were driven until they sank only $\frac{1}{20}$, or .05 of an inch per blow from a hammer of 1700 lbs, falling 16 ft. Here Maj Sanders' rule gives 2.4 tons as the perfectly safe load; which there is no reason to doubt is greatly in excess. It is the writer's opinion, however, that the piles did not actually sink, as was (and always is, in such cases) taken for granted by the observers; but that they were merely compressed or partially crushed by overdriving. Most of the piles were driven until they sank (?) only an inch under 150 blows; but we doubt whether they were any safer, or farther in the ground, than when they had received only one of them; and consider such extreme precaution worse than useless.

Our own rule is as follows. Mult together the cube rt of the fall in ft; the wt of hammer in lbs; and the decimal .023. Divide the prod by the last sinking in ins, + 1. The quot will be the extreme load that will be just at the point of causing more sinking. For the safe load take from one sixth to one half of this, according to circumstances. Or, as a formula,

$$\text{Extreme load in tons} = \frac{\text{Cube rt of fall in feet} \times \text{Wt of hammer in pounds} \times .023}{\text{Last sinking in inches} + 1}$$

Example. The same as the foregoing at Chestnut St Bridge. Here the cube rt of 20 ft fall is 2.714 ft. Hence we have

$$\text{Extreme load in tons} = \frac{2.714 \times 1200 \times .023}{.75 + 1} = \frac{74.9}{1.75} = 42.8 \text{ tons.}$$

Or say half of this, or 21.4 tons, the load for a safety of 2. Maj Sanders' rule also makes the safe load 21.4.† The actual one is 18 tons.

In some very recent experiments (1873) at Philada, a trial pile was driven 15 ft into soft river mud, by a 1600 lb hammer; its last sinking being 18 ins under a fall of 36 ft. Only 5 hours after it was driven it was loaded with 6.5 tons, which caused a sinking of but a very small fraction of an inch. Our rule gives 6.4 tons as the extreme load. Under 9 tons it sank .75 of an inch: and under 15 tons, 5 ft. By Maj Sanders rule its safe load would be 2.14 tons. By Mr. McAlpine's rule (variously quoted) the extreme load would be either 86.6 or 67.9 tons; or 11 to 14 times what ours gives. A German rule quoted by Gen Barnard gives for the extreme load, 13.7 tons, or fully twice the fact in this instance. At the Hull docks, England,

* *They were driven 30 to 40 ft into sand and gravel, in some cases wet. Fine was first tried; but it broomed and split so badly under the hard driving, that American elm was substituted, with success. The total height of masonry above them is 126 ft.; semicircular arches of 63½ ft span.*

† *But although Maj Sanders' rule and ours agree very well in this instance, they differ widely in some others. Thus, in the Hull docks, we are as 17 to 12; and much farther apart in the following*

e, driven 16 ft into alluvial mud, by a 1500 lb monkey, falling 24 ft, ch blow at the end of the driving. They support fully 20 tons; or se statements, 25, each. Our rule gives 33.2 tons for the extreme : a safety of 2. Maj Sanders gives for perfect safety 12.06 tons. For bridge, our rule gives 98.5 tons extreme load if there was no sink- s if there was an *actual* sinking of .05 inch per blow. One half of as, is our safe load; they actually bear 70.

ers consider a pile safe for a load of 25 tons, when it is driven to the s, falling 4 ft; our rule gives a safe load of 23.5 tons. They estimate not sinking more than .4 of an inch under 30 blows. At Neuilly s heaviest hammer weighed 2000 lbs: fall 5 ft. The piles sustain ur rule gives 39.3 tons safe load, if we take one half as safe.

proper load for safety, we think that not *more* than one half the by our rule should be taken for piles *thoroughly* driven in *firm* soils; nor more n in river mud or marsh; assuming as we have hitherto done, that their feet do

eat difference in the penetrability of different sands. Thus, in the pecial difficulty was found in driving piles 35 ft into deep wet sand; vet localities, piles of very tough wood, well shod with iron, cannot to sand, without being battered to pieces. The same difference has he case of screw-piles. At the Brandywine light-house these could ore than 10 ft into the clean wet sand. Stiff wet clay (and clean er very much in this respect. Generally they are penetrable to any with comparative ease; but we have seen stout hemlock piles bat- driving 6 ft through wet gravel; and Mr Rendel found that at Ply- d not by any force drive screw-piles more than about 5 ft into the as stiff as the London clay," on which the forementioned new London bridges were founded; and into which even ordinary wooden piles ; without special difficulty.

l with the sand or gravel facilitates driving very much; but before beginning an f piling, a few experimental ones should be driven, to remove doubt as to that may be anticipated. Mere boring will often be but a poor substitute for this. a heavy hammer with a low fall, drives more pleasantly than a light one with a hammer of ½ ton (1500 lbs) falling 25 ft, in a very strong ground, shattered the , (4500 lbs.) with 7 ft fall, drove them satisfactorily. More blows can be made in a low fall; and this gives less time for the soil to compact itself around the piles At times a pile may resist the hammer after sinking some distance; but start rest; or it may refuse a heavy hammer, and start under a lighter one. It may , and more rapidly afterward, from causes that may be difficult to discover. The etimes causes adjacent ones previously driven, to spring upward several feet. A favorable position when its foot rests upon rock, after its entire length has been irm soil, which affords perfect protection against its bending like an overloaded same time creates great friction against its sides; thus assisting much in sus- d thereby relieving the pressure upon the foot. A pile may rest upon rock, and for if driven through very soft soil, all the pressure is borne by the sharp point; e merely a column in a worse condition than a pillar with one rounded end. See ength of Iron Pillars. In such soils the piles need very little sharpening; indeed, without any; or even butt end down.

soil, especially if stony; or if the piles are pointed, and to rock; their feet should shoes of either wrought nd *b*, Figs 13; spiked to the f the iron straps *n*, forged ast iron, as at *c*, where the inverted cone, the wide flat ords a good bearing for the he pile-point. The dotted wrought-iron spike, well se- ne, which is cast around it; hoe to the pile. Regular

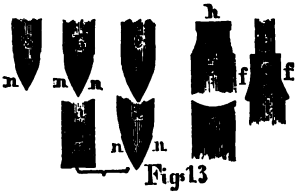


Fig 13

bridges, &c, the French engineers drive until there is no sinking under an 800 lb . Our rule here gives 31.5 tons as the extreme load; and 15.7 for a safety of 2. rs' safe load is only 2.68 tons.

f a floating steam pile driver, in Philada, scow 24 ft by 50 ft, e engine for driving, and one (to save time) for getting another pile ready; with about $6000; and $500 more will add a circular saw, &c. for sawing off piles at any s engineman, cook, and 4 or 5 others. Will burn about half a ton of coal per day. gravel, and sawing off, will average from 15 to 20 piles per day of 10 hours. In *many. On land about half* as many as in water. A gunpowder driver, scow, &c. *ere; but will do about twice* as much work as a steam driver.— To drive a pile *ses about one third* lb *of* powder: into gravel 4 times as much. One for use o *The U. S. Gunpowder* Pile Driving Co. No. 10 S. Delaware Avenue, Philad *s of machines; also for* dredging and piling.

wrought-iron shoes will generally weigh 18 to 30 lbs; but sheet iron s when the soil is but moderately compact; plate iron when more so; an or steel points, from 2 to 4 ins square at the butt, and 4 to 8 ins long compact and stony. Holes may be drilled in rock for receiving the poi and thus preventing them from slipping; by first driving down a tube to the drill, after the earth is cleaned out of the tube. To preserve t some extent from splitting under the blows of the hammer, they are rounded by a hoop *h*, Fig *d*; from ½ to 1 inch thick; and 1½ to 3 ins w are, however, sometimes but imperfect aids; for in hard driving the head split, and bulge out on all sides, frequently for many feet below the l over, the hoops often split open. The heads, therefore, often have to l pared off several times before the pile is completely driven; and allowa made for this loss in ordering piles for any given work; especially i Capt Turnbull, U S Top Eng, states that at the Potomac aqueduct, h were preserved from injury by the simple expedient of dishing them ou of about an inch, and covering them by a loose plate of sheet iron; s section at *e*, Figs 13. A very slight degree of brooming or crushing c materially diminishes the force of the ram. Piles may be driven thr loose rubble without much labor. Shaw's driver does not injure the head

When the head of a pile is to be driven beneath the water, a wooden punch, shown i Figs 13, may be used. The foot of this punch fits into the upper part of a casting square, according to the shape of the pile; and having a transverse partition *o o*. of the casting is fitted to the head of the pile *t*; and the hammer falls on top of the p driving piles vertically in very soft soil, to support retaining-walls, or other structur horizontal or inclined forces, care must be taken that these forces do not push over th selves; for in such soils piles are adapted to resist vertical forces only, unless they be inclination corresponding to the oblique force.

Iron piles and cylinders; Mitchell's screw-pile. various shapes has been much used in Europe for sheet-piles; especial tended to remain as a facing for the protection of concrete work, fille and against them.* Cast-iron cylinders, open at both ends, may be used piles; and may be cleaned out, and filled with concrete, if required. Th driving is greater than in solid piles, inasmuch as it takes place along bo and the outer surfaces. This may be diminished by gradually extractin soil as they go down. They require much care, and a lighter hammer than wooden ones, to prevent breaking; to which end a piece of wood interposed between the hammer and the pile; or the ram may be of w is better to use them in the shape of **screw cylinders,** which, mor them the advantage of a broad base.

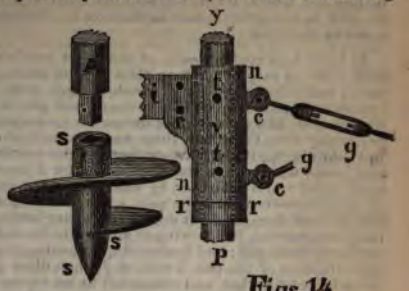
Mr Brunel's process. Mr Brunel experimented with an open cast-iron cylinder, 3 1½ ins thick; in lengths of 10 ft, connected together by internal socket and joggle join pins, and run with lead. It had a sharp-edged hoop or cutter at bottom; and a little turn of a screw, with a pitch of 7 ins, and projecting one foot all around the outside o By means of capstan bars and winches, he screwed this down through stiff clay and s rock, on the bank of a river. In descending this distance the cylinder made 142 revolu on an average about 5 ins at each. The time occupied in actually screwing was 48¼ h $1\frac{2}{10}$ ft per hour. There were, however, many long intervals of rest, for cleaning aw the inside. After resting, there was no great difficulty in restarting.† The next fig wi of the arrangement of the screw.

The screw-pile of Alexander Mitchell, Belfast, consists usually of a shaft, A, Figs 14, from 3 to 8 ins diam; and having at its foot a cast S S S, with a blade of from 18 ins to 5 ft diam. The screws used for l exposed to moderate seas, or heavy ice-fields, are ordinarily about 3 f 1½ turns or threads, and weigh about 600 lbs. The round rolled sha 5 to 8 ins diam. They are screwed down from 10 to 20 ft into clay, sand, about 30 to 40 men, pushing with 6 to 8 capstan bars, the ends of whic circle of about 30 to 40 ft diam. For this purpose a platform on piles ha to be prepared. In quiet water, this may be supported on scows; o moored may be used when the driving is easy; or the deck of a large well-hole in the center for the pile to pass through. Roughly made cribs, filled with stone and sunk, might support a platform in some pos platform must evidently be able to resist revolving horizontally und pushing force of the men at the capstan bars; and on this account i to drive screws to a sufficient depth, in clean compact sand, by means platform. The feet of the piles must be firmly secured to the screws

* *Cast iron, for any purpose, when intended to resist sea-water, should be close-white metal. In such, the small quantity of contained carbon is chemically combined; but in the darker or mottled irons it is mechanically combined, and such iron soo (somewhat like plumbago,) when exposed to sea-water. Hard white iron has been for at least 40 years without any deterioration; whether constantly under water, e and dry.*

† *See Minutes of the Proceedings of the Inst Civ Eng, vol 7, p 138.*

ifted out of them by the upward force of waves against the super-
t *y p*, Figs 14, is shown a mode of splicing or uniting the different
tions of a pile. The point of junction is at *v*; *r r* is a stout iron ring
the lower pile *p*,
r 18 ins below its
g cylindrical cast-
osing the ends of
rests on this ring,
through the piles,
this casting are
ections *c c c*, for at-
g, and beams *i*, &c,
bracing the struc-
to pile. The time
ired for driving a
2 to 10 hours, in
umstances.
Brandywine light-
sand-bank of very
vered 6 or 8 ft at
d from 11 to 13 ft
could not be forced
a fixed platform,
10 ft. At other places 20 ft in sand is reached without much trouble,
ud contains a good deal of mud.



Figs 14

es have been screwed about 40 feet into a mixture of clay and sand, with screws
y pass through small broken stone and coral rock without much difficulty; and will
ders of moderate size. Ordinarily, clay or sand will present no great obstruction;
either of them will do so. Perfectly pure clean sand, as a general rule, gives most
he Brandywine shoal the driving was aided by a spur and pinion placed as low as the
l; and the levers were worked by 30 men. The danger of twisting off the shaft is
ewing them. They are much used for the anchoring of chains for mooring buoys, &c.
screws, with short hollow shafts, make good durable supports for depot pillars, cranes,
ph poles, station signals in marine surveying, &c, &c. They can readily be unscrewed
horses or oxen may be used in driving large screws. The Brandywine light-house
rw-piles, which are surrounded by 30 others of 5 ins diam, as fenders. They have to
moderate seas, but immense fields of floating ice, miles in extent. An unfinished
estroyed by ice.

es Brunlee's process, first used for supporting (like trestles)
spans of the Levan viaduct, is well adapted to soft soils. At the site,
t was a deposit of a sandy detritus of calcareous rock, broken shells, &c;
70 ft below was a finely comminuted marl-dust, which when dry was
by the wind; and when wet, as it was here, formed quicksand of un-
. Upon this material the viaduct is founded by means of hollow cast-
unk about 20 ft into it. The piles, Fig 15, are 10 ins outer diam, 8½
½ in thick. At the base is a cast circular disk *d d*, or B T, 2 ft 6 ins
thick; strengthened by flanges *s s* on top of it. T is a top view, and
ne, of the disk: which forms a base for the pile,
at bottom, with the exception of a hole *t*, 3 ins
center. On the lower face B of the disk, are cast
jections *c c*, about an inch deep, called *cutters*.
inking of a pile, water is forced through the hole
f a long wrought-iron pipe, 2 ins diam, extending
he pile; its upper end being connected by means
tube with a force-pump driven by steam. The
ed the soft soil so as to allow the piles to descend
the first 9 or 10 ft through sand; but below this
was very slow, until the idea was adopted of adding
c; and at the same time giving the piles an alter-
motion. By this means the soil was loosened;
led the water forced through *t*, to either wash or
; and a pile could be sunk in 3 or 4 hours. Within
he wet sand would then settle and compact itself
utside of the pile, and hold it firm.
ion of sinking was much facilitated by keeping a
part of the weight of the pile hanging on the
hains. Without this precaution the piles some-

Fig 15

regularly; and occasionally so fast as to overpower the pumps. After
nally, each pile was driven about 2 ins deeper by short blows from a
Actual trial showed their *safe* sustaining power to be about 5 tons per

ering the pumps" is probably meant that the sudden sinking of the pile so comp

sq foot of area of *base* or *disk*. Each pier consists of 6 piles in a single row. Owing to their small diameter, which prevented men from entering them, the flanges for bolting the several lengths or sections together are on the outside. The longest piles are 20 ft in the ground; and 22 ft above it. Rubble stone was deposited to prevent scour by the tides.

At Lock Ken viaduct, each of the piers consists of two plain cylinders, open at both ends; of cast iron, 8 ft in diam; 1⅛ ins thick; in lengths of 6 ft, weighing 4 tons each; and bolted together by inside flanges, with iron cement between them. The cylinders stand 8 ft apart in the clear; and are in 36 ft water. "A strong staging was erected; and 4 guide-piles driven for each cylinder. The several lengths being previously bolted together, these were lowered into their places. Each cylinder sank by its own weight one or two ft through the top-mud, and then settled upon the sand and gravel which form the substratum for a great depth. Into this last they were sunk about 8 or 9 ft farther, by excavating the inside earth under water, by means of an inverted conical screw-pan, scoop, or dredger, of ¼ inch plate iron. This was 2 ft greatest diam, and 1 ft deep; and to its bottom was attached a screw about 1 ft long, for assisting in screwing it down into the soil. Its sides had openings for the entrance of the soil; and leather flaps, opening inward, to prevent its escape. From opposite sides of the pan, 3 rods of ¾ inch diam projected upward 4 feet, and were there forged together, and connected by an eye-and-bolt joint to a long rod or shaft, at the upper end of which was a four-armed cross-handle, by which the pan was screwed down by 4 men on the staging.

"When a pan was full, a slide which passed over the joint at the bottom was lifted; and the pan was raised by a tackle. This pan raised about 1 cub ft at a time. A smaller one of only 1 ft diam, and 1 ft deep, raising about ¼ cub ft, was used when the material was very hard. By this means the cylinders were sunk at the rate of from 2 to 18 ins per day. The slow rate of 2 ins was caused by stones, some of them of 50 lbs. These were first loosened by a screw-pick, which was a bar of iron 3 ft long, with circular arms 12 ins long projecting from the sides. After being loosened by this, the stones were raised by the pan. The expense of all this apparatus was very trifling; and the excavation was done easily and cheaply. After the excavation was finished, and the cylinder sunk, before pumping out the water, concrete (gravel 4, hydraulic cement 1 measure) was filled in to the depth of 12 feet, by means of a large pan with a movable bottom; and about 12 days were left it to harden. The water was then pumped out, and the masonry built in open air. In some of the cylinders, however, the water rose so fast, notwithstanding the 12 ft of concrete, that the pumps could not keep them clear; and 6 ft more of concrete had to be added in those. Finally random-stone, or rough dry rubble, was thrown in around the outsides of the cylinders, to preserve them from blows and undermining." * The masonry extends 20 ft above the cylinders, and above water.

The vacuum; and the plenum processes. We can barely allude to the general principles of these two modes of sinking large hollow iron cylinders. In the vacuum invented by Dr. Lawrence Holker Potts, of London, the cylinder *c*, Fig 16, while being sunk, is closed air-tight at top, by a trap-door, opening upward. A flexible pipe *p*, of India-rubber, long enough to adapt itself to the sinking of the cylinder, and provided with a stopcock *s*, leads from the cylinder to a vessel *v*; which may be placed on a raft, or a scow, or on land, as may suit circumstances. The cylinder being first stood up in position, as in the fig, the water is pumped out, and the interior soil removed if the cylinder has sunk some distance by its own weight. The cock *s* is then closed, and the air is drawn out from the vessel *v* by an air-pump. The cock is then opened, and most of the air in the cylinder rushes into the void vessel *v*; thus leaving the cylinder comparatively empty, and therefore less capable of resisting the downward pressure of the external air upon its top. This pressure, as is well known, amounts to nearly 15 lbs on every sq inch; or nearly 1 ton per sq ft of area of the top. Consequently the cylinder is forced downward in the bed of the river, by this amount of pressure, in addition to its own weight. At the same time, the pressure of the air upon the surface of the water is transmitted through the water to the soil around the open foot of the cylinder; so that if this soil be soft or semi-fluid, it will be pressed up into the nearly void cylinder, in which is no downward pressure to resist it. The descent varies from a few inches, to 4 or 5 ft each time. The process is then repeated, by admitting air again into the cylinder, opening the trap-door, removing the water and soil, as before, &c. Additional lengths of cylinder may be bolted on, by means of interior flanges, as the descent goes on.

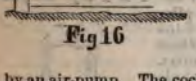


Fig 16

The Potts process is well adapted only to soft soils, and to wet sandy ones; but is not sufficiently powerful in very compact ones; nor does it answer where obstructions from bowlders, logs, &c, occur;

dated the soil immediately below it, that the water forced through the opening had not power to wash it away. The description (Min Proceedings Inst C E, vol 17, p 442) although full, is somewhat vague; and the language leaves it doubtful whether pile-drivers were actually used or not, though it is stated they were on hand. We cannot decide this point.

* *See Min Proc Inst C E, vol 21, p 258, for description by Edw'd I. Blythe, C E.*

oval of which requires men to enter the cylinder to its foot; which they cannot do in the rarefied
e pipe *p* should be of sufficient diam to allow the air to leave the cylinder rapidly, so that the
ressure may act upon the top as suddenly as possible.
e Goodwin Sands light-house, England, hollow cylinders 2½ ft in diam, were sunk 34 ft into
this process, in about 6 hours; where a steel bar could be driven only 8 ft by a sledge-ham-
thers, 12 ins in diam, have been sunk 16 ft into sand within less than an hour. In this last
e the air-pump had two barrels, 4½ ins diam, 16 inch stroke, worked by 4 men. The pipe *p*
lead, and only ½ inch diam.

e plenum process, invented by Mr John
s, England, consists in forcing air into the cylin-
, Fig 17, to such an extent as to force out the
compelling it to escape beneath the open foot,
he surrounding water. The interior of the cylin-
ing thus left dry to the bottom, men pass down it
en and remove the soil at and below its base. When
done, they leave; the compressed air is allowed to
; and the cylinder, being no longer sustained by
ward pressure of the compressed air beneath its
nks into the cavity, or the loosened material at its
Fig 17 shows the simple arrangement by which
men are enabled to enter or leave the cylinder,
ut allowing the compressed air to escape; as well
general principle of the entire process.

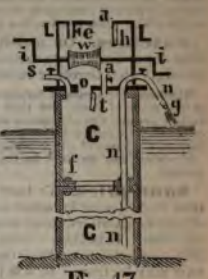


Fig 17

s a separate small upper chamber, called the *air-lock*, which
ved when a new length of pipe is to be added; and afterward
d and firmly bolted on. This chamber has a small air-tight
by which it can be entered from without; and another, *o*,
into the cylinder. The flaps, *f*, *h*, of both doors, open in-
r toward the cylinder. This chamber also has two stopcocks; one, *a*, in its floor, communi-
with the cylinder; and one *e*, above, communicating with the open air. At *s* is a bent tube,
th a cock, which passes air-tight through the side and the bottom of the air-lock. Through
ompressed air is forced into the cylinder, by an air force pump or condenser; and through it
e air is allowed to escape at a later period. A siphon is shown at *n n n*. A drum *w* is used
ting the excavated material from the bottom, to the air-lock; its axle *t t* passes air-tight through
-boxes in the sides of the lock; the hoisting being done by men outside. This is the general
ement employed by Mr W. J. McAlpine, C.E. of New York, at Harlem bridge; and from his
tion of it, ours has been condensed. The cylinders were there 6 ft diam, 1¼ ins thick, and in
of 9 ft, bolted together through inside flanges *f*, as the sinking went on. The air-lock is 6 ft
y nearly 6 ft high; with sides of boiler iron; and top and bottom of cast iron. See p 631.
suppose the cylinder C C to be let down, and steadied in position, as in the fig; and the air-
, to be adjusted on top of it. The next process is to force in air through the curved tube *s*;
f of the lower door *o*, and the cock *a*, being previously closed. As the compressed air accu-
s in the cylinder, it forces out the water; which escapes partly beneath the bottom of the cyl-
and partly by rising through the siphon *n n*, and flowing out at *g*. The door *o* being already
and that at *d* open, the air in the air-lock is in the same condition as that outside; so that
en can enter it readily. Having done so, they close the door *d*, and the cock *e*; and open the
through which condensed air from the cylinder rushes upward, soon filling the air-lock.
this is done, the flap *f* falls; and the workmen descend through the door *o* by a ladder, or by a
lowered by the drum *w*, to the bottom. Here they loosen and excavate the material as deep
can; and, filling it into a bucket or bag, they signal to those outside, who raise it to the air-
When done, they ascend to the air-lock, close the door *o*, and the cock *a*; and open the cock *e*,
h which the condensed air in the lock soon escapes, leaving the internal air the same as that
. The door *d* is then opened, the buckets of earth are removed, and the men go out. Finally
k at *s* is opened, the condensed air in the cylinder escapes through it to the outside air, and
inder sinks by its own weight into the cavity and loosened soil prepared for it at its base, and
is now forced up into the cylinder by the rush of the returning water. The process is then
d. The sinking will frequently amount to 10, 12 or more feet at one operation. Until depths
50 ft, most men can endure the pressure of the condensed air; but as the depth increases
comes more difficult, and positively dangerous to life. Cast-iron cylinders 15 ft diam; and
s of even 39 ft by 14 ft, have been thus sunk; but at times at great expense and trouble.

**llow cylinders, or other forms of brickwork or ma-
y,** with a strong curb or open ring of timber or iron beneath them, may be
ally sunk by undermining and excavating from the inside; and form very sta-
undations. Under water this may be done by properly shaped scoops, with or
ut the aid of the diving-bell, according to the depth, &c. On land it will often
most economical and satisfactory mode, especially in firm soils. The descent
e assisted by loading them, if, as sometimes happens, the friction of their sides
t the earth outside prevents their sinking by their own weight. A brick cyl-
46 ft outer diam, walls 3 ft thick, has been sunk 40 ft in dry sand and gravel,
ut any difficulty. It was built 18 ft high, (on a wooden curb 21 ins thick,) and
ed 300 tons before the sinking was begun. The interior earth was excavated
, so that the sinking was about 1 ft per day; the walls being built up as it sank.

e banks of the Rhine, for a coal shaft, a brick cylinder 25½ feet diam was first thus sunk by
weight 76 ft through sand and gravel; then an interior one, 15 ft diam, was sunk in the same
the depth of 256 ft below the surface; of which depth all the 180 ft below the first cylinder
ining quicksand. At 256 ft friction rendered the cylinder immovable. The quicksand wa
by boring; no pumping was done; but the water was permitted to keep the cylinder co
ll.

The entire foundation for a large pier of masonry has been sunk in this manner, in a single mass; a sufficient number of vertical openings being left in it for the workmen to descend, or for tools to be inserted for undermining. This is generally a very slow and tedious operation, especially under water. It may often be expedited by diving-bells or by diving-dresses. It will generally be better to make the mass wider at bottom than above it, so as to diminish friction against the outside earth. On land, water may at times be used for softening the bottom earth. By keeping the interior of such hollow masonry dry, it may even be *built downward* from the surface; by undermining only a portion of its circumference at a time, filling said portion with masonry, and then removing and filling the other portion; and so on in successive stages of 2 or 3 ft downward at a time. This mode may be adopted also when friction has stopped the sinking of a mass by its own weight when undermined.

Fascines. On marshy or wet quicksand bottoms, foundations may be laid by first depositing large areas of layers of fascines, or stout twigs and small branches, strongly tied together in bundles from 6 to 12 ft long, and from 6 ins to 2 ft in diam.

The layers or strata of bundles should cross each other. A kind of floating raft or large mattress is first made of these, and then sunk to the bottom by being loaded with earth, gravel, stones, &c. In this manner the abutments and piers of the great suspension bridge at Kieff, in Russia, with spans of 440 ft, were founded in 1852, on a shifting quicksand. There the fascine mattresses extend 100 ft beyond the bases of the masonry which rests upon them.

Fascines may be used in the same way for sustaining railway embankments, &c, over marshy ground.

Sand-piles. We have already alluded to the use of sand well rammed in layers into trenches or foundation pits; but it may also be used in soft soils, in the shape of piles. A short stout wooden pile is first driven 5 to 10 feet or more, according to the case. It is then drawn out, and the hole is filled with wet sand well rammed. The pile is then again driven in another place, and the process repeated. The intervals may be from 1 to 3 ft in the clear. Platforms may be used on these piles as on wooden ones. If the sand is not put in wet, it will be in danger of afterward sinking from rain or spring water. In this case, as with fascines, it is well to test the foundation by means of trial loads. Some settlement must inevitably take place until all the parts come to a full bearing; but it will be *comparatively* trifling. The same occurs in every large work to some extent; as in a roof or arch of great span, whether of wood, iron, or masonry; so also with all tall piers, walls, &c, &c. Sandy foundations under water should be surrounded by stout well-driven sheet-piling, to prevent the enclosed sand from running out in case the outer sand is washed away; and should also be defended by a deposit of random-stone.

On very treacherous bottoms under water, small artificial islands of good soil have been deposited; and the masonry founded upon them. Canal locks and other structures may at times be advantageously founded in this way in marshy soils. If necessary, a depth of several feet of the bad soil may be dredged out before the firmer soil is deposited; and the latter may be weighted by a trial load to test its stability.

The mode of laying a foundation under water, by building the masonry upon a timber platform above water, upheld by strong screws, and gradually lowered into the water as the work is finished in the open air, a course or two at a time, has of late been much employed with entire success, in large bridge-piers in deep water. It however is not new. It was suggested more than 100 years ago by Belidor.

Piles are driven 6 to 10 ft apart around the space to be occupied by the pier; having their tops connected by heavy timber cap-pieces. These last uphold the screws, which work through them. The whole is braced against lateral motion.

The shaded part of Fig 18 shows a transverse section of the caisson of yellow-pine timber and cement, for the foundation of the Brooklyn tower of East River (N Y) suspension bridge, of 1600 ft clear span. It is 168 ft long at bottom, and 102 ft wide. A longitudinal section resembles the transverse one, except in being longer, and in showing more shafts J. Of these there are 6, arranged in pairs, for expedition and as a precaution against accident. Namely, two water-shafts J, each 7 ft by 6½ ft across, for removing by buckets and hoisting apparatus, the material excavated beneath the caisson; together with such water as may accumulate at *o o*; two air-shafts of 21 ins diam, through which air is forced from above, to expel the water from the chamber C S S D below the caisson, so as to allow the laborers to work there at undermining; the expelled water escaping under the foot C D of the caisson, into the river; and two supply shafts of 42 ins diam, for admitting laborers, tools, &c. *The several shafts of course* have air-chambers on top, on the same principle as *Fig 17, to prevent the* escape of the compressed air in *s s*.

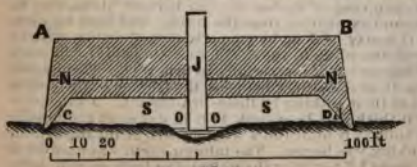


Fig 18

The shafts are of ¼ inch boiler iron. The foot C D, nine timbers high, is continuous, extending entirely around the caisson; its bottom is shod with cast iron; its four corners are strengthened by wooden knees 20 ft long.

From the bottom, up to the line N, N, 14 ft, the caisson is built of horizontal layers of timbers one foot square; the layers crossing each other at right angles; and the timbers of each layer touching each other well fitted and bolted together; and all the joints filled with pitch. To aid in preventing leakage, the nuts and heads of the screws have India-rubber washers; also all outside seams, as well as all the seams of the layer of timbers N, N, are thoroughly calked; and a layer of tin, enclosed between two layers of felt, is placed outside of each outer joint; and over the entire top of the layer next below N, N.

When the caisson was built up to N, N, on land, it was launched, floated into position, and anchored; after which were added for sinking it, fifteen courses of timbers one ft square; and laid one ft apart in the clear; with the intervals filled with concrete. The top course A B is of solid timber, to serve as a floor for supporting machinery, &c. It will be sunk some feet below the very bottom of the river, in order to avoid the teredo.

Cribs are sunk outside of the caisson, to form temporary wharves for boats carrying away excavated material; and for vessels bringing stone, &c.

When the caisson was sunk, and the water forced out from the chamber or space C S S D, workmen began to excavate uniformly the enclosed area of river bottom, so as to allow the caisson to descend slowly until it reached a firm substratum. The space C S S D, as well as the shafts, was then filled up solid with concrete masonry. A coffer-dam will be built on top of the caisson; and in it the regular masonry of the tower will be started. The total height of this tower, including the caisson, will be about 300 ft.

A CLUMP OF PILES WELL DRIVEN; and then enclosed by an iron cylinder sunk to a firm bearing, and filled with concrete, is an excellent foundation. The piles may extend to the top of the cylinder, and thus be enclosed in the concrete. Such an arrangement has been patented by S. B. Cushing, C. E., Providence, R. I.

COST OF DREDGING.

DREDGING is generally done by skilled contractors, who own the requisite machines, scows or lighters, &c; and who make it a specialty. It is necessary to specify whether the dredged material is to be measured in place before it is loosened; or after being deposited in the scow; because it occupies more bulk after being dredged. It was found, in the extensive dredgings for deepening the River St Lawrence through the Lake of St Peter, that on an average a cub yd of tolerably stiff mud in place, makes 1.4 yds in the scow; or 1 in the scow, makes .715 in place. Also stipulate whether the removal of bowlders, sunken trees, &c, is to constitute an extra. These often require sawing and blasting under water. The cost per cub yd for dredging varies much with the depth of water; the quantity and character of the material; the dist to which it has to be removed; whether it can be at once discharged from the machine by means of projecting side-shoots or slides; or must be discharged into scows, to be removed to a short dist by poling, or to a greater dist by steam tugs; whether it can be *dropped* or dumped into deep water by means of flap or trap doors in the bottom of the hoppers of the scows; or must be *shovelled* from the scows *into shallow water*, (at say 4 to 8 cts per yd;) or *upon land*, (at say from 6 to 10 or 20 cts for the shovelling alone, or shovelling and wheeling, as the case may be;) whether much time must be consumed in moving the machine forward frequently, as when the excavation is narrow, and of but little depth; as in deepening a canal, &c; whether many bowlders and sunken trees are to be lifted; whether interruptions may occur from waves in storms; whether fuel can be readily obtained, &c, &c. These considerations may make the cost per cub yd in one case from 2 to 4 times as great as in another. The *actual cost* of deepening a ship-channel through Lake St Peter, to 18 ft, from its original depth of 11 ft,* for several miles through moderately stiff mud, was 14 cts per cub yd in place, or 10 cts in the scows; including removing the material by steam tugs to a dist of about ½ a mile, and dropping it into deep water. This includes repairs of plant of all kinds, but no profit. It was a favorable case. When the buckets work in deep water they do not become so well filled as when the water is shallower, because they have a more vertical movement, and, therefore, do not scrape along as great a distance of the bottom. Hence one reason why deep dredging costs more per yard; in addition to having to be lifted through a greater height. Perhaps the following table is tolerably approximate for large works in ordinary mud, sand, or gravel; assuming the plant to have been paid for by the company; and that common labor costs $1 per day.

* *Since deepened to more than 20 ft.* At present (1873) prices of labor, &c, in the U. S., it would cost about twice as much, without allowing profit to contractor.

Table of actual cost of dredging on a large scale; including dropping the material into scows, alongside; or into side-shoots, on board. Common labor $1 per day. Repairs of plant are included; but no profit to contractor. (Original.)

Depth in Ft.	Cts per Yard, in place.	Cts per Yard, in scow.	Depth in Ft.	Cts per Yard, in place.	Cts per Yard, in scow.
Less than 10	8.4	6	25 to 30	18.2	13
10 to 15	9.8	7	30 to 35	25.2	18
15 to 20	11.2	8	35 to 40	35.0	25
20 to 25	14.0	10			

For towing of the scows by steam tugs to a dist of ¼ mile, and dropping the mud into deep water, add 4 cts per yard in the scow; for ½ mile, 6 cts; for ¾ mile, 8 cts; for 1 mile, 10 cts. Add profit to contractor. On a small scale work is done to a less advantage; and a corresponding increase must be made in these prices. Also, if the contractor himself furnishes the dredgers and plant, a still further addition must be made. It is evident that the subject admits of no great precision. Small jobs, even in favorable material, but in inconvenient positions, may readily cost two or three times as much per yd as the above; and in very hard material, as in cemented gravel and clay, four or five times as much for the *dredging*. The cost of *towing*, however, will remain as before, if wages are the same.

At present wages, 1873, if a contractor provides all the plant for dredging and towing, the total cost, including his profit, will be about 3 times the above table.

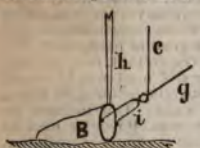
The cost of dredgers, tugs, &c, will vary of course with their capabilities, strength of construction, style of finish, whether having accommodations for the men to live on board or not, &c. When for use in salt water, the bottoms of both dredgers and scows should be coppered, to protect them from sea-worms; and if occasionally exposed to high waves, both should be extra strong. The most powerful machines on the St Lawrence cost about $45000 each; and removed in 10 working hours on an average about 1800 cub yds in place, or 2520 in the scows. Good machines, capable, under similar circumstances, of doing as much, may, however, be built for about $25000 to $30000. To remove this quantity to a dist of ½ to 1 mile, would require two steam tugs, costing about $6000 to $10000 each; and 4 to 6 scows, (some to be loading while others are away,) holding from 30 to 60 cub yds each; and costing from $800 to $1500 each at the shop. Scows with two hoppers are best. Such a dredger would require at least 8 or 10 men, including captain, engineer, fireman, and cook. Each tug 4 or 5 men; and each scow 2 men. The engineer should be a blacksmith; or a blacksmith should be added. In certain cases a physician, clerk, assistant engineer, &c, may be needed.

Dredgers are often built on the principle of the Yankee Excavator, with but a single bucket or dipper, of from 1 to 2 cub yds capacity. Hull about 25 by 60 ft. Draft 3 ft. Cylinder about 7 or 8 ins diam; 15 to 18 inch stroke; ordinary working pressure 50 to 80 lbs per sq inch, according to hardness of material. Cost $8000 to $12000. Will raise as an average days' work (10 hours) from 200 to 500 yds in place, or 280 to 700 in the scow, according to the depth, nature of the material, &c. Requires 5 or 7 men in all aboard, including cook. Burn ½ to 1 ton of coal daily. Tolerably large bowlders, and sunken logs, can be raised by the dipper.*

When the material is hard and compacted, the buckets of dredgers should be armed with strong steel teeth projecting from their cutting edge. On arriving at such material, every alternate bucket is sometimes unshipped. By arranging the buckets so as to dredge a few feet in advance of the hull, low tongues of dry land may be cut away; the machine thus digging its own channel. The daily work in such cases will not average half as much as in wet soil.

On small operations, dredgers may be worked by two or more horses, instead of by steam; and will answer very well in soft material; or even in moderately hard, by reducing the size and number of the buckets. A two-horse machine will raise from 50 to 100 yards of ordinary mud in place, or 70 to 140 in the scow, per day, at from 12 to 15 ft depth.

Soft material in small quantity, and at moderate depth, may be removed by the slow and expensive mode of the **bag-scoop,** or **bag-spoon.**



This is simply a bag B, made of canvas or leather, and having its mouth surrounded by an oval iron ring, the lower part of which is sharpened to form a cutting edge. It has a fixed handle *h*, and a swivel handle *i*. One man pushes the bag down into the mud by *h*, while another pulls it along by the rope *g*; and when filled, another raises it by the rope *c*, and empties it. If the bag is large, a windlass may be used for raising it. The men may work from a scow or raft properly anchored. Or a long-handled metal spoon, shaped like a deeply-dished hoe, may be used by only one man; or a larger spoon may be guided by a man, and dragged forward and backward by a horse walking in a circle on the scow, &c, &c.

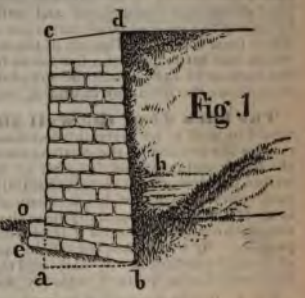
The weight of a cub yd of wet dredged mud, pure sand, or gravel, averages about 1½ tons; say 111 lbs per cub ft; muddy gravel, full 1½ tons; say 125 lbs per cub ft. Pure sand or gravel dredge easily; also beds of shells. Wet dredged clay *will slide down* a shoot inclined at from 1 to 5, to 1 to 3, according to its freedom *from sand, &c; but* wet sand or gravel will not slide down even 3 to 1, without a free *flow of water to aid it;* otherwise it requires much pushing.

* *The writer has seen cases in which a circular saw for sawing large logs in deep water, would have been a very useful addition to a dredger. It should be worked by steam; and be adjustable to different depths.*

RETAINING-WALLS.

. We here speak only of walls sustaining *earth;* for those sustaining e Hydrostatics. **A retaining-wall** is one for sustaining the pres sand, or other *filling* or *backing,** deposited behind it after it is built; in n to a **face-wall,** which is a similar structure for preventing the fall of ich is in its undisturbed natural position, but in which a vert or inclined been excavated. The earth is then in so consolidated a condition as to exert o lateral pres. Consequently a face-wall may generally be thinner than a one.

wever, will depend upon the nature and the strata in which the face is cut. If are of rock, with interposed beds of clay, nd; and if they dip or incline toward the y require to be of far greater thickness rdinary retaining-wall; because when the of earth become softened by infiltrating act as lubrics, like soap, or tallow, to fa- sliding of the rock strata; and thus bring us pres against the wall. Or the rock may otion by the action of frost upon the clay as sometimes occurs, by the tremor pro- assing trains. Even if there be no rock, strata of *soil* dip toward the wall, there s be danger of a similar result; and addi- autions must be adopted, especially when reach to a much greater height than the



rience, rather than theo- t be our guide in the building of ds of wall. We recommend that hickness *a b*, Fig 1, at the base of a nearly vert retaining-wall *c d b a*, stains a backing of either sand, gravel, or earth, level with its top *c d*, fig, should not be less than the following, in railroad practice.

n the backing is deposited loosely, as usual, *as when from carts, cars, &c,*

cut-stone, or of first-class large ranged rubble, in mortar..... .35 *of its entire vert height* d b.
ood common scabbled mortar-rubble, or brick. .4 " " " "
well-scabbled dry rubble..... .5 " " " "

n the backing is somewhat consolidated in hor layers, hese thicknesses *may be* reduced about $\frac{1}{10}$ part.

ffset, or **footing,** *o e*, in front of the wall, is not included in these thick-

however, the backing is a pure clean sand, or gravel, we should use only the larger dimen- smuch as the tremor, caused by passing trains, would neutralize any supposed advantage ning materials so devoid of cohesion. Such sand may be rammed with much advantage rpose of compacting it in foundations; but a diff principle is involved in that case. When ven with cohesive earths, with a view of saving masonry in retaining-walls, it is probable xpense will generally be found quite equal to that of the masonry saved. See Rem 4, p 339.

e *a b* in Fig 1, is $\frac{3}{10}$ of the height *b d*. In the foregoing thicknesses at base, the back *d b* l is supposed to be vert; and the face *c a* either vert, or battered (sloped or inclined back- n extent not exceeding about 1½ inches to a foot; which limit it is rarely advisable to *ex- ercise*, owing to the bad effect of rain, &c, upon the mortar when the batter is great. The vert wall (that is, one in which both face and back are vert) need not in fact be as thick as battered face; but when the batter does not exceed 1½ inches to a foot, the diff is very e Table 3, Art 7, p 338.

. **A mixture of sand, or earth, with a large proportion** BOWLDERS, paving pebbles, &c, will weigh considerably more than the materials ordinarily acking; and will exert a greater pres against the wall; the thickness of which should be say about ⅕th to ⅙th part, when such backing has to be used.

. The wall will be stronger if all the **courses of masonry be laid n inclination inward,** as at *o e b*; especially if of dry masonry, e cannot be allowed (as it always should be, when practicable) for the mor- t properly, before the backing is deposited behind it. The object of 'inclin-

eference to such walls, the terms filling and *backing* have another application. Thus, if th *e, c o, of the wall, is, for* the sake of appearance, built of cut-stone, while the other p *masonry, this last* is also called backing, or filling.

ing the courses, is to place the joints more nearly at right angles to the direction *f* P, Figs 6, 7, and 8, of the pres against the back of the wall; and thus diminish the tendency of the stones to slide on one another, and cause the wall to bulge. See Art 19 of Force in Rigid Bodies. When the courses are hor, there is nothing to prevent this sliding, except the friction of the stones, one upon the other, when of dry masonry; or friction and the mortar, when the last is used. But if, as is frequently the case, (especially in thick and hastily built walls,) this has not had time to harden properly, it will oppose but little resistance to sliding. But when the courses are inclined, they cannot *slide*, without at the same time being *lifted up* the inclined planes formed by themselves. In retaining-walls, as in the abuts of important arches, the engineer should place as little dependence as possible upon mortar; but should rely more upon the position of the joints, for stability.

An objection to this inclining of the joints in *dry* (without mortar) walls, is that rain-water, falling on the battered face, is thereby carried inward to the earth backing; which thus becomes soft, and settles. This may be in a great measure obviated by laying the outer or face-courses hor; or by using mortar for a depth of only about a foot from the face. The top of the wall should be protected by a coping *c d*, Fig 1, which had better project a few ins in front. After the masonry has been built up to the surface of the ground, the foundation pit should be filled up; and it is well to consolidate the filling by ramming, especially in front of the wall.

The back *d b* of the wall should be left rough. In brickwork it would be well to let every third or fourth course project an inch or two. This increases the friction of the earth against the back, and thus causes the resultant of the forces acting behind the wall to become more nearly vert; and to fall farther within the base, giving increased stability. It also conduces to strength not to make each course of uniform height throughout the thickness of the wall; but to have some of the stones (especially near the back) sufficiently high to reach up through two or three courses; even if it becomes necessary to place them on edge for this purpose. By this means the whole masonry becomes more effectually interlocked, or bonded together as one mass; and therefore less liable to bulge. Very thick walls may consist of a facing of masonry, and a backing of concrete.

REM. 3. It is the pres itself of the earth against the back, that creates the friction, which in turn modifies the action of the pres; as the wt or pres of a body upon an inclined plane produces friction between the body and the plane, sufficient, perhaps, to prevent the body from sliding down it. A retaining-wall is *overthrown* by being made to revolve around its outer toe or edge *e*, Fig 1, as a fulcrum, or turning-point; but in order thus to revolve, its back must first plainly rise; and in doing so must rub against the backing, and thus encounter and overcome this friction. The friction exists the same, whether the wall stands firm or not; as in the case of the body on an inclined plane; the only diff is that in one case it *prevents* motion; and in the other only *retards* it.

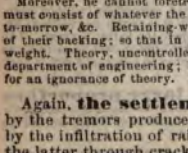


Fig 2

REM. 4. When the wall is too thin, it will generally fail by **bulging** outward, at about ⅓ of its height above the ground, as at *a*, in Fig 2. A slight bulging in a new wall does not *necessarily* prove it to be actually unsafe. It is generally due to the newness of the mortar, and to the greater pres exerted by the fresh backing; and will often cease to increase after a few months. It need not excite apprehension if it does not exceed ¼ inch for each foot in thickness at *a*. See Remark 3, Art 7, p 339.

Art. 2. The young engineer need not in practice concern himself particularly about the PRECIS SP GRAV OF HIS BACKING, or about the ANGLE OF SLOPE at which it will stand; for the material which he deposits behind his wall one day, may be dry and incoherent, so as to slope at 1½ to 1; the next day rain may convert it into liquid mud, seeking its own level, like water; the next it may be ice capable of sustaining a considerable load, as a vert pillar.

Moreover, he cannot foretell what may be the nature of his backing; for, as a general rule, this must consist of whatever the adjacent excavation may produce from time to time; sand to-day, rock to-morrow, &c. Retaining-walls are therefore usually built before the engineer knows the character of their backing; so that in practice, these theoretical considerations have comparatively but little weight. Theory, uncontrolled by observation and common sense, will lead to great errors in every department of engineering; but, on the other hand, no amount of experience alone will compensate for an ignorance of theory. The two must go hand-in-hand.

Again, **the settlement of the backing under its own wt,** aided by the tremors produced by heavy trains at high speed; its expansion by frost, or by the infiltration of rain; the hydrostatic pressure arising from the admission of the latter through cracks produced in the backing during long droughts; as well as *its lubricating* action upon it, (diminishing its friction, and giving it a tendency to *slide*,) &c, exert at times quite as powerful an overturning tendency as the legitimate *theoretical pres does*. The action of these agencies is gradual. Careful observation *of retaining-walls* year after year, will often show that their battered faces are becoming *vertical*. Then they will begin to incline outward; and eventually the wall *will fail*.

ning the theoretical views advanced by Professor Moseley to be correct as s, the thicknesses which we have recommended in Art 1, for mortar walls, ond to from 7 to 14 times; and for dry walls about 10 to 20 times, the pres d by him; and we do not consider them greater than experience has shown ecessary. See Table 3. Retaining-walls designed by good engineers, but in e accordance with theory, (which assumes that a resistance equal to twice oretical pres is sufficient,) have failed; and the inference is fair that many of hich stand have too small a coefficient of safety.

t is, (or at least so it appears to us,) there must be defects in the theoretical assumptions of the most prominent writers who give practical rules on this subject. Thus Poncelet, who is at their head, states that his tables, for practical use, give thicknesses of base for sus- $\frac{8}{10}$ times the theoretical pres; and this he considers amply safe. Yet, for a vert wall of cut his base for sustaining dry sand level with the top, as in Fig 1, is .35 of the vert height; rick, .45. But the writer found that *when not subject to tremor*, a wooden model of a vert ghing but 28 lbs per cub ft, and with a base of .35 of its height, balanced perfectly dry sand t 1½ to 1, and weighing 89 lbs per cub ft.

THE RESISTANCE OF SIMILAR WALLS, OF THE SAME DIMENSIONS, S THEIR SPECIFIC GRAVITIES; and, since granite weighs about 165 ub foot, or 6 times as much as our model, it follows, we conceive, all of that material, with a base of .35 of its height, must have nce of 6 times any *true theoretical* pres, instead of only 1.8 nd that his brick wall must have about 15 times the mere bal- esistance. Our experiments were made in an upper room of a built dwelling; and we found that the tremor produced by pass- les in the street, by the shutting of doors, and walking about t, sufficed to gradually produce leaning in walls of considerably n twice the mere balancing stability while quiet; and it appears t the injurious effects of a heavy train would be comparatively great upon an actual retaining-wall, supporting so incohesive al as dry sand.

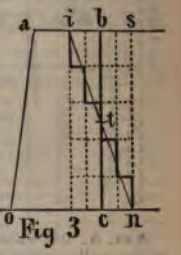


Fig 3

therefore, Poncelet's wall is in this instance sufficiently stable tice, it seems to us that his *theory*, which neglects the effect of &c, must be defective. He also gives $\frac{1}{5}$ of the height as a suf- safe thickness for a vert granite wall supporting *stiff earth;* but ct that very few engineers would be willing to trust to that pre- when, as usual, the earth is dumped in from carts, or cars; espe- ring a rainy period. If deposited, and consolidated in layers, ould scarcely assign any thickness for the wall; for the backing thus becomes, as it were, a unburnt brick, exerting no hor thrust; and requiring nothing but protection from atmospheric , to insure its stability without any *retaining*-wall. It is with great diffidence, and distrust pinions, that we venture to express doubts respecting the assumptions of so profound an in- r and writer as Poncelet; and we do so only with the hope that the views of more compe- sons than ourselves, may be thereby elicited. Our own have no better foundation than ex- ts with wooden and brick models, by ourselves; combined with observation of actual walls.

. 3. After a wall *a b c o*, Fig 3, with a vert back, has been proportioned by le in Art 1, it may be **converted into one, with an offsetted** , as *a i n o*. This will present greater resistance to overturning; and yet con- more material. Thus, through the center *t* of the back, draw any line *i n*; draw *n s*, vert; divide *i s* into any *even* number of equal parts; (in the fig re 4;) and divide *s n*, into *one more* equal parts; (in the fig there are 5.) From nts of division draw hor, and vert lines, for forming the offsets, as in the fig. e offsetted wall, the cen of grav is thrown farther back from the toe *o*, than other, thus giving it increased leverage and resistance; but within ordinary al limits, the diff is very small; and since the triangle of supported earth is than when the back is vert, its pres is also greater; so that probably no ap- le advantage attends *that* consideration. **The increase of thickness the base, diminishes, however, the age *v a*, Fig 8, of the pres *f* P, of the** against the back. The *center of pressure* of es is in both cases at ⅓ the vert height, meas- om the bottom; and it is therefore plain that her back from the front it is applied, the shorter *a* become. Moreover, in the offsetted back, the n of the pres becomes more nearly vert than he back is upright; for it will be at right angles line *i n*; see Remark 1, Art 6. It is to these rather than to the throwing back of the cent , that the offsetted wall owes its increase of y over one with a vert back.

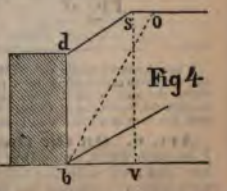


Fig 4

. 4. When, as in Fig 4, the backing is higher than the *and slopes away from its inner* edge *d*, at the natural slope d s, of 1½ to 1 *nfident that the following* thicknesses at base will at least be found sufficie

for vert walls with sand. They are deduced from the experiments just alluded to, and are but rude approximations, with no scientific basis. We should not have inserted them, but for the fact that we know of no others for this case.

The first column contains the vert height $s\,v$, of the earth, as compared with the vert height of the wall; which latter is assumed to be 1; so that the table begins with backing of the same height as the wall, as in Fig 1. These vert walls may be changed to others, with battered faces, by Art 8; or without any such proceeding, their faces may be battered to any extent not exceeding 1½ inches to a foot, or 1 in 8, without sensibly affecting their stability, without increasing the base.

TABLE 1. (Original.)

Total height of the earth, compared with the height of the wall.	Wall of Cut Stone, in Mortar.	Good Mortar Rubble, or Brick.	Wall of good dry Rubble.	Total height of the earth, compared with the height of the wall.	Wall of Cut Stone in Mortar.	Good Mortar Rubble, or Brick.	Wall of good dry Rubble.
1.	.35	.40	.50	2.	.58	.63	.73
1.1	.42	.47	.57	2.5	.60	.65	.75
1.2	.46	.51	.61	3.	.62	.67	.77
1.3	.49	.54	.64	4.	.63	.68	.78
1.4	.51	.56	.66	6.	.64	.69	.79
1.5	.52	.57	.67	9.	.65	.70	.80
1.6	.54	.59	.69	14.	.66	.71	.81
1.7	.55	.60	.70	25.			
1.8	.56	.61	.71	or more	.68	.73	.83

Art. 5. But when the slope $n\,m$, Fig 5, of 1½ to 1, starts from the *outer* edge n of the wall, greater thickness is required. Poncelet gives the following for this case, for *dry sand*.

TABLE 2.

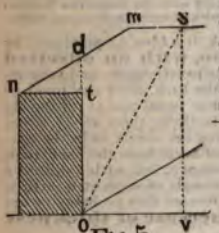


Fig 5

Total depth of earth, compared with height of wall.	Wall of Cut Stone in Mortar.	Wall of Brickwork.	Total depth of earth, compared with height of wall.	Wall of Cut Stone in Mortar.	Wall of Brickwork.
1	.35	.452	2.4	.762	1.02
1.1	.393	.498	3.0	.811	1.11
1.2	.439	.548	4.0	.852	1.18
1.3	.485	.604	6.0	.883	1.25
1.4	.532	.665	11.0	.909	1.28
1.5	.579	.726	21.0	.922	1.31
1.6	.617	.778	31.0	.926	1.32
1.7	.645	.824	Infinite.	.934	1.34
1.8	.668	.847			
1.9	.690	.903			
2.0	.707	.930			

When the earth reaches above the top of the wall, as in Figs 4 and 5, the wall is said to be SURCHARGED and the earth that is *above* the top, is called the SURCHARGE. When the surcharge is carefully deposited above the wall, so as to slope back at a steeper angle than 1½ to 1, as say at 1 to 1, theory does not require the wall to be as thick. Notwithstanding Poncelet's high position, the writer cannot imagine that the base of a brick wall need be so great as 1½ times its height for any height of sand whatever.

Art. 6. On the theory of retaining-walls. Let $b\,c\,a\,m$, Figs 6 and 7, be such walls, upholding backing or filling $c\,s\,m\,g$; the upper surf $c\,s$ of which is *hor, and level with the top* $b\,c$ of the wall; and let $m\,s$ represent the nat slope of the *earth which composes* the backing; $m\,g$ being hor.

Abundant experience on public works shows that this slope, whether for sand, gravel, or earth *when dry*, may be practically taken at 1½ to 1; that is, 1½ hor, to 1 of vert measurement; which *corresponds to an angle* $s\,m\,g$ of 33° 42′ with the hor; which is also about the angle at which brick *and roughly dressed masonry* begin to slide on each other. This angle, however, varies considerably

r; being greatly influenced by the degree of dryness, or dampness, of the material; so that modetely damp sand or earth will stand at a slope of 1 to 1, or at an angle of 45°. Whatever it may be, is called THE ANGLE OF NAT SLOPE of the material under consideration. In theoretical calculations r walls, it is safest to assume (as we have done throughout) that the backing is perfectly dry, since

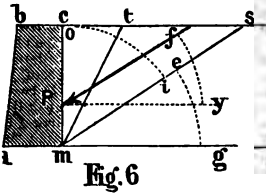


Fig. 6

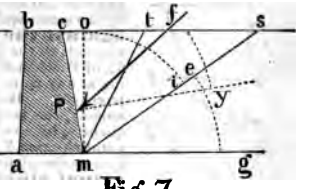


Fig. 7

i pres is then greatest; unless it be supposed to be so wet as to possess some degree of fluidity. The iangle *c m s* of earth above the nat slope *m s*, tends to slide down said slope, but is prevented from doing by the wall.

It is assumed in all cases, that the wall is secured from *sliding* along its base, see Art 9; that it is ick enough to prevent failure by *bulging;* and that it will fail only by *overturning*, by rotating ound its toe, *a*, as a fulcrum. The thickness necessary to insure safety against the last will also be fficient to prevent bulging.

If the angle *o m s*, contained between the natural slope *m s*, and a vert line *m o*, drawn from the ner bottom edge *m* of the wall, be divided by a line *m t*, into two equal angles, *o m t*, *t m s*, then the igle *o m t* is called THE ANGLE, and *m t* THE SLOPE, OF MAXIMUM PRESSURE. The triangular prism of rth, of which *o m t* is a section, or an end view, is called THE PRISM OF MAX PRES; because, if condered as a wedge acting against the back of the wall, it would produce a greater pres upon it than ould the entire triangle *c m s* of earth, considered as a single wedge. For although the last is the eaviest, yet it is more supported by the earth below it. Calculation shows that if we consider the rth *o m s* to be thus div into wedges by *any* line *m t*, the wedge that will press most against the wall that formed when *m t* divides the angle *c m s*, or the arc *o t*, into two equal parts.

Since *m g* is hor, and *m o* vert, the two form an angle of 90°; consequently the angle of max pres plainly found by taking the angle *s m g* of nat slope from 90°, and div the rem by 2. Thus a nat ope of 1½ to 1, or 33° 42′, taken from 90°, leaves 56° 18′; and $\frac{56° 18'}{2} = 28° 9$, the corresponding igle *o m t* of max pres.

For ease of calculation, only one foot of the *length* of the wall, and of its backing, is usually conidered. The number of *cub* ft of wall, or of backing, is then equal to that of the *square* feet in eir respective profiles, or cross-sections.

Now, according to Moseley, if we assume the particles of earth composing the ıcking to be perfectly dry, and devoid of cohesion, (or tendency to stick to each ;her,) which is very nearly the case in pure sand; and if we suppose the wall to be ıddenly removed, then the triangle of earth *c m t*, comprised between the slope *m t* ' max pres, and the back *c m* of the wall, (whether vert as in Fig 6, or battered forard as in Fig 7,) would slide down, under the influence of a force which may be ıpresented by *y* P, acting in a *direction y* P, at right angles to the face *c m* of the triigle of earth; (or in other words, at right angles to the back of the wall, whether ırt or inclined;) its *cent-r of force* being at P, distant ⅓ way between *m* and *c*, easured from the bottom; and its *amount* equal to either of the following:

1st. $$\frac{\textit{Perp pres}}{\text{y P}} = \frac{\textit{Wt of the triangle of earth } \text{c m t} \times \text{o t}}{\textit{vert depth } \text{o m}}; \text{ or}$$

2d. $$\frac{\textit{Perp pres}}{\text{y P}} = \frac{\textit{Wt of triangle}}{\textit{of earth } \text{c m t}} \times \frac{\textit{nat tang of the angle}}{\text{o m t } \textit{of max pres}}; \text{ or}$$

3d. $$\frac{\textit{Perp pres}}{\text{y P}} = \frac{\frac{\textit{Wt of a single cub}}{\textit{ft of the backing}} \times \frac{\textit{sq of the}}{\textit{vert depth } \text{o m}} \times \frac{\textit{sq of nat tang of}}{\textit{angle } \text{o m t } \textit{of max pres}}}{2}; \text{ or}$$

4th. $$\frac{\textit{Perp pres}}{\text{y P}} = \frac{\frac{\textit{Wt of a single cub}}{\textit{ft of the backing}} \times \textit{sq of } \text{o t}}{2}$$

Some appear to assume this perp pres to be the only one acting against the back ' the wall; and hence arrive at erroneous practical conclusions. For when, in ·der to prevent this force from causing the triangle of earth to slide, we place a reining-wall in front of it, then, instead of *motion*, the force will produce pres of the *ırth against the wall; (see* Article 3, of Force, page 445). But in producing pres *necessarily produces* the new force of *friction*, between the pressed surfaces ' *earth and wall. To* ascertain the amount and effect of this friction, let y P,

8, represent *by scale* the force perp to the back $c\,m$; and supposed to have been previously calculated by either of the foregoing formulas. Make the angle $y\,P\,f$ equal to the angle $r\,m\,w$ of nat slope; draw $y\,f$ at right angles to $y\,P$, or parallel to $m\,c$; make $P\,x$ equal to $y\,f$, and complete the parallelogram $P\,y\,f\,x$. Then will $x\,P$ represent by the same scale, **the amount of the friction against the back of the wall.** Since the friction acts in the direction of the back $c\,m$, (see end of Art 62, of Force in Rigid Bodies,) it may be considered as acting at any point P, in that line; (see Art 18, of Force at page 452.) Hence we have acting at P, two forces; namely, the perp force $y\,P$, and the friction $x\,P$; consequently, by comp and res of forces, the diag $f\,P$ of the parallelogram $P\,y\,f\,x$, if measured by the same scale, will give us the amount of their resultant; **which is the actual single theoretical force, both in amount and in direction, which the wall has to resist.**

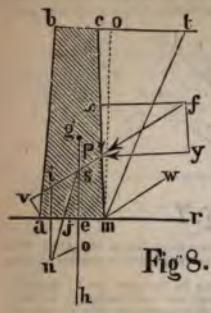


Fig 8.

But this force, $f\,P$, is also always equal to the perp force $y\,P$, mult by the nat sec of the angle $r\,m\,w$ of nat slope of the backing; and of course may be ascertained thus:

1st. $$\textit{True theoretical pres } f\,P = \frac{\textit{wt of triangle } c\,m\,t \times o\,t \times \textit{nat sec of angle } r\,m\,w \textit{ of nat slope}}{\textit{vert depth } o\,m}; \text{ or}$$

2d. $$\textit{True theoretical pres } f\,P = \textit{wt of triangle } c\,m\,t \times \textit{nat tang ang } o\,m\,t \textit{ of max pres} \times \textit{nat sec ang of nat slope}; \text{ or}$$

3d. $$\textit{True theoretical pres } f\,P = \frac{\textit{wt of a cub ft of the backing} \times \textit{sq of the vert depth } o\,m \times \textit{sq of nat tang of ang of max pres} \times \textit{nat sec of angle of nat slope}}{2}; \text{ or}$$

4th. $$\textit{True theoretical pres } f\,P = \frac{\textit{wt of a cub ft of the backing} \times \textit{sq of } o\,t \times \textit{nat sec of angle } r\,m\,w \textit{ of nat slope}}{2.}$$

But the nat slope of dry sand, or earth, is 1½ to 1, corresponding to an angle of 33° 42′; and the nat sec of 33° 42′ is 1.202; therefore, for perfectly dry sand or earth, the first formula becomes

$$\textit{True theoretical pres } f\,P = \frac{\textit{wt of triangle } c\,m\,t \times o\,t \times 1.202}{\textit{vertical depth } o\,m.}$$

Again, for a nat slope of 1½ to 1, the nat tang ang max pres is .535; and .535 × 1.202 = .643. Therefore, the second formula becomes,

$$\textit{True theoretical pres } f\,P = \textit{wt of triangle } c\,m\,t \times .643.$$

Again, the angle $o\,m\,t$ of max pres, corresponding to a nat slope of 1½ to 1, being 28° 9′, its nat tang is .535; and $.535^2$ is .2863. Therefore, in the third formula, the

Square of the nat tang of angle of max pres × nat sec of angle of nat slope, becomes .2863 × 1.202 = .344; and $\frac{.344}{2} = .172$.

Therefore, the third formula becomes,

$$\textit{True theoretical pres } f\,P = \textit{wt of a single cub ft of backing} \times \textit{sq of vertical depth } o\,m \times .172.$$

And the fourth formula becomes,

$$\textit{True theoretical pres } f\,P = \textit{wt of a single cub ft of backing} \times \textit{sq of } o\,t \times .601.$$

REM. 1. **When the back of the wall is offsetted or stepped,** as in Fig 3, instead of being simply battered, as in Figs 7 and 8, the direction of the pres of the earth will be the same as if the back had the batter $i\,n$, on the principle given in Art 34, Fig 17, of Force in Rigid Bodies, p 464.

REM. 2. **For calculating the overturning moment, or tendency of the pres of the earth, through its leverage** $v\,a$, Fig 8; as well as the resisting moment of the wall by means of its weight, (supposed to be concentrated at its cen of grav g,) mult by its leverage $e\,a$; see Arts 35 and 41, of Force in Rigid Bodies. Or if a vert line $g\,h$, Fig 8, be drawn through the cen of grav g, of a retaining-wall; and if the direction $f\,P$, of the pres against the back of the wall, be continued past said line as toward i or v; and if then any dist $s\,o$, by scale, be laid off on the vert line from s to represent the wt of the wall; and $s\,i$ by the same scale to represent the amount of the pres $f\,P$; and the parallelogram $s\,i\,n\,o$ of forces be completed; then will its diag $s\,n$ be the resultant of the two, both in amount and

in direction. For safety this should fall inside of the base a dist $a j$, not *less* than about ½ of the base $a m$.

REM. 3. According to theory, when as in Fig 9, there is a surcharge $b a o$ of backing, sloping away from o, at its nat slope $a b$, the max pres against the wall is attained when the earth reaches to the level of d, where the slope $n c d$ of max pres intersects the face of the nat slope $a b$; so that if afterward the earth is raised to b, or to any greater height, no additional pres is thereby thrown against the back of the wall. So also if the earth slopes from x, or from between a and x. Our own experiments induce us, however, to believe that this is not true in fact, owing to tremors, &c.

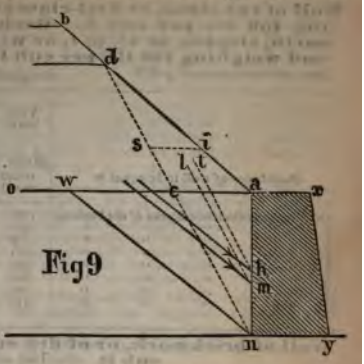


Fig 9

According to Moseley, the *amount* of the theoretical oblique pres f P, Figs 6, 7, and 8, when the wall is surcharged, (as in any of the Figs 4, 5, 9,) may be found on the same principle as when the earth is level with its top; namely, instead of the *triangle* $c m t$ of earth, find the wt of *all* the earth $d z o b$, Fig 4; $d m s o$, Fig 5, or $a d n$, Fig 9; included between the back of the wall and the slope of max pres; then, as in the 2d of the foregoing formulas, the

$$\text{True theoretical pres f P} = \text{wt of the earth} \times \frac{\text{nat tang ang}}{\text{max pres}} \times \frac{\text{nat sec ang}}{\text{nat slope.}}$$

Or when the nat slope is 1½ to 1, as with perfectly dry sand or earth,

$$\text{True theoretical pres f P} = \text{weight of the earth} \times .643.$$

When the earth extends to the outer edge of the wall, the small triangle $n d t$, Fig 5, *which rests upon the top of the wall*, is neglected. The *direction* of the pres thus found will be the same as when the earth is level with the top. But Moseley has omitted to state the very important fact, that it will not be applied at ⅓ of the height of the wall from the bottom of the earth, except when, as at $d n a$, Fig 9, the sustained earth forms a complete triangle. In other cases, the point of application of the center of pres will be at that point h, in the back of the wall, which is intersected by a line, $h t$, parallel to the slope ($n d$, Fig 9, or $m t$, Figs 6, 7, and 8) of max pres, and passing through the cen of grav of the sustained earth, whose weight was found at first. This change in the position of the point of application, serves to explain the great increase of thickness required to meet *the first small* surcharges of earth above the top of the wall. These *raise* the point of application considerably more than subsequent ones do. Thus, in Fig 9, suppose the surcharge to reach only to $s i$; then the line $h t$, drawn through the cen of grav of the earth $s i a n s$, and parallel to $n d$, will plainly intersect the back $a n$ at a higher point h, than one $l m$ drawn through the cen of grav of the triangle of earth $a c n$, or of $a d n$; which latter line intersects it at m, at one-third of its height from the base $n y$. So that with the small surcharge $s i$, we have not only the pres of a greater quantity of earth; but a greater *leverage* is given to it for overthrowing the wall. But when the surcharge is made to reach d, then the earth $d a n$, being, like $a c n$, a triangle, the line through its cen of grav will strike the back $a n$ at m, at the same height as that through $a c n$; or lower down, and with a shorter leverage, than that through $s i a n s$.

Art. 7. The following table gives the theoretical thickness at base both for vert walls and for those with vert backs and battered faces, and required to present resistances having different proportions to the oblique *theoretical* pres f P of earth level with the top of the wall. We have already remarked in Art 2, that other pressures besides these theoretical ones must be guarded against. Any of these walls may be converted into others with offsetted backs, as directed in Art 3. **Notwithstanding the results in this table, we recommend not to use bases less than those given in Art 1.**

TABLE 3.

Wall of cut-stone, or first-class rubble, in mortar, and weighing 150 lbs per cub ft. Backing of dry sand, gravel, or earth, sloping at 1½ to 1, or with a natural slope of 33° 42′, and weighing 100 lbs per cub ft, level with top of wall.
See last sentence of p 337. (Original.)

Resistance of wall to be equal to	Vert Wall.	Face batter. 1 in 12.	Face batter. 2 in 12.	Face batter. 3 in 12.	Face batter. 4 in 12.	Triangular Wall.
	Mult the vert height of the wall by the numbers below, which are the bases for a wall 1 ft high.					
1½ *times the theoretical pres of the backing*	.228	.235	.248	.268		.279
2 " " " "	.252	.258	.273	.291		.309
3 " " " "	.286	.291	.304	.323	.348	.330
4 " " " "	.311	.317	.330	.347	.367	.381
6 " " " "	.344	.350	.359	.375	.396	.421
8 " " " "	.369	.372	.382	.396	.419	.452
10 " " " "	.384	.388	.396	.412	.430	.470
15 " " " "	.412	.416	.424	.438	.455	.505

Wall of brickwork, or of dry rubble, weighing 115 lbs per cub ft. See last sentence of p 337.

1½ *times the theoretical pres of the backing*	.250	.256	.270	.290		.306
2 " " " "	.274	.280	.290	.310	.334	.336
3 " " " "	.310	.317	.328	.345	.367	.380
4 " " " "	.332	.339	.349	.365	.386	.407
6 " " " "	.368	.372	.382	.396	.418	.451
8 " " " "	.388	.393	.402	.416	.435	.475
10 " " " "	.404	.410	.417	.432	.451	.495
15 " " " "	.427	.431	.439	.452	.470	.523

The thicknesses for the vert walls in the table were found by trial, by means of a drawing, and are approximate enough for practice. The writer does not know whether they can be obtained at once by any simple process of calculation, when the true theoretical *oblique* pres *YP* is introduced. With the erroneous assumption used by some writers, of a single hor pres against a vert back, it is very easy, but useless. In that case, the leverage of the earth does not change with the thickness of the wall. The thicknesses for the triangular and battered walls were deduced from the vert ones by the original process given in Art 8.

Moseley and others quote Gadroy, for A DRY SAND SLOPING AT 21°. It would be better to cease from circulating such evident mistakes. Dry sand will stand at no less angle for a savant than for anybody else. For ordinary practical purposes, we may say that dry sand, gravel, and earths, slope at 1½ to 1; as abundant experience on railroad embkts proves. Poncelet gives tables for walls to support dry earth sloping at 1 to 1, or 45°; but as we do not believe in the existence of such earth, we omit such tables. Sand, gravel, and earths may be moistened to diff degrees, so as to stand at any angle between hor and vert; and by moistening and ramming, the earths may be converted into compact masses, exerting little or no pres; and may even so continue after they become dry; being then, in fact, a kind of air-dried brick. It is sometimes difficult to know whether earth or sand is perfectly dry or not; and an *exceedingly* small degree of moisture will cause them to stand at 1 to 1, in *small heaps*, such as have probably been observed by the authorities on the subject. The writer found that fine sand from the sea-shore, and under cover, would stand at 1½ to 1 during warm dry weather, and at 1 to 1 when the air was damp. Yet no diff whatever in its degree of moisture was perceptible to the feeling. Its susceptibility to dampness was of course owing to salt. A few handfuls of dry earth may perhaps be coquetted into standing at 1 to 1 on a table; but so far as our observation extends, when it is dumped in large quantities from carts and wheelbarrows, its slope is about 1½ to 1; and this we consider the proper one to be used in practical calculations, where safety is the consideration of paramount importance.

The less the nat slope, the greater is the pres; and since the slope is least when the backing is perfectly dry, (omitting of course its condition when so absolutely *wet* as to become partially fluid,) we have, on the score of safety, confined our tables to dry backing. As stated in Art 1, we cannot recommend dimensions less than those there given, when we consider the rough treatment to which masonry is exposed on public works.

In carrying a road along dangerous precipices, we should rather be tempted at times to make them greater. We imagine, for instance, that the centrifugal force of a heavy train, whirling around a sharp curve, convex on the *dangerous side, should* not be overlooked in designing walls for such localities. This *force is hor; and is applied* near the top of the wall; and, consequently, its leverage *may be considered as equal* to the height; whereas the theoretical pres of the earth *is oblique; and is applied* at ⅓ of the height from the bottom; so that its leverage *about the toe of the* wall is very short. Moreover, the simple weight of the train, pro*uces pres against the* wall; as well as that of the backing. All such consideratio*ns omitted by theorists.* The dangerous pres caused by tremors, &c, cannot

to be applied at ⅓ of the height from the bottom; nor indeed, can it be l at all.

Wharf walls are an instance where the thickness should be increased, anding that the pres of the water in front helps to sustain them. The earth ch walls, is not only liable to be very heavily loaded when vessels are dis- ; but is apt to become saturated with water, especially below low-water d thus to exert a very great pres against the walls. Moreover, the water er the wall; and by its upward pressure virtually reduces its weight, and ntly its stability. The same cause of course diminishes the friction of the n its base. Such walls are, therefore, very liable to slide, if the foundation , and horizontal; and have done so even when the foundation had a con- inclination backward, as in Fig 1. See Art 9.

A retaining-wall is usually in greater danger for a few months after its completion, than has been allowed for the mortar to harden perfectly; and for the backing to settle. When uspicions of the safety of a new wall, it would be well to place strong temporary shores at about ⅓ to ½ of its height above ground. In some cases, permanent buttresses of ay be built for the purpose. They should be well bonded into the wall.

The pres of the earth backing will be much reduced, if the first few feet of its height be thin hor layers, to be consolidated by being used by the masons instead of scaffolding; as , Fig 1. Frequently this can be done without inconvenience; and at very trifling cost.

5. To change a vert retaining-wall, into one with a ed face, which shall present an equal resistance t overturning; although requiring less masonry. metimes termed a **transformation of profile.** (Original.)

b o i, Fig 10, be the vert wall. Mult its by 1.225; the prod will be the base *o e*, of a r wall *b o e*, possessing the same stability; ot requiring much more than half the ma- the vertical one. See Rem 1. This being pose a wall to be desired with a face batter, ins to a ft; or 1 in 4. From the point *n*, e face of the triangular wall intersects e vert one, step off vert any 4 short equal and from the upper one *m*, step off one r, to *v*. Through *v* and *n* draw the dotted which evidently will batter 1 in 4. Then is proximately the reqd wall; but a little han necessary. To reduce it, from *t* draw d line *t b*. Mark the point *c*, where this rsects the face *a i*, of the vert wall; and *c* draw *d l*, parallel to *s t*. Then is *b d l o* wall. Our fig is drawn in an exaggerated so as to avoid confusion in the lines. The f the triangular wall, would not in reality o great as it is represented.

Fig 10

be observed that as the base increases, the quantity of masonry diminishes.

THE FOLLOWING FACT MAY AT FIRST SIGHT APPEAR PARADOXICAL to a novice; we do not remem- e ever seen it alluded to. Although the wall, after being transformed from a rectangular with a battered face, (and consequently with a greater base,) has the same moment of sta- efore; and although the amount of pres of the earth against it, also remains unchanged, ew wall is better able to resist this pres than before. In other words, the *moment* or *t n-* e pres to upset the wall, has become less. For let *a b m n*, Fig 11, represent a rectangular *f o* the *amount* and direction of pres behind it. Now, the leverage with which this pres erturn the wall around its toe *m*, is the dist *m s*, from the toe or fulcrum *m*, and at right angles to on *f o s c* of the pres; and this leverage mult by *f o*, gives the overturning tendency or moment of See leverage, in Force in Rigid Bodies. Again, epresent a triangular wall of the same stability er, as found by our rule. Here we still have the nt *f o*, and direction *f o s c*, of pres force against but it now acts to overturn the wall *a n y* toe *y*: and therefore, with the reduced leverage equently, its overturning tendency is less than herefore, in ordinary language, we may say that stronger than before, although its moment of r standing tendency, has in itself undergone no

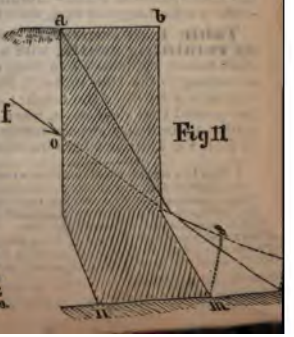


Fig 11

pres *f o* were hor, then its leverage would evi- the same in both walls; and the proportion be- overturning moment of the pres, and the moments *of the two walls, would* remain unchanged.

In attempting to reduce the masonry by adopt- *o b e*, Fig 10, of a triangular section; or of one *aching a triangle*, special attention should be *quality of the* masonry near the thin toe e; *therwise* be apt to crack, or fail under the pres.

REM 3. MOREOVER, WHEN COMMON MORTAR IS USED WITHOUT AN ADMIXTURE OF CEMENT, which it never should be, in retaining-walls, where durability is an object, a great batter is objectionable; inasmuch as the rain, combined with frost, &c, soon destroys the mortar. In such cases, therefore, the batter should not exceed 1 or 1½ ins to a ft; and even then, at least the pointing of the joints, and a few feet in height of both the upper and the lower courses of masonry, should be done with cement, or cement-mortar. We have observed a most marked diff in the corrosion of the mortar, where, in the same walls, with the same exposure, one portion has been built with a vert face; and another with a batter of but 1½ inch to a foot. Common mortar will never set properly, and continue firm, when it is exposed to moisture from the earth. This is very observable near the tops and bottoms of abuts, retaining-walls, &c; the lime-mortar at those parts will generally be found to be rendered entirely worthless. A profile somewhat like Fig 12, may at times prove serviceable, instead of the triangular. This is the form of the Gothic buttress; which probably had its origin in the cause just spoken of.

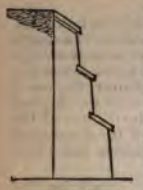


Fig 12

Art. 9. A retaining-wall may slide, without losing its verticality; and, indeed, without any danger of being overturned. This is very apt to occur if it is built upon a hor wooden platform; or upon a level surf of rock, or clay, without other means than mere friction to prevent sliding. This may be obviated by inclining the base, as in Fig 1; by founding the wall at such a depth as to provide a proper resistance from the soil in front; or in case of a platform, by securing one or more lines of strong beams to its upper surf, across the direction in which sliding would take place. On a bare surf of moist clay, friction may become as low as ⅓ the wt of the wall; on dry earth, it is about ½ to ⅔; and on sand or gravel, about ⅔ to ¾. The friction of masonry on a wooden platform, is about $\frac{9}{10}$ of the wt, if dry; and ¾ if wet.

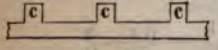


Fig 13

Counterforts, shown in plan at c c c, Fig 13, consist in an increase of the thickness of the wall, *at its back*, at regular intervals of its length. We conceive them to be but little better than a waste of masonry. When a wall of this kind fails, it almost invariably separates from its counterforts; to which it is connected merely by the adhesion of the mortar; and to a slight extent, by the bonding of the masonry. The table in Art 7 shows that a very small addition to the base of a wall, is attended by a great increase of its strength; we therefore think that the masonry of counterforts would be much better, and more cheaply employed in giving the wall an additional thickness, along its entire length; and for the lower third of its height. Counterforts are very generally used in retaining-walls by European engineers; but rarely, if ever, by Americans.

Buttresses are like counterforts, except that they are placed *in front* of a wall instead of behind it; and that their profile is generally triangular, or nearly so. They greatly increase its strength; but, being unsightly, are seldom used, except as a remedy when a wall is seen to be failing.

Land-ties, or long rods of iron, have been employed as a makeshift for upholding weak retaining-walls. Extending through the wall from its face, the land ends are connected with anchors of masonry, cast-iron or wooden posts; the whole being at some dist below the surf.

Retaining walls with curved profiles are mentioned here merely to caution the young engineer against building them. Although sanctioned by the practice of some high authorities, they really possess no merit sufficient to compensate for the additional expense and trouble of their construction.

Art. 10. Among military men, a retaining-wall is called a **revetment.** When the earth is level with the top, a **scarp revetment;** when above it, a **counterscarp** revetment, or a *demi-revetment*. When the face of the wall is battered, a *sloping;* and when the back is battered, a *countersloping* revetment. The batter is called the **talus.**

Art. 11. Our own experiments lead us to believe that (see Fig 9) the pres against the back of a wall, from dry sand, *a o*, level with the top, is not at all diminished by reducing the width *a w* of the sand, until it becomes about one-sixth of the dist *a w*, pertaining to the angle *a n w* of natural slope. With less than that limit, the pres was plainly diminished. It would therefore seem to be dangerous to make a retaining-wall thin, merely because the backing does not extend to c.*

Table 4, of contents in cub yards for each foot in length of retaining-walls, with a thickness at base equal to .4 of the vert height. Face batter, 1½ inches to a foot; or ⅛th of the height. Back either vert, or stepped according to the rule in Art 3, Fig 3. When not stepped, the thickness of such a wall at top is .275 of the height; and at the middle, .3385 of the height. Experience has proved that such walls, when composed of well-scabbled mortar rubble, are safe under all ordinary circumstances for earth level with the top. Steps or offsets, *o e*, at foot, Fig 1, are not here included.†

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* This is a singular fact, and the writer does not know that it has ever been noticed before.

We cannot understand how correct results are to be expected from experiments like those of Gen. *Pasley, and others,* who confined their backing in a box, placing their walls in front of it. The friction of the backing against the sides of the box must diminish the pres against the wall, and thus lead to adopting too slight a thickness. We conceive that an experimental wall should diminish in height and thickness, each way from its central portion, (preserving, however, the same proportion between the two,) until it terminates in a point at each end. Ours were made in this way.

† If the batter is only 1 inch to a foot, mult the cub yds in the table by 1.062; or approximately, add ⅛th part. If the batter is ½ inch to a ft, mult by 1.124; or approx, add ⅛th part. If the face is vertical, mult by 1.186; or roughly, add a trifle less than ⅕th part.

TABLE 4. (Original.)

Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.	Ht. Ft.	Cub. Yds.
1	.018	10½	1.38	20	5.00	29½	10.9	48	28.8	74	68.5
½	.028	11	1.51	½	5.25	30	11.3	49	30.0	76	72 2
2	.050	½	1.65	21	5.51	31	12.0	50	31.3	78	76 1
½	.078	12	1.80	½	5.78	32	12.8	51	32 5	80	80.0
3	.113	½	1.95	22	6.05	33	13.6	52	33.8	82	84.1
½	.153	13	2.11	½	6.33	34	14.5	53	35.1	84	88.4
4	.200	½	2.28	23	6.61	35	15.3	54	36.5	86	92.5
½	.253	14	2.45	½	6.90	36	16.2	55	37.8	88	96.8
5	.313	½	2.63	24	7.20	37	17.1	56	39.2	90	101.3
½	.378	15	2.81	½	7.50	38	18.1	57	40.6	92	105.8
6	.450	½	3.00	25	7.81	39	19 0	58	42.1	94	110.5
½	.528	16	3.20	½	8.13	40	20.0	59	43.5	96	115.2
7	.613	½	3.40	26	8.45	41	21.0	60	45.0	98	120.1
½	.703	17	3.61	½	8.78	42	22.1	62	48.1	100	125.0
8	.800	½	3.83	27	9.12	43	23.1	64	51.2	102	130.1
½	.903	18	4.05	½	9 45	44	24.2	66	54.5	104	135.2
9	1.01	½	4.28	28	9.80	45	25.3	68	57.8	106	140.5
½	1.13	19	4.51	½	10.2	46	26.5	70	61.3		
10	1.25	½	4.75	29	10.5	47	27.6	72	64.8		

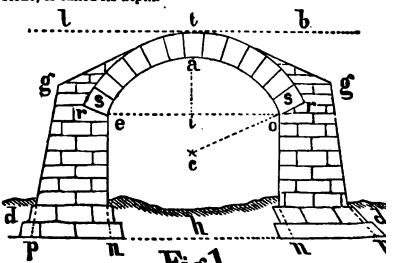
See preceding footnote.

STONE BRIDGES.

Art. 1. In an arch *s t s*, Fig 1, the dist *e o* is called its **span**; *t a* its **rise**; *t* it **crown**; its lower boundary line, *e a o*, its **soffit**, or **intrados**; the upper one *r t r*, its **back**, or **extrados.** The terms soffit and back are also applied to th entire lower and upper curved *surfaces* of the whole arch. The ends of an arch, o the showing areas comprised between its intrados and extrados, are its **faces**; thu the area *s t s a* is a face. The inclined surfaces or joints, *r e*, *r o*, upon which the *fee* of the arch rest, or from which the arch *springs*, are the **skewbacks.** Line level with *e* and *o*, at right angles to the faces of the arch, and forming the lowe edges of its feet, (see *n n*, Fig 2½,) are the **springing lines**, or **springs.** Th blocks of which the arch itself is composed, are the **arch-stones**, or **voussoirs**. The center one, *t a*, is the **keystone**; and the lowest ones, *s s*, the **springers**. The term *archblock* might be substituted for voussoir, and like it would apply t brick or other material, as well as to stone. The parts *t r*, *t r*, are the **haunches**; and the spaces *t r l*, *t r b*, above these, are the **spandrels.** The material deposited in these spaces is the **spandrel filling**; it is sometimes earth, sometimes ma sonry; or partly of each, as in Fig 1.

In large arches, it often consists of several parallel SPANDREL-WALLS, *l l*, Fig 2½, running lengthwis of the roadway, or astraddle of the arch. They are covered at top either by small arches from wall t wall, or by flat stones, for supporting the material of the roadway. They are also at times connecte together by vert cross-walls at intervals, for steadying them laterally, as at *t t*, Fig 2½. The part *g p e n*, *g p o n*, Fig 1, are the ABUTMENTS of the arch; *e n*, *o n*, the *faces*; *g p*, *g p*, the backs; an *p n*, *p n*, the *bases* of the abuts. The bases are usually widened by *feet*, *steps*, or *offsets*, *d d*, for dis tributing the wt of the bridge over a greater area of foundation; thus diminishing the danger of set tlement. The distance *t a* in any arch-stone, is called its *depth*.

The only arches in common use for bridges, are the circular, (often called segmental); and the elliptic.



Art. 2. To find the depth of keystone for cut-stone arches, whether circular or elliptic.* (Original.)

Find the rad *c o*, Fig 1, which will touch the arch at *o*, *a*, and *e*. Add together this rad, and *half* the span *o e*. Take the sq *rt of the sum. Div this sq rt by 4. To the quot add* $\frac{2}{10}$ *of a ft. Or by formula,*

$$\text{Depth of key in feet} = \left(\frac{\sqrt{Rad + half\ span}}{4}\right) + .2\ foot.$$

For second-class work, this depth may be increased about ⅛th part; or **for brick or fair rubble,** about ¼th. See table of Keystones, p 345.

In large arches it is advisable to increase the depth of the archstones toward the springs; but when the span is as small as about 60 to 80 or 100 feet, this is not at all *necessary* if the stone is good; although the arch will be stronger if it is done. In practice this increase, even in the largest spans, does not exceed from ¼ to ½ the depth of the key; although theory would require much more in arches of great rise. See Ex 2, p 468, also Ex 6, p 478, of Force in Rigid Bodies.

REM. **To find the rad *c o*,** whether the arch be circular or elliptic. Square *half* the span *e o*. Square the whole rise *i a*. Add these squares together; div the sum by *twice* the rise *i a*. Or it may be found near enough for this purpose by the dividers, from a small arch drawn to a scale.

Amount of pressure sustained by archstones. In bridges of the same width of roadway, if all the other parts bore to each other the same proportion as the spans, the *total* pres, as well as the *pres per sq ft*, in arches of the same proportionate rise, would increase as the squares of the spans. But in practice the depth of the archstones increases much less rapidly than the span; while the thickness of the roadway material, and the extraneous load per sq ft, remain the same whether the span be great or small. Hence the pressures increase less rapidly than the squares of the spans. Thus in two bridges of the same width, but with spans of 100 and 200 ft, with depths of archstones taken from our table page 345, and uniform from key to spring; supposed to be filled up solid with masonry of 160 lbs per cub ft, to a level of about 15 inches above the crown, (including the stone paving of the roadway); with an extraneous load of 100 lbs per sq ft; and the leverages *y m*, *y o*, Fig 52, p 479, measured from the center *y* of the skewback; the pressures will be approximately as follows:

	Span 100 ft.				Span 200 ft.			
	AT KEY.		AT SPRING.		AT KEY.		AT SPRING.	
Rise.	For 1 ft in width of its entire depth.	Per sq ft.	For 1 ft in width of its entire depth.	Per sq ft.	For 1 ft in width of its entire depth.	Per sq ft.	For 1 ft in width of its entire depth.	Per sq ft.
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
⅙	42¼	13¾	58	18¾	126	29¼	179	42
¼	36¾	12¼	57	19	112	27¼	181	44
⅓	31¼	11	57½	20	97	24½	188	47½
½	25	9	61½	22½	80¼	21	207	54¼
⅔	18	6¾	67½	25	57½	15½	230	61½

Here it is seen that the *entire* pres of the 200 ft span, with different rises, averages but little more than three times that of the 100 ft one; while the pres *per sq ft* of each stone averages but about 2¼ times that of the 100 ft one. It will also be seen that with the same span, the pres at the key becomes less, while that at the spring becomes greater, as the rise increases. Also that when the archstones are of uniform depth, the pres at either spring of a semicircular arch is about 4 times as great as at the key; whereas when the rise is but ⅙ of the span, the pres at spring averages but about ½ greater than at the key. These proportions vary somewhat in different spans.

The greater pres per sq ft at the springs may be reduced by increasing the depth of the archstones towards the springs. This however is not *necessary* in moderate spans, inasmuch as good stone will be safe even under this greater pres.

By using parallel spandrel walls, see Fig 2½, p 346, or by partly filling with earth instead of masonry, the pres on the archstones may be diminished, say, as a rough average, about ⅕ part.

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and calculations made by the writer, of lines of pres, &c, of arches from 1 to 300 ft span, and of every rise, from a semicircle to $\frac{1}{15}$ of the span. From these drawings he endeavored to find proportions *which, although* they might not endure the test of strict criticism, would still apply to all the cases *with an accuracy sufficient* for ordinary practical purposes. The result is here published for the first *time. How far we have* succeeded, we must leave to the decision of others. If we have committed *errors, no one will be* more pleased than ourselves to have them rectified.

We know of no book to which a practical man can refer for any such information as we here en*deavor to impart on this very* important subject.

In Force in Rigid Bodies, Art 72, Ex 2 of Art 38, and Ex 6 of Art 54, refer to stone bridges.

Table 1. Of some existing arches, with both their actual, and their calculated depths (by our rule) of keystone. Where the two depths are given in the last column, the smallest is for first class cut-stone, and the largest for good rubble, or brick. Those also which are not specified are of first-class cut-stone. C stands for circular, E for elliptic. For 2d class work, add about ⅓th part; and for brick, or fair rubble, about ¼th.

		Span.	Rise.	Radius.	Actual Key.	Key by our Rule.	Engineer.
		Ft.	Ft.	Ft.	Ft.	Ft.	
Cabin John. Washington Aqueduct	C	220	57.25	134.25	4 16	4.11	Meigs.
Grosvenor Bridge, Chester, England	C	200	42	140	4	4.07	Hartley.
London Bridge, England	E	152	29.5	112.6	4.75	3.62	Rennie.
Gloucester, across the Severn, England	E	150	35	98	4.50	3.49	Telford.
Dora Riparia, Turin, Italy	C	148	18	160.1	4.92	4.03	Mosca.
Pont-y-Prydd, Wales. Rough rubble	C	140	35	87.5	1.5	3.3 / 4.2	Edwards.
Pont de Alma, Paris, France. Small rough rubble, in cement. Carries a railroad	E	141.4	28.2	102.8	4.92	3.5 / 4.44	Darcel.
Soupes, France. An experimental arch of cut-stone in Portland cement mortar*	C	124	6.92	281 1	2.67	4.82	Vaudray.
Waterloo. Across the Thames at London	E	120	32	72.2	4.5	3.07	Rennie.
Tongueland, England. Turnpike	C	118	38	64.8	3.5	3.0	Telford.
Pont Napoleon. R. R. viaduct, Paris. Small rough rubble in cement	C	115.5	14.75	120	4	3.54 / 4.48	Couche.
Holy Trinity, Florence. Very slightly pointed		95.5	15.1	83 9	2.75	3.07	Ammanati.
Dean Bridge, Scotland. Turnpike	C	90	30	48.9	3	2.62	Telford.
Dunkeld " "	C	90	30	48.9	3.17	2.62	Telford.
Licking Aqueduct. Chesapeake & Ohio Canal	C	90	15	75	2.83	2.94	Fisk.
Royal Border Viaduct, England. Brick in cement	C	80	40	40	2.66	2.44 / 3.10	
Posen Viaduct, Germany. Brick in cement	C	80	16	58	4.66	2.68 / 3.40	
Allenton, England. Turnpike	C	75	11.5	67	2.5	2.75	Stevenson.
Falls Bridge, Philadelphia & Reading R R	C	78	25	43	3	2.46	Steele.
Staines, England. Turnpike	C	74	9.3	78	3	2.98	Rennie.
Brent R R Viaduct, England. Brick in cement	E	70	17.6	43.6	3	2.42 / 3.02	Brunel.
Bow Bridge. Turnpike. England	E	66	13.75	46.5	2.50	2.42	Walker.
Bewdly. " "	C	60	20	33	2.2	2.18	Telford.
Chestnut Street Bridge, Philadelphia. Brick in cement	C	60	18	34	2.5	2.2 / 2.8	Kneass.
Monocacy Aqueduct, Chesapeake & Ohio Canal	E	54	9	45	2.5	2.32	Fisk.
Llanrwast, Wales. Turnpike	C	58	17	33	1.5	2.17	Inigo Jones.
Avon Viaduct, England. Brick in cement	E	50	15	28.3	2	2.03 / 2.60	Vignoles.
James River Aqueduct, Virginia	C	50	7	47	2.66	2.32	Eliet.
Tenobway Culvert, under Ches & Ohio Canal. Rubble in cement	C	40	15	21	2	1.8 / 2.3	Fisk.
Philadelphia & Reading R R	C	44	8	34.3	2.5	2.08	Steele.
" " " "	C	31.2	5	26.8	1.66	1.83	Steele.
Edinburg & Dalkeith R R. Scotland	E	24	4	20	.75	1.62	Jardine.

* The width is not given in any account that we have seen, but probably it was not more than [illegible] ft. It was constructed by [illegible] distributed load of 380 tons, and by a weight of 5 tons falling 18 ins. [illegible] uniformed thereby. Total settlement before and after being loaded, about 1¼ ins. The depth [illegible] arches were increased gradually from 2.67 ft at key, to 3.6 ft at springs. Mr. [illegible] Yonne Rail[illegible] about 1/48 of the span.

The arch on the BOURBONNAIS RAILWAY, is probably the boldest;* and THE CABIN JOHN ARCH, by Capt. now Gen'l M. C. Meigs, U S Army, the grandest stone one in existence. PONT-Y-PRYDD, in Wales, is a common road bridge, of very rude construction; with a dangerously steep roadway. It was built entirely of rubble, in mortar, by a common country mason, in 1750; and is still in perfect condition. Only the outer, or *showing* arch-stones, are 2.5 ft deep; and that depth is made up of two stones. The inner arch-stones are but 1.5 ft deep; and but from 6 to 9 inches thick. The stone quarried with tolerably fair natural beds; and received little or no dressing in addition. The bridge is a fine example of that ignorance which often passes for boldness. PONT NAPOLEON carries a railroad across the Seine at Paris. The arches are of the uniform depth of 4 ft, from crown to spring. They are composed chiefly of *small rough quarry chips*, or *spawls;* well washed, to free them from dirt and dust; and then thoroughly bedded in good cement; and grouted with the same. It is in fact an arch of cement-concrete. The PONT DE ALMA, near it, *and built in the same way*, has elliptic arches of from 126 to 141 ft span; with rises of $\frac{1}{5}$ the span. Key 4.9 ft. These two bridges, considering the want of precedent in this kind of construction, on so large a scale, must be regarded as very bold; and as reflecting the highest credit for practical science, upon their engineers, Darcel and Couche. Some trouble arose from the unequal contraction of the different thicknesses of cement. They show what may be readily accomplished in arches of moderate spans, by means of small stone, and *good hydraulic cement* when large stone fit for arches is not procurable. In Pont Napoleon the depth of arch is barely what our rule gives for second class cut-stone.

REM. **Our engineers are usually too sparing of cement.** It should be freely used, not only in the arches themselves, and in the masonry above them, as a protection from rain-soakage; but in abuts, wing-walls, retaining-walls, and all other important masonry exposed to dampness. The entire backs of important brick arches should be covered with a layer of good cement, about an inch thick. The want of it can be seen throughout most of our public works. The common mortar will be found to be decayed, and falling down from the soffits of arches; and from the joints of masonry generally, within from 3 to 6 ft of the surface of the ground. The moisture rises by capillary attraction, to that dist above the surf of the nat soil; or descends to it from the artificial surf of embankments, &c; therefore, cement-mortar should be employed in those portions at least. The mortar in the faces of battered walls, even when the batter is but 1 to $1\frac{1}{2}$ inches per foot, is far more injured by rain and exposure, than in vert ones; and should therefore be of the best quality. See MORTAR, &c.

We have, however, seen a quite free percolation of surface water through brick arches of nearly 3 ft in depth, even when cement was freely used. In aqueduct bridges, we believe that cement has not been found to prevent leaks, whether the arches were of brick, or even of cut-stone. May not this be the effect of cracks produced by settlement of the arch; or by contraction and expansion under atmospheric influence? Cement at any rate prevents the joints from crumbling.

Art. 3. The keystones for large elliptic arches by the best engineers, appear generally to be about $\frac{1}{3}$ part deeper than our rule requires; or than is considered necessary for circular ones of the same span and rise. We do not, however, see any sufficient reason for this. The elliptic arch, with its spandrel filling, has slightly less wt; and that wt has a trifle less leverage than in a circular one; and consequently it exerts less pres both at the key, and at the skew-back. See London, Gloucester, and Waterloo bridges, in the preceding table.†

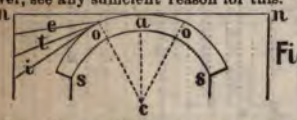

Fig. $1\frac{1}{2}$

REM. Young engineers are apt to affect shallow arch-stones; but it would be far better to adopt the opposite course; for not only do deep ones make a more stable structure, but a thin arch is as unsightly an object as too slender a column. According to our own taste, arch-stones fully $\frac{1}{3}$ deeper than our rule gives for first-class cut-stone, are greatly to be preferred when appearance is consulted. Especially when an arch is of rough rubble, which costs about the same whether it is built up as arch, or as spandrel filling, it is mere folly to make the arches shallow. Stability and durability should be the objects aimed at; and when they can be attained even to excess, without increased cost, it is best to do so.

TABLE of keystones for arches of first-class cut-stone; drawn up from the foregoing rule. For 2d class cut-stone, add about $\frac{1}{6}$ part, and for good rubble or brick, about $\frac{1}{4}$ part. In spans of less than about 10 ft, however, such increase is unnecessary.

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* Built like the experimental one at Soupes, in the preceding table.

† *A possible reason* is suggested a few pages forward; namely, that by making the archstones very *deep from the key to the spring*, the line of pressure in elliptic and semicircular arches will not fall *entirely below the arch* at any point. This may have been the reason of the great depth of the old *Roman archstones. It is*, however, unnecessary.

TABLE 2. (Original.)

Span. Feet.	Rise, in parts of the span. $\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$
	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.	Key. Ft.
2	.55	.56	.58	.60	.61	.64	.68
4	.70	.72	.74	.76	.79	.83	.88
6	.81	.83	.86	.89	.92	.97	1.03
8	.91	.93	.96	1.00	1.03	1.09	1.16
10	.99	1.01	1.04	1.07	1.11	1.18	1.26
15	1.17	1.19	1.22	1.26	1.30	1.40	1.50
20	1.32	1.35	1.38	1.43	1.48	1.59	1.70
25	1.45	1.48	1.53	1.58	1.64	1.76	1.88
30	1.57	1.60	1.65	1.71	1.78	1.91	2.04
35	1.68	1.70	1.76	1.83	1.90	2.04	2.19
40	1.78	1.81	1.88	1.95	2.03	2.18	2.33
50	1.97	2.00	2.08	2.16	2.25	2.41	2.58
60	2.14	2.18	2.26	2.35	2.44	2.62	2.80
80	2.44	2.49	2.58	2.68	2.78	2.98	3.18
100	2.70	2.75	2.86	2.97	3.09	3.32	3.55
120	2.94	2.99	3.10	3.22	3.35	3.61	3.88
140	3.16	3.21	3.33	3.46	3.60	3.87	4.15
160	3.36	3.44	3.58	3.72	3.87	4.17	
180	3.56	3.63	3.75	3.90	4.06	4.38	
200	3.74	3.81	3.95	4.12	4.29		
220	3.91	4.00	4.13	4.30	4.48		
240	4.07	4.15	4.30	4.48			
260	4.23	4.31	4.47	4.66			
280	4.38	4.46	4.63				
300	4.53	4.62	4.80				

Art. 4. To proportion the abuts for an arch of stone or brick, whether circular or elliptic. (Original.)

The writer ventures to offer the following rule, in the belief that it will be found to combine the requirements of theory with those of economy and ease of application, to perhaps as great an extent as is attainable in an endeavor to reduce so complicated a subject, to **a simple and reliable working rule for practical bridge-builders.** This is all that he claims for it. Notwithstanding its simplicity, it is the result of much labor on his part. It applies equally to the smallest culvert, and to the largest bridge; whatever may be the proportions of span and rise; and to any height of abut whatever. It applies also to all the usual methods of filling above the arch; whether with solid masonry to the level *v f*, Fig 2, of the top of the arch; or entirely with earth; or partly with each, as represented in the fig: or with parallel spandrel-walls extending to the back of the abut, as in Fig 2½. Although the stability of an abut cannot remain precisely the same under all these conditions, yet the diff of thickness which would follow from a strict investigation of each particular case, is not sufficient to warrant us in embarrassing a rule intended for popular use, by a multitude of exceptions and modifications which would defeat the very object for which it was designed. We shall not touch upon the theory of arches, except in the way of incidental allusion to it. Theories for arches, and their abuts, omit all consideration of passing loads; and consequently are entirely inapplicable in practice when, as is frequently the case, (especially in railroad bridges of moderate spans,) the load bears a large ratio to the wt of the arch itself. Hence the theoretical line of thrust has no place in such cases. Our rule is intended for common practice: and we conceive that no error of practical importance will attend its application to any case whatever; whether the arch be circular or elliptic.

It gives a thickness of abut, which, without any backing of earth behind it, is safe in itself, and in all cases, against the pres, when the bridge is unloaded. Moreover, in very large arches, in which the greatest load likely to come upon them in practice is small in comparison with the wt of the arch itself, and the filling above it, our abuts would also be safe from the *loaded* bridge, without any dependence upon the earth behind them; but as the arches become less, and consequently the wt of the load becomes greater in proportion to that of the arch, and of the filling above it, we must depend more and more upon the resistance of the earth behind the abuts, in order to avoid the necessity of giving the latter an extravagant thickness. It will therefore be understo[illegible] *throughout, that our rules suppose that after the bridge is finished*, earth will be depos[illegible] *behind the abuts, and to the height of the roadway*, as usual; except when par[illegible] *spandrel walls are used.*

To proportion the abutments of a stone bridge.

RULE. Find, as in Art 2, the rad c *o*, Fig 2, in ft, which will touch the arch at *o*, *a*, and *e*. Div this rad by 5. To the quot add $\frac{1}{10}$ of the rise, and 2 ft. The sum will be the thickness *o n* or *e y* of each abut at the springing line, or the level from which the arch starts, for any abut whose height *o s* does not exceed $1\frac{1}{2}$ times the base *s p*. Or by formula,

$$\left.\begin{array}{c}\textit{Thicks o n of abut at spring}\\ \textit{in ft, when the height o s}\\ \textit{does not exceed } 1\tfrac{1}{2} \textit{ times the}\\ \textit{base s p}\end{array}\right\} = \frac{\textit{Rad in ft}}{5} + \frac{\textit{rise in ft}}{10} + 2\,\textit{ft.}$$

Mark the points *n* and *y* thus ascertained. Next, from the center *i*, of the span or chord *e o*, lay off *i h*, equal to $\frac{1}{24}$ part of the span. Join *a h*; and through *n*, and parallel to *a h*, draw the indefinite line *g n p* of the abut. Do the same with the other abut. Make *y m* and *n g* each equal to half the entire height *i t* of the arch; and from *g* draw a straight line *g x*, touching the back of the arch as high up as possible; or still better, as shown at *t m*, with a rad *d t* or *d m*, (to be found by trial,) describe an arc *t m*. Then *g x* or *t m* will be the top of the masonry filling above the arch.*

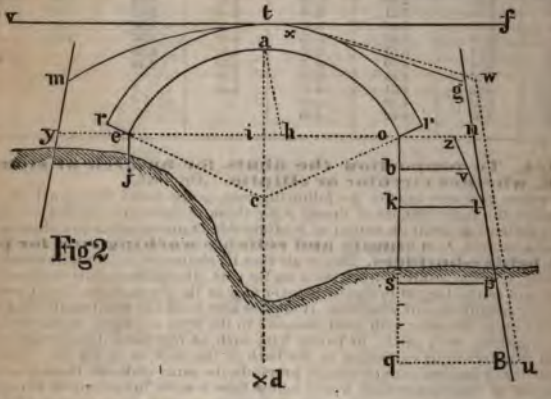

Fig 2

Now find by trial the point *s*, Fig 2, at which the thickness *s p* is equal to two-

Fig $2\frac{1}{2}$

* EXCEPT WHEN THE RISE IS BUT ABOUT $\frac{1}{5}$ OF THE SPAN, OR LESS; in which case carry the masonry up solid to the level *v t f*, of the top of the arch. Or if the arch is a large one, exceeding say about 60 ft span; and especially if its rise is greater than about $\frac{1}{5}$ of its span, it is better to economize masonry by the use of parallel interior spandrel-walls, *l l*, Fig $2\frac{1}{2}$, carried up to *v t f*, Fig 2. Indeed, such interior walls may often be advantageously introduced in much smaller arches. When high, they are steadied by occasional cross-walls, as *t t*, Fig $2\frac{1}{2}$. Their feet should be *spread by offsets, as shown at o o o, so as to bear upon the whole surf of the back of the arch; thus equalizing the pres upon it. On top of the walls flagstones may be laid, or small arches may be turned from wall to wall, for supporting the ballast, &c. of the roadway. The spaces below are left hollow. In Fig $2\frac{1}{2}$ the dark part w w is supposed to be a section across an abutment; but omitting a second cross-wall, similar to t t.*

thirds of the corresponding vert height $o\,s$, and draw $s\,p$. Then will the *thickness* $o\,n$ or $e\,y$ be that at the springing line of the given circular or elliptic arch of any rise and span; and the line $g\,p$ will be the *back* of the abut; provided its height $o\,s$ does not exceed $1\frac{1}{2}$ times $s\,p$: or in other words, provided $s\,p$ is not less than $\frac{2}{3}$ of $o\,s$. In practice, $s\,p$ will rarely exceed this limit; and only in arches of considerable rise. But if it should, as for instance at $o\,q$, then make the base $q\,u$ equal to $s\,p$, added to one-fourth of the *additional* height $s\,q$; and draw the back $u\,w$, parallel to $g\,p$; and extending to the same height, &c, as in Fig 2. If, however, this addition of $\frac{1}{4}$ of $s\,q$ should in any case give a base $q\,u$, less than *one-half* the total height $o\,q$, (which will very rarely happen in practice,) then make $q\,u$ equal to half said total height; drawing the back parallel to $g\,p$, and extending it to the same height as before. The additional thicknesses thus found below $s\,p$, have reference rather to the pres of the earth behind the abut, than to the thrust of the arch. In a very high abut, the inner line $g\,p$ would give a thickness too slight to sustain this earth safely.

When the height $o\,b$, Fig 2, of the abut is less than the thickness $o\,n$ at spring, a small saving of masonry (not worth attending to, except in large flat arches) may be effected by reducing the thickness of the abut throughout, thus: Make $o\,k$ equal to $o\,n$, and draw $k\,l$. Make $o\,z$ equal to $\frac{3}{4}$ of $o\,n$, and draw $l\,z$. Then, for any height $o\,b$ of abut less than $o\,n$, draw $b\,v$, terminating in $l\,z$. This $b\,v$ will be sufficient base, if the foundations are firm. The back of the abut will be drawn upward from v, parallel to $g\,p$, and terminating at the same height as g or w.

Rem. 1. All the abuts thus found will (with the provisions in Art 6) be safe, without any dependence upon the wing-walls; no matter how high the embkt may extend above the top of the arch. If the bridge is narrow, and the inner faces of the wing-walls are consequently brought so near together as to afford material assistance to the abuts, the latter may be made thinner; but to what extent, must depend upon the judgment of the engineer.

We, however, caution the young practitioner to be careful how he adopts dimensions less than those given by our rule. There are certain practical considerations, such as carelessness of workmanship; newness of the mortar; danger of undue strains when removing the centers; liability of derangement during the process of depositing the earth behind the abuts, and over the arch; &c, which must not be overlooked; although it is impossible to reduce them to calculation.

Whenever it can be done, the centers should remain in place until the embkt is finished; and for some time afterward, to allow the mortar to set well.

Rem. 2. A good deal of liberty is sometimes taken, in reducing the quantity of masonry above the springing line of arches of considerable rise, and of moderate spans. When care is taken to leave the centers standing until the earth filling is completed above the arch, and behind its abuts, so that it may not be deranged by accident during that operation; and when good cement is used instead of common mortar, such experiments may be tried with comparative safety; especially with culvert arches, in which the depth of arch-stones is great in proportion to the span. They must, however, be left to the judgment of the engineer in charge; as no specific rules can be laid down for them. They can hardly be regarded as legitimate practice, and we cannot recommend them. We have known nearly semicircular arches, of 30 to 40 ft span, to be thus built successfully, with scarcely a particle of masonry above the springs to back them. Such arches, however, are apt to fall, if at any future period the earth filling is removed, without taking the precaution to first build a center or some other support for them. Even when the embkt can be finished before the centers are removed, we cannot recommend (and that only in small spans) to do *less* than to make $n\,g$, Fig 2, equal to $\frac{1}{6}$ of the total height $i\,t$ of the arch; and from g so found, to draw a straight line touching the back of the arch as high up as possible.

Rem. 3. We have said nothing about **battering the faces of the abuts,** because in the crossing of streams, the batter either diminishes the water-way; or requires a greater span of arch. Such a batter, however, to the extent of from $\frac{1}{2}$ to $1\frac{1}{2}$ ins to a ft, is useful, like the offsets, for distributing the wt of the structure, and its embkt, over a greater area of foundation; especially when the last is not naturally very firm; or when the embkt extends to a considerable height above the arch. In our tables, Nos 3 and 5, of approximate quantities of masonry in semicircular bridges of from 2 to 50 ft span, the faces are supposed to be vert.

Art. 5. Abutment-piers. When a bridge consists of several arches sustained by piers of only the usual thickness, if one arch should by accident of flood, or otherwise, be destroyed, the adjacent ones would overturn the piers; and arch after arch would then fall. To prevent this, it is usual in important bridges to make some of the piers sufficiently thick to resist the pres of the adjacent arches, in case of such an accident; and thus preserve at least a portion of the bridge from ruin. Such are called abutment-piers.

Our formula of $\frac{\text{Rad}}{5} + \frac{\text{Rise}}{10} + 2$ ft, for the thickness at spring; with the back battering as before, at the rate of $\frac{1}{24}$ of the span to the rise; face vert; will of itself (*without any modification for great heights*) give a perfectly safe abut-pier, for any unloaded bridge; and to any height whatever; dw *regard being had, however, to the consideration alluded to in the next Art.* Thus, for an abut-pi *as high as e p, Fig 2; or of any greater height;* it is only necessary first to find the thickness o n *spring as before; and then draw the battered back g n p;* extending it down to the base at B; *out adding ⅙ of the additional height s g.* This addition is made in the case of abuts, that

may be secure from the pres of the *earth* behind them; as well as from the pres of the arch; a consideration which does not apply to abut-piers; in which only the pres of the arch is to be resisted.*

Fig 3

But although the abut-pier thus found by our formula, would be abundantly safe, yet its shape *a b c o*, Fig 3, is inadmissible. In practice it would be changed to one somewhat like that shown by the dotted lines; having an equal degree of batter on both faces. This of course requires more masonry, with but little increase of stability; but that cannot be avoided.

WHEN AN ABUT-PIER IS BUILT IN DEEP WATER, or in a shallow stream subject to high freshets, care must be taken that water cannot find its way *under* the pier, and thus produce an upward pres, which will either diminish, or entirely counteract its efficiency as an abut. See Remark 2, Art 4, of Hydrostatics, p 525.

Art. 6. Inclination of the courses of masonry below the springs of an arch. Although our foregoing rule gives a thickness of abut which cannot be *overturned*, or *upset*, by the pres of the arch, yet if the arch be of large span, and small rise, its great hor thrust may produce a *sliding* outward of the masonry near the level of the springs, if the stones are laid in *hor* courses; especially if the mortar has not set well.

This danger, it is true, could be avoided by confining the courses together by iron bolts and cramps; or by increasing considerably the thickness of the abuts; but the expense of doing either of these, leads to the cheaper expedient of inclining the masonry, as shown between *o* and *n*, Fig 4; the courses near *o* being steeper; and gradually becoming less steep near *n*. By this process the arch is virtually prolonged into the body of the abut, so far that when the inclination of the lower masonry ceases, as at *n*, the direction of the theoretical *line of thrust*, or *of pres* of the arch, (rudely represented by the dotted curved line *o n*) is nearly at right angles to the joints of the hor masonry below *n*; and consequently, said thrust is unable to produce sliding at that point. Between *o* and *n*, the line of pres is everywhere so nearly at right angles to the variously inclined joints, as to preclude the possibility of sliding in that interval also. See Art 63 of Force in Rigid Bodies. The abut being thus safe throughout from both overturning and sliding, can fail only from defective foundations; or from the inferiority of the stone of which it is built; and which, if soft, may be crushed.

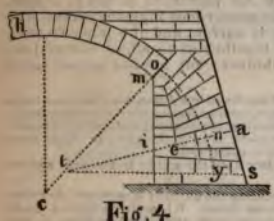

Fig. 4

This inclination of the masonry is as necessary in an elliptic arch, Fig 4½, as in a circular one. **The direction of the line of thrust *n a o u m* in an elliptic arch,** differs but slightly from that in a circular one of the same span and rise. The *amount* of thrust is also nearly the same in both; consequently, the same precautions for resisting it, must be adopted in each. The dotted line *n a o u m*, in Fig 4½, gives a tolerable *general* idea of the theoretical line of thrust in an elliptic arch, when the filling above the arch, and abuts, is either as represented in the fig; or when it is wholly of earth, or

Fig. 4½

* An intelligent engineer to whom this formula was shown in manuscript, has drawn the writer's attention to two bridges in the immediate vicinity of Philadelphia, which might at first sight appear to show that the formula does not give a sufficient abut-pier. In one case each of the two piers sustains on one side, the thrust of a nearly semicircular arch of brick, of 22 ft span; and on the other side, the end of an iron girder bridge of 55 ft span, heavily paved with granite. In the other case, each of the two piers sustains on one side, the thrust of a semicircular brick arch of 60 ft span; and on the other side, the end of an iron railroad bridge of the Pratt system, of 290 ft span; which rests about four feet upon each pier. The sides of the piers batter nearly ½ an inch to a foot. Their height below the springs of the arches is 35 ft; and 43 ft below the bridge-seat, or bottom chord of the truss. Thickness at springs 12 ft; and at base 14 ft 8 ins; or somewhat in excess of what the formula requires. *Each of these two piers stands on wooden cribwork resting on an inclined rock-bottom on each side of the river; the inclination being so great that the height of the cribs is some feet greater under the bridge-seats than under the springs of the arches.* In both bridges, both piers now lean several inches; as if pushed outward by the thrust of the brick arches. Now, in the case of the larger bridge, it is plain that the weight of the iron span resting but 4 ft upon the piers, *combines with the thrust of the 60 ft arches, to throw a great excess of pres near that edge of the base of the piers which rests* upon the highest part of the wooden crib foundation. Hence, a greater amount *of compression must* take place in the cribwork on that side than on the other; and a leaning of the *piers is inevitable.* In the small bridge, the base of the pier is considerably less than the formula *requires; therefore,* it need not be considered. It is evident that in all such cases, if the foundation *is compressible,* or if the mortar-joints have not had time to harden fully, the piers are in the same *danger of leaning* outward than if they were abut-piers only; and a very slight degree of settlement *one edge of the base* will produce a considerable leaning in a tall pier.

wholly of solid masonry; although both its *amount*, and its *direction*, will diffe somewhat according as one or the other of these modes of filling is adopted.*

The elliptic form is plainly unfavorable for uniting the arch-stones with the inclined masonry ne the springs, so as to receive the line of thrust properly; or at right angles to its direction. In ord nary cases this difficulty may be overcome by making the joints of only the outside or showing arc stones to conform to the elliptic curve; as between *s* and *u*; while the joints of the inner or hidd ones, may have the directions shown between *g* and *u*, nearly at right angles to the line of thrust. will rarely happen, however, that the young engineer will have to construct elliptic arches of suf cient magnitude to require either this, or any equivalent expedient. For spans less than 50 ft, with ris not less than about $\frac{1}{5}$ of the span, nothing of the kind is actually necessary, if the mortar is good, ar has time to harden.†

In order to incline the masonry of any abut with scrupulous accuracy, it woul be necessary first to trace the curved line of pres of the given arch, as directed i Art 72 of Force in Rigid Bodies, so as to arrange the beds of the stones at right angl to it at every point of its course; but we offer the following process as sufficing for a ordinary practical purposes; while its simplicity places it within the reach of the con mon mason. In actual bridges the direction of the line of thrust changes as the lo is passing; therefore, in practice no given degree of inclination of the abut masoni can conform to it precisely during the entire passage. Consequently, any excess refinement in this particular, becomes simply ridiculous; especially in small span

Rule for inclining the beds of the masonry in the abut Add together the rad *c m*, Fig 4; and the span of the arch. Div the sum by 5. the quot add 3 ft. Make *o t*, on the rad, equal to the last sum. Then is *t* a centr point, toward which to draw the directions of the beds, as in the fig. Draw *t s* ho and from *t* as a center, describe the arc *o y*; *o* being the center of the uniform dep of the arch. From *y* lay off on the arc the dist *y n*, equal to one-sixth part of *t* draw *t n a*. It will never be *necessary* to incline the masonry below this *t n* Neither *need* the inclination extend entirely to the face *m i* of the abut; but ma stop at *e*, about half-way between *i* and *n*. From *e* upward, the inclination ma extend forward to the line *e m*.

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* It will be seen that in consequence of the more sudden curvature of an elliptic arch, near t parts *a* and *u*, the dotted line of thrust in Fig 4½ may pass entirely out from the arch-stones at tho points, and enter again below. This causes a tendency in the parts of the arch in that vicinity rise; and thus permit the portions near the crown *o*, to descend. If the arch in Fig 4½ were a se ment of a circle, the direction of the line of thrust would become but little changed; and the ar would consequently coincide with it more nearly than in the elliptic one; and hence would be mo stable than this last. If the arch is *a full semicircle*, with stones of ordinary depth, the line thrust will pass out of it, and into it again, as in the elliptic one; producing the same deranging te dency. See Rem 3, p 489, of Force in Rigid Bodies. So long as the intrados is elliptic, or a full sem circle, with keystones of the ordinary proportions, no increase in the depth of the arch-stones fro the key toward the springs, will prevent the line of thrust from thus falling below the arch; or ev materially change its direction in an actual bridge. Thus, if in Fig 4½, the depth of the arch-ston were increased toward the springs, even as much as is shown by the dotted line *w s*, the line of thru in the finished bridge would still be along the dotted line *o a n*; the greater depth serving merely distribute the thrust over a greater area. If, instead of solid masonry, a hollow vault were construct at *w* to support the roadway, the line of thrust at *a* would rise a little; and by thus building so parts of the spandrels hollow, and other parts solid, much may be done in the way of keeping the li of thrust within the arch-stones. The same could be accomplished by *greatly* increasing the dep of *all* the arch-stones; for in that case a line parallel to *o a n*, and passing through the center of t depth of the new keystone, as well as that of the new springer, would everywhere fall *within* the arc stones; although not to the same extent in all of them. This may, in fact, be the reason for the gre depth of key in elliptic arches, referred to a few pages back. In cases where appearance may be sa rificed to strength, we might first find (by Rem 3, p 489, of Force in Rigid Bodies,) the line of thrust pressure of a required arch; and if, as in Fig 4½ above, it does not fall well within the arch-stone we may arrange the latter so as to coincide with it. Such a change in the arch-stones would of cour in turn slightly change the direction of the line of thrust; but in ordinary practice this might i neglected. As a general rule, the change in the shape of the arch would be detected by but few pe sons; nor indeed, does it follow that the new curve must be at all ungraceful.

As a segment arch becomes more flat, the line of thrust coincides more nearly with the arch-stone The writer draws the following inference, from the results with many of his own diagrams. Fro the center *c*, Fig 1½, from which any segmental arch whose rise does not exceed .4 of its span, described, draw two lines *c o*, *c o*, forming angles of 25° with *c a*. Then supposing the depth of t keystone to be according to our rule, make the depths of the arch-stones between *o* and *s*, everywhe equal to 1¼ times that of the key. From the key increase the depth gradually to *o* and *o*. Then wi the theoretical line of thrust of the arch (supposed to be filled up to the level of *n n*) everywhere fa within the actual arch-stones; nowhere approaching their extrados or intrados nearer than about of the depth of the arch at the same point; and this will be the case whether the filling be either eart or solid masonry; or of parallel spandrel walls; or partly of masonry, and partly of earth; both di posed as indicated by the line *s*, *t*, or *i*, &c; although both the direction and intensity of the line thrust will of course vary *slightly* under these different conditions. This appears to the writer be an important fact in practice, as regards segmental arches of great spans.

† The feet of both elliptic and semicircular arches are always made hor; but it is plain from F 4½, that this practice is at variance with correct principles of stability in the case of the ellipse. *is the same in the semicircle. In ordinary bridges of the latter form, the vert pres, or weight re on each skewback, is (roughly speaking)* usually about from 3¼ to 4 times the hor pres on the *and the total pres is about 4 times as great as the pres on the keystone. Therefore, theoretical skewback should* usually be about 4 times as deep as the keystone; and its bed, instead of be *should be inclined at the rate of* about 1 vert to 4 hor. See Example 2, p 468, of Force in Rig

When the arch is flat, this inclination may become so steep, especially in the upper parts, that struts, or shores of some kind, must be used for preventing the masonry from sliding down, until the completion of the arch secures it from doing so. The hor courses between the face *m i*, and the line *o e*, will aid somewhat in this respect.

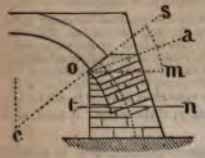


Fig 5

The method thus described, should be applied to *all very large arches* whose rise is ⅓, or less, of the span. As before remarked, it is not actually necessary in arches not exceeding about 50 ft span, and not flatter than $\frac{1}{5}$ of the span. Indeed, if the earth filling can be deposited before the centers are removed, these limits may be considerably extended without danger. Still, since a certain degree of inclination is attended with very little trouble or expense, we would recommend for even such arches, a process somewhat like the following: From *half* the span take the *rise*. Div the rem by 3. Make *o t*, Fig 5, equal to the quot. Draw *t n*, and *o m*, hor. Div the angle *s o m* into two equal parts, by the line *o a*. Incline the masonry so as to be parallel to *o a*, as far down as *t n*. The inclined courses may extend out to the face *o t*, or not, at pleasure.

Rem. 1. The necessity for inclining the courses becomes greater as the span increases; because the pres from arches of the same transverse width, but otherwise of the same proportions, and of diff spans, increases nearly as the squares of the spans; whereas the thickness of the abut at the springs increases only nearly in the same proportion as the span itself. Hence, in the larger span, the abut at spring presents a less proportion of mortar-joint, and of friction, to prevent sliding, than in the smaller one. In practice, the pres of arches does not increase quite as rapidly as the squares of the spans, because the depth of the arch-stones does not increase as fast as the spans; thus reducing the relative wt of the large one considerably.

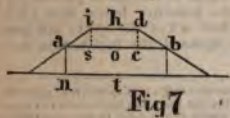


Fig 7

Rem. 2. **To find the length** (*a b*, Fig 7) **from face to face of a culvert.** From the height *h i* of the embkt, take the *above-ground* height *n s* of the culvert; the rem will be the height *h o* of the embkt *above* the culvert. Then the reqd length *a b* is plainly equal to the top width *i d* of the embkt, added to the two dists *a s*, *c b*, which correspond to its steepness of side-slopes. Thus, if the side-slope is, as usual, 1½ to 1, then *a s* and *c b* will each be equal to 1½ times *o h*; or the two together will be 3 times *o h*. So that if the width *i d* is 14 ft, and *h o* 5 ft, the length *a b* will be

$$14 + (5 \times 3) = 14 + 15 = 29 \text{ ft.}$$

Art. 7. The following tables, 3, 4, and 5, of quantities, will be found useful for expediting preliminary estimates; for which purpose chiefly they are intended; hence no pains have been taken to make them scrupulously correct, but rather a little in excess of the truth. The first column of Table 3 contains the total vert height *o c*, Fig 6, from the crown *o* of a semicircular arch, to the foundation or base *g m* of its abut. The other columns give approximately the number of cub yds contained in each running foot, or foot in length of the culvert or bridge, measured from end to end (face to face) of the arch proper; and including only the arch and its abuts, as shown in Fig 1; or in the half section *a p m g y* in Fig 6; including footings to the abuts, but omitting the wing-walls (*w n*), and the spandrel-walls (*s*), Figs 6 and 2½. At the foot of each column is the approximate content in cub yds of the two spandrel-walls by themselves; one over each face of the arch.

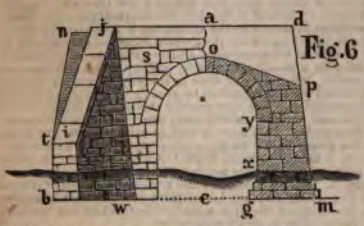


Fig.6

These spandrel-walls are calculated on the supposition that their thickness at base, at their junction with the wing-walls, where their height is greatest, is equal to $\frac{4}{10}$ of their height at that point: except where that proportion gives a less thickness at top than 2½ ft; and that they extend 2 ft (*o a*) above the top *o* of the arch. At the top of the arch, they are all supposed to be 2½ ft thick at top; that being assumed to be about the least thickness admissible in a rubble wall in such a position. Both the back and the face are supposed to be vert. The contents of these spandrel-walls will vary somewhat, however, even in the same span, with the height of the abut and the arrangement of the wings. They, however, constitute so small a proportion of the entire contents given in Table 5, that *this consideration may be neglected in preliminary estimates. They are so firmly bonded into the masonry of the wings* at their highest points, and so strongly connected by mortar with the backing *of the arch at their bases*, that they require no greater thickness even when the embkt extends to *an unlimited height* above them.

The contents of the four wing-walls, of which *n j w b*, Fig 6, is one, *will be found in a table* (No. 4) immediately following that for the body of the cul*vert. We have also added a table* (No. 5) for complete semicircular culverts *rious lengths*, including their spandrel and wing walls.

REM. 1. Although the thickness of wing-walls increases in all parts with their ght, they are not made to *show* thicker at *n j* than at *t i*, Fig 6; but (as seen in the are offsetted at their back *t n*, a little below their slanting upper surf *i j*, so as give a uniform width for the steps or flagstones, as the case may be, with which y are covered. In the fig the covering is supposed to be of flagstones; but steps preferable, being less liable to derangement. To prevent the flagstones from ling down the inclined plane *j i*, the lower stone *i* should be deep and large, and l with a hor bed. The flags are sometimes cramped together with iron, and bolted vn to the wall. Steps require nothing of that kind.

REM. 2. **The tables show the inexpediency of too much contracting the width of water-way,** with a view to economy, by adopting mall span of arch, when a culvert of greater span can be made, *of the same total ght.*

or the wings must be the same, whether the span be great or small, provided the total height is same in both cases; and since the wings constitute a large proportion of the entire quantity of sonry, in culverts of ordinary length, the span itself, within moderate limits, has comparatively le effect upon it. Thus, the total masonry in a semicircular culvert of 3 ft span, 8 ft total height, 60 ft long between the faces of the arch, is, by Table 5, 151¼ cub yds; while that of a 5 ft span, the same height and length, is 152.4. A semicircular bridge of 25 ft span, 24 ft total height, and t between the faces of the arch, contains 1031 cub yds; while one of 35 ft span, of the same height l length, contains 1134 yds; so that in this case we may add nearly 50 per cent to the water-way, increasing the masonry of the bridge but $\frac{1}{10}$th part.

REM. 3. Partly for the same reason, and partly because **the culverts for a uble-track road are not twice as long as those for a single-ack one,** the quantity of culvert masonry for the former will not average more an about from ¼ to ⅙ part more than that for the latter; so that it frequently comes expedient to *finish* the culverts at once to the full length required for a uble track, although the embkts may at first be made wide enough for only a gle one, with the intention of increasing them at a future time for a double one.

Thus, the average size of culverts for a single track may be roughly taken at 6 ft span, 30 ft long m face to face, and 10 ft total height; and such a one contains, by Table 5, 140 cub yds. For a uble track, it would require to be about 12 feet longer; and we see by Table 3 that this will add 7 × 12 = 32 cub yds; making a total of 172 yds instead of 140; thus adding rather less than ¼ t. When the culverts are under very high embkts, and consequently much longer, the addition a double track becomes comparatively quite trifling.

Table 3, of approximate numbers of cub yds of masonry er foot run, contained in the arches and abutments only, as own in Fig 1 (omitting wings, and the spandrel-walls over the faces of the arches) semicircular culverts and bridges, of from 2 to 50 ft span, and of different total ights, *h t*, Fig 1, or *o c*, Fig 6. It will be seen that in many cases, a bridge of larger an contains less masonry than one of smaller span, *when their total heights are the me.* There is a liberal allowance for footings or offsets at the bases of the abuts. e quantities given in this table are generally slightly *in excess* of the truth.

TABLE 3. (Original.)

Total Height.	Span 2 ft.	Span 3 ft.	Span 4 ft.	Span 5 ft.	Span 6 ft.	Span 8 ft.	Span 10 ft.	Span 12 ft.	Span 15 ft.
Feet.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.	Cub. y.
2	.42	.....	.....	.....	.....	.....	.....	.....	.....
3	.60	.63	.67	.....	.....	.....	.....	.....	.....
4	.79	.83	.87	.92	.97	.....	.....	.....	.....
5	.99	1.04	1.08	1.15	1.21	.....	.....	.....	.....
6	1.28	1.28	1.28	1.37	1.46	1.58	1.69	.....	.....
7	1.62	1.59	1.55	1.64	1.72	1.85	1.97	2.12	.....
8	2.01	1.96	1.91	1.95	1.99	2.13	2.26	2.38	.....
9	2.45	2.38	2.31	2.29	2.27	2.42	2.56	2.65	3.02
10	2.94	2.85	2.76	2.72	2.67	2.77	2.87	2.93	3.34
11	.....	3.38	3.26	3.19	3.12	3.16	3.19	3.23	3.67
12	.....	3.98	3.82	3.72	3.62	3.57	3.52	3.55	4.01
13	.....	.....	4.42	4.29	4.17	4.10	4.02	3.86	4.36
14	.....	.....	5.08	4.90	4.77	4.67	4.57	4.41	4.72
15	.....	.....	.....	5.57	5.42	5.30	5.17	5.01	5.09
16	.....	.....	.....	6.30	6.12	5.97	5.82	5.56	5.69
17	.....	.....	.....	.....	6.87	6.70	6.52	6.26	6.34
18	.....	.....	.....	.....	7.69	7.48	7.27	7.01	7.04
19	.....	.....	.....	.....	.....	8.32	8.07	7.71	7.69
20	.....	.....	.....	.....	.....	9.20	8.92	8.56	8.49
21	.....	.....	.....	.....	.....	.....	9.82	9.46	9.34
22	.....	.....	.....	.....	.....	.....	10.8	10.3	10.2
23	.....	.....	.....	.....	.....	.....	.....	11.3	11.1
24	.....	.....	.....	.....	.....	.....	.....	12.3	12.1
25	.....	.....	.....	.....	.....	.....	.....	.....	13.2
26	.....	.....	.....	.....	.....	.....	.....	.....	14.2
Contents of the two spandrel-walls, over the two ends of the arch, in cub yds.	2.9	3.7	4.4	5.2	5.8	7.9	9.8	12.	16.

TABLE 1. Continued.

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Feet.	Cub. yds.
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	30.9
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	33.4
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	34.6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	36.3
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	38.1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	45.5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	47.4
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	49.6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	51.6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	53.7
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	55.9
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	58.1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	60.4
[illegible]	[illegible]		[illegible]		[illegible]	62.7
[illegible]		[illegible]			[illegible]	65.1

Contents of the two wing-walls, [illegible] in cub yds.

	[illegible]	[illegible]		[illegible]		196.

Art. 9. The following table of approximate contents of wing-walls, or [illegible] preliminary estimates. The [illegible] of 120°, wit [illegible] are all assumed to [illegible] at E. Thickness at base at every part [illegible] when that proportion be [illegible] to be 2.5 ft; in which case it [illegible] See Remark 2. This happen

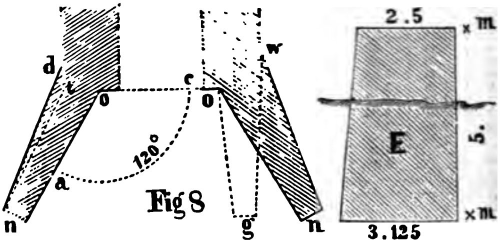


Fig 8

when the height *m m*, Fig E, of the wing, becomes less than 9 ft. Batter of fac one to a ft, or 1 in 8. Back vert; but offsetted, if necessary, for a short dist the top, so as to give a uniform *showing* top thickness of 2½ ft. The mas *supposed to be good* well-scabbled mortar rubble. The height given in the *column is the greatest* one; or that at o o, (or w j, Fig 6,) where the wing join *face of the culvert.* In the table no allowance is made for footings (offsets or *at the base of the* wings) as these are frequently omitted in wings on good

In taking out quantities from the table, bear in mind that the height of the is usually a little greater than that of the culvert itself.

le 4, of approximate contents, in cub yds, of the four wing-walls of a culvert, or bridge. (Original.)

The heights are taken where greatest; as at *j w*, Fig 6

Height of wing.	Length of one wing.	Cub. yds. in 4 wings.	Height of wing.	Length of one wing.	Cub. yds. in 4 wings.
Feet.	Feet.		Feet.	Feet.	
6	1.73	4.04	30	43.3	818
7	3.46	8.85	32	46.8	997
8	5.20	14.6	34	50.3	1192
9	6.93	21.5	36	53.7	1414
10	8.66	30.2	38	57.2	1661
11	10.4	40.9	40	60.7	1928
12	12.1	53.7	42	64.2	2220
14	15.6	85.2	44	67.6	2552
16	19.1	128	46	71.1	2912
18	22.5	183	48	74.6	3306
20	26.0	247	50	78.0	3741
22	29.5	329	55	86.7	4942
24	32.9	426	60	95.3	6404
26	36.4	541	65	104	8131
28	39.8	672	70	113	10155

To reduce cub yds to perches of 25 cub ft, mult by 1.080.
To reduce perches to cub yds, mult by .926, or div by 1.08.

ontents for heights intermediate of those in the table may be found approximately by simple ion.

t. 1. It is not recommended to actually prolong all wings until their dimen- become as small as shown at E, in Fig 8. In large ones it will generally be economical to increase their end height *m m*, a few feet. The contents, how- may be readily found by the table in that case also. Thus suppose the height wings at one end to be 30 ft, and at the other end 8 ft; we have only to sub- the tabular content for 8 ft high, from that for 30 ft high. Thus, 818 — 14.6 = ub yds required content.

t. 2. It might be supposed that inasmuch as the wings of arches often have to n the pressure from embankments reaching far above their tops, they should, rdinary retaining-walls, be made much thicker in that case. But the fact that lerive great additional stability from being united at their high ends to the of the bridge or culvert, renders such increase unnecessary when proportioned r rule; no matter how far the earth may extend above them; as shown by lant experience.

ing upon this aid, we may indeed, when the earth does not extend above the top, reduce the *e* to ⅓ of the height, as shown at *o t*; and by the dotted line *t s*. Experience shows that we so do the same even when the earth reaches to a great height above the top; provided that ngs, instead of being splayed or flared out, as at *e m*, *e m*, merely form straight prolongations abutments of the arch, as shown by the dotted lines at *e g w*. In this case the pressure of the against the wings is less than when they are splayed. We have known the thickness at *e* duced in such cases to rather less than ⅓ the height, when the wings were 15 ft high, and ght of the embankment *above their tops* 16 feet in one case, and 36 ft in another. In another se, similar wings 25½ ft high, and with 29 ft of embankment above their top, had their bases ther less than $\frac{1}{5}$ of the height. In all these cases, the uniform thickness at top was 2.5 feet; vertical. We mention them because this particular subject does not seem to be reducible to netical rule. The last wall appears to us to be too thin; especially if the earth is not deposited rs; and after allowing the mortar full time to set. The labor, however, required in compact- earth carefully in layers, may cost more than is thereby saved in the masonry. The young loner must bear this in mind when he wishes to economize masonry by such means; and also e thin wall may bulge, or fail entirely, if the earth backing is deposited while the mortar is ostly set.

Table 5. Approximate contents in cubic yards, of complete semicircular culverts and bridges of from 2 to 50 feet span; including the 2 spandrel walls; and the 4 wings; all proportioned by the foregoing directions: and taken from the two preceding tables. The height in the second column, is from the top of the *keystone* to the bottom of the foundation. The wings are calculated as being 2 ft higher than this, including the thickness of the coping. The wings are frequently carried only to the height of the top of the arch, thus saving a good deal of masonry. Table 4, of wings alone, will serve to make the proper deduction in this case.

The several lengths are from end to end, or from face to face, of the *arch proper*. The contents for intermediate lengths may be found exactly; and those for intermediate heights, quite approximately, by simple proportion. In this table, as in No. 3, it will be observed that *when the heights are the same in both cases*, a larger span frequently contains less masonry than a smaller one. A semicircular culvert or bridge contains less masonry than a flatter one, when the total height is the same in both cases therefore, the first is the most economical as regards cost; but it does not afford as much area of water-way or width of headway.

(Original.)

Span.	Height.	Length. 15 Ft.	Length. 20 Ft.	Length. 30 Ft.	Length. 40 Ft.	Length. 60 Ft.	Length. 80 Ft.	Length. 100 Ft.	Length. 120 Ft.	Length. 140 Ft.	Length. 160 Ft.	Length. 180 Ft.	Length. 200 Ft.
Ft.	Ft.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.
2	5	27	32	42	52	72	92	112	132	152	172	192	212
	6	37	43	56	69	94	120	146	171	197	222	248	274
	7	49	57	73	89	122	154	187	219	251	284	316	349
	8	63	73	93	113	153	193	233	273	313	353	393	433
	10	101	116	145	175	234	291	351	410	469	527	586	645
3	5	28	34	44	54	75	96	117	138	158	179	200	221
	6	38	44	57	70	95	121	146	172	198	223	249	275
	7	49	57	73	89	121	153	184	216	247	280	312	343
	8	63	73	93	112	152	191	230	269	308	348	387	426
	10	101	115	143	172	229	286	343	400	457	514	571	628
	12	149	169	208	248	328	407	487	567	646	726	806	885
4	5	30	35	46	57	78	100	122	143	165	186	208	229
	6	38	45	58	70	96	122	147	173	198	224	250	275
	7	49	57	73	88	119	150	181	212	243	274	305	336
	8	63	73	92	111	149	188	226	264	302	340	379	417
	10	100	114	141	169	224	279	335	390	445	500	555	611
	12	147	166	204	243	319	395	472	548	625	701	777	854
	14	209	234	285	336	437	539	641	742	844	945	1047	1149
5	6	41	47	61	75	102	130	157	184	212	239	267	294
	7	52	60	76	93	125	158	191	224	257	289	322	355
	8	65	75	94	114	153	192	231	270	309	348	387	426
	10	100	114	141	168	223	277	331	386	440	495	549	603
	12	146	165	202	239	314	388	463	537	611	686	760	835
	14	207	231	280	329	427	525	623	721	819	917	1015	1113
6	7	53	62	79	96	131	165	200	234	268	303	337	372
	8	66	76	96	116	156	196	236	276	316	356	396	436
	10	100	113	140	167	220	274	327	380	434	487	541	594
	12	146	164	200	236	308	381	453	526	598	670	743	815
	14	206	219	277	325	420	516	611	706	802	897	993	1088
	16	281	311	373	434	556	679	801	923	1046	1168	1291	1413
8	7	57	67	85	104	141	178	215	252	289	326	363	400
	8	70	81	102	124	166	209	251	294	337	379	422	464
	10	104	118	145	173	228	284	339	395	450	505	561	616
	12	147	165	200	236	308	379	450	522	593	664	736	807
	14	206	230	276	323	416	510	603	696	790	883	977	1070
	16	281	310	370	430	549	669	788	908	1027	1146	1266	1385
	18	367	405	480	554	704	854	1003	1153	1302	1452	1602	1751
10	8	74	85	108	131	176	221	266	311	357	402	447	492
	10	107	121	150	179	236	294	351	408	466	523	581	638
	12	148	166	201	236	306	377	447	518	588	658	729	799
	14	207	229	275	321	412	504	595	686	778	869	961	1052
	16	280	309	368	426	542	659	775	891	1008	1124	1241	1357
	18	366	402	475	548	693	839	984	1129	1275	1420	1565	1711
12	10	110	125	154	183	242	301	359	418	476	535	594	652
	12	151	168	204	239	310	381	452	523	594	665	736	807
	14	206	228	272	317	405	493	581	669	758	846	934	1022
	16	279	306	362	418	529	640	751	862	974	1085	1196	1307
	18	364	399	469	540	680	820	960	1100	1241	1381	1521	1661
	20	470	512	598	684	855	1026	1197	1368	1540	1711	1882	2053

Table 5—(Continued.) (Original.)

Span.	Height.	Length. 15 Ft.	Length. 20 Ft.	Length. 30 Ft.	Length. 40 Ft.	Length. 60 Ft.	Length. 80 Ft.	Length. 100 Ft.	Length. 120 Ft.	Length. 140 Ft.	Length. 160 Ft.	Length. 180 Ft.	Length. 200 Ft.
Ft.	Ft.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.
15	12	162	182	222	262	342	422	502	583	663	743	823	903
	14	215	239	286	333	427	522	616	711	805	899	994	1088
	16	285	313	370	427	541	654	768	882	996	1110	1223	1337
	18	369	404	474	545	686	826	967	1108	1249	1390	1530	1671
	20	473	515	600	685	855	1024	1194	1364	1534	1704	1873	2043
	22	595	646	748	850	1054	1258	1462	1666	1870	2074	2278	2482
20	14	237	264	317	371	478	586	693	801	908	1015	1123	1230
	16	304	335	397	458	582	706	829	953	1076	1200	1324	1447
	18	381	416	486	556	697	838	978	1119	1259	1400	1541	1681
	20	479	520	601	682	844	1007	1169	1332	1494	1656	1819	1981
	22	598	646	741	837	1028	1220	1411	1603	1794	1985	2177	2368
	24	739	785	908	1021	1247	1473	1699	1925	2151	2377	2603	2829
25	16	327	360	428	496	631	766	901	1036	1173	1307	1442	1577
	18	403	441	517	594	746	896	1050	1202	1355	1507	1659	1811
	20	500	543	629	715	887	1059	1231	1403	1575	1747	1919	2091
	22	614	663	760	857	1051	1246	1440	1635	1829	2023	2218	2412
	24	751	807	919	1031	1255	1479	1703	1927	2151	2375	2599	2823
	26	909	974	1104	1234	1494	1754	2014	2274	2534	2794	3054	3314
	28	1085	1160	1310	1460	1760	2060	2360	2660	2960	3260	3560	3860
35	22	685	743	859	975	1207	1439	1671	1903	2135	2367	2599	2831
	24	817	880	1007	1134	1388	1642	1896	2150	2404	2658	2912	3166
	26	969	1033	1181	1309	1585	1861	2137	2413	2689	2965	3241	3517
	28	1130	1205	1356	1507	1809	2111	2413	2715	3017	3319	3621	3923
	30	1327	1408	1571	1734	2060	2386	2712	3038	3364	3690	4016	4342
	32	1549	1639	1820	2001	2363	2725	3087	3449	3811	4173	4535	4897
	35	1946	2054	2271	2488	2922	3356	3790	4224	4658	5092	5526	5960
50	30	1494	1594	1795	1996	2398	2800	3202	3604	4006	4408	4810	5212
	32	1711	1819	2035	2251	2683	3115	3547	3979	4411	4843	5275	5707
	34	1956	2071	2302	2533	2995	3457	3919	4381	4843	5305	5767	6229
	36	2228	2350	2597	2844	3338	3832	4326	4820	5314	5808	6302	6796
	38	2519	2650	2913	3176	3702	4228	4754	5280	5806	6332	6858	7384
	40	2835	2975	3255	3535	4095	4655	5215	5775	6335	6895	7455	8015
	42	3197	3347	3647	3947	4547	5147	5747	6347	6947	7547	8147	8747
	45	3818	3991	4337	4683	5375	6067	6759	7451	8143	8835	9527	10219
	50	5063	5281	5717	6153	7025	7897	8769	9641	10513	11385	12257	13129

Art. 9. Especial pains should be taken to **secure an unyielding foundation for culverts and drains under high embkts;** otherwise the superincumbent weight, especially under the middle of the embkt, may squeeze them into the soil below, if soft or marshy; and thus diminish the area of waterway, or at least cause an ugly settlement at the midlength of the culvert. Also, in soft ground, the embkt may press the side walls closer together, narrowing the channel. This may be prevented by an inverted arch, or bed of masonry, between the walls. A stratum from 3 to 6 ft thick, of gravel, sand, or stone broken to turnpike size, will generally give sufficient foundation for culverts in treacherous marshy ground; or quicksand, with but a moderate height of embkt. It shoul extend a few feet beyond the masonry in every direction, and should be rammed the sand or gravel being thoroughly wet, if possible, to assist the consolidation. Piling will sometimes be ecessary. If the masonry is built upon timber platforms, or smooth surface of rock, care must be taken to prevent it from sliding, from the pres of the earth behind t. This same pres may even overthrow the piles, if they are not properly secured against it.

Art 10. Drains. Drains of the dimensions in Fig 11, contain 1 perch, of 25 cub ft; or .926 of a cub yd, per ft run.

They are frequently built of dry scabbled rubble, and *paved with spawls. When there is much wash through them, with a considerable slope, it is better to continue the foundation*

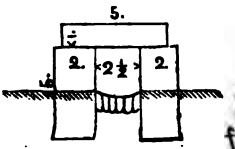
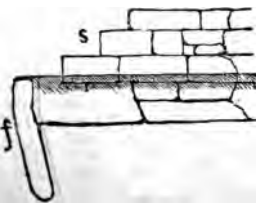


Fig 11



Test of Board Measure used of different

THICKNE

1	¾	½	¼	
Ft. B.M.	Ft. B.M.	Ft. B.M.	Ft. B.M.	Ft. B
.0208	.0260	.0313	.0365	.041
.0417	.0521	.0625	.0729	.0833
.0625	.0781	.0938	.1094	.1250
.0833	.1042	.1250	.1458	.1667
.1042	.1302	.1563	.1823	.2083
.1250	.1563	.1875	.2188	.2500
.1458	.1823	.2187	.2552	.2917
.1667	.2083	.2500	.2917	.3333
.1875	.2344	.2813	.3281	.3750
.2083	.2604	.3125	.3646	.4167
.2292	.2865	.3438	.4010	.4583
.2500	.3125	.3750	.4375	.5000
.2708	.3385	.4063	.4739	.5416
.2917	.3646	.4375	.5104	.5833
.3125	.3906	.4688	.5469	.6250
.3333	.4167	.5000	.5833	.6667
.3542	.4427	.5312	.6198	.7083
.3750	.4688	.5625	.6563	.7500
.3958	.4948	.5938	.6927	.7917
.4167	.5208	.6250	.7292	.8333
.4375	.5469	.6563	.7656	.8750
.4583	.5729	.6875	.8020	.9167
.4792	.5990	.7188	.8385	.9583
.5000	.6250	.7500	.8750	1.000
.5208	.6510	.7813	.9115	1.042
.5417	.6771	.8125	.9479	1.083
.5625	.7031	.8438	.9844	1.125
.5833	.7292	.8750	1.021	1.167
.6042	.7552	.9063	1.057	1.208
.6250	.7813	.9375	1.094	1.250
.6458	.8073	.9688	1.130	1.292
.6667	.8333	1.000	1.167	1.333
.6875	.8594	1.031	1.203	1.375
.7083	.8854	1.063	1.240	1.417
.7292	.9115	1.094	1.276	1.458
.7500	.9375	1.125	1.313	1.500
.7708	.9635	1.156	1.349	1.542
.7917	.9896	1.188	1.385	1.583
.8125	1.016	1.219	1.422	1.625
.8333	1.042	1.250	1.458	1.667
.8542	1.068	1.281	1.495	1.708
.8750	1.094	1.313	1.531	1.750
.8958	1.120	1.344	1.568	1.792
.9167	1.146	1.375	1.604	1.833
.9375	1.172	1.406	1.641	1.875
.9583	1.198	1.438	1.677	1.917
.9792	1.224	1.469	1.714	1.958
1.000	1.250	1.500	1.750	2.000
1.021	1.276	1.531	1.786	2.042
1.042	1.302	1.563	1.823	2.083
1.063	1.328	1.594	1.859	2.125
1.083	1.354	1.625	1.896	2.167
1.104	1.380	1.656	1.932	2.208
1.125	1.406	1.688	1.969	2.250
1.146	1.432	1.719	2.005	2.292
1.167	1.458	1.750	2.042	2.333
1.188	1.484	1.781	2.078	2.375
1.208	1.510	1.813	2.115	2.417
1.229	1.536	1.844	2.151	2.458
1.250	1.563	1.875	2.188	2.500
1.271	1.589	1.906	2.224	2.542
1.292	1.615	1.938	2.260	2.583
1.313	1.641	1.969	2.297	2.625
1.333	1.667	2.000	2.333	2.667
1.354	1.693	2.031	2.370	2.708
1.375	1.719	2.063	2.406	2.750
1.396	1.745	2.094	2.443	2.792
1.417	1.771	2.125	2.479	2.833
1.438	1.797	2.157	2.516	2.875
1.458	1.823	2.188	2.552	2.917
1.479	1.849	2.219	2.589	2.958
1.500	1.875	2.250	2.625	3.000
1.521	1.901	2.281	2.661	3.042
1.542	1.927	2.313	2.698	3.083
1.563	1.953	2.344	2.734	3.125
1.583	1.979	2.375	2.771	3.167
1.604	2.005	2.406	2.807	3.208
1.625	2.031	2.438	2.844	3.250
1.646	2.057	2.469	2.880	3.292
1.667	2.083	2.500	2.917	3.333
1.688	2.109	2.531	2.953	3.375
1.708	2.135	2.563	2.990	3.417
1.729	2.161	2.594	3.026	3.458
1.750	2.188	2.625	3.063	3.500
1.771	2.214	2.656	3.099	3.542
1.792	2.240	2.688	3.136	3.583
1.813	2.266	2.719	3.172	3.625
1.833	2.292	2.750	3.208	3.667
1.854	2.318	2.781	3.245	3.708
1.875	2.344	2.813	3.281	3.750
1.896	2.370	2.844	3.318	3.792
1.917	2.396	2.875	3.354	3.833
1.938	2.422	2.906	3.391	3.875
1.958	2.448	2.938	3.427	3.917
2.000	2.500	3.000	3.500	4.000

BOARD MEASURE.

Remark on following table. The table extends to 12 ins by 24 ins, but it is easy to find for greater sizes; thus, for example, the board measure in a piece of 19 by 22, will be twice that of a piece of 19 by 11, or 17.42 × 2 = 34.84 ft board meas; or that of 19¼ by 22, will be that of 10¼ by 22 added to that of 9 by 22, or 18.79 + 16.50 = 35.29. A foot of board meas is equal to 1 foot square and 1 inch thick, or to 144 cub ins. Hence 1 cub ft = 12 ft board meas.

Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.)

Width in Inches.	THICKNESS IN INCHES. 1	1¼	1½	1¾	2	2¼	2½	2¾	3	Width in Inches.
	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	Ft. Bd. M.	
¼	.0208	.0260	.0313	.0365	.0417	.0469	.0521	.0573	.0625	¼
½	.0417	.0521	.0625	.0729	.0833	.0938	.1042	.1146	.1250	½
¾	.0625	.0781	.0938	.1094	.1250	.1406	.1563	.1719	.1875	¾
1.	.0833	.1042	.1250	.1458	.1667	.1875	.2083	.2292	.2500	1.
¼	.1042	.1302	.1563	.1823	.2083	.2344	.2604	.2865	.3125	¼
½	.1250	.1563	.1875	.2188	.2500	.2813	.3125	.3438	.3750	½
¾	.1458	.1823	.2187	.2552	.2917	.3281	.3646	.4010	.4375	¾
2.	.1667	.2083	.2500	.2917	.3333	.3750	.4166	.4583	.5000	2.
¼	.1875	.2344	.2813	.3281	.3750	.4219	.4688	.5156	.5625	¼
½	.2083	.2604	.3125	.3646	.4167	.4688	.5208	.5729	.6250	½
¾	.2292	.2865	.3438	.4010	.4583	.5156	.5729	.6302	.6875	¾
3.	.2500	.3125	.3750	.4375	.5000	.5625	.6250	.6875	.7500	3.
¼	.2708	.3385	.4063	.4739	.5416	.6094	.6771	.7448	.8125	¼
½	.2917	.3646	.4375	.5104	.5833	.6563	.7292	.8021	.8750	½
¾	.3125	.3906	.4689	.5469	.6250	.7031	.7813	.8594	.9375	¾
4.	.3333	.4167	.5000	.5833	.6667	.7500	.8333	.9167	1.000	4.
¼	.3542	.4427	.5312	.6198	.7083	.7969	.8854	.9740	1.063	¼
½	.3750	.4688	.5625	.6563	.7500	.8438	.9375	1.031	1.125	½
¾	.3958	.4948	.5938	.6927	.7917	.8906	.9896	1.086	1.188	¾
5.	.4167	.5208	.6250	.7292	.8333	.9375	1.042	1.146	1.250	5.
¼	.4375	.5469	.6563	.7656	.8750	.9844	1.094	1.203	1.313	¼
½	.4583	.5729	.6875	.8020	.9167	1.031	1.146	1.260	1.375	½
¾	.4792	.5990	.7188	.8385	.9583	1.078	1.198	1.318	1.438	¾
6.	.5000	.6250	.7500	.8750	1.000	1.125	1.250	1.375	1.500	6.
¼	.5208	.6510	.7813	.9115	1.042	1.172	1.302	1.432	1.563	¼
½	.5417	.6771	.8125	.9479	1.083	1.219	1.354	1.490	1.625	½
¾	.5625	.7031	.8438	.9844	1.125	1.266	1.406	1.547	1.688	¾
7.	.5833	.7292	.8750	1.021	1.167	1.312	1.458	1.604	1.750	7.
¼	.6042	.7552	.9063	1.057	1.208	1.359	1.510	1.661	1.813	¼
½	.6250	.7813	.9375	1.094	1.250	1.406	1.563	1.719	1.875	½
¾	.6458	.8073	.9688	1.130	1.292	1.453	1.615	1.776	1.938	¾
8.	.6667	.8333	1.000	1.167	1.333	1.500	1.667	1.833	2.000	8.
¼	.6875	.8594	1.031	1.203	1.375	1.547	1.719	1.891	2.063	¼
½	.7083	.8854	1.063	1.240	1.417	1.594	1.771	1.948	2.125	½
¾	.7292	.9114	1.094	1.276	1.458	1.641	1.823	2.005	2.188	¾
9.	.7500	.9375	1.125	1.313	1.500	1.688	1.875	2.062	2.250	9.
¼	.7708	.9635	1.156	1.349	1.542	1.734	1.927	2.120	2.313	¼
½	.7917	.9895	1.188	1.385	1.583	1.781	1.979	2.177	2.375	½
¾	.8125	1.016	1.219	1.422	1.625	1.828	2.031	2.234	2.438	¾
10.	.8333	1.042	1.250	1.458	1.667	1.875	2.083	2.292	2.500	10.
¼	.8542	1.068	1.281	1.495	1.708	1.922	2.135	2.349	2.563	¼
½	.8750	1.094	1.313	1.531	1.750	1.969	2.188	2.406	2.625	½
¾	.8958	1.120	1.344	1.568	1.792	2.016	2.240	2.463	2.688	¾
11.	.9167	1.146	1.375	1.604	1.833	2.063	2.292	2.521	2.750	11.
¼	.9375	1.172	1.406	1.641	1.875	2.109	2.344	2.578	2.813	¼
½	.9583	1.198	1.438	1.677	1.917	2.156	2.396	2.635	2.875	½
¾	.9792	1.224	1.469	1.714	1.958	2.203	2.448	2.693	2.938	¾
12.	1.000	1.250	1.500	1.750	2.000	2.250	2.500	2.750	3.000	12.
½	1.042	1.302	1.563	1.823	2.083	2.344	2.604	2.865	3.125	½
13.	1.083	1.354	1.625	1.896	2.167	2.438	2.708	2.979	3.250	13.
½	1.125	1.406	1.688	1.969	2.250	2.531	2.813	3.094	3.375	½
14.	1.167	1.458	1.750	2.042	2.333	2.625	2.917	3.208	3.500	14.
½	1.208	1.510	1.813	2.115	2.417	2.719	3.021	3.322	3.625	½
15.	1.250	1.563	1.875	2.188	2.500	2.813	3.125	3.438	3.750	15.
½	1.292	1.615	1.938	2.260	2.583	2.906	3.229	3.552	3.875	½
16.	1.333	1.667	2.000	2.333	2.667	3.000	3.333	3.667	4.000	16.
½	1.375	1.719	2.063	2.406	2.750	3.094	3.438	3.781	4.125	½
17.	1.417	1.771	2.125	2.479	2.833	3.188	3.542	3.896	4.250	17.
½	1.458	1.823	2.187	2.552	2.917	3.281	3.646	4.010	4.375	½
18.	1.500	1.875	2.250	2.625	3.000	3.375	3.750	4.125	4.500	18.
19.	1.583	1.979	2.375	2.771	3.167	3.563	3.958	4.354	4.750	19.
20.	1.667	2.083	2.500	2.917	3.333	3.750	4.167	4.583	5.000	20.
21.	1.750	2.188	2.625	3.063	3.500	3.938	4.375	4.812	5.250	21.
22.	1.833	2.292	2.750	3.208	3.667	4.125	4.583	5.042	5.500	22.
23.	1.917	2.396	2.875	3.354	3.833	4.313	4.792	5.270	5.750	23.
24.	2.000	2.500	3.000	3.500	4.000	4.500	5.000	5.500	6.000	24.

[illegible]—Continued.

[illegible]

[illegible]

[illegible]	¼	½	¾	[illegible]	¼	½	¾	5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.042
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.146
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.250
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.354
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.458
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.563
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.667
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.771
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.875
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.979
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.083
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.188
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.292
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.396
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.375	2.500
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.474	2.604
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.573	2.708
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.672	2.813
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.771	2.917
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.870	3.021
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.969	3.125
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.906	3.068	3.229
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.000	3.167	3.333
¼	2.234	2.406	[illegible]	[illegible]	[illegible]	3.094	3.266	3.438
½	2.302	2.479	2.656	[illegible]	[illegible]	3.188	3.365	3.542
¾	2.370	2.552	2.734	[illegible]	[illegible]	3.281	3.464	3.646
9.	2.438	2.625	2.813	3.000	[illegible]	3.375	3.563	3.750
¼	2.505	[illegible]	2.891	3.083	3.276	3.469	3.661	3.854
½	2.573	2.771	2.969	3.167	3.365	3.563	3.760	3.958
¾	2.641	2.844	3.047	3.250	3.453	3.656	3.859	4.063
10.	2.708	2.917	3.125	3.333	3.542	3.750	3.959	4.167
¼	2.776	2.990	3.203	3.416	3.630	3.844	4.057	4.271
½	2.844	3.063	3.281	3.500	3.719	3.938	4.156	4.375
¾	2.911	3.135	3.359	3.583	3.807	4.031	4.255	4.479
11	2.979	3.208	3.438	3.666	3.896	4.125	4.354	4.583
¼	3.047	3.281	3.516	3.750	3.984	4.219	4.453	4.688
½	3.115	3.354	3.594	3.833	4.073	4.313	4.552	4.792
¾	3.182	3.427	3.672	3.916	4.161	4.406	4.651	4.896
12	3.250	3.500	3.750	4.000	4.250	4.500	4.750	5.000
½	3.385	3.646	3.906	4.167	4.427	4.688	4.948	5.208
13	3.521	3.792	4.063	4.333	4.604	4.875	5.146	5.417
½	3.656	3.938	4.219	4.500	4.781	5.063	5.344	5.625
14	3.792	4.083	4.375	4.667	4.958	5.250	5.542	5.833
½	3.927	4.229	4.531	4.833	5.135	5.438	5.740	6.042
15	4.063	4.375	4.688	5.000	5.313	5.625	5.938	6.250
½	4.198	4.521	4.844	5.166	5.490	5.813	6.135	6.458
16	4.333	4.667	5.000	5.333	5.667	6.000	6.333	6.667
½	4.469	4.813	5.156	5.500	5.844	6.188	6.531	6.875
17	4.604	4.958	5.313	5.667	6.021	6.375	6.729	7.083
½	4.740	5.104	5.469	5.833	6.198	6.563	6.927	7.292
18	4.875	5.250	5.625	6.000	6.375	6.750	7.125	7.500
19	5.146	5.542	5.938	6.333	6.729	7.125	7.521	7.917
20	5.417	5.833	6.250	6.667	7.083	7.500	7.917	8.333
21	5.688	6.125	6.563	7.000	7.438	7.875	8.313	8.750
22	5.958	6.417	6.875	7.333	7.792	8.250	8.708	9.167
23	6.229	6.708	7.188	7.667	8.145	8.625	9.104	9.583
24	6.500	7.000	7.500	8.000	8.500	9.000	9.500	10.00

Table of Board Measure — (Continued.)

Width in Inches.	Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.) THICKNESS IN INCHES.									Width in Inches.
	7¾	8	8¼	8½	8¾	9	9¼	9½	9¾	
	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	Ft.Bd.M.	
¼	.1615	.1667	.1719	.1771	.1823	.1875	.1927	.1979	.2031	¼
½	.3229	.3333	.3438	.3542	.3646	.3750	.3854	.3958	.4063	½
¾	.4844	.5000	.5156	.5313	.5467	.5625	.5781	.5938	.6094	¾
1.	.6458	.6667	.6875	.7083	.7292	.7500	.7708	.7917	.8125	1.
¼	.8073	.8333	.8594	.8854	.9115	.9375	.9635	.9896	1.016	¼
½	.9688	1.000	1.031	1.063	1.094	1.125	1.156	1.188	1.219	½
¾	1.130	1.167	1.203	1.240	1.276	1.313	1.349	1.385	1.422	¾
2.	1.292	1.333	1.375	1.417	1.458	1.500	1.542	1.583	1.625	2.
¼	1.453	1.500	1.547	1.594	1.641	1.688	1.734	1.781	1.828	¼
½	1.615	1.667	1.719	1.771	1.822	1.875	1.927	1.979	2.031	½
¾	1.776	1.833	1.891	1.948	2.005	2.063	2.120	2.177	2.234	¾
3.	1.938	2.000	2.063	2.125	2.188	2.250	2.313	2.375	2.438	3.
¼	2.099	2.167	2.234	2.302	2.370	2.438	2.505	2.573	2.641	¼
½	2.260	2.333	2.406	2.479	2.552	2.625	2.698	2.771	2.844	½
¾	2.422	2.500	2.578	2.656	2.734	2.813	2.891	2.969	3.047	¾
4.	2.583	2.667	2.750	2.833	2.917	3.000	3.083	3.167	3.250	4.
¼	2.745	2.833	2.922	3.010	3.099	3.188	3.276	3.365	3.453	¼
½	2.906	3.000	3.094	3.188	3.281	3.375	3.469	3.563	3.656	½
¾	3.068	3.167	3.266	3.365	3.464	3.563	3.661	3.760	3.859	¾
5.	3.229	3.333	3.438	3.542	3.646	3.750	3.854	3.958	4.063	5.
¼	3.391	3.500	3.609	3.719	3.828	3.938	4.047	4.156	4.266	¼
½	3.552	3.667	3.781	3.896	4.010	4.125	4.240	4.354	4.469	½
¾	3.714	3.833	3.953	4.073	4.193	4.313	4.432	4.552	4.672	¾
6.	3.875	4.000	4.125	4.250	4.375	4.500	4.625	4.750	4.875	6.
¼	4.036	4.167	4.297	4.427	4.557	4.688	4.818	4.948	5.078	¼
½	4.198	4.333	4.469	4.604	4.740	4.875	5.010	5.146	5.281	½
¾	4.359	4.500	4.641	4.781	4.922	5.063	5.203	5.344	5.484	¾
7.	4.521	4.667	4.813	4.958	5.104	5.250	5.360	5.542	5.688	7.
¼	4.682	4.833	4.984	5.135	5.286	5.438	5.590	5.740	5.891	¼
½	4.844	5.000	5.156	5.313	5.469	5.625	5.782	5.938	6.094	½
¾	5.005	5.167	5.328	5.490	5.651	5.813	5.975	6.135	6.297	¾
8.	5.167	5.333	5.500	5.667	5.833	6.000	6.167	6.333	6.500	8.
¼	5.328	5.500	5.672	5.844	6.016	6.188	6.359	6.531	6.703	¼
½	5.490	5.667	5.844	6.021	6.198	6.375	6.552	6.729	6.906	½
¾	5.651	5.833	6.016	6.198	6.380	6.563	6.745	6.927	7.109	¾
9.	5.813	6.000	6.188	6.375	6.563	6.750	6.938	7.125	7.313	9.
¼	5.974	6.167	6.359	6.552	6.745	6.938	7.130	7.323	7.516	¼
½	6.135	6.333	6.531	6.729	6.927	7.125	7.323	7.521	7.719	½
¾	6.297	6.500	6.703	6.906	7.109	7.313	7.516	7.719	7.922	¾
10.	6.458	6.667	6.875	7.083	7.292	7.500	7.708	7.917	8.125	10.
¼	6.620	6.833	7.047	7.260	7.474	7.688	7.901	8.115	8.328	¼
½	6.781	7.000	7.219	7.438	7.656	7.875	8.094	8.313	8.531	½
¾	6.943	7.167	7.391	7.615	7.839	8.063	8.286	8.510	8.734	¾
11.	7.104	7.333	7.563	7.792	8.021	8.250	8.479	8.708	8.938	11.
¼	7.266	7.500	7.735	7.969	8.203	8.438	8.672	8.906	9.141	¼
½	7.427	7.667	7.906	8.146	8.386	8.625	8.865	9.104	9.344	½
¾	7.589	7.833	8.078	8.323	8.568	8.813	9.057	9.302	9.547	¾
12.	7.750	8.000	8.250	8.500	8.750	9.000	9.250	9.500	9.750	12.
½	8.073	8.333	8.594	8.854	9.115	9.375	9.635	9.896	10.16	½
13.	8.396	8.666	8.938	9.208	9.479	9.750	10.02	10.29	10.56	13.
½	8.719	9.000	9.281	9.563	9.844	10.13	10.41	10.69	10.97	½
14.	9.042	9.333	9.625	9.917	10.21	10.50	10.79	11.08	11.38	14.
½	9.365	9.666	9.969	10.27	10.57	10.88	11.18	11.48	11.78	½
15.	9.688	10.000	10.31	10.63	10.94	11.25	11.56	11.88	12.19	15.
½	10.01	10.33	10.66	10.98	11.30	11.63	11.95	12.27	12.59	½
16.	10.33	10.67	11.00	11.33	11.67	12.00	12.33	12.67	13.00	16.
½	10.66	11.00	11.34	11.69	12.03	12.38	12.72	13.06	13.41	½
17.	10.98	11.33	11.69	12.04	12.40	12.75	13.10	13.46	13.81	17.
½	11.30	11.66	12.03	12.40	12.76	13.13	13.49	13.85	14.22	½
18.	*11.63*	*12.00*	12.38	12.75	13.13	13.50	13.88	14.25	14.63	18.
19.	*12.27*	*12.67*	13.06	13.46	13.85	14.25	14.65	15.04	15.44	19.
20.	*12.92*	*13.33*	13.75	14.17	14.58	15.00	15.42	15.83	16.25	20.
21.	*13.56*	*14.00*	14.44	14.88	15.31	15.75	16.19	16.63	17.06	21.
22.	*14.21*	*14.66*	15.13	15.58	16.04	16.50	16.96	17.42	17.88	22.
23.	*14.85*	*15.33*	15.81	16.29	16.77	17.25	17.73	18.21	18.69	23.
24.	*15.50*	*16.00*	16.50	17.00	17.50	18.00	18.50	19.00	19.50	24.

Table of Board Measure—(Continued.)

Width in Inches.	Feet of Board Measure contained in one running foot of Scantlings of different dimensions. (Original.) THICKNESS IN INCHES.									Width in Inches.
	10	10¼	10½	10¾	11	11¼	11½	11¾	12	
	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	Ft. Bd.M.	
¼	.2083	.2135	.2188	.2240	.2292	.2344	.2396	.2448	.2500	¼
½	.4167	.4271	.4375	.4479	.4583	.4688	.4792	.4896	.5000	½
¾	.6250	.6406	.6563	.6719	.6875	.7031	.7188	.7344	.7500	¾
1.	.8333	.8542	.8750	.8958	.9167	.9375	.9583	.9792	1.000	1.
¼	1.042	1.068	1.094	1.120	1.146	1.172	1.198	1.224	1.250	¼
½	1.250	1.281	1.313	1.344	1.375	1.406	1.438	1.469	1.500	½
¾	1.458	1.495	1.531	1.568	1.604	1.641	1.677	1.714	1.750	¾
2.	1.667	1.708	1.750	1.792	1.833	1.875	1.917	1.958	2.000	2.
¼	1.875	1.922	1.969	2.016	2.063	2.109	2.156	2.203	2.250	¼
½	2.083	2.135	2.188	2.240	2.292	2.344	2.396	2.448	2.500	½
¾	2.292	2.349	2.406	2.464	2.521	2.578	2.635	2.693	2.750	¾
3.	2.500	2.563	2.625	2.688	2.750	2.813	2.875	2.938	3.000	3.
¼	2.708	2.776	2.844	2.911	2.979	3.047	3.115	3.182	3.250	¼
½	2.917	2.990	3.063	3.135	3.208	3.281	3.354	3.427	3.500	½
¾	3.125	3.203	3.281	3.359	3.438	3.516	3.594	3.672	3.750	¾
4.	3.333	3.417	3.500	3.583	3.667	3.750	3.833	3.917	4.000	4.
¼	3.542	3.630	3.719	3.807	3.896	3.984	4.073	4.161	4.250	¼
½	3.750	3.844	3.938	4.031	4.125	4.219	4.313	4.406	4.500	½
¾	3.958	4.057	4.156	4.255	4.354	4.453	4.552	4.651	4.750	¾
5.	4.167	4.271	4.375	4.479	4.583	4.688	4.791	4.896	5.000	5.
¼	4.375	4.484	4.594	4.703	4.813	4.922	5.031	5.141	5.250	¼
½	4.583	4.698	4.813	4.927	5.042	5.156	5.270	5.385	5.500	½
¾	4.792	4.911	5.031	5.151	5.271	5.391	5.510	5.630	5.750	¾
6.	5.000	5.125	5.250	5.375	5.500	5.625	5.750	5.875	6.000	6.
¼	5.208	5.339	5.469	5.599	5.729	5.859	5.990	6.120	6.250	¼
½	5.417	5.552	5.688	5.823	5.958	6.094	6.229	6.365	6.500	½
¾	5.625	5.766	5.906	6.047	6.188	6.328	6.469	6.609	6.750	¾
7.	5.833	5.979	6.125	6.271	6.417	6.563	6.708	6.854	7.000	7.
¼	6.042	6.193	6.344	6.495	6.646	6.797	6.948	7.099	7.250	¼
½	6.250	6.406	6.563	6.719	6.875	7.031	7.188	7.344	7.500	½
¾	6.458	6.620	6.781	6.943	7.104	7.266	7.427	7.589	7.750	¾
8.	6.667	6.833	7.000	7.167	7.333	7.500	7.667	7.833	8.000	8.
¼	6.875	7.047	7.219	7.391	7.563	7.734	7.906	8.078	8.250	¼
½	7.083	7.260	7.438	7.615	7.792	7.969	8.146	8.323	8.500	½
¾	7.292	7.474	7.656	7.839	8.021	8.203	8.385	8.568	8.750	¾
9.	7.500	7.688	7.875	8.063	8.250	8.438	8.625	8.813	9.000	9.
¼	7.708	7.901	8.094	8.286	8.479	8.672	8.865	9.057	9.250	¼
½	7.917	8.115	8.313	8.510	8.709	8.906	9.104	9.302	9.500	½
¾	8.125	8.328	8.531	8.734	8.939	9.141	9.344	9.547	9.750	¾
10.	8.333	8.542	8.750	8.958	9.167	9.375	9.583	9.792	10.00	10.
¼	8.542	8.755	8.969	9.182	9.396	9.609	9.823	10.04	10.25	¼
½	8.750	8.969	9.188	9.406	9.625	9.844	10.06	10.28	10.50	½
¾	8.958	9.182	9.406	9.630	9.854	10.08	10.30	10.53	10.75	¾
11.	9.167	9.396	9.625	9.854	10.08	10.31	10.54	10.77	11.00	11.
¼	9.375	9.609	9.844	10.08	10.31	10.55	10.78	11.02	11.25	¼
½	9.583	9.823	10.06	10.30	10.54	10.78	11.02	11.26	11.50	½
¾	9.792	10.04	10.28	10.53	10.77	11.02	11.26	11.51	11.75	¾
12.	10.00	10.25	10.50	10.75	11.00	11.25	11.50	11.75	12.00	12.
½	10.42	10.68	10.94	11.20	11.46	11.72	11.98	12.24	12.50	½
13.	10.83	11.10	11.38	11.65	11.92	12.19	12.46	12.73	13.00	13.
½	11.25	11.53	11.81	12.09	12.38	12.66	12.94	13.22	13.50	½
14.	11.67	11.96	12.25	12.54	12.83	13.13	13.42	13.71	14.00	14.
½	12.08	12.39	12.69	12.99	13.29	13.59	13.90	14.20	14.50	½
15.	12.50	12.81	13.13	13.44	13.75	14.06	14.38	14.69	15.00	15.
½	12.92	13.24	13.56	13.89	14.21	14.53	14.85	15.18	15.50	½
16.	13.33	13.67	14.00	14.33	14.67	15.00	15.33	15.67	16.00	16.
½	13.75	14.09	14.44	14.78	15.13	15.47	15.81	16.16	16.50	½
17.	14.17	14.52	14.88	15.23	15.58	15.94	16.29	16.65	17.00	17.
½	14.58	14.95	15.31	15.77	16.04	16.41	16.77	17.14	17.50	½
18.	15.00	15.38	15.75	16.13	16.50	16.88	17.25	17.63	18.00	18.
19.	15.83	16.23	16.63	17.02	17.42	17.81	18.21	18.60	19.00	19.
20.	16.67	17.08	17.50	17.92	18.33	18.75	19.17	19.58	20.00	20.
21.	17.50	17.94	18.38	18.81	19.25	19.69	20.13	20.56	21.00	21.
22.	18.33	18.79	19.25	19.71	20.17	20.63	21.08	21.54	22.00	22.
23.	19.17	19.65	20.13	20.60	21.08	21.56	22.04	22.51	23.00	23.
24.	20.00	20.50	21.00	21.50	22.00	22.50	23.00	23.50	24.00	24.

Average price of lumber in Phila in 1873: Spruce joists, $28 to $30 per 1000 ft board meas. k joists, $20 to $23. Yellow pine floor boards, $40 to $60. White pine boards, $30 to $70. t to quality, degree of seasoning, &c. Sawed W pine timbers $40 to $50. Heart Y pine $5 ulock $20 to $25.

TABLE OF WEIGHT OF CAST IRON.*

The weight of a pattern of perfectly dry white pine, by 20, will give approximately the wt of the casting. If well seasoned, but perfectly dry, mult by 19, or by 18.

Assuming 450 lbs to a cub ft, a pound contains 3.8400 cubic inches; a ton 5 and a cubic inch weighs .2604 lbs.

Thickness or Diameter in Inches.	Thickness or Diam. in decimals of a foot.	Wt. of a *Square Foot.* Lbs.	Wt. of a *Square* bar, 1 ft. long. Lbs.	Wt. of a *Round* bar, 1 ft. long. Lbs.	Wt. of *Balls.* Lbs.	Thickness or Diameter in Inches.	Thickness or Diam. in decimals of a foot.	Wt. of a *Square Foot.* Lbs.	Wt. of a *Square* bar, 1 ft. long. Lbs.	Wt. of a *Round* bar, 1 ft. long. Lbs.
1-32	.0026	1.175	.003	.002		3⅛	.2604	117.3	30.52	23.97
1-16	.0052	2.344	.012	.010		¼	.2708	121.8	33.01	25.93
3-32	.0078	3.516	.027	.021	.0001	⅜	.2813	126.5	35.60	27.95
⅛	.0104	4.687	.048	.038	.0003	½	.2917	131.2	38.28	30.07
5-32	.0130	5.861	.076	.060	.0005	⅝	.3021	135.9	41.07	32.25
3-16	.0156	7.032	.110	.086	.0009	¾	.3125	140.6	43.95	34.51
7-32	.0182	8.203	.150	.118	.0014	⅞	.3229	145.3	46.93	36.85
¼	.0208	9.375	.195	.154	.0021	4.	.3333	150.0	50.01	39.27
9-32	.0234	10.54	.247	.194	.0030	⅛	.3438	154.7	53.18	41.77
5-16	.0260	11.73	.305	.240	.0042	¼	.3542	159.3	56.46	44.33
11-32	.0287	12.89	.370	.290	.0056	⅜	.3646	164.0	59.82	46.99
⅜	.0313	14.06	.440	.346	.0072	½	.3750	168.7	63.33	49.71
13-32	.0339	15.24	.516	.406	.0092	⅝	.3854	173.4	66.86	52.52
7-16	.0365	16.41	.598	.470	.0114	¾	.3958	178.1	70.52	55.39
15-32	.0391	17.56	.687	.540	.0140	⅞	.4063	182.8	74.28	58.34
½	.0417	18.75	.781	.616	.0170	5.	.4167	187.5	78.12	61.37
9-16	.0469	21.10	.989	.777	.0243	⅛	.4271	192.2	82.10	64.47
⅝	.0521	23.44	1.221	.959	.0334	¼	.4375	196.9	86.14	67.65
11-16	.0573	25.79	1.478	1.161	.0444	⅜	.4479	201.6	90.29	70.52
¾	.0625	28.12	1.758	1.381	.0575	½	.4583	206.2	94.54	74.26
13-16	.0677	30.47	2.064	1.621	.0732	⅝	.4688	210.9	98.89	77.66
⅞	.0729	32.81	2.393	1.880	.0913	¾	.4792	215.6	103.3	81.16
15-16	.0781	35.16	2.747	2.158	.1124	⅞	.4896	220.3	107.9	84.72
1.	.0833	37.50	3.125	2.455	.1363	6.	.5000	225.0	112.5	88.36
1-16	.0885	39.84	3.528	2.771	.1636	¼	.5208	234.4	122.1	95.89
⅛	.0938	42.19	3.955	3.107	.1942	½	.5417	243.8	132.0	103.7
3-16	.0990	44.53	4.407	3.461	.2284	¾	.5625	253.1	142.4	111.9
¼	.1042	46.87	4.883	3.835	.2664	7.	.5833	262.5	153.2	120.2
5-16	.1094	49.22	5.384	4.229	.3084	¼	.6042	271.9	164.2	129.0
⅜	.1146	51.57	5.909	4.640	.3546	½	.6250	281.3	175.8	138.1
7-16	.1198	53.91	6.461	5.073	.4058	¾	.6458	290.7	187.7	147.4
½	.1250	56.26	7.033	5.523	.4603	8.	.6667	300.0	200.1	157.0
9-16	.1302	58.60	7.632	5.993	.5204	¼	.6875	309.4	212.7	167.0
⅝	.1354	60.94	8.253	6.484	.5852	½	.7083	318.8	225.8	177.3
11-16	.1406	63.28	8.900	6.991	.6555	¾	.7292	328.2	239.3	187.9
¾	.1458	65.63	9.572	7.518	.7310	9.	.7500	337.4	253.1	198.8
13-16	.1510	67.97	10.27	8.064	.8122	¼	.7708	346.8	267.4	210.0
⅞	.1563	70.32	10.99	8.630	.8991	½	.7917	356.2	282.1	221.5
15-16	.1615	72.66	11.73	9.215	.9920	¾	.8125	365.6	297.0	233.3
2.	.1667	75.01	12.50	9.821	1.073	10.	.8333	375.0	312.5	245.5
⅛	.1771	79.70	14.11	11.09	1.308	¼	.8542	384.4	328.4	257.8
¼	.1875	84.40	15.83	12.43	1.554	½	.8750	393.7	344.5	270.6
⅜	.1979	89.07	17.63	13.85	1.827	¾	.8958	403.1	361.2	283.7
½	.2083	93.75	19.54	15.34	2.131	11.	.9167	412.5	378.2	297.0
⅝	.2188	98.44	21.54	16.56	2.467	¼	.9375	421.9	395.5	310.6
¾	.2292	103.2	23.64	18.56	2.835	½	.9583	431.2	413.3	324.6
⅞	.2396	107.8	25.84	20.29	3.241	¾	.9792	440.6	431.4	338.8
3.	.2500	112.6	28.13	22.10	3.682	12.	1 Foot.	450.	450.	353.4

The weights of balls are as the cubes of their diams.

To find the weight of a spherical shell. From the weight *which has the outer diam of the shell,* take the wt of one which has its inn

* ***For Copper,*** mult by 1.2; Lead, mult by 1.6; Brass, add 1-7th; T by .97. *All approximate.*

WEIGHT OF CAST-IRON PIPES per running foot,

Assuming the weight of cast-iron at 450 lbs per cub ft, or .2604 lb per cub inch. No allowance is here made for the spigot and faucet-joints used in water-pipes. As these are now commonly made, (see Hydraulics, Art 39,) they add to the weight of each length or section of pipe of any size, about as much as that of 8 inches in length of the plain pipe as given in the table. See Hydraulics, Art 16; also next table. For lead-pipe mult by 1.6; copper, mult by 1.2; brass, add 1-7th.

Inner diam. or bore, in inches.	THICKNESS OF PIPE IN INCHES.												
	¼	⅜	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½	1¾	2
	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.	Wt in Lbs.
1.	3.07	5.07	7.38	9.99	12.9	16.2	19.7	23.5	27.7	32.1	36.9	47.4	59.1
¼	3.69	6.09	8.61	11.5	14.8	18.3	22.2	26.3	30.8	35.5	40.6	51.7	64.0
½	4.30	6.92	9.84	13.1	16.6	20.5	24.6	29.1	33.8	38.9	44.3	56.0	68.9
¾	4.92	7.84	11.1	14.6	18.5	22.6	27.1	31.8	36.9	42.3	48.0	60.3	73.8
2.	5.53	8.76	12.3	16.2	20.3	24.8	29.5	34.6	40.0	45.7	51.7	64.6	78.7
¼	6.15	9.69	13.5	17.7	22.2	26.9	32.0	37.4	43.1	49.0	55.4	68.9	83.7
½	6.76	10.6	14.8	19.2	24.0	29.1	34.5	40.1	46.1	52.4	59.1	73.2	88.6
¾	7.37	11.5	16.0	20.8	25.9	31.2	36.9	42.9	49.2	55.8	62.7	77.5	93.5
3.	7.98	12.5	17.2	22.3	27.7	33.4	39.4	45.7	52.3	59.2	66.4	81.8	98.4
¼	8.60	13.4	18.5	23.8	29.5	35.5	41.8	48.4	55.4	62.3	70.1	86.1	103.
½	9.21	14.3	19.7	25.4	31.4	37.7	44.3	51.2	58.4	65.9	73.8	90.4	108.
¾	9.83	15.2	20.9	26.9	33.2	39.8	46.8	54.0	61.5	69.3	77.5	94.7	113.
4.	10.3	16.1	22.2	28.5	35.1	42.0	49.2	56.7	64.6	72.7	81.2	99.0	118.
¼	11.1	17.1	23.4	30.0	36.9	44.1	51.7	59.5	67.7	76.1	84.9	103.	123.
½	11.7	18.0	24.6	31.5	38.8	46.3	54.1	62.3	70.7	79.5	88.6	108.	128.
¾	12.3	18.9	25.8	33.1	40.6	48.5	56.6	65.0	73.8	83.9	92.3	112.	133.
5.	12.9	19.8	27.1	34.6	42.5	50.6	59.1	67.8	76.9	87.2	96.0	116.	138.
¼	13.5	20.8	28.3	36.1	44.3	52.8	61.5	70.6	80.0	90.6	99.6	121.	143.
½	14.2	21.7	29.5	37.7	46.1	54.9	64.0	73.3	83.0	94.0	103.	125.	148.
¾	14.8	22.6	30.8	39.2	48.0	57.1	66.4	76.1	86.1	97.4	107.	129.	153.
6.	15.4	23.5	32.0	40.8	49.8	59.2	68.9	78.9	89.2	99.8	111.	134.	158.
¼	16.6	25.4	34.5	43.8	53.5	63.5	73.8	84.4	95.3	107.	118.	142.	167.
7.	17.8	27.2	36.9	46.9	57.2	67.8	78.7	89.4	102.	113.	126.	151.	177.
¼	19.1	29.1	39.4	50.0	60.9	72.1	83.7	95.5	108.	120.	133.	159.	187.
8.	20.3	30.9	41.8	53.1	64.6	76.4	88.6	101.	114.	127.	140.	168.	197.
¼	21.5	32.8	44.3	56.1	68.3	80.7	93.5	107.	120.	134.	148.	177.	207.
9.	22.8	34.6	46.8	59.2	72.0	85.1	98.4	112.	126.	140.	155.	185.	217.
¼	24.0	36.4	49.2	62.3	75.7	89.3	103.	118.	132.	147.	163.	194.	226.
10.	25.1	38.3	51.7	65.3	79.4	93.6	108.	123.	138.	154.	170.	202.	235.
¼	26.4	40.1	54.1	68.4	83.0	97.9	113.2	129.	145.	161.	177.	211.	245.
11.	27.6	42.0	56.6	71.5	86.7	102.	118.	134.	151.	168.	185.	220.	255.
¼	28.8	43.8	59.1	74.6	90.4	107.	123.	140.	157.	174.	192.	228.	265.
12.	30.0	45.7	61.5	77.7	94.1	111.	128.	145.	163.	181.	199.	237.	275.
13.	32.5	49.4	66.4	83.8	102.	120.	138.	156.	175.	195.	214.	254.	294.
14.	35.0	53.1	71.4	89.4	109.	128.	148.	168.	188.	208.	229.	271.	314.
15.	37.4	56.7	76.3	96.1	116.	137.	158.	179.	200.	222.	244.	289.	334.
16.	39.1	60.4	81.2	102.	124.	145.	167.	190.	212.	235.	258.	306.	354.
17.	42.3	64.1	86.1	108.	131.	154.	177.	201.	225.	249.	273.	323.	373.
18.	44.8	67.8	91.0	115.	139.	163.	187.	212.	237.	262.	288.	340.	393.
19.	47.3	71.5	96.0	121.	146.	171.	197.	223.	249.	276.	303.	357.	412.
20.	49.7	75.2	101.	127.	153.	180.	207.	234.	261.	289.	317.	375.	432.
21.	52.2	78.9	106.	133.	161.	188.	217.	245.	274.	303.	332.	392.	452.
22.	54.6	82.6	111.	139.	168.	196.	227.	256.	286.	316.	347.	409.	471.
23.	57.1	86.3	116.	145.	175.	206.	236.	267.	298.	330.	362.	426.	491.
24.	59.6	89.9	121.	152.	183.	214.	246.	278.	311.	343.	375.	444.	511.
25.	62.0	93.6	126.	158.	190.	223.	256.	289.	323.	357.	391.	461.	531.
26.	64.5	97.3	131.	164.	198.	231.	266.	300.	335.	370.	406.	478.	550.
27.	66.9	101.	135.	170.	205.	240.	276.	311.	348.	384.	421.	495.	570.
28.	69.4	105.	140.	176.	212.	249.	286.	323.	360.	397.	436.	512.	590.
29.	71.8	109.	145.	182.	220.	257.	295.	334.	372.	411.	450.	530.	609.
30.	74.2	112.	150.	188.	227.	266.	305.	345.	384.	424.	465.	547.	629.
31.	76.7	116.	155.	195.	234.	275.	315.	356.	397.	438.	480.	564.	649.
32.	79.1	120.	160.	201.	242.	283.	325.	367.	409.	451.	495.	581.	668.
33.	81.6	123.	165.	207.	249.	292.	335.	378.	421.	465.	509.	598.	688.
34.	84.1	127.	170.	213.	257.	300.	345.	389.	434.	479.	524.	616.	708.
35.	86.5	131.	175.	219.	264.	309.	354.	400.	446.	492.	539.	633.	726.
36.	89.0	134.	180.	225.	271.	318.	364.	411.	458.	506.	554.	650.	746.
42.	104.	156.	210.	262.	315.	370.	423.	478.	532.	588.	644.	753.	864.
48.	119.	178.	239.	298.	359.	422.	482.	544.	605.	669.	733.	858.	982.

The weight in lbs per foot run, of cast-iron pipes, may be found thus: to the inn lam or bore of the pipe, add its single thickness; and multiply the sum by 10 th id thickness.

WEIGHT OF CAST-IRON WATER-PIPES,

As used in Phila, and tested by hydraulic press before delivery, to an internal pres of 300 lbs per sq inch. This table includes spigots and faucets, or bells. The pipes are required to be made of remelted strong tough gray pig iron, such as may be readily drilled and chipped; and all of more than 4 ins diam to be cast vertically, with the bell end down. Deviations of 5 per cent above or below the theoretical weights, are allowed for irregularities in casting, which it seems impossible to avoid.

Diam.	Length.		Weight.	Thickness of body.	Thickness of bell.	Thickness of lead joint.
Ins.	Ft.	Ins.	Lbs.	Ins.	Ins.	Ins.
3	9		135	$\frac{5}{16}$	⅜	¼
4	9		190	⅜	$\frac{11}{16}$	$\frac{5}{16}$
4	12	3	220	⅜	$\frac{11}{16}$	$\frac{5}{16}$
6	12	3½	370	$\frac{7}{16}$	$\frac{11}{16}$	⅜
8	12	3½	500	$\frac{7}{16}$	$\frac{13}{16}$	⅜
10	12	3½	640	½	$\frac{13}{16}$	⅜
12	12	3½	850	$\frac{9}{16}$	⅝	$\frac{7}{16}$
16	12	3½	1260	⅝	1	$\frac{7}{16}$
20	12	3½	1815	$\frac{11}{16}$	$1\frac{5}{16}$	$\frac{7}{16}$

Price in Phila, 1873, about $56 to $67 per ton; or say 2½ to 3 cents per lb; depending on size, and quantity ordered. Elbows, connections, &c, about 6 to 15 per ct more. In ordering anything by the ton, be careful to specify the number of lbs (2240). This prevents *misunderstandings*.

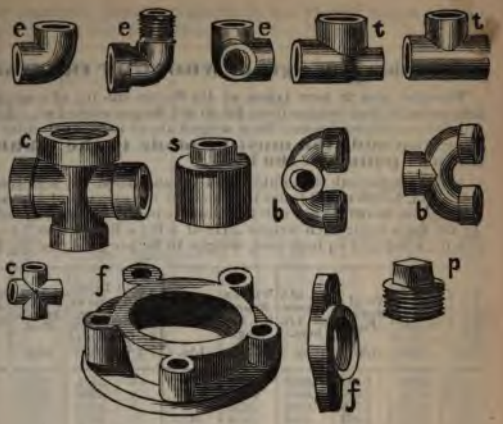
Approximate average prices in cents per lb, currency, of iron and steel in Philada, (1873, when prices ranged high). **Iron bars** of ordinary sizes (¾ inch to 2 ins diam or square,) best Norway 8; Swedish 7½; American common, round or square, 3½ to 3¾; refined, 3¾ to 4; increasing by degrees, for both larger and smaller sizes, to about 40 or 50 per ct more for bars of either ¼ inch or of 4 inches diam or square. **Extra** refined, 1 ct per lb more. **Hoop iron** 5½ cts for 1 inch or more wide; to 8 or 9 for very thin ½ inch wide. **Sheet iron,** common, Nos 10 to 16, 5¼; Nos 16 to 25, 8; best, about 50 per cent more. Best Russian, 24 cts. **Plates,** common, 5; best, 7¼. **Angle,** 5¼. **T,** 6¼. **Phœnix beams,** 5¾ to 6½. **Phœnix columns,** 7¾ to 10. **Rails,** 3¾ to 4. **Pig iron,** common, 2 to 2¼; best, 2½. **Cast iron house fronts,** fitted and put up, 6½ to 7. **Ordinary castings,** 3 to 4 cts.

Steel bars, ordinary sizes, common, 12 to 13; best, 15 to 16; machinery, 11. Very small, or large sizes, up to 40 or 50 per ct more. **Sheet steel,** common, 8 to 10; best, 15 to 17. **Plates,** homogeneous, 11½. **Steel rails,** 5¼ to 6¾. **Ingot crucible steel,** 7 to 9. Bessemer, 4 to 5. **Crucible steel castings,** 12 to 14. Other prices will be found under the proper heads.

Weight per foot run of welded wrought-iron tubes, as made at the Pascal Iron Works of Morris, Tasker & Co, Phila; usually in lengths of 20 ft. Other sizes and lengths made to order.

Inner Diam.	Outer Diam.	Ths.	Wt. per Foot.	Inner Diam.	Outer Diam.	Ths.	Wt. per Foot.	Inner Diam.	Outer Diam.	Ths.	Wt. per Foot.
Ins.	Ins.	Ins.	Lbs.	Ins.	Ins.	Ins.	Lbs.	Ins.	Ins.	Ins.	Lbs.
.270	.405	.068	.243	.928	1.660	.366	5.065	2.331	2.855	.262	7.126
.320	.540	.110	.505	.995	1.315	.160	1.976	2.468	2.876	.204	5.773
.320	.840	.260	1.613	1.048	1.315	.134	1.670	3.067	3.500	.217	7.547
.364	.540	.088	.422	1.108	1.900	.396	6.370	3.548	4.000	.226	9.055
.437	.675	.119	.708	1.294	1.660	.183	2.891	4.026	4.500	.237	10.728
.478	1.050	.286	2.338	1.380	1.660	.140	2.258	4.508	5.000	.247	12.492
.494	.675	.091	.561	1.459	2.375	.458	9.390	5.045	5.563	.259	14.564
.580	.840	.130	.987	1.504	1.900	.198	3.604	6.065	6.625	.280	18.767
.622	.840	.109	.845	1.611	1.900	.145	2.694	7.023	7.625	.301	23.410
.675	1.315	.320	3.405	1.807	2.855	.524	13.656	7.982	8.875	.322	28.348
.764	1.050	.143	1.387	1.917	2.375	.229	5.400	9.003	9.688	.344	34.371
.824	1.050	.113	1.126	2.067	2.375	.154	3.967	10.019	10.750	.366	40.583

Inner and outer, or male and female screws are cut at the ends to order; either to show a flush joint, or, by means of enlarged ends, so as not to diminish either the thickness or the bore. This firm, and others which make iron roofs, &c, furnish also the necessary detailed fixtures; some of which, of either wrought or cast iron, resemble much the following gas-fittings: Elbows, *e e e*; *Ts*, *t t*; crosses, *c c*; sockets, *s*; bends, *b b*; flanges, *f f*; plugs, *p*, &c, much used with wrought-iron gas-pipe.



Seamless Drawn Brass and Copper Tubes of the American Tube Works, Boston, Mass. Approximate sizes and weights.*

Although this is the Co's own table, it is not consistent in itself; for where two thicknesses are given, the wts in some cases correspond with the least, and in others with the greatest; and there is no way to discriminate between them.

Outer Diam. Ins.	Lgth. Ft.	Ths. Inch.	Wt. in lbs. per ft. Brs.	Wt. in lbs. per ft. Cop.	Outer Diam. Ins.	Lgth. Ft.	Ths. In.	Wt. in lbs. per ft. Brs.	Wt. in lbs. per ft. Cop.	Outer Diam. Ins.	Lgth. Ft.	Ths. In.	Wt. in lbs. per ft. Brs.	Wt. in lbs. per ft. Cop.
⅝	10	.049	.375	.375			.083					.095		
¾	12	.058	.500	.500	1¾	13	to	1.75	1.80	2⅝	12	to	3.	3.13
⅞	10	.058	.625	.625			.109					.120		
1	10	.065	.750	.750	1⅞	12	"	1.88	1.94	2¾	12	"	3.13	3.25
1⅛	10	.065	.875	.875	2	15	"	2.2	2.25	3	12	"	3.33	3.5
		.083			2⅛	13	"	2.25	2.38	3⅛	10	"	3.5	3.63
1¼	15	to	1.25	1.25	2¼	14	"	2.38	2.50	3¼	10	"	3.88	4.13
		.109			2⅜	13	"	2.50	2.67	3½	10	"	4.25	4.38
1⅜	10	"	1.375	1.375			.095			4	10	"	5.	5.25
1½	14	"	1.50	1.60	2½	13	to	2.75	3.	5	10	"	7.	8.
1¾	12	"	1.631	1.70			.120							

These tubes are furnished either plain, or tinned with pure tin. The Co also make the connections, elbows, bends, &c. of the same metals, usually required in plumbing. The brass tubes have been used in Boston, &c, as a substitute for lead service-pipes in dwellings for several years; and are much approved of. They have also been introduced in the City Hospital, and in some of the hotels.

Approx prices of plain tubes, either brass or copper, per foot: Diam ⅝, 25 cts; ¾, 30 cts; 1 inch, 60 cts; 1¼, 70 cts; 1½, 80 cts. If tinned, add about one-seventh part to these p ral discounts on large sales. 1873.

Table of Weight of WROUGHT IRON* and STEEL.

Wrought iron is here taken at 485 lbs per cub ft; or a sp gr of 7.77. Very pure soft wrought iron weighs from 488 to 492 lbs per cubic foot. Light weight indicates impurities, and weakness. Steel weighs about 490 lbs per cubic foot; therefore, **for steel, an addition must be made to the tabular amounts, of about 1 pound in 100 lbs.**

At 485 lbs per cub ft, a cubic inch weighs .28067 lb; a lb contains 3.5629 cub ins; and a ton, 4.6186 cub ft; and this is about the average of *hammered* iron. The usual assumption is 480 lbs per cub ft; which is nearer the average of ordinary *rolled* iron. At 480 lbs, a cubic inch weighs .2778 of a lb; a lb contains 3.6 cub ins; a ton, 4.667 cub ft; a rod of 1 sq inch area, weighs 10 lbs per yard; or 3½ lbs per foot, exactly.

Thickness or Diameter in Inches.	Thickness or Diam. in decimals of a foot.	Wt. of a *Square Foot.* Lbs.	Wt. of a *Square* bar, 1 ft. long. Lbs.	Wt. of a *Round* bar, 1 ft. long. Lbs.	Wt. of *Balls.* Lbs.	Thickness or Diameter in Inches.	Thickness or Diam. in decimals of a foot.	Wt. of a *Square Foot.* Lbs.	Wt. of a *Square* bar, 1 ft. long. Lbs.	Wt. of a *Round* bar, 1 ft. long. Lbs.	Wt. of *Balls.* Lbs.
1-32	.0026	1.263	.0033	.0026		3⅛	.2604	126.3	32.89	25.83	4.484
1-16	.0052	2.526	.0132	.0104		¼	.2708	131.4	35.57	27.94	5.046
3-32	.0078	3.789	.0296	.0233	.0001	⅜	.2813	136.4	38.37	30.13	5.649
⅛	.0104	5.052	.0526	.0414	.0003	½	.2917	141.5	41.26	32.41	6.291
5-32	.0130	6.315	.0823	.0646	.0005	⅝	.3021	146.5	44.26	34.76	7.000
3-16	.0156	7.578	.1184	.0930	.0009	¾	.3125	151.6	47.37	37.20	7.750
7-32	.0182	8.841	.1612	.1266	.0015	⅞	.3229	156.6	50.57	39.72	8.550
¼	.0208	10.10	.2105	.1653	.0023	4.	.3333	161.7	53.89	42.33	9.405
9-32	.0234	11.37	.2665	.2093	.0033	⅛	.3438	166.7	57.31	45.01	10.32
5-16	.0260	12.63	.3290	.2583	.0045	¼	.3542	171.8	60.84	47.78	11.28
11-32	.0287	13.89	.3980	.3126	.0060	⅜	.3646	176.8	64.47	50.63	12.31
⅜	.0313	15.16	.4736	.3720	.0078	½	.3750	181.9	68.20	53.57	13.39
13-32	.0339	16.42	.5558	.4365	.0098	⅝	.3854	186.9	72.05	56.59	14.54
7-16	.0365	17.68	.6446	.5063	.0123	¾	.3958	192.0	75.99	59.69	15.75
15-32	.0391	18.95	.7400	.5813	.0151	⅞	.4063	197.0	80.05	62.87	17.03
½	.0417	20.21	.8420	.6613	.0184	5.	.4167	202.1	84.20	66.13	18.37
9-16	.0469	22.73	1.066	.8370	.0262	⅛	.4271	207.1	88.47	69.48	19.79
⅝	.0521	25.26	1.316	1.033	.0359	¼	.4375	212.2	92.83	72.91	21.28
11-16	.0573	27.79	1.592	1.250	.0478	⅜	.4479	217.2	97.31	76.43	22.82
¾	.0625	30.31	1.895	1.488	.0620	½	.4583	222.3	101.9	80.02	24.45
13-16	.0677	32.84	2.223	1.746	.0788	⅝	.4688	227.3	106.6	83.70	26.16
⅞	.0729	35.37	2.579	2.025	.0985	¾	.4792	232.4	111.4	87.46	27.94
15-16	.0781	37.89	2.960	2.325	.1211	⅞	.4896	237.5	116.3	91.31	29.80
1.	.0833	40.42	3.368	2.645	.1470	6.	.5000	242.5	121.3	95.23	31.74
1-16	.0885	42.94	3.803	2.986	.1763	¼	.5208	252.6	131.6	103.3	35.88
⅛	.0938	45.47	4.263	3.348	.2093	½	.5417	262.7	142.3	111.8	40.36
3-16	.0990	48.00	4.750	3.730	.2461	¾	.5625	272.8	153.5	120.5	45.19
¼	.1042	50.52	5.263	4.133	.2870	7.	.5833	282.9	165.0	129.6	50.40
5-16	.1094	53.05	5.802	4.557	.3323	¼	.6042	293.0	177.0	139.0	56.00
⅜	.1146	55.57	6.368	5.001	.3820	½	.6250	303.1	189.5	148.8	62.09
7-16	.1198	58.10	6.960	5.466	.4365	¾	.6458	313.2	202.3	158.9	68.40
½	.1250	60.63	7.578	5.952	.4960	8.	.6667	323.3	215.6	169.3	75.24
9-16	.1302	63.15	8.223	6.458	.5606	¼	.6875	333.4	229.3	180.1	82.52
⅝	.1354	65.68	8.893	6.985	.6306	½	.7083	343.5	243.4	191.1	90.25
11-16	.1406	68.20	9.591	7.533	.7062	¾	.7292	353.6	247.9	202.5	98.43
¾	.1458	70.73	10.31	8.101	.7876	9.	.7500	363.8	272.8	214.3	107.1
13-16	.1510	73.26	11.07	8.690	.8750	¼	.7708	373.9	288.2	226.3	116.3
⅞	.1563	75.78	11.84	9.300	.9688	½	.7917	384.0	304.0	238.7	126.0
15-16	.1615	78.31	12.64	9.930	1.069	¾	.8125	394.1	320.2	251.5	136.2
2.	.1667	80.83	13.47	10.58	1.176	10.	.8333	404.2	336.8	264.5	146.9
⅛	.1771	85.89	15.21	11.95	1.410	¼	.8542	414.3	353.9	277.9	158.2
¼	.1875	90.94	17.05	13.39	1.674	½	.8750	424.4	371.3	291.6	170.1
⅜	.1979	95.99	19.00	14.92	1.969	¾	.8958	434.5	389.2	305.7	182.6
½	.2083	101.0	21.05	16.53	2.296	11.	.9167	444.6	407.5	320.1	195.6
⅝	.2188	106.1	23.21	18.23	2.658	¼	.9375	454.7	426.3	334.8	209.2
¾	.2292	111.2	25.47	20.01	3.056	½	.9583	464.8	445.4	349.8	223.5
⅞	.2396	116.2	27.84	21.87	3.492	¾	.9792	474.9	465.0	365.2	238.4
3.	.2500	121.3	30.31	23.81	3.968	12.	1 Foot.	485.	485.	380.9	253.9

To find the weight of a spherical shell. From the weight of a ball which has the outer diam of the shell, take the wt of one which has its inner diam.

* ***For Copper,*** add ⅛th part. Lead, mult by 1.47. Brass, mult by 1.06. Zinc, *mult by .9. Tin, mult by .95.* All approximate.

ht of one square foot of Rolled, or Sheet Iron, Steel, Copper, or Brass Plates. (From Haswell.)

ss by the Birmingham gauge for n wire, sheet iron, and steel.

lled lead, multiply copper by 1.3; and multiply wrought iron by the deci

ck-s.	Iron.	Steel.	Copper.	Brass.
.	Lbs.	Lbs.	Lbs.	Lbs.
4	18.35	18.54	20.5662	19.4372
5	17.18	17.35	19.2525	18.19
	15.36	15.51	17.214	16.264
	13.74	13.87	15.402	14.552
	12.13	12.25	13.59	12.84
4	11.48	11.59	12.8652	12.1552
9	10.47	10.57	11.7327	11.0852
8	9.619	9.715	10.7814	10.1864
	8.892	8.981	9.966	9.416
3	8.205	8.287	9.1959	8.6884
	7.275	7.348	8.154	7.704
5	6.669	6.736	7.4745	7.062
8	5.981	6 041	6.7044	6.3344
4	5.416	5.470	6.0702	5.7352
	4.850	4.899	5.436	5.136
9	4.405	4.449	4.9377	4.6652
5	3.840	3.878	4.3035	4.066
3	3.555	3.588	3.7599	3.5524
2	2.910	2.939	3.2616	3.0816
5	2.627	2.653	2.9445	2.782
8	2.344	2.367	2.6274	2.4824
9	1.980	1.999	2.2197	2.0972
2	1.697	1.714	1.9026	1.7976
5	1.415	1.429	1.5855	1.498
2	1.293	1.305	1.4496	1.3696
8	1.132	1.143	1.2684	1.1984
5	1.010	1.020	1.1325	1.07
2	.8892	.8981	.9966	.9413
	.8083	.8164	.906	.856
8	.7225	.7348	.8154	.7704
6	.6467	.6532	.7428	.6848
4	.5658	.5715	.6342	.5992
3	.5254	.5307	.5889	.5564
2	.4850	.4899	.5436	.5136
0	.4042	.4082	.453	.428
0	.3638	.3674	.4077	.3852
8	.3233	.3265	.3624	.3424
7	.2829	.2857	.3171	.2996
5	.2021	.2041	.2265	.214
4	.1617	.1633	.1812	.1712

Thickness by the American gauge.

For rolled lead, multiply copper by 1.3; and for zinc, multiply wrought iron by the decimal .9. Silver is ¼ heavier than steel.

No. of Gauge.	Thickness.	Iron.	Steel.	Copper.	Brass.
	Ins.	Lbs.	Lbs.	Lbs.	Lbs.
0000	.46	18.63	18.87	20.838	19.688
000	.40964	16.58	16.80	18.5567	17.5326
00	.3648	14.77	14.96	16.5254	15.6134
0	.32486	13.15	13.32	14.7162	13.904
1	.2893	11.70	11.86	13.1053	12 382
2	.25763	10.43	10.57	11.6706	11.0266
3	.22942	9.291	9.415	10.3927	9.8192
4	.20431	8.273	8.384	9.2552	8.7445
5	.18194	7.366	7.462	8.2419	7.787
6	.16202	6.561	6.648	7.3395	6.9345
7	.14428	5.842	5.920	6.5359	6.1752
8	.12849	5.203	5.272	5.8206	5.4994
9	.11443	4.633	4.695	5.1837	4.8976
10	.10189	4.125	4.180	4.6156	4.3609
11	.090742	3.672	3.723	4.1106	3.8838
12	.080808	3.272	3.315	3.6606	3.4586
13	.071961	2.916	2.952	3.2598	3 0799
14	.064084	2.592	2.629	2.903	2 7428
15	.057068	2.311	2.341	2.5852	2.4425
16	.05082	2.052	2.085	2.3021	2.1751
17	.045257	1.825	1 857	2.0501	1.937
18	.040303	1.631	1.653	1.8257	1.725
19	.03589	1.452	1.468	1.6258	1.5361
20	.031961	1.293	1.311	1.4478	1.3679
21	.028462	1.152	1.166	1.2893	1.2182
22	.025347	1.026	1.040	1.1482	1.0849
23	.022571	.913	.925	1.0225	.96604
24	.0201	.814	.824	.91053	.86028
25	.0179	.724	.734	.81087	.76612
26	.01594	.644	.653	.72208	.68223
27	.014199	.574	.582	.64303	.60755
28	.012641	.511	.518	.57264	.54103
29	.011257	.455	.471	.50994	.4818
30	.010025	.405	.410	.45443	.42907
31	.008928	.360	.366	.40444	.38212
32	.00795	.321	.326	.36014	.34026
33	.00708	.286	.290	.32072	.30302
34	.006304	.254	.258	.28557	.26981
35	.005614	.226	.230	.25431	.24028
36	.005	.202	.205	.2265	.214
37	.004453	.180	.182	.20172	.19059
38	.003963	.159	.162	.17961	.1697
39	.003531	.142	.144	.15995	.15113
40	.003144	.127	.128	.14242	.13456

roximate prices in 1873 for copper or brass sheets, Nos 0 to 25, from 8 cts per lb. Ingots, 30 to 35 cts. Castings, 35 to 45 cts. Sheet iron, com- os 10 to 16, 5¼; Nos 16 to 25, 8 cts; best, about 50 per ct more. Sheet steel, n, 8 to 10 cts; best, 15 to 17 cts.

et two cols vary considerably in different books. Ours are from a table prepared by an Eng- er of the gauges themselves. See "Gauge," in Tomlinson's Cyclopedia.

Among trade-stupidities, none is more thoroughly senseless than the adherence to the various ham, Lancashire, &c. gauges; instead of at once denoting the thickness and diameter of ire, &c. by the parts of an inch; as has long been suggested. Thus, No. ¼, or No. $\frac{1}{16}$ wire, *metal of any kind, should* be understood to mean ¼ or $\frac{1}{16}$ of an inch diam. or thickness. To *takes, which* are very apt to occur from the number of gauges in use; and from the absur *applying the same No.* to different thicknesses of different metals, in different towns, i *ore them all; and* in giving orders, to define the diameter of wire, and the thickness *, by parts of an* inch. Or the weight per hundred ft for wire; or per sq ft for s *ployed. We believe* that the *foregoing* Birmingham gauge applies to zinc, copper,

and lead; although it is generally stated to be for iron and steel only. Another Birmingham gauge is used for sheet brass, gold, silver, and some other metals; but we have never seen it stated what those others are. There are different gauges even for wire to be used for different *purposes*; and various firms have gauges of their own; not even according among themselves.

The American gauge differs from all others, and is equally senseless. We often hear of "Stubs' (England) gauge; but as Mr. Stubs makes various English gauges, the term *by itself* means nothing. We believe, however, in our machine-shops, it applies to the Birmingham gauge of the preceding table.

Weight of one foot in length of Wire, of Iron, Steel, Copper, or Brass. (From Haswell.)

Diameters by the Birmingham gauge for iron wire, sheet iron, and steel.

No. of Gauge.	Diam.	Iron.	Steel.	Copper.	Brass.
	Ins.	Lbs.	Lbs.	Lbs.	Lbs.
0000	.454	.546207	.55136	.623913	.589286
000	.425	.478656	.483172	.546752	.516407
00	.38	.38266	.38627	.437099	.41284
0	.34	.30634	.30923	.349921	.3305
1	.3	.2385	.24075	.27243	.25731
2	.284	.213738	.215755	.244146	.230596
3	.259	.177765	.179442	.203054	.191785
4	.238	.150107	.151523	.171461	.161945
5	.22	.12826	.12947	.146507	.138376
6	.203	.109204	.110234	.12474	.117817
7	.18	.08586	.086667	.098075	.092632
8	.165	.072146	.072827	.08241	.077836
9	.148	.058046	.058593	.066303	.062624
10	.134	.047583	.048032	.054353	.051336
11	.12	.03816	.03852	.043589	.04117
12	.109	.031485	.031782	.035964	.033968
13	.095	.023916	.024142	.027319	.025802
14	.083	.018256	.018428	.020853	.019696
15	.072	.013738	.013867	.015692	.014821
16	.065	.011196	.011302	.012789	.012079
17	.058	.008915	.008999	.010183	.009618
18	.049	.006363	.006423	.007268	.006864
19	.042	.004675	.004719	.00534	.005043
20	.035	.003246	.003277	.003708	.003502
21	.032	.002714	.002739	.0031	.002928
22	.028	.002078	.002097	.002373	.002241
23	.025	.001656	.001672	.001892	.001787
24	.022	.001283	.001295	.001465	.001384
25	.02	.00106	.001070	.001211	.001144
26	.018	.0008586	.0008687	.0009807	.0009263
27	.016	.0006784	.0006848	.0007749	.0007319
28	.014	.0005194	.0005243	.0005933	.0005604
29	.013	.0004479	.0004521	.0005116	.0004832
30	.012	.0003816	.0003852	.0004359	.0004117
31	.01	.000265	.0002675	.0003027	.0002859
32	.009	.0002147	.0002167	.0002452	.0002316
33	.008	.0001696	.0001712	.0001937	.000183
34	.007	.0001299	.0001311	.0001483	.0001401
35	.005	.00006625	.00006688	.00007568	.00007148
36	.004	.0000424	.0000428	.00004843	.00004574

Diameters by the American gauge.

No. of Gauge.	Diam.	Iron.	Steel.	Copper.	Brass.
	Ins.	Lbs.	Lbs.	Lbs.	Lbs.
0000	.46	.56074	.56603	.640513	.605176
000	.40964	.444683	.448879	.507946	.479908
00	.3648	.352659	.355986	.40283	.380666
0	.32486	.279665	.282303	.319451	.301816
1	.2893	.221789	.223891	.253342	.239353
2	.25763	.175888	.177548	.200911	.189818
3	.22942	.13948	.140796	.159323	.150522
4	.20431	.110616	.11166	.126353	.119376
5	.18194	.08772	.088548	.1002	.094666
6	.16202	.069565	.070221	.079462	.075075
7	.14428	.055165	.055685	.063013	.059545
8	.12849	.043751	.044164	.049976	.047219
9	.11443	.034699	.035026	.039636	.037437
10	.10189	.027512	.027772	.031426	.029687
11	.090742	.02182	.022026	.024924	.023549
12	.080808	.017304	.017468	.019766	.018676
13	.071961	.013722	.013851	.015674	.014809
14	.064084	.010886	.010989	.012435	.011746
15	.057068	.008631	.008712	.009859	.009313
16	.05082	.006845	.006909	.007819	.007387
17	.045257	.005427	.005478	.006199	.005857
18	.040303	.004304	.004344	.004916	.004643
19	.03589	.003413	.003445	.003899	.003684
20	.031961	.002708	.002734	.003094	.00292
21	.028462	.002147	.002167	.002452	.002317
22	.025347	.001703	.001719	.001945	.001838
23	.022571	.00135	.001363	.001542	.001457
24	.0201	.001071	.001081	.001223	.001155
25	.0179	.0008491	.0008571	.0009699	.0009158
26	.01594	.0006734	.0006797	.0007692	.0007265
27	.014195	.000534	.0005391	.0006098	.0005762
28	.012641	.0004235	.0004275	.0004837	.0004571
29	.011257	.0003358	.0003389	.0003835	.0003624
30	.010025	.0002663	.0002688	.0003042	.0002874
31	.008928	.0002113	.0002132	.0002413	.000228
32	.00795	.0001675	.0001691	.0001913	.0001808
33	.00708	.0001328	.0001341	.0001517	.0001434
34	.006304	.0001053	.0001063	.0001204	.0001137
35	.005614	.00008366	.00008445	.0000956	.0000903
36	.005	.00006625	.00006687	.0000757	.0000715
37	.004453	.00005255	.00005304	.00006003	.00005671
38	.003965	.00004166	.00004205	.00004758	.00004496
39	.003531	.00003305	.00003336	.00003775	.00003566
40	.003144	.0000262	.00002644	.00002992	.00002827

Birmingham gauge for sheet Brass, Silver, Gold, and all metals except iron and steel.

No.	Thickn.	No.	Thickn.	No.	Thickn.	No.	Thickn.	No.	Thickn.	No.	Thickn.
	Inch.		Inch		Inch		Inch		Inch.		Inch.
1	.004	7	.015	13	.036	19	.064	25	.095	31	.133
2	.005	8	.016	14	.041	20	.067	26	.103	32	.143
3	.008	9	.019	15	.047	21	.072	27	.113	33	.145
4	.010	10	.024	16	.051	22	.074	28	.120	34	.148
5	.012	11	.029	17	.057	23	.077	29	.124	35	.158
6	.013	12	.034	18	.061	24	.082	30	.126	36	.167

Price of brass and copper wire, approximate for 1873. **Ansonia Brass & Copper Co.,** Nos 19 & 21 Cliff St., N. York. For 100 lbs or more.

Nos 0 to 25 Copper..... 54 cts per lb.
" " " " High brass..... 44 " " "
" " " " Low brass..... 48 " " "

Table of iron wire made by the Trenton Iron Co;* Trenton, N. J. Charles Hewitt Esq, President.

The numbers of the wires in the first column are those of the Trenton Iron Co; and correspond to somewhat smaller diams than those of the Birmingham gauge.

No. by Tr. Ir. Co's G.	Diam. Ins.	Area. Sq. ins.	No. to a sq. in. of metal.	Wt. of 100 yds. Lbs.	Breakg pull. Lbs.	No. by Tr. Ir. Co's G.	Diam. Ins.	Area. Sq. ins.	No. to a sq. in. of metal.	Wt. of 100 yds. Lbs.	Breakg pull. Lbs.
0	.305	.07306	13.69	73.96	5926	10	.130	.01327	75.36	13.43	1233
1	.285	.06379	15.68	64.58	5226	11	.1175	.01084	92.25	10.97	1010
2	.265	.05515	18.13	55.83	4570	12	.105	.00866	115.5	8.767	810
3	.245	.04714	21.21	47.72	3948	13	.0925	.00672	148.8	6.804	631
4	.225	.03976	25.15	40.25	3374	14	.080	.00503	198.8	5.092	474
5	.205	.03301	30.29	33.42	2839	15	.070	.00385	259.7	3.897	372
6	.190	.02835	35.27	28.70	2476	16	.061	.00292	342.5	2.956	292
7	.175	.02405	41.58	24.35	2136	17	.0525	.00216	462.9	2.187	222
8	.160	.02011	49.73	20.36	1813	18	.045	.00159	629.0	1.610	169
9	.145	.01651	60.57	16.71	1507	20	.033	.00086	1162.8	.870	96

The wire in this table is supposed to be hard, bright, or unannealed. Annealing renders wire more pliable, but less elastic; and reduces its strength perhaps 20 or 25 per cent. **The strengths in the last column** are at the rate of 81000 lbs per sq inch for No. 0; 93000 for No. 10; and 111000 for No. 20; and are say from 15 to 25 per ct greater than those of ordinary market wire. The wires in the table are all made of the same specially prepared iron; and the increased strength per sq in of the smaller ones is due to their more frequent passage through the draw-plates. The choice Swedish and Norway irons would not yield wire of much greater strength; while the cost would be about 60 per ct more.

Hard steel wire averages about twice the strength of iron wire.

Unannealed or hard **brass wire** has about ¾ths the strengths of the above table, and about $\frac{1}{9}$ more weight. If annealed, only full half the strength.

Hard copper wire may be taken at ⅔ of the tabular strengths, and full $\frac{1}{9}$ more weight.

To find approximately the number of straight wires that can be got into a cable of given diameter.

Divide the diameter of the cable in inches, by the diameter of a wire in inches. Square the quotient. Multiply said square by the decimal .78. The result will be correct within about 2 or 3 per cent at most.

The solidity, or metal area of all the wires in a cable, will be to the area of the cable itself, about as 1 to 1.282. Or, **to find the united areas of all the wires in a cable,** multiply the area of the cable by the decimal .78; or, **for the area of all the voids,** or vacant spaces between the wires of a cable, multiply the area of the cable by the decimal .22. In other words, the area of the voids is nearly ¼ that of the cable; while that of the wires is fully ¾ that of the cable. All approximate.

***Approximate wholesale prices of the Trenton Iron Co's wire in 1873.**

	Number of wire by Trenton Iron Co's gauge.					
	0 to 9	10 to 13	14 to 17	18 to 20	23	28
	Approximate price in cents per pound.					
Iron wire of quality of the above table.	6½	8	10	12½	14	16
Best Cast Steel wire.	19½	21½	26½	30½	60	

Tinned wire is about one third higher price.
The prices of annealed and unannealed wire are the same.
The Co make other wire of specified quality to order.

Weights in lbs per sq ft, of galvanized sheet iron, both and corrugated. The Nos and thicknesses are those of the iron before galvanized; but the weights refer to the galvd sheets. When a flat sheet (the nary size of which is from 2 to 2½ ft in width, by 6 to 8 ft in length) is conv into a corrugated one, with corrugations 5 ins wide from center to center, and an inch deep (the common sizes,) its width is thereby reduced about $\frac{1}{10}$th pa from 30 to 27 ins; and consequently the weight per sq ft of area covered is incr about ⅓th part. When the corrugated sheets are laid upon a roof, the overla of about 2½ ins along their sides, and of 4 ins along their ends, diminishe covered area about ⅓th part more; making their weight per sq ft of roof about part greater than before. Or the weight of corrugated iron per sq ft, in place roof, is about ⅓ greater than that of the flat sheets of above sizes of which it is m

GALVANIZED IRON.

Weights per square foot.

No. by Birmingham wire gauge.	Thickness in inches.	Flat. Lbs.	Corrugated. Lbs.	Corrug on roof. Lbs.	No. by Birmingham wire gauge.	Thickness in inches.	Flat. Lbs.	Corrugated. Lbs.	Cor on L
30	.012	.806	.896	1.08	21	.032	1.63	1.81	2
29	.013	.857	.952	1.14	20	.035	1.75	1.94	2
28	.014	.897	.997	1.20	19	.042	2.03	2.26	2
27	.016	.978	1.09	1.30	18	.049	2.32	2.58	3
26	.018	1.06	1.18	1.41	17	.058	2.68	2.98	3
25	.020	1.14	1.27	1 52	16	.065	2.96	3.29	3
24	.022	1.22	1.36	1.62	15	.072	3.25	3.61	4
23	.025	1.34	1.49	1.79	14	.083	3.69	4.10	4
22	.028	1.46	1.62	1.95	13	.095	4 18	4.64	5

Nos. 20 to 23 are commonly used for covering roofs.

The galvanizing is simply a thin film of zinc on both sides o sheet, as in what is known as "tinned plates," or "tin;" which are in reality sheet iron sim coated with tin. Zinc, like tin, resists corrosion from ordinary atmospheric influences, much than iron; and hence the use of these metals as a protection to the iron. A well galvanized of a good pitch, will suffer but little from 5 to 6 years' exposure without being painted. It wil take paint readily, and should be painted. It is better, however, always to paint tin ones at o

Paint does not adhere well to new zinc, and this is the princ reason why new galvanized roofs are not painted; but this may be remedied by first brushin zinc over with the following: One part of chloride of copper, 1 part nitrate of copper, 1 part o ammoniac. Dissolve in 64 parts of water. Then add 1 part of commercial hydrochloric acid. brushed with this solution, the zinc turns black; dries within 12 to 24 hours, and may then be pai

Paint of some mineral oxide of a brown color is generally used: one coat being applied to sides in the shop; and the other after being put on the roof. Repainting every 3 or 4 years will afterward. Ungalvanized iron (called BLACK IRON, for distinction) is also very enduring for ro well painted every 1 or 2 years. The chief advantage of galvanized roofing is that it does not re painting so often as the black. The galvanizing adds about ¼ of a ℔ per square foot of surfa about ¼ ℔ per sq ft of sheet as coated on both sides; without regard to the thickness of the On this principle the above table has been prepared. Those in text books generally, are incorr regards the thinner sheets. Paint for roofs should not have much *dryer*. See Painting, p 517.

The sulphurous fumes from coal are very corrosive EITHER GALVANIZED OR BLACK IRON; as may be seen in shops, railroad bridges, or engine h roofed with either; if efficient means are not provided for carrying off the smoke; and the same other metals. THE ACID OF OAK TIMBER is said to destroy the zinc of galvanized iron. See Ti Zinc. Flat iron is usually nailed upon a sheeting of boards; but the strength of corrugated obviates the necessity for this, and enables it to stretch 5 or 6 ft from purlin, to purlin, without mediate support. The corrugated sheets are riveted together on the roof, by rivets of galva wire about ¼ inch thick, 300 to a pound, well driven (so as to exclude rain) at intervals of inches, all around the edges. The rivet-holes are first punched by machinery, so as to insure c dence in the several sheets; and the rivets are driven by two men, one above, and one benea roof. For black iron, ungalvanized nails, boiled in linseed oil as a partial preservative from ru commonly used; as also in shingling or slating. Galvanized ones, however, would be better *these cases;* or even copper ones for slating because good slate endures much longer than *shingles or iron, and therefore* it becomes true economy to use durable metals for fastening i *those of these cases, however,* are the nails fully exposed to the weather.

The sheets of flat iron are put together by overlapping *FOLDING THE EDGES, much the same as shown by the fig under the head Tin: the joints w up and down the roof being the same as at a a, and the horizontal ones except that inasmuch as these are not soldered in the iron sheets, the jo about ¾ to 1 inch wide, instead of ¼ inch, the better to provide agai Cleats are used as in tin, with 2 nails to a cleat. The iron plates are sheeting boards; but in sheds, &c, are sometimes laid directly on raf than about 18 ins apart in the clear; the plates being allowed to sag a*

s, so as to form shallow gutters. In such cases it is well to bevel off the tops of the rafters as in this fig.

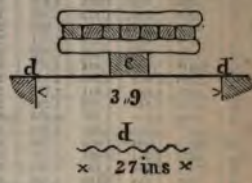
rious objection to iron as a roof covering, is its rapid con- of atmospheric moisture; which falls from the iron in drops like rain, and may do injury s, floors, or articles in the apartments immediately beneath the roof. Painting does not bly diminish this; it may, however, be obviated by plastering, as shown at 11, of Figs 21½, s, page 265. Corrugated iron makes an excellent permanent street or other awning.

e of corrugated iron in 1873, 7 to 9 cts per lb; or 12 to 14 cts if galvanized, 14 to 26. Does not require sheets of best iron, for roofs.

CORRUGATED IRON.

eriments by the writer, on the strength of corrugated iron.

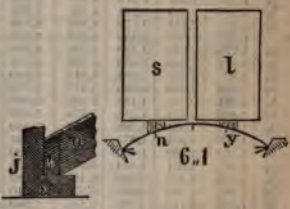
st. A sheet *d d*, of No. 16 iron, $\frac{1}{15}$ inch thick,) 27 ins wide, by 4 ft long, e complete corrugations of 5 ins by 1 inch, l on supports 3 ft 9 ins apart. A block of 9 ins wide, by 7 ins thick, and 30 ins long, ced across the center, and gradually loaded stings weighing 1600 lbs.



used a deflection at the center of precisely ¼ an e the removal of the load after an hour, no perma- was appreciable. The severity of the test was pur- ncreased by applying the several castings very jolting the whole as much as possible.* The sus- rea of the sheet was 8.44 sq ft; and since the actual *center* load of 1600 lbs is about equiva- 000 lbs *equally distributed*, it amounts to $\frac{3000}{8.44} = 355$ lbs per sq ft distributed. But 3000 lbs ed would produce a deflection of but about full ¼ of an inch. Again, 355 lbs per sq ft 4 times the weight of the greatest crowd that could well congregate upon a floor. Conse- this iron, at 3' 9" span, is safe in practice for any ordinary crowd. Moreover, such a crowd oduce a center deflection of only the ¼th part of ¼ of an inch; or $\frac{1}{16}$ of an inch; or $\frac{1}{720}$ ar span; which is but two-thirds of Tredgold's limit of $\frac{1}{480}$ of the span. See Art 26 of of Materials, p 196.

experiment the ends of the sheets rested upon supports dressed so as to present undulations ding tolerably closely with the shape of the corrugations; but in the other the supports and each end of the sheet rested only upon the lower points of the corrugations. No ap- difference was observed in the results

ond. An arch of No. 18 ($\frac{1}{20}$ **ron,** corrugated like the foregoing, depth of corrugation increased to by the process of arching the sheet; an 6 ft 1 inch; rise 10 ins; breadth 27 which, however, only 25 ins bore the abutments.)



oot *o* of the arch abutted upon a casting *j*, portion *t* of which was undulated on top, to d with the corrugations of the arch, which on it. At *y*, (one-fourth of the span,) two blocks were placed, occupying a width of 9 nd extending across the arch; on them was ad. *l*, of castings, to the extent of 4480 lbs, . Under this load the arch descended about h at *y*, becoming flatter along that side, and more curved upward along the unloaded side *n*. Two similar blocks were then placed at *n*, tons of load, *s*, were piled upon them, in addition to the 2 tons at *l*; making a total of 8960 tons. This brought the arch more nearly back to its original shape; but still slightly ued at both *n* and *y*, and a little more curved in the center. The load was then increased to , and left standing for several days. Two tie-bars, each ½ by 1¾, which were used for pre- he abutment castings *j* from spreading, were found to have stretched nearly ¼ of an inch. al ones were inserted, and the load increased to a total of 6 tons, or 13440 lbs; parts of it on nd part in the shape of long broad bars of iron at the center of the arch, below the loads *s* d between *n* and *y*. So far as could be judged by eye, the shape of the arch was now almost *The loads s and l did not touch each other.* After standing more than a week, the load lentally overturned, crippling the arch. The load was equal to about 1000 lbs per sq ft of

ed roofs of 30 to 45 ft span, rising about ¼ span, may be made *ry corrugated iron of Nos 16 to 13, riveted as usual; and having no acce- ept tie-rods a few feet apart; continuous angle-iron skewbucks; and th ds to prevent the ties from sagging.*

however, allowing the deflection to exceed the ½ inch; which was effected by m be sheet.

Weight of 1 ft in length of FLAT ROLLED IRON, at 480 lbs per cubic foot. For cast iron, deduct $\frac{1}{16}$ part; for steel, add $\frac{1}{48}$; for copper, add $\frac{1}{7}$; for cast brass, add $\frac{1}{18}$; for lead, add $\frac{1}{2}$; for zinc, deduct $\frac{1}{12}$.

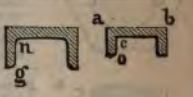
Width in Ins.	THICKNESS IN INCHES.											
	1-16	⅛	3-16	¼	5-16	⅜	7-16	½	⅝	¾	⅞	1
1.	.2083	.4166	.6250	.8333	1.042	1.250	1.458	1.666	2.083	2.500	2.916	3.333
⅛	.2344	.4688	.7033	.9375	1.172	1.406	1.640	1.875	2.344	2.812	3.280	3.75
¼	.2605	.5210	.7810	1.042	1.303	1.562	1.823	2.083	2.605	3.125	3.646	4.166
⅜	.2865	.5730	.8595	1.146	1.432	1.719	2.006	2.292	2.864	3.438	4.012	4.583
½	.3125	.6250	.9375	1.250	1.562	1.875	2.188	2.500	3.125	3.750	4.375	5.000
⅝	.3385	.6771	1.015	1.354	1.692	2.031	2.370	2.708	3.384	4.062	4.740	5.416
¾	.3646	.7292	1.094	1.458	1.823	2.188	2.550	2.916	3.646	4.375	5.105	5.833
⅞	.3906	.7812	1.172	1.562	1.953	2.344	2.735	3.125	3.906	4.688	5.470	6.25
2.	.4166	.8333	1.25	1.666	2.083	2.500	2.916	3.333	4.166	5.000	5.833	6.666
⅛	.4427	.8855	1.328	1.771	2.214	2.656	3.098	3.542	4.428	5.312	6.196	7.083
¼	.4688	.9375	1.406	1.875	2.344	2.812	3.281	3.750	4.688	5.624	6.562	7.500
⅜	.4948	.9895	1.484	1.979	2.474	2.968	3.463	3.958	4.948	5.936	6.926	7.916
½	.5210	1.042	1.562	2.083	2.605	3.125	3.646	4.166	5.210	6.250	7.291	8.333
⅝	.5470	1.094	1.641	2.187	2.735	3.282	3.829	4.375	5.470	6.564	7.658	8.750
¾	.5730	1.146	1.719	2.292	2.865	3.438	4.011	4.583	5.730	6.876	8.022	9.166
⅞	.5990	1.198	1.797	2.396	2.995	3.594	4.193	4.792	5.990	7.188	8.386	9.584
3.	.625	1.250	1.875	2.500	3.125	3.750	4.375	5.000	6.250	7.500	8.750	10.00
⅛	.6515	1.303	1.954	2.605	3.257	3.908	4.560	5.210	6.514	7.816	9.120	10.42
¼	.6770	1.354	2.031	2.708	3.385	4.062	4.739	5.416	6.770	8.124	9.478	10.83
⅜	.7031	1.406	2.109	2.812	3.516	4.218	4.921	5.625	7.032	8.436	9.842	11.25
½	.7291	1.458	2.188	2.916	3.646	4.375	5.105	5.833	7.291	8.750	10.21	11.66
⅝	.7555	1.511	2.266	3.021	3.777	4.533	5.288	6.042	7.554	9.066	10.58	12.08
¾	.7812	1.562	2.343	3.125	3.906	4.686	5.468	6.25	7.812	9.372	10.94	12.50
⅞	.8070	1.614	2.421	3.229	4.035	4.842	5.65	6.458	8.070	9.684	11.30	12.92
4.	.8333	1.666	2.500	3.333	4.166	5.000	5.833	6.666	8.333	10.00	11.66	13.33
⅛	.8595	1.719	2.578	3.438	4.297	5.156	6.016	6.875	8.594	10.31	12.03	13.75
¼	.8855	1.771	2.656	3.542	4.427	5.312	6.198	7.083	8.854	10.62	12.40	14.16
⅜	.9115	1.823	2.734	3.646	4.557	5.468	6.380	7.291	9.114	10.94	12.76	14.58
½	.9375	1.875	2.812	3.750	4.687	5.624	6.562	7.500	9.374	11.25	13.12	15.00
⅝	.9636	1.927	2.891	3.854	4.818	5.782	6.745	7.708	9.636	11.56	13.49	15.42
¾	.9895	1.979	2.968	3.958	4.947	5.936	6.926	7.917	9.894	11.87	13.85	15.83
⅞	1.016	2.031	3.048	4.062	5.080	6.096	7.112	8.125	10.16	12.19	14.22	16.25
5.	1.042	2.083	3.125	4.166	5.210	6.25	7.291	8.333	10.42	12.50	14.58	16.66
⅛	1.068	2.136	3.204	4.271	5.340	6.408	7.476	8.542	10.68	12.81	14.95	17.08
¼	1.094	2.188	3.282	4.375	5.470	6.564	7.658	8.750	10.94	13.13	15.31	17.50
⅜	1.120	2.240	3.360	4.479	5.600	6.720	7.840	8.958	11.20	13.44	15.68	17.92
½	1.146	2.292	3.438	4.584	5.730	6.876	8.022	9.167	11.46	13.75	16.04	18.33
⅝	1.172	2.344	3.516	4.687	5.859	7.032	8.204	9.375	11.72	14.06	16.40	18.75
¾	1.198	2.396	3.594	4.791	5.990	7.188	8.386	9.583	11.98	14.37	16.77	19.16
⅞	1.224	2.448	3.672	4.896	6.120	7.344	8.568	9.792	12.24	14.69	17.13	19.58
6.	1.250	2.500	3.750	5.000	6.250	7.500	8.750	10.00	12.50	15.00	17.50	20.00
⅛	1.276	2.552	3.828	5.104	6.380	7.656	8.932	10.21	12.76	15.31	17.86	20.42
¼	1.302	2.604	3.906	5.208	6.510	7.812	9.114	10.42	13.02	15.62	18.23	20.83
⅜	1.328	2.657	3.984	5.313	6.640	7.968	9.297	10.63	13.28	15.93	18.59	21.25
½	1.354	2.708	4.063	5.417	6.770	8.126	9.480	10.83	13.54	16.25	18.96	21.66
⅝	1.381	2.761	4.143	5.521	6.906	8.286	9.658	11.04	13.81	16.57	19.33	22.08
¾	1.406	2.813	4.218	5.625	7.030	8.436	9.843	11.25	14.06	16.87	19.69	22.50
⅞	1.432	2.864	4.296	5.729	7.160	8.592	10.02	11.46	14.32	17.18	20.04	22.92
7.	1.458	2.916	4.375	5.833	7.291	8.750	10.20	11.66	14.58	17.50	20.42	23.33
⅛	1.484	2.969	4.452	5.938	7.420	8.904	10.39	11.87	14.84	17.81	20.78	23.75
¼	1.511	3.021	4.533	6.042	7.555	9.066	10.58	12.08	15.11	18.13	21.16	24.16
⅜	1.536	3.073	4.608	6.146	7.680	9.216	10.75	12.29	15.36	18.43	21.50	24.58
½	1.562	3.125	4.686	6.250	7.810	9.372	10.93	12.50	15.62	18.74	21.86	25.00
⅝	1.588	3.177	4.764	6.354	7.940	9.528	11.12	12.71	15.88	19.05	22.24	25.42
¾	1.615	3.229	4.845	6.458	8.075	9.690	11.31	12.92	16.15	19.38	22.62	25.83
⅞	1.641	3.281	4.923	6.562	8.205	9.846	11.48	13.13	16.41	19.69	22.96	26.25
8.	1.666	3.333	5.000	6.666	8.333	10.00	11.66	13.33	16.66	20.00	23.33	26.66
⅛	1.693	3.386	5.079	6.771	8.455	10.15	11.85	13.54	16.91	20.30	23.70	27.08
¼	1.719	3.438	5.157	6.875	8.595	10.31	12.03	13.75	17.19	20.61	24.06	27.50
⅜	1.745	3.489	5.235	6.979	8.725	10.47	12.21	13.96	17.45	20.94	24.42	27.92
½	1.771	3.542	5.313	7.083	8.855	10.63	12.40	14.17	17.71	21.26	24.80	28.33
⅝	1.797	3.594	5.391	7.188	8.985	10.78	12.58	14.37	17.97	21.56	25.16	28.75
¾	1.823	3.646	5.469	7.292	9.115	10.94	12.76	14.58	18.23	21.88	25.52	29.17
⅞	1.849	3.698	5.547	7.396	9.245	11.09	12.94	14.79	18.49	22.18	25.88	29.58
9.	1.875	3.750	5.625	7.500	9.375	11.25	13.12	15.00	18.75	22.50	26.24	30.00
⅛	1.901	3.802	5.703	7.604	9.505	11.41	13.31	15.21	19.00	22.81	26.62	30.42
¼	1.927	3.854	5.781	7.708	9.635	11.56	13.49	15.42	19.27	23.12	26.98	30.83
⅜	1.953	3.906	5.859	7.812	9.765	11.72	13.67	15.62	19.53	23.44	27.34	31.25
½	1.979	3.958	5.937	7.916	9.895	11.87	13.85	15.84	19.79	23.74	27.70	31.67
⅝	2.005	4.010	6.015	8.021	10.02	12.03	14.04	16.04	20.04	24.06	28.08	32.08
¾	2.031	4.062	6.093	8.125	10.16	12.18	14.21	16.25	20.32	24.36	28.42	32.50
⅞	2.057	4.114	6.171	8.229	10.29	12.34	14.40	16.46	20.58	24.68	28.80	32.92
10.	2.083	4.166	6.250	8.333	10.41	12.50	14.58	16.66	20.82	25.00	29.16	33.33
⅛	2.109	4.219	6.327	8.438	10.55	12.65	14.76	16.87	21.10	25.30	29.52	33.75
¼	2.135	4.270	6.405	8.541	10.67	12.81	14.94	17.08	21.34	25.62	29.88	34.17

ght of 1 ft in length of FLAT ROLLED IRON, at 480 lbs per cubic foot—(Continued.)

THICKNESS IN INCHES.

1-16	⅛	3-16	¼	5-16	⅜	7-16	½	⅝	¾	⅞	1
2.162	4.323	6.486	8.646	10.81	12.97	15.13	17.29	21.62	25.94	30.26	34.58
2.188	4.375	6.564	8.750	10.94	13.13	15.31	17.50	21.88	26.26	30.62	35.00
2.214	4.427	6.642	8.854	11.07	13.28	15.50	17.71	22.14	26.56	31.00	35.42
2.239	4.479	6.717	8.958	11.20	13.43	15.67	17.92	22.40	26.86	31.34	35.83
2.266	4.531	6.798	9.062	11.33	13.59	15.86	18.12	22.66	27.18	31.72	36.25
2.291	4.583	6.875	9.166	11.46	13.75	16.04	18.33	22.90	27.50	32.08	36.66
2.318	4.636	6.954	9.271	11.59	13.91	16.22	18.54	23.18	27.82	32.44	37.08
2.344	4.688	7.032	9.375	11.72	14.06	16.40	18.75	23.44	28.12	32.80	37.50
2.370	4.740	7.110	9.479	11.85	14.22	16.59	18.96	23.70	28.44	33.18	37.92
2.395	4.791	7.185	9.582	11.97	14.37	16.76	19.16	23.94	28.74	33.52	38.33
2.422	4.844	7.266	9.688	12.11	14.53	16.95	19.37	24.22	29.06	33.90	38.75
2.448	4.896	7.344	9.792	12.24	14.68	17.13	19.58	24.48	29.36	34.26	39.16
2.474	4.948	7.422	9.896	12.37	14.84	17.32	19.79	24.74	29.68	34.64	39.58
2.500	5.000	7.500	10.00	12.50	15.00	17.50	20.00	25.00	30.00	35.00	40.00

is table may be used for finding the ht of L, T, U, H, &c, iron. Thus, if the width channel-iron be 6 ins; each arm or flange, *c o*, 3 ins; and the thickness $\frac{7}{16}$ inch; its weight per foot run is evidently equal of a flat bar of 12 by $\frac{7}{16}$; or 17.5 lbs per table. When some angles are rounded, as at *g* and *n*, what is taken from one, as ally added to another, as *n*; thus producing no change in the weight.

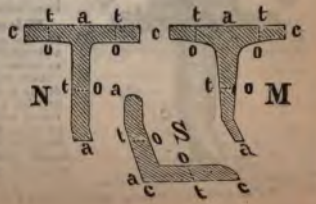


T, AND ANGLE IRON.

roximate weight of one foot in length of rolled angle, F iron. The thickness measured at the middle, *o t*, of each flange. See Re- k below.

es s.	Thicks. in ins.	Weight in lbs.	Flanges in ins.	Thicks. in ins.	Weight in lbs.	Flanges in ins.	Thicks. in ins.	Weight in lbs.
⅝	⅛	0.6	2 × 2¼	⅜	5	3½ × 4	½	12.
⅞	3-16	1.	2¼ × 2¼	5-16	4.5	3¾ × 4	½	12.5
1	3-16	1.2	2 × 2⅝	⅜	5.3	4 × 4	⅝	9.6
1¼	3-16	1.4	2½ × 2½	5-16	5.	4 × 4	½	12.7
1¼	¼	2	2½ × 2½	⅜	6.	4 × 4	¾	19.
1½	¼	2.2	2½ × 2⅞	⅜	6.5	4 × 4	1	25.
1½	3-16	1.8	2½ × 3	⅜	6.7	4 × 5	½	14.3
1½	¼	2.3	3 × 3	⅜	7.3	5 × 3	½	12.7
1¾	¼	2.8	3 × 3½	⅜	7.7	5 × 6	½	17.7
2	¼	2.8	3 × 3½	½	10.4	5½ × 3	½	14.
2	5-16	3.4	3 × 4	⅜	8.5	5½ × 3½	½	14.4
2	¼	3.	3 × 4	½	11.3	6 × 3½	⅝	19
2	¼	3.3	3½ × 3½	⅜	8.5	6 × 4½	⅝	21
2	5-16	3.9	3 × 3¾	½	10.8	7½ × 3¼	⅝	22
2	⅜	4.7	3½ × 3½	7-16	9.7			

RK. THE DIMENSIONS IN THIS TABLE ARE FROM OUT EACH WAY, AS *a a*, *c c*. When these, and kness *o t*, measured at the center of each arm, ame in M, N, and S, there will scarcely be an able difference in the weights of the three. nensions in the table are MARKET SIZES; that can be bought ready-made. The young en- should bear this in mind in such matters eighting; and not introduce sizes that have ecially made; *for this requires* the manufac- *prepare a new set of rolls; the* expense of *ould not be warranted by* an order less than *usands of dollars.*



Weight of heads and nuts of iron bolts.* The weights include both a head, and a nut, which are supposed to be neatly finished off. Rough machine-made ones frequently weigh from ¼ to ⅓ more.

Diameter of bolt, in inches.												
¾	⅞	½	⅝	¾	⅞	1	1¼	1½	1¾	2	2½	3
Weight of an hexagonal head and nut in lbs.												
.017	.057	.128	.267	.43	.73	1.10	2.14	3.78	5.6	8.75	17.	28.8
Weight of a square head and nut in lbs.												
.021	.069	.164	.320	.55	.88	1.31	2.56	4.42	7.0	10.5	21.	36.4

An hexagonal head and nut together, weigh about as much as 5 diams of length of the bolt; or as 6 diams, if they are square. Machine-made bolts have the head in the same solid piece with the shank. They are frequently so made by the smith also; but at times the head consists of a separate ring, welded upon the end of the bolt. For *washers*, see following.

The thickness *y t*, of a nut N; or *m g*, of a head, H, should be *at least* equal to the diam *c o*, of the *screw*. And the width *a b*, between two parallel sides of the hexagon, is 1½ times the same diam. Some machinists add ⅛ inch to this width, for all diams. If square, each side is made equal to 1½ diams of the screw. The following are the usual numbers of screw-*threads* cut per inch of length at the ends of bolts. The depth to which the threads are cut is a little less than their *pitch*, or distance apart from center to center.

Diameter of bolt, in inches.	¾	5/16	⅜	7/16	½	9/16	⅝	¾	⅞	1	1¼	1½	1¾	1½	1¾	1¾	1⅞
Number of threads per inch of length.	20	18	16	14	13	12	11	10	9	8	7	7	6	6	5½	5	5
Diameter of bolt, in inches.	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4½	4½	4¾	5	5¼	5½	5¾	6
Number of threads per inch in length.	4½	4½	4	4	3½	3½	3¼	3	3	2¾	2¾	2¾	2½	2½	2¾	2¾	2¾

These may be termed the American standard dimensions. With them a bolt will not yield by stripping off its threads; but will generally break off just below the nut, where the diminution of metal by the cutting of the threads commences. To allow for this virtual reduction of diam, we must first calculate what diam is necessary for sustaining the load, or strain, which will come upon it; and then add enough to cover this reduction. That is, we must add twice the depth to which the threads are cut. For this purpose, it will suffice to increase the neat calculated diam about ¼ part, for bolts from 1 to 2½ ins diam; ⅕ for 3 ins diam; ⅙ for 6 ins diam.*

This of course involves a considerable increase in the quantity of iron; but is necessary when the bolt is required to fit close in the hole through which it passes. But where this is not the case, the screw end may be UPSET, or made so much thicker than the other part of the shank, that the threads may be cut into it, without injury to the strength of the bolt; as in the next fig. THE FOLLOWING TABLE GIVES THE DIAM FOR EITHER CASE.

In carpentry, as well as in ties for masonry, *washers*, *w w*, of either cast or wrought iron, are placed between the timber, or stone, and the head and nut; in order to distribute the pressure over a greater surface, and thus prevent crushing; especially in timber.

When much strained against wood, the side of a square wrought-iron washer; or the diam *w w* of a circular one, should not be less than 4 diams of the *screw*, as in the fig; and its thickness, *t w*, ½ diam *at least*.

Two such square washers will together weigh as much as 18 diams in length of a round rod of the same diam as the screw; or with a square head and nut, as much as 24 diams. Two round washers will weigh together as much as 14 diams of rod of same diam as screw; or with a square head and nut, as much as 20 diams. Cast-iron washers, being more apt to split under heavy strains, may be made about twice as thick as wrought ones. When the strain is very great, the diam of

*Price of screw-bolts and nuts in Philadelphia, in 1873, about 6 to 7 cts per lb.

the washer may be 5 or 6 times that of the screw; and its thickness equal to diam; but 4 diams will suffice for most practical purposes.

When the screw end of the bolt is *upset* as at *s e*, for a length *s e* of 6 diams of the shank *g y*, we must add to the weight of the entire length of the bolt, an allowance sufficient to cover the additional thickness. This allowance will equal 3½ diams of length of the body or shank *e g* of the bolt. But when thus upset, the head, nut, and washers must be enlarged to suit the thicker screw. For bolts of from 1 to 3 ins diam, the total allowance must be increased to the weight of a length equal to 35 diams *y g* of the *shank*, or *body* of the bolt, supposing the washers to be round; and the head and nut square. When a neat finish is required, the upper edges of the washers are shaped as shown by the dotted lines near *w w*; and frequently the upset length *s e* need not exceed 2 or 3 diams; both of which slightly diminish the foregoing allowance. But in making preliminary estimates of all such details, it is better to err a little on the safe side.

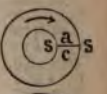
When the bolt is not to be much strained, or when the timber is hard, the washers may be but 3 diams of the screw in width, or diam; and about .4 of a diam in thickness; and this will reduce their weight fully one-half. For ease of reference we recapitulate the allowances advisable to be made in preliminary estimates; as measured by the weight of a rod of the same diam as the *shank e g*; the length of the rod being given in its own diameters.

BOLT NOT UPSET.	Square head and nut together	6 diams.
	Hexagon " " " "	5 "
	2 round washers: ½ diam thick; 4 diams in diam; wrought	14 "
	Square head, and nut; and 2 round washers	20 "
BOLT UPSET	Upsetting	3.5 "
	Square head and nut together	9.5 "
	Hexagon " " " "	7.9 "
	2 round washers; ½ diam thick: 4 diams of *screw* in diam	22.0 "
	Upsetting: square head; and nut: and 2 round washers	35.0 "

If the washers are of the smaller dimensions given above, deduct half their weight.

Lock-nut washers.

When bolts are subjected to much rough jolting, as at rail-joints, &c, the *nuts are liable to unscrew themselves in time*. On railroads this is a source of great annoyance. SHAW'S LOCK-NUT WASHER is intended to prevent this. It is a simple circular washer made of steel; with a slit *s s* cut through it, leaving sharp edges. On one side, *a*, of the slit, the metal is pressed upward about ¼ inch; and that on the other side, *c*, downward, the same distance; so that a perspective view would be somewhat as at *t*. Now, when the nut is screwed down over the washer, in the direction of the arrow, the slit offers no obstruction; but if the nut afterward tends to unscrew itself, the sharp upper edge of the slit, along *a*, presents friction against the bottom of the nut, sufficient, it is said, to prevent it from so doing. ANOTHER DEVICE is to cut at the end of the screw a few threads of a screw of less diam than the main one, and in the opposite direction. The nut is then screwed upon the larger diam; and after it the lock-nut is screwed in the other direction upon the smaller diam, until it comes into contact with the main nut. At Figs 16 and 17 of rail-joints, other methods will be seen. p 396.



THE BILLINGS LOCK-NUT WASHER is also said to be effective. It is a thin hollow cup of tempered spring steel, *n n*, made by pressure, from a flat circular piece previously heated. It is made by the Wharton Safety Switch Co, of Philada. Diam 2¼ ins; height ¾ inch; thickness ⅛ inch; weight 3½ ounces.*

Table of diameters, weights, and approximate breaking strains, for round rolled iron bolts, ties, or bars; assuming the breaking strain per square inch of average quality of rolled iron to be as follows: Up to 1 inch square, or 1 inch diam, 20 tons, or 44800 lbs; from 1 to 2 ins sq or diam, 19 tons; 2 to 3 ins, 18 tons; 3 to 4 ins, 17 tons; 4 to 5 ins, 16 tons; 5 to 6 ins, 15 tons.† The first 4 columns of the table are to be used when the screw end of the bolt is enlarged or upset, so that the shank or body of the bolt shall not be weakened by the cutting of the screw threads. But when the shank is so weakened, the diam and weight of the bolt must be taken from the last 2 cols.

EXAMPLE 1. To find the diam of a bolt, that shall just break under a strain, or a load of 52.5 tons, we see by the table and opposite 52.5 tons, that it will be 1¾ ins if the screw end is enlarged, and 2.3 ins if it is not. In the first case, the weight of the bolt will be 9.3 lbs per foot run; and in the second, 13.8 lbs.

EXAMPLE 2. What must be the diam in order to sustain a strain of 52.5 tons, with a safety of 3? Here 52.5 × 3 = 157.5 tons. In the table, the nearest we find to 157.5 tons, is 163.6; opposite which we find the diam 3½ ins. A diam a trifle less than this will *break* under a strain of 157.5 tons; and consequently will have a safety of 3 for 52.5 tons.

The breakg strains in this table will also answer for square BARS, by merely increasing them ¼ part; for a round bar has very approximately $\frac{4}{5}$ of the strength of a square one whose side is equal to the diam of the round one; or the square one has 1¼ times the strength of the round one; or, more correctly, as 1 to .7854. **For the strength of COPPER bolts,** multiply the tabular ones by the decimal .8; and for their weight, increase, that of iron ones $\frac{1}{7}$ part. Heads, nuts, and washers, are not included in the table.

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* Price in 1873, $5 to $6 per 100.

† There is no difficulty in procuring American rolled iron with a breaking strain of 21 tons, or 60000 lbs per sq inch; or say, ½ greater than that assumed in this table. With such stronger iron, the sectional areas of bolts will be but ⅔ of those of the weaker; and consequently the diams may be nearly one-seventh part less than those in the table; or more correctly, mult the diams in the table by the dec .87. With 65000 lb iron (which also can be procured) the diams will be but .83 of those in our table.

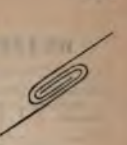
WEIGHT AND STRENGTH OF IRON BOLTS. (Original.)

Ends enlarged, or upset.				Ends not enlarged.		Ends enlarged, or upset.				Ends not enlarged.	
Diam. of shank	Weight per foot run.	Breaking strain.	Breaking strain.	Diam. of shank	Weight per foot run.	Diam. of shank	Weight per foot run.	Breaking strain.	Breaking strain.	Diam. of shank	Weight per foot run.
Ins.	Pds.	Tons.	Pds.	Ins.	Pds.	Ins.	Pds.	Tons.	Pds.	Ins.	Pds.
1/8	.0414	.245	549			1 3/4	8.10	45.7	102368	2.14	12.0
3-16	.093	.553	1239			13-16	8.69	49.0	109760	2.22	12.9
1/4	.165	.983	2202	.35	.321	7/8	9.30	52.5	117600	2.30	13.8
5-16	.258	1.53	3427	.43	.452	15-16	9.93	56.0	125440	2.38	14.7
3/8	.372	2.21	4950	.50	.654	2.	10.6	59.7	133728	2.45	15.7
7 16	.506	3.00	6720	.58	.897	1/8	12.0	63.8	142912	2.59	17.5
1/2	.661	3.93	8803	.66	1.14	1/4	13.4	71.6	160384	2 73	19.5
9-16	.837	4.97	11133	.73	1.41	3/8	14.9	79.7	178528	2.88	21.6
5/8	1.03	6.14	13754	.80	1.67	1/2	16.5	88.4	198016	3.02	23.9
11-16	1.25	7.42	16621	.88	2.03	5/8	18.2	97.4	218176	3.16	26.1
3/4	1.49	8.83	19779	.96	2.41	3/4	20.0	106.9	239456	3.30	28.5
13-16	1.75	10.4	23296	1.04	2.81	7/8	21.9	116.8	261632	3.45	31.1
7/8	2.03	12.0	26880	1.12	3.26	3.	23.8	127.2	284928	3.60	33.9
15-16	2.33	13.8	30912	1.20	3.77	1/4	27.9	141.0	315840	3.86	39.1
1 in.	2 65	15.7	35168	1.27	4.27	1/2	32.4	163.6	366464	4.12	44.4
1-16	2.99	16.8	37632	1.35	4.77	3/4	37.2	187.7	420448	4.41	51.0
1/8	3.35	18.9	42336	1.42	5.28	4.	42.3	213.6	478464	4.70	57.8
3-16	3.73	21.1	47264	1.49	5.81	1/4	47.8	227.0	508480	4.98	65.2
1/4	4.13	23.3	52192	1.55	6.39	1/2	53 6	254.5	570080	5.25	72.9
5-16	4.56	25.7	57568	1.64	7.04	3/4	59.7	283.5	635040	5.53	80.5
3/8	5.00	28.2	63168	1.72	7.74	5.	66.1	314.2	703808	5.80	88.1
7-16	5.47	30.8	68992	1.80	8.48	1/4	72.9	324.7	727328	6.08	97.0
1/2	5.95	33.6	75264	1.87	9.20	1/2	80.0	356.4	798336	6.36	106.
9-16	6.46	36.4	81536	1.94	9.88	3/4	87.5	389.5	872480	6.63	116.
5/8	6.99	39.4	88256	2.00	10.6	6.	95.2	424.1	949984	6.90	126.
11-16	7.53	42.5	95200	2.07	11.3			See p 375			

ROLLED LEAD, COPPER, and BRASS: Sheets and Bars.

Thickness or Diameter, or side, in Inches.	LEAD.			COPPER.			BRASS.			Thickness or Diameter, or side, in Inches.
	Sheets, per Square Foot.	Square Bars; 1 Foot long.	Round Bars; 1 Foot long.	Sheets, per Square Foot.	Square Bars; 1 Foot long.	Round Bars; 1 Foot long.	Sheets, per Square Foot.	Square Bars; 1 Foot long.	Round Bars; 1 Foot long.	
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	
1-32	1.86	.005	.004	1.44	.004	.003	1.36	.004	.003	1-32
1-16	3.72	.019	.015	2.89	.015	.012	2.71	.014	.011	1-16
3-32	5.58	.044	.034	4.33	.034	.027	4.06	.032	.025	3-32
1/8	7.44	.078	.061	5.77	.060	.047	5.42	.056	.044	1/8
5-32	9.30	.121	.095	7.20	.094	.074	6.75	.088	.069	5-32
3-16	11.2	.174	.137	8.66	.135	.106	8.13	.127	.100	3-16
7-32	13.0	.237	.187	10.1	.184	.144	9.50	.173	.136	7-32
1/4	14.9	.310	.244	11.5	.240	.189	10.8	.226	.177	1/4
5-16	18.6	.485	.381	14.4	.376	.295	13.5	.353	.277	5-16
3/8	22.3	.698	.548	17.3	.541	.425	16.3	.508	.399	3/8
7-16	26.0	.950	.746	20.2	.736	.578	19 0	.691	.543	7-16
1/2	29.8	1.24	.974	23.1	.962	.755	21.7	.903	.709	1/2
9-16	33.5	1.57	1.23	26.0	1.22	.955	24.3	1.14	.900	9-16
5/8	37.2	1.94	1.52	28.9	1.50	1.18	27.1	1.41	1.11	5/8
11-16	40.9	2.34	1.84	31.7	1.82	1.43	29.8	1.70	1.34	11-16
3/4	44.6	2.79	2.19	34.6	2.16	1.70	32.5	2.03	1.60	3/4
13-16	48.3	3.27	2.57	37.5	2.55	1.99	35.2	2.38	1.87	13-16
7/8	52.1	3.80	2.98	40.4	2.94	2.31	37.9	2.76	2.17	7/8
15-16	56.0	4.37	3.42	43.3	3.38	2.65	40.6	3.18	2.49	15-16
1.	59.5	4.96	3.90	46.2	3.85	3.02	43.3	3.61	2.84	1.
1/8	66.9	6 27	4.92	52.0	4.87	3.82	48.7	4.57	3.60	1/8
1/4	74.4	7.75	6.09	57.7	6.01	4.72	54.2	5.64	4.43	1/4
3/8	81.8	9.37	7.37	63.5	7.28	5.72	59.6	6.82	5.31	3/8
1/2	89.3	11.2	8.77	69.3	8.65	6.80	65.0	8.12	6.38	1/2
5/8	96.7	13.1	10.3	75.1	10.2	7.98	70.4	9.53	7.49	5/8
3/4	104.	15.2	11.9	80.8	11.8	9.25	75.9	11.1	8.66	3/4
7/8	112.	17 5	13.7	86.6	13.5	10.6	81.3	12.7	9.97	7/8
2.	119.	19.8	15.6	92.3	15.4	12.1	86.7	14.4	11.3	2.

Roof copper is usually in sheets of 2½ ft × 5 ft: or 12½ square feet weighing 10 to 14 lbs per sheet; and is laid on boards.* No solder is used in the horizontal joints as it is in tin roofs; but both the horizontal and the sloping joints are formed by only overlapping and bending the sheets, much as shown by the figs under the head "Tin;" except that the horizontal joints are bent or locked together, as in this figure; and then flattened down close.



Roof lead generally weighs 4 to 5 lbs per sq ft. Its great contraction and expansion, however, render it unfit for this purpose, as it is liable to crack and leak. It may be used for flashings. Price in 1873 about 10 to 12 cts per lb.

WEIGHT OF BALLS.

Diameter in Inches.	Cast Lead.	Cast Copper.	Cast Brass.	Cast Iron.	Diameter in Inches.	Cast Lead.	Cast Copper.	Cast Brass.	Cast Iron.
	Lbs.	Lbs.	Lbs.	Lbs.		Lbs.	Lbs.	Lbs.	Lbs.
¼	.026	.021	.019	.017	5¼	30.1	24.1	21.5	19.8
½	.068	.070	.063	.058	½	34.7	27.7	24.7	22.7
1.	.209	.167	.148	.136	¾	39.6	31.7	28.3	25.9
¼	.408	.325	.290	.266	6.	45.0	36.0	32.0	29.4
½	.705	.562	.501	.460	½	57.2	45.8	40.8	37.4
¾	1.12	.893	.795	.731	7.	71.5	57.2	50.9	46.8
2.	1.67	1.33	1.19	1.07	½	88.0	70.3	62.6	57.5
¼	2.38	1.90	1.69	1.55	8.	106.	85.3	76.0	69.8
½	3.25	2.60	2.32	2.13	½	127.	102.	91.2	83.7
¾	4.34	3.47	3.09	2.83	9.	151.	121.	108.	99.4
3.	5.63	4.50	4.01	3.68	½	178.	143.	127.	117.
¼	7.15	5.72	5.10	4.68	10.	208.	167.	148.	136.
½	8.94	7.14	6.36	5.85	½	241.	193.	172.	158.
¾	11.0	8.79	7.83	7.19	11.	277.	222.	198.	182.
4.	13.4	10.7	9.50	8.73	½	317.	253.	226.	207.
¼	16.0	12.8	11.4	10.5	12.	360.	288.	257.	236.
½	18.9	15.2	13.5	12.4					
¾	22.7	17.9	15.9	14.6					
5.	26.0	20.8	18.6	17.0					

The weights of balls are as the cubes of their diams.

LEAD PIPES; weight of, per Foot run.

Bore, or inner diam. in inches.	THICKNESS OF METAL, IN INCHES.											
	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1 inch.
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
¼	.305	.724	1.28	1.95	2.74	3.65	4.53	5.84	8.52	11.7	15.3	19.5
5-16	.366	.845	1.47	2.20	3.05	4.02	4.96	6.33	9.14	12.4	16.2	20.5
⅜	.427	.967	1.65	2.44	3.35	4.38	5.39	6.82	9.76	13.2	17.0	21.5
7-16	.488	1.09	1.83	2.69	3.66	4.75	5.82	7.31	10.4	13.9	17.9	22.4
½	.548	1.21	2.01	2.93	3.96	5.11	6.24	7.79	11.0	14.6	18.7	23.4
⅝	.670	1.46	2.38	3.42	4.57	5.85	7.10	8.77	12.2	16.1	20.4	25.4
¾	.791	1.70	2.74	3.90	5.18	6.58	7.96	9.75	13.4	17.6	22.1	27.3
⅞	.911	1.95	3.11	4.39	5.79	7.31	8.82	10.7	14.6	19.1	23.9	29.3
1.	1.03	2.19	3.47	4.88	6.40	8.04	9.67	11.7	15.8	20.5	25.6	31.2
⅛	1.16	2.44	3.84	5.37	7.01	8.77	10.5	12.7	17.1	22.0	27.3	33.2
¼	1.28	2.69	4.21	5.85	7.62	9.50	11.4	13.7	18.3	23.4	29.0	35.1
⅜	1.40	2.94	4.58	6.34	8.23	10.3	12.3	14.7	19.5	24.9	30.7	37.1
½	1.52	3.18	4.94	6.83	8.84	11.0	13.1	15.6	20.7	26.3	32.4	39.0
⅝	1.64	3.43	5.31	7.32	9.47	11.7	14.0	16.6	22.0	27.8	34.1	41.0
¾	1.76	3.67	5.67	7.81	10.1	12.4	14.8	17.6	23.2	29.3	35.8	42.9
⅞	1.89	3.92	6.04	8.30	10.7	13.2	15.7	18.6	24.4	30.8	37.6	44.9
2.	2.01	4.16	6.40	8.78	11.3	13.9	16.5	19.5	25.6	32.2	39.3	46.8
¼	2.25	4.65	7.13	9.76	12.5	15.4	18.2	21.5	28.1	35.1	42.7	50.7
½	2.49	5.14	7.86	10.7	13.7	16.8	20.0	23.4	30.5	38.0	46.1	54.6
¾	2.73	5.63	8.59	11.7	14.9	18.3	21.7	25.4	32.9	41.0	49.5	58.5
3.	2.98	6.12	9.32	12.7	16.1	19.7	23.4	27.3	35.4	43.9	52.9	62.4
¼	3.22	6.61	10.1	13.7	17.4	21.2	25.1	29.3	37.8	46.8	56.4	66.4
½	3.46	7.10	10.8	14.6	18.6	22.7	26.8	31.3	40.3	49.7	59.8	70.3
¾	3.71	7.59	11.5	15.6	19.8	24.1	28.5	33.2	42.7	52.7	63.2	74.2
4.	3.95	8.08	12.2	16.6	21.0	25.6	30.2	35.2	45.2	55.6	66.6	78.1
¼	4.19	8.57	13.0	17.6	22.2	27.1	32.0	37.1	47.6	58.5	70.0	82.0
½	4.43	9.06	13.7	18.5	23.5	28.5	33.7	39.1	50.0	61.4	73.5	85.9
¾	4.68	9.55	14.4	19.5	24.7	30.0	35.4	41.0	52.5	64.4	76.9	89.8
5.	4.92	10.0	15.2	20.5	25.9	31.4	37.1	43.0	54.9	67.3	80.4	

Lead service pipes for single dwellings are usually from ½ to ¾ inch bore; scant $\frac{3}{16}$ inch and weigh 2 to 2.2 lbs per foot run. In Philada, 1873, about 10 cts per lb. Tin-lined, 15 cts. a large scale by Messrs *Tatham & Brothers*, 226 S. Fifth St, Philada.

***To which it is held by copper cleats**; as at Fig y, next page. Price *roofing copper in 1873, about 45 cts*; and copper nails 50 cts per lb.

BRAZED COPPER PIPES; weight of, per Foot run.

Thickness in Inches.	INNER DIAMETER IN INCHES.													
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3½	4	4½	5	6
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
1-16	.800	.989	1.18	1.37	1.56	1.75	1.93	2.12	2.31	2.69	3.07	3.45	3.82	4.58
3-32	1.24	1.52	1.80	2.08	2.36	2.64	2.92	3.20	3.48	4.05	4.61	5.17	5.73	6.85
¼	1.70	2.08	2.46	2.84	3.22	3.61	3.99	4.37	4.75	5.51	6.27	7.03	7.80	9.32
3-16	2.70	3.27	3.84	4.41	4.98	5.55	6.12	6.69	7.26	8.40	9.54	10.7	11.8	14.1
⅜	3.79	4.55	5.31	6.07	6.83	7.59	8.35	9.11	9.87	11.4	12.9	14.4	16.0	19.0
5-16	4.97	5.92	6.86	7.81	8.76	9.70	10.6	11.6	12.5	14.4	16.3	18.2	20.1	23.9

TIN AND ZINC.

The pure metal is called block tin. When perfectly pure, (which it rarely is, being purposely adulterated, frequently to a large proportion, with the cheaper metals lead or zinc,) its sp grav is 7.29; and its weight per cub ft is 455 lbs. It is sufficiently malleable to be beaten into tin foil, only $\frac{1}{1000}$ of an inch thick. Its tensile strength is but about 4600 lbs per sq inch; or about 7000 lbs when made into wire. It melts at the moderate temperature of 442° Fah. Pure block tin is not used for common building purposes; but thin plates of sheet iron, covered with it on both sides, constitute the *tinned plates*, or, as they are called, the *tin*, used for covering roofs, rain pipes, and many domestic utensils. For roofs it is laid on boards.

The sheets of tin are united as shown in this fig. First, several sheets are joined together in the shop, end for end, as at *tt*; by being first bent over, then hammered flat, and then soldered. These are then formed into a roll to be carried to the roof; a roll being long enough to reach from the peak to the eaves. Different rolls being spread up and down the roof, are then united along their sides by simply being bent as at *a* and *s*, by a tool for that purpose. The roofers call the bending at *s* a *double groove*, or *double lock;* and the more simple ones at *t*, a *single groove*, or lock.

To hold the tin securely to the sheeting boards, pieces of the tin 3 or 4 ins long, by 2 ins wide, called cleats, are nailed to the boards at about every 18 ins along the joints of the rolls that are to be united, and are bent over with the double groove *s*. This will be understood from *y*, where the middle piece is the cleat, before being bent over. The nails should be 4-penny slating nails, which have broader heads than common ones. As they are not exposed to the weather, they may be of plain iron.

Much use is made of what is called **leaded tin,** or **tarns,** for roofing. It is simply sheet-iron coated with lead, instead of the more costly metal tin. It is not as durable as the tinned sheets, but is somewhat cheaper.

The best plates, both for tinning and for tarns, are made of charcoal iron; which, being tough, bears bending better. Coke is used for cheaper plates, but inferior as regards bending. In giving orders, it is important to specify whether charcoal plates or coke ones are required;* also whether *tinned* plates, or *tarns.*

There are also in use for roofing, certain compound metals which resist tarnish better than either lead, tin, or zinc; but which are so fusible as to be liable to be melted by large burning cinders falling on the roof from a neighboring conflagration.

A roof covered with tin or other metal should, if possible, slope not much less than five degrees, or about an inch, to a foot; and at the eaves there should be a sudden fall into the rain-gutter, to prevent rain from backing up so as to overtop the double-groove joint *s*, and thus causing leaks. Where *coal is used for fuel, tin roofs* should receive two coats of paint when first put up, and a coat at every

* *Prices in Philada. 1873:* Coke and charcoal iron, tinned plates, $15 to $25 per box; according to *weight and quality*, and tarns from $9 to $16.

fter. Where wood only is used, this is not necessary; and a tin roof, with a good pitch, 30 years.

on plates are sold by the box. These boxes, unlike glass, have *not* equal ntents. They may be designated or ordered either by their names or ıy makers, however, have their private brands in addition; and some of a much higher reputation than others.

ntents of a box of either tinned plates or tarns.

Names.	No. in a box.	Sizes. Ins.	Wt. of a box. Lbs.	Marks on the boxes.
No. 1	225	13¾ × 10	112	CI
No. 2	"	13¼ × 9¾	105	CII
No. 3	"	12¾ × 9½	98	CIII
1	"	13¼ × 10	140	XI
s *No.* 1	"	"	161	XXI
ıss *No.* 1	"	"	182	XXXI
ıs *No.* 1	"	"	203	XXXXI
Doubles	100	16¾ × 12¼	98	CD
ibles	"	"	126	XD
s *Doubles*	"	"	147	XXD
ıss *Doubles*	"	"	182	XXXD
ıs *Doubles*	"	"	203	XXXXD
Small Doubles	200	15 × 11	168	CSD
all Doubles	"	"	188	XSD
s *Doubles*	"	"	209	XXSD
ıss *Doubles*	"	"	230	XXXSD
ıs *Doubles*	"	"	251	XXXXSD
r *Wastes, Common No.* 1	225	13¾ × 10	112	WCI
'ross *No.* 1	"	"	140	WXI
	112	14 × 20	112	
	112	14 × 20	140*	

ırger size may be made to special order; those of tinned iron, in England; but leaded ie in Philada also, and elsewhere. Sheets of 36 ins by 33, 39, 45, 48, 52, or 58, on hand. up to 3 ft by 7 ft, and up to No. 26 gauge, (.018 inch.) made to order.
5 sheets of 13¾ by 10, contains 214.84 sq ft; but, allowing for overlapping, it will cover sq ft of roof; even without any allowance for the waste which occurs in cutting away der to fit at angles, &c.
area of roof covered by any sheet, first deduct 2 ins from its width, and 1 inch from its

n sheets, and laid in the same manner as slates, is much used in some parts of ofing. By exposure to the weather, it soon becomes covered by a thin film of white protects it from further injury, and renders the roof very durable.† Corrugated sheet ed. See Galvanized Sheet Iron, page 370.

r IN ZINC VESSELS is said to become injurious to health; and recently an outcry has on ırisen against galvanized-iron service-pipes in dwellings. Yet such have been in use rs in New England, Philada, and elsewhere, without as yet any deleterious effects. bly owing to the fact that service-pipes being short, the water is usually all drawn several times a day; and hence does not remain in contact with the zinc or lead long uire a poisonous character. In taking possession of a house in which the water has ;nant in the service-pipes for some considerable time, such water should all be run to vise sickness may ensue from its use.

other sizes, as 10 × 20, &c.

ırice of sheet zinc does not ordinarily differ much from that of sheet lead, ada in 1873 was about 10 to 12 cts per ℔; or in pigs, from 7 to 9 cts.

ice of block tin, made into either pipes or sheets, about 56 to 60 cts per ℔. n & Bros, 226 South Fifth St, make both. In bars 40 to 45 cts, in 1873.

Table of Wire Rope, manufactured by John A. Roebling' Sons, Trenton, N. J. Prices in 1873, 10 per ct more than table.

Rope of 133 Wires.

Trade Number.	Circumference in Inches.	Price per Foot, in cents.	Ultimate Strength in tons of 2000 lbs.	Circumference of Hemp Rope of equivalent strength, in inches.
1	6¾	1 20	74 00	15½
2	6	1 05	65 00	14¾
3	5¼	91	54 00	13
4	5	78	43 60	12
5	4⅝	65	35 00	10¾
6	4	53	27 20	9½
7	3½	41	20 20	8
8	3⅛	34	16 00	7
9	2¾	28	11 40	6
10	2¼	25	8 64	5
10¼	2	24	5 13	4½
10½	1⅝	23	4 27	4
10¾	1½	22	3 48	3¾

Tiller Rope, ⅝ in diam, 26 cts.

Ropes from No. 8 to No. 10¾ are specially adapted for hoisting-rope.

Steel Rope of all sizes made to order.

All kinds of shackles, sockets, swivel hooks, and fastenings, put on, and splices made.

Rope of 49 Wires.

Trade Number.	Circumference in Inches.	Price per Foot, in cents.	Ultimate Strength in tons of 2000 lbs.	Circumference of Hemp Rope of equivalent strength, in inches.
11	4⅝	54	36 00	10¾
12	4¼	47	30 00	10
13	3¾	41	25 00	9½
14	3⅜	35	20 00	8¼
15	3	29	16 00	7¼
16	2⅝	23	12 30	6¼
17	2⅜	18	8 80	5½
18	2¼	15	7 60	5
19	1⅞	13	5 80	4¾
20	1⅝	11	4 09	4
21	1⅜	9	2 83	3¼
22	1¼	8	2 13	2¾
23	1⅛	7	1 65	2½
24	1	6½	1 38	2¼
25	⅞	6	1 03	2
26	¾	5½	0 81	1¾
27	⅝	5	0 56	1½
27½	½	4		
28		3	Large Sash Cord.	
29		2	Small " "	

Copper Rope, corresponding to the above sizes, made to order.

Notes on the use of Wire Rope, by Mr. Roebling.

"Two kinds of Wire Rope are manufactured; the larger sizes, as also the most pliable, are compo of 133 wires, and are generally used for hoisting or running rope. Those of 49 wires are stiffer, are better adapted for standing rope, guys, and rigging. Orders should state the use of Rope, advice will be given. Ropes up to 3 inches diameter are made upon special application.

For safe working load, allow 1-5 to 1-7 of ultimate strength, according to speed and vibration. W substituting Wire Rope for hemp rope, it is good economy to allow for the former the same rate foot run which experience has approved of for the latter.

Wire Rope is as pliable as new hemp rope of the same strength; the former will therefore run the same sized sheaves and pulleys which are used for the latter. But the greater the diamete the sheaves, pulleys, or drums, the longer Wire Rope will last. In the construction of machinery Wire Rope it will be found good economy to make the drums and sheaves as large as possible. size of drum is as follows: The same figure which expresses the circum in inches in the sec column of the table is also the minimum diam of drum in feet; doubling that figure will give the m imum. The diameter of drum should be no less than the minimum, nor is it necessary to exceed maximum. As an example, take a No. 4 rope, circumference 5 inches; therefore the minimum di eter of drum is 5 feet; and the maximum 10 feet. Or a No. 10¼ rope, circumference 2 inches; th fore minimum diam is 2 feet; and maximum 4 feet. A smaller diameter of drum may answer, the short bending will result in a much more rapid wear. In most cases the Rope will wear twic long on a max diam as on a minimum.

Experience has also demonstrated that the wear increases with the speed. It is better to incre the load than the speed.

Wire Rope is manufactured either with a wire or hemp centre. The latter is more pliable than former, and will wear better where there is short bending. Orders should specify.

Wire Rope must *not* be coiled or uncoiled like hemp rope. When mounted on a reel the latter sh be turned on a spindle to pay off the rope. When forwarded in a coil without reel, roll it over ground like a wheel, and run off the rope in that way. All untwisting must be avoided.

To preserve Wire Rope apply raw linseed oil with a piece of sheepskin, wool inside; or mix th *with equal parts of* Spanish brown and lampblack.

To preserve Wire Rope under water or under ground, take mineral or vegetable tar, add 1 bu *of fresh slacked lime to* 1 barrel of tar, (which will neutralize the acid,) and boil it well, then satu *the rope with the boiling tar.*

The grooves of cast-iron pulleys and sheaves should be filled with well-seasoned blocks of *wood, set on end, to be* renewed when worn out. This end wood will save the rope and increa *beaion. The small pulleys* or rollers which support the ropes on inclined planes should be cons *on the same plan.* When large sheaves run with a very great velocity, the grooves must be line

with leather set on end; with cork; or with India rubber. This is done in the case of all sheaves used in the *transmission of power* between distant points by means of ropes; which frequently run at the rate of 4000 feet per minute. Full information given, if desired, on the size of rope, and the size and speed of sheaves to be used for transmitting power. Rope ¾ ins diam will transmit 100 horse power to a great dist.

Notes on Rigging.

"Wire Rope for shrouds and stays is now universally superseding hemp rope, for the following reasons: it is much cheaper than hemp rope; it is more durable, and it will not stretch permanently under great strains, as is the case with hemp rigging, thus saving much labor in setting up; and it is fully as elastic as hemp rope of equivalent size. The rope with 49 wires is best adapted for rigging.

The great economy in using wire in place of hemp rigging is the large reduction in size and weight. The bulk of wire rigging is only one-sixth that of hemp, while the weight is only one-half. The advantages of lightness are apparent at a glance to every seaman; the removal of several tons of weight from the height occupied by the standing rigging must increase both the steadiness and the stability of the ship. Again, on account of its smaller bulk, less resistance is offered to the wind, a fact fully appreciated by captains of steamers, when running against the wind.

For protection from rust, wire rigging is galvanized; when not galvanized, apply good paint with a brush or a piece of sheepskin, once or twice a year, and a good set of wire shrouds and stays will outlast the best built sailing ship or steamer. The best and cheapest paint is linseed oil mixed with equal parts of lampblack and Spanish brown or Venetian red, or any other preparation of oxide of iron.

All vessels in the U S Navy are now rigged with Roebling's Wire Rope exclusively, it having proved the best in the test made by the Government at the Washington Navy Yard."

The foregoing is a copy of a circular by Messrs Roebling.

On inclined planes in Schuylkill Co. &c, a wire rope generally lasts long enough to raise one million of tons of coal up a plane half a mile long, and rising 1 in 16. The ordinary duration on inclined planes throughout the country is from 1½ to 4 years, according to the amount of service; and also greatly to the care taken of them, and of the sheaves and rollers upon which they move.

On the Mt Pisgah plane, 2500 ft long, rising 600 ft, for raising empty coal-cars, and lowering loaded ones, thin iron bands, 7 inches wide, and about ¼ inch thick, have been used instead of ropes. They scarcely exhibit any sign of wear in several years. They should be riveted; being apt to break if welded. Steel would probably be the best material in many cases.

WEIGHT AND STRENGTH OF IRON CHAINS.

The links of ordinary iron chains are usually made as short as is consistent with easy play, in order that they may not become bent when wound around drums, sheaves, &c; and that they may be more easily handled in slinging large blocks of stone, &c.

When so made, their weight per foot run is quite approximately 3½ times that of a single bar of the round iron of which they are composed. Since each link consists of two thicknesses of bar, it might be supposed that a chain would possess about double the strength of a single bar; but the strength of the bar becomes reduced about $\frac{3}{10}$, by being formed into links; so that the chain really has but about $\frac{7}{10}$ of the strength of two bars. As a thick bar of iron will not sustain as heavy a load in proportion as a thinner one, so of course, stout chains are proportionably weaker than slighter ones. In the following table, 20 tons *per sq inch*, is assumed as the average breaking strain of a single straight bar of ordinary rolled iron, 1 inch in diam; or 1 inch square; 19 tons, from 1 to 2 ins; and 18 tons, from 2 to 3 ins. Deducting $\frac{3}{10}$ from each of these, we have as the breaking strain of the two bars composing each link, as follows: 14 tons *per sq inch*, up to 1 inch diam; 13.3 tons, from 1 to 2 ins; and 12.6 tons, from 2 to 3 ins diam; and upon these assumptions the table is based.*

Table of strength of chains.

Chains of superior iron will require ¼ to ⅓ more to break them. (Original.)

Diam of rod of which the links are made.	Weight of chain per ft run.	Breaking strain of the chain.		Diam of rod of which the links are made.	Weight of chain, per ft run.	Breaking strain of the chain.	
Ins.	Pds.	Pds.	Tons.	Ins.	Pds.	Pds.	Tons.
3-16	.325	1731	.773	1	9.26	49280	22.00
¼	.579	3069	1.37	1⅛	11.7	59226	26.44
5-16	.904	4794	2.14	1¼	14.5	73114	32.64
⅜	1.30	6922	3.09	1 3/16	17.5	88301	39.42
7-16	1.78	9408	4.20	1½	20.8	105280	47.00
½	2.31	12320	5.50	1⅝	24.4	123514	55.14
9-16	2.93	15590	6.96	1¾	28.4	143293	63.97
⅝	3.62	19219	8.58	1⅞	32.6	164505	73.44
11-16	4.38	23274	10.39	2	37.0	187152	83.55
¾	5.21	27687	12.36	2¼	46.9	224448	100.2
13-16	6.11	32301	14.42	2½	57.9	274088	122.7
⅞	*7.10*	*37632*	*16.80*	2¾	70.0	335928	149.7
15-16	*8.14*	*43277*	*19.32*	3	83.3	398944	178.1

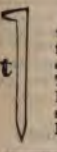
Price in 1873 of chains made by H. J. Mitchell & Co. Philada Chain Works, about 10 cts [illegible] *⅝; 2¼ for ⅝; 6 for ¾; 5½ for 1⅛.* Office S. W. corner Walnut and Fourth Sts.

Table of Manilla rope.

Diam. Ins.	Circ. Ins.	Wt per foot. lbs.	Breaking load. Tons.	Breaking load. lbs.	Diam. Ins.	Circ. Ins.	Wt per foot. lbs.	Breaking loa Tons.	Breaking loa lbs.
.239	¾	.019	.25	560	1.91	6	1.19	11.4	2553
.318	1	.033	.35	784	2.07	6½	1.39	13.0	2912
.477	1½	.074	.70	1568	2.23	7	1.62	14.6	3270
.636	2	.132	1.21	2733	2.39	7½	1.86	16.2	3628
.795	2½	.206	1.92	4278	2.55	8	2.11	17.8	3987
.955	3	.297	2.73	6115	2.86	9	2.67	21.0	4704
1.11	3½	.404	3.81	8534	3.18	10	3.30	24.2	5420
1.27	4	.528	5.16	11558	3.50	11	3.99	27.4	6137
1.43	4½	.668	6.60	14784	3.82	12	4.75	30.6	6854
1.59	5	.825	8.20	18368	4.14	13	5.58	33.8	7571
1.75	5½	.998	9.80	21952	4.45	14	6.47	37.0	8288

The strength of Manilla ropes, like that of bar iron, is very varial and so with hemp ones. The above table supposes an average quality. Rope good *Italian* hemp are considerably stronger than Manilla; but their cost excl them from general use. **The tarring of ropes** is said to lessen their stren and, when exposed to the weather, their durability also. We believe that the us it in standing rigging is partly to diminish contraction and expansion by alter wet and dry weather. **The common rules** for finding the strength of by multiplying the square of the diam or circumf by a given coefficient are enti erroneous. **Prices** in Philada, in 1873, Manilla 17 to 18 cts per lb; Italian he 25 cts; American hemp, 15 cts; Sisel hemp, 16 cts; jute (E. Indies,) 10 cts.

WEIGHT OF RAILROAD SPIKES.*



The hook-headed spikes *t*, commonly used for confining rail the cross-ties, vary within the limits of the following table; the lightest for light rails on short local branches; and the heaviest ones for heavy on first-class roads. The table is from the Phœnix Iron Co of Philadelp The spikes are sold in kegs usually of 150 lbs. For the weight of spike larger dimensions, we may near enough take that of a square bar of same length. What is saved at the point, suffices for the addition at head.

Size in ins. Length.	Side.	No. per keg of 150 lbs.	No. per lb	Size in ins. Length.	Side.	No. per keg of 150 lbs.	No. per lb.
4½ ×	7/16	526	3.5	5½ ×	½	350	2.33
4½ ×	½	400	2.66	5½ ×	9/16	289	1.93
5 ×	⅝	705	4.7	5½ ×	⅝	218	1.46
5 ×	7/16	488	3.25	6 ×	⅝	310	2.07
5 ×	½	390	2.6	6 ×	9/16	262	1.75
5 ×	9/16	295	1.97	6 ×	⅝	196	1.30
5 ×	⅝	257	1.71				

A size in very common use, is 5½ × 9/16; which weighs about ½ lb spike. A mile of single-track road, with 2112 cross-ties, 2½ feet apart from ce *to center;* and with rails of the ordinary length of 24 feet, or 10 ties to a rail; t

* *The price of spikes,* and of cut nails, in Philada, in 1873, about 5 cts per lb. Rivets 16 cts.

ng 440 rail-joints per mile; with 4 spikes to each tie, except at the rail-joints, at of which there will be 4 spikes;* will require at a neat calculation 9328 spikes.

an allowance must be made for rail guards at road-crossings, which we may assume to be 24 ft or the length of a rail. A guard will usually consist of 4 extra rails for protecting the track- and spiked to the 11 ties by which said track rails are sustained. Consequently, such a crossing es 11 × 6 = 88 spikes. For turnouts, sidings, loss, &c, we may roughly average 584† spikes per mile; thus making in all if we assume one road-crossing per mile, 9328 + 88 + 584 = 10000 per mile; or 5000 lbs, or 33⅓ kegs of 150 lbs.

dhesion of spikes. Professor W. R. Johnson found that a plain spike or ⅜ inch square, driven 3⅜ ins into seasoned Jersey yellow pine, or unseasoned nut, required about 2000 lbs force to extract it; from seasoned white oak, about and from well-seasoned locust, about 6000 lbs. Bevan found that a 6-penny driven one inch, required the following forces to extract it: Seasoned beech, 667 oak, 507; elm, 327; pine, 187.

ecent careful experiments in Hanover, Germany,

GINSER FUNK, give from 2465 to 3940 lbs. (mean of many experiments, about 3000 the force necessary to extract a plain ½ inch square iron spike, 6 ins long, wedge- d for one inch, (twice the thickness of the spike,) and driven 4½ ins into *white or pins.* When driven 5 ins, the force reqd was about $\frac{1}{10}$ part greater. Similar $\frac{9}{16}$ inch square, 7 ins long, driven 6 ins deep, reqd from 3700 to 6745 lbs to ex- them from pine; the mean of the results being 4873 lbs. In all cases, *about twice as force was reqd to extract them from oak.* The spikes were all driven *across* the of the wood. Experience shows that when driven *with* the grain, spikes or nails hold with much more than half as much force.

ed spikes, or twisted ones, (like an auger,) or those which were either swelled or diminished he middle of their length, all proved inferior to plain square ones. When the length of the wedge was increased to 4 times the thickness of the spike, the resistance to drawing out was a trifle less. n the length of the spike is fixed, there is probably no better shape than the plain square cross- , with a wedge point twice as long as the width of the spike, as per this fig.

ards of oak or pine, nailed together by from 4 to 16 tenpenny n cut nails, and then pulled apart in a direction lengthwise of the boards, and across the nails, g to break the latter in two by a shearing action, averaged about 300 to 400 lbs per nail to sepa- em; as the result of many trials.

WEIGHT OF NAILS.‡

Name.	Length. Inches.	No. per lb.	Name.	Length. Inches.	No. per lb.
3 penny.	1	557	8 penny.	2½	101
4 "	1¼	353	10 "	2¾	68
5 "	1¾	232	12 "	3	54
6 "	2	175	20 "	3½	34
7 "	2¼	141			

sizes and weights vary considerably with different makers. The above are machine-made, or AILS; in distinction to the WROUGHT NAILS made by the blacksmith.

SPECIFIC GRAVITY.

E sp grav of a body, is its weight as compared with that of an equal bulk of some r body, which is adopted as a standard of comparison. For other substances than nd gases generally, pure water is the usual standard; and since the weight of a n bulk of water varies somewhat with its temperature; and also with the state e air, the former is assumed to be 60° Fah; and the latter at 30 ins, at sea-level. where extreme scientific accuracy is not aimed at, all these considerations may eglected; and any clear fresh water, at any ordinary temperature, say from 60° °, may be used. At 60° pure water weighs 62.331 lbs avoir per cub ft.

o find the sp grav of a body, heavier than water. Weigh it in the air; and then in water; and find the diff. The diff is what the body loses ater: and is the weight of a bulk of water equal to the bulk of the body. Then s this, Diff : wt in air : 1 :: sp grav of body.

his supposes the joint and chair to rest upon a tie: but when long chairs are used with a view cing the rail-joint between two ties laid near each other, there will be 8 spikes to a joint; or per mile *more than above; equal to* 880 lbs; making in all, per mile single track, say 12000 *, or 6000 lbs, or 40 kegs.*

ds allows that turnouts and sidings amount to about 1 mile of extra track on 15 miles of road ces *in Philada, 1873, about 5 cts* per lb. Roofing nails of tinned iron, 12 cts. Copper na

The weight of a given bulk of a substance which is either porous, or absorbent of water, cannot be inferred from its sp gr. Thus pure river sand, is pure quartz; and of course has the same sp gr; yet, a solid cub ft of quartz, weighs nearly twice as much as a cub ft of sand; on account of the interstices of the latter. A brick, some sandstones, &c. absorb water; so that their sp gr will not furnish the weight of a dry mass of the same. In such cases, the engineer will generally first measure the contents of a piece of the substance, if a solid; and then weigh it; thus ascertaining its weight per cub ft, &c. If it is in grains, or dust, he will measure, and then weigh, a cub ft of it.

To find the sp grav of a liquid. First carefully weigh some solid body, as a piece of metal, in the air. Then weigh it in water, and note the loss, say L. Then weigh it in the other liquid; and note the loss, say *l*. Then as loss L, is to loss *l*, so is 1, or the sp gr of water, to the sp gr of the liquid. Or, if the sp gr, and weight of the solid body, are already known, merely weigh it in the liquid. Then as its weight in air, is to its loss in the liquid, so is its sp gr, to that of the liquid.

Timber, when first purchased from lumber yards, even under shelter, is rarely, if ever, perfectly dry; but its weight, if tolerably seasoned, will be about ¼ part greater than given in our tables, or about ¼ to ⅓ part, if green.

Table of specific gravities, and weights.

In this table, the sp gr of air, and gases also, are compared with that of water, instead of that of air; which last is usual.

Names of Substances.	Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
Air, atmospheric; at 60° Fah, and under the pressure of one atmosphere or 14.7 lbs per sq inch, weighs $\frac{1}{815}$ part as much as water at 60°	.00123	.0765
Alcohol, pure	.793	49.43
" of commerce	.834	52.1
" proof spirit	.916	57.2
Ash, perfectly dry ... average	.752	47.
1000 ft board measure weighs 1.748 tons.		
Ash, American white, dry ... "	.61	38.
1000 ft board measure weighs 1.414 tons.		
Alabaster, falsely so called; but really Marbles	2.7	168.
" real; a compact white plaster of Paris ... average	2.31	144.
Aluminium	2.6	162.
Antimony, cast, 6.66 to 6.74 ... average	6.70	418.
" native ... "	6.67	416.
Anthracite, 1.3 to 1.84. Of Penn'a, 1.3 to 1.7 ... usually	1.5	93.5
A cubic yard solid, averages about 1.75 cub yds, when broken to any market size; and loose.		
Anthracite, broken, of any size. Loose ... average	.....	52 to 56
" " moderately shaken ... "	.....	56 to 60
" heaped bushel, loose, 77 to 83.		
A ton, loose, averages from 40 to 43 cub ft ... "		
at 54 lbs per cub ft, a cub yard weighs .651 ton.		
Asphaltum, 1 to 1.8 ... "	1.4	87.3
Basalt. See Limestones ... "	2.9	181.
Bath Stone, Oolite ... "	2.1	131.
Bismuth, cast. Also native ... "	9.74	607.
Bitumen, solid. See Asphaltum.		
Brass, (Copper and Zinc,) cast, 7.8 to 8.4 ... "	8.1	504.
" rolled ... "	8.4	524.
Bronze. Copper 8 parts; Tin 1. (Gun metal.) 8.4 to 8.6 ... "	8.5	529.
Brick, best pressed	.....	150.
" common hard	.....	125.
" soft, inferior	.....	100.
Brickwork. See Masonry.		
Boxwood, dry ... "	.96	60
Calcite, transparent ... "	2.722	169.9
Carbonic Acid Gas, is 1½ times as heavy as air ... "	.00187	
Charcoal, of pines and oaks ... "	.....	15 to 30
Chalk, 2.2 to 2.8. See Limestones ... "	2.5	156.
Clay, potter's, dry, 1.8 to 2.1 ... "	1.9	119.
" dry, in lump, loose ... "	.....	63.
Coke, loose, of good coal ... "	.....	23 to 32
" a heaped bushel, loose, 35 to 42 lbs ... "		
" a ton occupies 80 to 97 cub ft ... "		
In coking, coals swell from 25 to 50 per cent.		
Equal weights of coke and coal, evaporate about equal wts of water; and each abt twice as much as the same wt of dry wood.		
Corundum, pure, 3.8 to 4	3.9	
Cherry, perfectly dry ... average	.672	42.
1000 ft board measure weighs 1.562 tons.		
Coal, bituminous, 1.2 to 1.5 ... "	1.35	84.
" " broken, of any size; loose ... "	.....	47 to 52
" " moderately shaken ... "	.....	51 to 56
" " a heaped bushel, loose, 70 to 78 lbs.		
" " a ton occupies 43 to 48 cub ft.		
A cubic yard solid, averages about 1.75 yards when broken to any *market size*, and loose.		

Table of specific gravities, and weights — (Continued.)

Names of Substances.	Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
Chestnut, perfectly dry ..... average ..	.66	41.
1000 board measure weighs 1.525 tons.		
Cement, hydraulic. American, Rosendale; ground, loose ..... average ..	.....	56.
" " " " U.S. Struck bush, 70 lbs		
" " " Louisville, " " 62	.....	49.6
" " " Copley, " " 67	.....	53.6
" " English Portland, U.S. struck bush, by Gillmore, 100 to 128	.....	81 to 102
" " " " Various, weighed by writer, 95 to 102	.....	76 to 81.6
" " " " a barrel 400 to 430 lbs.		
" " French Boulogne Portland, struck bush, 95 to 110	.....	76 to 88
Differences of 4 or 5 pounds either more or less than we here give per loose struck U.S. bush, often occur in the cement from the same manufactory, owing not only to the difficulty of measuring exactly, but to the want of uniformity in the composition of the stone, degree of burning, grinding, dryness, &c. Moreover, the term "loose" is indefinite. We mean by it the average looseness which it has when thrown by a scoop into a half bushel when measuring that quantity for sale. By shaking it may easily be compacted about ⅛ part, so as to weigh ⅛ more per bush, or cub ft. And by ramming, about ¼ part, so as to weigh about ⅓ more. So with lime, plaster, &c.		
Copper, cast ..... 8.6 to 8.8	8.7	542.
" rolled ..... 8.7 to 8.9	8.8	548.
Crystal, pure Quartz. See Quartz.		
Cork	.25	15.6
Diamond, 3.44 to 3.55; usually 3.51 to 3.55	3.53	
Earth; common loam, perfectly dry, loose	.....	72 to 80
" " " " " shaken	.....	82 to 92
" " " " " moderately rammed	.....	90 to 100
" " " slightly moist, loose	.....	70 to 76
" " " more moist, "	.....	66 to 68
" " " " shaken	.....	75 to 90
" " " " moderately packed	.....	90 to 100
" " " as a soft flowing mud	.....	104 to 112
" " " as a soft mud, well pressed into a box	.....	110 to 120
Ether	.716	44.6
Elm, perfectly dry ..... average ..	.56	35.
1000 ft board measure weighs 1.302 tons.		
Ebony, dry ..... " ..	1.22	76.1
Emerald, 2.63 to 2.76 ..... " ..	2.7	
Fat ..... " ..	.93	58.
Flint ..... " ..	2.6	162.
Feldspar, 2.5 to 2.8 ..... " ..	2.65	166.
Garnet, 3.5 to 4.3; Precious, 4.1 to 4.3 ..... " ..	4.2	
Glass, 2.5 to 3.45 ..... " ..	2.98	186.
" common window ..... " ..	2.52	157.
" Millville, New Jersey. Thick flooring glass ..... " ..	2.53	158.
Granite, 2.62 to 2.76. See Limestone ..... " ..	2.69	168.
Gneiss, common, 2.62 to 2.76 ..... " ..	2.69	168.
" in loose piles ..... " ..	.....	96.
" Hornblendic ..... " ..	2.8	175.
" " quarried, in loose piles ..... " ..	.....	100.
Gypsum, Plaster of Paris, 2.24 to 2.30 ..... " ..	2.27	141.6
" in irregular lumps ..... " ..	.....	82.
" ground, loose, per struck bushel, 70 ..... " ..	.....	56.
" " well shaken, " " 80 ..... " ..	.....	64.
" " Calcined, loose, per struck bush, 65 to 75 ..... " ..	.....	52 to 60
Greenstone, trap, 2.8 to 3.2 ..... " ..	3.	187.
" " quarried, in loose piles ..... " ..		107.
Gravel, about the same as sand, which see.		
Gold, cast, pure, or 24 carat ..... " ..	19.258	1204.
" native, pure, 19.3 to 19.34 ..... " ..	19.32	1206.
" " frequently containing silver, 15.6 to 19.3 ..... " ..		
" pure, hammered, 19.4 to 19.6 ..... " ..	19.5	1217.
Gutta Percha ..... " ..	.98	61.1
Hornblende, black, 3.1 to 3.4 ..... " ..	3.25	203.
Hydrogen Gas, is 14½ times lighter than air; and 16 times lighter than oxygen ..... average ..	.....	.00527
Hemlock, perfectly dry ..... " ..	.4	25.
1000 feet board measure weighs .930 ton.		
Hickory, perfectly dry ..... " ..	.85	53.
1000 feet board measure weighs 1.971 tons.		
Iron, *cast, 6.9 to 7.4* ..... " ..	7.15	446.
" " *usually assumed at* ..... " ..	7.21	450.

At 450 lbs, a cub inch weighs .2604 lb; 3601.6 cub inches a ton; and a lb = 3.8400 cub ins.

Table of specific gravities, and weights—(Continued.)

Names of Substances.	Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
Iron, wrought, 7.6 to 7.9; the purest has the greatest sp gr.....average..	7.77	485.
" large rolled bars..... " ..	7.6	474.
" " usually assumed at..... " ..	7.69	480.
" sheet..... " ..	.....	485.
At 480 lbs, a cub inch weighs .2778 lb; and a lb = 3.6000 cub ins. Light iron indicates impurity.		
Ivory.....average..	1.82	114.
Ice..... " ..	.94	58.7
India rubber..... " ..	.93	58.
Lignum vitæ, dry..... " ..	1.33	83.
Lard..... " ..	.95	59.3
Lead, 11.35 to 11.47..... " ..	11.41	711.
Limestones and Marbles, 2.65 to 2.85.....	2.75	172.
" " " ordinarily about.....	2.7	168.
" " " quarried in irregular fragments, 1 cub yard solid, makes about 1.9 cub yds perfectly loose; or about 1¾ yds piled. In this last case, .571 of the pile is solid; and the remaining .429 part of it is voids.....piled..	.....	96.
Lime, quick, of ordinary limestone and marbles 92 to 98 lbs per cub ft....	1.5	93.
" " either in small irregular lumps; or ground, loose 50 to 58....	.....	53.
In either case 1 *solid* measure makes about 1.8 meas loose; and then .555 of the mass is solid, and .445 is voids. To measure correctly, none of the lumps should exceed about ¼ or $\frac{1}{10}$ of the smallest dimension of the vessel used for measuring.		
Lime, quick, ground, loose, per struck bushel 62 to 70 lbs.....	.....	53.
" " " well shaken, " " ....80 "	.....	64.
" " " thoroughly shaken, " ....93¾ "	.....	75.
Mahogany, Spanish, dry *.....average..	.85	53.
" Honduras, dry..... " ..	.56	35.
Maple, dry..... " ..	.79	49.
Marbles, see Limestones.		
Masonry, of granite or limestones, well dressed throughout.....	.....	165.
" " " well-scabbled mortar rubble. About $\frac{1}{5}$ of the mass will be mortar.....	.....	154.
" " " well-scabbled dry rubble.....	.....	138.
" " " roughly scabbled mortar rubble. About ¼ to ⅓ part will be mortar.....	.....	150.
" " " roughly scabbled dry rubble.....	.....	125.
At 155 lbs per cub ft, a cub yard weighs 1.868 tons; and 14.45 cub ft, 1 ton.		
Masonry of sandstone; about ¼ part less than the foregoing.		
" " brickwork, pressed brick, fine joints .....average..	.....	140.
" " " medium quality ..... " ..	.....	125.
" " " coarse; inferior soft bricks ..... " ..	.....	100.
At 125 lbs per cub ft, a cub yard weighs 1.607 tons; and 17.92 cub ft, 1 ton.		
Mercury, at 32° Fah.....	13.62	849.
" 60° "	13.58	846.
" 212° "	13.38	836.
Mica, 2.75 to 3.1	2.93	183.
Mortar, hardened, 1.4 to 1.9.....	1.65	103.
Mud, dry, close.....	.....	80 to 110
" wet, moderately pressed.....	.....	110 to 130
" wet, fluid	.....	104 to 120
Naphtha.....	.848	52.9
Nitrogen Gas is about $\frac{1}{35}$ part lighter than air.....	.....	.0744
Oak, live, perfectly dry, .88 to 1.02 * .....average..	.95	59.3
" white, " " .78 to .88..... " ..	.83	51.8
" red, black, &c..... " ..	.....	32 to 45
Oils, whale; olive..... " ..	.92	57.3
" of turpentine ..... " ..	.87	54.3
Oolites, or Roestones, 1.9 to 2.5..... " ..	2.2	137.
Oxygen Gas, a little more than $\frac{1}{10}$ part heavier than air.....	.00136	.0848
Petroleum.....	.878	54.8
Peat, dry, unpressed.....	.....	20 to 30
Pine, white, perfectly dry, .35 to .45 *.....	.40	25.
1000 ft board measure weighs .930 ton.		
" *yellow, Northern*, .48 to .62.....	.55	34.3
1000 ft board measure weighs 1.276 tons.		
" " *Southern*, .64 to .80.....	.72	45.
1000 ft board measure weighs 1.674 tons.		

* *Green timbers* usually weigh from one-fifth to nearly one-half more than dry.

le of specific gravities, and weights—(Continued.)

Names of Substances,	Average Sp Gr.	Average Wt of a Cub Ft. Lbs.
long-leafed Southern yellow, unseasoned	1.04	65.
1000 ft board measure weighs 2.418 tons.		
.....	1.15	71.7
is; see Gypsum.		
ly shaken	1.	62.3
5 to 2.8	2.73	1.79
..... 21 to 22	21.5	1342.
e, in grains..... 16 to 19	17.5	
on, pure..... 2.64 to 2.67	2.65	165.
finely pulverized, loose		90.
" " well shaken		105.
" " well packed		112.
d, loose. One measure solid, makes full 1¾ broken and led		94.
phire, 3.8 to 4.0	3.9	
.....	1.1	68.6
er struck bushel; Syracuse, N. York..... 56 lbs		45.
" " " Turk's Island; Cadiz; Lisbon. 76 to 80		62.
" " " St. Barts ..... 84 to 90		70.
" " " some well-dried West India.... 90 to 96		74.
" " " Liverpool ..... 50 to 55		42.
l fine, for table use..... 60 to 62		49.
quartz, perfectly dried, and loose, usually 112 to 133 lbs per ushel	2.65	90 to 106*
erage of 98 lbs per cub ft, a struck bushel weighs 122½ lbs; 9 bushels, 1 ton; a cub yd = 1.181 tons; 22.86 cub ft, 1 ton.		
ken. 123 to 147 lbs per struck bushel		99 to 117
ked		101 to 119
wet, but the excess of water drained off		120 to 140
per cub foot, perfectly wet, 17.23 cub ft weigh 1 ton; and a = 1.567 tons.		
fine, even-grained sand, perfectly dry, may weigh as little 80 lbs per cub ft.		
inary pure natural sand be sifted into 2 or 3 or more parcels ently sized grains, a measure of any of these parcels will onsiderably less than an equal measure of the original sand. sand weighing 98 lbs per cub foot, may give others weighing e than 70 to 80 lbs. At 98 lbs per cub ft. 1 bulk of pure quartz, le 1.68 bulks of sand; of which the solid occupies .6; and the . But if this same sand be compacted to 110 lbs per cub ft. measure of solid quartz makes 1½ measures of sand; of which olid, and ½ voids. Sand is very retentive of moisture; and large bulks, is rarely as dry as that above in this table. But natural moisture, and loose, it is lighter than when dry, its weight then not exceeding about 85 to 90 lbs per cub ft; or 112½ lbs per struck bushel.		
t for building, dry, 2.1 to 2.73	2.41	150.
uarried, and piled, 1 measure solid, makes about 1¾ piled	.....	86.
ood..... 2.5 to 2.65	2.6	162.
ilien		5 to 12
ed, and compacted by rain		15 to 50
fectly dry	.59	37.
1000 ft board measure weighs 1.376 tons.		
black..... 2.4 to 2.8 ..... average	2.6	162.
ed, in piles ..... "	.....	92.
..... 2.7 to 2.9 ..... "	2.8	175.
..... "	10.5	655.
Steatite..... 2.65 to 2.8 ..... "	2.78	170.
9. The heaviest contains least carbon ..... "	7.85	490.
ot heavier than the iron from which it is made; unless the i impurities which were expelled during its conversion into		
..... average	2.	125.
tly dry ..... "	.4	25.
1000 ft board measure weighs .930 ton.		
ic..... 6.8 to 7.2 ..... "	7.00	437.5
d Ruby, 3.8 to 4 ..... "	3.9	
..... "	.94	58.6
..... "	1.	62.4
t, 2.8 to 3.2 ..... "	3.	187.
l; in piles ..... "	.....	107.
3.65 ..... "	3.55	

er cub foot, .57 of the mass is solid quartz; and .43 is void; or 1 measure of solid quartz sures of loose rounded sand. On voids see note p 504.

Table of specific gravities, and weights — (Continued.)

Names of Substances.		Average Sp Gr.	A W C
Tin, cast, 7.2 to 7.5	average..	7.35	4
Turf, or Peat, dry, unpressed		.....	3
Water, pure rain, or distilled, at 32° Fah. Barom 30 ins.		.....	62
" " " " " 60° " " "		1.	62
" " " " " 80° " " "		.....	62
At 60°, a cub inch weighs .03607 lb; or .57712 oz avoir. And a lb contains 27.724 cub ins; equal to a cube of 3.0263 inches on each edge.			
Water, sea, 1.026 to 1.030	average ..	1.028	64
Although the weight of fresh water is in practice almost invariably assumed as 62½ lbs per cub ft, yet 62¼ would be nearer the truth, at ordinary temperatures of about 70°; or a lb = 27.759 cub ins; and a cub in = .5764 oz avoir; or .4323 oz troy; or 252.175 grains. The grain is the same in troy, avoir, and apoth.			
Wax, bees	average ..	.97	60
Wines, .993 to 1.04	" ..	.998	62
Walnut, black, perfectly dry	" ..	.61	38
1000 ft board measure weighs 1.414 tons.			
Zinc, or Spelter, 6.8 to 7.2	" ..	7.00	437
Zircon, 4.0 to 4.9	" ..	4.45	

Table of grades per mile; or per 100 feet measured zontally.

Grade in ft. per mile.	Grade in ft. per 100 ft.	Grade in ft. per mile.	Grade in ft. per 100 ft.	Grade in ft. per mile.	Grade in ft. per 100 ft.	Grade in ft. per mile	G i pe
1	.01894	39	.73864	77	1.45833	115	2
2	.03788	40	.75758	78	1.47727	116	2
3	.05682	41	.77652	79	1.49621	117	2
4	.07576	42	.79545	80	1.51515	118	2
5	.09470	43	.81439	81	1.53409	119	2
6	.11364	44	.83333	82	1.55303	120	2
7	.13258	45	.85227	83	1.57197	121	2
8	.15152	46	.87121	84	1.59091	122	2
9	.17045	47	.89015	85	1.60985	123	2
10	.18939	48	.90909	86	1.62879	124	2
11	.20833	49	.92803	87	1.64773	125	2
12	.22727	50	.94697	88	1.66666	126	2
13	.24621	51	.96591	89	1.68561	127	2
14	.26515	52	.98485	90	1.70455	128	2
15	.28409	53	1.00379	91	1.72348	129	2
16	.30303	54	1.02273	92	1.74242	130	2
17	.32197	55	1.04167	93	1.76136	131	2
18	.34091	56	1.06061	94	1.78030	132	2
19	.35985	57	1.07955	95	1.79924	133	2
20	.37879	58	1.09848	96	1.81818	134	2
21	.39773	59	1.11742	97	1.83712	135	2
22	.41667	60	1.13636	98	1.85606	136	2
23	.43561	61	1.15530	99	1.87500	137	2
24	.45455	62	1.17424	100	1.89394	138	2
25	.47348	63	1.19318	101	1.91288	139	2
26	.49242	64	1.21212	102	1.93182	140	2
27	.51136	65	1.23106	103	1.95076	141	2
28	.53030	66	1.25000	104	1.96969	142	2
29	.54924	67	1.26894	105	1.98864	143	
30	.56818	68	1.28788	106	2.00758	144	
31	.58712	69	1.30682	107	2.02652	145	
32	.60606	70	1.32576	108	2.04545	146	
33	.62500	71	1.34470	109	2.06439	147	
34	.64394	72	1.36364	110	2.08333	148	
35	.66288	73	1.38258	111	2.10227	149	
36	.68182	74	1.40152	112	2.12121	150	
37	.70076	75	1.42045	113	2.14015	151	
38	.71970	76	1.43939	114	2.15909	152	

Remark on preceding Table.

If the grade per mile should consist of feet *and tenths*, add to the grade per 100 feet in the foregoing table, that corresponding to the number of tenths taken from the table below; thus, for a grade of 81.7 feet per mile, we have .81439 + .01326 = .82765 feet per 100 feet.

Ft. per Mile.	Per 100 Feet.	Ft. per Mile.	Per 100 Feet.	Ft. per Mile.	Per 100 Feet.
.05	.00094	.4	.00758	.7	.01326
.1	.00189	.45	.00852	.75	.01420
.15	.00283	.5	.00947	.8	.01515
.2	.00379	.55	.01041	.85	.01609
.25	.00473	.6	.01136	.9	.01705
.3	.00568	.65	.01230	.95	.01799

Table of grades per mile, and per 100 feet measured horizontally, and corresponding to different angles of inclination.

Deg.	Min.	Feet per mile.	Feet per 100 ft.	Deg.	Min.	Feet per mile.	Feet per 100 ft.	Deg.	Min.	Feet per mile.	Feet per 100 ft.	Deg.	Min.	Feet per mile.	Feet per 100 ft.
0	1	1.536	.0291	0	45	69.11	1.3090	1	58	181.3	3.4341	3	26	316.8	5.9994
	2	3.072	.0582		46	70.64	1.3381	2	0	184.4	3.4924		28	319.8	6.0579
	3	4.608	.0873		47	72.18	1.3672		2	187.5	3.5506		30	322.9	6.1163
	4	6.144	.1164		48	73.72	1.3963		4	190.6	3.6087		32	326.0	6.1747
	5	7.680	.1455		49	75.26	1.4254		6	193.6	3.6669		34	329.1	6.2330
	6	9.216	.1746		50	76.80	1.4545		8	196.7	3.7250		36	332.2	6.2914
	7	10.75	.2037		51	78.33	1.4837		10	199.8	3.7833		38	335.3	6.3498
	8	12.29	.2328		52	79.87	1.5128		12	202.8	3.8416		40	338.4	6.4083
	9	13.82	.2619		53	81.40	1.5419		14	205.9	3.8999		42	341.4	6.4664
	10	15.36	.2909		54	82.94	1.5710		16	208.9	3.9581		44	344.5	6.5246
	11	16.90	.3200		55	84.47	1.6000		18	212.0	4.0163		46	347.6	6.5832
	12	18.43	.3491		56	86.01	1.6291		20	215.1	4.0746		48	350.7	6.6418
	13	19.96	.3782		57	87.54	1.6583		22	218.1	4.1329		50	353.8	6.7004
	14	21.50	.4073		58	89.08	1.6873		24	221.2	4.1911		52	356.8	6.7583
	15	23.04	.4364		59	90.62	1.7164		26	224.3	4.2494		54	359.9	6.8163
	16	24.58	.4655	1		92.16	1.7455		28	227.4	4.3076		56	363.0	6.8751
	17	26.11	.4946		2	95.23	1.8038		30	230.5	4.3659		58	366.1	6.9339
	18	27.64	.5237		4	98.30	1.8620		32	233.5	4.4242	4		369.2	6.9926
	19	29.17	.5528		6	101.4	1.9202		34	236.6	4.4826		5	376.9	7.1384
	20	30.72	.5818		8	104.5	1.9784		36	239.7	4.5409		10	384.6	7.2842
	21	32.26	.6109		10	107.5	2.0366		38	242.8	4.5993		15	392.3	7.4300
	22	33.80	.6400		12	110.6	2.0948		40	245.9	4.6576		20	400.1	7.5767
	23	35.33	.6691		14	113.6	2.1530		42	248.9	4.7159		25	407.8	7.7234
	24	36.86	.6982		16	116.7	2.2112		44	252.0	4.7742		30	415.5	7.8701
	25	38.40	.7273		18	119.8	2.2694		46	255.1	4.8325		35	423.2	8.0163
	26	39.94	.7564		20	122.9	2.3277		48	258.2	4.8908		40	431.0	8.1625
	27	41.47	.7855		22	126.0	2.3859		50	261.3	4.9492		45	438.7	8.3087
	28	43.01	.8146		24	129.1	2.4441		52	264.3	5.0075		50	446.5	8.4554
	29	44.54	.8436		26	132.1	2.5023		54	267.4	5.0658		55	454.2	8.6021
	30	46.08	.8727		28	135.2	2.5604		56	270.5	5.1241	5		461.9	8.7489
	31	47.62	.9018		30	138.3	2.6186		58	273.6	5.1824		5	469.6	8.8951
	32	49.16	.9309		32	141.3	2.6768	3		276.7	5.2407		10	477.4	9.0413
	33	50.69	.9600		34	144.4	2.7350		2	279.7	5.2990		15	485.1	9.1875
	34	52.23	.9891		36	147.4	2.7932		4	282.8	5.3573		20	492.9	9.3347
	35	53.76	1.0182		38	150.5	2.8514		6	285.9	5.4158		25	500.6	9.4819
	36	55.30	1.0472		40	153.6	2.9097		8	289.0	5.4742		30	508.4	9.6292
	37	56.83	1.0763		42	156.6	2.9679		10	292.1	5.5326		35	516.1	9.7755
	38	58.37	1.1054		44	159.7	3.0262		12	295.1	5.5909		40	523.9	9.9218
	39	59.90	1.1345		46	162.8	3.0844		14	298.2	5.6493		45	531.6	10.068
	40	61.44	1.1636		48	165.8	3.1427		16	301.3	5.7077		50	539.4	10.215
	41	62.97	1.1927		50	169.0	3.2010		18	304.4	5.7660		55	547.2	10.362
	42	64.51	1.2218		52	172.0	3.2592		20	307.5	5.8244	6		555.	10.510
	43	66.04	1.2509		54	175.1	3.3175		22	310.5	5.8827				
	44	67.57	1.2800		56	178.2	3.3758		24	313.6	5.9410				

An angle of 1° 58′, or about 1 in 35, or 151 ft per mile, is about the greatest slope that shou ren to a turnpike road, to allow horses to trot down rapidly with safety. In crossing mou s is often increased to 3, or even to 5°. It should never exceed 2½°, except when ab ssary.

TABLE OF ACRES REQUIRED per mile, and per 100 feet, for different widths.

Width. Feet.	Acres per Mile.	Acres per 100 Ft.	Width. Feet.	Acres per Mile.	Acres per 100 Ft.	Width. Feet.	Acres per Mile.	Acres per 100 Ft.	Width. Feet.	Acres per Mile.	Acres per 100 Ft.
1	.121	.002	26	3.15	.060	52	6.30	.119	78	9.45	.179
2	.242	.005	27	3.27	.062	53	6.42	.122	79	9.58	.181
3	.364	.007	28	3.39	.064	54	6.55	.124	80	9.70	.184
4	.485	.009	29	3.52	.067	55	6.67	.126	81	9.82	.186
5	.606	.011	30	3.64	.069	56	6.79	.129	82	9.94	.188
6	.727	.014	31	3.76	.071	57	6.91	.131	½	10.	.189
7	.848	.016	32	3.88	.073	¾	7.	.133	83	10.1	.190
8	.970	.018	33	4.00	.076	58	7.03	.133	84	10.2	.193
¼	1.	.019	34	4.12	.078	59	7.15	.135	85	10.3	.195
9	1.09	.021	35	4.24	.080	60	7.27	.138	86	10.4	.197
10	1.21	.023	36	4.36	.083	61	7.39	.040	87	10.5	.200
11	1.33	.025	37	4.48	.085	62	7.52	.142	88	10.7	.202
12	1.46	.028	38	4.61	.087	63	7.64	.145	89	10.8	.204
13	1.58	.030	39	4.73	.090	64	7.76	.147	90	10.9	.207
14	1.70	.032	40	4.85	.092	65	7.88	.149	¾	11.	.209
15	1.82	.034	41	4.97	.094	66	8.	.151	91	11.0	.209
16	1.94	.037	¼	5.	.094	67	8.12	.154	92	11.2	.211
½	2.	.038	42	5.09	.096	68	8.24	.156	93	11.3	.213
17	2.06	.039	43	5.21	.099	69	8.36	.158	94	11.4	.216
18	2.18	.041	44	5.33	.101	70	8.48	.161	95	11.5	.218
19	2.30	.044	45	5.45	.103	71	8.61	.163	96	11.6	.220
20	2.42	.046	46	5.58	.106	72	8.73	.165	97	11.8	.223
21	2.55	.048	47	5.70	.108	73	8.85	.168	98	11.9	.225
22	2.67	.051	48	5.82	.110	74	8.97	.170	99	12.	.227
23	2.79	.053	49	5.94	.112	¼	9.	.170	100	12.1	.230
24	2.91	.055	½	6.	.114	75	9.09	.172			
¾	3.	.057	50	6.06	.115	76	9.21	.174			
25	3.03	.057	51	6.18	.117	77	9.33	.177			

RAIL-JOINTS AND CHAIRS.

A railroad track being weakest at the joints between the rails, where they are deprived of their vertical strength, has of course a greater tendency to bend at those points; and this bending produces an irregularity in the movement of the train, which is detrimental to both rolling-stock and track. Moreover, that end of a rail upon which a loaded wheel is moving, bends more than the adjacent unloaded end of the next rail; so that when the wheel arrives at said second rail, it imparts to its end a severe blow, which injures it. Thus, the ends of the rails are exposed to far more injury than its other portions. Numerous devices have been resorted to for strengthening the joints of the rails, with a view of preventing this bending entirely; or, at least, of causing the two adjacent rail-ends to bend equally, and together; so as to avoid the blows alluded to. None of these joint-fastenings, known as chairs, fish-plates, wooden blocks, &c, have proved entirely satisfactory.

Much of the deficiency ascribed to the fastenings, is, however, really due to want of stability in the cross-ties at the joints; and more attention must be directed to this latter consideration, before an efficient fastening can be obtained. Observation shows that when the joint-ties are very firmly bedded, almost any of the ordinary fastenings will, (if the joint is placed *between* two ties, instead of resting *upon* a tie,)* answer very well; whereas, when the cross-ties are so insecurely bedded as to play up and down for half an inch or more under the driving-wheels of the engines, the strongest and most effective fastenings soon become comparatively inoperative. All the parts of the best of them will in that case become gradually loosened, warped, bent, or broken. This remark applies to all the fishing-splices, chairs, long wooden blocks, bolts, spikes, &c, in present use.

Experience has established the superiority of suspended joints, over supported *ones.* On a portion of a track carrying a heavy business, with joint-fastenings closely *resembling that in Figs* 10, (with long wooden blocks B; and long fish-pieces C, *alternate joints* were suspended between two ties; while the intermediate one rested *upon a center tie*; the blocks, however, extending over three ties. The ends of the *rails were more injured* by crushing and brooming in the latter than in the former. *And so with a number of other patterns of short joint-fastenings of wrought iron. Long fastenings,* perhaps, possess but little superiority over short ones, where *track is not kept in good repair*; for the great bearing of the former, although im-

* *In the first case the joint is called a* SUSPENDED ONE; *in the last a* SUPPORTED ONE.

ing increased firmness on a good track, becomes converted into a powerful leverage, by which it accelerates its own destruction, in a bad one. An element in the injury of joints, is the omission of proper fastenings at the center of the rails. Each rail should be so firmly attached to the cross-ties at and near its center, as to compel the contraction and expansion to take place equally from that point, toward each end.

It would probably be somewhat difficult to accomplish this perfectly. The attempts hitherto made have failed.

One of the earliest suggestions for a joint fastening, was the **fish-joint, or fish-splice, or fish-plates,** Fig 1, introduced upon the Newcastle and Frenchtown R.R. in Delaware, by Robt. H. Barr, in 1843.*

This consists of two strips of iron, *z z*, called *fishes*, rolled to fit the sides of the rail; and bolted together and to the rails, by either keybolts, or screwbolts. Usually 4 bolts are used; sometimes 6. The fishes are usually about ½ to ⅝ inch thick. The curved shape of the inner top and bottom edges of these splices; and of the top and bottom of the stem of the rail, creates a tendency to yield under the bending of the joint under heavy loads, as shown in an exaggerated manner at Figs 2 and 3. Either of these brings a great strain upon the bolt, and is apt to pull it apart. At a later period, Edward Miller, C E, of Philada, in order to remedy these defects, introduced the rolling of the rails with square shoulders, as in Fig 4.

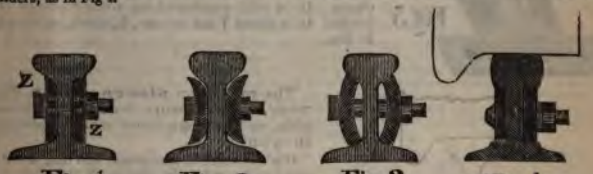


Fig 1 Fig 2 Fig 3 Fig 4

Fish-splices are frequently made with a shallow groove about ⅛ inch deep, on their outer side, as at *n n*, and F, Figs 10. This groove receives the square or oblong head of the bolt; and is intended to prevent the bolt itself from turning; and the nut at its other end from unscrewing itself; which it does under the action of passing trains, *even with this precaution.* See Lock-nut washers, p 375. The groove weakens the splice materially. Fish-plates from 16 to 24 ins long, and from ⅝ to ⅞ inch thick, slightly curved as in Fig 3, with 4 bolts, and suspended between ties about 12 ins apart in the clear, appear to be the favorite joint-fastening both in Europe and in the U S. In this country they are commonly used without chairs. The bolt-holes in the plates, or else in the rails, are about ¼ inch longer than high, to allow for expansion of the rails.

Under the extremes of temperature in the United States, bar iron expands or contracts about 1 part in 916; or 1 inch in 76½ feet; consequently, a rail 30 ft long will vary $\frac{7}{16}$ inch; and one 29 feet long fully ¼ inch.

Beside this, the rails are very liable to move or creep bodily in the direction of the heaviest trade; and by this process also the joint-fastenings are exposed to additional strain and derangement.

The patent **stop-chair, of Mr John A. Wilson, C E,** is said to prevent this. It consists simply of a piece of plate-iron, bent to fit to the fish-plate, and to the outer top part of the base of the rail. It is about 3 ins wide, by 7 ins long. Its lower end is held to the tie by two common rail-spikes; and its upper end is bolted to the outside of a fish-plate by one of the bolts which confine the fish to the rail. Two chairs are used at each joint; one at each end of a fish. Made by the Wharton Switch Co, Phila. Wt, 1 lb each.

All rails appear to become elongated very slightly at their ends by use; and this renders a full allowance for contraction and expansion the more necessary.

It is a remarkable fact, not satisfactorily accounted for, that when lengths of from 100 yards, to some miles, of rails have been perfectly welded, or riveted together tightly, and spiked to the ties as usual, no elongation or contraction by heat or cold could be detected.

What is called the compensating fish-joint† is in use on many roads.

Its novelty consists in a peculiar *cup-washer*, enclosing a ring of India-rubber 2 inches diam, and ⅝ inch thick; and placed under the nut of each of the four bolts. It is claimed that this prevents the nuts from unscrewing themselves; and diminishes the jar and noise of passing trains.

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* *The fish-joint is, we believe, claimed at present to be an English invention of 1846. It was in use on the N. C. and F. railroad three years prior to that date.*

Average price in Philada, in 1873, of wrought-iron chairs, about 4 to 5 cts per lb. Common fish-*plates the same. Bolts and nuts 6 to 7 cts.*

This and other fish-joints, are extensively manufactured by Verree & Mitchell, 939 North Beln-*ue, Philada. They also sell machines for drilling the bolt-holes in the rails.* Price per joint *e dimensions, with its cup-washers, &c,* about $1.50 to $2.50. With rails 24 ft long, a sin-*gle-track road will require 440 joints;* which, at $2 each, will cost $880. Ordinary ones *size, and perhaps not much inferior,* about half as much.

Each of the two fish-plates of each joint, is usually from 16 to 24 ins long, by ⅝ to ¾ ins thick. The weight of a complete joint, with its four ¾-inch bolts, cup-washers, &c, from 16 to 24 lbs. They are generally used without chairs.

Fig 4½

A new form of fish-plates, contrived by Mr. S. E. Pettiss, is shown at Fig 4½.

They are about 18 ins long. Near the bottom they are connected by two bolts *c*; and at top by four bolts *v*. These last have the Billings lock-nut washer *a*; for which see page 375. This joint seems to be much more effective than the common fish, against both vertical and lateral force. It is of course a suspended joint; and a quadrant-shaped piece is cut out from the ends of the vertical flanges *c c*, to prevent interference with the cross-tie. It has been tried on the Reading RR with good results, but not sufficient to displace the common fish-plates.


Fig 5

Fig 5 was an early form of supported wrought-iron chair. It is still employed on some roads of light traffic. It is about 7 ins square, ¾ thick, and weighs 10 lbs.

Fig 6.

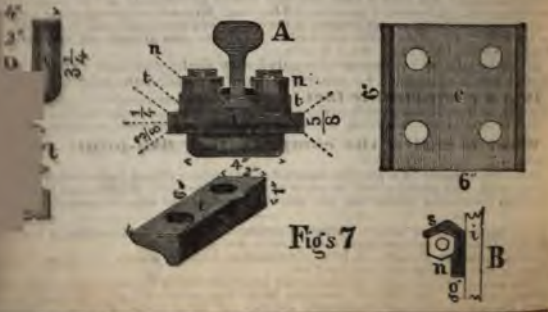
The rolled-iron **sleeve-chair**, Fig 6, made by the Phœnix Iron Co, of Philadelphia, and by some other establishments, is in extensive use.

It is first rolled in long pieces; and then sawed into such lengths as may be ordered. When supported, it is usually either 9 or 10 inches long; requiring 4 spikes; and when suspended, about 2 ft long; with 8 spikes. It is chiefly used as a supported chair; and as such, is a favorite on many roads of moderate traffic. Under heavy traffic, it is deficient in vertical strength, especially when suspended. The curved lips then frequently break off at the ends; and occasionally the chair breaks entirely across at the spike-holes. An objection to *long* chairs of this pattern, is the difficulty of sliding them upon the rails; and the still greater one of sliding them off, when either a chair, or a rail, is to be removed. See Fig 8.

The sleeve-chair, Fig 6, weighs 1½ lbs per inch of length; so that chairs of 9, 10, or 24 ins long, weigh 13½, 15, or 36 lbs. With rails 20, 24, or 30 feet long, there would in one mile of single-track road, be 528, 440, or 352 chairs.

The 9-inch ones weigh 7128, 5940, or 4752 lbs; the 10-inch, weigh 7920, 6600, or 5280 lbs; the 24-in, weigh 19008, 15840, or 12672 lbs.* But an addition should be made to these weights and costs, as 6 per cent, or $\frac{1}{16}$ part, or ¼ mile in every 5 miles, for turnouts, sidings, &c. Allowing four 7-pound spikes to a chair, there are 2112, 1760, 1408 chair-spikes; or 1056, 880, or 704 lbs per mile.

Figs 7 show **Fisher's wrought-iron rail-joint**, highly esteemed on the Lehigh Valley road; where it is extensively used under a heavy traffic of coal, mer-



Figs 7

* Price in Philada, 1873, 3 to 6 cts per lb.

chandise, and passengers. In this, the disadvantages before alluded to of the long sleeve-chair, are entirely got rid of. At A is shown a transverse section of the fastening, with the rail in its place: while the other portions of Figs 7 show its details separately. It consists of a simple rolled chair *c*, 6 ins square, and ⅜ inch thick; with two of its sides turned up ½ of an inch. In its base are 4 bolt-holes, through which are passed from below, two curved double screw-bolts, of round iron 1 inch diam; one of which is shown at D. Over the tops of these bolts are let down the two rolled pieces or bars *t t*, each as long as the chair, or 6 ins; and shaped to fit the top of the base of the rail. The nuts *n n* bind firmly together these bars *t t*, the rail, and the chair *c*. To prevent the nuts *n* (see Fig B) from unscrewing themselves, a small rod *s g*, of ⅜ inch square iron, notched on the side next to the nut, is inserted between the nut and the rail; one end of it, *s*, being bent to clasp the nut. In Fig A, the ends *g* of these rods are shown by 2 small black squares. The whole is suspended between two cross-ties, 7 inches apart in the clear; on each of which the rail is simply held by two spikes, as on the other ties; without any chair. The chairs *c*, when suspended, very seldom, if ever, break. Those of them first used on this road were but ¼ inch thick, and were supported *upon* the ties; only one double bolt being used instead of two. They frequently broke across the bolt-holes.

This excellent fastening was patented by Mr Mark Fisher, of the iron firm of Fisher & Norris, of Trenton, N Jersey. The same firm make also a longer chair, with 3 double bolts.

Notwithstanding the fact that the cross-ties are but from 7½ to 8 ft long, (gauge 4 ft 8½,) the road is a very pleasant one to ride upon: there is but little jolting, or rattling at the joints; and the ends of the rails appear to be as well protected as by any joint fastening we have seen. The track is well watched, and carefully kept in line; a precaution especially necessary in a road doing the heavy business of the Lehigh Valley R R. The success of this joint confirms our opinion that a great length of fastening, or of cross-tie, is not in itself essential to a good track.

The weight of this fastening complete is about 13½ lbs. With rails 20, 24, or 30 ft long, there would in one mile of single-track road, be 528, 440, or 352 chairs, weighing 7128, 5940, or 4752 lbs; or, allowing 5 per cent for turnouts and sidings, say 7484, 6237, or 4990 lbs.

A modification of the sleeve-chair. Fig 6, has been introduced upon the Hudson River R R, involving, to some extent, the same principle as that on the Lehigh Valley. It is shown at Fig 8.

Fig 8

As but one bar is here used, and as it is placed on the *outside* of the rail, its nuts are not in danger of being struck by the flanges of the wheel when a low rail is employed; or when the wheel tires become worn, thus bringing the flanges lower down. It is probable that a suspended chair of this kind, not more than 7 inches long, and with its lips considerably thicker than they are usually made, would equal the Lehigh Valley one in efficiency.

Figs 9 show a remarkable **suspended fastening, called the Ring-joint;** highly approved of at one time on the Camden & Amboy road, on which it was employed for many years, under a heavy traffic; to the almost entire exclusion of others, except for experimental comparison.

Figs 9

It was invented in 1851, by Edwin A Stevens, President of the road. The chief engineer, Col Cook, informed the writer that after several years' trial of a great variety of fastenings, he gave a decided preference to this. It has, however, one very serious defect: namely, that when, in consequence of bad foundation, the cross-ties play too freely up and down, the ends of the rails frequently split either at top, as shown by the line l; or at bottom, as shown by k. We have never learned, but that this was productive of accident to trains. When the break occurs above the slot we of course gives notice of the fact, and a new rail is put in; but when below the slot and its wedges continue to uphold the broken piece; at times probably for days before it is observed. Nearly all the fractures are below the slot.

This fastening consists of a simple welded triangular ring *t c a*, (in the end view; or *m* in the side view,) ½ an inch thick, and 3¼ ins wide. This ring passes through a slot *v v*, (see middle fig,) 4 ins long, cut into the adjacent rail-ends. Two cast-iron wedges *w w*, 6 to 8 ins long, of a shape to fit the ring and the rail, are inserted between them; and a thinner one *s s*, of plate iron, below the rail. The first are cast around a cylindrical rod of rolled iron, (*w* of the end view, *i i* of the side view,) about ⅜ inch diam; and a little longer than the wedges; for increasing their strength, and for preventing them from falling out from the chair in case they should break, which they sometimes do.

The joint is suspended between two cross-ties, 1 ft apart in the clear.

The writer examined it carefully for many consecutive years; and so far as regards protecting the ends of the rails from *wear*, and freedom from jolting, it certainly appears to him to be fully as effective as any other. He has seen joints laid alternately with it; with heavy cast-iron supported chairs; and with long splices somewhat similar to the one next to be described, but extending over 3 ties. With the exception of the breaking alluded to, the ring appeared to be decidedly superior to the supported chair; and by no means inferior to the long splice.

We have also repeatedly compared it with the heavy rolled joint-fastening Fig 13, which is 5 feet long, and rests on three cross-ties. Under similar conditions of road-bed, whether good or bad, we have been unable to perceive that the ring-joint (when the ties are but a foot apart in the clear) yielded, under heavy trains, any more than it.

The weight of a ring itself is 5½ lbs; the three wedges about 7½ lbs; total 13 lbs; or nearly the same as the Lehigh Valley fastening, Fig 7. The gauge of the C & A road is 4 ft, 10 ins; and the cross-ties average about 8½ ft long.

Without pretending to advocate this joint in its present shape, we have thought proper to describe it, as furnishing useful hints on this important subject. It was the first that led us to doubt whether some long fastenings did not furnish the elements of their own destruction, by the long leverage which they afforded for bending and breaking themselves to pieces, when the ties are badly supported. Since the death of Col Cook, a gradual substitution of long combined wood and iron fastenings, in place of the ring-joint, has been commenced; and is applied wherever the latter is found to have fractured a rail.

Figs 10 represent the combined suspended joint-fastening introduced upon the Philada & Reading railroad by **J. Dutton Steele,** C E.

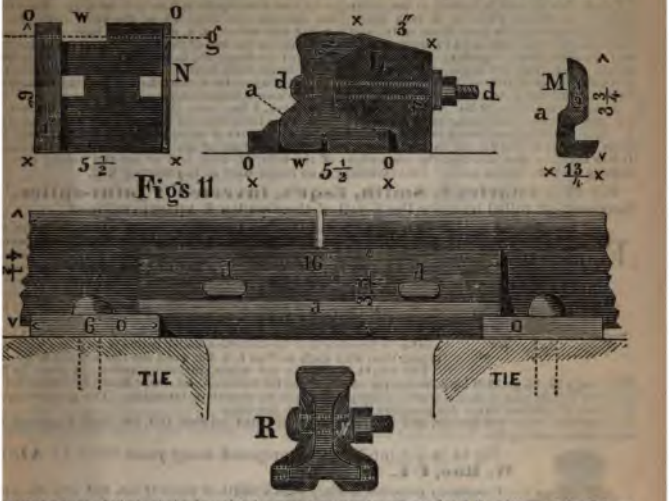
The very heavy tonnage of this road, exceeding perhaps that of any other in existence, demands particular attention to the joints; and Mr Steele's combination of Trimble's long wooden splice, with a long wrought-iron fish-splice, and a longer rolled-iron chair under the whole, certainly proved itself, for many years, superior to any other of the numerous cast and wrought iron chairs previously tried on that road. The joint is suspended between 2 cross-ties, placed 1 ft apart in the clear. B is a block (known as Trimble's splice) of oak, 3 by 3 ins, and 3 ft long, dressed on one side to fit the outside of the rail; and *c* is a rolled fish, 17 ins long, (shown more in detail at F,) placed on the *inside* of the rail. This fish and the oak block are bolted together, through the rail, by two ¾-inch screw-bolts *a a*, 13 inches apart. Under the rail is a rolled chair *d*, 2 ft 8 ins long, 5 ins wide, and ½ inch thick; turned up ¼ inch along each of its 2 sides, and fastened to the 2 wooden cross-ties by 4 hook-headed spikes, ½ inch square, by 5½ ins long. The heads of the two screw-bolts are made somewhat oblong, (about ¾ inch by 1¼,) for fitting into the groove *n n*, seen along one side of the fish; so as to prevent the tendency of the bolts to revolve under the action of the trains, and thus unscrew the nut at the other end. The nuts, however, unscrew themselves, notwithstanding this precaution. We consider 13 ins to be too far apart for the two screw-bolts, inasmuch as it allows the fish in time to become bulged out from the rail quite perceptibly, between the spikes, under the downward pressure on the ends of the rails. This impairs its efficiency. Four bolts would be better. The strain on the screw-bolts is great, both vert and hor; and it becomes greater as the wooden blocks in time lose (as they do) their close fit to the sides of the rails. The blocks then cease to act in perfect unison with the other parts of the fastening, in sustaining passing loads; and when the track is not kept in good order, the various parts may plainly be seen to yield and move in various directions, independently of each other. The bolt-nuts then loosen; and the fish pieces, long chairs, and long bolts, become bent; and sometimes split, or break entirely. The long wooden blocks B crush and decay soonest near where their tops are in contact with the rail. They, however, have an average life of 6 to 8 years, *upon roads kept in tolerable order.*

These remarks are more or less applicable to all joint-fastenings with long splices, when the ties are allowed to become unstable. With the care taken to prevent this on the Reading road, Mr Steele's *fastening has sustained the enormous traffic of the line very well for many years. It will, however, be supplanted by better ones more recently introduced.*

The iron in one of these joints, including bolts and spikes, weighs about 32 lbs; and with rails 20, 24, or 30 ft long, there would be 528, 440, or 352 joints; or 16896, 14080, or 11264 lbs, per mile single-track road.

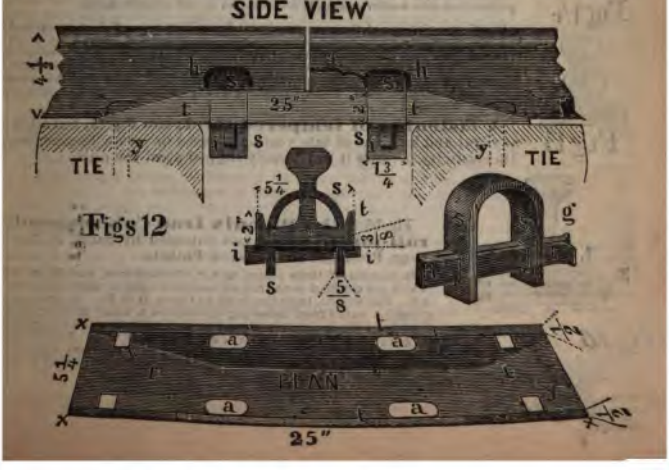
Figs 11 show a modification of the foregoing, in use on part of Pennsylvania Central. It has not proved a success.

L is a wooden block 2½ ft long, resting on two ties, 12 ins apart in the clear. M or *a* is a rolled fish. lice, 16 ins long, bolted to the rail by the same two bor bolts *d*, 1 inch diam, and 8½ ins apart. lich confine the wooden block to the rail. At each end of this fish, and spiked to a tie by two ½ ch square bolts, 5½ ins long, is a chair *o o*, 6 ins square, by ½ inch thick, shown in plan at N.



Figs 11

rt of the base of these chairs is omitted, as at *w*, to receive the ends of the bottom of the fish, as own at *w* in the transverse section L; which section is supposed to be made across the chair at the e *g g* in the plan N. That side of the base of the rails which is covered by the fish, is cut away for width of ¾ of an inch, and for a length equal to that of the fish. The fish fits into the notch 16½ hes long, thus formed in the base of the rails; which aids, as does also the notch *w* in the base of chairs, to keep the fish in position. By way of experimental comparison, the wooden block L has n omitted at some of the joints; and a second iron splicing-piece has been substituted for it, as n at R; the two end chairs being retained. The rails often separate an inch with both these joints.



Figs 12

Figs 12 show the **Lamborn and Justice suspended joi ening,** tried upon a portion of the Philada & Reading R R.

It consists of a rolled trough-chair *t t*, 25 ins long, 5¼ ins wide, and ⅝ inch thick; tur ins on each long side. The upper corners of these sides are cut off, as seen in the figs. its ends this trough is spiked to the cross-ties by two five-eighth spikes, *y*, 5 ins long. yokes, *s, s*, 1¾ ins wide, by ⅝ inch thick, pass through two holes *h h* in the stem of through 4 holes in the base of the rails; and through four others, *a, a, a, a*, in the base o Beneath the rails, split keys *i i* are passed through holes in the feet of the yoke; thus unit and the chair firmly together. These keys are about 7 ins long; 1¼ ins wide at one end, at the other; and are made of iron a scant ¾ inch thick, bent double. After they are two thicknesses of iron at the smaller end may be forced apart for a short dist, to prevent working out; see *g*, where a yoke and its key are shown on a larger scale.

A chair, *t t*, weighs 22 lbs; its two yokes, and two keys, 7 lbs; total, 29 lbs. Therefore, per mile for rails of diff lengths will be a little more than twice that for the Lehigh Valley 7. This fastening was found to be liable to occasional fractures of the end of a rail, as at a same cause as in Fig 9;) and rarely to a cracking across the base of the long chair, *t t*, at It was found in time also that the lower edges of the keys, *i i*, wore into or cut the yoke causing them to become slack. This fastening is no longer used on the Reading R R.

Fig 13 is **Charles E. Smith, Esqr's, inverted T joint-sp** fastening, of rolled iron, for the U rail of the Camden & Atlantic railroad.

Fig 13

It is 5 ft long, and rests on 3 cross-ties, one of which is rail-joint. Its base is 4¾ ins wide, by ½ inch thick. The 2 ins high, and 1 inch thick where it joins the base. This jo riveted loosely to the rails by 8 rivets in pairs; two pairs bei each two ties. Eight hook-headed spikes, 5½ inches long, square, in pairs, (2 pairs on the center tie,) confine both rails to the cross-ties. This splice keeps the rails in position ver it would be easier upon the ends of the rails if it rested on t as to suspend the rail-joint between two of them. The writ unable to perceive that the deflection under trains is any le splice, than with such as Figs 7, 9, &c, when the ties are no

The weight of iron in one splice, including spikes, is a With rails 20, 24, or 30 ft long, with 5 per ct allowance for turnouts, &c, there would be 3 or 24640 lbs per mile of single track; consequently it is an expensive fastening. The gre of spikes driven into three ties prevents the rails from creeping.

A SIMILAR JOINT, but shorter, and inverted, and suspended between ties, has been su edge rails; see Fig 17.


Fig 14

Fig 14 is a joint-fastening proposed many years since b **W. Rae, C E,** of Penna.

It certainly possesses merit. With a length of about 16 ins, and wit tion of a thin wedge, as well as of 2 more bolts, all below the rail, in th shown by the dotted lines, it would probably constitute as effective a joint-fastening as has yet been tried. There would be very little strain o If 16 inches long, and ½ inch thick, the weight per joint would be abou cluding 6 bolts, and a wedge beneath the base of the rail. As in all o some device should be used to prevent the nuts from unscrewing thems jolting of passing trains. See *Star Washer*, Fig 17; also, Billings wa The star washer being of very thin iron, becomes in time so brittle fro require renewal. The Billings washer occasionally breaks under great perfect lock-nut-washer of simple construction is still a desideratum.


Fig 15

Fig 15 was also one of the numerous joint-fastenings suggested at an but, like the foregoing, it never came into use. In lengths of about 8 i probably make an efficient fastening; especially with the addition of a wedge between the bottom of the rail and the foot of the chair. This w ish the difficulty of sliding the chairs on or off of the rail; and would t easy to employ larger ones; besides insuring a firm bearing for the bas upon the fastening.

A fastening of tempered steel, like Fig 15, bu any bolt and key, has of late years been used on some Engl Its elasticity permits it to be easily slipped upon the rails.

Fig 16

Fig 16 is **the Phœnix Iron Co's susp rail-joint,** devised and patented by Sam'l Esqr, President of the Co, of Philada.

It consists of three pieces, *a, b, c*, of rolled iron, each which are confined to the rail by two bolts; one of which *n n*. The weight, including bolts and nuts, is 35 lbs. To six-sided nuts t from loosening themselves, a small strip sheet iron, fits around them; and its ends are bent square and bottom of the piece *a*; or a star washer is used. T has a "stop," for preventing the ends of the rails from ther; but it is not only ineffective, but decidedly injur tire fastening.

17 is a fish-fastening, ved of on the Reading or a time, but not now used.

ds of the rails rest on a rolled chair as long; which has a bottom web *t*, 3 for 10 ins of its length; and tapering deep at each end. The two fishes *y y*, s long, are curved ¾ inch. They are by 4 bolts, *h g*, ¾ inch diam; the ads *h*, of which fit into a recess in the er the bolts are pushed through the he fishes and rails, a STAR-WASHER* *k*, $\frac{1}{20}$ inch) sheet iron, is laid over the s *g* of the two bolts on each side of The nuts *n* are then screwed down n this washer; and in whatever po- ey may step, some one or more of of the stars, coincide nearly with one of the sides of the nut, so as to admit bent up tight against them; thus pre- the nut from unscrewing itself. Four kes *o o*, with very coarse threads, and ng, fasten the chair *c c* to the cross- e weight of this fastening is:

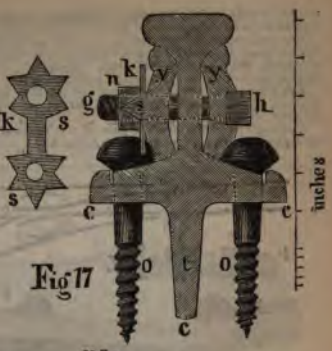


Fig 17

Chair c c	33 lbs.
2 fishes *y y*	14.5
4 bolts *h g*, and nuts	4.
4 screw-spikes *o*	3.
2 star-washers	0.1
Total	54.6 lbs.

fore remarked, much, if not most of the trouble with joint-fastenings, is due rfect foundation and drainage under the ties. No joint can possibly compen- these defects. Of all the joint fastenings in use, the writer would adopt either sh-plates, or the Fisher, Figs 7; placing the joint ties 9 ins apart in the clear.

TURNOUTS.

. 1. To enable an engine and train to pass from one track A B, Fig 1, to C D, a turnout is introduced.† This consists essentially of two lines of rails d *s y*, laid so as to form a short reversed curve for uniting the two tracks; of ovable rails *g o*, and *n s*, called **switch-rails**; a **frog**, *e*, (shown more ly by Figs 2 and 6;) and two **guide-rails**, *c c*, *a a*. The ends *g* and *n*, of tch-rails, form the fulcrum or center about which they move; and are called **eels**; and the ends *o* and *s*, at which the motion is greatest, are their **toes.** t *x o* reqd for this motion of the toe of each switch-rail, is called its **throw. ening.** It is shown more distinctly at Fig 5. It must plainly be equal at the width *a o* of the top of the rail, in addition to a width *x a*, sufficient to he flanges of the wheels on the track A B, to pass along readily between the and *w*. The tops of rails are generally between 2 and 2½ ins wide; while 1¾ to 2 ins will usually suffice for the flanges. The throw, *x o*, however, is nly about 5 ins; that is, the toes *o* and *s* are moved 5 ins from their original n. (shown by the dotted lines), when the train is intended to leave the track d to pass along the turnout, to the track C D. The motion is given by means **witch-lever,** which will be described by-and-by. The entire triangular Fig 2, is the **tongue** of the frog; and its sharp end is the **point.**

2 is a plan; and Figs 3 and 4 are side views of a frog as usually made, either of cast iron, or of steel, or of the two combined. Fig 6 is a frog, as sometimes y merely bending two pieces *f g*, and *h i*, of ordinary rails, for forming the **s**; and by cutting and bolting together two other pieces *c* and *c*, for the *tongue nt*. This, however, is rarely done, except as a temporary expedient. Should ermanence be required, stout iron chairs must be introduced at the cross-ties.

2, the entire frog was formerly made of cast iron; hardened by chilling; so as better to e *action of passing wheels*; but even with this precaution they wore out so much more *han the rails, that* the wings and tongue are now capped with the best steel. It is used in *s of from ¾ to 1 inch or more*; and is firmly bolted down to the cast-iron portion; besides *rwise secured to it in the best makes.* It is not necessary to steel the entire length of t

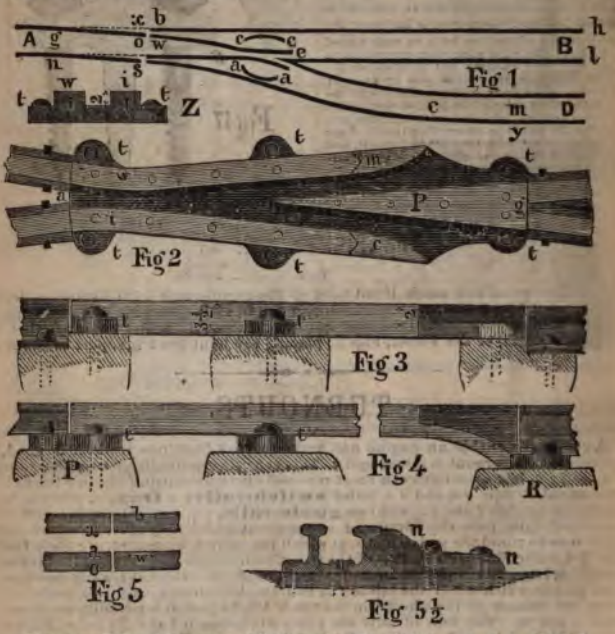
ion patent stop-washer, made at Coatesville, Penna; A. Gibbons, patentee.
business on the Pennsylvania Central increased, it was found expedient to increa
sidings to one mile.

wings; a very trifling economy is gained by leaving their ends *m* and *c* unprotected; inasmuch as the wheels do not run upon those parts. All the tongue P, however, should be steeled.

The projections *t t* are merely for bolting the frog down to the wooden cross-ties. The wings and tongue must evidently be raised above the bottom plate of the frog sufficiently to prevent the flanges of wheels from touching this last. About 1¾ to 2 ins will suffice, as shown at Fig 2; which is a transverse section of Fig 2, taken across *t i w t*.

The channel is called the MOUTH of the frog at *a*, Figs 2 and 8; and its THROAT, at the narrowest part *w i*, Fig 2; or *z z*, Fig 8. That part of the tongue which is back of *u*, Fig 8, or between *u* and *y*, is called its HEEL.

Although the frog forms a part of the turnout curve, still its shortness warrants us in making it straight from *f* to *i*; and from *s w t*, Fig 8.



To explain the use of the guide-rails, *c c*, *a a*, Fig 1.

Suppose wheels to be rolling from A toward B, Fig 1, on the main track; the switch-rails being in the dotted positions. On arriving opposite the frog, some irregularity of motion might cause the flanges of the wheels running along the rails *n i*, to press hard against said rails. Consequently, after passing the *throat w i* (Fig 2) of the frog, they would tend to press against the wing *i c c* and might, by that means, strike the sharp end of P: breaking it; and endangering the train. To prevent this, the guide-rail *c c* is placed so near the rails *g h*, Fig 1, (generally about 1¾ to 2 ins,) that the flanges at those rails, while passing between them and the guide-rail, not only prevent the flanges at the opposite rails from pressing against the wing *i c*; but guide them safely along their proper channel from *i* to *m*, Fig 2, without striking against the sharp end of the point.

In like manner, if the switch-rails be in the positions *g o*, *n s*, (or *switched*,) and if wheels be rolling along the turnout, Fig 1, from A toward D, when they arrive at the frog, the centrifugal force (being on a curve) would cause the flanges to press against the rails *w m*; and against the wing *w m*, Fig 2, of the *frog*; thus rendering liable the same kind of accidents as in the preceding case. This is prevented in the same manner as before, by the guide-rail a a; which keeps the flanges in their proper channel from *w* to *c*, Fig 2.

The narrow flange-way between the guide-rail c c, Fig 1, and the rail h h, need not extend further than from the end *a*, Fig 2, of the frog, to about one foot, toward *g*, from the sharp end of the point. In a distance of at least about 2 ft more at each of its ends, the guide-rail should flare out to allow ins from the rail *h h*; so as to guide the flanges into the narrower part of the flange-way. The rails with *a a*. Guide-rails should be very firmly confined to their wooden cross-ties; inasmuch as they have to resist a strong side pressure. This is usually done by bolting against them two or more blocks *n n*, Fig 5½, of wood, or cast iron; along the narrowest, or most strained portion of flange-way.

Fig 3 is a side view of a frog of uniform depth. This depth may be about 3¼ ins; namely, about as for the wings and tongue; and 1½ for the base plate.

In Fig 4, which is also a side view, the projections *t t*, the number of which will vary with the ngth of the frog, are cast in one piece with the frog; but are entirely below it. The end one at P, sses also under the end of the adjoining rails; thus forming a chair for them, as well as for the g. At the end R is shown a mode of proceeding when the rails are high; and are to rest upon the ds of the base of the frog, as in the fig. Whichever mode may be employed, it will of course be ed at both ends of the frog. Such details are pretty much a matter of whim; and vary with the tions of the designers.

Art. 2. Fig 7 is the plan of a SUBSTITUTE FOR A FROG. It is a bar, either of steel, or of cast on, 2 to 4 inches deep, with a steel *top* bolted to it. It is merely a short movable switch, which may made to accommodate either line of rails, by means of a switch-lever attached to it. It of course quires, like other switches, a person to attend to it; which the frog does not.

Fig 6

Fig 7

The length *a g*, Fig 8, of a frog, usually varies from 4 to 8 feet; and epends upon the obliquity at which the rails *w m*, Figs 1 and 8, cross the rails *n l*.

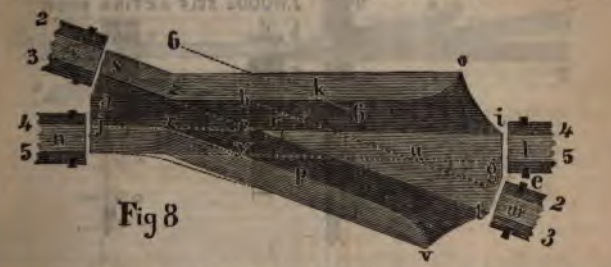


Fig 8

The switch rails *g o*, *n s*, Fig 1, when opened for the turnout, form the tangts, at the ends *o* and *s* of which the turnout curve *w m* commences. The angle at which this curve crosses the rails *n l*, Fig 1, determines how great must be the *frog-angle i c t*, Fig 8, of the frog; or, how many times the length *g c* of the tongue, or rather, how many times the length of a line from *c* to the center of a line from *t* to *i*, exceeds the breadth *t i*. The frog is called a No. 3, 4, 10, &c, according as this is 3, 4, or 10, &c, times greater than *t i*; thus, Fig 8 is a No. 3; and Fig 2, No. 5. They are usually made of Nos. 4 to 10; without any fractional numbers. Having first drawn two paral- *lel lines 22, 23, for the top of the* rails *w m*; and 4 4, 5 5, for the top of the rails n l, *rossing each other at the reqd* angle, or No, we get the theoretical point c of the *ng; and the dist from c to i* or *t*, need be no greater than will give a width *t i*, suff *nt to accommodate the ends of the* rails *l* and *m*; with room for the heads of *spikes at e, which confine them to their cross-ties.* Inasmuch as the theore *t c, would be too narrow and weak for service,* it is in practice termi *e the tongue is about ½ inch wide, as at r.* Draw *z k* parallel to *c*

[illegible] is about [illegible] feet or thereby sufficient to allow the ready passage of th[illegible] of the wheels. Also in the manner, these rails, parallel to $x\,t$. Now, the lengt[illegible] of the rail $y\,t$, or from x to t, need be no greater than will suffice for rounding off th[illegible] angle at x and t, so as to prevent their being struck by the flanges of wheel[illegible] The curve will answer the thing, thus giving the dotted curve $y\,y$, about one fo[illegible] The same will be done at x. If these three angles at y and x were left shar[illegible] it is apparent having shown that they become ground away by the flanges, they a[illegible] rounded off, and trains passing more gently in consequence.

[illegible] the length of the wing, [illegible] $x\,y$, and di[illegible] about six inches more than the entire width of the widest tires (probably 6[illegible] inches) on the track. There will be the least dist[illegible] from x, to which the wing shou[illegible] extend; from x to y, say not less than [illegible] inches, more, if the length of th[illegible] allows. In the three edge of the wing should be flared outward, (so as to guid[illegible] the wheel, the rest well has arrived beyond it). Inasmuch as the tracks of the whee[illegible] are run in the path $x\,y$. The same process and remarks apply to the wing $z\,v$.

The object in reducing the width of the channels

[illegible] the rail way from the point x to y, [illegible] the flanges freely, in to allow the trac[illegible] wheels to have a much bearing as possible upon the wings, and upon the narrow point of th[illegible] rails, leaving now the broader part of the dotted line $x\,y$ and z. In frogs shorter th[illegible] this, it is [illegible] difficult to secure a sufficient bearing for the tracks, even with the ston[illegible] of the channel, when the width of the tires on the road is, as usual, about 6 in[illegible] In the long frog, and frogs on railroads, the wheels do not run upon their tracks, while passi[illegible] the point of the frog, but upon their flanges. To effect this, that portion of th[illegible] channel was gradually widened, so that its depth below the top of the wings was [illegible] the depth of the flanges. In this manner, even in shorter possible frogs, or those in which th[illegible] rails cross each other at right angles, sufficient a satisfactory bearing for wheels passing along [illegible] This arrangement is still simpler of the well-constructed frogs, which are used when two trac[illegible] rails cross each other at right angles. A great objection to it, however, and one which exclude[illegible] it where circumstances necessitate its use, is that the flanges very soon cut gutters in the bottoms of th[illegible] channels, so as to make the tracks on the wings, and to jolt in crossing the channels. Sa[illegible] however, [illegible] very frequent occurrence.

Mr. J. Wood's self-acting frog. Fig. $8\frac{1}{2}$ was contrived with a view t[illegible] [illegible] this defect. It is made entirely of common T rails. The wings $a\,b\,c$, an[illegible] $w\,x$, are confined together by a bolt at x, which passes loosely through the tongue [illegible] as shown more fully in the cross-section p. Also by a clamp d; shown in cross[illegible] section at k. The ends rails are fast in the chairs, which is bolted to the cross-tie [illegible]

The wings thus united, are movable: the ends c and e being the center of motion; so that either [illegible] may be brought into contact with the tongue x, a very small play of the ends c and e, in the fra[illegible] which $c\,e$, being sufficient to admit of this. The clamp d moves with the wings; the motions of which are caused by the flanges of the wheels. The mode of action is this: Suppose the wings to be in the position shown by the fig. Then it is ready for a train moving from y toward k; or from k toward [illegible] and to either case the tracks of the wheels have a full bearing throughout the entire length of the frog; and there is no channel to be crossed. Again, if a wheel on $w\,x$ be moving toward i, its flange will push a b way from the tongue; and, consequently, will draw m a close up to it; so that self wheel will pass along the frog under the same circumstances as the preceding one did; and the frog will remain in position for the wheels which follow; or for others moving from i toward w. An iron plate o is bolted to the cross-tie, for the wings to slide on. Again, suppose the frog to be as in the fig[illegible] and an engine to be moving from i toward w. Now, in order to open the wing $c\,b\,a$, and to close the [illegible] wing $o\,m\,w$ up to the tongue $x\,a$, a guide-rail on the opposite line of the main track, and not shown in [illegible] drawn the wheels which are moving from i toward w, over against the wing $c\,b$, forcing it open; [illegible] engine closing $o\,m\,w$. But in this case, it will be observed that the motion of the wings has [illegible] While the weight of the engine itself is partly resting on the wing $m\,w$; and, consequently, [illegible] transversely of the track, is attended with great friction. This, however, does not in[illegible] to interfere with the efficiency of the frog. It has been in extensive use for many years [illegible] & Amboy R. R., where it is daily traversed in both directions by numerous trains.

high speed. When the frog is very long, one or two additional connecting clamps *d*, and slide-plates *g*, should be introduced. A little attention is necessary to prevent the accumulation of dirt and stones between the wings and the tongue. The whole should be of steel if possible. Of course, two guide-rails are necessary, as *c c*, *a a*, Fig 1.

Art. 3. With rails whose bases are, as usual, not more than 4 ins wide, the lengths of frogs which do not extend under the ends of the rails, need not be *greater* than the following, in ft and ins. But if they do so extend, an addition of from 8 to 12 ins must be made to the bottom of the frog for that purpose.

No. of Frog.	Length. Ft. Ins.	No. of Frog.	Length. Ft. Ins.	No. of Frog.	Length. Ft. Ins.
3	2 " 9	6	5 " 1	9	7 " 2
4	3 " 7	7	5 " 10	10	7 " 10
5	4 " 4	8	6 " 6	11	8 " 6

But we may shorten a frog considerably by cutting away a portion of the bases of the ends of the rails *l* and *m*, as shown by the line *g y*, Fig 9; so that the tops *l* and *m*, of the rails, shall come into contact with each other at *g*; thus diminishing the width necessary for *d*; and of course the distance *c g*. When this is done, the length will not *exceed* as follows, even with rails 2½ ins wide at the head.

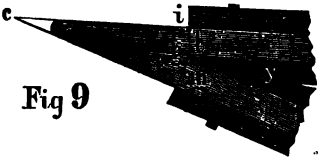


Fig 9

No. of Frog.	Length. Ft. Ins.	No. of Frog.	Length. Ft. Ins.	No. of Frog.	Length. Ft. Ins.
3	2 4	6	4 1	9	5 10
4	2 11	7	4 8	10	6 5
5	3 6	8	5 3	11	7 0

In long frogs especially, this reduction is advisable, not only on the score of economy; but of ease of handling; and less liability to be broken if the foundation becomes infirm.

Art. 4. The following table shows the **radii of the curves of turnouts.** When the turnout is to be traversed by passenger trains at ordinary travelling speed, the rad should not be *less* than about 800 ft. Nor should the length of each switch be less than about 20 feet. Short switches produce violent jolts at *g* and *n*, Fig 1.

The table gives three ordinates for laying out each of the turnout curves on the ground. These ords suppose the dist from the toe of the switch, to the theoretical point of the frog, to be divided into four equal parts; so that one ord is at the center of this dist, and the two equal side-ords at half-way between it and the ends of said dist.

It is not necessary that the frog should have precisely the angle given in the last column, in order

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The making of frogs has become one of the specialties of the day. In ordering them, it is not necessary to furnish more than the frog-angle, or No; and an exact cross-section of the rail used on the road. The same frog will answer for turning out either to the right hand, or to the left.

Excellent ones, called the Mansfield frog, are made by the American Railway Frog Co, at Boston, Mass. The wings and tongues are of the best steel; 1¾ ins thick; and 2¼ ins wide; bolted down to a base consisting of two parallel plates of ½ inch wrought iron, with a plate of India-rubber, ¼ inch thick between them, the whole supported on a bed-plate of oak. The writer has seen one of these laid for a test, at a point on the Reading R R, at which ordinary frogs of cast iron, capped with steel, wore out in a year. The test frog was in nearly perfect condition, after the wear of more than 3 years; equal to that of 6 or 8 years on most of our railroads.*

The Mansfield Frog Co, of Chicago; also of New Haven, Conn, make a similar frog. Prices about as follows: Length 7 ft 7 ins, $140; 5 ft 9 ins, $115; 5 ft, $105; 4 ft, $95.

The William Butcher Steel Co, of Philadelphia, made a reversible frog, entirely in one piece of fine annealed cast steel.† When one set of wings, and their tongue, become worn by use, the frog may be renewed by simply turning it upside down. The following are the weights of these frogs, of a pattern to suit rails 4½ ins high.

No. 4,	400 lbs.	No. 10,	710 lbs.
6,	460 "	12,	820 "
8,	560 "		

Although somewhat lighter than cast-iron frogs, their first cost is fully twice as great; owing to the greater cost of the material. There is difficulty however in keeping them firmly in place; and they are no longer laid on the Reading R R, which gave them a fair trial.

The following are the weights of some of the ordinary frogs of cast iron, steel-capped, used on the *Reading, and many other roads.*† Their lengths are about as in the last foregoing table of lengths.

No. 6,	475 lbs.	No. 8,	660 lbs.
No. 7,	600 "	No. 10,	820 "

This frog has since failed, owing to the decay and crushing of its wooden base. It has been replaced by a Butcher steel reversible frog, which costs less; and is pronounced by the officers to be preferable as to durability; although they speak highly of the Mansfield in other respects.

*212 " " " 6 to 7 " 28

[illegible] corresponding arcs: or observations of any $\frac{1}{10}$ part, will be almost inapprecia[illegible] [illegible] $\frac{\cdot 05}{\cdot 3} = \cdot 15$; therefore, a frog with [illegible] $= \cdot 3 = \cdot 3$, will answer very well. The first frog in t[illegible] $\frac{1}{0} = \cdot 1$ therefore we may use a frog of $12 - 1\frac{1}{2} = 10.63$; [illegible] [illegible] between those limits.

[illegible] are intermediate of those in the tab[illegible] [illegible] intermediate. Sometimes accuracy is not nec[illegible] [illegible] almost always use to as just in the in order [illegible] [illegible] the middle one, which, altho[illegible] [illegible] the latter the barometer is suppo[illegible] [illegible] from a [illegible] [illegible].

TABLE OF THEODOLITE. (Original.)

Switch.	Number of Frog.	Frog angle.		Frog distance.	Radius.	Middle Ordinate.	Side Ordinates.
		Deg.	Min.	Feet.	Feet.	Feet.	Feet.
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.573
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.580
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.606
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.623
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.640
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.657
	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	.674
			[illegible]	[illegible]	[illegible]	[illegible]	.544
[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	.562
	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	.580
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.600
	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	.619
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.638
	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	.656
			[illegible]	[illegible]	[illegible]	[illegible]	.541
[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	.564
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.585
[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	.606
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.625
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.646
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.666
			[illegible]	[illegible]	[illegible]	[illegible]	.542
[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	.525
			[illegible]	[illegible]	[illegible]	[illegible]	.547
[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	.571
			[illegible]	[illegible]	[illegible]	[illegible]	.595
[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	.621
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.645
	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	.444
[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	.471
			[illegible]	[illegible]	[illegible]	[illegible]	.495
			[illegible]	[illegible]	[illegible]	[illegible]	.525
			[illegible]	[illegible]	[illegible]	[illegible]	.552
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.582
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.609
			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	.430
			[illegible]	[illegible]	[illegible]	[illegible]	.453
			[illegible]	[illegible]	[illegible]	[illegible]	.476
[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	.500
			[illegible]	[illegible]	[illegible]	[illegible]	.526
			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	.330
			[illegible]	[illegible]	[illegible]	[illegible]	.372
			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	.463

[illegible] 5. On Given the Frog Alignements. Having the [illegible] e.g. Fig 10. of [illegible] the angle of the frog: the [illegible] a diagram, thus: Take a [illegible] dist apart equal [illegible] with a rod equal to the [illegible]

umber of times the throw $d\,g$, describe an arc $t\,t$. From a lay off the dist $a\,i$, equal o $a\,b$, and ending in the circum of the arc. Draw $a\,i$; which will *cut* the arc. Draw ny $s\,y$, making the angle $n\,s\,y$ equal to the frog angle, or No. Parallel to $s\,y$, draw nes like the dotted ones, from the line $m\,n$ to the line $a\,i$. This done, find by trial

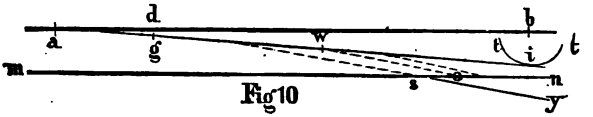


Fig 10

hich of these lines (here it is $w\,o$) has its length, $w\,o$, equal to the dist ($w\,g$) of its pper end (w), from the toe (g) of the switch. Then is $g\,o$ the reqd frog dist.

Or this dist may be calculated exactly, by the formula of Uriah Hunt, E; thus,

$$\textit{Frog dist} = 2\,(\textit{gauge of track} - \textit{throw of switch}) \times \frac{\textit{Nat cotang of switch angle} \times \textit{nat cotang of frog angle}}{\textit{Nat cotang of switch angle} + \textit{nat cotang of frog angle.}}$$

We add the following tables for facilitating the use of this formula.

No. of frog.	Frog angle.		Cotang.	No. of frog.	Frog angle.		Cotang.	No. of frog.	Frog angle.		Cotang.
	Deg.	Min.			Deg.	Min.			Deg.	Min.	
12	4	47	11.950	8½	6	44	8.470	5	11	25	4.952
11½	4	59	11.468	8	7	10	7.953	4½	12	42	4.437
11	5	12	10.988	7½	7	38	7.462	4	14	15	3.938
10½	5	27	10.481	7	8	12	6.940	3½	16	16	3.427
10	5	44	9.960	6½	8	48	6.460	3	18	56	2.915
9½	6	2	9.461	6	9	32	5.954	2½	22	38	2.398
9	6	22	8.962	5½	10	23	5.458	2	28	4	1.875

When the gauge is the ordinary one of 4 ft 8½ ins, and the throw of the switch 5 ns, then 2 (gauge of track — throw of switch) becomes 8.582 ft.

Length of Switch.	Switch Angle.		Cotang.	Length of Switch.	Switch Angle.		Cotang.	Length of Switch.	Switch Angle.		Cotang.
Ft.	Deg.	Min.		Ft.	Deg.	Min.		Ft.	Deg.	Min.	
30	0	48	71.615	20	1	12	47.740	10	2	23	24.026
28	0	51	67.402	18	1	20	42.964	8	3	0	19.081
26	0	55	62.499	16	1	30	38.188	6	3	59	14.361
24	1	0	57.290	14	1	43	33.366				
22	1	5	52.882	12	2	0	28.636				

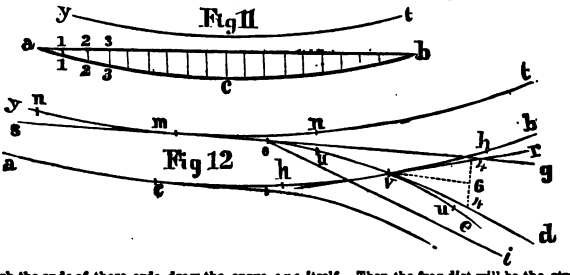
Art. 6. To find the rad of a turnout curve, having the frog angle, he switch angle, and the frog dist. From the frog angle take the switch angle; he rem will be the angle at the center of the circle; which angle call c. Subtract his angle from 180°, and div the rem by 2. Call the quot, angle a.

Then, as *Nat sine of angle* c : *nat sine of angle* a :: *frog dist* : *rad.*

Art. 7. It is frequently necessary to start a turnout from a curved line of road. n such cases the frog dist can generally be found near enough for practice, from a rawing on a scale of about ¼ inch to a foot. And so in the numerous cases where urnouts cross tracks in various directions, in and about stations, depots, &c.

Figs 11 and 12 are intended merely to furnish a few general hints with reference to such drawings. *or instance, the curves of a main track, as well as those of a turnout, generally have radii too large admit of being drawn on a scale of ¼ inch to a foot, by a pair of dividers, or compasses. But the y be managed thus: Draw any straight line a b, Fig 11, to represent by scale a 100 ft chord of re, divide it into twenty 5 ft parts, a 1, 1 2, 2 3, &c, and lay off by scale the 19 correspond ates, 1 1, 2 2, 3 3, &c, taken from the table on page 631. By joining the ends of these n the reqd curve a c b, of the main track; and of course can draw the inner line y t, ft by scale the width of track, say 4 ft 8½ ins. Now let a c b, and y t, Fig 12, be a curve so drawn: and let any point m be taken as the starting point of the turnout m o n, &c. m measure off any two equidistant points n and m, in the same curve; and throug*

$s\,g$, parallel to $n\,n$. Then is $m\,g$ a tang to the curve $y\,m\,t$, at the point m. From m, and on $m\,g$, off $m\,o$, equal to the switch rail. Having determined on the rad of the turnout curve $o\,v\,e$, pres to draw that curve by the same process as before; first laying off the angle $g\,o\,t$, equal to the tang tial angle of the curve, taken from the table in Railroad Curves. Then, beginning at o, lay off 5 dists along $o\,t$; and from them, as in Fig 11, draw the ords corresponding to the turnout cu

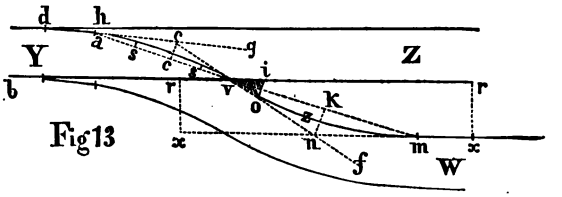


Through the ends of these ords, draw the curve $o\,v\,e$ itself. Then the frog dist will be the stra dist from o to v, and can be measured by the scale, within a few inches; or near enough for prac The middle ord of the arc $o\,v$ cannot be found correctly by so small a scale as ¼ inch to a foot. should be calculated thus: From the square of the rad, take the square of half the chord $o\,v$. T the sq rt of the rem. Subtract this sq rt from the rad. If two other ords should be desired, way between o and v, and the center one, they may each be taken as ¼ of the center one.

The frog angle at v will be equal to the angle $r\,v\,d$, formed between the tang $v\,r$, to the curve a and the tang $v\,d$, to the curve $o\,v\,a$. These tangs are found in the same way as $m\,g$; namely, fo tang $v\,r$, lay off from v two equidistant points h and h, on the curve $a\,c\,b$; and through v dra parallel to $h\,h$. Also, for $v\,d$, lay off from v any equidistant points u and u, on the curve $o\,v\,e$, through v draw $v\,d$ parallel to them. This angle may be measured by a protractor. Or if on the tangs we make $v\,4$ and $v\,4$, equal to each other, and draw the dotted line 4 4; and from its center draw $6\,v$; then $6\,v$ divided by 4 4 will give the No of the frog. With care, and a little ingenuity, young student will be able, by similar processes, to solve graphically any turnout case that may cent itself. The method by a drawing has great advantages over the tedious and complicated c lations which otherwise become necessary in cases where curved and straight tracks intersect other in various directions. The drawing serves as a check against serious errors, which woul detected at once by eye. None of the graphical measurements will be strictly accurate; but care, none of the errors need be of practical importance. The ordinates for bending rails so suit turnout curves, can be found from the table, p 418.

Art. 8. An experienced track-layer, with a good eye, can place his own gu stakes by trial on the ground; and by them lay his turnouts with an accurac practically useful as the most scrupulous calculations of the engineer can secur

The following example, Fig 13, of a turnout from a straight track Y Z, exhibits a common cas which all the work may be performed on the ground, without previous calculation. Let $i\,v\,o$ be tongue of a frog, with which the assistant has been directed to make a turnout from Y Z; and he has received no instructions more than that the turnout must start at d, with a switch $d\,a$, 26 long, having a throw $h\,a$, of 5 ins; and must terminate in a track W, to be laid parallel to Y Z, distant from it $r\,x$ or $r\,x$, equal to 6 ft.

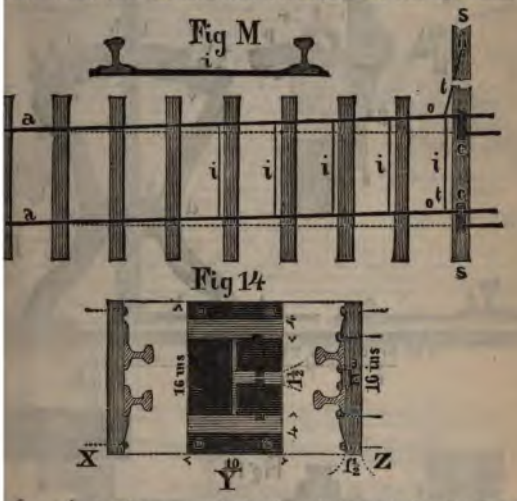


First lay off $d\,a$ on the ground, with its 5 ins throw; placing a stake, with a tack in its top, Stretch a piece of twine along $d\,a$, and extending some distance beyond, as to g; confining its e Then place the tongue of the frog by guess near where it must come, having its edge $v\,i$ *precisel line with the inner*, or flange edge, of the rail $b\,r$. Then stretch another piece of twine along the $o\,v$ *of the frog, and extending* to the twine $d\,g$. Try by measure whether $v\,e$ is then equal to $o\,a$: *If it is not, move the frog* along the line $b\,r$, until those two dists become equal. Then is v the pr *place for the point of the frog*; $a\,v$ is the frog dist; one-half of $c\,e$ is the length of the middle *the turnout curve* $a\,v$; and if two intermediate ords are needed at s and s, each of them will b *said middle one.*

The frog being now placed, proceed thus: Place two stakes and tacks, x and x, at the re *tack dist*, $r\,x$ and $r\,x$, of 6 ft from the rails $b\,r$. Then range by pieces of twine $x\,x$ and *e point n of intersection*. Then measure $n\,v$, and make $n\,m$ equal to it. Then is m the *verse curve* $v\,m$ of the turnout. The ords for this curve may be found as before; com *ng the middle one, &c.*

ay frequently be of use to remember that in any arc, as *v m*, of a circle; *v m* and *m n* rom the ends of the arc; one-half of the dist *k n* is the middle ord *k* Z of the curve; near ost practical purposes, *whenever the length of the chord* v m *of the arc is not greater f the rad of the circle of which the arc is a part.* Or within the same limit, vice versa, n equal to twice *k* Z, then will *n* be very approximately the point at which two tangs of the arc will meet. Also, the middle ord of the half arc *v z*, or *z m*, may be taken as die ord *k z* of the whole arc.

Switches, or switch rails. The arrangement of these, and the man- ing them, is very simple. Although the minor details admit of much general principle is the same. Fig 14 shows a portion of a track, on *o*, are the switch rails; usually from 20 to 26 ft long on main lines. At *a*, *a*, they are merely held by an ordinary joint-chair; or more simply, rail spikes. Either mode admits, without trouble, the slight motion place at that end. At their toes, *o*, *o*, they should rest upon firm stout

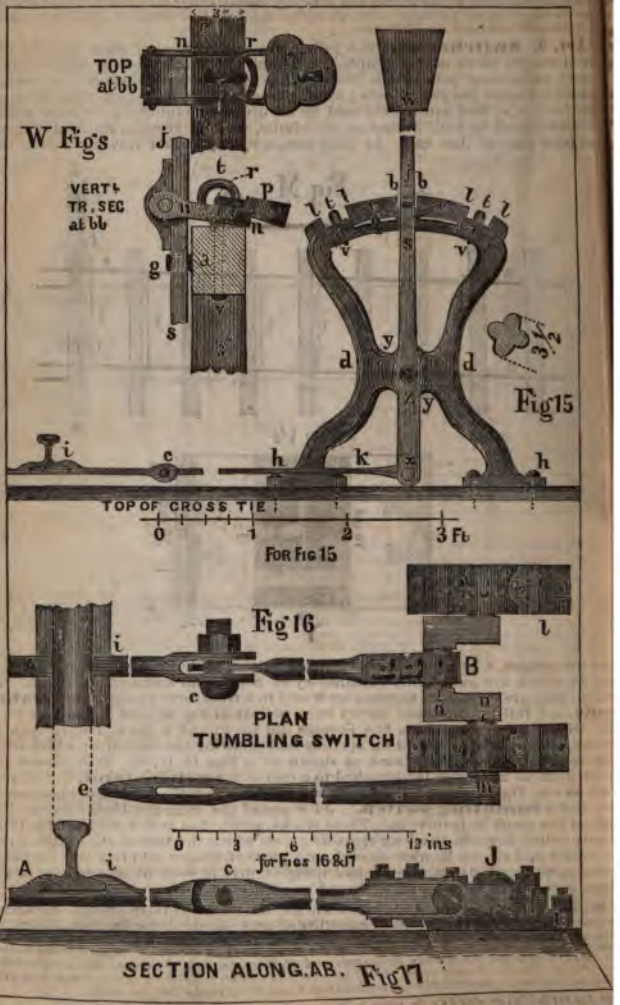


airs, *c*, *c*; of which an approved pattern is shown in detail at Figs X, Y, ch are sufficiently explanatory to require no written description. The are connected together by from 3 to 5 transverse wrought-iron **clamp-** full 1¼ ins diam; shown more plainly at Fig M; and at *i* in Figs 15, 16, these clamp-bars, *f t n*, is near the toe *o*; and is made longer than the much as it is used for opening and shutting the switches; which opera- s it also to be jointed, as shown at *c*, Figs 15, 16, 17. It is moved by a as *x s w*, Fig 15; attached to a cast-iron **switch-stand,** *l*, *d*, *h*, *y*: or Fig 16, about 2 ft long, by aid of the crank B. This last arrangement **umbling switch.** J *l* is one of the plummer blocks for the jour- crank to revolve in. These blocks, as also the switch-stand of Fig 15, are d down to the long cross-tie *s z*, on which the toes of the switches rest; aid tie not be wide enough, to a piece of timber securely laid for the ex- se; or the switch-rod, as *t n*, Fig 14, may be bent at an angle, or at a re- so as to bring its outer end *n* over the cross-tie.

-stand may be used wherever it will not be in the way of passing trains. Its lever *w x* sed also as a vane-rod, for supporting a vane *w*, about a foot or two high, painted white, *to about 7 ft above the ground.* In the upright position of the lever, as in the fig, the *to an approaching engine* that the main track is clear. If the rod leans to the left, *t to the left is ready for travel*; and if there be two turnouts, one to the right, and *left of the main track.* (In which case the switch is called a three-throw one,) then th *tion of the rod shows the right-hand one to be ready for the train.*

bling switch may be used in the space between t
its lever c m is always lying flat upon the cross-tie, except durin
it for moving the switch; consequently it is not in the way of the t

The oblong hole near the end of the lever is for allowing it to fit over a staple bolted to the tie. The hasp of a padlock is then attached to the staple, for preventing mischievous persons from misplacing the switch. A second staple receives the lever when it is reversed or turned back. The dist n n, Fig 16, must plainly be equal to one-half of the throw of the switch-rail.



The vane-rod, or lever, *w i x*, Fig 15, is held in position at *b b*, *l l*, or ns of a hinged collar *n*, Figs W, fitting over the staple *t*, to which it is hasp *r*, of a padlock *p*. When the lever is to be shifted from *b b*, *p* is

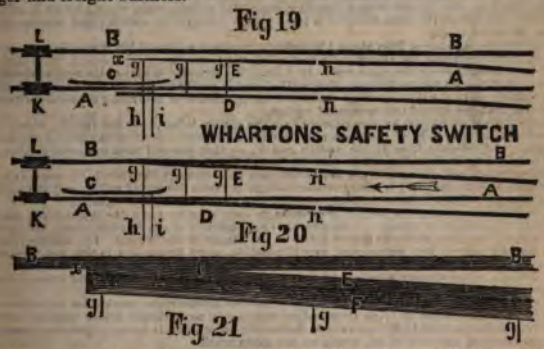
and removed; the collar n is raised up to clear the staple t; and the lever is first drawn back, or toward the reader, about an inch, in order to clear the projecting bead g. The elasticity of the long lever permits this last to be done readily. The lever is then shifted to ll, where its elasticity again brings it flat against the face a, of the switch-stand, where the bead g retains it. The collar n then falls into its place, on each side of the staple t, and the padlock is replaced. Figs W will make the details plain. These admit, of course, of much variety; what we have given are perhaps as good as any.

Usually, pieces of flat iron about 3 ins wide by ½ inch thick, are spiked to the tops of all the cross ties which sustain the switches; in order to facilitate the sliding of the latter. These may be very short, toward the heel ends a; and increase to about 15 inches in length at the toe ends. Sometimes these are omitted entirely.

Fig 18 is a side view of another arrangement which may be used between tracks. It is called the Monkey Switch. A stand ee is cast in one with the hollow cone nn. The latter is smoothly bored, to admit the cylindrical pin so, about 2 ins diam; with a square head s, for the handle or lever hh, about 3 feet long. A wrought collar ee is fastened to so by a pin through both, and turns with so. The hor piece oi is fast to so; and near its end i is fastened the short vert round pin iv, about 1 inch diam, which passes *loosely* through an opening in the end c of the long rod ttn, Fig 14, which opens and shuts the switch. A small pin passes through iv just below c, to prevent the latter from falling. For a throw of 5 ins, the hor dist between the centers of so and iv must plainly be 2½ ins, or half the throw. In the fig, oi, &c, is in the position of only half open. When the switch is a three-throw one, this dist between centers must be 5 ins; and the length of the piece oi, and the length tt of the stand, must be increased to suit. In a 3 throw, the position of oic as seen in the fig, is after oi has described a quadrant, thus opening one turnout, and closing another. In this position it must be held for the time by a bolt passing through the top of the standard, and through oi near the end i; otherwise a passing train would be apt to move it, thus partially closing the turnout, and throwing the train off the track. By turning oi around through another quadrant, the remaining third track becomes passable. In a 2 throw, oi is never left in the position shown in Fig 18, but describes an entire semicircle each time the turnout is opened or closed. It therefore requires no bolt for confining it temporarily. The breadth of the stand tt does not show in the fig; it is usually 6 ins at every part.

MONKEY SWITCH

Art. 10. Wharton's patent safety switch, Figs 19, 20, 21, devised by William Wharton, Jr, of Philada, is being introduced on many roads doing a heavy passenger and freight business.



A and B are the rails of the main track. E and D, are the switch rails; with their heels at n; the place of the toe of ordinary ones. These switch rails are connected together by the usual clamp bars, gg; and slide on the ties, like others. Fig 21 shows more in detail the toe of the outer switch E; which is tapered nearly to a point at x, *in order that it may fit close to the* main rail B, for the dist x t, when in place for *the turnout. These switches* have been in use on the Reading railroad for about *two years; and have there given* such proofs of superiority over others, that the com*pany have purchased the right* to use them.* The following description is nearly *the inventor's own words:*

*Mr. Wharton has established at Philada, a manufactory of his switches. Since the abo*ve, Mr. Wharton has greatly improved his switch. It now bears but little resemblance to *it; it must therefore be regarded merely as an illustration of a principle.*

THE CHIEF FEATURE of this switch is, that by its use the continuity and solidity of both rails of the main track are entirely undisturbed; and that trains moving in either direction cannot be thrown off the railroad by any possible misplacement of the switches. The worst that can happen is that a train might pass on the turnout track, instead of keeping on the main one; and thus lose the few minutes required to return to its proper position. The saving of wear and tear upon both the rolling stock and the rails themselves, renders this invention of importance to railroad companies, in addition to the consideration of safety.

Both rails A and B, Figs 19 and 20, of the main track, remain the same at the switch as at other parts of the road; being firmly spiked to the cross-ties. The main track is, therefore, *absolutely unbroken, or continuous;* the usual shifting rails of the *main track* are dispensed with; and the battering of the ends of the rails at the open joints necessary in common switches, together with the severe blows received by every wheel and axle passing over those open joints, are entirely avoided. Neither can any accident arise from the breaking of the clamp bars or switch rod, as now happens to cars moving upon the main track when any of the apparatus holding the shifting rails of the usual switches gets out of repair.

The switch rails D and E (connected together by clamp bars *g g g*); and the movable guard rail C, are so arranged that when the switch is not in use, they can be simultaneously moved away from the main rails, to the position shown in Fig 19, where they do not in any way act upon, or touch against, the car-wheels. When, however, it is desired to transfer cars from the main track to the siding, they are moved to the position shown in Fig 20; in which case the guard rail C draws the wheel K gently to that side, so that all of the projecting portion of its tread shall be caused to run upon the switch rail D. This, at its end, is of the same height as the rail A of the main track; but it gradually inclines upward, so that the advancing wheel is quietly raised above, and becomes free from, the rail A; and is then supported only by the switch rail D; which in a distance of about four feet from its end has sufficient rise to enable the flange of the wheel K to pass over and above the main rail A, without touching it.

The guard rail C, at the same time draws the opposite wheel L a little distance away from the other main rail B; so that its flange is insured to run inside of the point of the switch rail E; and can never touch, or strike against, the point itself.

The wheels are thus guided on to the siding in the most easy and gentle manner. Of course, the cars can pass out from the siding, on to the main track, with equal facility.

The *point, x*, is not subjected to any lateral wear; neither can it receive the full weight of cars passing on to the siding; for the wheels also bear upon the main rail B, until they have advanced so far upon the siding, that the pointed rail has become of its full width, and strength.

The outer switch rail E, Fig 21, is provided with a guard rail F; (not shown in Figs 19 and 20,) either bolted to it, or rolled with it in one rail, as may be preferred; the duty of which is to steady and otherwise assist in guiding the car-wheels.

The inner switch rail D, after being elevated along a moderate length, is gradually lowered, until it is of the same level as the other rails.

The whole arrangement is constructed of rolled T rails; and no castings are required.

All cars travelling upon the main track, use the main-track rails alone; and the switch rails are never used, except by such cars as are taken into, or out of the siding.

It will be evident that cars moving along the main track, in the direction of the arrow, will, in case the switch has been left wrong, continue upon their proper course, by merely pressing the switch rails D and E to a sufficient distance aside, by means of the flanges, to allow the wheels to pass through. This is done by the simple bending backward of the switch rod A; which is effected with the utmost facility, as the first wheel of a train passes through the switch; and without giving any check or jar to the train.

The switch rod is made of iron about 1¼ inches diameter; a size which is readily bent; but which is stiff enough for moving the switch rails backward and forward.

But if cars moving along the main track, in a direction opposite to that of the arrow, should happen to find the switches set for the turnout, they will of course be deflected from their intended route; but will pass safely on to the turnout; and the slight inconvenience of returning to the main track, is the greatest one that can attend this system of switch.

The lateral pressure caused by cars passing into the siding, is borne by the rail E; which is in turn supported against the side of the main rail B; which, therefore, receives the lateral pressure of the car-wheels; and relieves from it the switch rod A; so that this rod need not be of greater strength than is required for moving the switch rails backward and forward.

The other switch rod, *i*, does not get bent when cars pass through the switch, in case it is left wrong for the main track; for there is always room enough between the guard rail C, and the main rail A, for the flanges of wheels to pass through, whether the switch is closed or open.

This switch, therefore, gives perfect protection to cars moving upon the main track, whether the switch is fastened right or wrong; or whether it is not fastened at all, but left loose; for, in the latter case, it will all the more easily be moved aside by the flanges of the wheels. The accidents caused by leaving an ordinary switch unfastened, are, if possible, more disastrous to life and destructive to property, than when the switch is fastened in the wrong position; since the switch rails are apt to move sideways under the car-wheels, and thus cause some to go on one track and some on the other; which brings about an immediate and certain crash.

Upon railroads with single tracks, where it is of course necessary to provide for the safety of trains moving in both directions, it is so arranged by means of a most simple device, that the switch rails will not remain in position to form a switch, unless the switchman is actually present; therefore, the switch can never by accident be left wrong for the main track.

Platform weigh-scales. Since the making of these is a specialty, we shall not describe their construction. Large ones from 30 to 150 ft long, for weighing from 30 to 150 tons of loaded cars at once, cost at the shop from $25 to $30 per ft of length. The preparation of the pit under the track, with its lining, foundations &c, and putting the scales into place, will cost about 35 to 40 per ct in addition. Superior ones are made by Messrs Riehlé, 9th St below Coates, Philada, and by Fairbanks & Co, St Johnsbury, Vermont; office, 715 Chestnut St, Philada.

***Testing* machines** for either tension or compression, up to some hundreds of tons, are *also specialties of* Messrs Riehlé.

RAILROADS.

otal annual expenses, on U States railroads, usually ı 45 to 75 per cent of the receipts; the mean being 60 per cent. Few fall lower limit; while few exceed the higher one. The average of all the n England and Wales, previous to 1865, was about 48 per ct; in 1866, they and in 1867, 50.6 per ct.

f average per centages of the various general items ich constitute the annual expenses of railroads.

Items.	Approximate Average of U. S. Roads.	Average of English Roads.
	Per ct.	Per ct.
*ce of way and works**	25	19
ver	30†	28
id renewals of cars‡	10	9
y transportation§	30	28
penses‖	5	
Government duties	.....	6½
o persons and goods	.....	1½
iament, miscellaneous	.....	8
	100	100

e proportions which the several items of expense on the Penna Central, between Phila ·g, bore to the total expense, were very nearly as follows: Conducting transportation, motive power, 30.4; maintaining roadway, 27.3; maintaining cars, 12; general ex- total, 100.

ese items is, however, subject to great variation, not only on diff roads, but on the same ear to year. A road with many bridges, deep cuts, high embkts, &c, to keep in repair, avier maintenance of way than one which has but few; and this item may be but small d twice as great the next. Fuel may be cheap on one road, and dear on another; thus ffecting the item of motive power. And so with the other items. Sometimes mainte- y exceeds motive power and cars together; at others, conducting transportation is fully l expense.

ross receipts per mile of road in 1868, according to Poor's Rail- ıal, were, in Massachusetts, $15400; New York, $13142; Pennsylvania, erage of the whole United States, about $10000. And the earnings from ·raged about 2½ times that from passengers.

ɔnnage on the N York roads was 3625 tons per mile; Massachusetts, Pennsylvania, 8000 tons; average of U S, about 2500 tons.

ɔtal annual expenses on railroads in the United States nge between 65 and 130 cents per train mile; that is, per mile actually ins. Also, between 1 and 2 cents per ton of freight, and per passenger e mile. When a road does a very large business, and of such a character ains may be heavy, and the cars full, (as in coal-carrying roads,) the ex- train mile becomes large; but that per ton or passenger, small; and vice

g bridges, depots, stations, and other buildings, roadway, superstructure, fences, &c.
0 per ct of total expenses, about 11 (7 to 14) is for fuel; about 7 (5 to 10) for repairs of 1 12 for pay of engine drivers, firemen, cleaners, oil, waste, &c; as averages of a great ɔads of every character as to length, traffic, &c. Fuel ranges from 5 to 12 cts per train- y 6 to 9 cts. The repairs of engines from 6 to 14 cts: usually about 8 to 12.
ıal repairs of a large 8-wheel passenger car, ranges between about $300 and $600; mail cars, from $150 to $300; freight cars, from $75 to $150; coal cars, from $20 to $30, (4

vages of clerks, ticket-sellers, conductors, brakemen, bridge and switch-tenders, tele- rters, weighers, &c. &c; lighting and warming of cars, depots, stations, &c.
of the higher officers of the Company; law, stationery, advertising, office rents, &c, &a aries from 2½ to 7 per ct.

Table of Annual Expenses of some U S Railroads.*

Names of Companies.	Per Mile of Road. $	Per Train Mile. cts.
Lehigh Valley, 1860	.....	103
" " 1862	.....	178
" " 1868 and 1869 ..... about	.....	.....
" " 1872	.....	.....
Baltimore & Ohio, main stem, 1859	4434	54
" " " " 1860	4254	49
" " " " 1865	.....	.....
" " " " 1866	.....	.....
" " " " 1872	.....	.....
East Tennessee & Georgia, 1872	.....	.....
Memphis & Charleston, 1860	2617	89½
Georgia Central, 1872	4180	105
Penna Central, main line from Phila to Pittsburg, 358 miles, 1859, exclusive of State tonnage tax	7848	87
" " 1860, " " " "	.....	.....
" " 1861, " " " "	.....	.....
" " 1868, tonnage tax repealed	.....	.....
" " 1869, " " " ..... about	32000	.....
" " 1872, " " "	.....	.....
Phila & Reading, 1859	.....	.....
" " 1860	.....	91
" " 1868, 365 miles of main road and branches	17200	144
" " 1869	.....	.....
" " 1872	.....	.....
North Pennsylvania, 1860, 54 miles long	3213	76
" " 1862	3240	68
" " 1867	9534	.....
" " 1868	.....	.....
" " 1872	.....	.....
Connecticut; average of all the railroads, 1861	3781	95
Massachusetts; " " " " 1861	3785	85
" averages of 19 years previous	2700 to 4300	70 to 110
" 1867 ..... average	.....	.....
Galena & Chicago, 1859	.....	93
" " 1860	3102	73
Phila, Wilmington & Baltimore, main stem, 1859	4586	116
" " " " " 1860	7100	167
" " " " " 1861	7785	.....
" " " " " 1867	17380	187
New York; all the R R in the State, average,† 1859	4964	106
" " " " " " 1861	5100	.....
" " " " " " 1867	13856	.....
New Jersey R R and Transportation, 1861	12213	84½
Louisville & Nashville, 1861	.....	85
Phila & West Chester, 1861, 27 miles	2274	59
" " 1862	2282	.....
" " 1872	7030	104
Phila, Germantown & Norristown, 1861, 20 miles	6405	56
" " " 1862	6405	55¾
" " " 1867	18208	128
New York & Erie, 1861	6461	.....
" " 1867, with its branches, 784 miles in all	14545	177
New York Central, 1861	8360	.....
" " 1867, with its branches, 696 miles in all	15620	170
English R R, averages for 1856–7–8	.....	66
Scotch " " " " " "	.....	58
rish " " " " " "	.....	52

* Annual reports often omit the lengths of the roads and branches; and as these fre
m year to year, it is possible that the table may contain some errors in the first col
2528 miles in operation. Total exps equalled 1.56 cts per passenger or ton carried
ght of cars, equal to 1.19 tons per passenger; and to 1.14 tons per ton of freight.

Fuel for locomotives. As nearly as the writer can judge from a mass of ery conflicting testimony, a ton of good anthracite, or of bituminous coal, is equal o 1½ cords of good dry hard mixed woods, (chiefly white oak;) or to from 1¾ to 2¼ ords* of such soft ones as hemlock, white, and common yellow pine. Much of the nferior bituminous coal of Illinois is hardly equal to a cord of average wood. **On he Illinois Central** in 1860, the engines run about 200,000 miles, with average rains of 10½ loaded 8-wheel cars. The wood-burners averaged 39 miles per cord; r 2¼ cords per 100 miles. The coal-burners, with this inferior coal, 36½ miles per on; say 2¾ tons per 100 miles; making 1 cord of wood equal to $1\frac{1}{10}$ tons of coal. **On the Chicago, Burlington & Quincy R R.** a very full trial (about 1000 miles with each kind) was made with this coal, and with another of good quality from another locality. The trains averaged 28½ eight-wheel cars, with their daily loads; and the result was 3.5 tons of the bad coal, and 2.2 of the good, per 100 miles; making 1 ton of the good coal equal to 1.45 cords mixed woods. **On the Great Western R R of Mass.** the same trains required 2½ tons of anthracite per 100 miles; or 4 cords of hemlock, (an inferior fuel;) or 1 ton anth = 1.6 cords hemlock.

On the Philada & Reading, passenger trains of 5 large cars, and weighing 54 ons, exclusive of engine and tender, (or about 90 tons with them,) used 2.7 cords of good mixed woods, or 1.5 tons of anth per 100 miles. Here 1 ton anth=1.8 cords of wood. In 1867, passenger trains of 66 tons, (about 102 with engine and tender,) are reported to use 2.22 tons per 100 miles. This is much greater in proportion to the increased weight of train than in the preceding case; and there would seem to be some discrepancy. The freight trains use 3.45 tons, and the very heavy coal trains 3.75 tons per 100 miles. These last consist of about 96 four-wheel coal-cars, weighing 2½ tons each, or 241 tons in all; 96 loads of coal, of nearly 4¾ tons each, or 443 tons in all; and engine and tender about 50 tons; making the entire weight of the train 734 tons. The returning empty trains up the road weigh 291 tons in all, and consume about the same quantity of fuel as the descending loaded ones. The grade of the 95 miles of road in the direction of the loaded trains is descending; most of it at less than 6 feet per mile; and none exceeding 14 ft. except 3 continuous miles of 22.4 ft. and a stretch of less than 2 miles of 35 ft per mile *ascending;* on which last auxiliary power is used. When the coal trains burned wood they used about 1½ cords to the present ton of coal.

The mean weight of the loaded coal trains down, and of the empty ones up, is 512 tons; and that of the passenger trains each way is 102 tons, or only $\frac{1}{5}$ as much; yet the quantity of fuel consumed by the first is but little more than twice as great as by the last. The first run about 10 miles per hour; the last, 25 miles.

On the Penn'a Central, experiments made 1 ton of Pittsburg bituminous coal equal o 1.49 cords of wood (chiefly white oak) weighing 5215 lbs, or 2.328 tons; or 3500 lbs per cord. Trains of 4 eight-wheel freight cars, weighing 30.8 tons, and loaded with 33 tons; engine and tender 41.56 ons, making 105.36 tons in all; in ascending 12 miles of continuous grade of 95 ft per mile, at a speed of 20½ miles per hour, consumed either 1073 lbs (.479 ton of coal; or 2483 lbs (1.108 ton, or .7094 cord) of wood. This in 100 miles gives 3.992 tons of coal, or 5.91 cords of wood; or say 4 tons, or 6 cords; or 1 ton coal = 1½ cords wood. And this accords closely with the comparative consumption of anthracite and wood, in hauling heavy coal trains, on the Reading R R. The passenger engine used in the foregoing experiments weighed 28½ tons, of which 18.16 tons rested on 4 drivers, 5½ ft diam. It was arranged to be equally well adapted to coal and to wood. The trials show the effect of steep up-grades in increasing the consumption of fuel. Thus, on the Reading road, an engine with 20 tons on drivers, 46 ins diam, habitually drew down at 12 miles an hour, trains of 80 loaded cars, weighing with engine and tender about 625 tons; consuming at the rate of 4¾ tons of anthracite, or 7.1 cords of wood per 100 miles; which in 12 miles would be .57 ton, or .84 cord; against the .48 ton, or .71 cord, required on the steep grade, with a train weighing but ⅛ as much.

The performances on the Balt & Ohio appear remarkable. on all the divisions of that road, (except the mountain one,) although the grades are much heavier than on the Reading, passenger trains as heavy as those on the last road are reported to average but about 1 ton of bituminous coal per 100 miles; and freight trains of about 400 tons gross weight, only 2 tons. Here the experience with passenger trains would make 1 ton of Cumberland bituminous on the B & O, equal to at least 2.2 tons of anthracite on the Reading; which is quite incredible. Actual trial on the B & O made 1 ton of best Cumberland equal to 1¼ tons of anth; or to 2⅛ cords of pine, weighing 2.55 tons.

The run of freight and passenger trains throughout the U States is 20 to 40 miles per cord, and 30 to 60 miles per ton. Much depends upon the adaptation of the engine to the kind of fuel used. A good coal-burner may be bad for wood, and vice versa; so that trials with the same engine may give very erroneous results as to the comparative merits of the two kinds of fuel. When wood is used, about 1 cord; or when coal, about ¼ cord of wood, must be used for kindling, and getting up steam ready for running; and this item is the same for a long run as for a short one; so that long roads have in this respect an advantage over short ones, in economy of fuel. Wood has the disadvantage of emitting sparks; and is, moreover, nearly twice as heavy as coal, for the performance of equal duty; and is, therefore, more expensive to handle. It also occupies 4 or 5 times as much space as coal.

To find the tractive power of a locomotive, mult together the

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* *A cord is 4 × 4 × 8 ft, or 128 cub ft.* A cord of good dry white oak, (next to hickory, the best *wood for fuel,*) *weighs about 3500,* or 1.563 tons. Dry hemlock, white, or common yellow pine, (*them inferior for fuel,*) about .9 ton. Perfectly green woods generally weigh about ½ to ¾ ton *when partially dried* for locomotive use; in other words, a cord of wood, in its partial dry *from ½ to ¾ ton of water,* and still contains a large quantity of it. Since this water *waste of heat,* green wood should never be used as fuel. The values of woods as fuel *the same proportion* as their weights per cord when perfectly dry.

[illegible] and the [illegible] "[illegible]"

Table of approximate greatest weight of train exclusive of engine and tender [illegible] **weighing 27** [illegible] **can take up dif**[illegible] **; say 8 to 12** [illegible] **in which case the** [illegible] **For other** [illegible] **in proportion to the weight on their** [illegible]

Grade in ft. per Mile.	Tons.	Grade in ft. per Mile.	Tons.	Grade in ft. per Mile.	Tons.	Grade in ft. per Mile.	Tons.
Level	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	118
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	105
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	98
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	91
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	84
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	71
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	60
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	51
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	44
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	33
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	22
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7

In practice, trains usually [illegible], as a general rule, weigh more than from ½ to ⅔ of those in the table. This allowance is necessary in order to have the trains under more complete control; to admit greater speed in case of deviations; to allow for curves, slippery rails, head winds, &c, &c.

The annual expense of running a locomotive and tende[illegible] *running, say, 75 miles a day, for 267 days in the year, or 20000 miles annual*[illegible] *which is common, may be found in the following manner:*

Fuel, say 2½ cords per 75 miles, at $2.50 per cord; $7.87½ per day.	$21[illegible]
Repairs, at 9 cts per mile run	180[illegible]
Engineer, or driver, 12 months, at $90	108[illegible]
Fireman, 12 months, at $50	60[illegible]
Oil and waste, at 2 ct per mile run	20[illegible]
Sawing, and loading-up wood; 1½ cts per mile run	30[illegible]
Supplying water ..... 1 ct " "	20[illegible]
Putting away, cleaning, and getting out, say	12[illegible]
Locomotive superintendence "	10[illegible]
Total	$6.[illegible]

Equal to, say 34 cts per train mile; or $24.35 per running day; or $17.81, for ev[illegible] *day in the year.*

This is all that is usually stated in annual reports of expenditures; but inasmuch as an engi[illegible] active service, even under a judicious system of repairs, generally becomes worthless, (except a[illegible] iron,) in say 16 years on an average, an additional allowance of about 6 per ct on the first cos[illegible] about $500 to $600, should be made annually for DEPRECIATION OF EACH ENGINE.

For **Evaporation by Locomotives,** see p 434.

* The cylinder press is always less than the boiler pres; and the disproportion increases wit[illegible] speed. Thus, at 8 or 10 miles an hour, the boiler pres may be about 110 lbs per sq inch; and [illegible] cylinder pres from 90 to 100 lbs; while at a speed of 30 or 40 miles, the proportion may be as 110 [illegible] to 95.

† Baldwin's 8-driver engines of 35 tons, all on the drivers, draw 40 empty coal cars, weighing [illegible] net, on a continuous grade of 175 ft per mile for 3½ miles, with many curves and reversed curve[illegible] [illegible] 100 to 600 ft grad, at 8 miles an hour, as an every-day operation.

On the Penn'a Central, engines of 29 tons, all of it on 8 drivers, take 280 tons of cars and tra[illegible] continuous grade of 95 ft per mile for 9¾ miles. The passenger engines take up 2 loads, [illegible] s, (about 60 tons,) at 24 miles per hour.

ength of locomotives in feet, from out to out, of everything, is usually about
eir weight in tons, and their extreme width for 4 ft 8½ inch track, 9 ft.*

eight of the pipe, or smoke stack, is usually 13 to 15 ft above the rails.

stout steel driving-tires,† bearing about 3 to 3½ tons, will run about 30000
e becoming worn sufficiently uneven to require turning-down, about ⅛ inch in thickness,
n diam; and will bear 3 or 4 turnings-down before wearing too thin for safety. Driving
ournals are about 5½ to 6 ins diam.

ers of passenger engines, usually weigh 5 to 6 tons empty; and 12 to
ater and fuel; those of freight engines, 6 to 8 tons empty; and 15 to 20, with water and fuel.
th from out to out, is usually about ¾ that of the engine. Extreme width, 9 ft.

enger engines usually carry fuel and water sufficient for 20
; rarely for 40 or 50. Freight trains, enough for 10 to 15 miles. Roads, or divisions with
s, require the fuel and water stations to be nearer together than where the grades are easy.
f passenger engines usually contain from 1200 to 1800 gallons; and those of freight engines,
o 2200. Twelve hundred galls = 160 cub ft = 4.47 tons; and 2200 galls = 294 cub ft = 8.15
tons for fuel and water, as well as those for passengers, &c, should, as far as possible, be
e summit of two up-grades, for convenience of both stopping and starting.

enger cars. Eight wheels; for 56 passengers; with water-closet, and
a stove, are about 46 to 48 ft extreme length from out to out of bumpers. On
½ inch gauge, the extreme width, out to out of the slightly projecting roof-
s usually 8½ to 10 feet; the last especially on sleeping-cars. On the 6 ft gauge
York & Erie, 11 ft. About 9¼ ft is ample. Weight empty about 16 tons; or
ed with passengers, about 20 tons.

e, even when the cars are full, there is a *dead* load of about 4 tons, to 1 *paying* ton of
. But as a general rule, passenger trains are not *more* than one-half filled; thus making
ion at least 8 to 1. The annual reports of the State of N. York, show there about 18 to 1!
ton of dead load for each passenger! In England, with their heavier passenger-cars, the
on some lines is from 20 to 30 times the paying one! A ton of paying *freight*, with the cars
lly loaded as usual, requires on an average about 1½ of dead car-weight; and since freight
t a less expensive speed than passengers, it is seen that the carrying of a single passenger
y cost as much as that of a ton of freight; especially when, as is too often the case, a con-
ortion of the passengers are *dead-heads;* a class which travel very often.

platforms of passenger cars are usually 3⅓ to 3½ ft above the rails.

ht of cars. Passenger cars for about 50 to 60 persons, 14 to 18
aggage and mail cars, 8 wheels; length from out to out of bump-
t 40 to 45 ft; width 8½ to 9½ ft out and out of all, 10 to 14 tons. **Freight**
ouse or box-cars.) length 30 to 36 ft out to out; width 7½ to 9 ft; 8 wheels,
ns. **Platform,** and **gondola cars,** for stone, lumber, iron, &c, 8
5 to 30 ft long out to out; width 7 to 9 ft, 6 to 8 tons. **Coal cars,** single,
els; 2½ to 2¾ tons; and carry about 5 tons of coal. Size about 12 feet
6½ ft wide out to out of everything. Double ones, on 8 wheels, are most in
ly weigh from 5 to 6 tons; and carry about 10 tons of coal. **Ordinary**
g cars are about the same size as day-cars; and weigh from 20 to 25
t some recent ones, to accommodate 56 persons, are nearly 70 ft long; by
9 out to out; with 16 wheels; and weigh, empty, about 33 tons.‡

wheels for passenger and freight cars, are usually about
ins diam; and those of coal cars 26 to 28. The cast-iron wheels made by
. Whitney & Sons, of Philada, weigh as follows, per single wheel.§ Combined
tread and flange, 5⅝ ins. Cone, 1 in 21.

Diam. ins.	36.	33.	30.	28.	26.	24.
Weight, lbs.	585	500	440	400	350	335

ights by other makers do not differ from these materially.
am of car or engine wheels does not include the flanges; but is the least
m tread to tread. Good chilled cast wheels will run about 100,000 miles;
ich more durable than steel ones.

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st of locomotives is about $400 to $500 per ton of their weight; the tender being supposed
n ins.
n Philada, 1871, 15 to 18 cts per lb.
MATE AVERAGE PRICES OF CARS in the Eastern States in 1871. Passenger-cars for abo
ons, $5000 to $6000. Drawing-room cars, from $6000 to $10000. Sleeping cars, $800
e very extravagant ones, for 60 persons, have cost from $20000 to $25000. Mail
$3000. Freight cars, (house, or box-cars,) $1000. Gondola, and platform, cars
the foregoing on 8 wheels. Coal cars, single, 4 wheels, wooden bodies, $200; w
iladelphia, in 1871, about 3½ cts per lb. Wrought-iron axles, hammered, s
led, 4½ cts.
K 9

The railroads of Pennsylvania, with their equipments, have cost on an a rage, about $58000 per mile; and have about

1	Locomotive to every	4	miles of road.	
1	Engine-house to "	20	"	"
1	Passenger car to "	8	"	"
1	Baggage, mail, or express car to every	18	"	"
2½	Freight cars, and trucks, to "	1	"	"
6	Coal cars to "	1	"	"
1	Depot, or station "	7	"	"
1	Fuel and water station to "	9	"	"

The average cost per mile of the railroads of New England, has about $40500; of the Middle States, $55000; of the Southern States, $30000.

Miles of railroad in the world, at the close of 1872, about 150000; or full times the circumf of the earth. In North America, 70000; Europe, and entire Eastern hemisp 78000; South America, 2000.

The size of axles for passenger or freight-cars, on a 4 ft 8½ track, averages ab ft 10 ins long; 4¼ in diam. Journals 5½ to 7 ins long; and 3¼ ins diam. About the same for form and gondola cars. Average wt about 320 lbs each axle.

Table of cubic yards of ballast per mile of road.

Side-slope of the ballast 1 to 1. Width in clear between tracks of double-track r 6 feet.

Depth in Ins.	Top width, Single Track.			Top width, Double Track.		
	10 Ft.	11 Ft.	12 Ft.	21 Ft.	22 Ft.	23 Ft
	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y.	Cub. Y
12	2152	2347	2543	4303	4499	4695
18	3374	3667	3960	6600	6894	7188
24	4694	5085	5474	8996	9388	9780
30	6111	6600	7087	11490	11980	12470

A skilful workman can break about 3 to 4 cub yds per day, of hard quarried stone, to a size suit for ballast; say averaging cubes of 3 ins on an edge. Where other ballast cannot be had, hard-b clay is a good substitute. The slag from iron furnaces is excellent. The ties decay more rap when gravel or sand is used instead of broken stone, because these do not drain off the rain, but the ties damp longer.

Cross-ties of 8½ ft, by 9 ins, by 7 ins, contain 3.719 cub ft each; and if placed 2½ ft a from center to center, there will be 2112 of them per mile, amounting to 291 cub yds. Therefor they are completely embedded in the ballast, they will diminish its quantity by that amount. A apart there will be 2640 of them, occupying 364 cub yds; and at 3 ft apart, 1760 of them; 243 cub

Cubic feet contained in cross-ties of different sizes.

Dimensions.		Dimensions.		Dimensions.	
Ft. Ins. Ins.	Cub. Ft.	Ft. Ins. Ins.	Cub. Ft.	Ft. Ins. Ins.	Cub
8 by 8 by 6	2.667	8½ by 8 by 6	2.833	9 by 8 by 6	3.0
8 9 6	3.000	8½ 9 6	3.188	9 9 6	3.3
8 9 7	3.500	8½ 9 7	3.719	9 9 7	3.9
8 10 6	3.333	8½ 10 6	3.542	9 10 6	3.7
8 10 7	3.889	8½ 10 7	4.132	9 10 7	4.3
8 10 8	4.444	8½ 10 8	4.722	9 10 8	5.0
8 12 8	5.333	8½ 12 8	5.667	9 12 8	6.0

The average life of cross-ties does not exceed about 6 to 8 years; and the expense for renewing is serious; amounting probably to not less than ten millions of dollars annually in the United S alone. The Robbins process of preserving timber by subjecting it to the hot vapors of coal-tar, iron chamber, also increases its hardness and strength in every respect, and to an important ex *and there* can be no doubt that its employment will in many cases be true economy. This re *applies as well to timber* for bridges, and for engineering and architectural purposes generally. *process is said to preserve* timber from the attacks of the Teredo navalis, or sea-worm; but i *opinion, there is, as yet,* no good foundation for this statement. No remedy is, we believe, yet k *for this evil, although* saturation with creasote certainly delays their attacks.

The writer believes that most of the fault usually ascribed to cross-ties, as well as to rail-j *in reality due to imperfect* drainage of the roadbed. Hence, he does not agree with those w *cate* VERY LONG TIES; but considers that with good ballast, on a well-drained roadbed, 8½ fe *and* that 8½ ft, by 9 ins, by 7 ins; and 2½ ft apart from center to center, is suffic *as more;* and that 8½ ft, by 9 ins, by 7 ins; and 2½ ft apart from center to center, is suffic *heaviest traffic.* On many important roads they are but 8 ft; and on some only 7½ ft. *ft 8½.* The actual COST OF CUTTING DOWN THE TREES, topping off the branches, and ha *eady for hauling away to be laid,* is about 6 to 9 cts per tie, at $1.75 per day per hewer.

narrow bases of rails resting immediately on the cross-ties, without chairs, frequently produce e such an amount of crushing in the ties as to injure them materially even before decay begins. H. Whitman, of Portland, Maine, has patented a mode of obviating this by letting into the tie a block of hard wood under each rail. He sells machines for cutting the grooves for receiving blocks. Burnetised ties rust the spikes away rapidly.

ost-and-rail fences, in panels 8¼ ft long; 5 rails: usually cost between 100 cents per panel, including the putting up; or from $512 to $1280 per mile ad fenced on both sides, with 1280 panels.

orm fences seven rails high, with two rails on end at each angle, cost about less. Labor $1.75 per day. The scarcity or abundance of timber chiefly inces the price; as is also the case with ties.

RAILS.

amber of rails of different lengths, to a mile of single-ck road. Length 36 ft, 293½ rails; 30 ft, 352; 25 ft, 422.4; 20 ft, 528: 18 ft, ; 16 ft, 660; 15 ft, 704. Every sq inch of sectional area of rail, corresponds to s per yard run of a single rail; or to 15.7143 tons per mile of single-track road. equently,

$$\frac{\text{Wt in lbs per yd of rail,}}{10} \text{ or } \frac{\text{Wt in tons per mile of single-track}}{15.7143} = \text{Area of rail in sq ins.}$$

is, a rail of 100 tons per mile of single track, will have a section of 6.364 sq ins; and will weigh lbs per yd of single rail. Add for turnouts, sidings, road-crossings, and a trifle for waste in g. Steel must inevitably take the place of iron for rails, or at least for their tops.* When the ties place, and the rails distributed in piles at short intervals, a gang of 6 men can lay ½ a mile of per day, of single track; or after the ballast is in place, a gang of 15 men will lay from ¾ to 1 of complete single-track superstructure per week.

pproximate average estimate for a mile of single-track railway. Labor $1.75 per day.

bing and clearing, (average of entire road,) 3 *acres at* $50	$ 150
ing; 20000 *cub yds of earth excavation, at* 35 *cts*	7000
" 2000 *cub yds of rock excavation, at* $1.00	2000
mry of culverts, drains, abutments of small bridges, retaining-walls, &c; 0 *cub yds, at* $8, *average*	3200
ast; 3000 *cub yds broken stone, at* $1.00	3000
-ties; 2112, *at* 60 *cts, delivered*	1267
s; (60 *lbs to a yard;*) 96 *tons, at* $100, *delivered*	9600
es	275
-joints, or chairs	525
delivery of materials along the line	300
ing track	600
ing; (average of entire road,) supposing only ¼ of its length to be fenced	450
ll wooden bridges, trestles, sidings, road-crossings, cattle guards, &c, &c	1000
l damages	1000
ineering, superintendence, officers of Co, stationery, instruments, rents, inting, law expenses, and other incidentals	2033
Total	$32400

l for Depots, Offices, Machine-shops, Engine-houses, Passenger and Freight Stations, Platforms, l Sheds, Water Stations with their tanks and pumps, Telegraph, Engines, Cars, Weigh Scales, , &c, &c. Also for large bridges, tunnels, &c.† Engines and cars for starting business will cost to $10000 per mile; increasing in time with the traffic.

ler peculiarly favorable circumstances, such as a level region requiring very light grading; and furnishes plenty of sand or gravel convenient for ballasting; and by the use of a lighter rail, good road may be built for $25000, or even considerably less, per mile. And, on the other hand, nstances must be somewhat unfavorable that raise the cost to more than $40000 per mile, exclu- f buildings and outfit, at prices in 1873.

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he writer infers from his own observations, that the STEEL-CAPPED RAIL of J. L. Booth, of Ro- r, New York, is likely to supersede those wholly of either iron or steel. It is an admirable tion; and we think it possesses advantages over one entirely of steel. When the steel cap wears it may be renewed without renewing the other portions of the rail.

ce of steel rails in Philada, in 1871, $95 to $110 per ton. Iron ones, about $75.

The cost of a complete set of shops of brick, for the thorough repair of *20 locomotives, and of the corresponding number of passenger and other cars; together wi e smith shop, foundry, car shop, boiler shop, copper and brass shop, paint shop, store roo sheds, offices, &c; completely furnished with steam power, lathes, planing machines, sc other necessary tools and appliances, will be about from $15000 to $100000, exclusi*

A large yard, of at least an acre, should adjoin the buildings. A moderate establisl pairs of a few engines only, may be built and furnished for $25000.

HOUSES of brick will cost about $1500 to $2000 per engine stall.

substantial WAY-STATION HOUSE, 30 × 60 ft, surrounded by a platform 12 ft wide, ng roof; for passengers and freight; will cost from $6000 to $10000, according tions, at eastern city prices.

[illegible] Curves. Chord 100 feet.

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Angle of Deflection.			Int. Ord.
						Ver. Dis. in ft.	Hor. Dis. in ft.		
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
39	[illegible]	1.192	.581	.145	54	[illegible]	[illegible]	[illegible]	[illegible]
40	[illegible]	1.191	.595	.149	56	[illegible]	[illegible]	[illegible]	[illegible]
41	[illegible]	1.221	.610	.152	58	[illegible]	[illegible]	[illegible]	[illegible]
42	[illegible]	1.250	.625	.156	3°	[illegible]	[illegible]	[illegible]	[illegible]
43	[illegible]	1.279	.639	.159	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
44	[illegible]	1.308	.654	.163	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
45	[illegible]	1.337	.668	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
46	[illegible]	1.366	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
47	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
48	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table of Radii, Middle Ordinates, &c, of Curves. Chord 100 feet. (Continued.)

The Tangential Angle is always one-half of the Angle of Deflection.

Ang. of Defl.	Rad. in ft.	Defl. Dist. in ft.	Tang. Dist. in ft.	Mid. Ord.	Ang. of Defl.	Rad. in ft.	Defl. Dist. in ft.	Tang. Dist. in ft.	Mid. Ord.
°					° ′				
4 25	1298	7.708	3.854	.963	10 15	559.7	17.87	8.935	2.238
30	1274	7.853	3.927	.981	30	546.4	18.30	9.150	2.292
35	1251	7.998	3.999	.999	45	533.8	18.73	9.365	2.347
40	1228	8.143	4.071	1.017	11	521.7	19.17	9.585	2.401
45	1207	8.289	4.145	1.036	15	510.1	19.61	9.805	2.456
50	1185	8.432	4.216	1.054	30	499.1	20.05	10.03	2.511
55	1166	8.577	4.288	1.072	45	488.5	20.50	10.25	2.566
5	1146	8.722	4.361	1.091	12	478.3	20.94	10.47	2.620
5	1127	8.869	4.434	1.109	15	468.7	21.36	10.69	2.675
10	1109	9.014	4.507	1.127	30	459.3	21.79	10.90	2.730
15	1092	9.159	4.579	1.146	45	450.3	22.21	11.12	2.785
20	1074	9 304	4.652	1.164	13	441.7	22.64	11.34	2.839
25	1058	9.449	4.724	1.182	15	433.4	23.07	11.56	2.894
30	1042	9.595	4.798	1.200	30	425.5	23.51	11.77	2.949
35	1026	9.740	4.870	1.218	45	417.7	23.94	11.99	3.000
40	1011	9.885	4.942	1.236	14	410.3	24.37	12.21	3.058
45	996.8	10.03	5.015	1.255	15	403.1	24.81	12.43	3.113
50	982.7	10.18	5.090	1.273	30	396.2	25.24	12.65	3.168
55	969.0	10.32	5.160	1.291	45	389.6	25.67	12.86	3.222
6	955.4	10.47	5.235	1.309	15	383.1	26.11	13.08	3 277
5	947.5	10.62	5.310	1.327	15	376.9	26.52	13.30	3.332
10	939.7	10.76	5.380	1.345	30	370 8	26.94	13.52	3.387
15	917.0	10.99	5.450	1.364	45	365.0	27.37	13.73	3.442
20	905.0	11.04	5.520	1.382	16	359.3	27.83	13.95	3.496
25	893.5	11.20	5.600	1.400	30	348.4	28.70	14.38	3 606
30	882.0	11.34	5.670	1.419	17	338.3	29.56	14.82	3.716
35	870.7	11.48	5.740	1.437	30	328.7	30.43	15.25	3.826
40	859.5	11.63	5.815	1.455	18	319.6	31.29	15.69	3.935
45	849.3	11.78	5.890	1.473	30	311.0	32.15	16.12	4.045
50	838.9	11.92	5.960	1.491	19	302.9	33.01	16.56	4.155
55	828.9	12.06	6.030	1.509	30	295.3	33.87	16.99	4.265
7	819.0	12.21	6.105	1.528	20	287.9	34.73	17.43	4.375
5	813.3	12 36	6.180	1.546	21	274.4	36.44	18.30	4.595
10	807.4	12.50	6.250	1.564	22	262.9	38.15	19.17	4.815
15	790.8	12.64	6.320	1.582	23	250.8	39.87	20.02	5.035
20	781.9	12.79	6.395	1 600	24	240.5	41.58	20.91	5.255
25	773.2	12.94	6.470	1.618	25	231.0	43.28	21.77	5.476
30	764.5	13.08	6.540	1.637	26	222.3	44.98	22.64	5.697
35	756.1	13.22	6.610	1.655	27	214.2	46.68	23.51	5.918
40	748.0	13.37	6.685	1.673	28	206.7	48.38	24.37	6.139
45	739.9	13.51	6.755	1.692	29	199.7	50.07	25 24	6.361
50	732 0	13.66	6.830	1.710	30	193.2	51.76	26.11	6.582
55	724.3	13.80	6.900	1.728	31	187.1	53.45	26.97	6.804
8	716.8	13.95	6.975	1.746	32	181.4	55.13	27 83	7.027
15	695.1	14.38	7.190	1 801	33	176.0	56.80	28.70	7.249
30	674.6	14.81	7.405	1.855	34	171.0	58.47	29.56	7.472
45	655.5	15.25	7.625	1.910	35	166.3	60.14	30.42	7.694
9	637.3	15.68	7.840	1.965	36	161.8	61.80	31.29	7.918
15	620.2	16.12	8.060	2.019	37	157.6	63.46	32.15	8.143
30	603.8	16.55	8.275	2.074	38	153.6	65.11	33.01	8 367
45	588.4	16.99	8.495	2.128	39	149.8	66.76	33.87	8 592
10	573.7	17.43	8.715	2.183	40	146.2	68.40	34.73	8.816

For ordinates 5 ft apart, for Chords of 100 ft, see p 633.

To find tangential and deflection angles for any given rad and hord. Divide *half* the chord by the rad. The quot will be nat sine of the tangl ng. Find this tangl ang in the table of nat sines; and mult it by 2 for the def ang.

To find the def dist for chords 100 ft long. Div 10000 by the rad n feet.

To find the def dist for equal chords of any given length. Div *half* the given chord by the rad. Mult the quot by 2. Mult the prod by the hord.

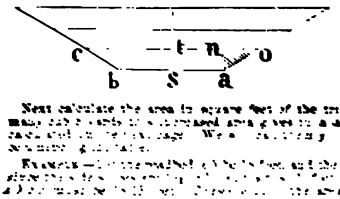
To find the tangl dist for equal chords of any given length. *First find the tangl ang as above.* Divide it by 2. Find in the table of nat sines the *at sine of the quot. Mult this* nat sine by the given chord.

To find the rad to any given def ang, for equal chords of any leng *te the ang from 180°.* Find the nat sine of *half* the rem. Mult this nat sine *chord. Div the prod by* nat sine of def ang.

Table of Middle Ordinates, to be used for the bending of rails of different lengths, so as to form portions of curves of different radii. Ordinates for lengths or radii intermediate of those in the table, may be found by simple proportion.

Def. Ang.	Radius.	LENGTHS OF RAILS.												
		30	28	26	24	22	20	18	16	14	12	10	8	6
Deg.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
.5	11460.	.010	.008	.006	.005	.004	.004	.003	.002	.002	.001	.001	.000	.000
1.	5730.	.020	.016	.013	.011	.009	.008	.006	.005	.004	.003	.002	.001	.001
1.5	3820.	.029	.026	.021	.018	.016	.013	.010	.008	.006	.004	.003	.002	.001
2.	2865.	.038	.034	.029	.025	.021	.017	.014	.011	.008	.006	.004	.003	.001
2.5	2292.	.049	.043	.037	.031	.027	.022	.018	.014	.010	.007	.005	.003	.002
3.	1910.	.058	.051	.044	.037	.031	.026	.022	.017	.012	.009	.006	.004	.002
3.5	1637.	.070	.061	.052	.043	.037	.031	.025	.020	.015	.011	.008	.005	.003
4.	1433.	.079	.069	.060	.050	.042	.035	.029	.023	.018	.013	.009	.006	.003
4.5	1274.	.088	.077	.067	.056	.047	.039	.032	.026	.020	.015	.010	.007	.004
5.	1146.	.099	.086	.074	.063	.053	.044	.035	.029	.022	.016	.011	.007	.004
5.5	1042.	.108	.094	.082	.070	.059	.048	.039	.032	.024	.018	.012	.008	.004
6.	955.4	.117	.102	.088	.076	.064	.052	.042	.034	.026	.019	.013	.008	.005
6.5	882.	.128	.112	.097	.082	.069	.057	.046	.037	.028	.021	.014	.009	.005
7.	819.	.137	.120	.104	.088	.074	.061	.049	.039	.030	.022	.015	.010	.005
7.5	764.5	.146	.127	.111	.094	.079	.065	.053	.042	.032	.024	.016	.010	.006
8.	716.8	.158	.137	.119	.100	.083	.070	.056	.045	.034	.025	.017	.011	.006
8.5	674.6	.166	.145	.126	.106	.090	.074	.060	.048	.036	.027	.018	.012	.007
9.	637.3	.175	.153	.133	.112	.095	.078	.063	.050	.038	.029	.019	.012	.007
9.5	603.8	.187	.163	.141	.119	.101	.083	.067	.054	.042	.031	.021	.013	.008
10	573.7	.196	.171	.148	.125	.106	.087	.071	.057	.045	.032	.022	.014	.008
11	521.7	.216	.188	.163	.139	.117	.096	.078	.063	.049	.036	.024	.016	.009
12	478.4	.236	.206	.179	.151	.128	.105	.085	.069	.053	.039	.026	.017	.010
13	441.7	.254	.222	.192	.163	.138	.113	.092	.073	.057	.042	.028	.019	.010
14	410.3	.275	.239	.207	.175	.148	.122	.099	.080	.061	.045	.030	.020	.011
15	383.1	.295	.257	.223	.188	.159	.131	.106	.085	.065	.048	.033	.021	.012
16	359.3	.317	.273	.238	.200	.170	.140	.113	.091	.070	.052	.035	.023	.013
17	338.3	.338	.290	.252	.213	.180	.148	.120	.096	.074	.055	.037	.024	.014
18	319.6	.351	.306	.265	.225	.190	.156	.127	.102	.078	.058	.039	.025	.014
19	302.9	.371	.324	.280	.238	.201	.165	.134	.108	.082	.061	.041	.027	.015
20	287.4	.392	.341	.296	.250	.212	.174	.141	.114	.087	.065	.044	.028	.016
21	273.4	.413	.357	.310	.262	.222	.182	.148	.120	.091	.068	.046	.030	.017
22	260.	.433	.375	.325	.275	.233	.191	.155	.126	.096	.072	.048	.031	.018
23	250.8	.450	.390	.338	.287	.243	.199	.162	.131	.100	.075	.050	.033	.019
24	240.5	.469	.408	.354	.299	.253	.208	.169	.137	.104	.078	.052	.034	.019
25	231.	.488	.424	.367	.311	.263	.216	.176	.142	.108	.081	.054	.035	.020
26	222.3	.501	.441	.382	.323	.274	.225	.183	.148	.112	.084	.056	.037	.021
27	214.2	.524	.457	.396	.335	.284	.233	.190	.153	.116	.087	.058	.038	.022
28	206.7	.545	.475	.411	.348	.294	.242	.197	.158	.120	.090	.060	.039	.022
29	199.7	.564	.491	.425	.361	.305	.250	.203	.164	.124	.093	.062	.041	.023

To prepare a Table, T, next page, of Level Cuttings, for every $\frac{1}{10}$ of a foot of height, or depth.



Let the diagram represent the section [illegible] inverted, the [illegible] which the [illegible] are supposed to be [illegible] feet wide. First calculate the area in square feet of the [illegible] adjoining the [illegible]. Then find how many cable yards that area gives in a [illegible] of [illegible] feet. These cable yards [illegible] Y may form the first [illegible] in the Table T.

Next calculate the area in square feet of the [illegible] Find how many cable yards the increased area gives in a distance of [illegible] feet. [illegible] ready [illegible] page. We [illegible] before [illegible]

Example — [illegible] of the [illegible] of [illegible] feet [illegible] the [illegible] of [illegible] in a [illegible] = [illegible] square ft. [illegible] = [illegible] cubic yards. [illegible] Y and [illegible] in the [illegible] which [illegible] in [illegible] the [illegible] should be [illegible] by [illegible]

full depth arrived at; multiply it by 100, and divide the product by 27 for the cubic yards rards thus found should agree with the table.

..... 6.7222	..... Y. 6.722	.1	
..... .1111			
6.8333	6.8333		
..... .1111			
	13.5555	.2	
6.9444	6.9444		
..... .1111			
	20 5000	.3	
7.0555	7.0555		
..... .1111			
	27.5555	.4	
7.1666	7.1666		
..... .1111			
	34.7222	.5	
7.2777	7.2777		
	42.0000	.6	

TABLE T.

Height. Feet.	Cub. Yds.
.1	6.72 Y.
.2	13.6
.3	20.5
.4	27.6
.5	34.7
.6	42.0
&c.	

lowing table contains y, ready calculated for different side-slopes. It plainly he same for all widths of roadbed.

-slope.	y	Side-slope.	y
o 1	..... .0185	1¾ to 1	..... .1296
o 1	..... .0370	2 to 1	..... .1482
o 1	..... .0556	2¼ to 1	..... .1667
o 1	..... .0741	2½ to 1	..... .1852
o 1	..... .0926	3 to 1	..... .2222
o 1	..... .1111	4 to 1	..... .2963

Table of Long Chords.

of Chord in ft, required to subtend from 1 to 4 stations of 100 ft each.

Sta.	2 Sta.	3 Sta.	4 Sta.	Ang. of Defl.	1 Sta.	2 Sta.	3 Sta.	4 Sta.
100	200.0	300.0	400.0	¾	100	199.7	298.9	397.5
100	200.0	300.0	399.9	6°	100	199.7	298.8	397.3
100	200.0	300.0	399.9	¼	100	199.7	298.7	397.0
100	200.0	300.0	399.8	½	100	199.7	298.6	396.7
100	200.0	299.9	399.7	¾	100	199.6	298.5	396.5
100	200.0	299.9	399.6	7°	100	199.6	298.4	396.2
100	200.0	299.8	399.5	¼	100	199.6	298.3	396.0
100	200.0	299.8	399.4	½	100	199.6	298.2	395.7
100	200.0	299.7	399.3	¾	100	199.6	298.1	395.4
100	200.0	299.7	399.2	8°	100	199.6	298.0	395.1
100	200.0	299.6	399.1	¼	100	199.5	297.9	394.8
100	200.0	299.6	399.0	½	100	199.5	297.8	394.5
100	199.9	299.6	398.9	¾	100	199.4	297.7	394.3
100	199.9	299.5	398.7	9°	100	199.4	297.5	394.1
100	199.9	299.4	398.5	¼	100	199.4	297.4	393.7
100	199.9	299.3	398.3	½	100	199.3	297.3	393.2
100	199.9	299.2	398.0	¾	100	199.2	297.2	392.8
100	199 8	299.1	397.8	10°	100	199.2	297.0	392.4
100	199.8	299.0	397.6					

llowing tables of **level cuttings** are from the author's book *surement of Excavations* and **Embankments.**

Table I. Level Cuttings.

Roadway 14 feet wide, side-slopes 1½ to 1.

For single-track embankment.

Height in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		5.36	10.6	16.1	21.6	27.3	33.1	39.0	45.0
1	57.4	63.9	70.2	76.8	83.5	90.3	97.2	104.2	111.3
2	125.9	133.4	140.9	148.6	156.4	164.4	172.4	180.5	188.7
3	196.9	214.1	222.8	231.6	240.5	249.5	258.7	267.9	277.3
4	296.3	306.0	315.8	325.7	335.7	345.8	356.1	366.4	376.9
5	398.1	408.9	419.9	430.9	442.0	453.2	464.6	476.1	487.6
6	511.1	523.0	535.0	547.2	559.4	571.8	584.2	596.8	609.5
7	622.2	635.2	648.3	661.6	675.0	701.4	714.9	728.6	742.4
8	770.3	784.5	798.7	813.1	827.5	842.1	856.8	871.6	886.5
9	901.7	916.9	947.3	962.7	978.3	994.0	1010	1026	1042
10	1074	1090	1107	1123	1140	1157	1174	1191	1208
11	1243	1260	1278	1295	1313	1331	1349	1367	1385
12	1422	1441	1459	1478	1497	1516	1535	1554	1574
13	1613	1653	1652	1672	1692	1712	1733	1753	1773
14	1815	1835	1856	1877	1898	1920	1941	1962	1984
15	2028	2050	2072	2094	2116	2138	2161	2183	2206
16	2252	2275	2298	2321	2344	2368	2391	2415	2439
17	2487	2511	2535	2559	2584	2608	2633	2658	2683
18	2733	2759	2784	2809	2835	2861	2886	2912	2938
19	2991	3017	3044	3070	3097	3124	3151	3178	3205
20	3259	3287	3314	3342	3370	3398	3426	3454	3482
21	3539	3567	35.6	3625	3654	3683	3712	3741	3771
22	3830	3859	3889	3919	3949	3979	4009	4040	4070
23	4132	4162	4193	4224	4255	4287	4318	4349	4381
24	4444	4476	4508	4541	4573	4605	4638	4670	4703
25	4769	4802	4835	4868	4901	4935	4968	5002	5036
26	5104	5138	5172	5206	5241	5275	5310	5345	5380
27	5450	5485	5521	5556	5592	5627	5663	5699	5735
28	5807	5844	5880	5917	5953	5990	6027	6064	6101
29	6176	6213	6251	6289	6326	6364	6402	6440	6479
30	6556	6594	6633	6672	6711	6750	6789	6828	6867
31	6946	6986	7026	7066	7106	7146	7186	7226	7267
32	7348	7389	7430	7471	7512	7553	7595	7636	7678
33	7761	7803	7845	7887	7929	7972	8014	8057	8099
34	8185	8228	8271	8315	8358	8401	8445	8489	8532
35	8620	8664	8709	8753	8798	8842	8887	8932	8976
36	9067	9112	9157	9203	9248	9294	9340	9386	9432
37	9524	9570	9617	9663	9710	9757	9804	9851	9898
38	9993	10040	10088	10135	10183	10231	10279	10327	10375
39	10472	10521	10569	10618	10667	10716	10765	10815	10864
40	10963	11013	11062	11112	11162	11212	11263	11313	11364
41	11465	11516	11567	11618	11669	11720	11771	11823	11874
42	11978	12029	12081	12134	12186	12238	12291	12343	12396
43	12502	12555	12608	12661	12715	12768	12822	12875	12929
44	13037	13091	13145	13200	13254	13309	13363	13418	13473
45	13583	13639	13694	13749	13805	13861	13916	13972	14028
46	14141	14197	14254	14310	14367	14424	14480	14537	14595
47	14709	14767	14824	14882	14940	14998	15056	15114	15172
48	15289	15347	15406	15465	15524	15583	15642	15701	15761
49	15880	15939	15999	16059	16119	16179	16239	16300	16360
50	16481	16542	16603	16664	16725	16787	16848	16909	16971
51	17094	17156	17218	17280	17343	17405	17467	17530	17593
52	17719	17782	17845	17908	17971	18035	18098	18162	18226
53	18354	18418	18482	18546	18611	18675	18740	18805	18870
54	19000	19065	19131	19196	19262	19327	19393	19459	19525
55	19657	19724	19790	19857	19923	19990	20057	20124	20191
56	20326	20393	20461	20529	20596	20664	20732	20800	20869
57	21005	21074	21143	21212	21280	21349	21419	21488	21557
58	21696	21766	21836	21906	21976	22046	22116	22186	22257
59	22398	22469	22540	22611	22682	22753	22825	22896	22968
60	23111	23183	23255	23327	23399	23472	23544	23617	23689

For continuation to 100 feet, see TABLE I.

Table 2. Level Cuttings.

Roadway 24 feet wide, side-slopes 1½ to 1.

For double-track embankment.

.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
	8.94	18.0	27.2	36.4	45.8	55.3	64.9	74.7	84.5
94.4	104.5	114.7	124.9	135.3	145.8	156.4	167.2	178.0	188.9
200.0	211.2	222.4	233.8	245.3	256.9	268.6	280.5	292.4	304.4
316.6	328.9	341.2	353.7	366.3	379.0	391.9	404.8	417.8	431.0
444.4	457.8	471.3	484.9	498.6	512.4	526.4	540.4	554.6	568.8
583.3	597.8	612.4	627.1	642.0	656.9	671.9	687.1	702.3	717.7
733.3	748.9	764.7	780.5	796.4	812.5	828.7	844.9	861.3	877.8
894.4	911.2	928.0	944.9	962.0	979.2	996.4	1014	1031	1049
1067	1085	1102	1121	1139	1157	1175	1194	1212	1231
1250	1269	1288	1307	1326	1346	1365	1385	1405	1425
1444	1465	1485	1505	1525	1546	1566	1587	1608	1629
1650	1671	1692	1714	1735	1757	1779	1800	1822	1845
1867	1889	1911	1934	1956	1979	2002	2025	2048	2071
2094	2118	2141	2165	2189	2213	2236	2261	2285	2309
2333	2358	2382	2407	2432	2457	2482	2507	2532	2558
2583	2609	2635	2661	2686	2713	2739	2765	2791	2818
2844	2871	2898	2925	2952	2979	3006	3034	3061	3089
3117	3145	3172	3201	3229	3257	3285	3314	3342	3371
3400	3429	3458	3487	3516	3546	3575	3605	3635	3665
3694	3725	3755	3785	3815	3846	3876	3907	3938	3969
4000	4031	4062	4094	4125	4157	4189	4221	4252	4285
4317	4349	4381	4414	4446	4479	4512	4545	4578	4611
4644	4678	4711	4745	4779	4813	4846	4881	4915	4949
4983	5018	5052	5087	5122	5157	5192	5227	5262	5298
5333	5369	5405	5441	5476	5513	5549	5585	5621	5658
5694	5731	5768	5805	5842	5879	5916	5954	5991	6029
6067	6105	6142	6181	6219	6257	6295	6334	6372	6411
6450	6489	6528	6567	6606	6646	6685	6725	6765	6805
6844	6885	6925	6965	7005	7046	7086	7127	7168	7209
7250	7291	7332	7374	7415	7457	7499	7541	7582	7625
7667	7709	7751	7794	7836	7879	7922	7965	8008	8051
8094	8138	8181	8225	8269	8313	8356	8401	8445	8489
8533	8578	8622	8667	8712	8757	8802	8847	8892	8938
8983	9029	9075	9121	9166	9212	9259	9305	9351	9398
9444	9491	9538	9585	9632	9679	9726	9774	9821	9869
9917	9965	10012	10061	10109	10157	10205	10254	10302	10351
10400	10449	10498	10547	10596	10646	10695	10745	10795	10845
10894	10945	10995	11045	11095	11146	11196	11247	11298	11349
11400	11451	11502	11554	11605	11657	11709	11761	11812	11865
11917	11969	12021	12074	12126	12179	12232	12285	12338	12391
12444	12498	12551	12605	12659	12713	12766	12821	12875	12929
12983	13038	13092	13147	13202	13257	13312	13367	13422	13478
13533	13589	13645	13701	13756	13813	13869	13925	13981	14038
14094	14151	14208	14265	14322	14379	14436	14494	14551	14609
14667	14725	14782	14840	14899	14957	15015	15074	15132	15191
15250	15309	15368	15427	15486	15546	15605	15665	15725	15785
15844	15905	15965	16025	16085	16146	16206	16267	16328	16389
16450	16511	16572	16634	16695	16757	16819	16881	16942	17005
17067	17129	17191	17254	17316	17379	17442	17505	17568	17631
17694	17758	17821	17885	17949	18013	18076	18141	18205	18269
18333	18398	18462	18527	18592	18657	18722	18787	18852	18918
18983	19049	19115	19181	19246	19313	19379	19445	19511	19578
19644	19711	19778	19845	19912	19979	20046	20114	20181	20249
20317	20385	20452	20521	20589	20657	20725	20794	20862	20931
21000	21069	21138	21207	21276	21346	21415	21485	21555	21625
21694	21765	21835	21905	21975	22046	22116	22187	22258	22329
22400	22471	22542	22614	22685	22757	22829	22901	22972	23
23117	23189	23261	23334	23406	23479	23552	23625	23698	2
23844	23918	23991	24065	24139	24213	24286	24361	24435	
24583	24658	24732	24807	24882	24957	25032	25107	25182	
25333	25409	25485	25561	25636	25713	25789	25865	25941	

For continuation to 100 feet, see TABLE 7.

Table 3. Level Cuttings.

Roadway 18 feet wide, side-slopes 1 to 1.

For single-track excavation.

Depth in Ft.	.0	.1	.2	.3	.4	.5	.6	.7
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		6.70	13.5	20.3	27.3	34.3	41.3	48.5
1	70.4	77.8	85.3	92.9	100.6	108.3	116.1	124.0
2	148.1	156.3	164.6	172.9	181.3	189.8	198.4	207.0
3	233.3	242.3	251.3	260.3	269.5	278.7	288.0	297.4
4	325.9	335.6	345.3	355.1	365.0	375.0	385.0	395.1
5	425.9	436.3	446.8	457.4	468.0	478.7	489.5	500.3
6	533.3	544.5	555.7	567.0	578.4	589.8	601.3	612.9
7	648.1	660.0	672.0	684.0	696.1	708.3	720.6	732.9
8	770.4	783.0	795.7	808.5	821.3	834.3	847.3	860.3
9	900.0	913.4	926.8	940.3	953.9	967.6	981.3	995.1
10	1037	1051	1065	1080	1094	1108	1123	1137
11	1181	1196	1211	1226	1241	1256	1272	1287
12	1333	1349	1365	1380	1396	1412	1428	1444
13	1493	1509	1525	1542	1558	1575	1592	1608
14	1659	1676	1693	1711	1728	1745	1763	1780
15	1833	1851	1869	1887	1905	1923	1941	1960
16	2015	2033	2052	2071	2089	2108	2127	2146
17	2204	2223	2242	2262	2281	2301	2321	2340
18	2400	2420	2440	2460	2481	2501	2521	2542
19	2604	2624	2645	2666	2687	2708	2729	2751
20	2815	2836	2858	2880	2901	2923	2945	2967
21	3013	3056	3078	3100	3123	3145	3168	3191
22	3259	3282	3305	3328	3352	3375	3398	3422
23	3433	3516	3540	3564	3588	3612	3636	3660
24	3733	3758	3782	3807	3832	3856	3881	3906
25	3981	4007	4032	4057	4083	4108	4134	4160
26	4237	4263	4289	4315	4311	4368	4394	4420
27	4500	4527	4553	4580	4607	4634	4661	4688
28	4770	4798	4825	4853	4881	4908	4936	4964
29	5048	5076	5105	5133	5161	5190	5218	5247
30	5333	5362	5391	5420	5449	5479	5508	5537
31	5626	5656	5685	5715	5745	5775	5805	5835
32	5926	5956	5987	6017	6048	6079	6109	6140
33	6233	6264	6296	6327	6358	6390	6421	6453
34	6548	6580	6612	6644	6676	6708	6741	6773
35	6870	6903	6936	6968	7001	7034	7067	7100
36	7200	7233	7267	7300	7334	7368	7401	7435
37	7537	7571	7605	7640	7674	7708	7743	7777
38	7881	7916	7951	7986	8021	8056	8092	8127
39	8233	8269	8305	8340	8376	8412	8448	8484
40	8593	8629	8665	8702	8738	8775	8812	8848
41	8959	8996	9033	9071	9108	9145	9183	9220
42	9333	9371	9409	9447	9485	9523	9561	9600
43	9715	9753	9792	9831	9869	9908	9947	9986
44	10104	10143	10182	10222	10261	10301	10341	10380
45	10500	10540	10580	10620	10661	10701	10741	10782
46	10904	10944	10985	11026	11067	11108	11149	11191
47	11315	11356	11398	11440	11481	11523	11565	11607
48	11733	11776	11818	11860	11903	11945	11988	12031
49	12159	12202	12245	12288	12332	12375	12418	12462
50	12593	12636	12680	12724	12768	12812	12856	12900
51	13033	13078	13122	13167	13212	13256	13301	13346
52	13481	13527	13572	13617	13663	13708	13754	13800
53	13937	13983	14029	14075	14121	14168	14214	14260
54	14400	14447	14493	14540	14587	14634	14681	14728
55	14870	14918	14965	15013	15061	15108	15156	15204
56	15348	15396	15445	15493	15541	15590	15638	15687
57	15833	15882	15931	15980	16029	16079	16128	1717
58	16326	16376	16425	16475	16525	16575	16625	166
59	16826	16876	16927	16977	17028	17079	17129	17
60	17333	17384	17436	17487	17538	17590	17641	1

For continuation to 100 feet deep, see Table 7.

Table 4. Level Cuttings.

Roadway 18 feet, side-slopes 1½ to 1.

For single-track excavation.

.1	.2	.3	.4	.5	.6	.7	.8	.9
Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
6.72	13.6	20.5	27.6	34.7	42.0	49.4	56.9	64.5
80.1	88.0	96.1	104.2	112.5	120.9	129.4	138.0	146.7
164.5	173.5	182.7	191.9	201.3	210.8	220.4	230.1	240.0
260.0	270.1	280.4	290.8	301.3	311.9	322.6	333.4	344.5
366.7	378.0	389.4	400.9	412.5	424.2	436.0	448.0	460.0
484.5	496.9	509.4	522.0	534.7	547.6	560.5	573.6	586.7
613.4	626.9	640.5	654.2	668.1	682.0	696.1	710.2	724.5
753.4	768.0	782.7	797.6	812.5	827.6	842.7	858.0	873.4
904.5	920.2	936.1	952.0	968.1	984.2	1001	1017	1033
1067	1084	1101	1118	1135	1152	1169	1187	1205
1240	1258	1276	1294	1313	1331	1349	1368	1387
1425	1444	1463	1482	1501	1521	1541	1560	1580
1620	1640	1661	1681	1701	1722	1743	1764	1785
1827	1848	1869	1891	1913	1934	1956	1978	2000
2045	2067	2089	2112	2135	2158	2181	2204	2227
2273	2297	2321	2344	2368	2392	2416	2440	2465
2513	2538	2563	2588	2613	2638	2663	2688	2713
2765	2790	2816	2842	2868	2894	2921	2947	2973
3027	3054	3081	3108	3135	3162	3189	3217	3245
3300	3328	3356	3384	3413	3441	3469	3498	3527
3585	3614	3643	3672	3701	3731	3761	3790	3820
3880	3910	3941	3971	4001	4032	4063	4094	4125
4187	4218	4249	4281	4313	4344	4376	4408	4440
4505	4537	4569	4602	4635	4668	4701	4734	4767
4833	4867	4901	4934	4968	5002	5036	5070	5105
5173	5208	5243	5278	5313	5348	5383	5418	5453
5525	5560	5596	5632	5668	5704	5741	5777	5813
5887	5924	5961	5998	6035	6072	6109	6147	6185
6260	6298	6336	6374	6413	6451	6489	6528	6567
6645	6684	6723	6762	6801	6841	6881	6920	6960
7040	7080	7121	7161	7201	7242	7283	7324	7365
7447	7488	7529	7571	7613	7654	7696	7738	7780
7865	7907	7949	7992	8035	8078	8121	8164	8207
8293	8337	8381	8424	8468	8512	8556	8600	8645
8733	8778	8823	8868	8913	8958	9003	9048	9093
9185	9230	9276	9322	9368	9414	9461	9507	9553
9647	9694	9741	9788	9835	9882	9929	9977	10025
10120	10168	10216	10264	10313	10361	10409	10458	10507
10605	10654	10703	10752	10801	10851	10901	10950	11000
11100	11150	11200	11251	11301	11352	11403	11454	11505
11607	11658	11709	11761	11813	11864	11916	11968	12020
12125	12177	12229	12282	12335	12388	12441	12494	12547
12653	12707	12761	12814	12868	12922	12976	13030	13085
13193	13248	13303	13358	13413	13468	13523	13578	13633
13745	13800	13856	13912	13968	14024	14081	14137	14193
14307	14364	14421	14478	14535	14592	14649	14707	14765
14880	14938	14996	15054	15113	15171	15229	15288	15347
15465	15524	15583	15642	15701	15761	15821	15880	15940
16060	16120	16181	16241	16301	16362	16423	16484	16545
16667	16728	16789	16851	16913	16974	17036	17098	17160
17285	17347	17409	17472	17535	17598	17661	17724	17787
17913	17977	18041	18104	18168	18232	18296	18360	18425
18553	18618	18683	18748	18813	18878	18943	19008	19073
19205	19270	19336	19402	19468	19534	19601	19667	19733
19867	19934	20000	20068	20135	20202	20269	20337	20405
20540	20608	20676	20744	20813	20881	20949	21018	21087
21225	21294	21363	21432	21501	21571	21641	21710	21780
21920	21990	22061	22131	22201	22272	22343	22414	22485
22627	22698	22769	22841	22913	22984	23056	23128	23200
23345	23417	23489	23562	23635	23708	23781	23854	23927
24073	24147	24221	24294	24368	24442	24516	24590	24665

For continuation to 100 feet deep, see Table 7.

Table 5. Level Cuttings.

Roadway 28 feet wide, side-slopes 1 to 1.

For double-track excavation.

Depth in Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
0		10.4	20.9	31.4	42.1	52.8	63.6	74.4	85.3	96.3
1	107.4	118.6	129.8	141.1	152.4	163.9	175.4	187.0	198.7	210.4
2	222.2	234.1	246.1	258.1	270.2	282.4	294.7	307.0	319.4	331.9
3	344.4	357.1	369.8	382.6	395.4	408.3	421.3	434.4	447.6	460.8
4	474.1	487.4	500.9	514.4	528.0	541.7	555.4	569.2	583.1	597.1
5	611.1	625.2	639.4	653.7	668.0	682.4	696.9	711.4	726.1	740.8
6	755.6	770.4	785.4	800.4	815.5	830.6	845.8	861.1	876.5	891.9
7	907.5	923.0	938.7	954.5	970.3	986.2	1002	1018	1034	1050
8	1067	1083	1099	1116	1132	1149	1166	1182	1199	1216
9	1233	1250	1267	1285	1302	1319	1337	1354	1372	1390
10	1407	1425	1443	1461	1479	1497	1515	1534	1552	1570
11	1589	1607	1626	1645	1664	1682	1701	1720	1739	1759
12	1778	1797	1816	1836	1855	1875	1895	1914	1934	1954
13	1974	1994	2014	2034	2055	2075	2095	2116	2136	2157
14	2178	2199	2219	2240	2261	2282	2304	2325	2346	2367
15	2389	2410	2432	2454	2475	2497	2519	2541	2563	2585
16	2607	2630	2652	2674	2697	2719	2742	2765	2788	2810
17	2833	2856	2879	2903	2926	2949	2972	2996	3019	3043
18	3067	3090	3114	3138	3162	3186	3210	3234	3259	3283
19	3307	3332	3356	3381	3406	3431	3455	3480	3505	3530
20	3556	3581	3606	3631	3657	3682	3708	3734	3759	3785
21	3811	3837	3863	3889	3915	3942	3968	3994	4021	4047
22	4074	4101	4128	4154	4181	4208	4235	4263	4290	4317
23	4344	4372	4399	4427	4455	4482	4510	4538	4566	4594
24	4622	4650	4679	4707	4735	4764	4792	4821	4850	4879
25	4907	4936	4965	4994	5024	5053	5082	5111	5141	5170
26	5200	5230	5259	5289	5319	5349	5379	5409	5439	5470
27	5500	5530	5561	5591	5622	5653	5684	5714	5745	5776
28	5807	5839	5870	5901	5932	5964	5995	6027	6059	6090
29	6122	6154	6186	6218	6250	6282	6315	6347	6379	6412
30	6444	6477	6510	6543	6575	6608	6641	6674	6708	6741
31	6774	6807	6841	6874	6908	6942	6975	7009	7043	7077
32	7111	7145	7179	7214	7248	7282	7317	7351	7386	7421
33	7456	7490	7525	7560	7595	7631	7666	7701	7736	7772
34	7807	7843	7879	7914	7950	7986	8022	8058	8094	8130
35	8167	8203	8239	8276	8312	8349	8386	8423	8459	8496
36	8533	8570	8608	8645	8682	8719	8757	8794	8832	8870
37	8907	8945	8983	9021	9059	9097	9135	9174	9212	9250
38	9289	9327	9366	9405	9444	9482	9521	9560	9599	9639
39	9678	9717	9756	9796	9835	9875	9915	9954	9994	10034
40	10074	10114	10154	10194	10235	10275	10315	10356	10396	10437
41	10478	10519	10559	10600	10641	10682	10724	10765	10806	10847
42	10889	10930	10972	11014	11055	11097	11139	11181	11223	11265
43	11307	11350	11392	11434	11477	11519	11562	11605	11648	11690
44	11733	11776	11819	11863	11906	11949	11992	12036	12079	12123
45	12167	12210	12254	12298	12342	12386	12430	12474	12519	12563
46	12607	12652	12696	12741	12786	12831	12875	12920	12965	13010
47	13056	13101	13146	13191	13237	13282	13328	13374	13419	13465
48	13511	13587	13603	13649	13695	13742	13788	13834	13881	13927
49	13974	14021	14068	14114	14161	14208	14255	14303	14350	14397
50	14444	14492	14539	14587	14635	14682	14730	14778	14826	14874
51	14922	14970	15019	15067	15115	15164	15212	15261	15310	15359
52	15407	15456	15505	15554	15604	15653	15702	15751	15801	15850
53	15900	15950	15999	16049	16099	16149	16199	16249	16299	16350
54	16400	16450	16501	16551	16602	16653	16704	16754	16805	16856
55	16907	16959	17010	17061	17112	17164	17215	17267	17319	17370
56	17422	17474	17526	17578	17630	17682	17735	17787	17839	17892
57	17944	17997	18050	18103	18155	18208	18261	18314	18368	18421
58	18474	18527	18581	18634	18688	18742	18795	18849	18903	18957
59	19011	19065	19119	19174	19228	19282	19337	19391	19446	19500
60	19556	19610	19665	19720	19775	19831	19886	19941	19996	20052

For continuation to 100 feet, see Table 7.

Table 6. Level Cuttings.

Roadway 28 ft wide, side-slopes 1½ to 1.

For double-track excavation.

h	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.	Cu. Yds.
		10.4	21.0	31.6	42.4	53.2	64.2	75.3	86.5	97.9
	109.3	120.8	132.5	144.3	156.1	168.1	180.2	192.4	204.8	217.2
	229.6	242.3	255.0	267.9	280.9	294.0	307.2	320.5	334.0	347.5
	361.2	374.9	388.8	402.8	416.9	431.1	445.4	459.9	474.4	489.1
	503.7	518.6	533.6	548.6	563.9	579.3	594.7	610.2	625.8	641.6
	657.5	673.4	689.5	705.7	722.1	738.5	755.0	771.7	788.4	805.3
	822.2	839.3	856.5	873.8	891.2	908.8	926.4	944.2	962.0	980.0
	998.1	1016	1035	1053	1072	1090	1109	1128	1147	1166
	1185	1204	1224	1243	1263	1283	1303	1322	1343	1363
	1383	1403	1424	1445	1465	1486	1507	1528	1549	1571
	1592	1614	1635	1657	1679	1701	1723	1745	1767	1790
	1812	1835	1858	1881	1904	1927	1950	1973	1997	2020
	2044	2068	2092	2116	2140	2164	2189	2213	2238	2262
	2287	2312	2337	2362	2387	2413	2438	2464	2489	2515
	2541	2567	2593	2619	2645	2672	2698	2725	2752	2779
	2806	2833	2860	2887	2915	2942	2970	2997	3025	3053
	3081	3109	3138	3166	3195	3223	3252	3281	3310	3339
	3368	3397	3427	3456	3486	3516	3546	3576	3606	3636
	3667	3697	3728	3758	3789	3820	3851	3882	3913	3944
	3976	4007	4039	4070	4102	4134	4166	4198	4231	4263
	4296	4328	4361	4394	4427	4460	4493	4527	4560	4594
	4627	4661	4695	4729	4763	4797	4832	4866	4900	4935
	4970	5005	5040	5075	5111	5146	5181	5217	5253	5288
	5324	5360	5396	5432	5469	5505	5542	5578	5615	5652
	5689	5726	5763	5800	5838	5875	5913	5951	5989	6027
	6065	6103	6141	6179	6218	6257	6295	6334	6373	6412
	6451	6491	6530	6570	6609	6649	6689	6729	6769	6809
	6850	6890	6931	6971	7012	7053	7094	7135	7176	7217
	7259	7300	7342	7384	7426	7468	7510	7552	7594	7637
	7680	7722	7765	7808	7851	7894	7937	7981	8024	8067
	8111	8155	8199	8243	8287	8331	8375	8420	8464	8509
	8554	8598	8643	8688	8734	8779	8824	8870	8915	8961
	9007	9053	9099	9145	9191	9238	9284	9331	9378	9425
	9472	9519	9566	9613	9661	9708	9756	9804	9851	9900
	9948	9997	10045	10093	10142	10190	10239	10288	10337	10386
	10435	10484	10534	10583	10633	10683	10732	10782	10832	10882
	10933	10983	11034	11084	11135	11186	11237	11288	11339	11391
	11443	11494	11546	11598	11649	11701	11753	11806	11858	11910
	11963	12016	12068	12121	12174	12227	12281	12334	12387	12441
	12494	12548	12602	12656	12710	12764	12819	12873	12928	12982
	13037	13092	13147	13202	13257	13312	13368	13423	13479	13535
	13591	13647	13703	13759	13815	13872	13928	13985	14042	14099
	14156	14213	14270	14327	14385	14442	14500	14558	14615	14673
	14731	14790	14848	14906	14965	15024	15082	15141	15200	15259
	15318	15378	15437	15497	15556	15616	15676	15736	15796	15856
	15917	15977	16038	16098	16159	16220	16281	16342	16403	16465
	16526	16587	16649	16711	16773	16835	16897	16959	17021	17084
	17146	17209	17272	17335	17398	17461	17524	17587	17651	17714
	17778	17842	17905	17969	18033	18098	18162	18226	18291	18356
	18420	18485	18550	18615	18680	18746	18811	18877	18942	19008
	19074	19140	19206	19272	19339	19405	19472	19538	19605	19672
	19739	19806	19873	19940	20008	20075	20143	20211	20279	20347
	20415	20483	20551	20620	20688	20757	20826	20894	20963	21032
	21102	21171	21241	21310	21380	21450	21519	21589	21659	21730
	21800	21870	21941	22012	22082	22153	22224	22295	22366	22438
	22509	22581	22652	22724	22796	22868	22940	23012	23085	23157
	23230	23302	23375	23448	23521	23594	23667	23741	23814	23888
	23961	24035	24109	24183	24257	24331	24405	24480	24554	24629
	24704	24779	24854	24929	25004	25079	25155	25230	25306	25381
	25457	25533	25609	25686	25762	25838	25915	25992	26068	26145
	26222	26299	26376	26454	26531	26609	26686	26764	26842	26920

For continuation to 100 feet, see Table 7.

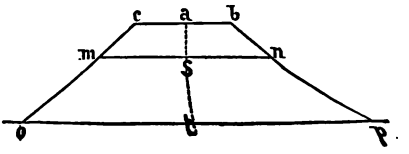
Table 8,

Of Cubic Yards in a 100-foot station of level cutting or filling, to be added to, or subtracted from, the quantities in the preceding seven tables, in case the excavations or embankments should be increased or diminished 2 feet in width.

Cubic Yards in a length of 100 feet; breadth 2 feet; and of different depths.

Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.	Height or Depth in Feet.	Cubic Yards.
.5	3.70	.5	152	.5	300	.5	448	.5	596
1	7.41	21	156	41	304	61	452	81	600
.5	11.1	.5	159	.5	307	.5	456	.5	604
2	14.8	22	163	42	311	62	459	82	607
.5	18.5	.5	167	.5	315	.5	463	.5	611
3	22.2	23	170	43	319	63	467	83	615
.5	25.9	.5	174	.5	322	.5	470	.5	619
4	29.6	24	178	44	326	64	474	84	622
.5	33.3	.5	181	.5	330	.5	478	.5	626
5	37.0	25	185	45	333	65	481	85	630
.5	40.7	.5	189	.5	337	.5	485	.5	633
6	44.4	26	193	46	341	66	489	86	637
.5	48.1	.5	196	.5	344	.5	493	.5	641
7	51.9	27	200	47	348	67	496	87	644
.5	55.6	.5	204	.5	352	.5	500	.5	648
8	59.3	28	207	48	356	68	504	88	652
.5	63.0	.5	211	.5	359	.5	507	.5	656
9	66.7	29	215	49	363	69	511	89	659
.5	70.4	.5	219	.5	367	.5	515	.5	663
10	74 1	30	222	50	370	70	519	90	667
.5	77.8	.5	226	.5	374	.5	522	.5	670
11	81.5	31	230	51	378	71	526	91	674
.5	85 2	.5	233	.5	381	.5	530	.5	678
12	88.9	32	237	52	385	72	533	92	681
.5	92.6	.5	241	.5	389	.5	537	.5	685
13	96.3	33	244	53	393	73	541	93	689
.5	100	.5	248	.5	396	.5	544	.5	693
14	104	34	252	54	400	74	548	94	696
.5	107	.5	256	.5	404	.5	552	.5	700
15	111	35	259	55	407	75	556	95	704
.5	115	.5	263	.5	411	.5	559	.5	707
16	119	36	267	56	415	76	563	96	711
.5	122	.5	270	.5	419	.5	567	.5	715
17	126	37	274	57	422	77	570	97	719
.5	130	.5	278	.5	426	.5	574	.5	722
18	133	38	281	58	430	78	578	98	726
.5	137	.5	285	.5	433	.5	581	.5	730
19	141	39	289	59	437	79	585	99	733
.5	144	.5	293	.5	441	.5	589	.5	737
20	148	40	296	60	444	80	593	100	741

REMARK. **The foregoing tables of level cuttings may also be used for widths of roadway greater than those at the heads of the tables.** Thus, suppose we wish to use Table 1, for a roadbed *m n*, 16 ft wide, instead of *c b*, which is only 14 ft, and for which the table was calculated. It is only necessary first to find the vert dist *s a*, between these two roadbeds; and to add it *mentally* to each height *t s*, of the given embkt, when taking out from the



ıe numbers of cub yds corresponding to the heights. By this means we
tents of the embkt *c b o p*, for any required dist. Next, from these co
t that corresponding to the height *s a*, for the same dist. The remainde
be the embkt *m n o p*.

ctice it will be sufficiently correct, to take *s a* to the nearest tenth of a foot, which w
ı adding it mentally to the heights in the tables.

roadbed is *narrower* than that at the head of any table, as, for instance, if *m n* be the
, but we wish to find the contents for the width *c b*, then first find *s a*, and calculate the
length of *c b m n*. Then, in taking out the cub yds from the table, first subtract *s a* f
h height; and to the cub yds taken out for each 100 ft, opposite this reduced height,
h 100 ft of *c b m n*.

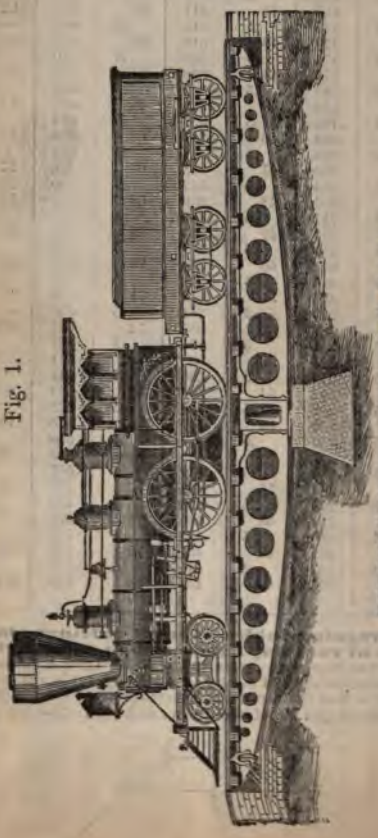


Fig. 1.

SELLERS' CAST-IRON TURNTABLE.

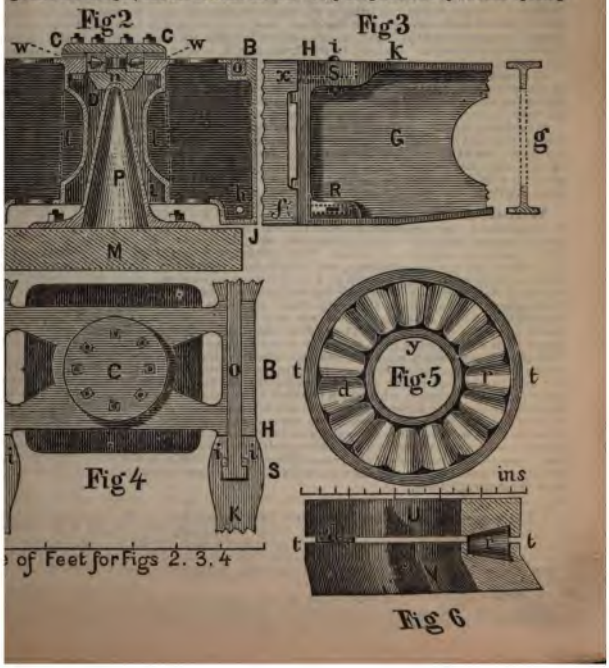
BY WILLIAM SELLERS & CO., PHILADA.

TURNTABLES.

ntable is a platform, usually from 40 to 60 ft long, and about 6 , (see Fig 1,) upon which a locomotive and its tender may be run, and then be turned izontally through any portion of a circle; and thus be transferred from one track to ning any angle with it. The table is supported by a pivot under its center; and by llers under its two ends. Frequently other rollers are added between the center and ends. platform is excavated a circular pit about 4 or 5 ft deep, having its circumf lined with sonry or brick about 2 ft thick, capped with either cut stone or wood. The diam of the of this lining is about 2 ins greater than the length of the turntable. The lining is gen- with a step, as seen in Fig 1, for supporting the circular rail on which the end rollers travel; f this step, a detached support may be used for this circular rail, as at *w*, in Figs 7. At the e pit is a solid well-founded mass of masonry or timber, for the pivot to rest on, as seen his, as well as the step for the end rollers, should be very firm, and perfectly level; other- tform will be hard to work. The platform is frequently floored across for a width of 6 to nish a pathway across the pit, without stepping down into it; especially when under alding. At first they were floored over so as to cover the entire circular pit; but this in- only their cost, but their wt, so as to make them difficult to turn; besides causing much repairs; with greater trouble in making them. It is therefore rarely done at present, e want of space sometimes renders it necessary in indoor turntables.

le should be several feet longer than is necessary for merely allowing the engine and nd on it; for the increased length enables the engine-men to move them a little backward so as to balance them chiefly upon the central support; and thus relieve the end rollers, ns the friction while turning is confined as much as possible to the center of motion; and more readily overcome than if it were allowed to act at the circumf. The engine-men by feeling, the proper spot for stopping the engine so as thus to balance the platform.

ellers cast-iron turntable, of the well-known firm of Wm. Sel- achinists, Philada, shown in Figs 1 to 6, is probably the most perfect one that has been is expensive in first cost, but economical in the long run. One man can readily turn it aid of machinery, when loaded with a heavy engine and tender. It is extensively used U. S.; and the principle is also largely applied to great turning bridges. It consists of ast-iron girders, one of which is seen in Fig 1; and parts of one in Figs 3 and 4. The girders are straight, and the bottoms curved. They are perforated by circular openings



to save metal. A transverse section of one is shown at *g*, Fig 3. The metal averages about 1⅜ ins thick. Each of these girders is in two separate pieces, which are fastened to a kind of central or low cast-iron boxing, a transverse section of which is shown by A B J J D L, Fig 2; and a top view by A B C, Fig 4. This boxing is all cast in one piece. Its four vertical sides, about 1⅜ ins thick, are solid; the opening shown in Fig 1 does not exist, but was inserted in the cut in order to show the central pivot. The top and bottom have openings, as in Figs 2 and 4. The girders are fastened to the central boxing, by means of heavy bars 3¾ ins square, of rolled iron, *o*, *o*, Figs 2 and 4, or *z z*, Fig 3, fitting into sunk recesses on top of the boxing, and tightened in place by wedges *i*, *i*, screw-bolted beneath, as shown below S, Fig 3; and by two 2¼-inch key-bolts *f*, Fig 3, which pass through the bolt-holes *h*, *h*, Fig 2; and are confined to the girders by horizontal keys just below E, Fig 1, where the keyhole is seen.

The central portion D L, Fig 2, of the boxing, is a hollow cone, open at top and bottom, and loosely surrounding the hollow conical pivot-post P. To save material, two openings *ll* are left in its circumf. The post P is about 1¾ ins thick; and is firmly bolted down to the large block of cut stone M, which caps the supporting masonry of the post. On top of the post rests loosely a rough cast-iron cap *n*; which permits a slight rocking motion of the entire platform, as the engine enters and leaves it, without at all straining the friction rollers. This cap supports the steel box, seen above it in Fig 2, or at *tt*, Fig 5, or U V, Fig 6, which contains the steel **friction rollers**, *r*, *d*, *kk*, Figs 5 and 6. There are about 15 of these; each about 2¾ ins, both in length, and in greatest diam. They have no axles; but merely lie loosely in the lower part of the box; filling its circumf with the exception of about ⅛ an inch left for play. In the direction of their axes they have but ⅛ inch play in the box. The lid U, Fig 6, of the roller-box, rests on top of the rollers themselves; and does not come down to the lower part V of the box by about ⅛ inch. Both the rollers themselves, and the insides of the box in contact with them, are finished with mathematical accuracy, so as to insure a perfect bearing between them. The rollers are kept constantly well oiled, as much of the ease of turning the platform depends upon it. To oil them, the screws of the cap C are loosened; and a mixture of oil and tallow sufficient for several months is put into the roller-box. On top of the roller-box is the cast cap C C, which is bolted down by 8 screw-bolts of 1¾ diam. These bolts sustain all the weight of the entire platform and its load, except what little may rest on the two rollers at each of its ends in case the engine is not perfectly balanced upon the rollers alone. By partly unscrewing them, all the platform is lowered around the post; only *n*, the rollers above it, and the cap C C, remaining unmoved on top of the post. This furnishes the means of adjusting the height of the platform above the bottom of the pit, so as to bring the end rollers to their due bearing upon the circular rail upon which they travel. These end rollers should barely touch lightly on their rail, because the object in the Sellers table, and one of its important features, is to throw all the weight upon the friction rollers. The friction being thus kept near the center of motion, has but little leverage with which to resist the turning of the table. The diam of the roller-box being as great as 15 inches, it is not difficult to balance the engine and tender upon it alone; but this cannot be done upon a central pivot only about 6 ins diam: and more especially because the foot of such a pivot should be convex, as at *f*, Figs 7, so as to adjust itself to the slight tilting of the platform when the load enters or leaves it. Under C C, Fig 2, are blocks, *w w*, of hard wood, which are driven between the eight screws, so as to hold all in place after this adjustment is completed.*

All turntables should have the means of making such adjustment. In Figs 7 (of another design of platform) it is attained by placing the central pivot, *f*, at the foot of a large screw, by turning which the whole platform can be raised or lowered at pleasure. After thus making the adjustment the screw is keyed fast to the platform, so as to revolve with it without screwing or unscrewing itself.

The ends of the two girders of the Sellers are firmly connected transversely by heavy cast-iron beams; the ends of which project sideways beyond the girders; and carry the cast iron end rollers, 9 ins diam; two at each end of the platform. Intermediate transverse connection is secured by the wooden cross-ties notched upon the girders to support the rails, and frequently 10 or 12 feet long, for giving a wide footway across the pit. A lever 8 or 10 ft long, fitting into a staple, is used for turning these platforms, not on account of friction, but to afford a handhold to the man who turns them.

Wooden turntables, with none but two common wheel rollers at each end of the platform, are sometimes resorted to from motives of original cost. They are, however, much harder to turn, generally requiring two men, aided by wheelwork; and are more liable to get out of order; and more expensive to repair. They are made of a great variety of patterns, both as regards the girders, and the central pivots, end rollers, &c. Frequently an addition is made of 8 to 12 small rollers travelling on a circular rail of 6 to 12 ft diam, around the pivot as a center. These are intended to sustain the whole weight; the end rollers being so adjusted as to touch their rail only when the platform rocks or tilts as the engine enters or leaves it. Therefore, there is less resistance from friction than when, as in Figs 7, there are only the end rollers *r*. In this last case, the engine and tender cannot be balanced so precisely upon the slender central pivot, as to prevent a great part of the wt from being thrown upon the end rollers; thus materially increasing the frictional resistance.

In plan, these wooden platforms are generally in shape of a cross; that is, in addition to the main platform for the engine, there is another transverse or at right angles to it, also extending across the pit; and having end rollers travelling on the circular rail. Thus, in Figs 7, (which show one of the many modes of framing a table which has only a central pivot *f*, and end rollers *r*,) the main platform rests on the girders *e*, which are strengthened below by braces *a*; while the transverse one rests on the timbers *o*, *o*, which must be imagined to extend across the pit. One half, or one arm, of this transverse platform is intended to carry the wheelwork R *x x*, for turning the platform; and the other arm serves merely as a balance to it; therefore, neither of them requires to be very strong. It is important to connect the four ends of the two platforms by four beams, as the whole structure is thereby materially stiffened. In the figs the wheelwork R *x x* is for convenience improperly shown as if it stood *upon the main* platform.

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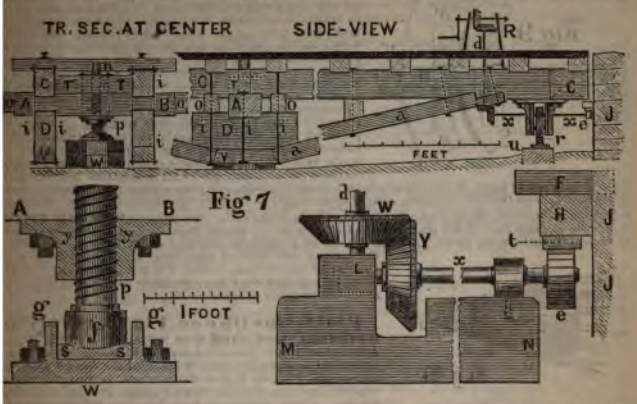
* ***The price of the Sellers turntable,*** at the shop in Philada, in 1871, *is about as follows;* 30 ft long, $900; 40 ft, $1300; 45 ft, $1500; 50 ft, $1600; 54 ft, $2000; 60 ft, $2200. *A 45-ft one weighs about* 23000 lbs; a 50-ft, 24700 lbs; a 54-ft, 33000; thus making the average in the *shop about 7 cts per* lb. These prices do not include cross-ties and other woodwork; nor the circular *railway for the end rollers.* Machinery for turning, being considered unnecessary, is not attached *unless specially ordered.* Its cost is extra.

The entire cost of excavating and lining the pit; foundation for pivot; circular rail for end rollers, &c. complete, for a 56-ft turntable, will vary from $1200 to $2500 in addition, depending on the *materials* and workmanship; and whether the bottom of the pit is paved or not.

he Figs 7 need but little explanation. They represent an actual 45 ft platform, which has been in for some years. The convex foot *f* of the central pivot, about 6 ins diam, should be faced with el; and should rest on a steel *step s s*. This should be kept well oiled; and protected from dust by eather collar around *p*, and resting on *g g*. Its upper part, about 4 ins diam, is cut into a screw h square threads about ¾ inch thick, for a distance of about 15 ins. It works in a female screw in



e strong cast-iron nut *y y*; and serves for raising the whole platform when necessary. When not use for this purpose, it is keyed tight to the platform, (by a key at its head *n*,) so as to revolve with Strong screw bolts *i i* connect the several timbers at the center of the platform.

R is a light cast-iron stand supporting two bevel wheels about 1 foot diam, which give motion by ans of an axle *d*, 1¾ ins diam to two similar ones below, shown more plainly at W and Y. These last e motion by the axle *x* to the pinion *e*,(6 ins diam, and 2½ ins face,) which turns the platform by work- g into a circular rack *t*, (teeth horizontal, 1 inch pitch; 3½ inches face,) which surrounds the entire . This rack is spiked to the under side of a continuous wooden curb H, which is upheld by pieces F, few feet apart, which are let into the wall J J, which lines the pit. The short beam M N, (about 6 which carries the lower wheelwork, is suspended strongly from the beams of the transverse plat- m above it. Instead of the two lower bevel wheels W Y, and the horizontal axle *x*, a more simple rangement is to place the pinion *e* at the lower end of the vertical axle *d*; and let it work into a k with *vertical* teeth at *u*, on the inner face of the stone foundation of the circular rail. For this rpose the stand R should be directly over *u*. There are two cast-iron rollers *r*, 2 ft diam, 3 inch e, under each end of the main platform; and one under each end of the secondary one.

lthough this kind of platform necessarily has much friction, yet one man can generally turn a 45- one by means of the wheelwork, when loaded with a heavy engine and tender. Indeed, he may do with some difficulty by hand only, while all is new and in perfect order; but when old, and the cir- ar railway uneven and dirty, it requires two men at the winches to do it with entire ease.

s before remarked, the resistance to turning is di- nished by employing a set of from 8 to 12 rollers or eels *r*, Figs 8, about a foot to 15 ins in diam, so ar- ged as to form a circle 8 to 12 ft diam around the ot. When this is done, the main girders of the plat- m are placed 8 to 12 ft apart; and long cross-ties used for supporting the railway track. Also, the in girders are sometimes trussed by iron rods, as in swing bridge on page 271; but instead of one post , it is best to have two, 6 or 8 ft apart at foot, and eting at top. The width of platform must then be ficient to allow the engine to pass the posts on either e of it. Ten feet will suffice.

ig 8 shows the arrangement of these rollers *r*, ich revolve upon a circular track *s*; while the plat- m rests on their tops by the track *u*. The rollers *r* are held between two wrought-iron rings *o*, *o*, out 3 ins deep, by ½ inch thick, which also are carried by the rollers. From each roller a radial rod *t*, 1 inch diam, extends to a ring *n n*, which surrounds the pivot *p*, closely, but not tightly, so to revolve independently of it. These tie-rods keep the rings *o o* at their proper dist from the pivot, that the rollers cannot leave the rails *s* and *u*. Between each two rollers, the rings o o should be engthened *by some arrangement like a*, to prevent change of shape. The pivot p may be as in Figs *There must, of course, be the usual* two rollers under each end of the platform, for sustaining the *ine as it goes on or off*; but during the act of turning the platform, the whole wt should rest on *central rollers. Such a platform* of 50 ft length can, if carefully made, be turned, together with *ngine and tender, by one* man, by means of a wooden lever 12 to 15 ft long, inserted in a staple *at purpose; and therefore* may dispense with the transverse platform for sustaining wheelwo *however, is usually added.*

rollers as have just been described, in connection with the friction rollers, Fig 5, is per **best arrangement for a large turning bridge.** At least o

of a platform must be provided with a **catch** or **stop** for arresting its motion at the moment it has reached the proper spot. A common mode is shown at Figs 9. It consists of a wrought-iron bar m n 1 ft long, 3 ins wide, and ¾ thick; hinged at its end m, which is confined to the top of the platform. Its outer end n is formed into a ring v for lifting it. A strong casting c a (in longitudinal section at t t,) about 15 ins long, 3 ins wide, and 1 inch thick, is also firmly bolted to the top of the platform; and the stop-bar m n rests in its recess r, while the platform is being turned. A similar casting a a is also bolted to the wooden or stone coping c c, which surrounds the top of the lining wall of the pit. When the stop-bar reaches this last casting, as the platform revolves, it rises up one of its little inclined planes t t, and falls into the recess of a a, bringing the platform to a stand. When the platform is to be started again, the bar is lifted out of its recess by the ring v, until it passes the casting; when it is again laid upon the coping c c, and moves with the platform; or, if required, the hinge at m allows it to be turned entirely over on its back. When there is a transverse platform, the proper place for the stop is at the end which carries the turning gear; as it is there handy to the men who do the turning. If there is only a main platform, the stop may be placed midway of the rails. Sometimes a **spring catch** is placed at each end of the platform; and each catch is loosened from its hold at the same instant by a long double-acting lever. All the details of a platform admit of much variety. Sometimes the girders are made of plate iron, like Figs 22 to 24, p 23. Instead of the friction *rollers*, Fig 5, friction *balls* 5 or 6 ins diam, of polished steel, are sometimes used. The pivots also are made in many shapes.

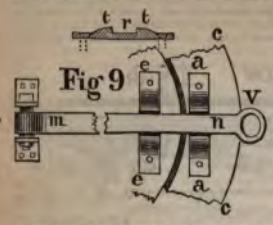


Fig 10

Platforms like *o a*, Fig 10, revolving around one end *o* as a center of motion, are sometimes useful. The shaded space is the pit. If an engine approaching along the track W, is intended to pass on to any one of the tracks 1, 2, 3, 4, the platform is first put into the required position, and the engine passes at once without detention. If the platform is long, it will be necessary to have roller-wheels not only under the moving end *a*, but at one or two other points, as indicated by the roller rails c c.

WATER STATIONS.

Water stations are points along a railroad, at which the engines stop to take in water. Their distance apart varies (like that of the fuel stations, which accompany them,) from about 6 miles, on roads doing a very large business; to 15 or 20 miles on those which run but few trains. Much depends, however, upon where water can be had. It has at times to be conducted in pipes for 2 or 3 miles or more. The object in having them near together is to prevent delay from many engines being obliged to use the same station. To prevent interruption to travel, they are frequently placed upon a side track. A supply of water is kept on hand at the station usually in large wooden tanks or tanks, enclosed in frame tank-houses. The tank-house stands near the track, leaving only about 2 to 4 ft clearance for the cars. It is two stories high; the tank being in the upper one; and having its bottom about 10 or 12 feet above the rails. In the lower story is usually the pump for pumping up the water into the tank; and a stove for preventing the water from freezing in winter.*

The tanks are usually circular; and a few inches greater in diam at the bottom than at the top, so that the iron hoops may drive tight. Their capacity generally varies from 6000 to 40000 gallons (rarely 80000 or more.) depending on the number of engines to be supplied. A tender-tank holds from 1200 to 2400 gallons; and an engine evaporates from 20 to 150 gallons per mile, depending on the class of engine; weight of train; steepness of grade, &c. Perhaps 40 gallons will be a tolerably full average for passenger, and 80 for freight engines. The following are the contents of tanks of different inner diams, and depths of water. U. S. galls of 231 cub ins; or 7.4805 galls to a cub ft. See p. 43

Diam.	Depth.	Gallons.	Cub. Ft.	Diam.	Depth.	Gallons.	Cub. Ft.
Ft.	Ft.			Ft.	Ft.		
12	8	6767	905	24	12	40607	5429
14	9	10363	1385	26	13	51628	6902
16	9	13535	1810	28	14	64481	8621
18	10	19034	2545	30	15	79310	10603
20	10	23499	3142	32	16	96253	12868
22	11	31277	4181	34	17	115451	15435

Cypress or any of the pines answer very well for tanks. The staves may be about 2½ ins thick for the smaller ones; to 4 or 5 ins for the largest. The bottoms may be the same. The staves should be planed by machinery to suit the curve precisely. Nothing is then needed between the staves to produce tightness. A single wooden dowel is inserted between each two near the top, merely to hold them in place while being put together. The bottom is dowelled together; and simply inserted into a very accurately cut, about an inch deep, around the inner circumf of the tub, at a few inches from the bottoms of the staves.

*A frame tank-house, 18 ft square, with stone foundations for both house and tank, will cost $400 to $600. A brick or stone one somewhat more.

One of 20 ft diam, and 12 ft deep, may have 9 hoops of good iron; placed several inches nearer together at the bottom of the tank than at the top. Their width 3 ins; the thickness of the lower two, ¼ inch; thence gradually diminishing until the top one is but half as thick. The lower two are driven close together. These dimensions will allow for the rivet-holes for riveting together the overlapping ends; and for a moderate strain in driving the hoops firmly into place.* Tarse rivets of ½ inch diam, and 3 ins apart, in line, are sufficient for a joint of a lower hoop. One of 34 ft diam, 17 deep, may have 12 hoops; the lower ones 4 ins by ½; with three ¾ inch rivets to a lower hoop-joint.

The bottom planks of the tank must bear firmly upon their supporting joists, or bearers.

A tank must have an inlet-pipe by which the water may enter it; a waste-pipe for preventing overflow; and a discharge or feed-pipe 7 or 8 ins diam. in or near the bottom; through which the water flows out to the tender. The inner end of the discharge-pipe is covered by a valve, to be opened at will by the engine man, by means of an outside cord and lever. To its outer end is generally attached a flexible canvas and gum-elastic hose about 7 o. 8 ins diam, and 8 or 10 ft long, through which the water enters the tender-tank. Or, instead of a hose, the feed-pipe may be prolonged by a metallic pipe, or nozzle, sufficiently long to reach the tender; and so jointed as, when not in use, to swing to one side, or to be raised to a vertical position, (in the last case it is called *a drop*,) so as not to be in the way of passing trains.

The same tank may supply two engines on different tracks, at once. The tanks are very durable.

The patent frost-proof tank of John Burnham, Batavia, Illinois, is simply an ordinary tank, in which the water is prevented from freezing by means, 1st, of a circular roof which protects a ceiling of joists, between which is a layer of sawdust; 2d, by an air-space obtained by a ceiling of boards beneath the timbers on which the tank rests. Although the sides are entirely unprotected, no house is necessary; but merely strong posts and beams on a stone foundation, for the support of the tank.†

Tanks are frequently made rectangular, with vertical sides of posts lined with plank, and braced across in both directions by iron rods. They are more apt to leak than circular ones. They have been made of iron; but wood seems to be preferred.

The water for supplying the tanks. may be pumped by hand, steam, horse, wind, hydraulic ram, or otherwise, from a running stream; from a pond made by damming the stream if very small or irregular; from a cistern below the tank; or from a common well. Many roads doing a business of 10 or 12 engines daily in each direction, depend entirely upon wells; and pump by hand; generally two men to a pump. Those doing a very large business, when the supply cannot be obtained by gravity, mostly use steam. The windmill is the most economical power; and when well made, is very little liable to get out of order. Of course it will not work during a calm; but this objection may be obviated in most cases by having the tanks large enough to hold a supply for several days.‡ Steam, however, is most reliable.

The following table will give some idea of the power reqd in a steam engine for the pumping. In ordering an engine, specify not its number of horse-powers, but the number of gallons it must raise in a given number of hours, to a given height; with a given steam pressure, (say about 60 to 80 lbs per sq inch.) The pump should be sufficiently powerful not to have to work at night; and should be capable of performing at least 25 per cent more than its reqd duty.

A fair average horse should pump in 8 hours the quantities contained in the first 3 cols; to the height in the 4th col; or sufficient to supply the number of locomotives in the 5th col, with about 2000 gals each. Two men should do about ⅓ as much.**

Cub. Ft.	Lbs.	Gals.	Ht. Ft.	No. of Locos.	Cub. Ft.	Lbs.	Gals.	Ht. Ft.	No. of Locos.
1600	100000	11968	100	6	4571	285714	34194	35	17
2000	125000	14960	80	7½	5333	333333	39893	30	20
2667	166666	19946	60	10	6400	400000	47872	25	24
3200	200000	23936	50	12	8000	500000	59840	20	30
3555	222222	26596	45	13¼	10667	666667	79787	15	40
4000	250000	29920	40	15	16000	1000000	119680	10	60

A reservoir, with a stand-pipe, or water column, is preferable to the ordinary tank, when the locality admits of it; being less liable than the pump to get out of order; and being cheaper in the end. The reservoir is supposed to be filled by water flowing into it by

* Such a tank, put up in its place, will cost about $400. Geo. J. Burkhardt & Co, 1341 Buttonwood Street, Philada, make tanks their specialty; and are provided with machinery which secures perfect accuracy of joint in every part. Their work is sought from great distances.

† The "Railway Frost-proof Tank Co," of Batavia, Ill, make a specialty of the construction and erection of these tanks, complete in every detail, ready for use. They also make windmills.

‡ The "Empire Windmill Manufacturing Co," of Syracuse, N. York, furnish excellent machines. *They also, when desired, provide pumps, &c. complete.* The cost of windmill alone, for railway station, *varies from about $400 for 18 ft diam*; to $1400 for 36 ft diam, at the factory. The company *may have an office at 76 John St, N. York.*

** ***The cost of a direct acting steam pump,*** with its boilers, &c. fixed *place, ready for work, and capable of the duty of the above table,* may be roughly set down at any *0; twice the duty, $600; 4 times,* $750; 6 times, $900; 10 times, $1350; 20 times, $2000. *of engine house. Made by* Henry R. Worthington, 61 Beekman St, N. York; the Philada *raulic Works, Eveline St, Philada;* and by many establishments in most of our large cities

gravity; and to have its bottom at least about 8 ft above the rails; or at any greater height whatever that the ground and the height of the water may require. It may be excavated in the ground, lined with brick or masonry in cement; with a bottom of concrete; or it may be built above ground, according to the locality. It may be roofed and covered in, or not; and it may be near the track, or at a considerable distance from them, according to circumstances.* From its bottom, an iron pipe from 8 to 12 ins diam, is carried (generally underground,) to within a few ft of the track. At this point it turns vertically upward to about 8 or 10 ft above the track, forming a **stand-pipe, or water-column**; from the upper end of which the water flows (through either a hose or a jointed nozzle,) as in the case of a tank. Several such pipes, or one larger one, may be laid, for the supply of two or more engines at once, through as many stand-pipes. Where the pipe makes its bend, and becomes vertical, is a valve for opening and closing it; and which may be worked by a hand-wheel placed at such a height as to be easily reached by the engine man.† A valve on the principle of those for street pipes, (page 572,) is best.

Fast trains sometimes scoop up water while in motion from a long trough laid between the rails. Length of trough about ¼ mile; width 18 ins; depth 4 ins.

Evaporation from Locomotives. In addition to what is said on page 432, 'in the passage preceding this table,' we may state that the evaporation is usually from 6 to 7 lbs of water to 1 lb of fair coal. Hence if we take the average at 6½ lbs, or say .8 of a gallon of water to 1 lb of coal, and assume, as on page 432, that a passenger engine evaporates an average of 40 gallons per mile, and a freight engine 80 galls, we shall have very nearly 7¾ tons of coal consumed per 100 miles by the former; and 4½ tons by the latter. The evaporation from a heavily tasked powerful engine may amount to 150 galls or more per mile; but such is an exceptional case.

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* An uncovered reservoir 50 ft diam by 12 ft deep, lined with brick or masonry, will usually cost from $2500 to $3500, according to circumstances.

† The price of a cast-iron water-column, of 6-inch bore; with bed-plate; holding-down bolts, and washers; connecting-pipes; swing-joint with copper arm 9 ft long; valve; hand-wheel; &c; complete, ready to set up, (by the Pascal Iron Works, Philada, in 1871,) is $475 at the shop. This company manufacture cast and wrought-iron pipes, stopcocks, valves, fireplugs, and a vast number of other iron articles; and have a high reputation for the excellence of their work. Office, No. 22 South Third Street.

COST OF EARTHWORK.

rwing is taken from the last edition of the writer's volume on the Measurement of Excava- Embankments.

1. It is advisable to pay for this kind of work by the cubic yard of *excavation* only; in- lowing separate prices for excavation and embankment. By this means we get rid of the f measurements, as well as the controversies and lawsuits which often attend the deter- f the allowance to be made for the settlement or subsidence of the embankments.

reover, our opinion that justice to the contractor should lead to the **English prac- paying the laborers by the cubic yard**, instead of by the day. s fully proves that when laborers are scarce and wages high, men can scarcely be depended three-fourths of the work which they readily accomplish when wages are low, and when s are waiting to be hired in case any are discharged. The contractor is thus placed at the his men. The writer has known the most satisfactory results to attend a system of task- mpanied by liberal premiums for all overwork. By this means the interests of the laborers ied with that of the contractor; and every man takes care that the others shall do their of the task.

Morris, C E, of Philadelphia, was, we believe, the first person who properly investigated ts of cost of earthwork, and reduced them to such a form as to enable us to calculate the a considerable degree of accuracy. He published his results in the Journal of the Franklin a 1841. His paper forms the basis on which, with some variations, we shall consider the d on which we shall extend it to wheelbarrows, as well as to carts. Throughout this paper f a cubic yard considered only as solid in its place, or before it is loosened for removal. It necessary to add that the various items can of course only be regarded as tolerably close ations, or averages. As before stated, the men do less work when wages are high; and more are low. A great deal besides depends on the skill, observation, and energy of the con- t his superintendents. It is no unusual thing to see two contractors working at the same precisely similar material, where one is making money, and the other losing it, from a want the proper distribution of his forces, keeping his roads in order, having his carts and bar- fitted, &c. &c. Uncommonly long spells of wet weather may seriously affect the cost of exe- thwork, by making it more difficult to loosen, load, or empty; besides keeping the roads in for hauling.

regate cost of excavating and removing earth is made up by the following items, namely:
oosening the earth ready for the shovellers.
oading it by shovels into the carts or barrows.
hauling, or wheeling it away, including emptying and returning.
preading it out into successive layers on the embankment.
keeping the hauling-road for carts, or the plank gangways for barrows, in good order.
ear, sharpening, depreciation, and interest on cost of tools.
superintendence, and water-carriers.
rofit to the contractor.

consider these items a little in detail, basing our calculations on the assumption that com- costs $1 per day, of 10 working hours. The results in our tables must therefore be in- diminished in about the same proportion as common labor costs more or less than this.

2. Loosening the earth ready for the shovellers. This is lone either by ploughs or by picks; more cheaply by the first. A plough with two horses, en to manage them, at $1 per day for labor, 75 cents per day for each horse, and 37 cents plough, including harness, wear, repairs, &c. or a total of $3.67, will loosen, of strong t, from 200 to 300 cubic yards a day, at from 1.83 to 1.29 cents per yard; or of ordinary 400 to 600 cubic yards a day, at from .97 to .64 of a cent per yard. Therefore, as an ordi- ge, we may assume the actual cost to the contractor for loosening by the plough, as fol- g heavy soils, 1.6 cents; common loam, .8 cent; light sandy soils, .4 cent. Very stiff pure rtinate cemented gravel, may be set down at 2.5 cents; they require three or four horses. ick, a fair day's work is about 14 yards of stiff pure clay, or of cemented gravel; 25 yards eavy soils; 40 yards of common loam; 60 yards of light sandy soils — all measured in ch, at $1 per day for labor, gives, for stiff clay, 7 cents; heavy soils, 4 cents; loam, 2.5 t sandy soil, 1.666 cents. Pure sand requires but very little labor for loosening; .5 of a over it.

3. Shovelling the loosened earth into carts. The amount er day depends partly upon the weight of the material, but more upon so proportioning r of pickers and of carts to that of shovellers, as not to keep the latter waiting for either carts. In fairly regulated gangs, the shovellers into carts are not actually engaged in for more than six-tenths of their time, thus being unoccupied but four-tenths of it; while, management, they lose considerably more than one-half of it. A shoveller can readily cart one-third of a cubic yard measured in place (and which is an average working cart- andy soil, in five minutes; of loam, in six minutes; and of any of the heavy soils, in seven This would give, for a day of 10 working hours, 120 loads, or 40 cubic yards of light sandy ads, or 33⅓ cubic yards of loam; or 86 loads, or 28.7 yards of the heavy soils. But from nts we must deduct four-tenths for time necessarily lost; thus reducing the actual work- ties to 24 yards of light sandy soil, 20 yards of loam, 17.2 yards of the heavy soils. When ers do less than this, there is some mismanagement.

g these as fair quantities, then, at $1 per day for labor, the actual cost to the contractor ing per cubic yard measured in place, will be, for sandy soils, 4.167 cents; loam, 5 cents; , clays, &c, 5.81 cents.

se, the carts are not usually loaded to any less extent with the heavier soils than with the . Nor, indeed, is there any necessity for so doing, inasmuch as the difference of weigh f one third of a cubic yard of the various soils is too slight to need any attention; ear the cart road is kept in good order, as it will be by any contractor who understands

[illegible] amount of any *slight inclinations* which [illegible] a ton.

Art. [illegible] Hauling away the earth: dumping, or emptying; and returning [illegible]. [illegible] average speed of horses in hauling is about 2½ miles [illegible] feet [illegible] each way or to 100 feet of *lead*, [illegible] allowance this more is a loss of [illegible] waiting to load, dumping, turning, &c. [illegible] 100 feet each in the lead; and [illegible] or any given average lead, we [illegible] to the number of 100 feet [illegible] that is,

$$\frac{\text{[illegible]}}{\text{[illegible]}} = \frac{\textit{the number of trips, or loads}}{\textit{removed per day, per cart.}}$$

[illegible] *Dumping*, [illegible] an average cart-load, the number [illegible] removed per day by each cart; and [illegible] will give the cost per cable yard for [illegible]

[illegible] earth, which, at $1 per day, is 25 cents per [illegible] 75 cents, and that of its [illegible] 1000 feet per cart $1.25. The [illegible] many days, as often as work; and this consid- [illegible] contractors employ a greater number of drivers, who also [illegible] in either case.

[illegible] be hauled by a horse and cart [illegible] of earth of lead of earth being 1000 feet, or [illegible] contractor for hauling, per cable foot, [illegible] $1.25

$$\frac{\text{[illegible] loads}}{\text{[illegible] cart}} = \frac{\text{[illegible]}}{4} = \text{[illegible] loads.} \quad \text{and} \quad \frac{43 \text{ loads}}{3} = 14.3 \text{ cubic yards.}$$

$$\text{[illegible]} = \frac{\text{[illegible] cents}}{\text{[illegible] cubic yard}} = \text{[illegible] cents per cubic yard.}$$

[illegible] following sums have been calculated.

Art. 3. Spreading, or levelling off the earth into regular [illegible] layers on the embankment. [illegible] man will spread from 50 to 100 cable [illegible] levelling on their dumps [illegible] as a fair average [illegible]

[illegible] earth [illegible] over the end of the embank- [illegible] allowed in either case for keeping in [illegible] which requires more time for ending say,

$$\frac{\text{[illegible]}}{\text{[illegible]}} = \frac{\textit{No. of carts removed,}}{\textit{per day, per cart.}}$$

Art. 4. Keeping the cart-road in good order for hauling. [illegible] be shifted [illegible] and the cost [illegible] per rurse yard for road repairs [illegible] of a mile.

Art. 7. Wear, sharpening, and depreciation of picks and shovels. [illegible] will cover this item.

Superintendence and water-carriers. [illegible] will vary with [illegible] under ordinary [illegible] and general cleaning [illegible] per day of rest.

Art. 8. Profit to the contractor. [illegible] expenses and other [illegible] to the public [illegible] to the interest.

Art. 9. A knowledge of the foregoing items enables us to calculate with tolerable accuracy the cost of removing earth. [illegible] average lead of 1000 feet; the [illegible] a day and a cart 25 cts. [illegible]

[illegible] the mean lead is the distance between the [illegible] and empty.

		Cents.
Here we have cost of loosening, say by pick,	*Art 2, per cubic yard, say,*	2.50
Loading into carts,	*Art. 3, " "*	5.00
Hauling 1000 feet, as calculated previously in example,	*Art. 4, "*	8.74
Spreading into layers,	*Art. 5, "*	1.50
Keeping cart-road in repair,	*Art. 6, 10 lengths of 100 ft,*	1.00
Various items in	*Art. 7,*	2.00
Total cost to contractor,		20.74
Add contractor's profit, say 10 per cent,		2.072
Total cost per cubic yard to the company,		22.812

ay to construct a table like the following, of costs per cubic yard, for different lengths of lead. 2 and 3 are first obtained by the Rule in Article 4; then to each amount in column 3 is added *ible* quantity of $\frac{1}{10}$ of a cent for every 100 feet length of lead, for keeping the road in order; *constant* quantity (for any given kind of soil) composed of the prices per cubic yard, for g, loading, spreading, or wasting, &c, either taken from the preceding articles; or modified articular circumstances. In this manner the tables have been prepared.

By Carts. Labor $1 per day, of 10 working hours.

	Number of cubic yards in place, hauled per day by each cart.	Cost per cubic yard in place, for hauling and emptying only.	Common Loam. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Strong Heavy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
	Cu. Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
)	47.0	2.66	13.69	12 44	11.99	10.74	16.00	14.75	13.50	12.25
)	44.4	2.81	13.86	12.61	12.16	10.91	16.17	14.92	13.67	12.42
)	42.1	2.97	14.05	12.80	12.35	11.10	16.36	15.11	13.86	12.61
)	40.0	3.12	14.22	12.97	12.52	11.27	16.53	15.28	14.03	12.78
)	36.4	3 43	14.58	13.33	12.88	11.63	16.89	15.64	14.39	13 14
)	33.3	3.75	14.95	13.70	13.25	12.00	17.26	16.01	14.76	13.51
)	28.6	4.37	15.67	14.42	13.97	12.72	17.98	16.73	15.48	14.23
)	25 0	5.00	16.40	15.15	14.70	13.45	18.71	17.46	16.21	14.96
)	22.2	5.63	17.13	15 88	15.43	14.18	19.44	18.19	16.94	15.69
)	20.0	6.25	17.85	16.60	16.15	14.90	20.16	18.91	17.66	16.41
)	18.2	6.87	18.57	17.32	16.87	15.62	20.88	19.63	18.38	17.13
)	16.7	7.48	19.28	18.03	17.58	16.33	21.59	20.34	19.09	17.84
)	15.4	8 12	19.92	18.67	18.22	16.97	22.23	20.98	19.73	18.48
)	14.3	8.74	20.74	19.49	19.04	17.79	23.05	21 80	20.55	19.30
)	13.3	9.40	21.50	20.25	19 80	18.55	23.81	22.56	21.31	20.06
)	12.5	10.0	22.20	20.95	20.50	19 25	24.51	23.26	22.01	20.76
)	11.8	10.6	22.90	21.65	21.20	19.95	25.21	23.96	22.71	21.46
)	11.1	11.2	23.60	22.35	21.90	20.65	25.91	24.66	23.41	22.16
)	10.5	11.9	24.40	23.15	23.70	21.45	26.71	25.46	24.21	22.96
)	10.0	12.5	25.10	23.85	23.40	22.15	27.41	26.16	24.91	23.66
)	9.52	13.1	25.80	24.55	24.10	22.85	28.11	26.86	25 61	24.36
)	9.09	13.7	26.50	25.25	24.80	23.55	28.81	27.56	26.31	25.06
)	8.70	14.4	27.30	26 05	25.60	24.35	29.61	28.36	27.11	25.86
)	8.33	15.0	28.00	26.75	26.30	25.05	30.31	29.06	27.81	26.56
)	7.54	16.6	29.85	28.60	28.15	26.90	32.16	30.91	29.66	28.41
)	6 99	18.1	31.60	30.35	29.90	28.65	33.91	32.66	31.41	30.16
le	6.58	19.0	32.64	31.39	30.94	29.69	34.95	33.70	32.45	31.20
)	5.88	21.2	35.20	33.95	33 50	32.25	37.51	36.26	35.01	33.76
)	5.48	22.8	37.05	35.80	35.35	34.10	39.36	38.11	36.86	35.61
)	5.13	24.3	38.80	37.55	37.10	35.85	41.11	39.86	38 61	37.36
)	4.82	25.9	40.65	39.40	38.95	37.70	42.96	41.71	40.46	39.21
)	4.54	27.5	42.50	41.25	40.80	39.55	44.81	43.56	42.31	41.06
)	4.30	29.1	44.35	43.10	42.65	41.40	46.66	45.41	44.16	42.91
)	4.08	30.6	46.10	44.85	44.40	43.15	48.41	47.16	45.91	44.66
)	3.88	32.2	47.95	46.70	46.25	45.00	50.26	49.01	47.76	46.51
)	3.70	33.8	49.80	48.55	48.10	46.85	52.11	50 86	49.61	48.36
le	3.52	35.5	51.78	50.53	50.08	48.83	54.09	52.84	51.59	50.34
	2.86	43.8	61.40	60.15	59.70	58.45	63.71	62.46	61.21	59.96
	2.40	52.1	71.02	69.77	69.32	68.07	73.33	72.08	70.83	69.58
	2.07	60.4	80.64	79.39	78.94	77.69	82.95	81.70	80.45	79.2
	1.82	68.7	90.26	89.01	88.56	87.31	92.57	91.32	90.07	88

By Carts. Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the earth is hauled, in feet.	Number of cubic yards in place, hauled per day by each cart.	Cost per cubic yard in place, for hauling and emptying only.	Pure stiff Clay, or cemented Gravel. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Light Sandy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
Feet.	Cu. Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
25	47.0	2.66	19.00	17.75	14.50	13.25	11.52	10.77	10.25	9.50
50	44.4	2.81	19.17	17.92	14.67	13.42	11.69	10.94	10.42	9.67
75	42.1	2.97	19.36	18.11	14.86	13.61	11.88	11.13	10.61	9.86
100	40.0	3.12	19.53	18.28	15.03	13.78	12.05	11.30	10.78	10.03
150	36.4	3.43	19.89	18.64	15.39	14.14	12.41	11.66	11.14	10.39
200	33.3	3.75	20.26	19.01	15.76	14.51	12.78	12.03	11.51	10.76
300	28.6	4.37	20.98	19.73	15.48	15.23	13.50	12.75	12.23	11.48
400	25.0	5.00	21.71	20.46	17.21	15.96	14.23	13.48	12.46	12.21
500	22.2	5.63	22.44	21.19	17.94	16.69	14.96	14.21	13.69	12.94
600	20.0	6.25	23.16	21.91	18.66	17.41	15.68	14.93	14.41	13.66
700	18.2	6.87	23.88	22.63	19.38	18.13	16.40	15.65	15.13	14.38
800	16.7	7.48	24.59	23.34	20.09	18.84	17.11	16.36	15.84	15.09
900	15.4	8.12	25.23	23.98	20.73	19.48	17.75	17.00	16.48	15.73
1000	14.3	8.74	26.05	24.80	21.55	20.30	18.57	17.82	17.30	16.55
1100	13.3	9.40	26.81	25.56	22.31	21.06	19.33	18.58	18.06	17.31
1200	12.5	10.0	27.51	26.26	23.01	21.76	20.03	19.28	18.76	18.01
1300	11.8	10.6	28.21	26.96	23.71	22.46	20.73	19.98	19.46	18.71
1400	11.1	11.2	28.91	27.66	24.41	23.16	21.43	20.68	20.16	19.41
1500	10.5	11.9	29.71	28.46	25.21	23.96	22.23	21.48	20.96	20.21
1600	10.0	12.5	30.41	29.16	25.91	24.66	22.93	22.18	21.66	20.91
1700	9.52	13.1	31.11	29.86	26.61	25.36	23.63	22.88	22.36	21.61
1800	9.09	13.7	31.81	30.56	27.31	26.06	24.33	23.58	23.06	22.31
1900	8.70	14.4	32.61	31.36	28.11	26.86	25.13	24.38	23.86	23.11
2000	8.33	15.0	33.31	32.06	28.81	27.56	25.83	25.08	24.56	23.81
2250	7.54	16.6	35.16	33.91	30.66	29.41	27.68	26.93	26.41	25.66
2500	6.90	18.1	36.91	35.66	32.41	31.16	29.43	28.68	28.16	27.41
¼ mile	6.58	19.0	37.95	36.70	33.45	32.20	30.47	29.72	29.20	28.45
3000	5.88	21.2	40.51	39.26	36.01	34.76	33.03	32.28	31.76	31.01
3250	5.48	22.8	42.36	41.11	37.86	36.61	34.88	34.13	33.61	32.86
3500	5.13	24.3	44.11	42.86	39.61	38.36	36.63	35.88	35.36	34.61
3750	4.82	25.9	45.96	44.71	41.46	40.21	38.48	37.73	37.21	36.46
4000	4.54	27.5	47.81	46.56	43.31	42.06	40.33	39.58	39.06	38.31
4250	4.30	29.1	49.66	48.41	45.16	43.91	42.18	41.45	40.93	40.18
4500	4.08	30.6	51.41	50.16	46.91	45.66	43.93	43.18	42.66	41.91
4750	3.88	32.2	53.26	52.01	48.76	47.51	45.78	45.03	44.51	43.76
5000	3.70	33.8	55.11	53.86	50.61	49.36	47.63	46.88	46.36	45.61
1 mile	3.52	35.5	57.09	55.84	52.59	51.34	49.61	48.86	48.34	47.59
1¼ m.	2.86	43.8	66.91	65.46	62.21	60.96	59.23	58.48	57.96	57.21
1½ m.	2.40	52.1	76.33	75.08	71.83	70.58	68.85	68.10	67.58	66.83
1¾ m.	2.07	60.4	85.95	84.70	81.45	80.20	78.47	77.72	77.20	76.45
2 m.	1.82	68.7	95.57	94.32	91.07	89.82	88.09	87.34	86.82	86.07

Art. 10. By wheelbarrows. The cost by barrows may be estimated in the same manner as by carts. See Articles 1, &c. Men in wheeling move at about the same average rate as horses do in hauling, that is, 2½ miles an hour, or 200 feet per minute, or 1 minute per every 100-foot length of lead. The time occupied in loading, emptying, &c (when, as is usual, the wheeler loads his own barrow,) is about 1.25 minutes, without regard to length of lead; besides which, the time lost in occasional short rests, in adjusting the wheeling plank, and in other incidental causes, amounts to about $\frac{1}{10}$ part of his whole time; so that we must in practice consider him as actually working but 9 hours out of his 10 working ones, at the rate of 2.25 minutes per 100 feet of lead. To find, then, the number of barrow-loads which he can remove in a day, multiply the number of minutes (600) in a working day by .9; and divide the product by the sum of 1.25, added to the number of 100-feet lengths in the lead; that is,

$$\frac{\textit{The number of minutes in a working day} \times .9}{1.25 + \textit{the number of 100-feet lengths of lead}} = \textit{the number of trips or of loads removed per day per barrow.}$$

See Remark, next page.

The number of loads divided by 14 will give the number of cub yards, since a cub yard, measured in place, averages about 14 loads. And the cost of a wheeler and barrow per day, (say $1 per man and 5 cents per barrow,) divided by the number of cub yards, will give the cost per yard for hauling, wheeling, and emptying.

MPLE. How many cubic yards of common loam, measured in place, will one man load, wheel, apty, per day of 10 working hours, (or 600 minutes:) the lead, or distance to which the earth is ed being 1000 feet, (or 10 lengths of 100 feet;) and what will be the expense per yard, supposing orer and barrow to cost $1.05 per day?

$$\textit{Here, } \frac{600 \textit{ minutes} \times .9}{1.25 + 10 \textit{ lengths}} = \frac{540}{11.25} = 48 \textit{ trips, or loads per day.}$$

$$\textit{And } \frac{48}{14} = 3.43 \textit{ cub yds per day.} \quad \textit{And } \frac{105 \textit{ cents}}{3.43 \textit{ cub yds}} = 30.6 \textit{ cents}$$

b yard for loading, wheeling away, emptying, and returning. This would be increased almost eciably by the cost of the shovel, which, in the following tables, however, is included in the tools.

. For *rock*, which requires more time for loading, say

$$\frac{\textit{No of minutes in a working day} \times .9}{1.6 + \textit{No of 100-feet lengths of lead}} = \frac{\textit{No of loads removed}}{\textit{per day, per barrow.}}$$

t. 11. The following tables are calculated as in the case of carts, by first finding columns 2 by means of the Rule in Art 4, and then adding to each sum in column 3, the variable quantity a cent per cubic yard per 100 feet of lead for keeping the wheeling-planks in order; and the of loosening, spreading, superintendence, water-carrying, &c, per cubic yard, as given in the ing Articles 2 to 7.

Wheelbarrows. Labor $1 per day, of 10 working hours.

	Number of cubic yards in place, loaded and wheeled per day; each barrow.	Cost per cubic yard in place, for loading, wheeling, and emptying.	Common Loam. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Strong, Heavy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
t.	Cu. Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
5	25.7	4.09	10.12	8.87	8.42	7.17	11.62	10.37	9.12	7.87
)	22.1	4.75	10.80	9.55	9.10	7.85	12.30	11.05	9.80	8.55
5	19.3	5.44	11.52	10.27	9.82	8.57	13.02	11 77	10.52	9.27
)	17.1	6.14	12.24	10.99	10.54	9.29	13.74	12.49	11.24	9.99
)	14.0	7.50	13.65	12.40	11.95	10.70	15.15	13.90	12.65	11.40
)	11.9	8.82	15.02	13.77	13.32	12.07	16.52	15.27	14.02	12.77
)	10.3	10.2	16.45	15.20	14.75	13.50	17.95	16.70	15 45	14.20
)	9.07	11.6	17.90	16.65	16.20	14.95	19.40	18.15	16.90	15 65
)	8.14	12.9	19.25	18.00	17.55	16.30	20.75	19.50	18 25	17.00
)	7.36	14.3	20.70	19.45	19.00	17.75	22.20	20.55	19.70	18.45
)	6.71	15.6	22.05	20.80	20.35	19.10	23.55	22.30	21.05	19.80
)	6.17	17.0	23.50	22.25	21.80	20.55	25.00	23.75	22.50	21.25
)	5.32	19.7	26.30	25.05	24.60	23.35	27.80	26.55	25.30	24.05
)	4.67	22.5	29.20	27.95	27.50	26.25	30.70	29.45	28.20	26.95
)	4.17	25.2	32.00	30.75	30.30	29.05	33.50	32 25	31.00	29.75
)	3.76	27.9	34.80	33.55	33.10	31.85	36.30	35.05	33.80	32.55
)	3.43	30.6	37.60	36.35	35.90	34.65	39.10	37.85	36.60	35.35
)	2.91	36.1	43.30	42.05	41.60	40.35	44.80	43.55	42.30	41.05
)	2.53	41.5	48.90	47.65	47.20	45.95	50.40	49.15	47.90	46.65
)	2.24	46.9	54.50	53.45	52.80	51.55	56.00	54.75	53.50	52.25
)	2.00	52.5	60.30	59.35	58.60	57.35	61.80	60.55	59.30	58.05
)	1 81	58.0	66.00	64.75	64.30	63.05	67.50	66.25	65.00	63.75
)	1.66	63.3	71.50	70.25	69.80	68.55	73.00	71.75	70.50	69.25
)	1.53	68 6	77.00	75 75	75.30	74.05	78.50	77.25	76.00	74.75
e.	1.39	75.5	84.14	82.89	82.44	81.19	85.64	84.39	83.14	81.89

By Wheelbarrows. Labor $1 per day, of 10 working hours.

Length of Lead, or distance to which the earth is wheeled.	Number of cubic yards in place, loaded, and wheeled, per day; each barrow.	Cost per cubic yard in place, for loading, wheeling, and emptying.	Pure Stiff Clay, or Cemented Gravel. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.				Light Sandy Soils. TOTAL COST PER CUBIC YARD, EXCLUSIVE OF PROFIT TO CONTRACTOR.			
			Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.	Picked and Spread.	Picked and Wasted.	Ploughed and Spread.	Ploughed and Wasted.
Feet.	Cu. Yds.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.	Cts.
25	25.7	4.09	14.62	13.37	10.12	8.87	8.79	8.04	7.52	6.77
50	22.1	4.75	15.30	14.05	10.80	9.55	9.47	8.72	8.20	7.45
75	19.3	5.44	16.02	14.77	11.52	10.27	10.19	9.44	8.92	8.17
100	17.1	6.14	16.74	15.49	12.24	10.99	10.91	10.16	9.64	8.89
150	14.0	7.50	18.15	16.90	13.65	12.40	12.32	11.57	11.05	10.30
200	11.9	8.82	19.52	18.27	15.02	13.77	13.69	12.94	12.42	11.67
250	10.3	10.2	20.95	19.70	16.45	15.20	15.12	14.37	13.85	13.10
300	9.07	11.6	22.40	21.15	17.90	16.65	16.57	15.82	15.30	14.55
350	8.14	12.9	23.75	22.50	19.25	18.00	17.92	17.17	16.65	15.90
400	7.36	14.3	25.20	23.95	20.70	19.45	19.37	18.62	18.10	17.35
450	6.71	15.6	26.55	25.30	22.05	20.80	20.72	19.97	19.45	18.70
500	6.17	17.0	28.00	26.75	23.50	22.25	22.17	21.42	20.90	20.15
600	5.32	19.7	30.80	29.55	26.30	25.05	24.97	24.22	23.70	22.95
700	4.67	22.5	33.70	32.45	29.20	27.95	27.87	27.12	26.60	25.85
800	4.17	25.2	36.50	35.25	32.00	30.75	30.67	29.92	29.40	28.65
900	3.76	27.9	39.30	38.05	34.80	33.55	33.47	32.72	32.20	31.45
1000	3.43	30.6	42.10	40.85	37.60	36.35	36.27	35.52	35.00	34.25
1200	2.91	36.1	47.80	46.55	43.30	42.05	41.97	41.22	40.70	39.90
1400	2.53	41.5	53.40	52.15	48.90	47.65	47.57	46.82	46.30	45.55
1600	2.24	46.9	59.00	57.75	54.50	53.25	53.17	52.42	51.90	51.15
1800	2.00	52.5	64.80	63.55	60.30	59.05	58.97	58.22	57.70	56.95
2000	1.81	58.0	70.50	69.25	66.00	64.75	64.67	63.92	63.40	62.65
2200	1.66	63.3	76.00	74.75	71.50	70.25	70.17	69.42	68.90	68.15
2400	1.53	68.6	81.50	80.25	77.00	75.75	75.67	74.92	74.40	73.65
¼ mile.	1.39	75.5	88.64	87.39	84.14	82.89	82.81	82.06	81.54	80.79

Art. 14. Removing rock excavation by wheelbarrows. A cubic yard of hard rock, *in place*, or before being blasted, will weigh about 1.8 tons, if sandstone or conglomerate, (150 lbs per cubic foot;) or 2 tons if good compact granite, gneiss, limestone, or marble, (168 lbs per cubic foot.) So that, near enough for practice in the case before us, we may assume the weight of any of them to be about 1.9 tons, or 4256 lbs per cubic yard, in place; or 158 lbs per cubic foot.

Now, a solid cubic yard of any of these, when broken up by blasting for removal by wheelbarrows or carts, will occupy a space of about 1.8, or $1\frac{4}{5}$ cubic yards; whereas average earth, when loosened, swells to but about 1.2, or $1\frac{1}{5}$ of its original bulk in place; although, after being made into embankment, it eventually shrinks into less than its original bulk. In estimating for earth, it is assumed that $\frac{1}{1\frac{1}{4}}$ cubic yard, in place, is a fair load for a wheelbarrow. Such a cubic yard will weigh on an average 2430 lbs, or 1.09 tons; therefore, $\frac{2430}{14} = 174$ lbs, is the weight of a barrow-load, of 2.31 cubic feet of loose earth. Assuming that a barrow of loose rock should weigh about the same as one of earth, we may take it at $\frac{1}{24}$ of a cubic yard; which gives $\frac{4256}{24} = 177$ lbs per load of loose rock, occupying 2 cubic feet of space.

In the following table, columns 2 and 3 are prepared on the same principle as for earth, as directed in Article 4. Column 4 is made up by adding to each amount in column 3, .2 of a cent for each 100 feet length of lead, for keeping the wheeling-planks in order; and 45 cents per cubic yard, in place, as the actual cost for loosening, including tools, drilling, powder, &c: as well as moderate drainage, and every ordinary contingency not embraced in column 3. Contractor's profits, of course, are not here included.

Ample experience shows that when labor is at $1 per day, the foregoing 45 cents per cubic yard, in place, is a sufficiently liberal allowance for loosening hard rock under all ordinary circumstances. *In practice it will generally range between 30 and 60 cents*; depending on the position of the strata, *hardness, toughness*, water, and other considerations. Soft shales, and other allied rocks, may frequently be loosened by pick and plough, as low as 15 to 20 cents; while, on the other hand, shallow *cuttings of very tough* rock, with an unfavorable position of strata, especially in the bottom of excavations, *may cost* $1, or even considerably more. These, however, are exceptional cases, of comparatively rare occurrence. The quarrying of average hard rock requires about ¼ to ½ lb of powder *per cubic yard, in place*; but the nature of the rock, the position of the strata, &c, may increase

, or more. Soft rock frequently requires more powder than hard. A good churn-driller will o 12 feet in depth, of holes about 2½ feet deep, and 2 inches diameter, per day, in average ck, at from 12 to 18 cents per foot. Drillers receive higher wages than common laborers.

Hard Rock, by Wheelbarrows.

Labor $1 per day, of 10 working hours.

n of dis- to the is ed.	Number of cubic yards, in place, wheeled per day by each barrow.	Cost per cubic yard, in place, for loading, wheeling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor.	Length of Lead, or distance to which the rock is wheeled.	Number of cubic yards, in place, wheeled per day by each barrow.	Cost per cubic yard, in place, for loading, wheeling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor.
t.	Cubic Yds.	Cents.	Cents.	Feet.	Cubic Yds.	Cents.	Cents.
)	12.2	8.64	53.7	600	2.96	35.5	81.7
)	10.7	9.81	54.9	700	2.62	40.1	86.5
)	9.58	11.0	56.2	800	2.34	44.8	91.4
)	8.66	12.1	57.3	900	2.12	49.5	96.3
)	7.26	14.5	59.8	1000	1.94	54.1	101.1
)	6.25	16.8	62.2	1200	1.65	63.6	115.0
)	5.49	19.1	64.6	1400	1.44	72.9	120 7
)	4.89	21.5	67.1	1600	1.28	82.2	130.4
)	4.41	23.8	69.5	1800	1.15	91.5	140.1
)	4.02	26.1	71.9	2000	1.04	100.8	149.8
)	3.69	28.5	7! 4	2200	.953	110.2	159.6
)	3.41	30 8	76.8	2400	.879	119.5	169.3

. 15. **Removing rock excavation by carts.** A cart-load of y be taken at $\frac{1}{5}$ of a cubic yard, in place. This will weigh, on an average, 851 lbs; or but 41 e than a cart load of average soil. Since the cart itself will weigh about ½ a ton, the total e very nearly equal in both cases. Columns 2 and 3 of the following table are prepared on the inciple as for earth, as directed in Art. 4. Column 4 is made up by adding to each amount in 3, the following items: For blasting, (and for everything except those in column 3; loading, airs of cart-road,) 45 cents per cubic yard, in place; for loading, 8 cents, per cubic yard, in ind for repairs of road, .2, or $\frac{1}{5}$ of a cent for each 100-feet length of lead. Contractor's profit nded.

Hard Rock, by Carts.

Labor $1 per day, of 10 working hours.

n of dis- to the is d.	Number of cubic yards, in place, hauled per day, by each cart.	Cost per cubic yard, in place, for hauling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor.	Length of Lead, or distance to which the rock is hauled.	Number of cubic yards, in place, hauled per day, by each cart.	Cost per cubic yard, in place, for hauling, and emptying.	Total cost per cubic yard, in place, exclusive of profit to contractor.
t.	Cubic Yds.	Cents.	Cents.	Feet.	Cubic Yds.	Cents.	Cents.
)	19.2	6.51	59.6	1800	5.00	25 0	81.6
)	18.5	6 77	59.9	1900	4.80	26.0	82 8
)	17.8	7.03	60.2	2000	4.62	27.1	84.1
)	17.1	7.29	60.5	2250	4.21	29.7	87.2
)	16.0	7.81	61.1	2500	3.87	32.3	90.3
)	15.0	8.33	61.7	½ mile	3.70	33.7	92.0
)	13.3	9.37	63.0	3000	3.33	37.5	96.5
)	12.0	10.4	64.2	3250	3 12	40.1	99.6
)	10.9	11.5	65.5	3500	2.92	42.8	102 8
)	10 0	12.5	66 7	3750	2.76	45.3	105.8
)	9.23	13.6	68.0	4000	2.61	47.9	108.9
)	8.57	14.6	69.2	4250	2.47	50.6	112.1
)	8.00	15.6	70.4	4500	2.35	53.2	115.2
)	7.50	16.7	71.7	4750	2.24	55.8	118.3
)	7.06	17.7	72.9	5000	2.14	58.4	121.4
)	6.67	18.7	74.1	1 mile	2.04	61.2	124.8
)	6.32	19.8	75.4	1¼ "	1 67	75.0	141.2
)	6.00	20.8	76.6	1½ "	1.41	88.8	157.6
)	5.71	21.9	77.9	1¾ "	1.22	102.5	174.0
)	5.45	22.9	79.1	2 "	1.08	116.3	190.4
)	5.22	24.0	80.4	2¼ "	.962	130.0	206.8

s omitted "loose rock," will cost about 30 cts per yd less; and even solid rock will never ts less than the tables.

[illegible]

[illegible]

[illegible] of the figure [illegible] angles, to the [illegible] the side opposite [illegible] measure it blindly.

If either a trapezoid, or a trapezoidal [illegible] By either of them [illegible] longest [illegible] of gravity will be [illegible]

Of a trapezoid similar. [illegible] other parallel [illegible] g; and make [illegible] opposite side [illegible] Then prolong the [illegible] side [illegible] opposite direction; make [illegible] Find the centre [illegible] Then [illegible] of gravity.

Of a semicircle. [illegible] by .4244; the [illegible] of gravity.

Of a segment of a circle, a d b c. Make twice the [illegible] the length [illegible] of Arcs, p 21, [illegible] of gravity.

Of a segment of a circle, a d b. Cube the chord [illegible] the area of the segment; [illegible] of segments, page 24. The [illegible] will be c n; and a [illegible] of gravity.

Of a parabola, [illegible] as [illegible] of its length from the base.

The [illegible] of gravity g, of two figures and [illegible] Find and their separate cen of gravity and [illegible] by a line st; and measure its length. Then add together the areas of the two figs: and say, as the sum is to the whole length st, so is the area of the longest fig a, to the longest division tg of the line. Then g is the cen of gravity of the two figs combined. The longest division most evidently be next to the smallest body b. If the common cen of gravity of the three bodies a, b, and c, is required, begin with any two of them as has just now been done. Then draw gc, and measure it. Then suppose the united areas of a and b to be placed at g, as if one fig at that point; and call it Fig g. Add the area of this fig to that of Fig c; and, as before, say as [illegible] of gc, so is the area of c (the greatest of them) to the [illegible] the least of them, or g. Then is y the common cen of gravity. [illegible] may now consider the combined areas of a, b, and c to be placed at y, and [illegible] in the same manner find the common cen of gravity of all four figs: and so on [illegible] If, instead of being merely plane figs, or areas, a, b, c, and d were [illegible] the process would be precisely the same, except that the weights [illegible] In other cases it is immaterial which two we begin with.

IN AN IRREGULAR FIG which may be divided into triangles, trapeziums, trapezoids, &c, the process is the same as with the figs *a*, *b*, *c*, and *d*.

The cen of grav of any plane fig may be found by drawing it to a scale on pasteboard; then cut out the figure; balance it in two or more positions over the edge of a table, or on a sharp knife-edge; and mark on it the directions of the edge. Where these directions intersect each other, will be the reqd point, near enough for most practical purposes. The paper on which the fig is prepared, must be so stiff that the fig will not bend when balanced.

Of a cube, parallelopiped, cylinder, prism, sphere, spheroid, ellipsoid; the cen of grav is in the cen of the body.

Of any frustum of either a right or oblique cylinder; *or of a right or oblique prism;* whether cut parallel to its base, or obliquely. In the center of the axis of the frustum.

Of a right pyramid, or cone; in its axis, at ¼ of its length from the base.

Of any pyramid, or cone; whether right, or oblique. In a line from its vertex, or top, to the cen of grav of its base; and at ¼ the length of this line from the base.

Of a frustum of a right cone, cut parallel to its base. Call the rad of the larger end R; and that of the smaller end r; and the height of the frustum measd on its axis, h. Then,

$$\frac{h}{4} \times \frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2} = \text{dist on axis from greater end, or base.}$$

Of a frustum of a right pyramid, cut parallel to its base. Call the area of the large end A; that of the small end a; and the height measd on the axis, h; then,

$$\frac{h}{4} \times \frac{A + 2\sqrt{Aa} + 3a}{A + \sqrt{Aa} + a} = \text{dist on axis from greater end, or base.}$$

Of any frustum of any pyramid, or cone; whether right or oblique; or whether the ends of the frustum are parallel to the base or not; it is in a line *t o*, drawn between the centers of grav of the two ends of the frustum; but its height *y c* above the lower end *a n*, and perp to it, must first be found upon the line *t y*, drawn from the cen of grav of the small end, to, and at right angles with, the large end *a n*. Having first found the line *t y*,

whether for a pyramid or a cone, use it instead of the height h, in the preceding formula, for a frustum of a right *pyramid*. The result will be *y c*; and by drawing *c s* parallel to *a n*, we find the reqd point *s*, in the line *t o*.

Of the curved or slanting surface only, of a right cone; in the axis; and at ⅓ of its length from the base.

Of a hemisphere; in its axis, at ⅜ of its length from the base.

Of a segment of a sphere. From twice the rad of the sphere, take the height of the segment. Square the remainder. Then from 3 times the rad of the sphere, take the height of the segment. Div the square just found by this last rem; and take ¾ of the quot, for the dist from the cen of the sphere to the cen of grav of the segment.

Of the curved surface only of a hemisphere; of a spherical segment; or of a zone: at the middle of its axis, or height.

Of a paraboloid: in its axis, and at ⅓ of its length from the base.

REM. We must not confound cen of *grav* with cen of *weight*. It does not follow when a body balances on a knife-edge, that there is equal wt on both sides of it; but merely that the wts of the several particles on one side, when mult by their respective leverages, or dists, at right angles from the knife-edge, have a united *moment* about the knife-edge, equal to that of the particles on the other side of it, when mult by *their* leverages. See Leverage, &c, in Force in Rigid Bodies.

MECHANICS. FORCE IN RIGID BODIES.

Art. 1. Mechanics (the very foundation of civil engineering) is that branch of science which treats of the effects of force upon matter. This broad application of the term necessarily includes hydrostatics, pneumatics, &c; but ordinarily it is restricted to the effects produced upon matter by the application of *outward, extraneous*, or *mechanical* force, from whatever source it may be derived; whether from steam, water, air, wind, gravity, animals, &c. Some of these effects consist in either *derranging, or separating the particles* of matter which compose a body, by pushing *them close together; or by pulling* them farther apart; and in all such cases the *extraneous force is to be considered* only as acting against the natural inherent *ves, or strength* of the materials of which the bodies consist. This class of effects

[illegible]

[illegible] *to put into it, or enter it*: and *produce an effect* [illegible]

[illegible] is any definite quantity of matter, as a [illegible]

[illegible] of a body is the amount of *vert* pres[illegible] while the body is *at rest*. In ordin[illegible] of matter in a body is measured by its wt; [illegible] but which is not adapted to

ıen pull or push, with equal forces, a body that is between them, their forces merely ɔunterbalance, or destroy (see Rem, Art 13) *each other;* but they produce no effect the body *as a whole; it* remains at rest; and has no more tendency to move in any if both forces were absent. The forces then merely strain (pull or push against,) each other, and keep each other at rest; but no practical error will arise from ımon phraseology, and saying they act upon the *body* to keep *it* at rest.

ı **measured by weight**; as by pounds, tons, &c. Its amount or ual to that of only one of the two equal opposing forces. Thus, if two ıinst each other at two ends of a rope, each with a force of 30 lbs, the rope is but 30 lbs; and it is equal throughout the length of the rope. forces strain each other, 30 lbs; as is made manifest if a spring-balance any part of the rope. If a rope passes over a pulley, and equal wts be each end of it, then the two equal forces of gravity of the two wts strain ɔther; and also strain the rope; to an amount equal to one of them.
ı Strain, see Art 13, p 449.
ng forces produce strain and a tendency to motion, among the *particles* se a body to which they are imparted; but exert no tendency to motion ly *as a whole;* because the particles are held together by their internal ; and this cohesive force reacts against our extraneous imparted forces. ıufficient to do so completely, the body is broken; and the remaining e imparted forces, becoming **motion,** scatters the fragments in all

as a whole, is isolated, detached matter, which is not adhering to anything by any in itself opposes no resistance to force; it is inert. Therefore, so far as the imparted ued with it, *as a whole,* to give it motion in any direction. it is merely a corpse, over ending forces are destroying *each other,* in their struggles to get possession of it. If orces are equal, they will strain against each other, until they are mutually destroyed; ill remain unmoved by either. If they are unequal, the surviving portion of the move on at a slackened pace, in its former course; moving the unresisting body ee Rem 3, &c, of Art 28, p 458.

Force is that principle of which, considered simply as a mechanical ɔw but little more than that when it is *imparted,* that is, *put into,* a body, ther motion alone; or strain, with or without motion. This is all that ıically considered, can do under any circumstances whatever. When it ion alone, it is called *moving* force. When it produces strain alone, or ɔn, *straining* force; or simply strain, pull, push, tension, or pres, as the As, for instance, when a body rests quietly on a table, or on a post, &c; ed by a chain, rope, &c; its effect is simply strain; a push or pres on ıble, and a pull or tension, on the rope. There can be no strain except are at least two forces to strain against each other. When it produces otion at the same time; or in other words, when a part of it produces part of it, motion; it is called *working* force: and its effect is *work.* man is lifting a wt, a part of his force is straining against the equal ty of the wt; while the other part of his force is giving motion to the ıll his force combined, is at work; or is working force; or performs rt 11, p 448.

different *kinds* of force; but diff *effects* of the same force; for there are no diff kinds of e; it is all the same: no matter from what source it may be derived. Indeed, we , and without fear of being misunderstood, call it simply motion, work, or strain, ich of the three effects it is producing at the time; and we shall frequently do so pages. The only means we have of measuring it, is by measuring the quantity of effects.

but force can resist force. When all the force imparted, or , is unresisted by opposing force; it produces motion only; when it is all resisted, it only; when a part of it is unresisted, and a part resisted, the first part produces ɔ last part strain; which two combined constitute *work.* A single force can produce least two are required to produce strain. See Art. 13, p 449.
ɔtimes defined to be that principle which either *does* produce, or *tends* to produce h either *does* prevent, or *tends* to prevent it. Friction, however, which is classed ıways tends to prevent motion; therefore, when the word force is used alone, it must that friction is not included.

infer from both reason and observation, that force, when once imparted, body, would, if not resisted by other force, remain in it, and move it ıtraight line; in the direction in which the force was imparted; and at ocity, or rate of speed.

re in practice, that motion continues longer in proportion as we remove resisting e infer, that if all resistances could be removed, it would continue forever. But in not remove all resisting forces, such as those of gravity, wt, friction, the air, &c. ainst, or resist the forces which we may impart; so that with the exception of a the force of gravity, in a vacuum, it is difficult to mention a case of motion alone,

***tter, in itself,* cannot resist** force. See Arts 5 and 11. Whe *motion alone,* without strain, it is often called Dynamic force; wh

continuous strain without motion. *Static;* when a sudden strain for an inst as a blow of a hammer, *Impulsive;* and its action, *an impulse.* See Art 9.

Rem 3. The **Living Force,** or **vis viva** of scientific writers, is noth than an expression referring to the quantity of work (motion and strain c which the force in a body at any given instant, could perform, if left to its out afterward receiving any additional force. See Art. 24, p 465.

Art. 4. When force is imparted in any direction which passes throug of grav of a homogeneous rigid body, *perfectly free to move,* it quickly diffu equally through every part; and gives to every atom composing the body, tendency to move onward in the same direction.

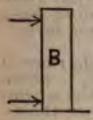

Fig 1

But although this diffusion takes place *rapidly,* especially in compact it requires *some* time; generally so short as not to be appreciable witho servation. If we put the body B, Fig 1, into motion by striking it a blow or bottom, the struck end will for an instant move faster than the other; both will move with equal velocity; because the great force which for a short time acted upon the top only, becomes equally diffused throughout We are not, however, now treating of cases of this kind, in which the dire force does not pass through the cen of grav of the body; for the body m tate, or whirl around as it moves forward.

This equal diffusion of force is owing to the inherent cohesive force of be holds their atoms together, somewhat as lime holds together the grains of piece of mortar; and will not permit one grain to move unless the others it. In producing this diffusion, the cohesive forces, or the lime, do not d *quantity* of the imparted force; but merely cause that instead of continuing to give a gr to that part only at which it was imparted, it shall give a slower, but equal one, to every

When a body breaks on the application of outward force, it indicates that the cohesive ment, was not sufficient to bear the rapid passage through it of the force, from one part to another; therefore, nearly all the force continues in the part to which it was imparted, it away from the remainder with great vel. See Art 27, p 456.

But if the rigid body to which we impart force in any direction, *is not free to move,* bein from so doing by opposing force, then our imparted force strains against the opposing one produces *strain,* instead of *motion,* in the body.

Art. 5. We must clearly distinguish between merel ing a body, and lifting it. The smallest imaginable force would sisted by other force, *move* the greatest imaginable body; but if we appl lift a body, then its wt, or force of gravity, resists. Force applied merely things without lifting them, is often called traction; as in the case of hors towing a canal boat, or hauling a vehicle along *level* ground. The same w ever, is generally used for hauling, even when the thing is partially lifted; a horse is hauling up hill; in which case he is lifting both himself and t with its load. See Traction, pp 603, 605.

Matter always *appears* to resist force; but this deception is caused by its gravity, or by some other resisting force acting upon it at the same instant.

A man cannot *lift* a wt of 20 tons; but if it be placed hor upon proper friction rollers, h it, as we see in some drawbridges, turntables, &c; and if friction and the air could be moved, he could move it by a single breath; and it would continue to move forever. It w ever, move very slowly, because the force of the single breath would have to diffuse itsel tons of matter. He can move it if it be placed in a suitable vessel in water, or if suspen long rope.

If we imagine two bodies, each as large and heavy as the earth, to be precisely balanc of scales without friction, it is plain that since all of their two opposing gravities strain a other, and thus neutralize each other's motions, a single grain of sand added to either would give motion to both bodies. It would of course be very slow, but still it would be the same proportion that the wt of the grain of sand bears to that of the two earths; or in portion to the unresisted force of gravity contained in the grain. If two grains were add of one, the two earths would move twice as fast.

The inherent cohesive force of matter, by which its particles are held in close union, causes the matter itself to appear to resist straining force; thus, a cake of ice may sust pres; but if we destroy its cohesive force by converting it into water, it will yield readily the metals and stones if reduced to dust. It is not the *material* that resists being broke inherent *cohesive force* which holds the particles of the material firmly together.

The fact that matter in itself opposes no resistance t is called its inertia, or inertness; and since nothing but force force, it follows that motion (which under all circumstances is the effect o resisted portion of force in a body) cannot be the result of an *effort* of th but is simply the manner in which said force *exhibits itself,* or *indicates its* It is as it were a notice hung out on the moving body, saying, "Unemplo stored up here; work wanted."

Gravity acting in a body falling freely in a vacuum, and consequently unresisted, exert *upon it;* it neither goes before, and pulls it along, nor behind, and pushes it; for there ca *or push except when there is some force* to pull or push against. But it simply as it wer *the body, or endows it* with the power of locomotion. As the body falls, the force of gra *gives it motion all remains* unimpaired, and stored up in it, ready to exert an effort agai *force which it may chance* to meet with. Therefore a body falling unresistedly, has no *gravity, which gives it weight* alone while at rest, now gives it motion alone.

In most kinds of machinery, nearly all the parts are supported by

ns, which **resist the action of gravity** of the parts; so that the working ower, has only to overcome the *friction;* and not to lift the *weight* of the expression, "**overcoming the inertia of a body,**" is founded al error; for there is no such resistance as inertia to be overcome. Inertia ie of a *fact;* not of a *force;* and that fact is simply, that matter does not e. What is called overcoming inertia, is simply *putting in force;* as when vater into a glass, we do not overcome any resistance of the glass against d; we simply apply the water to the glass, which receives it, and retains by applying force, we take it out again. "Overcoming the inertia" of a rain, &c, therefore, means nothing more than that if we wish the train to must put moving force into it. It is inert, and cannot move of itself. See 53.

. Care must be taken not to confound motion with vel. **Velocity is e of speed** of motion; or, in other words, it is the *dist* moved over *in ne*, without any regard to the wt of the body. Thus we say that it moves , or at the rate, of 10 ft per sec; 20 miles per hour, &c; whether it weighs or a ton. The vel of a body of given weight, is in direct proportion to the unresisted force; and in inverse proportion to the wt of the body; that is, ree times the unresisted force will give to the same body two or three times r if one body has two or three times the wt of another, the same amount ted force will give it but ½ or ⅓ the vel.*

. **We impart force to a body,** by **applying** to it (that is, by nto close contact with it) a second body in which the force already is. We riminate between *applied*, and *imparted* force; and the precaution is the ssary because writers generally appear not to recognize the distinction.

orce is merely carried *to* a body; imparted force is put *into* it. A heavy body containing may be applied to a small body containing none: and yet the former may impart but force to the latter. If we place a body on, that is, apply it to, a steep inclined plane, we he wt of the body, or its force of gravity; yet only a portion of this force is imparted, or plane; while another portion remains in the body; and being unresisted by the plane, e motion of sliding down it. Force may be applied to a body in any direction whatever; mparted to it only at right angles to the surf at the point of application. See Art 19. it of a body to which force is applied, is called the *point of application;* but in fact we ractice apply force to a *point*, according to the scientific meaning of that word; but have distributed over an appreciable area sometimes very large) of the surface of the body; lained in Art 57, it may, so far as regards its action upon the *entire mass* of a *rigid* body, ed as applied at one point. The expression, *direction of a force*, may refer either to the which it is applied, or to that in which it is imparted; or, in a word, to that direction, may be, in which it may happen to be acting at the moment under consideration.

. When diff forces act upon a body, it is absolutely essential, in consid- r effects upon it, to know whether they all act **in the same plane;** y do not, their effects become totally diff.

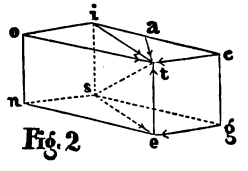

Fig. 2

e of paper is a plane, and if on it we draw any straight lines, in any direction whatever, they will many forces all acting in that same plane; that flat surface coincides with the directions of all of ill evidently do the same in whatever position this may be placed, whether hor, vert, or inclined. nes drawn on the floor of a room, will represent *at* same plane; lines on the ceiling, forces in *that* ; which of course is not the same plane of those ; so with lines on the sides of the room. All the *a t*, *c t*, Fig 2, are in the same plane *t o i c*. Although plane *t o i c*: and *t e* at the same time in the plane n the plane *t o n c*; and *s e* in the plane *s n e g*; and ane *i t s e*, still all of these, namely, *i t*, *t e*, *s e*, and ently in the same plane *i t s e*. Any two lines which meet, or would meet or intersect each ficiently extended in either direction, are in the same plane: as *o t*, *i t*; or *s t*, *i t*; or *t s*, wo lines may be in the same plane, and yet not meet if extended; as, for instance, the es *c t*, *g e*, in the plane *t c g e*. The lines *a t* and *g e*, being in parallel planes, *o i c t* and not meet if extended.

e must not confound acting *in*, with acting *on*, *upon*, or *against* the same plane. The om is a plane; and *upon* or *against* that same plane, forces *in* a thousand diff planes may istinction is so self-evident, that a bare allusion to it will prevent mistake. See Fig 53½, 1.

AR VELOCITY. When a body revolves around any center, it is plain that the parts which from said center, will move faster than those nearer to it. Therefore, we cannot assign in ft per sec, or miles per hour, that shall apply to *every part* of a revolving body. But plain that every part of the body revolves around an entire circle of 360° of angle, in the Hence, in many scientific investigations, the number of degrees per sec is used, (instead sec,) *to denote the vel of* such a body; and is called its angular vel. The hub of a wheel *e angular vel as the tire*: because both move through the same number of degrees per *tire has a greater actual* vel than the hub, because it moves through a greater numb a.

Art. 9. The **quantity**, or **amount, of any motion**, of any body, is measd by mult the wt of the body by the dist through which it is moved; thus, if a body of 4 lbs is moved 3 ft; or one of 6 lbs, 2 ft; or one of 120 lbs, $\frac{1}{10}$ of a ft; there is in each case an entire motion of 12 foot-pounds. If the body is *lifted*, there still is a *motion* of 12 ft-lbs; but there is also *work* of 12 ft-lbs performed. See Art 11. We remind the student that he must carefully avoid confounding motion alone, with motion and strain combined; that is, with *work*. *Motion alone* implies that the force which produces it, has no resistance to overcome. It merely causes the body to move, instead of standing still. The identical force, or portion of a force, which is producing motion, cannot at the same time be doing anything else.

The quantity of moving force in a body, is estimated or measured, by mult the wt of the body by its vel, or *rate* of motion; that is, by the dist to which the force could move it *in a certain given time;* as in one sec, hour, &c.

This is rendered necessary by the fact that the force would, if unresisted, move the body *for ever*, (see Rem 1, Art 3;) hence we could not assign to it any *total* dist. If a body of 4 lbs is moving with a vel of 3 ft per sec, we say it has a motion, or has in it a moving force of $4 \times 3 = 12$ foot-lbs per sec; that is, it has in it an amount of force sufficient, if unresisted by other force, to keep in motion for ever, 4 lbs at 3 ft per sec; or 6 lbs at 2 ft; or 12 lbs at 1 ft per sec; or any other number of lbs, at any other number of ft per sec, provided the two numbers when mult together make 12. The quantities of *motion per sec*, or of moving force, in all these cases, is the same; although the *vels* are diff. It never happens in practice that *all* the force imparted to a body, acts as motion alone; for some part of it has always to act as *strain* against some opposing force which we cannot get rid of; as gravity, friction, the resistance of the air, &c; and since these do not cease their opposition, they gradually resist, and destroy *all* the moving force also; and the body therefore finally comes to rest. If at any moment before it finally stopped, all resistance could be removed, it would move on for ever, with the vel it had at that moment.

Momentum is merely another name for moving force. Since the same force that will move a body of 1 lb with a vel of 1000 ft per sec, will impart to a body of 1000 lbs a vel of 1 foot per sec; and since the quantity of motion given by the force is the same in both cases, it follows that a body of 1000 lbs resists moving force no more than does a body of 1 lb. Hence, matter, as before remarked, does not resist moving force at all; but any given amount of force, unresisted by other force, will impart to an infinitely great body precisely the same amount of motion as to an infinitely small one; but at diff vels.

Art. 10. An impulse, impact, blow, stroke, or collision, is when force, before engaged in moving a body, *suddenly* encounters opposing force in another body; and thus *instantly* becomes changed, either wholly or in part, from motion, into strain.

It is the same identical force as before, but is engaged in producing a diff effect; in the first case this effect was motion; in the second, it is a blow; or pres, or push, for an instant. We cannot actually measure or estimate an impulse by lbs, tons, &c; that is, we cannot assign any number of pounds, tons, &c, of quiet pres or pull, which would produce an effect equal to what we call the force of the blow. Hence, those writers are in error who give rules for finding the number of tons of force with which a pile-hammer strikes the head of a pile. We can only say that the force of the stroke is that due to a moving force, or momentum of so many foot-lbs per sec; and this serves no other purpose than to indicate that if two rigid bodies, whether of equal or unequal wts, have in them the same quantity of moving force, they will strike *equally* hard blows; but *how* hard we cannot say. Thus, if a body of 100 lbs has a vel of 3 ft per sec; and another body of 5 lbs has a vel of 60 ft per sec, each has a moving force of 300 ft-lbs per sec; and if they meet each other from opposite directions, they will mutually destroy each other's moving forces, and bring each other suddenly to rest; their momentums, moving forces, *or blow-striking capabilities* being equal. The terms moving force, and momentum, may be, and generally are, used indifferently.

"In some careful experiments made at Portsmouth dock-yard, England, a man of medium strength, and striking with a maul weighing 18 lbs, the handle of which was 44 ins long, barely *started* a bolt about $\frac{1}{8}$ of an inch at each blow; and it required a quiet pres of 107 tons to press the bolt down the same quantity; but a small additional weight pressed it completely home."

Art. 11. Working force. When the force imparted to a body, not only moves it, but does so in spite of resisting forces; or in other words, when one part of it strains against, and balances the resistances equal to itself; while the other part of it moves the body, it is called working force. The quantity of working force expended; as well as the quantity of work thereby accomplished, is estimated by *mult the resistance* overcome, (which frequently consists of wt *lifted*,) by the dist *through which it is* overcome. Thus, if the resistance be a friction of 4 lbs, over*come at every point* along a dist of 3 ft; or if it be a wt of 4 lbs, lifted 3 feet high, *then either the* working force, or the work done by it, amounts to $4 \times 3 = 12$ ft-lbs. *The lifting of* wts; and the friction encountered in merely moving them by sliding *r rolling, constitute* the principal sources of resistance, and work, in practice. Th *uantity of* any work may evidently be considered by itself, without regard to *ne reqd to perform it*; but (as in motion) we generally require to know the qu *ich work* can be done; that is, how much can be done within a certain

Thus, in selecting a steam-engine, it is important to know how much it can do p minute, hour, or day. We therefore stipulate that it shall be of so many hor powers; which means nothing more than it shall be capable of overcoming resisti forces, at the rate of so many times 33000 ft-lbs per min.

Working force is often called *power;* thus, horse, steam, water power, &c. Since work, in a *stric scientific sense*, involves motion and strain combined, a man who is standing still is not considered be working, any more than a post or a rope sustaining a heavy load; although he may be support an oppressive burden; or holding a car-brake with all his strength; for in both cases his force *strains* only. The work done by a horse drawing a heavy load on a *level* road, consists entirely overcoming the *friction* at the axles and rims of the wheels; but in drawing it *up hill*, he partly l the *load*, and *himself* also. In the first case his work (scientifically) is not so much wt of load mo a certain dist; but so many pounds of rolling and axle-friction overcome through said dist. Up h his work consists of friction, overcome through one dist; namely, the *length* of the hill; and the wt of the load, vehicle, and himself, lifted to another dist; namely, the *vert height* of the hill. a perfect level road, and without friction, the horse would encounter no resistance in hauling load, however great it might be. He would have nothing to do but impart motion to unresist matter. Therefore, in such a case his hauling would not be *work*, in the scientific sense.

The ordinary unit of work, or standard for measuring it, is one lb of *lifted* one foot high; or one lb of any kind of *resistance*, overcome in any directi whatever, for the dist of one foot; and is called a *foot-pound*. Or, when more con nient, we may use foot-tons, &c, &c; as we use a two-ft rule, a yardstick, or a ta line, as may best suit our purpose. **The unit** of **RATE of work**, or the qu tity done in a *given time*, is *one ft-lb per sec.*

The same quantity of force which will overcome a given resistance through a given dist, in a gi time, will also overcome any other resistance through any other dist, in that same time, provi the resistance and dist when mult together give the same amount as in the first case. Thus, force that will lift 50 lbs through 10 ft in a sec, will lift 500 lbs, 1 ft; or 25 lbs, 20 ft; or 5000 $\frac{1}{10}$ of a ft in a sec; and in all these cases the amount of force expended, as well as of work done precisely the same. In practice, the adjustment of the speed to suit diff amounts of resistance usually effected by the medium of cog-wheels, or other species of levers. By means of these, engine, water-wheel, horse, or other motive power, may be made to overcome small resistances rapid or great ones slowly, by the same working force.

For more on working force, see Art 17, 18, 19, &c.

Art. 12. When vel undergoes no change, it is said to be *uniform;* so also w force, motion, strain, and work. When any of them becomes gradually greater, i said to be *accelerated;* when gradually less, *retarded*. If the acceleration, or retar tion is in exact proportion to the time; that is, when during any and every eq interval of time, the same degree of change takes place, it is *uniformly* accelerat or retarded. When otherwise, the words *variable*, and *variably* are used. Confusi arises from the frequent use by even the best writers, of the words "constant," a "uniform," instead of *uniformly accelerating* force.

Although the expressions are strictly correct, still they are inexpedient; for when causes and effe are so intimately connected, as moving force and motion, it is desirable that the same adjecti should be equally applicable to both; and we should not in the same sentence read of constan uniform force, producing inconstant, or ununiform motion.

Gravity is a uniformly accelerating force when it acts upon a body falling freely; for it then incres the vel at the uniform rate of .322 of a foot during every hundredth part of a sec; or 32.2 ft in every Also when it acts upon a body moving down an inclined plane; although in this case the incre is not so rapid, because it is caused by only a part of the gravity: while another part presses body to the plane; and a third part overcomes the friction. It is a uniformly retarding force, u a body thrown vert upward; for no matter what may be the vel of the body when projected upw it will be diminished .322 of a foot in each hundredth part of a sec during its rise; or 32 2 ft dur each entire sec. At least, such would be the case were it not for the varying resistance of the at diff vels. It is a uniformly straining force when it causes a body at rest, to press upon anot body; or to pull upon a string by which it is suspended. The foregoing expressions, like those momentum, strain, push, pull, lift, work, &c, do not indicate diff *kinds* of force; but merely diff ki of *effects* produced by the one grand principle, force. See footnote, p 455; also p 587.

The above 32.2 ft per sec is called the **acceleration of gravity**; and scientific writers is conventionally denoted by a small g; or, more correctly speaki since the acceleration is not precisely the same at all parts of the earth, g deno the acceleration per sec, whatever it may be, at any particular place. See note Art 25, p 445.

Art. 13. Reaction. Strain. Strain, as before said, in Art 2, is eithe pull or a push. The term may be applied equally to the *act*, or to its *effects;* that it may be said to be either the action of opposing forces against each other; or effects which that *action* produces upon the particles of the body in which they *A single force cannot produce strain*; for since nothing but force can oppose, *pull, or push, against force*, there must be at least two, to strain by pulling or *ing against each other*. This mutual opposition or straining is called also *tion of the forces*. *It can take place only between equal forces*, or equal po *unequal ones. When the forces are equal*, and meet from diametrically oppo *tions; that is, in the same straight line*, but in opposite directions al *become entirely converted* into reaction, or strain. If they are unequal

in the same straight line; but meet obliquely; then only equal portions of them will react against each other; while the remainder will continue as motion; unless some third force is present to prevent it; as when friction is generated. The reaction of the equal wholes or parts consists in their mutually resisting, opposing, arresting, balancing, equilibrating, straining, pulling, pushing, counteracting, or destroying each other.

All these words are equally applicable. As a matter of *convenience* only, we often say that the *bodies themselves* react. If a cannon-ball in its flight cuts a leaf from a tree, we say that the *leaf* has reacted against the *ball* with precisely the same degree of force that the ball acted against the leaf. That degree of force was sufficient to cut off a leaf, but not to arrest the ball; for, after a small portion of the moving force of the ball had been converted into straining force to react against the resistance of the leaf, the remainder was sufficient to carry it onward in its course. It, has however, lost precisely as much force as that which the leaf opposed to it. A ship of war, in ramming against a canoe, receives as violent a blow as it gives; but the same blow that will upset or sink a canoe, will not *appreciably* affect the motion of a ship. The fist of a pugilist striking his opponent in the face, receives as severe a blow as it gives; but the blow which may seriously damage a nose, mouth, or eyes, may have no such effect upon hard knuckles. The pain received in the one case, and not in the other, is of course no measure for force.

Rem. We have just said that strain is the mutual *destruction* of two equal amounts of force. This may readily be conceived in cases where two bodies come into sudden contact, and arrest each other's progress by a mutual blow; for we then see that their forces are lost, inasmuch as they no longer produce either motion, strain, or work. But in *continuous* strains (pulls, or pushes) the principle is not so evident, at first. For instance, when a wt rests upon a table, and continues to strain it day after day, it may be asked where is the loss of gravity force in the weight; inasmuch as it weighs as much after pressing for a long time, as it did at the beginning; or where is the loss of inherent cohesive force in the material of the table, which is as strong as at first?

The reply is that gravity, and the natural strengths, or inherent forces of matter, are being incessantly maintained or renewed, by unceasing streams, as it were, of those forces. The gravity of a wt which presses a table, or pulls on a rope, at one moment, is not identically the same that pressed or pulled it the moment before; and so with the cohesive force of the material of the table. If it were not so, a post or a rope which would be broken by a single force of 10 tons, would also be broken by sustaining one ton 10 consecutive times; for the one ton would each time react against, or destroy one ton of the inherent resisting force of the post or rope; and in ten applications would destroy it all. We must therefore conclude that these natural forces are being unceasingly supplied from that inexhaustible source of power "by which all things are upheld." When we lean forward against a strong wind, we are continuously exerting new force against the continuous stream of force which assails us; and in the same way does a post or a chain sustain its load. Continuous strains produced by the force of water, steam, animals, &c, we well know can only be maintained by a continuous supply of said force; to be as continuously reacted against, or destroyed, by whatever opposing force of grinding, pumping, &c, it is directed against. The strain of an impulse, blow, or stroke, lasts only for an instant, because new force is not supplied to make it continuous. Two equal forces, straining against each other, do not even keep a body at rest; but the body rests merely because the two forces destroy each other, and therefore cannot prevent it from resting. As a matter of convenience only, we may, however, say they keep it at rest. We might even assume strictly that force produces motion only; and that the destruction of force *produces* strain, and thus *restores* rest.

Art. 14. While a horse is hauling a boat on a canal, it is not the boat and its load which react against his force; because matter cannot react against force; it is the force of friction of the boat against the water; and the resistance of the water in front of the boat, produced by its cohesive force. So with an engine and train on a *level* railroad; the only resistance to the steam force of the engine, is the force of friction at the axles and tires of the wheels, and the pres force of the air in front. But on an up grade, the engine has also to partly *lift* the train; or, in other words, to react against its forces of friction and gravity combined.

Neither the horse nor the engine exerts any more force upon the opposing forces, than these last exert upon *them;* but both the horse and the engine possess a surplus of power beyond what is necessary to strain against or destroy the forces opposed to them: and this surplus, as moving force, enables them in addition to move forward, as in the case of the cannon-ball just alluded to; and since the *unresisting matter* of the boat, load, and train, is attached to them by the tow-rope and coupling-links, they of course must follow.

The resistance which an abutment opposes to the pres of an arch; or a retaining-wall to the pres of the earth behind it, is no greater than those pres themselves; but the abut and wall are, for the sake of safety, made *capable* of sustaining much greater pressures, in case accidental circumstances should produce such.

Art. 15. If two motions, or two strains, meet or come together, not in the same straight line, they will destroy equal portions of each other; and the remaining portions will unite to act in a direction somewhere between the original ones. But, on the other hand, if a motion and a strain come together, *in any direction*, they will not in the least degree interfere with each other in any way.*

To illustrate the case of two *motions*, or moving forces; suppose a huge falling rock, moving in a vert direction, to be met by the slightest breath of air moving in any direction that is not vert. Then *equal portions of these* motions would be lost by reacting against each other; and the remaining portions would unite to move the rock in a direction not exactly vert.

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* *The writer does not know* whether these views have been entertained by others or not. They are *at variance with the doctrine of the* "Independence of the simultaneous action of diff forces upon *the same point.*" (*Bennett's Morin*, p 132.) which we believe is maintained by the authorities gener*ally. See Art 28; especially* Rem 3. If wrong, we wish to be corrected.

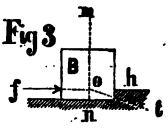


Fig 3

e of two *strains;* let the body B, Fig 3, weighing 1000 tons, a its hor base; and let a force *f*, of say only one ounce press nst the resisting obstacle h. Then we have two strains or ting in the body: the one of 1000 tons, vert, in the line *m n*; ounce, not vert, in the line *f o*. Equal parts of these strains other; and the remainders strain unitedly in a third direc-

e of one strain and one motion; let the 1000-ton body B be est on a hor base so smooth as to be absolutely without fric- ove the obstacle h. Then the body is affected by only the n; and if we again apply the one-ounce force *f*, which now le to press or *strain* against, it will act as *moving* force, and will move the body; which, stated, cannot by itself resist moving force. In practice, the force of friction at the base of ld resist a considerable amount of moving force like *f*.

strain of thousands of tons were produced by the jaws of a vise, in a body weighing an mense strain would not prevent, nor even tend in the smallest degree to prevent, the om falling down from the jaws of the vise. It is prevented by the third force, friction, s the one ounce of gravity force to become vert strain, instead of motion. The two forces of tons each, which produce the strain of the vise, are thereby *entirely destroyed*, as action upon the body *as a whole*. Hence *they* could not prevent the one ounce from pro- in it; nor could they affect it as a whole in any way; for all their action is actually *other*. It is on this principle alone that strains do not interfere with motions.

H, Fig 4, of 10 tons weight, is suspended from a long rope, its reaction equal opposing force at the other end of the rope, produces a continuous ng all the particles which compose the rope; but which does not in the the rope considered as a whole; inasmuch as it does not tend to move it ction. Now, in this case, there is no friction to e; and we know from daily experience that it is sy to move the unresisting body a little dist, by very small hor force *f*. We cannot move it far, nce, to *m*, because we then have not only to *move*, t, or overcome its gravity force, through the vert In doing this, it is true our force does not have the *entire* wt of the body; because most of it is y the rope. Still, if we move it *at all*, we have to *me* of its weight; otherwise, a mere breath would hough very slowly. If we attempt to move it by force *u*, we shall have still more of its wt to re- l if by a downward one *d*, we shall be resisted by e force of the rope. Therefore, in this case, we can move it more readily force *f*.

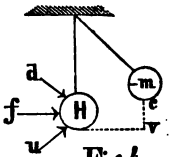


Fig 4

. If two unequal forces, which for illustration we will call 10 and 12, d, either as pulls or as pushes, in precisely opposite directions, to a rigid ich no other force is acting, then the small force 10, and 10 parts of the vill destroy each other as strain; after which, of course, they can produce any kind; but the remaining two parts of the large one, meeting with force to react against, will continue onward as motion, in the same di- before; taking the unresisting body along with them. In such a case, orce is said to *overcome* the small one; and such an expression is very con- reference to the entire original forces. But in a *strictly scientific* sense, annot overcome another.

e foregoing case, so far as the strain is concerned, the large force must be considered as 10 straining, and 2 moving parts. Neither of the two 10 forces which strained against ercame, or gained any advantage whatever over the other; for the two reacted equally , and mutually arrested, equilibrated, and destroyed each other. And this is plainly one force can do to another. The 2 force of the large body took no part whatever in the on the contrary, moved out of the way of it. As it had lost 10 twelfths of its previous , it now moves the body only by virtue of the remaining 2 twelfths; and consequently elfths of its former vel.

equal opposing forces, or equal portions of unequal ones, thus bring each other to a *equilibrate* each other, they are called *Static;* from the Latin "*Sto*, I stand;" and that science of force which treats only of cases in which all the applied forces keep each is called "Statics," or "Equilibrium."

. When force has once been put into a body, it can only be taken out e reaction of some opposing force. The same identical portion of any t produce both motion and strain at the same time. When continuous plied, as in mills, &c, to do both, (or, in other words, to work,) it must be to divide itself into two parts for those separate purposes. If, while at esistances to be overcome become less, the strain also becomes less, and becomes greater; and vice versa. Motion is diminished by converting nto strain; and strain, by converting it into motion.

motion or moving force in a cannon-ball is gradually converted into strain, by working *sisting force of the successive strata of air through which it passes. These gradual force by this process*, and thus permit it to rest. The moving force which remains *after steam is shut off*, is gradually destroyed by working against the resistance

the air; of friction at the axles and rims of the wheels; up-grades, curves, &c. The strain produced in a rope by two men pulling against each other at its opposite ends, is converted into motion if the rope breaks; throwing both men backward. So with the strain on a bent bow. If the string is cut; or if we cut the rope that holds a balloon, part of the force with which the balloon before strained the rope becomes moving force, and by it the balloon ascends. The other part balances gravity.

Art. 18. If force f be imparted to any rigid body, as N, Fig 5, at any point c; and if $f\,o$ represent the direction in which it was imparted, whether as a pull or a push, then the force would produce the same effect upon the body considered as an entire mass, as if it had been imparted as either a pull, or a push, in the same direction, at any other point of the body in said line; as at i, t, s, o, &c.

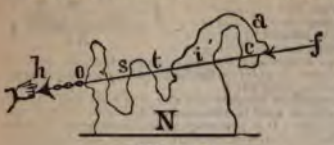


Fig 5

Under Composition and Resolution of Forces, it will be seen to be sometimes necessary to consider a push $f\,c$, to be changed to a pull $o\,h$; and vice versa; when we wish to ascertain the joint effect or resultant of a pull and push imparted to a body at the same time. See Remark 1, Art 29, p 459.

The foregoing important principle holds good, no matter how many diff forces may be acting upon the body at the same time, in diff directions; or how much the direction of their joint effect, or resultant, may differ from that of any one of them; the action of each force, considered *separately*, may be regarded as just stated. The tendencies of several forces, acting at the same moment, may therefore frequently be first investigated one by one; and these tendencies then combined into one; or the forces themselves may first be combined into one or more resultants, as directed under Composition and Resolution of Forces, and the effect of these resultants considered. The engineer has generally to divide all the forces acting upon his structures into two classes; namely, those whose tendency is to secure the stability he requires; and those which tend to impair that stability. He therefore first finds the resultant, or joint effort of each class separately; and then compares these two resultants with each other. The mode of doing this will be shown further on. See Arts 35, 72, &c.

It is plain that if, instead of regarding the body as rigid, we considered it as elastic, or as breakable, an entirely diff course would be necessary, as the question would then become one on *the strength of materials*; for the force f, applied at c or t, as a push, might break off the pieces c and t; and so with the same force as a pull at s or o. Although masonry, iron, timber, and other building materials are by no means absolutely rigid, yet generally they may be assumed to be so when we are investigating the effect of force to overthrow, or derange the structure as a whole.

Art. 19. The full amount of a given force cannot be imparted, or put into a resisting body B, Fig 6, except when applied in a direction at right angles to the surf of the body at the point of application. If it be applied in a direction at all oblique to the surf, then only a portion of it will be imparted to, or will enter, the body, and produce any effect on it. The remainder will continue to produce motion in that other body in which the applied force was carried to the body B; unless some third force be present to receive and react at right angles to that remainder also. Thus, the forces $m\,o$ and $n\,e$, being at right angles to the surf at the points of application o and e, will all enter B, *if the resistance of* B *is as great or greater than they*; if not, they will move B. But the force F g is not at right angles to the surf at its point of application g; therefore, only a portion of it will be transferred or imparted to the body B, however great may be the resistance of B.

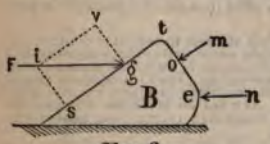


Fig 6

Under the head Composition and Resolution of Forces, it will be seen that when a force is thus applied obliquely to a surf, its action at the point g is precisely the same as that of two entirely separate forces; one of which, $v\,g$, is at right angles to the surf at g; and the other, $s\,g$, parallel to the same surf. Only $v\,g$ will enter the body B; while $s\,g$ will remain in the body F g, which carried the entire force to B; and (if F g also is rigid) will inevitably cause it to move in the direction $g\,i$, unless some third force be opposed at right angles to it also. The quantity of each of the forces $v\,g$ and $s\,g$, is very readily found thus: On the line F g measure off by any convenient scale a dist $g\,i$, to represent the amount in lbs, tons, &c, of the force F g; then on $g\,i$ as a diag draw a parallelogram $i\,v\,g\,s$, having two of its sides perp to the surf at g, and the other two parallel to it. Then $i\,s$, or $v\,g$, measd by the same scale as $g\,i$, will give the force imparted to the body B; and $i\,v$ or $s\,g$ will give that which remains in the body F g, and will move it from g toward t, unless prevented by some other force. If F g is not thus prevented from moving toward t, it is plain that the force $v\,g$, which is transferred to the body B, cannot remain stationary at g; but must move *along with* F g, and thus press upon the surf at every point between g and t.

This will appear in a clearer light on referring to a body placed upon an inclined plane so steep that the body will slide down. We know that a part of the force of *gravity, or wt of the body*, presses, as strain, upon the plane, and at right angles to it; *while a part remains in the body, as motion*; and causes it to slide down the

ane: see Art 60. As it slides, it is evident that the pressure part of the force also oves along with it; and is thus imparted at every point of the plane along the dist d over. The case is identically the same with the pres force $v\,g$ in Fig 6; this will de up the plane from g toward t; being carried along by the sliding force $s\,g$. But th in the case of $v\,g$, and of the body on an inclined plane, we may imagine this ding to be prevented by driving a spike into the surf $g\,t$, or into the surf of the ane, just in front of the sliding body. This will arrest its motion by converting it to strain against the spike. When this takes place, the entire force, F g, becomes ansferred to the body B; in the direction Fg. And in the case of an inclined plane, e entire wt of a body placed upon it, will press it in a *vert* direction: as always kes place when friction prevents the body from sliding down. It would *always* be cessary to devise some means for preventing sliding on an inclined plane, in case rfaces were *perfectly* smooth, and *entirely* devoid of friction; but that portion of e very pres itself which is at right angles to the surf, always in practice creates ore or less friction between the pressing body, and the surf of the plane; and this lction alone is frequently sufficient to react against the sliding portion of the ap- ied force, and thus keep the body at rest. Mortar is sometimes used for the same urpose in masonry; but we should be very cautious in depending upon it, if it leads a neglect of the proper positions of the joints.

This remark is particularly applicable to the case of the masonry ints in the abuts of stone arches; especially those of large span, with all rise. The pres which such an arch exerts upon its abuts is very eat; and its line of direction is curved, as shown by the dotted line s m, Fig 7. It therefore becomes necessary first to find the position this line, (see Art 72.) so as to know how to draw the varying inclitions of the joints nearly at right angles to it; otherwise, the upper arses, if hor, may slide outward upon the lower ones, as shown by s arrow. In small arches of considerable rise, the sliding portion this force may be safely resisted by good mortar or cement, if suffi- nt time be first allowed for it to harden properly; but in large ones, s direction of the joints must be relied on, unless we increase the pense by making the abuts unduly thick. The sliding, or hor pres, of ollar arches of diff sizes, vary as the squares of the span; while the resistance of the mortar varies ly as the thickness of the abut. Hence, if one arch has 4 times the span of another, it will exert times as much sliding pres; while the resistance of the mortar to this sliding will be less than n 4 times that of the small one; because the thickness of the abut does not increase quite as pidly as the span. See Stone Bridges; also Art 34.

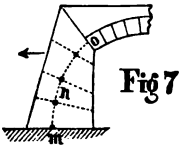


Fig 7

Fig 8 is added merely to illustrate more ikingly the necessity for clearly distin- lishing between *applied*, and *imparted* rces. Here the great force $a\,o$ is applied the body B B at the point o; but all of that is imparted to, or en- rs the body; or produces any kind of ect upon it, is the very small amount represented by $c\,o$, at right angles to the rf of B B at o.

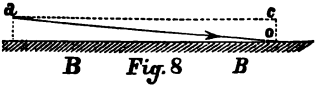


Fig. 8

All this will be better understood after studying Composition and Resolution of rces, Art 28, &c.

Art. 20. It rarely happens in practice that we impart very great force, or vel, a heavy body instantaneously; or by a single effort. In the machinery of mills, in ilroad trains, in steamships, &c, it is done very gradually; and indeed, few water, steam powers possess sufficient force to do it otherwise. The principle explained the following example, applies to the others. Thus, no locomotive, or marine en- ne, has sufficient power to start a heavy train, or a steamship, at once at a rapid eed. The first stroke of the piston imparts, or puts into the train, a quantity of rce sufficient not only to react or strain against, balance, or destroy (and be de- royed by) all the resisting forces of friction, the air, grade, curvature, &c, which esent themselves along the short portion of the road passed over during the first roke; but a small excess besides, which, being unresisted, is not destroyed; but ntinues in the train, as motion; giving a slow movement to the *unresisting matter* which it is composed. This last portion would remain in the train, and move it the same very slow vel forever, if only we could remove all resisting forces from fore it The next stroke in the same manner furnishes a new instalment of force; aich, like the first, divides itself into two parts; one of them as strain to destroy, id be destroyed by the new resistances met with in the next short dist along the ack; and the other as motion, to remain stored up in the train, united with the otion put there by the first stroke: thereby imparting increased vel to the unre- sting matter of the train; and this process is repeated during a great number of *rokes: the train moving* faster and faster. But after a while it is found that no *tter how powerful* the engine may be, or how light its train, the speed no longer *reases.*

The reason of this is that the resistances of the air, friction, &c, increase with the speed, but at a more rapid rate; so that after a certain vel has been attained, they require *all* the power the engine is capable of, in order as strain to react against, balance, or destroy them alone; without leaving any more surplus to be stored away, as motion, in the train. After this point is attained, the power of the engine no longer draws the train; but barely suffices to remove all the resistances which would otherwise impede it; and thus permits the moving force (which has *previously* been stored, and accumulated in the train, by small instalments) to continue to move the unresisting matter of the train at an undiminished vel. The train might now be said to "move itself;" and it would do so forever, without requiring any additional moving force, if the engine would only continue to destroy the resistances in its way. Should a stiff up-grade, sharp curvature, or other resistance present itself, so that the opposing forces actually *exceed* the *entire* power of the engine, for a short time, the train will actually come to its assistance, and by converting part of its own *motion* into *strain*, will, as it were, lend pressure-force to the engine, to help it push through its increased work; performing in fact the part of a fly-wheel in machinery. Frequently, when an engine appears to be pulling a heavy train up a short sharp grade, it is actually, so to speak, the train that is pushing itself up; the engine probably could not do it. This state of affairs can, however, continue for but a short time; otherwise all the motion of the train would be converted into strain; and being thereby destroyed, the whole would come to rest.

When a train is going at speed, and it becomes necessary to stop at a station some dist ahead, steam is shut off, so that the steam force of the engine shall no longer counterbalance, or destroy the resisting forces in front of it; and the number of the resistances themselves is increased by adding to them the friction of the brakes. Against all these combined, the train has now to "work" its way unaided; and it does so by the gradual conversion of its previously accumulated motion, or moving force, into straining force. Work, as before stated, consists in motion and strain combined. When the conversion has been completely effected, the train stops. Thus, we see, that up to the time of its stopping, the force which had gradually accumulated in it as *motion* before it had reached its greatest vel is gradually taken out of it as *work*, after steam is shut off. We may therefore speak of it as well under the head of *accumulated*, or *stored-up work*; as of accumulated *motion*, when we intend, in any kind of machinery, to convert such motion into work. Thus, the *motion* gradually accumulated in a fly-wheel, is also accumulated *work*, held in reserve until some extra strain on the machinery calls for its aid.

Art. 21. Up to the time that the vel ceased to increase because *all* the power of the engine became required as strain to react against resisting forces, and consequently could no longer spare any as motion to the train, the *work* of the *engine* was what is termed *variable*; being *gradually accelerated*. Also, when steam was shut off, the work of the train was variable; being gradually *retarded*. Such is the case in almost every kind of machinery, during the interval between starting, and ultimately attaining its maximum speed. When the latter point is reached, supposing the resistances afterward to remain the same, the work is termed *uniform*, or *steady*. All these remarks, as well as those of Art 20, apply alike to all kinds of heavy machinery; no matter by what kind of force or power it is driven; the machinery takes the place of the train just spoken of; and the friction of the cog-wheels, gudgeons, pivots, the grinding, sawing, or whatever the work may be, takes the place of the grades, curves, and friction of the train. All alike are simply cases of force in its various shapes of motion, strain, and work.

Art. 22. The entire quantity of any given work, considered by itself, without reference to the time reqd to perform it, is plainly to be measd by mult together the resistance in lbs, by the dist through which it is overcome in ft, as stated in Art 11.

After work becomes *uniform*, that is, when neither its strain nor its motion undergoes any change; its *rate* is of course measured by mult the resistance, or strain, in lbs, tons, &c, by the vel, or dist, in feet, &c, through which the resistance is overcome *in a given time*, as a sec, min, hour, &c.

Thus, if the resistance is 3300 lbs, and is overcome through a dist of 10 ft in every min; or if the resistance is 33 lbs, and is overcome through a dist of 1000 ft per min, the rate of the work is in each case the same, namely, 33000 ft-lbs per min, or one horse-power; for $\overset{\text{lbs}}{3300} \times \overset{\text{vel}}{10} = \overset{\text{lbs}}{33} \times \overset{\text{vel}}{1000} = 33000$ ft-lbs per min. The quantity of motion of a body (Art 9) is also estimated in ft-lbs; and under the head Levers, it will be seen that the tendencies (called *moments*) which the power and the weight respectively have to commence motion about the fulcrum as a center, are measured in the same term. Work, mere motion, and moments, are, however, effects of force so diff from each other, that confusion is no more likely to occur, than in applying the same measure, one foot, to materials as diff as cloth, bar iron, boards, &c.

In scientific phraseology, work done by machinery, &c, is divided into *useful*, and *prejudicial*; the latter being the quantity of force lost by friction, by the resistance of the air, &c. Thus, in pumping water, part of the applied force or power is lost in the friction of the diff parts of the pump; so that a steam or water power of 100 lbs, moving 6 ft per sec, cannot raise 100 lbs of water to a height of 6 ft per sec. Therefore machines, so far from *gaining power*, according to the popular idea, actually lose it, in one sense of the word. In the practical application of all machinery, the object is twofold; namely, to enable us conveniently to apply straining force, to balance, react against, or destroy, the resisting forces of friction, and the cohesive forces of the material which is to be operated on; and moving force, to give motion to the unresisting matter of the machine, and of the material operated on, after the resisting forces which had acted upon them have thus been rendered ineffective.

Art. 23. The total quantity of work that will be performed by the *moving force that is* in a body at any given moment, provided that after changing *from mere moving* force into working force, it is left to expend itself in uniformly *retarding work*, without receiving any additional force to aid it, (as in the case of the *moving force in the* locomotive in Art 20, after steam is shut off; and when said *force begins to work* against the resistances of the road,) is as, or in proportion to

not equal to,) the wt of the body, mult by the **square** of its vel at the moment it egins to work. For example, if a train at the time steam is shut off, has in it an ccumulated or stored-up moving force of 10 miles an hour; and if that force will y itself work the train against the resistances of the road for a dist of one-quarter f a mile, before coming to a stop; then, if steam is shut off while the train is noving at 2, 3, or 4 times that vel, and consequently with 2, 3, or 4 times the moving orce, it will work through 4, 9, or 16 times the dist of the first case, before coming o rest. If bullets of equal wt be fired with vels proportioned to each other as 1, 2, , they will respectively penetrate a plank to depths as 1, 4, 9. If an engine, water-heel, &c, works *steadily* in a mill, grinding at 2, 3, or 4 revolutions per min, it perorms only 2, 3, or 4 times as much *uniform* work per min, as when at but 1 rev per nin. But if steam or the water be suddenly shut off at 2, 3, or 4 revs per min, then he 2, 3, or 4 times quantity of *moving* force accumulated in the machinery at that noment, will, as *working* force, run the mill through 4, 9, or 16 times as many revs efore stopping, as if shut off at 1 rev. If a rolling ball, started against a row of ricks, will overcome their resistances, and knock them down for a distance of 4 ft; hen, if it be started at a vel 3, 4, or 5 times as great, it will overcome and knock hem down for dists of 9, 16, and 25 times 4 ft.

Hence it is seen that when the quantity of force in a body of *given wt*, is regarded as motion alone, r with respect to the vel alone which it imparts to the body, it is *directly as the vel*; so also when it s regarded as momentum, or the force of the single stroke, blow, or impulse, which it would give gainst a rigid body, which *instantaneously* arrests all its motion, by converting it into strain; and ually also when force is continuously supplied to the body for the purpose of performing *steady* or niform work. In all these cases the effect produced by the force in a body of given wt, is as its vel. ut when that same identical quantity of force is left to exhaust itself by performing *uniformly reerding* work, the quantity of effect which it will produce is as the *square* of the vel which it has at he commencement of the work.

But in all these cases the *rate* of the work done, that is, the quantity done *in any given time*, as ee see, is *directly* as the vels. Thus, the locomotive whose steam is shut off at 20, 30, or 40 miles a hour, will require but 2, 3, or 4 times as many seconds for running its 4, 9, or 16 dist before it omes to a stop; in other words, when its moving force is 2, 3, or 4 times as great, it will overcome ist 2, 3, or 4 times the resistances *in the same time*; although the *total* amount of resistances overome will be as 4, 9, and 16. And so with the other examples.

Art. 24. Vis viva, or living force. The preceding article serves as an ntroduction to this subject; of which we shall endeavor to give some idea in plain anguage. The term itself is merely one of those absurdities to which savants reort, in order to impart an air of mystery to their writings. We might with the ame propriety speak of a brickbat viva, or a living hod of mortar.

We have seen in Art 23, that if that portion of force in a body which is occupied in giving motion lone to the body, be suddenly converted into *working* force, the quantity of work which it would erform against *uniformly retarding* resistances, before being entirely destroyed, or coming to rest, ould be in proportion to the *square* of its vel at the time of beginning to work. Now, if "vis viva," r "living force," were merely the name given to this force; or to the quantity of work done by it, s measured in ft-lbs, by mult the resistances in lbs, by the dist in ft through which they were overome,) the expressions, although silly, would still convey an idea readily understood by practical en. We could then say, for instance, of a moving body, that its vis viva was 100 foot-lbs; meanig thereby that it would overcome a uniform resistance of 1 lb through a dist of 100 ft; or a resistnce of 100 lbs through a dist of 1 ft, &c. But scientific writers apply the terms to a purely *imagiary* quantity, equal to *twice* this: and which does not exist in any body, under any circumstances. he reason they do so is that it facilitates some of their calculations. But the practical engineer ed not concern himself with either this reason, or vis viva itself; the simple statement of facts ntained in the preceding and following articles, probably contains all that it is essential for him to ew on the subject of moving force, converted into uniformly retarded working force.

Art. 25. The actual total amount of work, in ft-lbs, that can e accomplished by a given moving force, when converted into unaided working orce, is found by div the square of the vel of the body in ft per sec, by 64.4; and hen mult the quot by the wt of the body in pounds; or, in shape of a formula,

$$\textit{Uniformly retarded working force} = \textit{wt of the working body, in lbs} \times \frac{\textit{sq of its vel in ft per sec}}{64.4.}$$

An *imaginary* force equal to double this, will be the vis viva, or living force of the avants:

$$\textit{Vis viva} = \textit{weight of body in lbs} \times \frac{\textit{sq of its vel in ft per sec}^{*}}{32.2.}$$

* For the purposes of abstract science, it is not sufficiently exact to measure the quantity of matter y its wt; because the wt or gravity of a body, as shown by a spring balance, varies somewhat in iff latitudes, and at diff heights above the level of the sea. Therefore, the vel with which it will all by that gravity, of course becomes a proper measure of the gravity; because it also varies in the ame proportion; all moving force being in proportion to the vel it imparts. Therefore, if the wt f a body at any place, be div by the vel imparted by gravity in one sec at the same place, (and called he acceleration of gravity of that place.) the quot will be the same at all places. And since the uantity of matter undergoes no change at diff places, the measure of that quantity should likew

[illegible] would not be uniform [illegible] the question, the solution would [illegible]

[illegible] power to overcome the frictio[illegible] motion. By the first stroke of t[illegible] number of strokes. For the m[illegible] the resisting force of fri[illegible] apply a second instalm[illegible] against it and thus destroy it;

[illegible] gravity acts a[illegible] and being continually destroyed [illegible] manner does it maintain a constant [illegible] It is plain that the rolling [illegible] along one mile of road, are not [illegible] the same as that of the next instant; [illegible] would destroy at least a portion of [illegible]

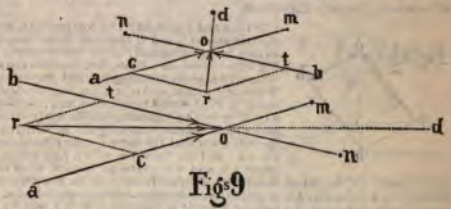
Art. 27. [illegible] rigid, any force imparted to it [illegible] other end, no matter how long it mi[illegible] would break it, no matter how thin it mig[illegible] and hence their inherent tensile or compre[illegible] force applied in excess. Ther[illegible] force through a body which woul[illegible] we must take *time*, and transmit i[illegible] of a long train of cars, would snap instantly [illegible] the train, *at one effort*, a moving force of [illegible] hours if an engine of such power existed. In practice we are therefore c[illegible] transmit the force from the engine to the train in instalments so small a[illegible] ceed the tensile force of the links. Rapid speeds are produced by the accumulated in the train by degrees. See Art 20.

The axes of wheels which transmit force from the motive power, to the working [illegible] chinery, are frequently broken if the power is applied too suddenly; that is, too much [illegible] If a pistol ball be thrown with very little vel. against a pane of glass, its force will ha[illegible] fuse itself over the whole pane, and will probably crack and shatter it in every direct[illegible]

A string may safely sustain a wt of 1 ℔ suspended from our hand; and if we wish
impart a great upward vel to the wt, we evidently can do so only by imparting to
a great force; and we may do this by jerking the string violently upward. But if
has not tensile force, or strength, sufficient to transmit this force all at once from
r hand to the wt, it will break. It plainly is not broken by the wt, but by the
cessive force which we endeavored to pass along it. We might safely give the wt
the vel or force we desire, by simply raising the string slowly at first, and more
d more rapidly by degrees; thus putting the force into the body gradually, in
stalments too small to break the string. Some imagine that the string is broken
the so-called inertia of the wt, which they say causes it to resist moving force;
d that we actually feel its resistance; and that it is made apparent by a spring
lance, if we hold the balance in our hand, with the string and wt attached to it.
ere is no doubt that when we jerk the string upward, the balance will indicate
at there is an increased strain upon its spring. But this strain arises not from
distance of the wt, but from the direct action of the force which, as *motion* only, we
ve imparted to the string, to be by it conveyed, as *work*, to the wt. *Work*, because,
en it reaches the wt, it has not only to impart *motion* to it; but also to *strain*
ainst its gravity force. The wt cannot receive great vel, unless we impart to it
eat force; this force is plainly imparted through the medium of the string; and if
attempt to impart too much *at once*, the string must break. The breaking of the
ing by the pull, is the same as the breaking or bending of a nail under too heavy
low of a hammer: in both cases the failnre is caused by the attempt to transmit
once a quantity of force which the inherent strength of the body is insufficient
sustain.

Springs of steel, &c, ease the violence of blows and jolts, because their elasticity *gives them time*, gradually yielding, to receive the whole action of the force, and react against, or destroy it, *by rees*, or part at a time. The imparting, and the receiving, of force are often attended by the ne effects. If we hold our hand still, and let a hard play-ball strike it, the hand will experience same sensation as if we first throw the ball upward; and afterward strike it with our open hand, the moment it is turning to fall, and is consequently still, or at rest. In the first case the ball im-ts its moving force, as strain, to the hand at rest; in the second, the hand imparts its moving ce as strain, to the ball at rest. In the first, we *received*; in the second, we *gave away* force; and both cases, the effect on the hand was the same.

Art. 28. Composition and resolution of forces. We have already d that when diff forces are *imparted* * (whether so *applied*, or not) in the same ection to a rigid body *free to move unresistedly*, they all act as motion alone, in st same direction. If two equal forces are imparted in diametrically opposite ections, they mutually destroy each other entirely, as strain (pull or push) against h other; thereby producing strain among the *particles* of the body; but having tendency to move the body, *as a whole*, in either direction. If unequal, and in metrically opposite directions, the whole of the small one, and a part of the large , equal to the small , destroy each other strain; while their diff ains as motion, (or noving of the whole ly,) in its original di-tion. But if two ces, *a o* and *b o*, Figs whether equal or un-al, are imparted at same time to an un-isting rigid body o, *directions either com-ging toward*; or *di-ging from, the same int* o, *at any angle whatever*; then the body *o* cannot possibly be kept at rest by m; or in other words, equilibrium cannot exist between them; or they cannot ance, or completely react against each other; the body must move. *Equal parts* each of the two forces will mutually destroy each other as strain among the *par-es* of the body; while the remaining portions will unite to constitute a single ce *r o*, which will move the *whole body* in a direction *o d*, in the line r o extended; l which direction *o d* will always be somewhere *between* those in which the separate ces *would have moved it.*



Figs 9

we lay off o a and t o by any convenient scale, to represent respectively the amount of the forces d b o, and then complete the parallelogram o c r t; the diag *r o*, measured by the same scale, wi ent both the direction and the amount, of the single remaining force.

* *absolutely necessary to keep distinctly in mind the diff between applied and imparted* carelessly confound the two very frequently. See Art 19.

The same process will answer also for forces which instead of motion, produce strain, not only in the particles of the body, but in the body itself considered as a whole; or, in other words, a tendency to press or pull the entire body in a certain direction. Thus, suppose that two men were either pulling or pushing with the forces $c\,o$ and $t\,o$; trying in vain to detach a piece o of rock, from a cliff of which it forms a portion; and which, by its inherent *force of cohesion* to the cliff, defies their efforts. Here we have a case of *extraneous forces*, resisted, or reacted against, or balanced, by *strength of material*.

As in the case of motion, the two forces partly destroy each other as strain among the particles of the body; and the remainders combine to form the single force $r\,o$, which tends to move the whole body toward d. The rock resists this single force, by a cohesive force precisely equal, and diametrically opposite to it; and so long as it does so, there is strain but no motion. The piece of rock may have strength enough to oppose a much greater resistance; but cannot actually *exert* it unless the men also exert more force.

In the matter of comp and res of forces, it must be remembered that when force is applied to a body in order to produce *motion*, care must be taken that there is no other force to prevent it; but when the force is intended to produce strain, it is equally necessary that other force should be present to oppose it; for strain is the *opposition* of forces.

The fig $o\,c\,r\,t$, Figs 9, is called the **parallelogram of forces.** The two original forces $c\,o$, $t\,o$, are called the *components* of the force $r\,o$; which *results* from their joint action; and the force $r\,o$ is called the *resultant* of the original ones which *compose* it. The principle of the parallelogram of forces, than which there is none more important in the whole range of mechanical science, may be expressed thus: If any two forces, (both motions, or both strains,) whose directions either converge toward, or diverge from, the same point, be represented both in quantity and in direction by two adjacent sides of a parallelogram: then will their resultant be similarly represented by the diag of the parallelogram.

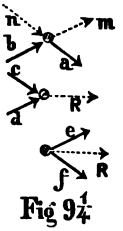


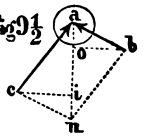
Fig 9¼

Rem. 1. If one of the forces, as a, Fig 9¼, is a pull; and the other, b, a push; then, in order to find the resultant, one of them must be imagined to be changed; so that both shall be pulls, as e and f; or both pushes, as c and d. This will not alter either the amount or direction of the resultant; for the result of the pull a, is precisely the same as that of the push n: and vice versa. See Rem 2, p 488. This fact should be well impressed on the mind, as it is very useful, and of frequent application.

Rem. 2. If the directions of the forces do not tend either to or from the same point, the principle stated in this Art does not apply to them; see Art 8; and their effect must be determined in a diff manner, which will be considered further on. If their directions are *parallel* to each other the resultant must be found as directed under Parallel Forces. Art 55, &c, p 481.

Rem. 3. We have alluded to equal parts of each component as being *lost*, or destroyed, by reacting against each other; thus producing within the body a straining of its particles; and therefore having no tendency to move, push, or pull, the body *as a whole*, in any direction.

Fig 9½



Let $b\,a$ and $c\,a$ be any two components, and $n\,a$ their resultant. From the two angles b and c, opposite to the diagonal, draw $b\,o$ and $c\,i$ at right angles to the diagonal; or to the diagonal extended, if necessary, as in Fig 9½. These two lines, $b\,o$, $i\,c$, will always be equal to one another; whatever may be the lengths and directions of the components $b\,a$, $c\,a$. When two forces, as $b\,a$, $c\,a$, are imparted at a, there occurs a loss of force equal to what would result from the reaction of two forces equal to $b\,o$ and $c\,i$. It is lost by becoming strain against the cohesive forces of the particles which compose the body a. In anticipation of what is said in Art 81, we will state that the force $b\,a$ may be regarded as made up of the forces $b\,o$, $o\,a$; and the force $c\,a$, of $c\,i$, $i\,a$; which act also in those directions, when $b\,a$ and $c\,a$ *converge* toward a, as in Fig 9½; or in the directions $a\,o$, $o\,b$, and $a\,i$, $i\,c$, when the forces *diverge* from a, as in Fig 9½. In either case, however, these forces, $b\,o$, $a\,o$, $c\,i$, $i\,a$, &c, must be considered as being imparted at a. This being supposed, it becomes plain that when $b\,a$ and $c\,a$ meet at a, inasmuch as $b\,o$ and $c\,i$ destroy each other as strain against the internal cohesive forces of the body, there remains nothing to act upon the body considered as a whole, except $o\,a$ and $i\,a$; which, being together equal to $n\,a$; (as seen in the fig.) are, in other words, equal to, or actually compose, the resultant $n\,a$ of the two components $b\,a$, $c\,a$. See Rem 5.

Rem. 4. It appears to the writer, that even some learned authorities have erroneous ideas upon this matter. We will illustrate by a familiar example, which is supposed to be conclusive in proof of their views. Let $a\,b\,n\,c$, Fig 9½, be a floating cake of ice. A man starts from a to walk across the ice to b on the shore; but during the time that he is walking from a to b, the ice floats down through the distance $a\,c$; so that he reaches n instead of b. Hence it is argued that no part of the force $a\,b$, *which he impressed* upon himself, has been lost, because it has carried him as far *across the ice as if the ice had been at rest.* And also that no part of the force $a\,c$, *impressed upon him by the ice*, has been lost, because it has carried him down stream *as far as it went itself.* Hence no force whatever has been lost; and, consequently, *the resultant $a\,n$ represents the joint action of all of $a\,b$ and $a\,c$. From this we dissent entirely. We contend that $a\,b$ and $a\,c$ lost their equal components $o\,b$ and $i\,c$*

ch reacted against, balanced, or destroyed *each other in the man;* and that con-
ıently he, considered as a whole, was moved from *a* to *n* by only their remaining
:ions *a o* and *a i*; which together are equal to or compose the resultant *a n*.
·in, under the head "Independence of the simultaneous action of many forces
n the same point," (a principle which appears to us to have no real foundation,)
ı of such cases, "it follows quite naturally that the forces which produce these
ions, exert actions independent of each other." In our opinion, so far from act-
independently of, they actually in great part *destroy* each other. See Art 15.
), STRAIN, in Art 2, p 444.

e conceive that each of the original forces endeavors as it were to compel the other to leave its
course, and follow that of its antagonist; and the struggle continues until they have succeeded
ıcing each other into *the same* direction. This is of course effected by their reactions against
other; and, as occurs in all cases of reaction, they expend equal parts of their forces on each
r. When the two forces act in diametrically opposite directions, where there is no neutral diag
ction that can be adopted, there is no alternative but for the larger force to react against or de-
y the smaller one entirely; thereby losing an equal amount of its own force. Its remains totter
lowly in their former unchanged direction. The writer can see no difference of principle between
reaction of opposite forces; that of oblique ones; and that of those at right angles to each other.

EM. 5. When the direction *a b*. Fig 9¾, of one of the forces, forms an angle *b a n*,
ater than 90°, with the diagonal, the shape of the parallelogram of forces becomes
h that the two equal lines *b o* and *c i*, cannot be drawn at right angles to the diag
itself; or within the parallelogram; in which case the diag must be extended
h way, as to *o* and *i*; and the lines *b o*, *c i*, must be drawn at right angles to the
ensions.

hen this occurs, the component forces *a o*, *a i*, cannot as in Fig
e measured on the diag *a n* of the parallelogram; because they
be greater than it; but must, like *b o*, *c i*, be measured outside
ne fig. And here it must be remembered that *a o* and *a i* no
er measure forces *acting* (like those in Fig 9½) *in the same di-*
ion. Thus the strain along *a b* may be considered (see Comp
Res of Forces, Art 31) to be made up of two forces imparted
; namely, a hor force equal to *o b*, and a vert one equal to *a o*,
ıg *upward*. And the strain along *a c*, as made up of one hor
ı equal to *i c*, and a vert one *a i*, (greater than the whole diag,)
ıg *downward*; both of them imparted at *a*. Hence, the re-
ant *a n* we find is equal to the diff between the two vert compo-
s *a o* and *a i*. Thus it is seen that this shape of the parallelo-
a in no way affects the principle laid down in Remark 3.* Fig
may illustrate a man walking from *a* to *b*, while the ice floats
ı *a* to *c*. He will land at *n*.

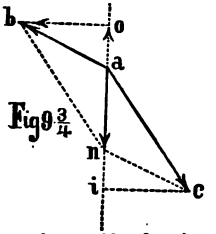


Fig 9¾

ART. 29. According to Art 18, the force *w e*, Fig 10, may be considered as im-
ted to the rigid body B at any point whatever in its line of direction *w c*; also,
force *x i*, at any point in its direction *x d*; conse-
ntly, both of them may be considered as imparted at
same point *a*; inasmuch as it is situated in both these
s. Hence, it is immaterial, so far as regards the effect
hose two converging forces upon the body considered
ne entire rigid mass, whether they are actually im-
ted like *x o* and *y o*, at the same point *o*; or like *w e* and
at diff points *i* and *e*. For in either case their result-
, or joint effect upon the body as a whole, is precisely
same; namely, a tendency to move the body in the
e line of direction *o a t*. This tendency will actually
duce motion if no opposing force prevents; otherwise
ill produce strain in the body.

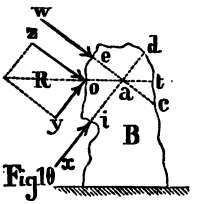


Fig 10

EM. 1. Hence the resultant R, of two converging forces *F f*, Fig 10¼; or of two
erging ones *F f*, Fig 10½, acting in the same plane, but imparted at diff points

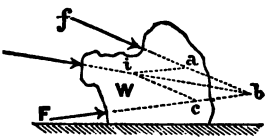


Fig 10¼

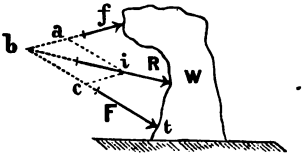


Fig 10½

ı *no book on mechanics*, or natural philosophy, have we ever seen an allusion to the componen
t, in cases like Fig 9¾. See Rem 5, Art 34.

of a rigid body W, may be found as readily as when imparted at the same point; as at *o*, Figs 9, or Fig 10.

Thus, produce their lines of direction, either forward as in Fig 10¼; or backward as in Fig 10½; as the case may require, until they meet, as at *b*. Make *b a* by any scale, equal to the force *f*; and *b c* equal to the force F. From *a* and *c*, draw lines respectively parallel to *b c* and *b a*; thus completing the parallelogram of forces, *b a i c*. The diag *b i* of this parallelogram, measured by the same scale, will represent the real resultant R both in quantity, and in direction. It is thus seen that it is not necessary that the point *b* shall be in the body itself.

Rem. 2. It is perhaps almost useless to again remind the young student that the bodies are all along assumed to be rigid; or inelastic, and incapable of being broken or bent by the imparted forces. For otherwise the force *f*, in Fig 10½, might split off the top of the body; or F might crush to dust its toe t; or both might penetrate it. But, assuming that the material is sufficiently strong to resist such splitting, crushing, and penetration, we at present confine ourselves to the effect of the forces, whether as motion, push, or pull, upon the body *as a whole*. The splitting, crushing, &c, is a matter that must be considered under the head of *Strength of Materials*. It is of course quite as necessary in practice to pay attention to these effects as to the others, but it must be done by a separate process.

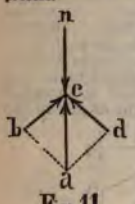


Fig 11

Art. 30. Since the effect produced upon a rigid body (considered as a whole) by the resultant (*a c*, Fig 11) of any two forces (*b c*, *d c*) tending to or from the same point, is the same as the joint effect of those two forces themselves, it follows that if we oppose to those two forces a third one (*n c*) equal to the resultant (*a c*), and diametrically opposite to it, that this third force will completely react against, balance, or destroy said two forces; or rather their remains. It is frequently necessary to consider such a third force, (*n c*,) equal and opposite to a resultant (*a c*); and inasmuch as we do not know that any specific name has been applied to it, although one is needed, we suggest to call it an *anti-resultant*. A resultant (*a c*) may be defined to be a single force which will produce upon a body considered as a whole, the same result that its components (*b c*, *d c*) produce. Or as a force which, if its direction were reversed, (thus making an anti-resultant,) would balance its components.

In the preceding Figs, the arrows represent *pressures*; if all the arrows be reversed, thus indicating *pulls*, the principle and processes remain precisely the same; for force is still only force; and its effect upon a rigid body, considered as a whole, is the same whether it act as a pull, or as a push. See Art 18.

When the forces diverge from the same point, their strain is a pull, or a tension; when they converge toward it, a push, or pres, or compression.

Art. 31. By a process the reverse of that in Art 28, any single force, *o d*, Fig 12, may be resolved into two component ones, *n d*, *m d*, one on each side of it, and in the same plane with it; which would produce the same effect as it upon a rigid body, *d*, (considered as a whole,) by merely drawing from *d*, 2 lines *d g*, *d t*, showing the *directions* of the two forces; and then, drawing from *o* two other lines *o n*, *o m*, respectively parallel to *d g*, *d t*; thus completing the parallelogram (*d n o m*) of forces, upon *o d* as its diag. Then measure *d n*, and *d m*, by the same scale as *o d*; and they will give the *amount* of each of those forces.

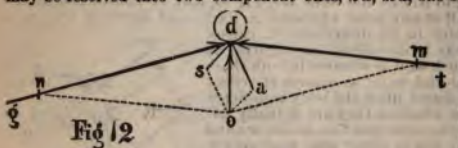


Fig 12

It is plain that an infinite number of differently proportioned parallelograms, such as *d n o m*, *d s o a*, &c, may be drawn upon any line *o d* as a diag; and in any one of them, two adjacent sides will represent components equal in effect to the single force *o d* represented by the diag. Thus the forces *n d*, *m d*, are equal to *o d*, as regards their effect upon a rigid body *d*, as a whole. So are also the forces *s d* and *a d*; consequently the effect of *s d*, and *a d*, is equal to that of *n d*, and *m d*. It will be observed that the longer any two components on the same diag are, (as *n d*, *m d*, longer than *s d*, *a d*,) the more nearly in a straight line, and more directly opposed to each other, do they become; and consequently the more nearly do they mutually destroy each other; leaving smaller portions of each to act upon the body. Thus the portion of the great forces *n d*, *m d*, left to act upon the body *d*, is no greater than that of the small forces *s d*, *a d*; this remainder being in both cases represented by the resultant *o d*.

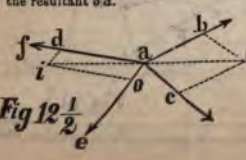
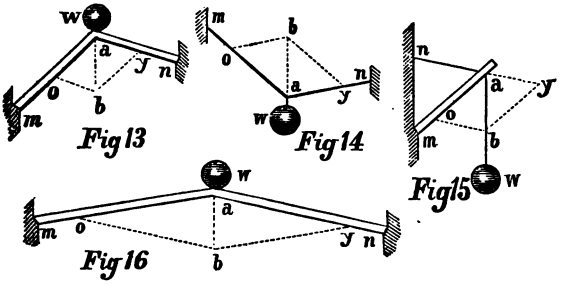


Fig 12 1/2

Rem. Hence, if we have two forces, as the two pulls *a b*, *a c*, Fig 12½, whose amounts and directions both are given; and which are counteracted, or held in equilibrium, by two other forces such as the two pulls *a f*, *a e*, whose *directions* alone are known, it becomes easy to find the amounts a d and a o of these last, thus: Complete the parallelogram h a c t; and draw its diag a t. Make *a i* equal to *a t*, and in a line with it. Complete the parallelogram a d i o; then plainly a d will be the amount of the force in the direction a f; and a o that in the direction a e.

Art. 32. It follows from the foregoing articles, that a single force cannot be olved into *two* components, one of which shall be in the same direction as that oe itself; for if a line representing that force be taken as a diag, it is self-evident at no parallelogram can be drawn upon it which shall have any of its sides parel to said diag.

herefore a rope, as *a b*, Fig 15, sustaining a wt *w*, so long as it remains perfectly vert, that is, presly in the direction of the force of gravity of the wt, will receive no assistance in upholding the by having added to it a single rope as *o b*, or *b y*; or one extending from the wt itself in any inted direction. In other words, a perfectly vert rope cannot sustain one part of a load, and ope inned rope another part. All this, indeed, is a result of the fact stated in Art 15; that any force, ever great, (as the vert force of an immense suspended weight *w*, Fig 15.) will be turned out of direction by any other force, however small, (as a slight pull from a rope *o b*, or *b y*,) unless there some third force to prevent it. In the present instance, this third force might be a third rope; the rope *a b* will be relieved, and still remain vert, if we employ *two* oblique ones to assist it, *provided they be exactly opposite each other;* or, in other words, that all three ropes, or forces, be in plane.



lo also in the case of a vert post sustaining a load; the pres from the load cannot pass vert through axis of the post, if the load at the same time is partly sustained by a single oblique brace pressing ainst the post. Indeed, such a brace, by turning away the direction of the strain from the axis the post, may very materially diminish the power of the latter to sustain the load; for it will be und under Strength of Materials, that if the strain along a post or column does not pass directly ough its axis, the column may in some cases lose two-thirds of its strength. The principle of urse applies to force in any other direction, as well as vert.

A resultant may be greater or less than either one of its two oblique components; t it can never be greater, or even quite equal, to both of them: on the plain prinple that any two sides of a triangle are greater than the third side. If the comnents are equal, and inclined to each other at an angle of 120°, the resultant will equal to one of them; therefore, the same weight that would break a single vert pe, or post, would break two ropes each of the same strength as the single one, or o posts, inclined 120° to each other. If the angle *o a b*, or *y a b*, which either of e forces form with the diag *a b*, exceeds 90°, see Rems 5, of Arts 28 and 34.

Art. 33. The principle of the parallelogram of forces is of constant applicaon in constructions of every kind; for instance, bridges, centers, roofs, retainingalls, &c. Figs 13, 14, 15, 16, show a few of the most simple cases of force (the load applied to produce strain; by reacting against opposing forces *y a*, *o a*, presented the walls. In all these, the load *w*, applied at *a*, is a single force of *gravity;* and nsequently acts in a *vert* direction downward. It is to be resolved into two comnent forces in the direction *a m*, *a n*, in order that we may find the strains which produces (according to the ordinary phraseology) along the pieces *a m*, *a n*, so that e may proportion their dimensions to resist those strains; which strains are in ct produced by the reactions of the *three* forces, of the load, and the two walls. To this, in all the figs, from *a* draw a vert line *a b*, to represent the *direction* of grav, of the force in the load *w*. On this line, lay off by any convenient scale, the dist *b* to represent the *amount* in lbs, tons, &c, of the load *w*. Also, from *a* draw the two nes *a m*, *a n*, in the directions of the reqd component forces. Then complete the rallelogram of forces, by drawing lines *b o*, *b y*, from *b*, respectively parallel to *m*, *a n*. Then will *a o*, measd by the same scale as *a b*, give the amount of strain, hether push or pull, which the load *w* produces along the piece *a m*; and in like anner will *a y give the amount* which it produces along the piece *a n*.

t must be especially borne in mind, that we here speak only of the amounts and directions of the *ins produced by the extraneous load w alone*; without reference to those produced by the weight *he pieces themselves. If the force acting at a is not vert*, but oblique, then the direction of *t of course be drawn oblique;* but if the force at *a* is gravity or wt, it must be vert.

A comparison of Figs 13 and 16, shows that the strains along the lengths, depend quite as much upon the degree of inclination of the pieces with regard to each other, as upon the actual loads which they support. Thus, in Fig 13, the strain *a y* along *a n* is less than the load *w*; while the strain *a o*, along *a m*, is somewhat greater; but in Fig 16, the strain along each piece is about 2½ as great as the load *w*; so that each piece would require to be at least 2½ times as strong as an upright post, or pillar that would sustain that load.

REM 1. Fig 16¼ will explain what we mean in saying that the strains *e l*, *e i*, are in reality produced by the walls; although they are usually ascribed to the load *l*, which is represented by the diag *e i*. We have said that a force cannot produce strain unless there is opposing force to strain against. Now, when we place the force of the load *l* at the point *e*, it is evident that it is upheld by the walls at A and B; or in other words, that it reacts against these walls; and the walls against it. The wall A furnishes the force indicated by the arrow A; and which may be considered as the resultant of the hor force *c*; and of the vert one *o*. So also the force B; as the resultant of *m* and *n*.

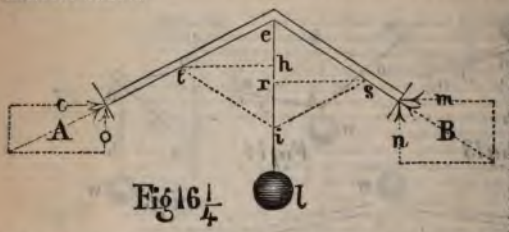


Fig 16¼

Now these forces A and B are applied at the point *e*, just as well as the load *l* is; for they pass up as pushes, along the rafters; as the force of *l* passes up as a pull, along the rope. The rafters and rope are merely the mediums through which the three forces reach *e*; and the forces in passing through them from end to end, of course produce in them strains respectively proportionate to the forces. Now, the forces *e l* and *e s*, which are usually said to be produced by the load, are nothing more or less than the two forces A and B, produced exclusively by the walls; and which, for convenience of drawing the parallelogram of forces in practice, are laid off each way from *e*. We have then three forces *l e*, *s e*, and *e i*, all acting at *e*, to produce *strain* alone; and this they must do by straining against each other.

The following is the manner in which they do so. The two hor components *m* and *c*, (which will always be equal to each other; no matter how different the slopes of the two rafters may be,) being diametrically opposite in direction, react or strain against, or balance, each other; thereby producing a hor strain, equal to one of them, throughout every part of each rafter. The two vert components *o* and *n*, (however unequal they may be,) will *together* be equal to the load *l*; or to its representative *e i*; and having a direction exactly opposed to it, they react against, or balance it; thereby producing in every part of the rafter *e s*, a vert strain equal to *n*; and in the rafter *e l*, one equal to *o*. Therefore, since *n*, is here greater than *o*, the rafter *e s* bears more of the load *l*, than the rafter *e l* does; and in the same proportion.

Thus, we see that every part of each of the three forces *e i*, *e l*, *e s*, produces strain, by balancing an equal part of one of the others. The walls really oppose to the load no force greater than its own; namely, *o* and *n*, against *e i*. With the hor components *m* and *c*, the walls react only against each other. Hence is seen the error of saying that the load *l* produces the forces *e s*, *e l*.

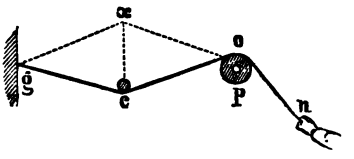
As it is difficult, however, to introduce a new phraseology, in place of one which, although erroneous, is in universal use, we also shall speak of component *strains* like *e l*, *e s*, as if they were really produced by the resultant, or load, *e i*. And in alluding to resultant *motion*, we shall probably often say they are the effects of *components*, instead of effects of their *remainders*, after the components have partially destroyed each other's moving forces by straining against each other to produce change of direction.

REM 2. The truth of such examples as Fig 14, with a rope or string, may easily be shown by means of two spring balances, to which the ends *m* and *n* of the string may be fastened. Suspend a weight *w* from the string, and the balances will show the strains along *a m* and *a n*. The balances must be held in inclined positions.

The student should try all such experiments. This one will show that in proportion as the two parts *a m*, *a n*, of the rope, approach nearer to one straight line, the greater will be the strain produced upon them by any given load, or force *w*; and so great will this be, that if the weight *w* be only one pound, two of the strongest men cannot strain the rope perfectly straight between them. Or if they stretch the rope alone to as nearly a straight line as possible, and if then a weight of a few lbs be suspended from it, this small weight will pull the men closer together. Or if the rope be stretched nearly straight between the two spikes so firmly driven as to require a great force to draw them, it will be found that a much smaller force applied as at *w*, will draw them readily. In other words, a rope so situated, and with force, or power *w*, applied to it in this manner, between its ends, and oblique to its direction, becomes a machine; for by it power may, (to use the

Fig 16½

expression,) *be gained.* It is called the **funicular machine**; or some-
ord. Fig 16½ shews the principle on which this machine is frequently employed
reat resistance, *r*, through a short distance, by a small power *p*. One end *c*, of a
y fixed. The rope passes over a pulley *d*; and its other end is tied to the resist-
y applying a small downward force *p*, at the center of the rope. drawing it down
ereby raised a short dist; for the same great strain which the small force *p* pro-
extends also down the rope, from *d* to *r*; except a slight loss produced by the
sy. Thus, the strain along the back-stays of a suspension bridge, is equal to that
just inside of the suspension piers; supposing the cables to rest upon rollers. In
sideration of ropes and chains, they are in most cases assumed not to stretch; to
s; without weight; and infinitely thin.
funicular machine, when one end is fixed or immovable, the other end being either
b some resistance intended to be overcome, such as a load to be raised; and when
wer is applied to it *at a point between its two ends; and in a direction across or*
of the rope.
e the two parts *s c*, *s d*, Fig 16½, are to be considered as two entirely distinct ties;
r as *a m* and *a n*, Figs 13 and 16, are two distinct struts. Each of these ties may
ifferent amount of strain, depending on their respective inclinations; which (*theo-*
r take place in the different parts of a rope to which all the power is applied *at one*
w many bends it may have between its two ends; or even whether it be wound any
ound a drum, pulley, post, &c. Power thus applied at one end, cannot be across or
l; because it necessarily pulls that end in the same direction that itself has. If a
one end of a straight rope, should move through a dist of 5 ft. it will evidently
of 10 lbs at the other end through a dist of 5 ft. It can (theoretically) do neither
nce, theoretically, the power at one end, and the resistance at the other, are equal;
ually strained throughout, to an extent equal to the power. No power is gained.
tated, this will be the case also if the rope be wound around a drum or post; or bent
of pulleys, &c. But in practice this by no means holds good; for an actual rope
e power may have to overcome this *weight* in addition to the resistance at the other
rope is by no means perfectly flexible; but requires a considerable loss of power to
um, post, &c. Again, it produces great *friction* around a post, &c; and this requires
to overcome it, so much so that the descent of a heavy body is often retarded,
ely taking a few turns of the rope around a post. These considerations render
alculations respecting ropes, almost useless in practice; but in many cases such
great value, and give results sufficiently accordant with facts.
the rope *g c o n* be fixed; a power of 9 tons at *n*; the rope passing over a pulley
t of line at *c* by a fixed pin. Make *c g* and *c o* by scale each equal to the power 9
the parallelogram; the diagonal *c x* of which is then found to be. say 6; or a result-
, in this case, theoretically the strain lengthwise of the rope is everywhere equal to
ns; and we have found that it produces also a strain *c x*, against the pin at *c*, of 6
duces a similar strain on
nount may be found in the
uring 9 tons by scale each
c and *n*; completing the
d measuring its diagonal
w let us use this rope as a
; and apply a power *x c* of
l that this 6 tons produces
of 9 tons along the rope;
g *c o* will pass along to *n*;
of 6 balances a resistance
n in the direction *n o*.



urfaces of contact of pieces of materials used in construction, are
When a piece is intended to resist compression, crush, or push, it is
or if inclined, it is often called a *brace;* or if vertical, a *post, pillar,*
en to resist tension or pull, a *tie.* When to resist both tension and
, a *tie-strut,* or a *strut-tie.* A strut should be stiff or inflexible; but a
hin rod, may answer for a tie.

listinguish a tie from a strut at a glance is sometimes
may be done thus. From the point *a*, Figs 16¾, at which the force
e *a c*, in the direction in which the force, if at liberty, would move
point. On any part,
as a diag, draw a paral-
es. Through the point
i, parallel to the *other*
all the pieces which
side of that line, that
s; while those on the
re ties. We may also
rmine, by imagining
a rope or chain, in-
; and seeing whether it would then bear the strain. If it would it is
trut.

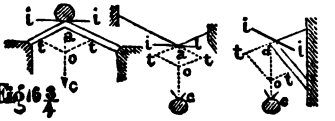


Fig 16¾

aterial is used to resist forces which tend to bend or break it crosswise, or trans-
h, as in Figs 47, 48, 49, 50, it is called a beam; such as joists, girders, &c. The
frequently acts at once, as a beam, and as a tie, &c. Its own weight strains
but in our present illustrations of comp and ten of forces, this strain, either
important one, could not be well considered at the same time.

Art. 34. Since any single force may be resolved into two oblique ones in the same plane with it, and which shall produce the same effect upon a rigid body considered as a whole, it follows that the single strain along any piece *a m* or *a n*, of the four preceding figs, may be thus resolved. In practice, it is frequently necessary to do this; and especially so for finding components at right angles to each other, in hor and vert directions.

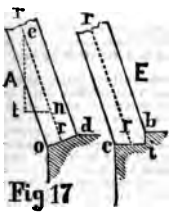


Fig 17

For instance, the joint *o d*, Fig 17, at the foot of the beam A, if made at right angles to the resultant *r r* of all the pressures along the beam, of course receives the whole of these pressures; which consequently are all imparted to the abutment; leaving no portion unresisted, so as to produce sliding; or even a tendency to slide along the joint *o d*. Consequently, this joint is perfectly adapted to its duty. See Art 19. But a joint of the form of *b i c*, which is equally effective, is sometimes reqd for receiving a single strain (like that along A) along a piece E; and in order to properly proportion the vert and hor faces *b i*, and *c i*, of the joint, we must find the proportion existing between the vert, and the hor components equal to the single strain *r r* along E. To do this is very easy; for we have only to lay off by scale, any length *e n* along *r r*, to represent the single strain in that direction; and on it as a diag, from *e* and *n* draw vert and hor lines *e t*, *n t*, meeting in *t*. Then *e t* measured by the same scale, will give the vert strain; while *n t* will give the hor one. The parts *b i*, *i c* of the joint, must consequently have the same proportion as these two components have to each other; bearing in mind, however, that since joints should be at right angles to the forces they have to sustain, the vert part *b i* must bear the hor strain; and the hor part *i c*, the vert one.

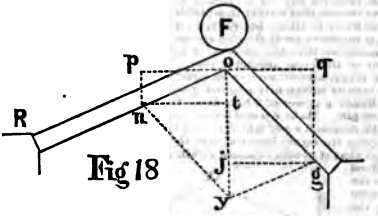


Fig 18

When, by Art 33, we are finding, by means of the parallelogram of forces *o n y g*, Fig 18, the total strains *o n*, *o g*, which an extraneous load F produces along two beams, PR, Fq, it is easy at the same time to find the vert and hor components also; by drawing the two hor lines *n t*, *g j*, and measuring them by the same scale used for the diag *o y*. Likewise measure *o t*, and *o j*, for the corresponding vert forces at the joints; because when *n t* and *g j* may be drawn inside of the parallelogram, (which is not always the case; as see Fig 18¼,) the component forces in the direction of any diag, whether vert or not, are measured respectively from the point *o*, where the extraneous force F is imparted to the beams; to those points *t* and *j*, where the diag is met by the equal lines *n t*, *g j*.

Rem. 1. It is an important fact that however diff may be either the inclinations, or the lengths of the two beams; or how diff the total strains in the directions of their respective lengths; the *hor* strains, caused both by the extraneous load and by the weights of the beams themselves, will always be equal on both of them. Thus, in Fig 18, *n t* is equal to *g j*; and in Figs 13 to 16, if hor lines be drawn from *o* and *y*, to *a b*, those in any one fig will be equal to each other.

Rem. 2. The beam *o* R, Fig 18, is not to be considered as acted upon at the same time by three distinct forces *o n*, *o t*, and *t n*; nor the beam *o g* by three forces *o g*, *o j*, *g j*; but each is acted upon by one force; thus *o* R is acted upon by *o n*; which produces upon it precisely the same effect as would be produced by its two components *o t*, *n t*. And so with the other beam. Each beam may be considered as receiving from the load F, either one force, or two components of that force.

The vert component *o j*, of the triangle *o g j*, being longer than *o t*, of the triangle *o t n*, shows that the beam *o g* bears more of the vert force or weight of the load F, than *o* R does; and in the same proportion as *o j* is to *o t*. The two components on the diagonal, (when inside of the parallelogram,) will always together equal the length of the diag, or the weight F. But as *n t* and *g j* are of the same lengths, they indicate that both beams are pressed *sideways*, or *hor*, to the same extent.

When we come to treat on trusses, we shall find this method of obtaining vert components, by means of *o t* and *o j*, very useful. The lengths of the beams *o* R, *o g*, do not affect the amount of strains produced upon them by the load F at their summits; but as their own wts must increase with their lengths, the strains arising from them must increase also; but we have not yet taken their own wts into consideration; neither are we yet prepared to do so. See Trusses, p 247 &c.

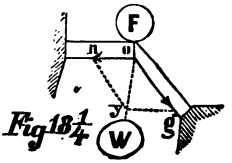


Fig 18¼

Rem. 3. If, as in Fig 18¼, one of the beams, as *n o*, is hor, it is plain that all of the diag *o y*, that is, all of the weight F or W, is borne by the other beam *o g*; and *n o* sustains hor strain only. The beam *o g* of course bears an equal hor strain also, as shown by *y g*, equal to *n o*.

Rem. 4. It is immaterial (Art 18) whether the load rests on top, as F; or is suspended below, as W; for in either case it is simply vert force imparted at *o*.

one of the forces, as
le *n o y*, greater than
o y, the positions of
become as in Fig 18½,
like Fig 9¾; that is,
ra, from the angles *n*
it angles to the diag,
nside of the parallelo-
we must extend the
a and *b*. If we wish
f the forces *on* and *og*
o components; then
e have *b n*, and *o b*;
og, we have *ag* and

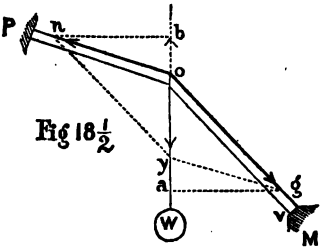


Fig 18½

hen the angle *n o a* exceeds 90°, the vert strain on *o g* is greater than the load
o the ordinary phraseology) produces it. But the part *a y* of the vert *o a*, has
id; but represents an upward vert force produced by the wall M, to balance a
ual to *b o* from the wall P. This excess *a y* over the diag, occurs only when
is an angle greater than 90° with the diag. We call the attention of the student
re do not remember to have met with it in any book. It will perhaps be a new
vert pres on the wall M, can be greater than the entire load.* See Rem 5, p 459.

a simple practical example of very common occurrence, of the ap-
egoing principle of finding the resultant of two forces in the same
to one point; let S, Fig 19, represent a block of stone weighing
ing on a *hor* base *m n*; but not attached to it in any way by ce-

ill be no force acting upon the body in such
ts being overturned around its toe *n* as a
its wt, or force of gravity, which always
By Arts 56, 57, all this force may be con-
ated at the cen of grav *i* of the stone; and
whatever in its vert line of direction *l g*.
s *f h*, of 2 tons, (which may be either one
sultant of many forces,) to be imparted to
plane with the force of gravity; which it
If its direction *f c* meets the direction of
oint; as, for instance, at *c*; because then a
incide with both directions. Art 8. Now
ly one (except gravity) that is acting upon
s in such a way as to tend to overturn the
is a turning point. The question is, which
ll prevail; the two tons of *f h*, to overturn
of gravity to prevent its being overturned?
may be considered to be imparted to the
int *a*, where their lines of direction meet;
equal to 2 inches, ft, &c, to represent the
ction of the 2 tons of pres of the force *f h*;
cale, for the direction, and 3 tons of pres of

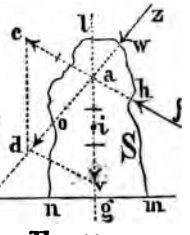


Fig 19

e. From c draw a line c *d* parallel to *a v*; and from *v* draw *v d* parallel to *a c*;
thus completing the parallelogram of forces *a c d v*. The diag *a d* of this par-
the same scale, will give about 2¾ tons for the single resultant force, which
e upon the rigid stone the same effect as gravity and *f h* combined. This force,
sidered as imparted to the stone at any point in the line of its direction (*w o*)
a *push* at *w*, as shown by the arrow *z w*; or as a *pull* at *o*, by means of a rope
ne at *o*. Since this resultant is supposed *to take the place* both of gravity and
ist of course be considered as annihilated; so that the stone becomes as it were
f matter without weight; and acted upon by one force *a d alone*. But a single
ter unresistedly, always produces motion in it; and since at *o* there is no force
b to *o*, to resist the resultant force *a d*, the body must be moved by *a d*; and
verthrown by overturning around *n* as a pivot.

e direction of the resultant *a d* struck the base of the stone *at n*,
outside of the base as at *b*, the stone would barely have stood;
resultant, on leaving the body at *n*, would have encountered the
he ground on which the stone stood, acting upon the body at that
irection struck *within n*, that is, between *n* and *m*, the stone would
mly; (see Rem 2, p 492;) and the more firmly in proportion as
where the direction *l g* of the gravity of the stone meets the base.
resultant may strike within the base; and the body remain firm,
verturning; but yet may *slide*. See Art 63; *very important.*

also the necessity for assuming at times that bodies are rigid, or unbreakable.
es of but little strength, the application of the great force of the resultant so
the body at that point; and might, besides, mash *n* into the yielding earth or

the writer is mistaken in this, he wishes to be corrected.

which it stood. A knowledge of the direction of the resultants of forces acting on bridge abutments, retaining-walls, &c, is therefore of use also by enabling us to guard against such accidents, by selecting the strongest stones for the most strained parts of the structure; as well as by adopting extra precautions in preparing those portions of the earth foundations, upon which those parts rest.

It must be remembered, however, in such cases as the foregoing, that with the exception of the point *o*, at which the resultant *leaves* the body, *a d* is not the direction which the resultant *actually follows in* the body; but is one which we may assume it to have, so long only as we assume the body to be *practically* rigid: that is, that it cannot be in any way broken, bent, or have its form changed, by the forces *actually imparted*. Frequently we cannot safely assume a mass of masonry to be thus rigid; for it may be composed of many separate pieces merely placed in contact with each other without mortar, as in dry masonry; or even if mortar or cement be used to unite these pieces, it may not have time to set or harden properly, before the deranging forces are brought to bear upon it. In that case, although the resultant might fall entirely within the body, and within the base, thus denoting perfect security to a *rigid* body; yet the structure might be completely destroyed by the sliding or other derangement of *its parts among one another*, under a force much less than would be required to overturn it. On this account, if we wish to obtain security at the least expense, we must frequently trace the actual curved direction of the resultant through its entire course; so that we may at every point of it place the joints of our masonry at right angles to it, as in Fig 1; or adopt other precautions to prevent the parts of the structure from separating. See Art 72, p 431.

Rem. 2. If in Fig 19 we suppose strong mortar or cement to exist between the base of the stone and a rigid foundation of masonry or rock, upon which we may assume it to stand, then it may not be overthrown, although the direction of the resultant *a d* falls outside of the base *m n*. For then a third force, namely, the cohesive strength of the mortar, is brought to act upon the stone; and the resultant of *all three forces* may fall within the base. In all cases where a body remains at rest, notwithstanding that the resultant of the forces falls outside of its base, (whether the base be hor, vert, or inclined,) we may be certain that it is because some other force, which we have neglected, is acting upon it at the same time; for when the direction of *all* the forces passes beyond its base, and is consequently *force unresisted*, the body *must move*. See Rem, Art 65; also see Art 72. When one body is thus cemented to another, the two become in fact *one* body, so long as the cement does not give way under the imparted forces; so that a problem which is one in *Statics*, if there is no cement, may become one in *Strength of Materials*, when there is cement. The cement takes the place of natural cohesive force between the bodies which it unites.

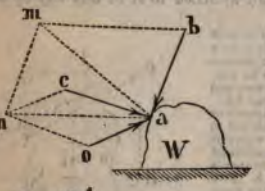


Fig 19¼

Art. 36. When the number of forces in the same plane, and tending to or from the same point, is greater than two, their resultant may be found in the manner already given for two. Thus, with the three forces *b a*, *c a*, *o a*, Fig 19¼, first find the resultant of any two of them; as, for instance, the resultant *n a*, of *o a* and *c a*. Then consider *o a* and *c a* as removed, and *n a* as taking their place; and then find the resultant *m a*, of *n a* and *a b*; then is *m a* the single resultant that will produce upon the rigid body W, the same effect as the three forces *o a*, *c a*, *b a*. If the three forces are imparted at diff parts of the body, proceed as in Figs 10¼ and 10½. If the number of forces be greater than 3, the process is precisely the same; find the 1st resultant of two of them; then the resultant of the 1st resultant and 3d force; then the resultant of the 2d resultant and 4th force; and so on to the end.

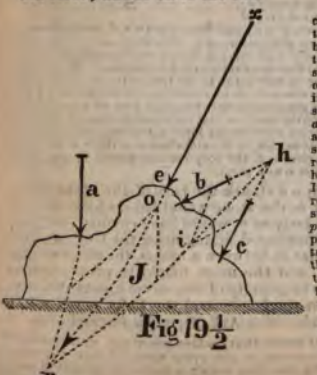


Fig 19½

Rem. Mistakes occur from supposing that this principle applies also when forces in one plane do *not* tend to or from the same point. Thus, let *a*, *b*, *c*, Fig 19½, be three such forces. Now if, as in Fig 10½, we prolong the directions of *b* and *c* backward, and find their resultant *h i*; it seems plausible to infer that we then have only two forces, *h i*, and *a*, in the same plane, and tending to the same point *n*; and which will produce the same effect upon the rigid body *J*, as the three forces, *a*, *b*, and *c*. Hence it is supposed that if we prolong *h i* and *a* forward to *n*, as in Fig 10¼, and find their resultant *o n*, Fig 19½, we shall thus have the single resultant *o n*, or *x e*, of all three of the forces; as we have *a m* in the foregoing Fig 19. But this is not so. It is true that this *o n* gives us the *length* of the actual resultant, and also its *direction;* that is, the actual resultant will be *equal* to *o n*, and *parallel* to it; but its *position*, or point of application, will *not* be at *e*, in the prolongation of *o n*, as in the fig; and as it would actually be, if the three original forces tended to or from the same point; but must be found afterward, by a totally different and distinct process. If we had first found the resultant of *a* and *b*, and afterward the resultant of that resultant and *c*, we should have found said final resultant to be equal to *o n*, and parallel to it, but removed a short distance to one side of it; which when the forces (however many) tend to or from a point, the resultant will always fall in the same line and position; no matter which what two of the forces number we may begin, or in what order we suppose them. See Art 40.

Art. 37. It sometimes happens, after having found the resultant of all the forces except the last one, that said resultant and remaining force are in the same straight line. Thus, with the forces *u*, *v*, *w*, Fig 20; the resultant *r* of *u* and *w*, is in the same straight line with the last remaining force *v*; and of course no parallelogram of forces can be drawn which shall have *v* and *r* for two of its sides. When this happens, if *v* and *r* are of diff lengths, we have the case given in Art 16, of two unequal forces meeting in the same straight line; but in opposite directions and keeping it. Consequently, the small one, and an equal part of the large one, mutually destroy each other; and the remainder of the large one is the resultant of the two.

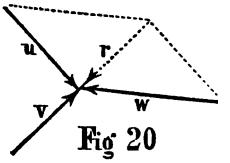


Fig 20

But if *v* and *r* are of the same length, then we have the case of two equal opposing forces, which mutually destroy each other entirely; and the body remains at rest. Consequently, there is no result in this case; for no single force can have the effect of keeping a body at rest; but will always move it.

Art. 38. The polygon of forces. The resultant of any number of forces in the same plane; and tending either to or from the same point, may also be readily found thus: Let *a*, *b*, and *c*, be three such forces; whose resultant R is to be found. Begin with any one of them, as *a*: and draw *a'*, parallel, and equal to it; and place an arrow-head at the proper end of it, to show its direction. From this arrow-head, draw *b'*, equal and parallel to *b*; placing an arrow-head at its end. From this second arrow-head, draw *c'*, equal and parallel to c; and so on with any number of forces; taking them in order. Finally, from the arrow-head *d* of the last of the forces, draw a line A to the butt-end, *n*, of the first one; thus closing the figure; and place an arrow head as on the others. Now, this closing line A, or *d n*, with its arrow, represent both in quantity and direction the antiresultant of the three given forces; or if its arrow be reversed, it will represent their resultant. Consequently, we have only to draw from *o* a line, *o* R, parallel to A; and to make R equal to A; but pointing in the opposite direction. Then is R the resultant.*

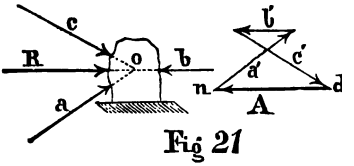


Fig 21

This process will give diff figures, according to which force we begin with: or whether we take the forces in right, or left-hand order; still, A will always come out the same in all of them. If the three given forces (or any greater number, as the case may be) had been in equilibrium with each other, that is, had mutually *destroyed* each other's tendency to cause motion, they of course could have no resultant, or single force that would produce an equal effect; because a single force, if the only one acting on a body, must produce motion. When this is the case, we shall find that the fig will have no lines crossing each other as *a* and *c* do; but will be a polygon; and will close by drawing the force alone, in order; thus, if *a*, *b*, and c had been in equilibrium with each other, then the arrow-head *d* of the third force *c*, would have come to *n*, the butt-end of the first force *a*; thus forming a complete triangle.

The foregoing depends upon the following, known as the **principle** of the **polygon of forces.** In a polygon, as *a b c d*, Fig 22, of any number, and lengths of sides whatever, if these sides be taken in their order, to represent so many consecutive forces, as denoted by the arrows; then will those forces be in equilibrium, or keep *each other* at rest, if they all be imparted in the same plane, to a rigid body B; and in directions respectively parallel to the sides of the polygon; and all tending to or from the same point, *s*. It follows, vice versa, that, if any number of such forces so situated, hold each other in equilibrium, they will, if drawn in a *consecutive order, (as in* N,) form a closed polygon.

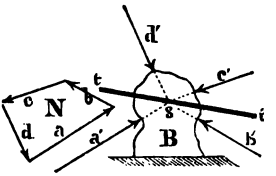


Fig 22

It is plain, that in any number of such forces, any one of them is the antiresultant of all the others; because it keeps them all in equilibrium: It is also equal, of course, to their resultant; but acts in the opposite direction. Also, if any straight line as t t' be drawn through the point s, the forces on one side of it, will balance those on the other side; thus, d' and c', will balance a' and b'.

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* *Some of even the best writers commit the oversight of saying that A, with its arrow-head, represents the resultant. It really represents the anti-resultant; and its direction must be reversed, for the resultant.*

REM. 1. It must not be inferred because forces balance *each other*, that therefore they balance the *body* to which they are imparted; for the body might move under the influence of other forces. Thus, the forces a', b', c', d', may be supposed to be the balancing forces of several persons holding a body at rest in a railroad car moving with great speed. Their forces prevent *each other* from giving motion to the body; but do not prevent the steam force of the engine from doing so. It is only when all the forces imparted to a body, including its own weight, are in equilibrium, that the *body itself* is also at rest. Or, if we hold a book, ruler, &c, vert between our thumb and forefinger; the opposite and equal pressures of the thumb and finger, hold each other in equilibrium, so that they cannot move the book either to the right or left, and the friction between them and the book, holds the gravity or weight of the book in equilibrium, so that it does not fall. So that so far as these forces are concerned, they produce no motion in the book; but we can move it vertically up and down, by introducing the third force of our wrist; or hor by stretching out our arm; or by walking; none of which will interfere with the equilibrium of the other forces; they only prevent *each other* from producing any motion.

REM. 2. A triangle being a polygon of 3 sides, if any 3 forces which form a triangle, be applied in one plane, to a body, and in directions parallel to the sides of the triangle; and tending either to or from one point; they will hold each other in equilibrium. And, vice versa, when we see a body kept at rest solely by the action of 3 forces which are not parallel to each other, we may be sure that those forces are proportional to the sides of a triangle drawn parallel to them; that they are in one plane; that they all tend either to or from one point; and that any one of them acts in the direction of an antiresultant to the other two. Moreover, each of the forces is proportionate to the sine of the angle included between the other two; so that if we know one of the forces, we can readily find the others if we have the angles. This is very often of use in practice; as in finding by calculation alone, the line of pressures through an arch; the pres of earth against retaining-walls, &c. It must be remembered that the wt of a body usually constitutes one of the forces to be considered as acting upon it. This REM is very important to the engineer.

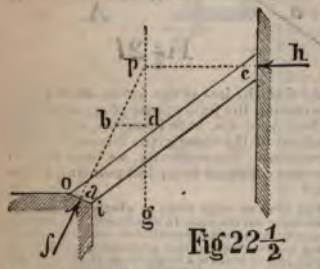


Fig $22\frac{1}{2}$

EX. 1. Let $a\,c$, Fig $22\frac{1}{2}$, be a beam; its foot resting on $o\,f$; and its head c merely *leaning against* a smooth *vert* wall; and whether $a\,c$ be unloaded; or whether it supports a load placed in any manner upon it, or suspended from it; let the vert line which passes through the cen of grav of the beam and its load (both of which are supposed to be known) be represented by $p\,g$. The beam and its load may be regarded as a single body, acted upon, and kept at rest, by three forces; namely, its own gravity or wt; the force h, at c; and the force f, at a. No other forces act on it. Now, gravity acts vert only; and in the case before us it may all be regarded as acting in the line $p\,g$. The force at c can act only at right angles to the surf or joint at that place, (see Art 19;) and since the joint is vert, the force h must be hor, or along $h\,p$. The question now is, how to find the direction of the third force f. To do this we must avail ourselves of the principle that when three forces, not parallel to each other, hold a body at rest, or in equilibrium, as these three forces hold the beam $a\,c$, their directions all tend to or from one point; which is either at the cen of grav of the body; or in a vert line passing through said cen. Hence, since the vert direction $p\,g$ of the force of gravity of the body; and the direction $h\,p$ of the force h, meet at p, therefore, the direction $f\,p$, of the force f, must also meet there. Hence we have only to draw a line $f\,p$, in order to find the reqd direction. A post intended to support the end a of the beam, should have the position $f\,a$; and the joint $o\,f$ should be at right angles to $f\,a$; and not to $a\,c$, as might at first be supposed from Figs 13 and 16, of Art 32; in which the wt of the beams is not considered.

Having found the directions of the three forces in Fig $22\frac{1}{2}$, it only remains to find their amounts. To do this, we already have one of them given, namely, gravity, or the wt of the beam and its load; and we know that they must be in proportion to the sides of the triangle drawn parallel to their directions. Consequently, if on the vert direction $p\,g$, we lay off by scale any portion whatever, as $p\,d$, to represent the force of gravity, then will the hor side of the triangle, $p\,d\,b$, represent by the same scale the hor pres at c; and the side $b\,p$, the oblique pres at a. The hor pres at the foot is equal to that at the head of the beam. It is of course included in the oblique pres f; which is compounded of said hor force, and of the vert force at a. The vert force is equal to the weight of the beam and its load; none of which is sustained at c; nor can be, so long as the joint at the head and wall is vert.

EX. 2. This is very similar to the preceding. Let $a\,b\,i\,j$, Fig $22\frac{3}{4}$, be the half of any arch bridge, loaded or unloaded equally throughout; and of which we know in either case the total wt; and that the cen of grav of said wt is somewhere in the vert line $g\,g$. Now this half bridge is, like the preceding beam, kept in equilibrium, or at rest, by three forces only; namely, the wt; a hor pres h, at the crown, arising from the other half of the arch; and an oblique pres o, at the springing, or skew-back $j\,i$. *To find the* directions and amounts of these forces, from a draw a hor line, *meeting the vert* $g\,g$ at c. From c draw a line to the center of $j\,i$: this is the direction *of the oblique* force o. From c measure down by scale any dist $c\,s$, on the vert *direction* $g\,g$, to represent the weight; and from s, draw $s\,t$ hor. Then $s\,t$, measured by *the same scale*, will be the hor pres h; and $c\,t$, the oblique one o. The joint $j\,i$, at *the spring of* the arch, bears all of $c\,s$; that is, all the wt of the half arch, and half

nd. No vert pres or wt is sustained at the center *ln* of the arch; nothing but the or pres.

The oblique force *ct* constitutes the total thrust exerted by the *entire* arch, gainst *each* of its two abuts; and the line *ct* shows the direction in which this hrust *enters* the abut at the skewback *fi*. After entering at that point, it begins to urve downward, on the principle explained in Art 72. Since *ct* is the hypothenuse f a right-angled triangle, of which the leg *cs* represents the half wt; and *st* the or pres of the arch, it follows that the total thrust *ct* of an arch may be found thus: *dd together the square of half its wt; and the square of the hor pres; and take the qrt of the sum.* This applies also to the arches in the Burr bridge, the Bowstring, c. The arch may *in such cases* be supposed to be uniformly loaded by the other imbers of the bridge, and by the greatest moving load of cars, &c, reaching from nd to end of it; and therefore we may *in such cases* consider the line *g g*, or *c s*, to e halfway between the center of the arch and the abutment.

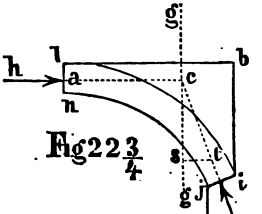


Fig 22 3/4

The joints of any arch which is a portion of a circle, are usually drawn toward the center of the circle; and, practically, this answers every purpose; but it is plain that strict theory would require the joint *if* to be at right angles to *ce*. e also the other joints of the archstones would be reqd to e perp to the pres which they have to sustain.

Moreover, since the pres *ct*, upon the joint *fi*, is much greater than the pres *st*, upon the joint *ln*, theory would equire the joint *fi* to be proportionally deeper than *ln*; herefore in practice they are usually made the same, except n very large arches. See Stone Bridges. P 349, footnote.

The last few lines of Ex 1, respecting the hor pres at the ot, and at the head, apply equally here. The young student should familiarize himself thoroughly with the principle illustrated by these two examples, as it is one of very frequent application in practice; as in retaining-walls, abutments, &c.

Here, as in the preceding example of the beam, we do not consider the strains produced along the length of the arch *nf i itself*; but merely the two forces which, acting at its center *ln*, and at its foot *fi*, keep each ther, and the wt of the half arch and its load, in equilibrium. For the others, see Stone Bridges.

Art. 39. A third mode of finding the resultant R, Figs 23, of any number of orces E, F, G, in one plane; and tending to or from one point *x*; is as follows:

Draw two lines, H H, and V V, at right angles to each other. From their point of intersection *o*, raw lines by any convenient scale, to represent the directions and amounts of the forces. By Art 4, resolve each of these forces into two component ones parallel to H H and V V. Thus F *o* is resolved into *u o* and *e o*; G *o*, into *m o* and *i o*; E *o*, into *t o* and *n o*. Then measure by the scale, and dd together, those components *i o* and *t o*, parallel to H H, and which tend to move the point *o*

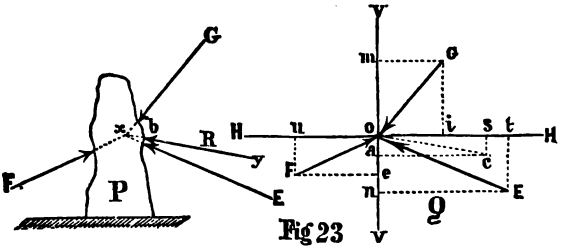


Fig 23

ward the *left* hand. Also add together those (in this case only *one*) *m o*, which tend to move *o* ward the *right* hand. Subtract the least sum from the greatest; their diff, equal to *s o*, will be the sultant of the two sets of forces which respectively tend to move *o* to the right, and to the left. In is instance, this *s o* must evidently be placed to the right of *o*, because the components on that side ive the greatest sum.

Next, add together these components *e o*, *n o*, parallel to V V, which tend to move the point *o* upward. In like manner add together those (in this case only *one*) components *m o*, parallel to V V, hich tend to move *o* *downward*. Subtract, as before, the least sum from the greatest; their diff, qual to *a o*, will be the resultant of the two sets of forces which respectively tend to move *o* upward nd downward. In this instance, *a o* must be mead off *below* *o*, because the upward tendency is the reatest. By this process, then, we first reduce *all* the original forces to two components, *s o* and *a o*. his being done, we have only to complete the parallelogram of forces *o s c a*, and draw its diag *c o*; hich will be the final single resultant of all the original forces. From *x* draw *x y* parallel to *c a*, nd make *b y* equal to *c o*; then is *b y*, or R, the reqd resultant; and *b* the point for imparting it to *he body P, so that its effect may be equal to that of the three original forces combined.*

The three foregoing processes for finding the resultants of any given number of forces in one plane, *d tending to or from one point*, are all exceedingly simple, and readily understood.

Art. 40. To find the resultant, (*e i*,) Fig 24, **of any number**

The forces in the fig may represent those of 4 diff persons pushing hor against each other, at the hor turnstile N; which in the fig is supposed to be seen from above; the point *i* being the pivot at its center. The fig is drawn to a scale; the forces, arms, and moments, being as follows:

Forces.	Arms.	Right-hand Moments.	Left-hand Moments.
$a = 3.4$	$i s' = 2.$	6.8	
$b = 2.$	$i t' = 3.5$	7.0	
$c = 3.$	$i u' = 3.2$	9.6	
$d = 6.5$	$i v' = 3.6$		23.4
		23.4 Total.	23.4 Total.

Both sets of moments being equal, the forces are in equilibrium; and of course have no resultant.

Art. 43. Forces in different planes; but tending to or from the same point. Such forces cannot, like those in one plane, be correctly represented together on one flat surf, such as a sheet of paper. Thus, let Fig 27 be a cube; and $t x$, $c x$, $i x$, three forces acting in the directions of its edges; and all tending to the same point x. It is plain that the relative positions of these forces are not correctly represented; for $t x c$, $t x i$, and $c x i$, are in reality right angles; whereas, in the fig, $t x c$ appears to be an acute one; $c x i$, a right angle; and $t x i$ an obtuse one.

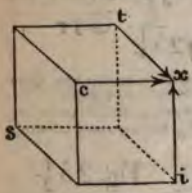


Fig 27

On this account the resultant of such forces cannot be had by measurement from a drawing. Recourse must therefore be had to calculation; which, however, will be facilitated by a drawing. The theoretical principle is very simple; being, in fact, the same as when the forces are all in one plane; namely, first find by Art 28 the resultant of any two of them, (for any two are really in one plane;) then find the resultant of this resultant and the third force; and so on to the end. It is easy to find the first resultant; but the others are more troublesome. Instances are comparatively rare, in which the resultants of such forces are reqd to be found; the attention of the engineer being generally confined to those in one plane; as when proportioning bridges, roofs, retaining-walls, &c.

Art. 44. To find the resultant of any number of such forces, knowing the angles which they form with one another.

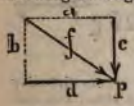


Fig 28

It is somewhat difficult to prepare a fig to illustrate this operation, on account of the number and interfering positions of the various lines required; but this may be avoided to a great extent, if we merely bear in mind that when a single force *f* is resolved into two components *c* and *d*, by drawing the parallelogram of forces *a b c d*, although the two sides *d* and *c* which converge to the same point *p*, with the force *f*, are those which really represent the components, yet, as the dotted sides *a* and *b* are of the same length, and in the same directions as the others, *they* may be considered to be the components, if we merely *imagine* them to converge at *p*; and by this means we may omit drawing *c* and *d* for completing the entire parallelogram, when their introduction renders the fig complicated. Thus, in the following Fig 29, the original forces being *b a*, *c a*, *d a*, the three sides *i a*, *e a*, *o a*, represent components; but *b i*, *c o*, *d e*, must all of them be *imagined* to be drawn *from* *a*, in order to be components. This being understood, let *b a*, *c a*, *d a*, be three forces in diff planes, and converging either at or toward the same point *a*; and let their angles *b a c*, *c a d*, be given.

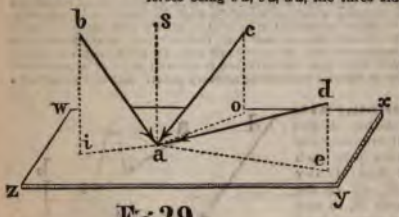


Fig 29

We must in the first place imagine any plane *w x y z*, Fig 29, to pass through the point *a*, in any direction; and must find the angles *b a i*, *c a o*, *d a e*, which the several forces form with said plane.*

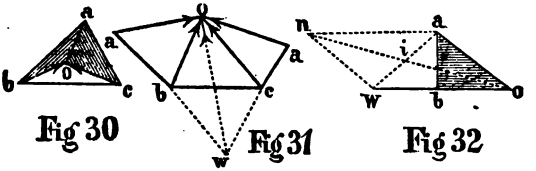
Also, resolve each of the forces into two, Art 31; one of which shall coincide with the plane, and the other be at right angles to it. This may readily

* By taking a perfectly smooth surface of a piece of board, or the top of a table, to represent the plane; and by sticking firmly into it straight pieces of wire, cut by a scale to represent the forces, taking care that the wires form the given angles with each other, we can perform the *entire operation* on this plane; using it as a drawing-board. We can thus find the components *in the plane without actually measuring* the angles *b a i*, &c, in degrees. We can also find their resultant, by *drawing the necessary lines* on the board itself. Then take away the wires representing the forces, *and substitute for them* a single wire at right angles to the board, and equal in length to *b i*, *c o*, *e d*. *Then we have* a model of the two resultants, from which we readily find their resultant; which may *also be represented* in the model by a piece of wire. By a little ingenuity, the engineer may in a *somewhat similar* way solve problems of force which would otherwise give him much trouble. See *Arts 45 and 46.*

ne by drawing each force separately on paper by scale; for instance. $b\,a$; at a lay off the angle previously found; and from b draw $b\,i$ at right angles to $a\,i$. By this the force $b\,a$ is resolved a, which lies in the plane; and into $a\,a$, at right angles to $i\,a$, or to the plane; but which is sented by $b\,i$. And so with the other two forces. This being done, draw $i\,a$, $e\,a$, $o\,a$, by tending to one point; and forming with each other angles equal to the angles $b\,a\,c$, $c\,a\,d$, d by the original forces. Now we have two distinct sets of forces; namely, First, $i\,a$, $e\,a$, $o\,a$, the same plane, and tending to the same point. Consequently, their resultant is easily found t 36. Second, the other set consists of three forces equal to $b\,i$, $c\,o$, $e\,d$; supposed to act at the a. Since these are all in the same line of direction, and all act in the same direction upon the point a, it is plain that their resultant is equal to their sum; in other words, it is found by y adding them together. Finally, then, we have thus found two separate resultants at right s to each other; and their resultant may be obtained by Art 28, p 437.

rt. 45. In cases where mathematical accuracy is not necessary, and the num- of forces only three, or four, the writer will venture to propose a method by *ts*; which, if open to the objection of empiricism, has the advantage of requiring time than other processes; is sufficiently correct for most practical purposes; and rs the resultant *in its actual position*, which is done by no method of calculation.

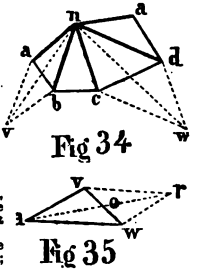
; $a\,o$, $b\,o$, $c\,o$, Fig 30, be the three forces, meeting at o; their angles with each other, $a\,o\,b$, $b\,o\,c$, (which alone are necessary in this method,) being of course known. Draw on pasteboard the



forces $a\,o$, $b\,o$, $c\,o$, as in Fig 31, with their *actual* angles $a\,o\,b$, $b\,o\,c$, $c\,o\,a$. By Art 28, draw arallelogram of forces for the middle pair $b\,o$, $c\,o$; and draw its diag $w\,o$, which will be the re- nt of those two; leaving the resultant of it, and $a\,o$, yet to be found. Cut away neatly the fig, $a\,o\,a\,c\,w\,b\,a$. Make deep knife-scratches along $o\,b$, $o\,c$, so that the two outer triangles may ore readily turned at angles to the middle one. Turn them until the two edges $o\,a$, $o\,a$, meet; hen paste a piece of thin paper along the meeting joint, to keep them in place. Stand the model its side $o\,b\,w\,c$ as a base; and we shall have the *slipper* shape $a\,o\,b\,w$, Fig 32; $o\,w$ being the sole, $o\,b$ the hollow foot.

now have the first resultant $w\,o$, and the third remaining force $a\,o$, in their *actual relative po-* s. Now, to find *their* resultant, also in its actual position, cut a separate triangular piece of paste- l of the size and shape of $w\,a\,o$. Find the center i, of the edge $w\,a$, and draw a line $i\,o$ on each f it. Finally, by means of the edges $a\,o$, $w\,o$, paste this piece to the inside of the model, along nter-line $w\,o$. This done, $i\,o$ represents *one half* of the reqd resultant, in its actual position.

reason why it represents but one-half of it is plain; for, as be- tated, we now have $a\,o$ and $w\,o$ in their actual positions in the l; consequently, if we complete the parallelogram of forces s, and draw its diagonal $n\,o$, this last will be their resultant. ince the two diags of every parallelogram divide each other into qual parts, the diag $a\,w$ thus divides the resultant; consequently one-half the resultant.

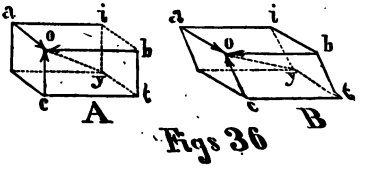


there be four forces, as $a\,n$, $b\,n$, $c\,n$, $d\,n$, Fig 34, draw them as in g, with their actual angles $a\,n\,b$, $b\,n\,c$, &c. Draw also the re- nts $n\,v$, of $a\,n$ and $b\,n$; and $n\,w$, of $n\,c$ and $n\,d$. Then cut out ntire fig, as before; and paste together the two edges $a\,n$, $a\,n$. we have the two resultants $a\,v$, $a\,w$, Fig 35, forming two simple s, in their *actual relative positions*; and we have only to measure dist apart from v to w; and thence find their resultant $a\,r$, h will evidently be that of the four original forces.

, as in the preceding case, cut out a separate piece of pasteboard, , Fig 35, and having drawn on each side of it a line from a to the r o of $v\,w$, paste it inside of the model. Then will $a\,o$ represent *half* of the resultant of the four forces, in its actual position.

ould the model be exposed to hard usage by workmen, it should be of wood; the triangles $a\,n\,b$, $b\,n\,c$, &c, being cut out separately; ining edges bevelled; and then glued together. See also Art 46.

rt. 46. The parallelopiped of forces. If any three forces, $a\,o$, $b\,o$, Figs 36, in diff planes, meet at one point o, whether they all be strains, or all ions, their resultant or joint ef- will be represented, both in ntity and in direction, by the y o, of a parallelopiped $a\,c\,t\,b\,i$, which three converging edges be assumed to represent the e converging forces.



is suggests another mode of showing resultant of three such forces by a ; for it is only necessary to prepare A or B, as the case may be; and yo present the reqd resultant.

Three converging forces in different planes cannot be in equilibrium.

Art. 47. Forces in different planes; and *not* tending to or from one point. It is but rarely that such forces have a resultant, or antiresultant; that is, no single force can usually be found either to produce an equal effect, or to balance them. It is so seldom that they present themselves to the engineer's attention, and their solution is so tedious, except in very simple cases, that we shall confine ourselves to one of that kind. As in Art 44, the resultants cannot be had by measurement from a drawing.

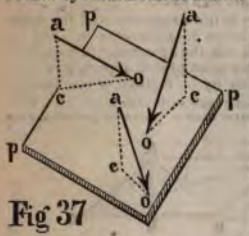


Fig 37

Let $a o$, $a o$, $a o$, Fig 37, be three such forces; and suppose them all to act *against* (see Remark, Art 8) the same plane $p p p$; and against the same side, or surf of it: that is, none of them pointing upward against the under side of the plane in the fig. Having the points o of application, and the relative positions of the forces themselves, as well as the angles $a o c$ which they form with the plane $p p p$, resolve each of the forces into two components; one of which, $c o$, coincides with the plane; while the other (parallel and equal to $a c$, but meeting $c o$ at o) is at right angles to $c o$, or to the plane. We then, as in Fig 29, Art 44, have two sets of forces; one set in the plane, and the other at right angles to it. Since those in the plane do not tend to or from one point, their resultant must be found by the principle of leverage, or moments, Art 40. While that of the several parallel components (equal to $a c$, but applied at o) may be obtained by Arts 56 and 59. These two resultants will rarely be in the same plane with each other, and consequently can have no joint resultant. If they should chance, however, to fall in the same plane, use Art 28 for finding their resultant. In simple cases, where the forces act against one plane, as in our fig, pieces of wire, cut to lengths to represent the forces; and stuck into a piece of smooth board, in their proper relative positions, (see Note, Art 44,) will greatly facilitate the finding of the resultant approximately enough for most practical cases.

The same general process must be used, no matter how great may be the number and directions of the forces. A plane must be assumed to pass somewhere through the system; and the directions of all the forces must be conceived to be so extended as to terminate at points of application in said plane. Each force must then be resolved into two, as in the foregoing example; and the resultants of the two sets of forces, as well as their joint resultant, if they have one, must be found as before. If any of the forces should be *parallel* to the assumed plane, but not *in* it, it evidently cannot be resolved into two, one of which shall be in the plane; for (Art 32) no force can have one of its components parallel to itself. Hence, in such a case, the resultant cannot be found by this process.

Art. 48. It is comparatively seldom that strict mathematical accuracy is reqd in finding the resultants of forces in engineering practice; therefore, the foregoing easy methods by measurement from a drawing, or model, will usually answer every purpose. Moreover, they appeal to the eye; and are therefore much less liable to serious errors than methods involving numerous calculations. But when more correct results are needed, they may be had by means of a table of nat sines, tangents, &c. Thus, in the case of two components and their resultant, calling the components, Fig 38, C and c; and the resultant R, *then if the angle* m b n *between the comps is* 90°,

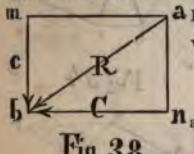


Fig 38

$R = \sqrt{C^2 + c^2}$; $C = R \times$ *cosine* a b n. *And* $c = R \times$ *sine* a b n.

When the angle is less than 90°, find its nat cosine, then

$$R = \sqrt{(C^2 + c^2) + (C \times c \times \textit{cosine} \times 2)}.$$

When the angle is greater than 90°, subtract it from 180°; and find nat cosine of remainder; then

$$R = \sqrt{(C^2 + c^2) - (C \times c \times \textit{cosine} \times 2)}.$$

Having thus obtained the *amount*, or *length* of the resultant, we must find its *direction*, by ascertaining the angle which it makes with *either* one of the components, thus; observing the distinction *either*, and *other*.

$$\text{Resultant} : \textit{other} \text{ component} :: \frac{\text{nat sine of whole angle } m\ b\ n}{\text{nat sine of reqd angle.}}$$

If the whole angle $m\ b\ n$ exceeds 90°, subtract it from 180°; and use the nat sine of the rem.

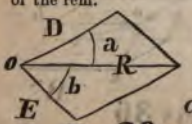


Fig 39

Or, having a resultant R, Fig 39; and the angles a, and b, which it forms with the components D and E, the components themselves may be found thus; using that angle which is *not* formed by the component sought for.

$$\textit{Comp } D = \frac{R \times \textit{nat sin of } b}{\textit{nat sin of } D\ O\ E}. \quad \text{And comp } E = \frac{R \times \textit{nat sin of } a}{\textit{nat sin of } D\ O\ E}.$$

If the angle D O E, or either of the others, should exceed 9

rom 180°; and use the nat sine of the rem, instead of that of the angle

. **Moments. Leverage.** If *a b*, sent any force acting in any direction nd if *o*, or *i*, be any point whatever, or out of the body on which the force nd if from said point a line *o s*, or *i c*, t *right angles* to said direction of the aid line *o s*, or *i c*, is called the **arm, ge** of the force *a b*, *about said point.* mount of the force in lbs, &c, be mult th of the arm or leverage in feet, &c, ft-lbs, &c, is called the **moment** of *out that point.* Thus, if the force *a b* tons; and the line *o s*, 6 feet, then the *a b* about *o* is 8 × 6 = 48 ft-lbs; or ft-

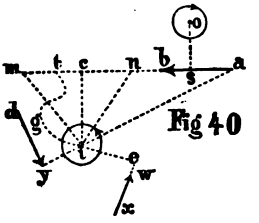


Fig 40

it represents the total tendency of the force to produce motion about the given point. id hor. between the ends of a thumb and forefinger, a piece of stick a foot long, which t the other end of it; because the tendency of the wt to produce motion is too great for r fingers to resist; but we can in that manner hold a stick two feet long, with a 3 lb wt we take it at the center. For although in this case there is twice as much moment as *at* our fingers; yet it is not now exerted *against them*; because we now have two equal pposite directions, reacting *against each other*; and leaving nothing for the fingers to except the mere vert wt of 6 lbs.

oment about *o* tends to produce motion at that point in the direction in which the hands ve, or from the left hand, toward the right, it is called a *right-hand* moment. But the same force, about the point *i*, tends to produce motion at that point, from right to left, e arrow-head on the small circle; hence it is called a *left-hand* moment. The moment *y*, with its leverage *y i*, about the point *i*, is a left-hand one; as is also that of *x w* ge *e i*.

rm *o s*, or *i c*, instead of being merely an imagined *dist*, is a rigid *bar*, at one end of *c*, the force is imparted; thus giving the bar a tendency to move around the point *o* or ter, it is frequently called a LEVER; and the point *o* or *i*, the FULCRUM of the lever.

er, instead of being like *c i*, at right angles to the direction *a m* of the d be oblique to it, as in *i m*, or *i q*; or should be curved, or bent in any ; this in no way affects the leverage, or moment of the force; for the **is always the perp dist.** *or in other words,* **the shortest dist e fulcrum, to the direction of the force**; and is entirely t of the length of the lever itself. **This is a grand funda- principle of all levers, and leverages**; and the young uld carefully impress it upon his memory, inasmuch as it is of constant in practice.

um is not always at one end of the lever, but may be between the two at there are two arms. Cog-wheels are merely continuous circular levers, crum at the center.

As a further illustration, let , be a bent lever, turning on *f*; and *m* and *n* two wts sus- n its ends, constituting two g in the vert directions *a n*, *c*, at right angles to the di- and *f a*, at right angles to n *a n*, are the arms, or lever- forces *m*, and *n*, about the

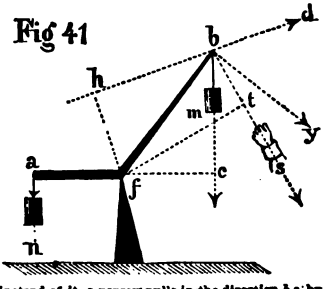


Fig 41

hese leverages are equal, say each each wt, *m* and *n*, be 100 lbs: then l moment of *m*, and the left-hand *f*, are each 6×100=600 ft-lbs; and forces, and *f*, are all in the same o forces, or the two opposite mo- s each other; although the actual ach longer than *f a*. See Art 42.

e the wt *m* to be removed: and that instead of it, a person pulls in the direction *b s*, by ing fastened at *b*. With what force must he pull in order to balance the wt *n*? First everage *f t*, from the fulcrum, and at right angles to the direction *b s* of the new force; is found to be 9 ft. Now we have already found the moment of *n* about *f* to be 600 ft- equire the same moment on the opposite side; so that all that is reqd is to find what

eet leverage must be mult by, in order to make 600. This is plainly $\frac{600}{9} = 66.66$ lbs.

ich the person must exert: because 9 × 66.66 = 600.

hat with the same length of lever, f b, we can have diff powers, (so called,) or lev

ages, according to the *direction* in which we apply our force to the lever. This, however, evidently has its limit; for the greatest power is gained (to use the popular expression) when we apply our force in the direction $b\,y$, at right angles to a line $f\,b$, drawn from the fulcrum to the outer end of the lever. If we apply it in the direction $b\,d$, we get only the leverage $f\,b$.

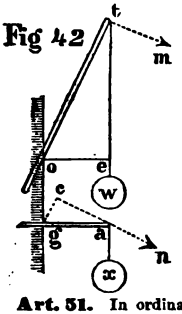


Fig 42

On the same principle as in the foregoing example, if $o\,t$ and $g\,a$, Fig 42, be two beams of equal scantling, but of diff lengths: with one end of each firmly fixed in a vert wall, and both sustaining equal suspended wts w, x: the moments of the wts about the points o and g will be equal, because the arms or leverages $o\,e$, $g\,a$, are equal. Therefore the wt o will have no more tendency to break off the long beam at o, than a ha to break the short one at g. The wts of the beams themselves are not here taken into consideration; and this is always the case in speaking of levers, unless otherwise expressed. In very many cases, the wt of levers of two arms does not affect the result aimed at, *provided the arms are so proportioned as to balance each other when unloaded;* no matter what their comparative lengths or wts may be.

If, in Fig 42, we apply pulls to the beams, in the parallel directions $t\,m$, $c\,n$, at right angles to $o\,t$, then the leverages become changed from $o\,e$ and $g\,a$, to $o\,t$ and $g\,c$; and since $o\,t$ measures 6 times the length of $g\,c$, it follows that the beam $o\,t$ would be broken off at o, by $\frac{1}{6}$ part as much force in this new direction, as $g\,a$ would; for the leverage being 6 times as great, must be mult by only $\frac{1}{6}$ as much wt, in order to have an equal breaking moment.

Art. 51. In ordinary phraseology, the load, or resistance of any kind, which we wish to move, overcome, or balance, by means of a lever, is called *the weight;* while the force of whatever kind which we apply to accomplish this, is called *the power.** Usually, but by no means always, the power is applied to the longer arm. Equilibrium, or balance, or equal moments in opposite directions, will plainly always take place, when the long leverage (not lever) has the same proportion to the short one, that the wt has to the power; because then only can the long leverage mult by the small power, have the same moment as the short leverage mult by the great wt.

This is seen in the common steelyard, Fig 43; which is merely an iron lever, turning on a fulcrum f, and having the wts of its two arms $f\,a$, $f\,b$, so proportioned as to balance each other when unloaded. Here the power, P 1, at the dist of two divisions from the fulcrum; balances the wt, W 2, at the dist of one division. If the wt W 2 were suspended at y, only half a division from f, it would balance the power P 1, suspended where W 2 is in the fig, at a whole division from f. If the power is req'd to *move*, or *overcome* the wt, it is plain that either the power itself, or the length of its arm, must be greater than when mere equilibrium is to be effected: in other words, besides the two *straining* forces, which by their mutual action balance or equilibrate each other, we need some *unresisted* force to impart *motion* to the inert matter.

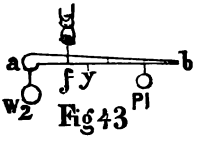


Fig 43

In the two levers of the same length, Fig 44, the leverage $f\,w$ of the wt w, is of the same length in both; namely, one division; but the leverage $f\,p$, of the power p, is but two divisions long in the upper one, and three divisions in the lower. Therefore a power of 1 lb will balance only a wt of 2 lbs in the upper one, and of three lbs in the lower. In the upper one, the power will move twice as fast as the wt; in the lower one, three times as fast. When the fulcrum is between the wt and the power, as in the upper one, the lever is said to be of the *first class;* when the fulcrum is at one end, and the power at the other, *second* class; fulcrum at one end, and wt at the other, *third* class. In all cases it is assumed that the fulcrum is in the same plane (Art 8) with the directions of both the wt and the power; otherwise the principles do not apply. When two weights balance each other on two arms of a lever, as a steelyard, or common weighing scales, &c, their directions are the vert lines passing through their centers of grav; and the same imaginary vert plane which coincides with these directions, coincides with, or passes through, the fulcrum also. When this is not the case, no equilibrium can exist. This may be readily proved by experiment; for we cannot balance a bow-shaped piece of stick or wire, so long as the bow is hor; for it will turn on the fulcrum, of its own accord, until the bow becomes vert; so that the same vert plane that passes through the fulcrum shall pass also through the cen of grav of each half of the bow. If all the forces acting on the lever are hor, or oblique, the imaginary plane must be so too. Moments, must not be confounded with momentum. See Art 10. Moment means much the same as tendency.

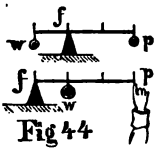


Fig 44

Rem. From what has been said respecting the lower Fig 44, it follows that when a load w is borne at any point of a beam $f\,p$ supported at both ends, then the portion of the load supported by each of these points f and p, is in the same proportion to the whole load that the respective portions $f\,w$ and $p\,w$ of the beam, are to the whole span $f\,p$ of the beam; *but inversely;* that is, the smallest portion of the load is borne by the support at p: and the largest portion by f. In the fig, $f\,w$ is one third, and $p\,w$ is two thirds of the clear length or span of the beam; hence, p supports $\frac{1}{3}$ of the load w; and f supports $\frac{2}{3}$.

* *The fact that by means of leverage a small power can be made to move a great wt, is in common parlance styled a gain of power.* Although in a scientific sense the expression is absurd, yet in practice it has by its universal use become very convenient; and we shall therefore employ it. When the lever, instead of merely *balancing* the power and the weight, has to be put into motion, it is plain that there must be *some* excess of force applied at the power end, to produce the motion.

52. Virtual velocities. Whenever the power and the wt balance
r, either in a single lever, or in a connected system of levers, or leverages,
nd whatever, then if we suppose them to be put into motion about the ful-
ir respective vels will be in the same proportion or ratio as their leverages;
the leverage of the power is 2, 5, or 50 times as long as that of the wt, the
ll move 2, 5, or 50 times as fast as the wt. Therefore, by observing these
nay determine the ratio of the leverages. The wt and the power are to
r, therefore, *inversely* as their vels, as well as inversely as their leverages;
s called *the principle of virtual velocities;* and is a very important one.

53. Neither the amount, nor the effect of leverage is changed, if the arms
er, (whether straight or crooked, or in whatever relative positions they may
ad of being in one piece, and supported by a single point or edge as a ful-
in Fig 44; should consist of two separate pieces *m* and *n*, Fig 45, firmly
a straight rigid axle *a x*,
ngth; (usually placed at
les to the levers *m*, *n*,) and
at two or more points
e moments are then about
or longitudinal center-line
le; any point of which
garded as the fulcrum. It
ver immaterial at what
ng the axle, the lever-arms
be attached to it; both
etween *f*, *f*; or both out-
one in each position. This

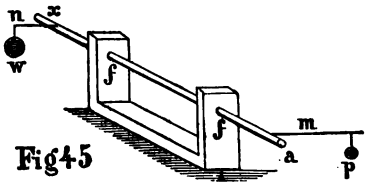


Fig45

ted by the common hoisting *wheel and axle;* in which the rad of the wheel,
of the axle, measd from their common axis, constitute two continuous
lso by series of cog-wheels, which we see placed indifferently at any points
tended shafts, or axles, whether vert, hor, or inclined.
evers *m*, *n*, are not at right angles to the length of the axle, then their lever-
not their actual *lengths*, (measd from the center line of the axle,) must evi-
used in calculating the moments of forces acting upon them. See Remark,
471.

e assumption of an imaginary axle, or axis, as in Rem to Art 41, enables to solve such
is. The entire load being known which is sustained by three vert supports; also the posi-
cen of grav; to find how much of it is
y each of the supports: Draw by scale,
a b c, Fig 46, showing in plan the correct
the supports; and of the cen of grav *g*,
ained load. Assume any side, as *a c*, to
; and from it draw the two perps: *i b*, to
e angle *b*; and *e g*. Now, it is plain that
b, may be considered as two leverages
upposed axle *a c*. At *g* is placed the en-
whose moment about the axis of the axle,
the load, (say 5 tons,) mult by the meas-
scale) length of *e g*, say 8 ft. And this
$i \times 8 = 40$ ft-tons) is balanced by that
ard force at *b*; which upward force must
that share of the load which rests upon
h as the two are in equilibrium. To find
it of the force at *b*, we have only to div

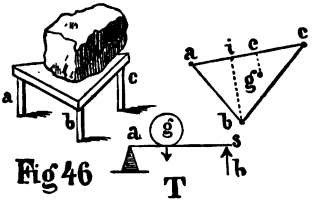


Fig46

it (40 ft-tons) of *g*, by the leverage *i b*, of the reqd force *b*. Suppose *i b* is found by the scale
et; then the upward force at *b*, is $\frac{40}{25} = 1.6$ tons; and we have its moment about the axis
$= 40$ ft-tons, the same as that of *g*. Therefore *b* supports 1.6 tons of the load. In pre-
same manner we may assume each side in turn to be an axis; and find how much is sus-
he opposite angle.

T, in which *a* represents one end of the axis; *a g* the leverage *e g* of the load *g*; and *a s*
f the force *b*, will make the principle more apparent.

**54. Ex. 1. The condition of a beam
g 47, may often be examined on the
ple of a lever.** Suppose it to be of uniform
thickness: its length *a b*, in the clear between its supports,
rt 600 lbs; and its position hor. In this case we know that
a wt, or 300 lbs, is borne by each support. To prove this, we
der its entire weight to be concentrated at its cen of grav,
is case will be at its center *t*. See Arts 56, 57. Then we suppose
supports, as o, to be removed; and an upward force *f* to be
a lever a b, 20 ft long, without wt; and sustaining a load of 600 lbs at 10 ft from the ful
lines the force and the load both act vert, and the beam is hor, or at right angles to the
therefore the dist a t and *a b* from the fulcrum, are the true leverages of said forces
the moment of the load, about the fulcrum, is 600 lbs mult by 10 ft. = 6000 ft-lbs; a

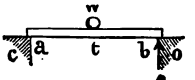


Fig47 f

balance this, the upward force f must be equal to $\frac{6000}{20} = 300$ lbs, or one-half of the wt of the beam; the proof reqd. When, instead of the *moments* of forces, we consider their *bending*, or *breaking effects* upon bodies, we cannot always suppose wts to be concentrated at the centers of gravity. See Art 58.

If, in addition to its own wt, the beam had actually sustained a load *w at its center*, we must add this load to the wt of the beam; and then proceed as before. So also if it sustains a load *uniformly* distributed over its length.

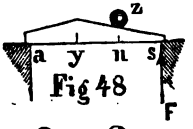


Fig 48

Ex. 2. If the cen of grav of the beam be at any point y, Fig 48, not at its center, we use the leverages $a\,y$ and $a\,s$, instead of $a\,t$ and $a\,b$; and if in addition it sustains a load z at any point n whatever, we first find as before the force reqd at F for the beam alone; and afterward, by using $a\,n$, and $a\,s$ as leverages, we find the force reqd by the load; and add the two forces together for the total F.

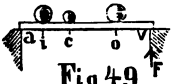


Fig 49

Ex. 3. If the beam sustains several loads at diff points, as in Fig 49, calculate for each of them separately, using the leverages $a\,i$, $a\,c$, $a\,o$, &c; and add all together for total F.

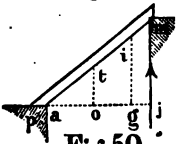


Fig 50

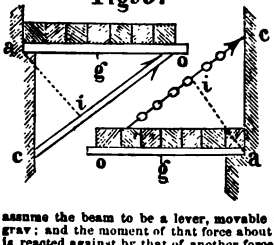
Ex. 4. If the beam in any such case, is inclined, as in Fig 50, the hor dist $a\,o$, $a\,g$, &c, must be taken as measured from the fulcrum a, instead of $a\,t$, $a\,i$, &c; because, since all the forces are vert in direction, only a hor line can be at right angles to them, and serve to measure their leverages from the fulcrum a. If the beam be *rigid*, and its ends cut hor, as shown in this fig, it will have no tendency to slide; because all the forces which through it are *applied* to the bodies m and p are vert; and since the joints are at right angles to those bodies at those points, the entire forces will be *imparted* also; no portion of them remaining *unresisted*, to act as *motion*, *so long as the beam remains rigid, and consequently straight*. But if it bends under either its own wt, or that of its load, new forces come into action, which will tend to push the supports outward from each other; as also in the foregoing cases.

It is only where we may practically regard a beam as rigid, or unchangeable under the forces, that the foregoing concentration of *entire* weights or forces at the cen of grav, can be safely assumed. It will not apply when we are investigating the *strength*, and *deflections* of beams; see Art 58.

After having thus obtained F, in any of these cases, or in other words, having found how much of the entire wt of beam and load bears upon one support, we have only to subtract it from the entire wt, to obtain that on the other support. It is plainly immaterial which end of the beam is assumed to be the fulcrum in any of these cases.

Ex. 5. Let $a\,o$, Figs 51, be a hor beam 10 ft long, projecting from a vert wall $a\,c$; and resting at one end on a step a; the other end being sustained by either a strut, or a tie $o\,c$, 12½ ft long. The beam, and its uniform load, weighing together 3 tons, what will be the pushing strain along the direction of the strut; or the pulling strain along the tie? Draw the Fig to scale; and measure $a\,i$ (which will be found to be 6 ft) at right angles to $c\,o$. Now, the weight of a rigid body, when considered only with regard to its effect in moving the entire unaltered body, or in straining it bodily against another body, acts the same as if it were all concentrated at its cen of grav; and since we are now about to consider it in that light, and not as tending to *bend* or *break* the beam $a\,o$ (in which case only *half* its uniform load, and wt must be assumed to be concentrated at its cen of grav); we consider the 3 tons wt to act at g, 5 ft, or half the length of the beam from a. Now, the 3 tons, being a force of grav, will act in a vert direction; and since the beam is hor, $a\,g$ is at right angles to this direction of the force exerted by the beam and its load. Consequently, if we

Figs 51

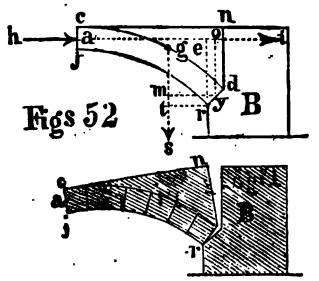


assume the beam to be a lever, movable about a, as a fulcrum, $a\,g$ is the leverage of that force of grav; and the moment of that force about a, consequently, is $3 \times 5 = 15$ foot-tons. But this moment is reacted against by that of another force in the direction $c\,o$; which acts at the point o of the lever $a\,o$, to uphold the beam and its load. The leverage $a\,i$ of this force, that is, the dist from the fulcrum a, and at right angles to the direction $c\,o$ of the force, has already been found to be 6 ft; consequently, the force itself, in order to have a moment of 15 ft-tons about a (as the beam and its load have) must evidently be $\frac{15}{6} = 2.5$ tons, the reqd strain along the strut, or along the tie, $o\,c$; for $2.5 \times 6 = 15$.

Ex. 6. The following is very important in its application to arches of any material.

Let $c\,n\,d\,r\,j$, Fig 52, represent one half of a bridge arch. If this half were not prevented by the *hor pres. $h\,a$, of the* opposite half, it would evidently fall, as in the shaded fig, by turning about the *point r as a fulcrum*. (See Rem 1.) Let us find what this hor pres amounts to in any case. To do *this, we may consider* the half bridge $c\,n\,d\,r\,j$ to be a lever. Suppose its wt to be 40 tons; and to *be concentrated at its cen of grav g*; the vert line $g\,s$ being of course the direction in which it would *act. And let its leverage $r\,t$*, about r, be 6 ft; $r\,t$ of course being at right angles to the direction

s its moment about r equal to $80 \times 6 = 480$ ft-tons. (Art 49.) Now, whatever may be of the hor force $h\,a$, which acts at the s lever, to counteract this moment of *a leverage* (Art 49) is plainly equal to re, m the fulcrum r, and at right angles to $h\,i$ of said force. Suppose we find by mea- m the drawing that $r\,e$ is 8 feet. Then the



Figs 52

ust necessarily be $\frac{480}{8} = 60$ tons; which

es which the opposite half of the bridge st the keystone a, of the arch; for ft-tons of moment.

ut so long as an arch is not deranged, firmly in position, the half arch, in- ing to revolve about the point r, presses the entire surf $r\,d$ of its skewback. e leverage with which the hor force h e skewback, is actually $y\,o$, measured ter of $r\,d$, and in practice it must be of $r\,e$. In the same manner, $y\,m$ be- verage for the wt, instead of $r\,t$.

The cen of grav of the h, can be found by making $n\,d\,r\,j$, about 4 to 6 ins long, on pasteboard, or on a stiff drawing-paper, to a scale. fig; and balance it flatways on a sharp straight-edge, or over the edge of a table, in ns or positions. Where these two directions intersect each other is the cen of grav. It this cen itself that is needed, but the line $g\,a$, of its direction; which may be found king care that the straight-edge is parallel to the back $n\,d$, while balancing the fig.

Under the head leverage, may be classed the tread-wheel; windlass and lever; capstan nd all axles turned by a winch or by a crank; such as the drum and winch with which a is raised from a well, &c. They are all merely continuous simple levers, of which the ulcrum; the rad of the circle described by the power is one arm, and the rad of that de- e shaft, drum, &c, is the other.

Compound levers. $a\,a$, $b\,b$, $c\,c$, Fig 52¼. may be used where t space for the arms of a single lever of sufficient power. They need not one line; but may be placed e other; or in such other po- may be convenient. Their uch greater than the combined he three simple levers, and is : As the product of the weight- $1 \times 3 = 6$; is to that of the s, $10 \times 8 \times 7 = 560$; so is the the weight; or, as $6 : 560 ::$ $\frac{1}{93}$. These arms are measd in from the *fulcrum*; which is at the *end* of one or more of when compounded; see Fig 44. ned effects of the three simple ld be but $20 + 8 + 21 = 49$; or P 1, W 49.

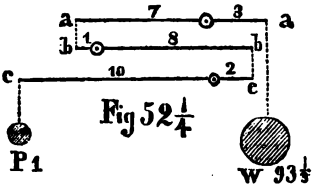


Fig 52¼

es or train of toothed pinions and wheels, working into each rely a series of continuous compound levers. These are generally set in motion by a e, the rad of which is the first leverage of the series; while at the other end of the train ally suspended from a drum, the rad of which is the last leverage. To find the effect, e prod the radii of the winch and of the wheels, and into another prod the radii of the of the drum; then, as the last of these prods is to the first, so is the power to the wt, ceding case.

e foregoing cases of compound leverage, as in all other cases whatever of leverage, the is to that of the power, as the power itself is to the wt; thus, in Fig 52¼, the wt will $\frac{1}{93\frac{1}{3}}$ part as fast as the power; or the power must descend 93⅓ inches, in order to raise inch; on the principle of virtual vels. See Art 52, p 477.

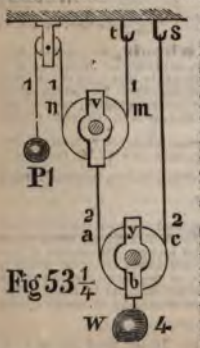
The Screw is a combination of leverage, with an inclined plane; a spiral inclined formed by the threads of the screw. While the power applied to the lever which turns oves around an entire circle, the body moves only the dist between the centers of two i since in all mechanical contrivances, the wt is to the power as the vel of the power is e wt, so in this case, *theoretically*, the wt is to the power, as the entire circumf of the bed by the power, is to the dist between the centers of two threads; but in practice, the e screw (which under heavy loads becomes very great) has also to be overcome by the this fact makes the calculations of but little use.

ulley, *also, when* a fixed one, is referable to leverage. In the fixed pulley A, Fig *no gain of power; for* here the diam $a\,b$ is a lever of two equal arms, revolving around *the center of the pulley.* Consequently, the wt and the power have equal leverages;

each equal to the ra circle; and in order to a wt W of say 1 ton, t P must also be 1 ton; of them moves, the ot plainly move with t vel. To *raise* the wt, must *exceed* the wt; it has also to over friction of the axl which the pulley rev the friction of the ro groove around its These frictions be great when many p combined, that theor culations of the pow little value. Althou pulley gives no *gain* it is very *convenie* lowing *change of di applying the power* by pulling downwar &c, we can cause rise vert. It is pla rope in this pulley strained at all poin retically, this is the any one single rope *g e*, Fig 52¾, passi any number of pulle fixed, as A or D, o as B; and all the calculations of the be based upon thi alone. They will be incorrect in p account of the fr

alluded to. In Fig 52¾, where only one rope is used, the lower pulley-block B *s*, to whic is attached, is directly upheld by the *two* parts *d f* and *e g* of the rope. Consequently ea parts is equally strained by a force equal to one *half* the wt; and since the whole single r retically strained to the same extent, that part of it to which the power is applied must equal to half the wt W; or, in other words, the power itself must be equal to half the move twice as fast, and twice as far.

In Fig 53, the lower pulley-block *t y* is sustained directly by the 4 parts *c c c c* of the s therefore, each part of the rope, and consequently the whole of it, is equally strained by a to ¼ of the wt W; and the power P must be equal to the same ¼, and will move 4 times 4 times as far, as the wt. It is immaterial whether the two pulleys in the lower block o placed one above the other, as shown, or (as usual, and more convenient) side by side; those in the upper block. If there were 3, 4, or 5, &c, pulleys in each block, then there 8, or 10 sustaining parts *c c*, &c, of rope, each stretched equal to $\frac{1}{6}$, $\frac{1}{8}$, or $\frac{1}{10}$ of the wt power would also be in the same proportions to the wt; in other words, to find the theo portion of the power to the wt, divide 1 by the number of parts *c c c c* of rope which dire the lower block. In our fig it is ¼. The same rule applies to Fig 52¾.



When more than one rope is used in a system of pulleys, become diff from the foregoing, on the principle illustrated Here the lower block *y b*, with its attached wt of say 4 tons sustained by the two parts *a* and *c* of one rope. Conseq part has a strain of ½ the wt, or 2 tons; which is uniform *that* rope. But all the 2 tons strain on the part *c* is susta hook *s*; while that on the part *a* is sustained by the two pa of the other rope; each of which plainly sustains one hal ton, which is uniform throughout this second rope to t Therefore the power also is 1 ton, or ¼ of the wt W. T proceeding is the same, whatever may be the number of n leys. To find the theoretical proportion of the power to t together continuously as many 2s as there are movable p div 1 by the prod. Thus, here we have two movable pulle 2 = 4; and ¼ = the answer. If there were 4 movable should have 2 × 2 × 2 × 2 = 16; and $\frac{1}{16}$ = answer.*

In all our figs, that end of the rope to which the power P is represented as hanging vertically, and parallel to the ot the rope; but this was done merely because the power is be a *weight*, and of course acting vert. But if the power i force, or any other kind that may act in any direction wha the power end of the rope, as *m n*, Fig 53, may have that which it is most convenient. The amount of power requi be thereby changed; for it is plain that leverage from the c pulley *o s* to *m*, when the power is at *n*, and acting in the rection of the rope, is equal to that from the same center to power is at P, and acting vert. But the theory of the sy

ropes requires the other portions of the rope or ropes to be parallel to each other; bes

* The rule given in text-books generally, is entirely incorrect.

s loads to be raised would act in directions oblique to the directions of the ropes, and principle would come into action. See Rem 2, Art 33; and its footnote, p 467.

edge is a kind of double inclined plane, (Art 60.) and is a very powerful machine. as it is usually worked by blows, the effect of which cannot be calculated; and inas- eoretical action is in practice totally changed by friction, no serviceable rules can be ng it.

. **Parallel forces** are those whose directions, as (whether opposite to one another, or not; or whether plane, or not,) are parallel. In Fig 53½, the forces, ting *upon* one plane, *o o o o*, are not *in* one plane, but

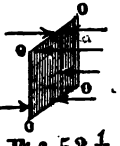


Fig 53½

uliarity of parallel forces *in one plane*, that all their verages with respect to any given point, are in the ht line. Thus, if *l, m, n, o*, Fig 54, be all supposed to me plane, then their leverages *p q, p r, p a*, about the all in the same straight line *p a*.

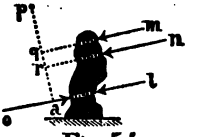


Fig 54

l forces are evidently always in the same plane; that is, surf could coincide with both of them; and their re- tiresultant will be in that same plane: which amounts other words, that if 3 parallel forces hold each other in hey are in the same plane. See Rem 2, page 468.

. **The resultant of any number of parallel forces,** the same plane, or in the same direction, or not, is always parallel to hey all act in the same direction, whether in the same plane or not, their s equal in amount to their sum; or, in other words, an antiresultant ent to balance them, must be equal to all dded together. But if they are in oppo- ons, their resultant will be equal to the those which act in one direction, and act in the opposite one; and its direction t of the greater sum. Thus, in Fig 53½, s pointing to the left amount to 10 tons, o the right 4 tons; then the resultant will = 6 tons; and it will point to the left.

Fig 55

l vert downward forces of gravity, upon the innu- rate particles, situated in the infinite number of rt planes, in any body, as W, Fig 55, is an illustra- If any such body be suspended by a string from a e B, the vert upward pull of the string will balance all these innumerable forces. Consequently, the nts their antiresultant, which is equal to their re- know that the vert pull on the string, as shown by lance, is equal to the wt of the body; which wt is made up of the innumerable parallel uded to. Thus we see that when any number of parallel forces, whether in the same act in the same direction, their antiresultant is parallel to them, and equal to their sum; their resultant must be so also. The same principle applies to parallel forces in any tever.

ly thus acted on by gravity is kept at rest, or balanced, as in the fig, then the direction nt or antiresultant, or of the string in the fig, passes through a certain point, called the *ity* of the body. The cen of grav of any body is a certain point, upon which when acted ity only, the body will balance itself, in whatever position it may be placed; and if the rav of the body could be concentrated into that single point, its effect, whether regarded e entire rigid body, or as producing strain (pull or push) between it and another rigid remain precisely the same as it actually is with the grav diffused throughout the entire

---

odies the cen of grav is also the center of the wt of the y frequently this is not the case. Thus, in a body , with its cen of grav at *c*, there is more wt on the side he side *c b*.

, Fig 55, suspended freely from any point *n*, is at rest, is directly under said point. If the body W be pushed side, and then left to itself, it will plainly tend of itself k to its *first position*; and when this is the case, it is *stable equilibrium*. But if the body, instead of being *balanced on top of a slim rod*, and if we then push it *ide*, it will not tend to return, but will fall over; and *uilibrium of a body so balanced is said to be unstable.*

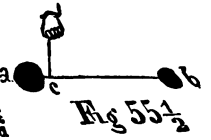
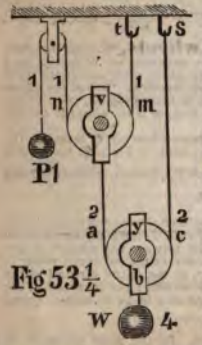


Fig 55½

each equal to the rad of circle; and in order to ba a wt W of say 1 ton, the p P must also be 1 ton; for of them moves, the other plainly move with the vel. To raise the wt, the must exceed the wt; be it has also to overcom friction of the axle which the pulley revolve the friction of the rope groove around its c These frictions becom great when many pull combined, that theoret culations of the power little value. Although pulley gives no *gain* o it is very *convenient* lowing *change of direc applying the power*; by pulling downward, &c, we can cause th rise vert. It is plain rope in this pulley is strained at all points retically, this is the any one single rope, *g e*, Fig 52¾, passin any number of pulleys fixed, as A or D, or as B; and all the f calculations of the p be based upon this alone. They will, be incorrect in pr account of the fri

alluded to. In Fig 52¾, where only one rope is used, the lower pulley-block B *z*, to which is attached, is directly upheld by the *two* parts *d f* and *e g* of the rope. Consequently each parts is equally strained by a force equal to one *half* the wt; and since the whole single ro retically strained to the same extent, that part of it to which the power is applied must b equal to half the wt W; or, in other words, the power itself must be equal to half the w move twice as fast, and twice as far.

In Fig 53, the lower pulley-block *t y* is sustained directly by the 4 parts *c c c c* of the sli therefore, each part of the rope, and consequently the whole of it, is equally strained by a f to ¼ of the wt W; and the power P must be equal to the same ¼, and will move 4 times a 4 times as far, as the wt. It is immaterial whether the two pulleys in the lower block of placed one above the other, as shown, or (as usual, and more convenient) side by side; s those in the upper block. If there were 3, 4, or 5, &c, pulleys in each block, then there w 8, or 10 sustaining parts *c c*, &c, of rope, each stretched equal to $\frac{1}{6}$, $\frac{1}{8}$, or $\frac{1}{10}$ of the wt W power would also be in the same proportions to the wt; in other words, to find the theor portion of the power to the wt, divide 1 by the number of parts *c c c c* of rope which direc the lower block. In our fig it is ¼. The same rule applies to Fig 52¾.



When more than one rope is used in a system of pulleys, become diff from the foregoing, on the principle illustrated b Here the lower block *y b*, with its attached wt of say 4 tons, sustained by the two parts *a* and *c* of one rope. Consequ part has a strain of ½ the wt, or 2 tons; which is uniform t *that* rope. But all the 2 tons strain on the part *c* is sustai hook *s*; while that on the part *a* is sustained by the two part of the other rope; each of which plainly sustains one half ton, which is uniform throughout this second rope to its Therefore the power also is 1 ton, or ¼ of the wt W. Th proceeding is the same, whatever may be the number of m leys. To find the theoretical proportion of the power to th together continuously as many 2s as there are movable p div 1 by the prod. Thus, here we have two movable pulley 2 = 4; and ¼ = the answer. If there were 4 movable p should have 2 × 2 × 2 × 2 = 16; and $\frac{1}{16}$ = answer.*

In all our figs, that end of the rope to which the power P i is represented as hanging vertically, and parallel to the oth the rope; but this was done merely because the power is s be a *weight*, and of course acting vert. But if the power is force, or any other kind that may act in any direction what the power end of the rope, as *m n*, Fig 53, may have that di which it is most convenient. The amount of power requir be thereby changed; for it is plain that leverage from the ce pulley *o n* to *m*, when the power is at *n*, and acting in the i rection of the rope, is equal to that from the same center to power is at P, and acting vert. But the theory of the sys

ropes requires the other portions of the rope or ropes to be parallel to each other; bec

* The rule given in text-books generally, is entirely incorrect.

to be raised would act in directions oblique to the directions of the ropes, and ple would come into action. See Rem 2, Art 33; and its footnote, p 462.

is a kind of double inclined plane, (Art 60.) and is a very powerful machine. is usually worked by blows, the effect of which cannot be calculated; and inas- cal action is in practice totally changed by friction, no serviceable rules can be

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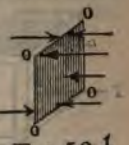

Fig 53½

ity of parallel forces *in one plane*, that all their es with respect to any given point, are in the e. Thus, if *l, m, n, o*, Fig 54, be all supposed to lane, then their leverages *p q, p r, p a*, about the n the same straight line *p a*.

Fig 54

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he resultant of any number of parallel forces, same plane, or in the same direction, or not, is always parallel to ll act in the same direction, whether in the same plane or not, their al in amount to their sum; or, in other words, an antiresultant o balance them, must be equal to all together. But if they are in oppo- heir resultant will be equal to the se which act in one direction, and in the opposite one; and its direction the greater sum. Thus, in Fig 53½, ting to the left amount to 10 tons, right 4 tons; then the resultant will ns; and it will point to the left.

Fig 55

downward forces of gravity, upon the innu- articles, situated in the infinite number of es, in any body, as W, Fig 55, is an illustra- y such body be suspended by a string from a e vert upward pull of the string will balance ese innumerable forces. Consequently, the eir antiresultant, which is equal to their re- that the vert pull on the string, as shown by is equal to the wt of the body; which wt is made up of the innumerable parallel o. Thus we see that when any number of parallel forces, whether in the same the same direction, their antiresultant is parallel to them, and equal to their sum; resultant must be so also. The same principle applies to parallel forces in any

s acted on by gravity is kept at rest, or balanced, as in the fig, then the direction antiresultant, or of the string in the fig, passes through a certain point, called the the body. The cen of grav of any body is a certain point, upon which when acted y, the body will balance itself, in whatever position it may be placed; and if the the body could be concentrated into that single point, its effect, whether regarded e rigid body, or as producing strain (pull or push) between it and another rigid precisely the same as it actually is with the grav diffused throughout the entire

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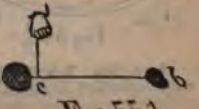


Fig 55½

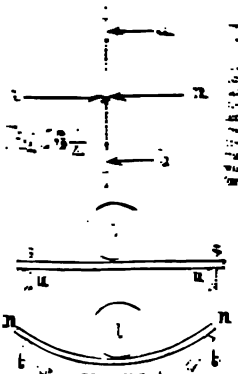


Fig 56 $\frac{1}{2}$

Art. 59. To find the point of impartatio of parallel forces. *Case* 1. Two parallel forces.

In such cases as that of a grindstone supported by its [illegible] *we cause it to revolve a short dist, and then leave it* [illegible] *turn, or to keep on revolving;* and its equilibrium is [illegible]

the same direction; and consequently in the same plane. Draw any straight line $a o$, uniting their directions $1 m$, $3 n$. Measure this line, and div it into two parts, $i o$, $i a$, proportioned like the forces; but placed *inversely* to the forces; that is, place the longest part near the smallest force, and vice versa. Through the point i draw the direction of the resultant R, parallel to the forces. Then is t the point of impartation. Make the resultant, $t 4$, equal to the sum of the two forces.

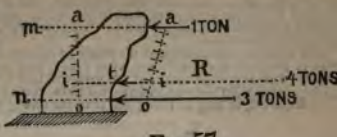


Fig 57

Ex. Let one force be 1 ton; and the other, 3 tons; and let $a o$ be 8 ft. Then, as the sum, 4 tons, of the two forces; is to the length, 8 ft, of $a o$; so is the large force, 3 tons; to the long part, $i a$, of $a o$. Or, $4 : 8 :: 3 : 6 = i a$. Consequently, $i o = 8 - 6 = 2$ ft; as shown in the fig.

The foregoing, as well as some other facts connected with parallel forces, will be more easily recalled to mind, by associating them with the idea of the common steel-yard, Fig 58. Here the two forces in Fig 57 are represented by the wt 1 lb at c, and the wt 3 lbs at d; one of them 3 times as great as the other. We know that these weights, when suspended at dists $f a$, $f b$, one of them 3 times as great as the other, from the fulcrum f, are balanced by their antiresultant $f h$, which is equal to $3 + 1 = 4$ lbs, or the sum of the two forces. This antiresultant has precisely the same tendency to pull the steel-yard upward, that the two weights have to pull it downward. Also $f s$, equal to $f h$, but acting in the opposite direction, is the resultant of the two forces; its effect to pull the steel-yard bodily downward, when applied to it at f, is precisely equal to that of the two wts applied at a and b. The effect of the two wts at a and b, to *bend* or *break* the steel-yard, would plainly be very diff from that of their resultant $s f$, applied at f. See Art 58.

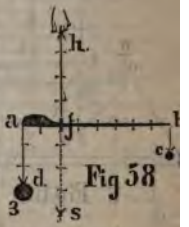


Fig 58

Case 2. Two *unequal* parallel forces, $d a$ of 3 tons, and $h f$ of 4 tons, Fig 59, imparted to a rigid body in opposite directions, but not in the same straight line.

Starting from the line of direction $d m$, of the small force, draw any line $m n$, passing through and *beyond* the direction $h f$ of the large one. Now find the *amount* of the resultant R. This, by Art 56, is equal to the diff between $h f$ and $d a$, since they are in opposite directions; and has the same *direction* as the large one; therefore it is equal to $4 - 3 = 1$ ton; and its direction is the same as that of $h f$. Then say, as the resultant is to the small force $d a$, so is the dist $o m$, to the dist $o s$, along the line $m n$. Through s draw $s c$, parallel to the two forces; and c will be the reqd *point of impartation* of the resultant R; the tendency of which to move the entire rigid body, will be equal to the joint tendency of $d a$ and $h f$. An antiresultant at t would of course balance the two forces $d a$, $h f$.

Fig 59

This case, like the preceding one, is illustrated by the steel-yard, Fig 58; where $a d$, and $f h$, represent the two forces in the last fig; while $b c$ represents their balancing antiresultant, corresponding to f

Case 3. **Couples.** Two *equal* parallel forces, a, and b, Fig 59 a, imparted to a rigid body in opposite directions, but not in the same straight line, are called *a couple*. The *force* of a couple, means simply *one* of the forces; the perp dist c, between the directions of the two forces, is called the *arm*, or *leverage* of the couple. If one of the forces be mult by this arm, the prod is the *moment* of the couple, in foot-lbs, &c. A couple has no tendency to move the entire body forward in the direction of either force; but merely to make it rotate around a point o, half-way between the points at which the two forces are imparted. A couple (as is also the case with two equal opposing forces in the same straight line) has no single resultant; only another couple can hold it in equilibrium.

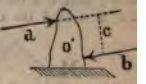


Fig 59 a

Case 4. Any number of parallel forces, whether in the same plane, or in the same direction, or not. The process in this case consists in a mere repetition of that in Case 1, Fig 57, as follows.

Namely, commencing with those forces which point in the same direction, as a, b, c, Fig 60, which all point downward; between the directions of any two of them, as a and b, draw any straight line $o i$, and div it into two parts $j o$, $j i$, proportioned like the forces, but placed inversely to the forces. Through j draw $m s$, parallel and equal to the forces a and b. Then is $m s$ the resultant of those forces. Next find the resultant of this resultant $m s$ and any third force, as c, in the same way. That is, draw any line $t w$, uniting their directions; div it into two parts $t i$ and $i w$, proportional to the forces, but placed inversely. Through i draw $n g$, and make it equal to $m s$, and c.

n g the resultant of the three forces a, b, c; and g is the cen of force of those forces, and consequently is the point of application of their resultant. So do with any number in the same direction. Then proceed in the same manner with those which point in the opposite direction, as d, e, x, y; and having found *their* resultant, find by Case 2, the resultant of the two resultant forces, now obtained in opposite directions.

It is not at all necessary that the forces be supposed to act upon a plane surf, as in Fig 60; the process is precisely the same when, as in Fig 61, they are imparted at points s, y, x, not in the same plane. This is a consequence of the principle laid down in Art 18, Fig 5; namely, that the effect produced upon a rigid body by an imparted force, remains the same, no matter at what point of the body it be imparted, so long as that point *is in the line of the direction of that force.*

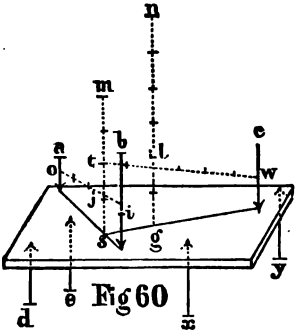


Fig 60

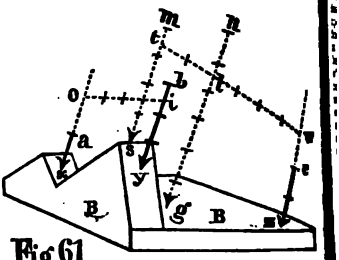


Fig 61

Although Figs 60 and 61 serve to illustrate the principle, they plainly do not give the actual positions of the forces and resultant; because they are necessarily drawn in a kind of perspective, so that all the parts cannot be measd by a scale. The *amounts* of the resultants are easily found by calculation; inasmuch as they are equal to the sums of the forces.

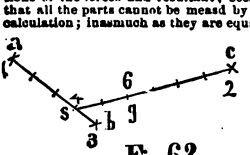


Fig 62

The points for imparting them can be found correctly from a drawing *in plan*, like Fig 62: where the stars represent by scale the *actual* dists apart of the directions of the forces. The *quantities* of the forces, instead of being shown by lines, are to be written in figures, as shown in the fig. This being done, it is easy to find the points s, g, &c, of the resultants.

Art. 60. The Inclined Plane is a rigid straight plane surf, as *a b*, Fig 63, not hor. If a vert line *b c* be drawn from the top *b* of the plane, to meet a hor line *a c*, drawn from its bottom *a*, then *b c* is called the *height* of the plane; *a c* its *base;* and *a b* its *length*. The angle *b a c*, which the plane forms with the hor line *a c*, is called its *inclination*, *slope*, or *steepness;* which, however, is frequently expressed also by the proportion which the base bears to the height; thus, if the length *a c* of the base be 1, 1½, 2, &c, times that of the height *b c*, the inclination or slope is said to be 1 to 1, 1½ to 1, 2 to 1, &c. The angle *b a c* is the *angle of inclination* of the plane.

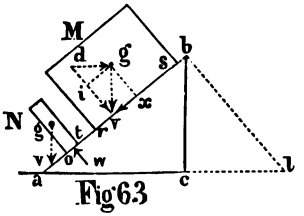


Fig 63

It follows from Arts 56, 57, that when one rigid body as N or M, Fig 63, is placed loosely upon another, as upon the rigid plane *a b*, the effect produced by its wt is the same as if all that wt were concentrated at its cen of grav *g*, and acted in the direction of a vert line *g v* drawn through said center. When we assume the wt to be thus concentrated at the point *g*, we must remember that *all other parts of the body must be considered to be without weight; although still retaining* their inherent cohesive force, or strength.

If, as in N, this vert line *g v*, which now represents the direction of the entire wt of the body, *passes beyond*, or outside of the base, the body must fall; because this wt meets with no opposing *force, in the direction v g*, to react against it; and thus prevent it from producing motion. See Remark, Art 65.

But if, as in the body M, the line *g v* falls within the base *r s*, the body will not upset; but we shall are (Art 19) a force *g v* equal to the wt of the body, and applied obliquely to a rigid surface *a b*,

and consequently resolvable into two components; namely, $i\,v$, perp to the surf $a\,b$, and parted to it as a pressure; and $x\,v$, parallel to the surf, and consequently not imparted ese lines may be drawn by scale, to represent their respective forces. When we consider e as $g\,v$ to be thus resolved into two components, with a view to ascertaining their effects, at said single force must then be considered as no longer existing; but as being replaced ponents. Now the component *force* $x\,v$ being parallel to the plane, it follows (Art 15) that r or *strain* $i\,v$, no matter how great it may be, cannot in the slightest degree oppose the cross moving force $x\,v$, no matter how small it may be; and $x\,v$ must therefore produce motion in using it to slide down the plane; unless some third force, not yet spoken of, shall present ed to $x\,v$. But any forces which press bodies together, always produce a new force, fric- joint, or surfs of contact of the bodies; and this friction acts in direct opposition to any y direction whatever that is in the plane of that joint or surfs. See Friction. Therefore produces fric between the surfs, $r\,s$, of the body M and of the inclined plane; and this the direction $r\,s$, or diametrically opposite to that of the force $x\,v$. The amount of fric on that of the pres; as also upon the nature of the bodies at whose surfs it is produced, gree of smoothness of those surfs, and upon whether they are lubricated or not. If the er than the force $x\,v$ in the opposite direction, the body of course cannot move; but if less, , under the action of a force equal to the excess of $x\,v$ over the fric. It must be remem- the pres component $i\,v$, which produces fric on an inclined plane, is not equal to the wt , but is less than it. It is equal only when the surf is hor, so that the vert force $g\,v$, rep- he entire wt of the body, is at right angles to the joint, and when, consequently, it *all* acts herefore, the steeper the plane becomes, the less is the fric; because then less of the wt acts as pres, and more of it as moving force. Hence, a locomotive has less adhesion on grade, than on a level; for the *so called* adhesion is in reality nothing but fric. But th the perp pres and the fric become less in amount as the plane becomes steeper, yet they retain the same *proportion* to each other; until pressures become so great that *abrasion* of contact takes place; the proportion of the fric to the pres then increases.

is evident that when we wish to push a body *up* an inclined plane, we must overcome c, and the parallel force $x\,v$; but in pushing it *down*, we are opposed only by the fric; for force assists us.

61. Experiment has determined the amount of fric which takes place be- e surfs of such materials as are employed in construction; that is, it has ed the proportion (or, more correctly, the ratio) between the pres and the y person may easily do this for himself, thus: A body as M, Fig 63, is on the plane surf $a\,b$; of which one end, as b, is gradually raised until the arely about to begin to slide. When this takes place, we know that the *has become barely equal to the fric;* and the angle $b\,a\,c$, which the plane then th the hor $a\,c$, is called the *angle of friction*, or *limiting angle of resistance, f repose*, for the particular kind of surf experimented on.

tle reflection will show that whatever may be this angle, $b\,a\,c$, of fric, the line $x\,v$, which ot only the parallel force, but also the *fric* existing at that moment, (and at no other one,) e $i\,v$, which measures the perp *pres*, (not the *wt* of the body;) as the vertical height $b\,c$ e at the same moment, is to its hor base $a\,c$. That is, *at the point of sliding*, as Frie- Pressure : : Height : Base; or, as Base : Ht. : : Perp Pres : Friction. Therefore, when ly begins to slide, measure $a\,c$ hor, and $b\,c$ vert; div the last by the first, and the quot proportion which the fric of the bodies experimented upon, bears to the pres which causes asure the angle $b\,a\,c$ in degrees, &c; the nat tang of this angle will be that same propor- proportion is called the *coefficient of friction* for those bodies; a table of which will be r Friction. A *hor* line $d\,g$, drawn from g, and terminating in $v\,i$ extended, will, when he same scale as $g\,v$, $i\,v$, $x\,v$, give a hor force which, without the aid of friction, would st the force $x\,v$, and prevent it from moving the body down the plane. Or if the length plane be taken by scale to represent the wt of a body, then $b\,l$, drawn from b, to meet $a\,c$ t l, will give that same hor force.

62. If the length $m\,n$, Fig 63½, of an inclined plane, be taken by a scale, ent the wt in lbs, tons, &c, of any body placed upon it; then the base $o\,n$ e same scale, give the perp pres in lbs, which the body imparts to the surf of ; and the height $m\,o$ will give the f force parallel to $m\,n$, and which tends the body down the plane, either by rolling. If the pres $o\,n$ be mult by r coeff of fric, the prod will plainly be l amount of fric in lbs, &c. If the fric ained proves to be greater than the rce $m\,o$, then the body will remain at he plane; but if less, then sliding or own the plane will be the result; and nt of force which *starts* or *begins* the will be equal to the excess of $m\,o$ over the fric.

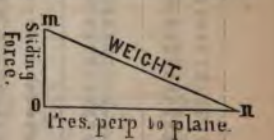


Fig 63 ½

otion continues, it will be accelerated by the accumulation of gravity. See Art 12, p 449. body is placed upon an inclined plane, whether it slides or not, the pres which it pro- ight angles to the surf of the plane, is equal to the wt × nat cosine of angle of slope; force parallel to the surface of the plane, = wt × nat sine of angle of slope; the natural *fric = wt × nat cosine angle of slope* × coeff of fric. For the fric does not vary as the *pe of the plane, but as the cosine* of that angle; in the same manner as the perp pres

rpose we wish to slide a wooden box M, Fig 63, filled with stone, and weighing in

Fig 64, has just that inclination, $y\,x\,w$, at which the fric of any given ced; and sliding is about to commence, from f any force $h\,o$, applied to the plane, through any direction $h\,o$, not perp to it; if from the plication, we draw a line $o\,p$, at right angles to he plane, then the angle $h\,o\,p$ will always be angle of fric $y\,x\,w$ of the body. If the plane hat sliding must take place, then the angle een the force $h\,o$, and a perp $o\,p$ to the plane, be- r than the angle of fric; but if the steepness is the body rests firmly on the plane, then said than the angle of fric.

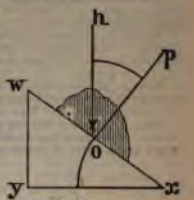


Fig 64

applications of this principle are very numerous; they ures in any directions whatever; and apply to plain ition whatever, whether inclined, vert, or hor; for any nces precisely the same amount of fric, whether we im- ling, the floor, or the wall of a room; provided they all material. The angle of fric of cut stone s about 32°; that is, one block of cut stone on another at a less slope than about 32°; ing full $\frac{6}{10}$ of the pres. Therefore, if the ceiling c, and walls w, w, of a room be of p, p, p, p, lines at right angles to them; we ece s of cut stone against them *with any* applied in the direction of the stone itself, of its sliding; provided only that the di- res along s does not form with the perp p ding 32°. But sliding will take place, s be great or small, if, as at o, o, o, o, said 2°. The angle of fric is, by some writers, ases, **the limiting angle of e.**

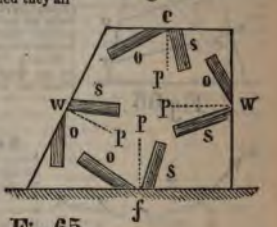


Fig 65

fric of oak endwise against hard limestone, is, according to herefore, if the walls, &c. of a room consisted of such lime- not press a piece of oak endwise against it without sliding, p exceeded 20¾°; and the legs of a wooden trestle, Fig 66, d, on the level surf of such limestone, under any wt w, if e less than 20¾°; but certainly would if it be greater, unless es besides fric at the feet be depended on. In this case the very nearly $\frac{4}{10}$ of the pres; that being the proportion cor- 9¾°. These two illustrations show how wide is the applica- ciple: for the announcement of which we are (the writer ed to Moseley.

Fig66

To find the effect of an extraneous force ($f\,g$, **Fig 67.**) **in any direction, to a rigid body** (**B**) **on an inclined** : when we know the angle of fric, and the wt of the body. The prin- wn in the preceding Art do this.

he cen of grav c, of the vert line $a\,w$; and extend $f\,g$ of the force, to meet t o. Make $o\,a$ by scale, to wt of the body; and $o\,z$ the amount of the force o a point at which we both these forces to be im- body. (Art 29.) Complete gram of forces $a\,x\,z\,o$, by , and $z\,x$, parallel and and $o\,a$. Draw the diag nd it to meet the plane, as

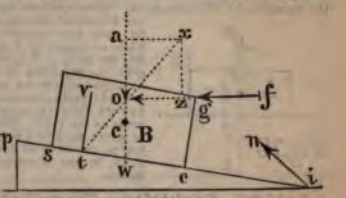


Fig 67

he line $t\,v$ perp to the surf of the plane. This done, we have a single al in effect upon the rigid body, to its wt, and $f\,g$ combined.

ce may (Art 18) be considered as imparted to the body at any point that lies in its $x\,t$; therefore, we will assume it to be imported at t, where it encounters the force the direction $s\,e$, of the joint formed between the body, and the plane. Now, if a base $s\,e$, $t\,v$ being at right angles to this joint, it follows from the last Art. the is less than the angle of fric corresponding to the nature of the materials whi

compose the body and plane, then the body will remain at rest on the plane. But if the angle $x\,t\,v$ be greater than said angle of fric. the body will slide *up* or *down* the plane, (according to circumstances, stated in the next paragraph;) if the angles be equal, the body will be just on the point of beginning to slide either up or down.

When the angle $x\,t\,v$ is on the *down hill* side of $v\,t$, as in the fig, the tendency of the body will evidently be to move *up* the plane; but if, in consequence of a diff direction of the force $f\,g$, (and consequently of the resultant $x\,o$,) the angle $x\,t\,v$ is on the *up hill* side of $v\,t$, then the tendency will be *down* the plane.

Rem. 1. If the direction of the resultant $x\,o$, or the point t, falls *outside* of its base $s\,s$, the body, instead of sliding, will upset. It will fall up hill, if t strikes $p\,t$ up hill from the base; and down hill, if t strikes down hill from the base. See Remark, Art 65.

Rem. 2. In order to draw the parallelogram of forces $a\,x\,z\,o$, and its resultant diag $x\,o$, the line $s\,a$, which represents the wt, may sometimes have to be regarded as *pulling* instead of *pushing* downward at the point o, where the other force meets it. See Art 28, Fig 9½; and Fig 69, Art 65.

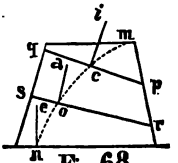


Fig 68

Rem. 3. It follows from the foregoing, that when at the joints $p\,q$, ri, Fig 68, of a mass of masonry; or at the joints of timbers in carpentry, iron work, &c, the fric alone is depended on to prevent sliding, the resultant as $m\,c$, $c\,o$, $o\,n$, &c, of all the forces acting at any joint, must not form an angle $m\,c\,i$, $c\,o\,a$, $o\,n\,s$, with a perp $c\,i$, $o\,a$, $n\,s$, to the joint, greater than the angle of fric corresponding to the nature of the materials whose surfaces constitute the joint.

Rem. 4. The extraneous force reqd to move a body up a plane, will be the least when its direction, $i\,n$, Fig 67, makes with surf $i\,p$, of the plane, an angle, $n\,i\,p$, equal to the angle of fric.

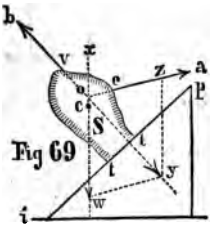


Fig 69

Art. 65. To find the force required to prevent a body S, Fig 69, from falling; when the direction, $o\,w$, of its wt. strikes outside of its base $t\,t$. Thus, suppose we wish to impart a pulling force at e, and in the direction $e\,a$, to prevent the body from upsetting down the plane. Through the cen of grav c, draw a vert line $x\,w$; and continue the line of direction of $a\,e$ to meet it at o. From o draw $o\,y$ at right angles to the plane $i\,p$. By scale make $o\,w$ equal to the wt of the body; and from w draw $w\,y$ parallel to $o\,a$. Make $e\,a$ equal to $w\,y$; then is $e\,a$ the reqd force, which will resist all tendency of the body to fall. For in the parallelogram of forces $o\,w\,y\,z$, we have the force $o\,w$ tending to make the body fall; and the force $o\,z$ (equal to $e\,a$) tending to prevent it from falling; and the resultant $o\,y$, of these two forces, equal in *their joint effect*, is at right angles to the surf of the plane; and is consequently (Art 19) all imparted to it as pres; no part being left unresisted, to produce motion in any direction. For as before said, when two forces, as $o\,w$, $o\,z$, are compounded into one resultant force $o\,y$, those two forces must be considered as no longer existing; thus, in this case, so long as we regard the joint effect of $o\,w$ and $o\,z$ as being concentrated in their resultant $o\,y$, we cannot of course, consider them as acting in other directions at the same time; so that there is, as it were, no longer any wt, $o\,w$, tending to make the body fall; nor any force $e\,a$, tending to uphold it; but only the single force $o\,y$, which presses the inert body against, and at right angles to, the surf $i\,p$; imparting to it a tendency to move only in the direction $o\,y$; which tendency is reacted against by the inherent cohesive force, or strength, of the plane.

Rem. If the resultant of all the forces of any kind, acting upon a body, does not strike inside of its base, no matter whether the base be inclined or hor, the body must evidently move; on the same principle as when grav is the only force acting upon it, in a direction which strikes outside of the base. See Art 35, Fig 19. Or, in other words, in all cases, if the direction of the resultant of all the forces acting upon a body, meets with no resisting force as it passes out of the body, the body must move in the direction of that resultant. Thus, suppose that the only forces acting upon S, were three, in the directions $a\,o$, $y\,o$, and $w\,o$, or the reverse of what they are in the fig; and that $o\,v$ was their resultant. Then, as this resultant meets with no opposing force at n, the body must move in the direction $v\,b$. Or, the body A, Fig 70, rests on a hor base. Being acted upon only by grav, the direction of the resultant is $g\,j$; which, as it leaves the body at j, encounters an equal opposing force from the resistance of the ground; and consequently, no motion takes place. But now suppose an upward force $t\,j$, to be imparted to the body; and let it be four times as great as the grav. Then the resultant will be a single force, three times as great as grav; and acting in the direction $t\,n$. As this direction leaves the body at o, it meets no opposing force; hence the body must move in the direction $o\,n$, since unopposed force always produces motion.

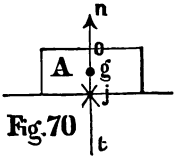


Fig.70

Art. 66. Stability. The stability of a structure, or of any body, is, strictly speaking, that resistance which its wt alone enables it to oppose against forces tending to change its position. Such resistance may be assisted by extraneous wts, or *by other forces* properly applied; but such must be distinguished from the stability *inherent in the* structure, or body itself. To insure the stability of a structure, the *disposition of its* parts, as well as that of the entire mass, must be such that neither *of them shall* move, either by *sliding*, or by *overturning*, under the action of the imparted forces. Stability is therefore a branch of Statics; or of forces at rest, or in *equilibrium* with each other; Art 16, p 451.

ity must not be confounded with strength. A structure strong, and yet very unstable. A block of stone is quite as strong while sliding down ne, or rolling down a steep bank, as when resting on a firm hor base: but it has stability ast case. A pyramid of weak chalk may have great stability; while a globe of granite has very little. We generally have to examine into the strength, as well as the stability ures; but it must be done by diff processes. The stability has reference to the structure s consisting of one or more *rigid* bodies, which may be *moved* as entire masses, but not *anged in form*, by the applied forces. See Remark 2, Art 29, p 459.

es which tend to impair the stability of a structure, are called *acting* ones; and those *s* maintain it, *resisting* ones. This distinction is merely a matter of convenience; for s act, and resist.

which affect the stability of a rigid structure considered as one mass, are its wt; extra- r strains, and the foundation, or support; which last reacts as an antiresultant (Art 30) thers. When these three balance each other, the structure is stable. When the struc- considered as composed of several rigid bodies, then the joints or surfaces of contact be- bodies must also be regarded as so many secondary foundations, and these also must re- slance the forces acting upon them; otherwise these parts may slide, or overturn, while nay remain firm.

7. In order to guard against accidents, a structure must generally be so designed as to of resisting much greater forces than those which it sustains under ordinary circum- e *proportion* which, with this object, we give to the resisting forces, in excess of the act- called the *coefficient of stability*; or simply the stability, or the safety, of the structure. make it capable of resisting 2, 3, or 6 times the amount of the ordinary acting forces, we stability, or a coeff of stability, or a safety, of 2, 3, or 6.

8. Since the stability of a structure, considered apart from its founda- sts entirely in the resistance which its several parts, as well as the entire present against both sliding and overturning, it follows that two precau- ady adverted to in previous articles, must be resorted to. Namely, 1st, *ding*, we must take care that the resultant of all the forces acting upon (including that between the base and the foundation,) shall act either at es to said joint; so as to be entirely imparted to it as strain, (press or pull,) part unresisted to tend to produce motion: or else that it shall not de- a right angle, to a greater extent than the angle of fric corresponding to als which compose the joint; so that the portion of it which is not in- right angles, shall be resisted by friction; and thus be prevented from a motion of sliding.

, instead of relying upon the *position* of the joints, resort must be had to the cohesive *joint-fastenings*, such as bolts, spikes, cramps, joggles, mortises and tenons, mortar, to prevent sliding. As to mortar and cement, however, it is important to remember that and especially in very massive work, they have not time to harden, or acquire their full fore the acting forces are brought to bear upon them; therefore, great care is necessary, them as substitutes for position. On this account we frequently cannot consider a mass to be a single rigid body, but must regard it as composed of several detached rigid stability of each of which must be separately provided for, before we can secure that of Therefore, in large massive structures of importance, we should, as far as possible, omit ation of the strength of the mortar, and rely for stability chiefly upon placing the joints at right angles to the forces acting upon them.

recaution, is that *against overturning*; where we must take equal care sultant of all the forces acting upon any single part, or upon the whole shall fall *within* the base-joint of that part, or whole. See Remark, Art ; Remark 1, Art 64; Remark, Art 65; Remark 2, Art 72.

9. Moment of stability. We have already stated (see Arts 41 at the resistance which any rigid body as B, Fig 71, opposes against being l about any given point *a*, is equal to that uld be produced if the entire wt of the concentrated at its cen of grav *g*; and e end *i* of a straight lever *a i*, of which *a* um; or at the end *o*, *z*, or *n*, of *any* straight 49) *a o*, *a z*, *a n*; or of any bent lever *a i n*, *a s o*, provided that in every case there is everage *a i*, measured from the fulcrum *a*, ht angles to the direction *m n* of the force the body. So far as regards tendency to turning, it is immaterial (Art 18) at what he body, *in this line of direction m n*, we he grav to act: or whether as a push at *o*, t *i*, as denoted by the arrows. We have hat the tendency, or moment, of this force

wt, to produce or to resist motion about the fulcrum *a*, through the me- *oy of these levers*, is found by mult the force or wt in lbs, by the leverage

The prod in ft-lbs is generally called the moment of the force about the t in cases like that before us, in which this moment becomes the measure *lity of the body*, it is called the *moment of stability* (or simply the st

[illegible] size and shape [illegible] in proportion to their [illegible]

[illegible] Thus, it would be [illegible] because the lever- [illegible] of the cen of grav [illegible]

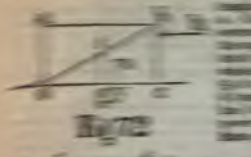


Fig 72

Fig 72½

[illegible] Now, suppose [illegible] will weigh [illegible] than that of the [illegible] the dist c g will be [illegible] the resulting moment will be [illegible] So that although the block [illegible] as great resisting [illegible] than the other to carry masonry, the [illegible] back.

[illegible] of two bodies, as A and B, Fig 72½, of [illegible] the same size, wt, and position; and [illegible] the same degree of stability; one may [illegible] expenditure of a greater amount of the [illegible] than the other, to overturn it. [illegible] the upper part c n, of A; and the lower [illegible] of B, the inside of wood; and the remain- [illegible] of stone, or brick. Then the cen of grav of [illegible] the body A, will be near the dist c; and that of B [illegible] near the dist n. The wt of both bodies being the [illegible] and the cen of grav of both, being at the [illegible] dist, as n, from the fulcrums a, a. Now [illegible] the bodies are to be overturned; they [illegible] of stability must also be the same. Consequently, both will require, in order to have [illegible] overturn them, precisely the same amount of force, [illegible] applied in any same given direction; and at the [illegible] same given point; as, for example, the equal force [illegible] applied in a hor direction, at the point [illegible] and B. But the body B will plainly require this force [illegible] to be maintained for a *longer time*, (or in other words, will require the expenditure of a larger amount of force,) in order to *actually* overthrow it.

[illegible] the body A will; for the body A will be overthrown when the force has acted for only the short time necessary to move it into the position of the dotted lines I. The cen of grav being then carried to n, which is beyond the base at a, the body must necessarily fall. But in the body B, the force must act, at least long enough to move it into the position B; for not until then will its cen of grav n be carried to the position C, so as to be beyond the base. When the stability of a body is considered only with regard to the degree of force necessary for its resistance to beginning to move, (which degree is the same in both A and B;) and is the force with which engineers are most concerned,) it is called the *static stability* of the body; and when considered with regard to the *total expenditure* of that more degree of force, necessary to complete the overthrow of the body, it is called its *dynamic*, or moving stability. The engineer rarely need concern himself about the last; his object being to secure his structures against beginning to move.

Rem. 2. It is not alone the wt of the body itself, which contributes to its stability in all cases; for this may be assisted by extraneous wts or loads. Thus, the wt of a pier P, Fig 73, gives it in itself a certain degree of stability; but when we add the wt of the two equal arches, its stability is thereby increased, supposing the foundation to be secure. And a passing load, when it is directly over the pier, increases it still more. It is true that the wt of the arches might crush the pier to fragments, if the stone be soft; but this is a matter of Strength of Materials; not of stability; and must be examined into by itself. If the two arches be of unequal sizes, or if there be but one arch, the stability of the pier may become either increased or diminished, according to circumstances; as will appear farther on.

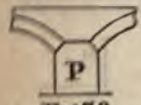


Fig 73

Whether the force acting for or against the stability of a structure, be gravity, or pushes, or pulls, produced from other sources, is, as in other cases, a matter of no importance: for force is simply force, no matter whence derived. We have, therefore, only to look at the diff forces acting upon our structures, as so many *tendencies to produce motions* in certain directions. If these tendencies are reacted against, or destroyed by others, they will not produce it; but if they meet no resistance, motion must take place. The only peculiarity we need assign to grav, is that the direction of its action is *always vert downward*; while other forces may be imparted in either that, or any other direction. The *resultant* of grav *combined with other force or forces*, may be in any direction.

Art. 70. Fig 73½. In the principal cases of stability that present themselves to the civil engineer, both the acting and resisting forces w x, f, f, f, &c, may all be considered to be imparted and acting in the same plane; which is a vert one, v l o r, passing through the cen of grav, v, of the structure, m n p q r s t u; and of course, coincidding with the line of direction w x, of its wt, or force of gravity. The planes v l o r, and all the forces, therefore, may be considered as coincidding with a line of [illegible] standing vert on one edge. This renders the calculations much more [illegible]

forces were in diff planes; in which case diagrams alone would not suf-

rmining the resultants, leverages, mo- See Art 44. Whereas, when they are plane, such diagrams, neatly drawn ient scale, will usually possess all the d in practice.

Fig 73½

mining the strains on the diff pieces composing the f, or of a bridge, &c, not only their wts, but all the es, may be assumed to act in a vert plane passing the truss from end to end; splitting it into two equal case of a structure of masonry, such as a retaining- e bridge B, Fig 74, we base our calculations upon a of it, having a *length* a o, *i i*, or *y t*, of only one ft; t may be the *height* y s; or the *thickness* y c, of the rough the center of this one ft of length, we suppose pass; splitting, as it were, the body B into two pre- parts; and all the forces actually diffused equally e whole one ft of length, are supposed to be concen- act upon one another, in this plane. See Remark,

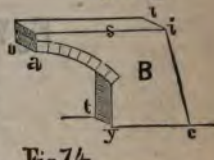


Fig 74

. **In order to ascertain the effect of diff forces to either sliding, or overturning,** of either an entire rigid one composed of several rigid bodies placed together without joint- we first find the direction of the resultant of those forces. If the body is not composed of diff parts; or if, being so composed, these parts are nited together by joint fastenings, as to constitute virtually but a single then we need do nothing more than (as in Art 35) find the resultant *a d*, ll the forces. If the direction of this resultant strikes *inside* of the base, ill not *overturn;* see Remark 2, Art 72; and if, besides striking inside of forms with a line *b x*, at right angles to the base, an angle *a b x*, less than f fric between the body and its support *n m*, then it cannot *slide*.

rt 64, the direction of the resultant *x o* strikes at *t*, inside of the base *s e*; therefore the t *upset;* whether it will *slide* or not, depends upon whether the angle *o t v* is greater or angle of fric corresponding to the materials composing the body, and the plane. If both y dressed granite, this angle must not *exceed* about 32°. See other angles, under head

n overturning, may also be solved on the principle of leverage. See Art 41, p 470,

. We will now consider a case in which the structure is assumed to be f several rigid bodies, merely placed together without joint-fastenings of uch as cramps, bolts, mortar, &c; but depending entirely upon their wt ns, for securing their stability. The process in this case is the same as ig 19, except that instead of assuming the body to have but the one joint nding the effect of the resultant *a d* with reference to this joint alone; it to have several joints, as P Z, F L, W X, &c, Fig 75; and then examine f the diff resultants which they must respectively sustain in consequence rts of the several parts N M P Z, N M F L, N M W X, resting on them.

N J T be one half of a stone arch; or rather a vert slice of k; and let N M W X be a similar slice of a dressed stone abut which has ned to sustain the thrust of the arch; and the fitness of which for the wish to ascertain.

e thrust of the slice of the arch, (that is, the resultant of its wt, and of its hor pres,) to eviously ascertained by Ex 2, Art 38, to be 30 tons; that this is concentrated at *o*, (the skewback;) and that its direction is *o b*. Also, suppose the wt of the part N M P Z be 0 tons, and to be concentrated at its cen of grav G; and, consequently, to act in the G *a*. Now (Arts 18 and 35) we may suppose the 30 tons resultant of the arch, and the of the part N M P Z, to act at the same point *c*, at which their directions G *a* and *o b* meet. scale equal 30 tons, and *c a* 10 tons; complete the parallelogram of forces *c d y a*, and *c y*, *which by the same* scale will give the resultant of all the forces acting upon the

that the direction c y of this resultant strikes at *i*, or within the base P Z; consequently *M P Z cannot upset*, no matter how great may be the pres *c y*; see Remark 2. From

draw *t t*, at right angles to the point P Z; and measure the angle *o t t*, which the resultant *o y* forms with it. This, we find, is greater than 32°; that is, it exceeds the angle of frie between surfaces of dressed stone. Therefore, the part N M P Z must *slide* along the joint P Z. This might possibly be prevented by good mortar, if time be allowed it to solidify properly, before the centers are used to

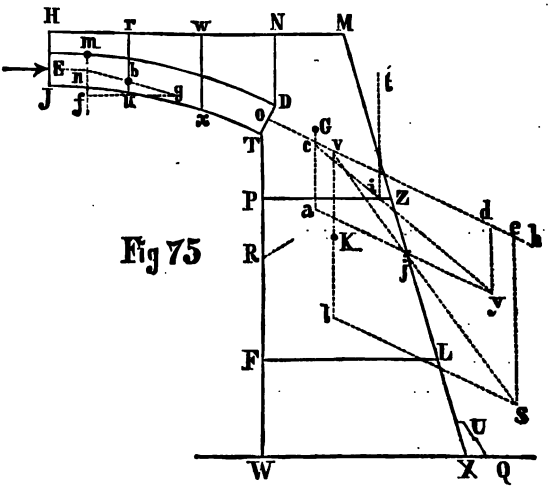


Fig 75

as to bring the pres of the arch upon the abut; or by iron cramps, stone joggles, *&c*; but these are expensive. The most obvious remedy, as well as the least expensive, is simply to incline the joint P Z into a direction somewhat like from E to Z: so as to receive the pres of the resultant *o y* more nearly at right angles; at least so nearly as to be fairly within the limits of the angle of frie. If this is done, *stability* is secured; for the part N M P Z, being now safe against both sliding and overturning, can move in no other way; unless the *strength* of the stone composing the masonry is insufficient to bear the pres, and may therefore crush to pieces under it. But this is a question of Strength of Materials. See Remark 2, Art 35. The point *t*, of Fig 75, comes much nearer to Z than would be desirable *in practice*; for it might cause crushing at Z. See Rem 2, following.

Having thus provided for both the sliding and the overturning stability of the abut as far down as the joint P Z, we will now examine as far down as the joint F L. Taking the entire part N M F L of the abut, we first find its weight, say 25 tons; and this we assume to act at its cen of grav K, and in the vert direction *v K l*. The amount and direction of the thrust of the arch at *o*, of course, remain as before. Therefore, from the point *v*, where the two directions meet, lay off *v e* to represent as before the 30 tons thrust of the arch; and *v l*, the 25 tons wt of the abut. Complete the parallelogram of forces *v e s l*, and draw its diag *v s*; which, measd by the same scale, will give the resultant of all the forces acting upon the part N M F L. Now we see that the direction of this resultant does not fall within the base F L; but, on the contrary, passes out of the body at *f*; outside of which it meets no force to resist it. Consequently, (Remark, Art 65.) since this resultant must be considered as an only force acting upon an inert body or abut, N M F L, *without wt*. (Art 35.) that body must upset around the point L; or around the nearest joint in the masonry between L and *f*; and cannot continue to stand of itself, unless its base be above *f*. It is true that by placing earth behind it, especially if well compacted by ramming, the abut of a small arch might be made to stand safely even upon the base W X; and in the case of arches of moderate spans, this aid may be resorted to for strengthening the abuts, when there is no danger that the earth may be washed away by floods or rains, and thus expose them to ruin; and this is generally and properly done.

Rem. 1. If in the same manner that the point *t* was found in the joint P Z, others between P Z and W X be determined also: then a curve, commencing at the skewback *o*, and drawn through them, will represent the *line of pressures* of the arch through the abut: or the varying curved resultant of the forces acting upon the body: see Fig 7. By means of such a line we see at once where, and to what extent, the joints should be inclined, so as to cross it at right angles, or nearly so. In Fig 75, such points evidently cannot be determined far below the joint P Z, because the line passes out of the body, and does not intersect the lower joints.

Rem. 2. The line of pres enables us to determine another very important point connected with the *stability* of a structure. It is not sufficient in practice that this line should strike *merely within the base: it must strike at a considerable dist* within. If the structure and its foundation were absolutely *rigid*, so that no conceivable force could bend or break them, this would not be necessary; but *all materials are more* or less weak, so that if great pressures come too near to their edges, there is *danger of splitting* or crushing at those points; or if near the edge of a base, an unequal settlement *of the soil beneath* may take place. Therefore, even in structures of but small size, the dist (Z, Fig 75, *of the line of* pres, from the outer point Z, should never be less at any joint than ⅓ of the width *of that joint*; except, perhaps, in a case like that of a small arch in which the parts sliding in temp

d the abuts before the centers are removed. In important works, it should not be less than f the width of the joint; and it is still better, when possible, at ½; or, in other words, at t of each joint. When, as at Q, a footing U is added at a base, W X should be taken as the t W Q.

3. **The line of pressure in an arch itself,** as H J N T, Fig 75, o be found much in the same way, thus: First divide the half arch H J N T, filling above it, by vert lines $r\,u$, $w\,x$, &c, which need not be at equal dists Four such lines will suffice for a flat arch, and about six for a semicircular We then consider in turn, and separately, each part, as $r\,u$ H J, $w\,x$ H J, H J, &c, which extends from these lines to the center H J of the half arch; t of these being the entire half arch. The cen of grav, and the wt, of each se parts, must be found; also, (Ex 6, p 478,) the hor pres at the keystone. each of these parts, like the beam in Ex 1, or the half arch in Ex 2, p 469, is acted upon, and equilibrium, by three forces; namely, the hor pres at the keystone, (see Ex 6, p 478;) its own ting vert; and the resulting force of the part next behind it. We proceed with each part sepa- as we did with the entire beam alluded to, thus: Beginning with the part $r\,u$ H J, from its grav, m, draw a vert line $m\,f$. From the center E of the keystone, draw a hor line E n, to meet From n lay off $n\,f$ by scale, to represent the wt of the part $r\,u$ H J; and from f lay off $f\,g$, hor ale, to represent the hor pres at the key. Draw the diag $n\,g$; which will give, by scale, the re- nt of these two forces. The point h, at which the diag $n\,g$ intersects the vert $r\,u$, is a point in ne of pres reqd. Next go to the part $w\,x$ H J; and in the same manner find another point in ne $w\,x$, using the cen of grav and the wt of that part. **The hor pres will be the ne in each part.*** Finally, treat the entire half arch N D T H J in the same way. The ltant diag of this last will pass through o, the center of the skewback, if the archstones have the e depth throughout. A curve drawn by hand through the points thus found, will be the reqd of pres. These points will not all fall equally well within the thickness of the archstones. In , if the intrados of the arch is a full semicircle, or semi-ellipse, some of them will even fall rely below the archstones, as shown at a and u, in the dotted line of pres in Fig 4½, p 348. en this is the case, the tendency of the line of pres is to bend the arch still more upward at a and and thus allow the parts about the crown o to descend. Each half, $o\,a$ and $o\,g$, of the arch is then the same condition that a crooked pillar or column would be if a heavy load were placed on top of The line of pres of the load would then not pass through the axis of the pillar, but outside of it, as the string of a bow; so that instead of being borne in safety, the load would bend the pillar ll more; as the tightening of the string would bend the bow. If the concave side of the pillar the bow, should crush under the undue strain upon it, the whole would fall. In like manner, if e stone of the arch along the concave intrados near a and u, is not strong enough to resist the undue ushing strain thus brought against it, it will crumble, and the arch will fail, by rising at its aunches, and falling at its crown. See footnotes to Art 6 of Stone Bridges, p 349.

Art. 73. The stability of bodies on inclined planes, as regards verturning, is measd in the same way as when the base is hor; namely, by mult their wt, by the perp dist ($a\,o$, or $c\,t$, at A, B, and D, Fig 76,) from the fulcrum, or turning-point a or c, to the vert line of direction ($g\,o$) drawn from the cen of grav

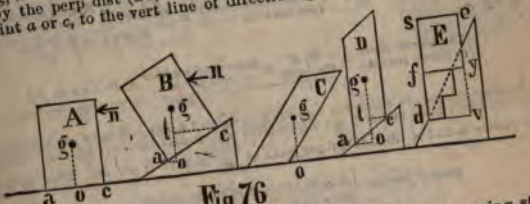


Fig 76

of the body. Hence, it is evident that the body B has less overturning stab about its toe a, than the similar body A has, when the force, n, tends to ups down hill. But it has more than A, when the force tends to upset it up hill, or n the toe c; for the leverage $t\,c$ of B is greater than that, $o\,c$, of A.

The body C, which would overturn upon a level base, because the line $g\,o$ strikes *outside* base; would be stable against overturning, if placed as D upon an inclination, where the v

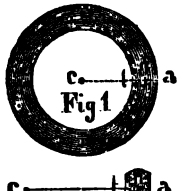
* That the hor pres throughout every part of the arch is equal to that at its center, may be stood by supposing one half of the arch to be changed into a bent column having the skewb its base; and the keystone for its top; and to be loaded on its top by a wt equal to the hor the center of the arch. Now it is plain that every part of a long bent hook has equally to bear pres of the load; on the same principle that every part of this column has equally to vert pull of any load upheld by it. In neither case will the amount of the vert force be ch any force we may apply to either the column or the hook in any other direction; so neither amount of hor force in any part of the arch be changed by our loading the arch with ear *sonry for forming a roadway.* The vert force of grav, or the wt of the loading, above eac *the arch, will combine with the hor pres* at the same point; and the two will form a res direction between them; thus producing the curved line of pres or thrust of the arc words, *the hor pres,* although equal at every part of the arch, is, as it were, pushed do the skewback, by the increasing vert pres of the greater height of masonry

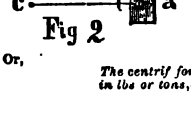
strikes *within* the base. Inasmuch as the leverage a o, is greater than the one c t, D would present more resistance to a force tending to upset it down hill, than up hill.

Structures built upon slopes are, however, liable to slide; that is, they are deficient in *frictional stability*. In practice this is remedied by cutting the slope into hor steps, as at E. But works so constructed are not as strong as if the base were a continuous hor line; because the vert faces of the steps *break the bond* of the masonry; and because the mortar in the higher portions *e d*, being in greater quantity than that in the lower portions *e y*, necessarily allows more settlement of the masonry in the former; and thus renders the work liable to crack, or split open vertically. The case is analogous to that of a foundation, firm in some parts, and compressible in others. Therefore, when circumstances permit, the foundation should be levelled off as at *d v*: or if the masonry has to sustain down-hillward pressures, *v* should be lower than *d*; and the courses of masonry be laid with a corresponding inclination.

CENTRIFUGAL FORCE.

Art. 1. CENTRIFUGAL force is that with which a body while revolving around a center, tends to fly off in a radius, or direct line drawn through said center and the body; and is entirely diff and distinct from the force which the body has in the curved direction in which it is moving. *Centripetal* force, on the contrary, (always precisely equal to the centrifugal, but in an opposite direction,) prevents the body from so flying off; and keeps it in the circle.* They are both called *central forces*. It is centrifugal force that pulls the string with which we whirl a stone; or which tends to burst a millstone, or a fly-wheel when revolving rapidly; or which presses a railway train against the outer rails in curves; while the cohesive force of the string, millstone, or fly-wheel; or the spikes at the rails, furnish the centripetal force. Both forces vary directly as the wt of the body; and also as the square of its vel; and inversely as the rad of the circle in which it moves. Let Fig 1 represent a uniform homogeneous circular ring like a fly-wheel, revolving by means of its arms, around its center of motion, c; and Fig 2, any isolated body *n*, revolving by means of a string, around its center of motion, c. In either case, if the dimension *a t* of the body, in the direction of a rad *a* c, does not exceed about ⅙ † part of the entire rad *a c*, we may near enough, for ordinary practice, consider c n as the rad *in feet* to be used in calculating the centrifugal force: *n* being the cen of grav of the entire body in Fig 2; and the cen of grav of its cross-section at *t a* in Fig 1.





In either case, if we know the wt of the body in lbs or tons, &c; the rad *c n*, in feet; and either the vel in ft per *sec* with which the point *n* revolves; or else the number of revs which the body makes per *min*; then

$$\textit{The centrif force, in lbs or tons, \&c, as the case may be,} = \frac{\textit{Wt of the body} \times \textit{Square of vel at n, in ft per sec,}}{\textit{Rad c n in ft} \times 32.2.}$$

Or,

$$\textit{The centrif force in lbs or tons, \&c.} = \frac{\textit{Wt of the body} \times \textit{Square of number of revs per min.} \times \textit{Rad c n in ft}\ ‡}{2935.}$$

Ex. Suppose the body in either Fig 1, or Fig 2, to weigh 1.2 tons; the rad c n to be 2.5 ft; (in which case the circumf will be 15.7 ft;) and suppose the body to make 100 revs per min. In this case its vel will plainly be $15.7 \times 100 = 1570$ ft per min; or $\frac{1570}{60} = 26.17$ ft per sec. Then by the first formula,

$$\textit{Centrif force} = \frac{1.2 \times 26.17^2}{2.5 \times 32.2} = \frac{1.2 \times 684.9}{2.5 \times 32.2} = \frac{821.88}{80.5} = 10.21 \text{ tons;}$$

or,

$$\textit{Centrif force} = \frac{1.2 \times 100^2 \times 2.5}{2935} = \frac{1.2 \times 10000 \times 2.5}{2935} = \frac{30000}{2935} = 10.22 \text{ tons.}$$

* Centrifugal and centripetal force being always exactly equal to each other, it may be asked why do rapidly revolving bodies sometimes break; and the fragments fly off? Does not this prove that the centrifugal force has become greater than the centripetal one? The answer is, No. When the centrifugal force becomes precisely equal to the ultimate strength of the string, or of the millstone, &c. so that the string, stone, &c, can furnish no greater resistance, then *that force which is moving the body forward*, causes the breakage; and at that instant both the centrifugal and centripetal forces *cease entirely*; and the fragments of the body do not fly off in the direction which the centrifugal force had; but in a tangent, or at right angles to it; being carried in that direction by said other force.

† When *a t* is equal to ⅙ of *a c*, the rad thus found will be but $\frac{1}{44}$ part too short; when ⅓ of *a c*, about $\frac{1}{19}$ part too short; and when ⅔ of *a c*, about ⅙ part too short; and the resulting centrif forces *will consequently* be too small in the same proportion. But when, as is usually the case, in fly-wheels, &c, *a t is much less* than even ⅙ of *a c*, the error is not worth notice in practice.

‡ These are not strictly correct, not only because n is not in reality at the cen of grav of *t a*; but *because the ring*, Fig 1, or the body, Fig 2, must be united to the center c, either by arms, or by a *string*, or *wire*, &c; and the weight of these arms, &c, will slightly shorten the rad of gyration. See *Art 2*. But in practice this effect is usually too small to be regarded; or a trifling allowance is made for it by *guess*.

, **2.** But in the case of a solid homogeneous uniform circular body, like a millstone of uni-
ckness, Fig 3, revolving around its center c, either vert or hor, the rad to be used is what is
e *radius of gyration*, c h. It is self-evident that in such a body the
arer the circumf move with greater vel and force than those nearer
er. But there is, in such cases, an imaginary interior circle
alled the *circle of gyration;* and it is such that if all the weight
tire body be supposed to be concentrated at the circumf of said
en the force of the revolving body would produce the same effect
nally does while distributed over the whole body. Any point as
l circle, where it is cut by any rad c w, is called the *center of gy-*
that rad. In a circular body of uniform thickness, like a mill-
is very easy to find the rad c h of gyration; for we have only to
actual out-to-out rad c w, by the decimal .707. This being done,
e resulting rad c h of gyr, instead of the rad c n of Figs 1 and 2;
recisely the same manner; using also the vel at h, instead of that
igs 1 and 2.

Fig 3

et Fig 3 be a millstone, 3 ft 8 ins, or 3.666 ft diam; its wt .5 of a ton; and making 326 revs
its circumf will then be 11.52, and will move 3755.5 ft per min; or 62.592 ft per sec. Its
$\frac{3.666}{2} = 1.833$ ft; consequently, its rad of gyr c h is $1.833 \times .707 = 1.296$ ft; and the diam
rcle of gyr is $1.296 \times 2 = 2.592$ ft; and its circumf h m s y is $2.592 \times 3.1416 = 8.143$ ft. And
stone makes 326 revs per min, the vel of the circle of gyr is $8.143 + 326 = 2655$ ft per min;
$= 44.25$ ft per sec. Hence the

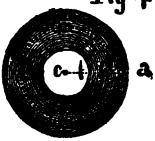
$$\textit{Centrif force} = \frac{\textit{Wt of the body} \times \textit{Square of vel at h in ft per sec}}{\textit{rad} \text{ c h, in ft} \times 32.2};$$

$$\text{or, } \frac{.5\ ton \times 44.25^2}{1.296 \times 32.2} = \frac{.5\ ton \times 1958}{41.73} = \frac{979}{41.73} = 23.47\ tons.$$

$$\text{Again, } \textit{Centrif force} = \frac{\textit{Wt of the body} \times \textit{Sq of number of revs per min} \times \textit{Rad} \text{ c h, in ft}}{2935};$$

$$\text{or, } \frac{.5\ ton \times 326^2 \times 1.296}{2935} = \frac{.5 \times 106276 \times 1.296}{2935} = \frac{68867}{2935} = 23.47\ tons.$$

Fig 4



, **3.** When, in a homogeneous revolving ring, Fig 4, of uniform
s, the dimension a t becomes so great, that it is not sufficiently ac-
assume the point h, of the rad of gyr c h, to be at the cen of grav
was done in Figs 1 and 2, the true rad c h may be found thus:
quare of the inner rad c t, to the square of the outer rad c a. Div
a by 2. Take the sq rt of the quot.

et c t be 3 ft; and c a, 7 ft. Then,

$$\textit{l of gyr}, \text{ c h} = \sqrt{\frac{3^2 + 7^2}{2}} = \sqrt{\frac{9 + 49}{2}} = \sqrt{\frac{58}{2}} = \sqrt{29} = 5.38\ ft.$$

d of gyr, or dist from the cen of revolution, to the circle of gyr, of a few other bodies, may
thus:

In a circular plate revolving around its centre,	Rad	$\times .707$
" " " " its diameter,	Rad	$\times .5$
" solid globe " " its diameter,	Rad	$\times .633$
" thin circular ring " " its diameter,	Rad	$\times .5$ *
" thin hollow globe " " its diameter,	Rad	$\times .817$ *
" solid cone " " its axis,	Rad	$\times .548$ †
" thin straight rod " " one end,	Rad	$\times .577$
" " " " revolving around any point between its two ends. Call the length of its two arms A and a.....		$\sqrt{\frac{A^2 + a^2}{3A + 3a}}$

ain from the last example, that any body may have any number of radii of gyration; de-
upon the position of the cen of rev.

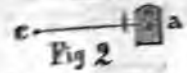
---

ectly correct, only when *infinitely* thin. When the thickness is appreciable, first measure the
e center of the thickness; then mult by .817, for a practical *approximation*.
rad means the rad of the circular base of the cone.

[illegible] built upon slopes are, however, liab[illegible]

CENTRIFUGAL FORCE.

Art. 1. Centrifugal force is that with which a body while revolvi[illegible]



The centrif force, in the or tons, &c, as the case may be, $= \dfrac{\textit{Wt of the body} \times \textit{Square of in ft}}{\textit{Rad c n in ft} \times}$

Or,

$$\textit{The centrif force in lbs or tons, \&c.} = \frac{\textit{Wt of the body} \times \textit{Square of number of revs per min.} \times \textit{Rad c n in}}{2935.}$$

Ex. Suppose the body in either Fig 1, or Fig 2, to weigh 1.2 tons; the rad c n to [illegible] case the circumf will be 15.7 ft.; and suppose the body to make 100 revs per m[illegible] vel will plainly be $15.7 \times 100 = 1570$ ft per min; or $\frac{1570}{60} = 26.17$ ft per sec. T[illegible]

[illegible] mula,

$$\textit{Centrif force} = \frac{1.2 \times 26.17^2}{2.5 \times 32.2} = \frac{1.2 \times 684.9}{2.5 \times 32.2} = \frac{821.88}{80.5} = 10.21 \text{ tons}$$

or,

$$\textit{Centrif force} = \frac{1.2 \times 100^2 \times 2.5}{2935} = \frac{1.2 \times 10000 \times 2.5}{2935} = \frac{30000}{2935} = 10$$

* Centrifugal and centripetal force being always exactly equal to each other, it [illegible] rapidly revolving bodies sometimes break; and the fragments fly off? Does n[illegible] centrifugal force has become greater than the centripetal one? The answer is, No [illegible] gal force becomes precisely equal to the ultimate strength of the string, or of the [illegible] the string, stone, &c, can furnish no greater resistance, then *that force which is* [illegible] *ward*, causes the breakage; and at that instant both the centrifugal and centri[illegible] *tirely*; and the fragments of the body do not fly off in the direction which the c[illegible] but in a tangent, or at right angles to it: being carried in that direction by said [illegible]

† *When a t is* equal to ⅓ of a c, the rad thus found will be but $\frac{1}{4\,4}$ part too s[illegible] *about* $\frac{1}{1\,0}$ part too short; and when ⅔ of a c, about ⅛ part too short; and the re[illegible] *will consequently* be too small in the same proportion. But when, as is usually [illegible] *&c, a t is much less* than even ⅓ of a c, the error is not worth notice in pract[illegible]

‡ *These are not* strictly correct, not only because n is not in reality at th[illegible] *because the ring*, Fig 1, or the body, Fig 2, must be united to the center [illegible] *string*, or wire, *&c*: and the weight of these arms, &c, will slightly shorten [illegible] *Art 2*. But in practice this effect is usually too small to be regarded; or [illegible] *for is by guess.*

l character of limestone. The moisture of the air causes it to swell, and become lighter, bulk k.

pproximate averages, (which will be too great, or too small, according to the character and de- burning of the limestone; and to the fineness or coarseness of the sand,) one measure of good me, either in lump, or ground; if wet with about ¼ a measure of water, will within less than r, slack to about 2 measures of dry powder. And if to this powder there be added about ¾ more es of water, and 3 measures of dry sand, and the whole thoroughly mixed, the result will be 1½ measures of mortar. Or the same slacked dry powder, with about 1 measure of water, and ures of sand, will make about 5⅔ measures of mortar. In both cases the bulk of the mortar, about ⅓ part greater than that of the dry sand alone. If ¾ of a measure of water be used cking, the result, instead of a dry powder, will be about 1⅓ measures of stiff paste; or with 1 measure of water for slacking, the result will be about 1¼ measures of thin paste, of about the consistence for mixing with the sand. Very pure, *fat* limes, slack quickly, and make consider- ore than 2 measures of powder; while poor, *meagre* ones, require more time, and swell less. lacking, and small swelling, in case the lime has been properly burnt, are not in general bad ties; but on the contrary, usually indicate that it is to some extent hydraulic. In this case it a better mortar; especially for works exposed to moisture, or to the weather. Very pure limes e worst of all for such exposures; or are bad *weather-limes*; and in important works, should be used without cement. They require a greater proportion of sand than poor ones, to expedite ardening, to diminish their shrinking, and lessen their liability to crack in drying.

l lime appears to be about the same as that from the purest limestones; but that from chalk is ore inferior, and will not bear more than about 1½ measures of sand: its mortar never becomes ard. Madrepores (commonly called coral) appear to furnish a lime intermediate between those k and limestone. They require to be but moderately burnt.

average weight of common hardened mortar is about 105 to 115 lbs per cub ft.

me burning, a cub yard of the broken stone requires about ½ cord of good wood; or a ton of nous coal will burn 10 or 12 tons of limestone; making 6 to 7 tons of quicklime.

mon grout is merely common mortar made so thin as to flow almost like cream. It is intended interstices left in the mortar-joints of rough masonry; but unless it contains a large amount ent, it is probably entirely worthless; since the great quantity of water injures the properties ; and moreover, its ingredients separate from each other; the sand settling below the lime. s this, it will never harden thoroughly in the interior of thick masses of masonry: indeed, the ay probably be said of any common lime mortar. In such positions, it has been found to be ly soft, after the lapse of many years.

the sand and the water for lime mortar, should be free from clayey particles, and from salt. y may be removed by thorough washing; but it is extremely difficult to get rid of the salt eashore sand, even by repeated washings. Enough will generally remain to keep the work and to produce efflorescences of nitre on the surface; whether with lime, or with cement mortar. nd, however, and even sea water, may be used with hydraulic cement mortar, if absolutely ry; although it materially weakens it, and ruins it as grout. Good limestones lose on an e about .4 of their weight in burning.

ar should not be mixed upon the surface of clayey ground; but a rough board, brick, or stone m should be interposed. Pit sand sifted from decomposed gneiss, and other allied rocks, is ex- for mortar: its sharp angles making with the lime a more coherent mass than the rounded of river or sea sand. Mortar should be applied wetter in hot than in cold weather; especially kwork; otherwise the water is too much absorbed by the masonry, and the mortar is thereby t. To prevent this still further, it is well to dip each brick, for an instant only, into a bucket er; especially if they are dusty: for dust also impairs the adhesion.

e tenacity, or cohesive strength, that is, the resistance to a pull, ienced good common lime mortar, according to careful trials by Gen. Totten, of the U. S. Army, ortar only 6 months old, varies from 6 to 34 lbs per sq inch; or from 864 to 4896 lbs, or from .39 tons per sq ft. About 15½ lbs per inch; or 1 ton per ft, is near the average of the general's nce; and is quite as high as should be assumed in practice. It is true that his experiments that at the end of 4½ years, some of the mortars had acquired from 2 to 3 times greater h; but others in that time had increased but ⅕ part. Moreover, in practice the working strains nerally brought upon masonry by the time that some of its mortar is not more than 6 months even less: therefore, we cannot avail ourselves of the increased strength given by time; but increase is needed speedily, we must add cement.

e crushing force may be taken at 50 tons per sq ft; or 777 lbs per sq inch.

e sliding resistance, or that which common mortar opposes to any ending to make one course of masonry slide upon another, is stated by Rondelet, to be but 5 lbs uare inch; or about ⅛ ton per sq ft, in mortar 6 months old.

ansverse strength. Prisms 6 ins long, 2 ins square, and 4 ins between pports, made from equal measures of quicklime and sand, will, when 6 months old, break under r load of about 200 lbs. With more sand they are weaker; and with less, stronger.

e adhesion to bricks, or stones, may ordinarily be assumed to be at least to its cohesive strength, in properly executed work. But in practice it should not be exposed to han about ⅓ part of the above loads, or strains, in any of the cases. It is the opinion of some ractical authorities that the mortar will be better if the lime is kept either slacked to a dry r, or made into a paste, for about a week before adding the sand. The delay gives time for the ous small refractory particles which occur in nearly all limes, to slack and soften. Otherwise n so after the mortar has been laid some time; and thus produce in it a partial disintegration. s account ground lime, if fresh, is preferable to lump; the grinding breaks down these particles; mortar may be well mixed for a few days before being used, and then reworked. The powder ked lime may be kept good for several months, if closely packed in barrels, and kept dry. The th of all mortars depends to a much greater extent than is generally supposed, upon the thorough- f the mixing. In important works on a large scale, the lime should be finely ground into paste *ater, by a mill; and the sand* and cement then added gradually, and also mixed with the paste *mill.*

lacked lime makes a weak mortar; and an excess of water in slacking (or what is called *g the lime) is also injurious.* The best mode of slacking, so far as regards the quality of the

mortar, is by *sprinkling* the loose lump, or ground lime, with about ¼ to ⅓, by trial, of its of water, and then covering it with sand. Water should not be added after the lime begins

Art. 2. Bricks. In England, the size of bricks is fixed by law; and t should be the case in this country. For want of such a provision, gross frauds are perpetra persons are aware of the loss they sustain in purchasing small bricks; not only in the be also in the laying by the thousand; and in the increased quantity of mortar required. Th standard size is 8¾ × 4½ × 2¾ inches; which gives a content of 105¼ cub ins; or 12 dry b cub ft. In our Eastern States a common size is 8¼ × 4 × 2 ins; making 66 cub ins; or le of the British standard; or a British one contains more than 1⅙ U. S. ones. But bricks a 8 × 3¾ × 1¾ are not unusual; containing only 52½ cub ins. One of 8¼ × 4 × 2 is abo larger than this last; which in turn is precisely half as large as the British. A good hand brick of 8¼ × 4 × 2, will weigh about 4½ lbs; or 118 lbs per cub ft; or 1.4223 tons per cub bricks to a ton. A machine-pressed one weighs about 5 lbs. If immersed in water, either will in a few minutes absorb from ⅛ to ¾ lb of water; the last being about $\frac{1}{7}$ of the w hand-moulded one: or ⅛ of its own bulk. Since the weight of hardened mortar averages less than that of good common brick, we may for ordinary calculations assume the weig brickwork at 1.4 tons per cub yard; 1.3 tons per perch of 25 cub ft; or 116 lbs per cub ft; chine-moulded, at 1.56 tons per cub yd; 1.44 tons per perch; or 129 lbs per cub ft.

Allowing for the usual waste in cutting bricks to fit corners, jambs, &c, the average 8¼ × 4 × 2, required per square foot of wall is as follows:

Thickness of Wall.	No. of Bricks.
8¼ ins, or 1 brick	14
12¾ " or 1½ "	21
17 " or 2 "	28
21¼ " or 2½ "	35
25¾ " or 3 "	42

As a rude approximation, a bricklayer, with a laborer to keep him supplied with materi common house walls, lay on an average about 1500 bricks per day of 10 working hours. In outer faces of back buildings, from 1000 to 1200; in good ordinary street fronts, 800 to 1000 very finest lower story faces used in street fronts, from 150 to 300, depending on the number &c. In plain massive engineering work, he should average about 2000 per day, or 4 cu in large arches, about 1500, or 3 cub yds.†

Since bricks shrink about $\frac{1}{13}$ part of each dimension in drying and burning, the mould about $\frac{1}{13}$ part larger every way than the burnt brick is intended to be.

Good well-burnt bricks will ring when two are struck together.

At the brick-yards about Philadelphia, a brick-moulder's work is 2333 bricks per day; o week. He is assisted by two boys, one of whom supplies the prepared clay, moulding water; while the other carries away the bricks as they are moulded. A fourth person arr in rows for drying. About ¾ of a cord, or 96 cub ft of wood, is allowed per 1000 for burn

Paving with brick. In our cities this is done over a 6-inch layer of gr should be free from clay, and well consolidated. With bricks of 8¼×4×2 ins, with joints to ¼ inch wide, a sq yard requires, flatwise, as is usual in streets, 38 bricks; edgewise, 73 149. An average workman, with a laborer to supply the bricks and gravel, will in 10 hours 2000 bricks; or 53 sq yds flat, 27 edgewise, 13 endwise. When done, sand is brushed into

Art. 3. The crushing strength of bricks of course varies g rather soft one will crush under from 450 to 600 lbs per sq inch; or about 30 to 40 tons per sq first-rate machine-pressed one will require about 6222 lbs per sq inch, or 400 tons per ft. about the crushing limit of the best sandstone; ⅔ as much as the best marbles or limestor as much as the best granites, or roofing slates. But masses of brickwork crush under mu loads than single bricks. In some English experiments, small cubical masses only 9 inch edge, laid in cement, crushed under 27 to 40 tons per sq ft. Others, with piers 9 ins squa 3 ins high, in cement, only two days after being built, required 44 to 62 tons per sq ft to c Another, of pressed brick, in best Portland cement, is said to have withstood 202 tons per with common lime mortar only ¼ as much.

It must, however, be remembered, that cracking and splitting usually commence under half the crushing loads. To be safe, the load should not exceed ⅓ or $\frac{1}{10}$ of the crushing so with stone. Moreover, these experiments were made upon low masses; but the strength with the proportion of the height to the thickness.

The pressure at the base of a brick shot-tower in Baltimore, 246 feet high, is estimated

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* Air-slacking (also called *spontaneous* slacking,) is caused by the gradual absorption o from the air only: and, according to the climate; the quality of the lime; and whether i layers or in heaps, occupies from a month or two, to more than a year. It is attended by er no heating: and the powder occupies much less space than when slacked by water. Th inclined to attach but little *practical* importance to some of the super-scientific remarks tions given by authorities on mortar. He has slacked fresh-burnt lime to a powder, and mortar of a portion of it immediately. Other portions he has allowed to remain exposed for two weeks: at the end of which time he has *completely drowned it* in water, to form *a th* This he has also left standing for a week, stirring it occasionally, until the water had ev that barely enough remained for forming an ordinary mortar. It may be that delicate t have shown some difference between the mortars of this lime, and those made immedi *slacking*; but none could be perceived by the writer in breaking them by hand; or trying *of a file* upon them. In all cases he found brickdust to improve the mortar. See note † p 5

† *Prices* in Philada in 1873. Bricks alone: Salmon, or soft, $8 per 1000. Hard brick, *stretchers*, (generally used for the facings of back buildings, &c,) $18. Paving brick, $1 (for lower stories of first-class fronts,) $30.

Bricklaying: including mortar and scaffolding; averaging an entire dwelling, $8 w *pressed bricks* in first-class fronts, $15 to $20.

; and in a brick chimney at Glasgow, Scotland, 468 feet high, at 9 tons. Professor Rankine
es that in heavy gales this is increased to 15 tons, on the leeward side. The walls of both are
e much thicker at bottom than at top. With walls 100 feet high, of uniform thickness, the
e at base would be 5.4 tons per sq ft.
our present imperfect knowledge on this subject, it cannot be considered safe to expose even
as pressed brickwork, *in cement*, to more than 12 or 15 tons per sq ft; or good hand-moulded,
than two-thirds as much.
lwaukee, Wisconsin, a yellow brick is made, which, either alone or in combination with red
oduces a very agreeable architectural effect. Such bricks require a clay containing little or

e English rod of brickwork is 306 cub feet, or 11⅓ cub yards; and requires
500 bricks of the English standard size; with about 75 cub ft of mortar. The English *hundred*
, is a cub yd.
been doubted whether machine-pressed brick are as well adapted as hand-moulded ones, *for*
y building purposes, where they are subjected to but moderate loads. The first are certainly
ngest, smoothest, handsomest, and less absorbent; but they are also heavier, and therefore more
ve to haul and handle; while their smoothness prevents their taking so good a hold on the

ozen mortar. There is risk in using common mortar in cold weather. If the cold
continue long enough to allow the frozen mortar to set well, the work may remain safe; but if
day should occur between the freezing and the setting of the mortar, the sun shining on one
the wall may melt the mortar on that side, while that on the other side may remain frozen
In that case, the wall will be apt to fall; or if it does not, it will at least always be weak; for
that has partially set while frozen, if then melted, will never regain its strength. By the
own trials hydraulic cements seemed not to be injured by freezing.

**periments for rendering brick masonry impervious to
r.** Abstract of a paper read before the American Society of Civil Engineers, May 4, 1870,
liam L. Dearborn, Civil Engineer, member of the Society.
face walls of the Back Bays of the Gate-houses of the new Croton Reservoir, located north
ty-sixth Street, in Central Park, were built of the best quality of hard-burnt brick; laid in
composed of hydraulic cement of New York, and sand mixed in the proportion of one measure
nt to two of sand. The space between the walls is 4 ft; and was filled with concrete. The face
ere laid up with great care, and every precaution was taken to have the joints well filled and
good work. They are 12 ins thick, and 40 ft high; and the Bays when full generally have 36 ft
r in them.
the reservoir was first filled, and the water was let into the Gate-houses, it was found to filter
these walls to a considerable amount. As soon as this was discovered, the water was drawn
the Bays, with the intention of attempting to remedy or prevent this infiltration. After care-
nsidering several modes of accomplishing the object desired, I came to the conclusion to try
ster's Process for Repelling Moisture from External Walls."
process consists in using two washes or solutions for covering the surface of brick walls; one
ed of Castile soap and water; and one of alum and water. The proportions are: three-quar-
a pound of soap to one gallon of water; and half a pound of alum to four gallons of water;
betances to be perfectly dissolved in the water before being used.
walls should be perfectly clean and dry; and the temperature of the air should not be below 50
Fahrenheit, when the compositions are applied.
irst, or soap wash, should be laid on when at boiling heat, with a flat brush, taking care not
a froth on the brickwork. This wash should remain twenty-four hours; so as to become dry
rd before the second or alum wash is applied; which should be done in the same manner as
t. The temperature of this wash when applied may be 60° or 70°; and it should also remain
-four hours before a second coat of the soap wash is put on; and these coats are to be repeated
tely until the walls are made impervious to water.
alum and soap thus combined form an insoluble compound, filling the pores of the masonry,
irely preventing the water from penetrating the walls.
e applying these compositions to the walls of the Bays, some experiments were made to test
orption of water by bricks under pressure after being covered with these washes, in order to
ine how many coats the wall would require to render them impervious to water.
this, a strong wooden box was made, put together with screws, large enough to hold 2 bricks;
the top was inserted an inch pipe forty feet long.
is box were placed two bricks after being made perfectly dry, and then covered with a coat of
the washes, as before directed, and weighed. They were then subjected to the pressure of a
of water 40 ft high; and, after remaining a sufficient length of time, they were taken out and
d again, to ascertain the amount of water they had absorbed.
bricks were then dried, and again coated with the washes and weighed, and subjected to press-
before; and this operation was repeated until the bricks were found not to absorb any water.
oatings rendered the bricks impenetrable under the pressure of 40 ft head.
mean weight of the bricks (dry) before being coated, was 3¼ lbs; the mean absorption was
lf pound of water. An hydrometer was used in testing the solutions.
his experiment was made in the fall and winter, (1865,) after the temporary roofs were put on
Gate-house, artificial heat had to be resorted to, to dry the walls and keep the air at a proper
ature. The cost was 10.06 cts per sq ft. As soon as the last coat had become hard, the water
into the Bays, and the walls were found to be perfectly impervious to water; and they still re-
o in 1870, after about 6½ years.
ARCH (FOOTWAY OF HIGH BRIDGE). The brick arch of the footway of High Bridge is the
a circle 29 ft 6 in radius; and is 12 in thick; the width on top is 17 ft; and the length covered
81 ft.
irst two courses of the brick of the arch are composed of the best hard-burnt brick, laid edge-
mortar composed of one part, by measure, of hydraulic cement of New York, and two parts
The top of these bricks, and the inside of the granite coping against which the two to
of brick rest, was, when they were perfectly dry, covered with a coat of asphalt, one-half
k, laid on when the asphalt was heated to a temperature of from 360° to 510° Fahrenh
of this was laid a course of brick flatwise, dipped in asphalt, and laid when the asphalt
the joints were run full of hot asphalt

On top of this a course of pressed brick was laid flatwise in hydraulic cement mortar, forming the paving and floor of the bridge. This asphalt was the Trinidad variety; and was mixed with 10 per cent. by measure, of coal tar; and 25 per cent of sand. A few experiments for testing the strength of this asphalt, when used to cement bricks together, were made, and two of them are given below.

Six bricks, pressed together flatwise, with asphalt joints, were, after lying six months, broken. The distance between the supports was 12 ins; breaking weight, 900 lbs; area of single joint, 22½ sq ins. The asphalt adhered so strongly to the brick as to tear away the surface in many places.

Two bricks pressed together end to end, cemented with asphalt, were, after lying 6 months, broken. The distance between the supports was 10 ins; area of joint, 8½ sq ins; breaking weight, 150 lbs.

The area of the bridge covered with asphalted brick, was 23065 sq ft. There was used 24200 lbs of asphalt, 33 barrels of coal tar, 10 cub yds of sand, 93800 bricks.

The time occupied was 109 days of masons, and 148 days of laborers. Two masons and two laborers will melt and spread, of the first coat, 1650 sq ft per day. The total cost of this coat was 5.5 cents per sq ft, exclusive of duty on asphalt. There were three grooves, 2 ins wide by 4 ins deep, made entirely across the brick arch, and immediately under the first coat of asphalt, dividing the arch into four equal parts. These grooves were filled with elastic paint cement.

This arrangement was intended to guard against the evil effects of the contraction of the arch in winter; as it was expected to yield slightly at these points, and at no other points; and then the elastic cement would prevent any leakage there.

The entire experiment has proved a very successful one, and the arch has remained perfectly tight.

"In proposing the above plan for working the asphalt with the brickwork, the object was to avoid depending on a large continued surface of asphalt, as is usual in covering arches, which very frequently cracks from the greater contraction of the asphalt than that of the masonry with which it is in contact; the extent of the asphalt on this work being only about one-quarter of an inch to each brick. This is deemed to be an essential element in the success of the impervious covering."

A cheap and effective process for preventing the percolation of water through the arches of aqueducts, and even of bridges, is a great desideratum. Many expensive trials with resinous compounds have proved failures. Hydraulic cement appears to merely diminish the evil. Much of the trouble is probably due to cracks produced by changes of temperature.

Art. 4. Hydraulic cements. Certain limestones, when burnt, will not slack with water; but when the burnt stone is finely ground, and made into a paste, it possesses the property of hardening under water; and is therefore called *hydraulic cement*. So long as the proportion of those ingredients which impart hydraulicity, is so small that the burnt stone will slack; but still make a paste or a mortar, which will harden under water, it is called *hydraulic lime*. This does not harden so promptly, or to so great a degree, as the cements. Hydraulic limes slack more slowly, and swell less, in proportion to their hydraulicity; some requiring many hours. Artificial hydraulic limes and cements, of excellent quality, may be made by mixing lime and clay thoroughly together; then moulding the mixture into blocks like bricks; which are first dried, then burnt, and finely ground. The celebrated artificial English Portland cement,* is made by grinding together in water chalk and clay. The fine particles are floated away to other vessels, and allowed to settle as a paste; which is then collected, moulded, dried, burnt, and ground.† Others use slacked lime and clay, similarly prepared; the proportions with different clays and limes, varying as much as from 1 measure of clay to 5 of lime, to 1 of lime to 2 of clay. Whatever may be the required proportion, ultimate success will depend entirely upon ascertaining by trial the proper degree of burning. The Vicat cement, much used in France, is of this kind. It is slow setting, and becomes very hard in a few months.

Almost any pure gray clay, free from sand, will answer. **Trass**, and **Pozzuolana**, found in many countries, consist of common rocks altered by volcanic heat. Their composition, and consequently their hydraulic properties, vary considerably. They require no burning, but grinding only; and must all be mixed with lime; without which they are useless. Sand also is generally added. When they are of good quality, equal measures of the powder, lime, and sand, make an excellent hydraulic mortar.

The "Natural Boulogne Portland cement," of the French, is made by burning and grinding a paste made from a peculiar natural mixed deposit of lime and clay, easy to excavate. It sets slowly; so that it may be made into mortar for several hours before it is to be used; and then restirred.

The French apply the name of cement also to fine dust of ground bricks, or burnt clay.‡ Like trass and pozzuolana, it must be mixed with lime; while other cements are best without it; although it is often expedient, for economy, to add it. This brickdust mortar is decidedly, but not very strongly, hydraulic; and in the absence of better, may be used in damp situations; or even in wet ones, especially if a day or two be allowed it to harden somewhat before the water is admitted. It is not, how-

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* **Price of Hydraulic cements** per barrel in Philada in 1873. French, Richards & Co, 1001 Market St, Philada, are large dealers in both foreign and American cements. English Portland, per barrel of 400 to 430 lbs net, $5 to $7, according to quantity ordered, fluctuations in market, &c. English Roman, averages about 350 lbs net, $4.50 to $6.50. Rosendale, about 300 lbs net, $2 to $2.25. Coplay, about 300 lbs net, $1.50 to $2. The choicest English Portland is known as "heavy best tested." J. B. White & Co; Knight, Bevan & Sturges, &c; are well known makers of English Portland.

‡ The writer's own experiments induce him to regard fine brickdust as a very good substitute for hydraulic cement in some cases, when the latter is not procurable. He made 1 measure of ground quicklime into mortar, with 5 measures of sand. A portion of this was formed into cakes about ½ inch thick, by 2 ins diam. To other portions were added very various proportions of brickdust, with water enough to still form a paste of about the consistency of soft putty. Similar cakes were made of these; and after being allowed to dry somewhat for two days, all were placed under water. All those containing brickdust, gradually hardened. At the end of a month it was difficult to break some of them with the fingers; and after being immersed 4 months, pieces of many of them, ½ inch thick, bore 1500 lbs per sq inch, without the slightest crumbling, crushing, or splitting. Those without brickdust had either gone to pieces in the water, or crumbled with perfect ease between the fingers. Much hydraulic cement gives worse results than our brickdust. We have no doubt that the addition of even as little as $\frac{1}{10}$ part as much brickdust as sand, in our common mortars, would prevent that disintegration so generally seen in the mortar of masonry along our public works; and which is a disgrace to those who permit it. Experiments should be made as to the proper degree of burning, and of the proper proportions to be used, &c.

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OOTWAY OF HIGH BRIDGE). The brick arch of the footway of High Bridge is the *ft 6 in radius; and is 12 in thick*; the width on top is 17 ft; and the length covered

rses of the brick of the arch are composed of the best hard-burnt brick, laid edge- posed of one part, by measure, of hydraulic cement of New York, and two par- f these bricks, and the inside of the granite coping against which the two r was, when they were perfectly dry, covered with a coat of asphalt one-half hen the asphalt was heated to a temperature of from 360° to 518° Fahren- laid a course of brick flatwise, dipped in asphalt, and laid when the asphal re run full of hot asphalt.

relied on.* Gen Gillmore says: "We infer from the foregoing results that a thin paste of Rosen cement is worthless for concrete if mixed up with sea-water; while, with fresh water, it will ha when injected under water, either fresh or salt: and affords a means of submarine constructions may be of great value under certain circumstances." Salt sand or water also produces on the su of the work, efflorescences of either nitre (saltpetre) or other injurious substances.

These occur indeed very generally, and to an injurious extent, even when the cement is pure, mixed with fresh water. This effect has received the technical name of *nitrification;* and is espe to be observed on the faces of brickwork in cement; particularly when exposed to the salt air the sea. Although most abundant at the joints, still it frequently disfigures seriously the entire face of brick walls. On masonry, especially of very hard compact stones, its bad effects are much apparent, and more confined to the joints. Bricks and porous stones, absorb from the morta salts which produce these efflorescences, and thereby become softened, and disintegrated. The jurious action on masonry resembles strongly that of frost. Gen Gillmore states, as the result own experiments, the important fact, that they may probably be prevented entirely, by mixing wit cement, lime with which *fat* has been incorporated during the process of slacking. The propor which he suggests, are 8 to 10 lbs of the fatty substance, to 100 lbs of quicklime, and 300 lbs (1 bu of dry cement; and in recommendation of this process, he adds: "We here take occasion to call attention to the fact that lime paste may be added to cement paste, in much larger quantities is usually practised in important works, without any considerable loss of tensile strength, or l ness. There is no material diminution of strength until the lime paste becomes nearly equal to of the cement paste; and it may be used within that limit without apprehension, under the mos favorable circumstances in which mortars can be placed." The lime, as before stated, shou thoroughly slacked for some days before it is added to the cement; and when procurable, fresh-gr quicklime is much to be preferred to lump lime Freezing does not seem to injure cement.

Protection from moisture, even that of the air, is very essential for preservation of cements, as well as of quicklime. On this account the barrels are generally with stout paper. With this precaution, aided by keeping the barrels stored in a dry place, r above the ground, the cement, although it may require more time to set, will not otherwise appreciably deteriorate for six months; but after 14 or 16 months, Gillmore says it is unfit for u important works. But in lumps, kept dry, it will remain good for 2 or 3 years; and may be gr as required for use.

Good Portland cement is stated by good authority rather to improve by free exposure to th under cover; but whether this is correct or not, we cannot say.

Restoration by reburning may be effected. If the injured ground ment is spread in a thin layer, on a red-hot iron plate, for about 15 minutes, its good qualities w in a great measure restored. The time should be ascertained by trial. If it has been actually and lumpy, or cemented into a mass, it should first be broken into small pieces, and then ground these pieces may be first kiln-burnt at a bright red-heat for about 1½ hours; and then ground.

Art. 5. For roughcasting, or stuccoing the outside of walls, very hydraulic cements are fit. Mr. Downing, in his work on "Country Houses," excepts that from R Connecticut, as the only one within his extended knowledge, that is suitable. Portland cem said to be good for that purpose. A wall with a northern exposure in Philada was coated with 1860; and appears to be in perfect condition in 1873.

The limestones from various parts of the State of New York, which furnish most of the cement in the United States, have a very fine grain: either little or no lustre: a great variety in color vary much in their composition; not only in distant localities, but even in the same quarry.

Carbonic acid	25 to 41	per ct	by weight.
Lime	25 to 40	"	"
Magnesia	12 to 22	"	"
Silica	9 to 28	"	"
Alumina	2 7 to 19	"	"
Peroxyd of iron	1.1 to 3.3	"	"

with traces of bituminous matter, and water. In some the iron is absent. Silica and alumin the constituents of clay: and the silica is usually considered to be the active agent in producin hydraulic property. Common quartz, and transparent sand, are nearly pure silica; but in the c tion of separate grains, or particles, it does not enter *chemically* into the composition of cem even when so found in the limestone.

Beck, in his "Mineralogy of N. York," says, the Rosendale limestones are broken into pieces the size of a fist; and calcined in perpetual kilns; in alternate thin layers with finely brok screenings of anthracite. The lime is drawn from the kiln at intervals of 12 hours; each having been calcined about 3 days. Gillmore says that about 35 lbs of anthracite are require barrel (300 lbs) of cement.

Cement mortar. If 1 measure, slightly compacted by shaking, of gr cement, be mixed with about ½ of a measure of water, it forms about ⅔ of a measure of pas for mortar, by itself. Perfectly fresh cements require a little more water than old; and cements among themselves as to the proper quantity of water. If sand is to be added, more water will of c be needed; but this should be added in very small quantities, as the mixing or tempering goe inasmuch as a much less quantity is required than would at first sight be supposed. So also, c addition of lime. As before remarked, the pure cement is stronger without any addition wha of either lime or sand; still, it will be quite strong enough for most ordinary purposes, espe when not exposed to water, even with considerable additions of both. But if it is to be expo absolute contact with water, lime should be added but sparingly, if at all, in the outer joints. the sand is in the proportion of 1 or more measures, to 1 of cement, the bulk of mixed mortar w about equal to, or a trifle less than that of the dry sand alone.

The cement mortar of the Croton Aqueduct of New York, was as follows:

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* *Recent* experiments in Prussia, showed that sea-water answered admirably for some cem *Also, that certain* cements which set rapidly with sea-water, afterward fell to pieces when pu *sea-water for a few* hours. Most of the cements experimented with were found to be adulter *ashes, clay, sand, &c.* Would it not be well for honorable manufacturers to publish certain *which* the purity of their cement could be ascertained by the purchaser?

For the brick inside lining of the aqueduct, 1 measure cement powder; 2 measures sand.
For the stone backing ..... 1 " " " 3 " "

When mortar is to be exposed to dampness only, we may use, cement 1, quicklime 1, sand 4 ; a lime should be thoroughly slacked, before it is added.

Quantity required. A barrel of cement, 300 lbs; and 2 barrels of sand (cubic, or 7½ cub ft;) mixed with about ½ a barrel of water, will make about 5 cub ft of mor Ekient for

192 sq ft of mortar-joint				½	inch thick.	
288 "	"	"	"	⅜	"	"
384 "	"	"	"	¼	"	"
768 "	"	"	"	⅛	"	"

r, to lay 1 cubic yard, or 522 bricks of 8½ by 4, by 2 ins, with joints ⅜ inch thick; or a cubic y roughly scabbled rubble stonework. The quantity of sand may be increased, however, to 3 measures for ordinary work.

Pointing mortar. Gen Gillmore recommends "1 part by *weight* of g ment powder, to 3 or 3½ parts of sand. To be mixed under shelter, and in quantities of only plats at a time, using very little water, so that the mortar, when ready for use, shall appear ra incoherent, and quite deficient in plasticity. The joints being previously scraped out to a dept t least ½ an inch, the mortar is put in by the trowel; a straight-edge being held just below the j f straight, as an auxiliary. The mortar is then to be well calked into the joint by a calk ron and hammer; then more mortar is put in, and calked, until the joint is full. It is then rul nd polished under as great pressure as the mason can exert. If the joints are very fine they sh n enlarged by a stonecutter, to about $\frac{3}{16}$ inch, to receive the pointing. The wall should be well efore the pointing is put in, and kept in such condition as neither to give water to, nor take it from mortar. In hot weather, the pointing should be kept sheltered for some days from the sun, so as to dry too quickly."

Art. 6. Concrete is merely a coarse mortar of lime, sand, and gravel, or bro stone. Engineers generally apply to it the French name of *Beton*, when cement is used inste common lime. The plain terms lime concrete, and cement concrete, appear to us preferable. Althe lime-concrete is certainly a useful material under certain circumstances, yet, when it was first i duced into England, exaggerated ideas of its merits became prevalent; which experience has b means sustained. Ground quicklime is first mixed with the sand, gravel, &c; then water is adde slack the lime; and the whole is quickly mixed together; and as soon as possible, while hot, thr from a height of several feet, into the trench prepared for it. Here it is left to harden, and thu form the foundation of the intended superstructure. When first mixed and deposited, the con occupies considerably less bulk than that of its dry materials; but in setting, it swells permane about $\frac{1}{30}$ part of its thickness. This last property has been supposed to render it peculiarly valu for underpinning; but as it also renders the concrete porous, and friable, the argument has little force.

A common proportion among English engineers is 1 measure of ground quicklime, 1½ of water, to 8 of gravel. Broken stone is often added; and, still better, fragments of brick. Every 11½ ds of gravel makes about 1 cub yard of concrete. In using concrete, the entire width of the fou ion trench should be filled with it, and it should be well rammed in layers about a foot thick, as eposited. By this means it is made to act as pure sand, gravel, or broken stone do; which it is known make a firm foundation when so treated, if not exposed to water. As lime concrete is st ut a weak material, it should extend well beyond the base of the masonry above it. It is doul whether in thick masses it is any better *as a foundation* than it would be without the lime; for f ations are almost invariably exposed to dampness, and therefore there is little probability that me in the interior ever becomes hard. It is also probable that the lime should be perfectly sla efore the other materials are added; and that the throwing of the concrete into its trench wi low to consolidate it, has rather the opposite effect; and is greatly inferior to ramming it in lay he rounded pebbles of gravel, although they answer very well, do not make as firm a concret mall angular fragments of stone; and fragments of brick, or burnt clay, are far better than eit

But when cement is used instead of lime, either in whole, or in great part, the chief objection concrete cease. It then becomes a truly valuable material; and may be substituted for masonr and, or in fresh or salt water; not only in foundations, but in many other positions in which ery would be much more expensive; such as arches, domes, &c.

The dome of the Pantheon at Rome. 142 feet in diam, and now nearly years old, is of concrete; also the arches of the railroad viaducts, Pont Napoleon, and Pont d A at Paris. The first has 5 segmental arches of 115 ft span, and 14¾ ft rise; while the central ell rch of the latter has 141.4 ft span, and 28.2 ft rise. Both are built of small rough quarry spa well bedded in pure good cement. Many smaller arched bridges have been built entirely of cor concrete, and with perfect success. Its most extended application, however, is as filling for th erior of massive masonry of brick, or stone; or, when moulded into large blocks, for breakwaters. For such purposes, not only small fragments, but large ones, may be mixed into the concrete thoroughly bedded into it as it is deposited, or moulded, as the case may be. As little water as sible should be used in the mixing; barely enough to render the mass coherent. When to be use ers water, common lime should not be mixed with it, even in small quantity. Marine works grand scale, have failed from an ignorance of this fact. There exists, however, a great differen opinion on this subject.

Since even concrete mixed with pure cement cannot withstand the action of waves, until it has time to harden, it should be formed into blocks for some months before being so exposed. "A harbor of refuge at Dover, England, Walker & Burgess, engineers, the exterior facing of the sea ouary was of stone; but the interior consisted of moulded blocks of 6 to 10 tons weight each, composed of 1 measure of dry Portland cement (White's), to 10 measures of mixed sand and gr This soon became very hard. Within 3 or 4 days they could be lifted and placed under cover. W of but 3 *or 4 tons, only 10 days after being* moulded, could be lifted by bowlines sunk only a inches *the mass. Generally, however, the bowline* holes were moulded in the blocks at the time of *them, and extended through ⅓ of their depth.* The blocks were left to harden for from 6 to 9 *were being put under water.* The large blocks of 112 cub ft (nearly 5 ft on each edge) c

cub ft of dry cement, and 136 cub ft of dry gravel. Hence their bulk was but little more than that of the dry gravel; or, in other words, the shrinkage was nearly $\frac{1}{5}$. At Alderney harbor, by same engineers, there being plenty of stone, the moulded blocks were composed of 1 mess of Wh Portland cement, to 2 of sharp sand, and 4 of coarse and fine gravel. These being mixed into a tar, were thrown into the moulds, 8 or 10 bushels at a time, so as to form a complete layer. flat spawls of stone, from 3 to 20 ins long, were imbedded; and so on, until the mould was full. quantity of stone thus incorporated, was about 40 per cent of the entire bulk; and the proporti dry cement, to the entire bulk, was about 1 to 10."

"**At the French harbor at Cherbourg,** the blocks of 12 ft, by 9 ft, by containing 712 cub ft, and weighing 52 tons, were *built* up of mortar and stone, like rubble mas without moulds. The mortar contained 1 mess of Portland cement powder, to 3 of sand; and pied from .3 to .4 of the entire mass. Blocks of this kind, 9 months old, showed a compressive str of 113 tons per sq ft; which is but little inferior to that of Portland stone. Their cohesive fors about 200 lbs per sq inch. After becoming hard, they were slung to pontoons, and thrown in into the sea. Some of the 52-ton blocks have in heavy gales been thrown up from the bottom sea, (30 ft deep,) and lodged on top of the breakwater, entirely uninjured."

In making concrete, the proportions of the several ingredients should be such that no voids (a cant spaces) shall exist between them. To ascertain the amount of each material to accomplish we may proceed as follows:* Suppose we are going to make a concrete of stone spawls, gravel,

* It is probable that mistakes have occurred from inadvertently assuming that because the vo a broken mass, constitute a certain proportion of the bulk of said mass; therefore, the original has swelled in only that same proportion. Thus, if a solid cubic yard of stone be broken into irregular pieces, which have among themselves about the same proportions of large and small ca usually occurs in quarrying, or in railroad rock-cuttings; and if these be loosely thrown into a the .47 of this heap, or rather less than half of it, will be voids. But it does not follow, therefore, the original solid cub yd has swelled only .47, or nearly one-half, or makes only 1½ cub yds of b stone; although many young engineers would probably consider this a very full allowance; and suppose that they were quite just to the company, if they counted for the contractor one solid of excavation for every 1½ yds of fragments. Now, it is plain that if .47 of the broken heap are the remaining .53 must be stone. But these .53 constituted the original solid cubic yard; and still remain equal to it in actual solidity. Hence we must say as follows: If .53 of the broken occupies one cub yd of actual space, how much space will the whole mass occupy; or,

Of the broken mass.		Cub yd of space.		Entire broken mass.		Cub yds of space.
.53	:	1	::	1	:	$\frac{1 \times 1}{.53} = 1.9$

Hence, we see that a solid cub yd of stone, when so broken, swells to 1.9, or nearly 2 cub yds; hence a proper allowance to a contractor, would be 1 cub yd solid, for every 1.9 cub yds of piece the yds of pieces must be divided by 1.9 for the solid yards.

If we know that a cubic yard of any stone, breaks to, say 1.9 yds, then to find the proportion voids, and solid, in the broken mass, proceed thus: The solid part of the broken mass must occu cub yd of space; and the question is what part of 1.9 yds does this 1 yd constitute. The answ $\frac{1}{1.9} = .53$; therefore, 53 hundredths of the broken mass is solid; and of course the remaining 47 dredths are voids.

If a cubic foot solid weighs N lbs; but when broken up, or ground, only *n* lbs *per cub ft*, th divided by N, will be the proportion of solid in the broken mass.

If the broken stone is loosely piled up, it will occupy a little less space, say about 1.8 cub yds which case the voids will be .44; and the solid, .56 of the mass. We will here venture to expres doubts whether hard rock when blasted and made into embankment, settles to *less* than 1½ yd every solid yd. Mr Ellwood Morris gives as the result of certain embankment of hard sandst made under his supervision, an increase of bulk of $\frac{5}{12}$; or in other words, that 1 cub yd of ro place, made 1 $\frac{5}{12}$, or 1.417 yds of embankment. This corresponds to very nearly .7 solid; a voids: while 1½ yds to 1 solid, corresponds to .6 solid; and .4 voids. The rough sides of rock exc tions, make it difficult to measure them with accuracy; and we cannot but suspect that somet of this kind has interfered with the results obtained by Mr Morris. He, however, may be right, we wrong.

By some careful experiments of our own, an ordinary pure sand from the sea shore, perfectly and loose, weighed 97 lbs per cub ft; and its voids were .41, and the solid .59 of the mass. By there shaking, and jarring, it could be settled the .1333 part, (halfway between $\frac{1}{7}$, and $\frac{1}{6}$,) and no m It then weighed 112 lbs per cub ft: and its voids were then .32; and the solid, .68 of the mass.

Another pure quartz sand, of much finer grain, perfectly dry and loose, weighed but 88 lbs per ft; the voids were .466; and the solid .534 of the mass. By thorough shaking and jarring it c be reduced; like the former, only the .1333 part; it then weighed 101.6 lbs per cub ft; and its v were .384; and the solid .616. Another, consisting of the finest sifted grains, of the last, weigh lbs per cub ft; so that its voids, and solid, each were very nearly .5 of the mass. This could be c pacted about ⅙ part; and then weighed 98½ lbs per cub ft.

The first, or coarsest of these sands, when quite moist, but not wet, perfectly loose, weighed b lbs per cub ft; or 11 lbs less than when dry. It could be rammed in thin layers, until it settled fifth part; and then weighed 107½ lbs per cub ft. Voids .348,† solid .652.

The second sand, similarly moist and loose, weighed but 69 lbs per cub ft; or 19 lbs less than w *dry*. It could be rammed in thin layers, until it settled ¼ part; and then weighed 103½ lbs per *ft. Voids .373,† solid .627.*

None of these sands when dry, and loose, if poured gently into water to a depth of 15 inches, *tled more than* about one-fifteenth part; the coarsest one, considerably less.

† *The voids* would in fact be a little greater, and the solid a little less, if we made allowan *the water necessary* to produce the dampness. The specific gravity of each of the sands wa *or that of pure quartz.*

cement or lime. Now a cubic yard of stone spawls contains about ⅓ a cub yd of voids. amount is a trifle less; but it is better for the concrete, to be a little in excess in this par- e cub yard of stone fragments, then, will require half a cub yard, or 13½ cub ft, of gravel, ids. But these 13.5 ft of gravel themselves contain voids equal to .37 (or more than ⅓ ir bulk. And these are to be filled with sand; of which evidently we must use 13.5 × .37 = 5 85 (or not quite $\frac{1}{2}$) of the entire cub yd of concrete. But again, about .333 (or ⅓ part) , or say 1.7 cub ft of it, are voids; and must be filled with the paste of either cement, or two mixed. If the paste is to be made of pure cement and water, it will occupy but about ace of the dry powder; or the dry powder required to form 1 meas of paste, will be 1½ at for 1.7 cub ft of paste, we require 1.7 × 1.5 = 2.55 cub ft of dry cement; or .095 (say entire cub yd of concrete. But if the paste is to be made of lime, one meas of such paste ly about .8 of a meas of quicklime; for while cement shrinks about ¼ part when made, quicklime swells about ¼ part. Therefore, for 1.7 cub ft of lime paste, we shall require 1.36 cub ft of quicklime; or .0504 (say $\frac{1}{19}$) part of the entire cub yd of concrete. Expe- s that it is better to increase somewhat the proportions of the smaller ingredients; espe- concrete is not to be rammed. Ramming greatly increases its strength.

7. Gen Totten, in his work on mortars, gives the following crete, which he used with perfect success where "springs of water flowed over the work , and were allowed to cover each day's work. The next morning the concrete was always , and perfectly set." It was rammed as it was deposited. When not to be rammed, he what increase the proportions of all the ingredients except the stone fragments; to insure f all the voids between these last.

measures of good New York hydraulic cement powder.
" of sand.
" of granite fragments of nearly uniform size, and about 5 ounces weight.
" of water, nearly.

e a little more than 4 meas of concrete; or about the same as the granite fragments alone; rrel of cement (300 lbs, or 3 packed bushels) made $16\frac{7}{10}$ cub ft, or nearly .62 cub yard of r a cub yd of the concrete required 1.61 barrels of cement. The General adds that if one- cement had been omitted, and its place supplied by quicklime in about the following pro- work would still have been very hydraulic, and very strong:

.6 measures of cement.
.4 " " quicklime.
2.0 " " sand.
4.0 " " granite fragments.
.5 " " water, nearly.

as of quicklime to be *thoroughly* slacked, before being mixed.
ives the following, as forming a very hard concrete, when rammed.

1 measure good N. York cement powder.
1¼ " sand.
2.3 " clean gravel.
.33 " water.

8 meas give nearly 2⅔ meas of concrete; or a trifle more than the gravel alone. rammed concrete "became very hard, but was rather too incoherent while fresh, to make ttious stone."

1 meas good N. York cement powder.
1.88 " sand.
3. " clean gravel.
0.38 " about water.

ortions of cement, sand, and water, should be slightly increased, for unrammed concrete. cement is quick setting, ramming (if done at all) should be done at once, and quickly. entire quantity is small and can all be applied at once, as in filling a small mould, it is but when the work is carried up in layers deposited at short intervals, it would be ruinous, very slow-setting cements. Lime retards the setting; and in that respect may at times ful. As concrete cannot well be rammed under water, lest the water should carry off the m the gravel, &c. the rake is chiefly depended on for levelling it, after it has been but the concrete will be much weaker than if rammed.

oncrete used on the Croton Aqueduct, N. York, con- of

1 meas of good N. York cement powder.
3 " clean sand.
3 " hard stone, broken to pass through a ring 1½ ins diam.

y good concrete is composed of

1 measure cement powder.
1¼ " clean sand.
2¾ " gravel.
0.35 about water.

measures give about 4½ of concrete."

ollowing brickdust hydraulic concrete has been adopted ess in some important French works:

1 meas quicklime, slightly hydraulic.
1¼ " brick, or tile dust.
1¼ " sand.
3 nearly, broken stone.
water, unknown.

measures gave about 5½ of concrete. This concrete was impervious to water.

Another, which, "after 3 months' immersion in salt water, sustained a pressure of 116 tons per sq ft, without impression."

1 meas of powder of common lime, *slacked*.
1 " brickdust.
1 " sea sand.

These were made into mortar with water; and this mortar was mixed with twice its own volume of gravel: giving a bulk of concrete equal to about $1\frac{1}{7}$ bulks of the gravel alone.

More extended experience shows, however, that brickdust mortars are not fit for sea water. They may harden, and stand well for a few years; but after that time they begin to disintegrate and crumble away.

Art. 8. The strength, &c. of cement. The strength, hardness, and adhesion to brick or stone, are much greater than in lime mortar. The cohesion, or strength to resist a pull, averages from 40 to 75 lbs per sq inch; or 2.5 to 4.8 tons per sq ft, in good Rosendale cement, without sand; 6 months after being laid. It is sometimes 30 to 50 lbs within a month. The Portland cement of Francis & White, of London, when fresh, and of choicest quality, is stated to have a cohesion of 100 lbs per sq inch within 7 days; and of 400 lbs within 3 months. Its *adhesion* to bricks and stones seems, however, to be far less. Many spurious and inferior articles are sold under the name of Portland cement.

Prisms 6 ins long; 2 ins square; and 4 ins between the supports; of good fresh Rosendale cement; 10 months after they were made, broke with the following loads suspended at their centers. The mortar of which they were made, was mixed with the smallest possible quantity of water. When more water was added so as to make a thin mortar, the strength of the prisms became much reduced.

Measures of Cement Powder.	Measures of Sand.	Load in Lbs.
1	2	450
1	1	600
2	1	750
2½	1	850

Crushing strength of bricks made of best English Portland, 3 and 9 months old. Pure, 243 and 385 tons per sq ft. With 1 part sand, 160 and 292 tons. With 5 parts sand, 61 and 108 tons.

Experiments by the writer lead him to believe that **cement is not injured by freezing** after being laid in a work.

Results by Gen Gillmore.

Adhesion of Rosendale cement to brick and granite.

The bricks were cemented together wet: and well pressed together by hand. They were wetted with fresh water every other day, for 29 days; and then submitted to trial. Each result is the average of 5 trials.

			Adhesion per square inch.	
			To Croton Bricks.	To very fine cut Granite.
			Lbs.	Lbs.
Pure cement paste	1 meas.	Sand 0 meas.	30.8	27.5
Cement powder...	1	1	15.7	20.8
" "	1	2	12.3	12.6
" "	1	3	6.8	9.3
" "	1	4	5.2	7.9
" "	1	5	4.3	
" "	1	6	3.3	

Transverse strength of cement mortars, made into prisms 2 ins square, by 8 ins long; in vertical moulds, under a pressure of 32 lbs per sq inch; and broken on supports 4 ins apart, by a pressure at the center. The prisms were kept in sea water after the first 24 hours; and were 320 days old when broken. The breaking weights are the averages of many trials. The cement was measured in powder.

Gen Gillmore.	Pure Cement	Cement 1. Sand 1.	Cement 1. Sand 2.
	Lbs.	Lbs.	Lbs.
English Portland, artificial	1536	1280	950
Cumberland, Maryland	954	920	556
Newark and Rosendale	841	540	500
Delafield & Baxter's Rosendale	896	692	532
Hoffman, Rosendale	849	607	
Lawrence, Rosendale	777		
Round Top, Maryland	.....	600	
Utica, Illinois	732	756	562
Shepherdstown, Virginia	747	618	450
Akron, New York	764	651	603
Kingston and Rosendale	720	556	500
Sandusky, Ohio	554	464	
James River, Virginia	.....	628	628
Roman cement, Scotland; probably injured	563	.....	360
The following were broken when one year old:			
Lawrenceville Manuf Co, Rosendale	.....	910	
Sandusky, Ohio	962		
Kensington, Connecticut	954	709	566
Lawrence Cement Co, Rosendale, Hoffman brand	875	911	
Round Top, Maryland	.....	840	

t be supposed that cement concrete attains the solidity and hardness of granite, or lime- f good quality, and properly deposited, either under water or in the air, it will become the medium quality of sandstones, or brick.

ete is good for bringing up an uneven foundation to before starting the masonry. By this means the number of horizontal masonry is equalized, and unequal settlement is thereby prevented.

ete may readily be deposited under water in the usual ring it, soon after it is mixed, in a V shaped box of wood or plate-iron, with a lid that d while the box descends. The lid however is often omitted. This box is so arranged hing bottom a pin may be drawn out by a string reaching to the surface, thus permitting oping sides to swing open below, and allow the concrete to fall out. The box is then refilled. In large works the box may contain a cubic yard or more, and should be sus- a travelling crane, by which it can readily be brought over any required spot in the concrete may if necessary be gently levelled by a rake soon after it leaves the box. Its and strength will of course be impaired by falling through the water from the box; and cannot be rammed under water without still greater injury. Still, if good it will in due sufficiently strong for all engineering purposes.

rea upon which it is deposited must previously be surrounded d of enclosure, to prevent the concrete from spreading beyond its proper limits; and to uild to give it its intended shape. This enclosure must be so strong that its sides may d outwards by the weight of the concrete. It will usually be a close crib of timber or ithout a bottom; and will remain after the work is done. If of timber it may require an cells, to be filled with stone or gravel for sinking it into place. Care must be taken to ecape of the concrete through open spaces under the sides of the crib or enclosure. To crib may be scribed to suit the inequalities of the bottom when the latter cannot readily ff. Or inside sheet piles will be better in some cases; or an outer or inner broad flap of ay be fastened all around the lower edge of the crib, and be weighted with stone or gravel place on the bottom. Broken stone or gravel or even earth (the last two where there is heaped up outside of a weak crib will prevent the bulging outwards of its sides by the the concrete. After the concrete has been carried up to within some feet of low water, off, the masonry may be started upon it by means of a caisson (page 316); or by men in es. Or if the concrete reaches very nearly to low water, a first deep course of stone may the work thus brought at once above low water without any such aids.

oncrete should extend out from 2 to 5 feet (according to yond the base of the masonry. **All soft mud should be removed** iting concrete. **Bags partly filled with concrete,** and merely thrown er may be useful in certain cases. If the texture of the bags is slightly open, a portion it will ooze out, and bind the whole into a tolerably compact mass. Such bags, by the aid ay be employed for stopping leaks, underpinning, and various other purposes, that may selves.

uslin may be spread over deep seams in rock to prevent oncrete; and in some cases, to prevent it from being washed away by springs.

t is much room for judgment in the various applications of specially under water.

r the head "Foundations" will be found some useful hints on ns, &c.

r Alexander's mode of using cement under water. Gen scribes a method of laying stone masonry in cement mortar, under water; suggested to r P. S. Alexander, of the U. S. corps of engineers. "It consists in protecting the mortar solving action of the water during the descent of the stone to its bed, by an envelope of ciently loose in texture to allow the mortar to ooze through between the fibres, and thus with the stone previously laid. It may be applied in the following manner, viz: A piece suitable quality, and somewhat larger than the bed of the block to be laid, is first spread rizontal surface, and covered with a coat of mortar of the thickness desired in the work; rea somewhat exceeding that of the bed of the stone. On this mortar the stone is then ced, and allowed to remain until the mortar begins to stiffen a little: the margin of the rbor to the stone having been folded up against the sides of the latter, and secured there ding over the top. The stone is then lowered through the water, to its position in the ed not again disturbed. Some test trials of this method, although giving discrepant w that if applied with care, it may be made to subserve a good purpose." The muslin ontact with the bricks, or masonry; and not in the center of the thickness of the mortar h the latter arrangement, the joint becomes very much weaker than when no muslin is a the former, the weakening effect of the muslin was but slight; and when the muslin in a very thin paste, or rather in a *cream* of pure cement, the joints were stronger than ortar, (which in all the trials was of pure cement) was used by itself, or without muslin. should be as thin and open as will answer the purpose.

ias in some cases been used in very large masses; as in the foundation of a graving dock rance; where it was deposited to a thickness of 15 feet, over an area of 400 ft by 100 ft; it were, a single artificial stone of that size. It was deposited under water; an immense ng first been prepared for its reception, by enclosing the area with close piling, lined in- rpaulin. On top of this foundation, were similarly built, likewise under water, the sides inside of great boxes, or enclosures of timber, conforming to the shape. The last deposits were then faced with masonry. Walls of buildings are also frequently built of cement con- ted between planks as a mould; and which are moved upward as the building goes on. e made in these walls by ramming concrete around a tube, which can afterward be lifted used for the next course above.

nama railroad, concrete deposited in high wooden moulds, which are afterward removed, *ible substitute for telegraph poles.* Its applications are innumerable.

ove remarks: "Neither the English method of lime-concrete, to be used while hot; nor of forming artificial blocks, to be hardened in the air, before being placed in the work, h employed in the United States. Natural hydraulic cement, with the addition in certain

[illegible], which, "after 3 months' immersion in salt water, sustained a pressure of 116 tons per sq [illegible]."

[illegible] of powder of common lime, slacked.
[illegible] " hydraulic.
[illegible] " sea-sand.

[illegible] were made into mortar with water, and this mortar was mixed with twice its own volume of [illegible], giving a bulk of concrete equal to about [illegible] bulks of the gravel alone.

[illegible] experiments show, however, that brickdust mortars are not fit for sea water. They [illegible] well for a few years, but after that time they begin to disintegrate and [illegible].

Art. 8. The strength, &c., of cement. The strength, hardness, and [illegible] are much greater than in lime mortar. The cohesion, or strength in resisting [illegible], averages from 40 to 70 lbs per sq inch; or 2.5 to 4.5 tons per sq ft. In good Rosendale cement, [illegible] 6 months after being laid. It is sometimes 30 to 50 lbs within a month. The Portland [illegible] of Francis B. White, of London, when fresh, and of choicest quality, is stated to have a cohesion of 120 lbs per sq inch within 7 days; and of 400 lbs within 3 months. Its *adhesion* to bricks and [illegible] varies, however, to be far less. Many spurious and inferior articles are sold under the name of Portland cement.

[illegible] long, 2 ins square, and 6 ins between the supports; of good fresh Rosendale cement; [illegible] after they were made, broke with the following loads suspended at their centers. The mortar of which they were made, was mixed with the smallest possible quantity of water. When more water was added as to be made a thin mortar, the strength of the prisms became much reduced.

Proportion of Cement Powder.	Proportion of Sand.	Load in Lbs.
3	2	450
1	1	600
2	1	750
3½	1	850

Crushing strength of bricks made of best English Portland, 3 and 9 months old. Pure, [illegible] and 300 tons per sq ft. With 1 part sand, 180 and 202 tons. With 5 parts sand, 61 and 108 tons.

Experiments by the writer lead him to believe that **cement is not injured by freezing** after being laid in a work.

Results by Gen Gillmore.

Adhesion of Rosendale cement to brick and granite.

The bricks were immersed together wet; and well pressed together by hand. They were wetted with fresh water every other day, for 28 days; and then submitted to trial. Each result is the average of 3 trials.

		Adhesion per square inch. To Croton Bricks.	To very fine cut Granite.
		Lbs.	Lbs.
Pure cement paste 1 meas.	Sand 0 meas.	30.8	27.5
Cement paste 1	1	15.7	20.8
" " 1	2	12.3	12.6
" " 1	3	6.8	9.2
" " 1	4	5.2	7.9
" " 1	5	4.3	
" " 1	6	3.3	

Transverse strength of cement mortars, made into prisms 2 ins square, by 8 ins long; in vertical moulds, under a pressure of 32 lbs per sq inch; and broken on supports 4 ins apart, by a pressure at the center. The prisms were kept in sea water after the first 24 hours; and were 320 days old when broken. The breaking weights are the averages of many trials. The cement was measured in powder.

Gen Gillmore.	Pure Cement	Cement 1. Sand 1.	Cement 1. Sand 2.
	Lbs.	Lbs.	Lbs.
English Portland, artificial	1536	1260	950
Cumberland, Maryland	954	920	[illegible]
Newark and Rosendale	841	560	500
Delafield & Baxter's Rosendale	836	692	552
Hoffman, Rosendale	849	607	
Lawrence, Rosendale	777		
Round Top, Maryland	.....	600	
Utica, Illinois	732	756	542
Shepherdstown, Virginia	747	618	450
Akron, New York	764	651	403
Kingston and Rosendale	720	556	500
Sandusky, Ohio	554	464	
James River, Virginia	.....	625	458
Roman cement, Scotland; probably injured	553	.....	380
The following were broken when one year old:			
Lawrenceville Manuf Co, Rosendale	.....	910	
Sandusky, Ohio	802		
Kensington, Connecticut	954	709	[illegible]
Lawrence Cement Co, Rosendale, Hoffman brand	875	911	
Round Top, Maryland	.....	840	

PLASTERING.

THE plastering of the inside walls of buildings, whether done on laths, bricks, or ne, generally consists of three separate coats of mortar. The first of these is called workmen the *rough* or *scratch coat;* and consists of about 1 measure of quicklime, 4 of sand; (which latter need not be of the purest kind;) and ½ measure of bul- k or horse hair; the last of which is for making the mortar more cohesive, and s liable to split off in spots. This coat is about ⅜ to ½ inch thick; is put on ghly; and should be pressed by the trowel with sufficient force to enter perfectly tween and behind the laths; which for facilitating this should not be nailed arer together than ½ an inch. In rude buildings, or in cellars, &c, this is often e only coat used. When this first coat has been left for one or more days, accord- g to the dryness of the air, to dry slightly, it is roughly *scored*, or *scratched*, (hence s name,) with a pointed stick, or a lath, nearly through its thickness, by lines run- ng diagonally across each other, and about 2 to 4 ins apart. This gives a better ld to the second coat, which might otherwise peel off. If the first coat has be- me too dry, it is well also to dampen it slightly as the second one is put on.

The second coat is put on about ¼ to ⅜ inch thick, of the same hair mortar, or *arse stuff.* Before it becomes hard, it is roughed over by a hickory broom, or me substitute, to make the third coat adhere to it better.

The third coat, about ⅛ inch thick, contains no hair; and for giving it a still hiter and neater appearance, more lime is used, say 1 of lime, to 2 of sand; and e purest sand is used. This mortar is by plasterers called *stucco;* which name is so applied to mortar when used for plastering the outsides of buildings. Or in- ead of stucco, the third coat may be, and usually is, of *hard finish*, or *gauge stuff;* hich consists of 1 measure of ground plaster of Paris, to about 2 of quicklime, thout sand. Hard finish works easier; but is not as good as stucco, for walls in- ded to be painted in oil. The plaster of Paris is for hastening the hardening.

Either of these third coats is smoothed or polished to a greater or less extent, according to whether s to show, or to be papered, painted, &c. The polishing tools are merely, the trowel; the hand- it, (a kind of wooden trowel;) and the water-brush, (a short-handled brush for wetting the surface t at a time with water, in order to polish more freely.) For finer polishing, a float made of cork red. The smooth piece of board about 10 to 12 ins square, with a handle beneath, on which the sterer holds his mortar until he puts it on to the wall with his trowel, is called a *hawk.*

The more thoroughly each coat is gone over with the water-brush and trowel, (which process is ied *hand floating,*) the firmer and stronger will it be. Frequently only two coats of plastering are t on in inferior rooms; or where great neatness of appearance is not needed. The first is of hair rtar, or coarse stuff; this is scratched with the broom, and then covered by the finishing coat of r mortar, (stucco.) If this last is nearly all lime, or with but very little sand, to make it work ier, it is called a *slipped coat.* Without any sand it is called *fine stuff.* Neither is as good as cco, if the wall is to be papered. When this is the case, the third coat also may have a little hair, give it more strength; but this is not absolutely necessary.

A very good effect may be produced in station-houses, churches, &c, by only two coats of plaster in ich fine clean screened gravel is used instead of sand. When lined into regular courses, it resem- s a buff-colored sandstone, very agreeable to the eye.

In purchasing plastering hair, care must be taken that it has not been taken from salted hides; smuch as the salt will make the walls damp. For the same cause sea-shore sand should not be d. It is almost impossible to wash it entirely free from salt.

In brick walls intended to be plastered, the mortar joints should be left very rough, to let the plas- adhere. If it is put on smooth walls, without first raking out the mortar to the depth of nearly inch, it is very apt to fall off; especially from outside walls; as can be seen daily in any of our ies. As this raking out of brick joints is tedious and expensive, it would generally be better to paint rather than plaster. The walls should also be washed clean from all dust; and should be ently dampened as the plaster is put on.

To imitate granite on outer walls: after the second or smooth coat of plaster is dry, it receives a t of lime wash, slightly tinted by a little umber, or ochre, &c. After this is dry, in case it appears dark, or too light, another may be applied with more or less of the coloring matter in it. Finally, wash of lime and mineral-black is *sprinkled* on from a flat brush, to imitate the black specks of anite. By this simple means, a skilful workman can produce excellent imitations. The horizontal d vertical joints of the imitation masonry, may be ruled in by a small brush, using the same black ish, and a long straight-edge.

The rough surfaces of all walls are more or less warped, or out of line; and it is not possible for e plasterer to rectify this perfectly by eye, as may be seen in almost every house. Even in what e called first-class ones, a quick eye can generally detect unsightly undulations of the plastered rfaces.

To prevent this, the process of *screeding* must be resorted to. Screeds are a kind of gauge or lde, formed by applying to the first rough coat, when partly dried, horizontal strips of the plas- ing mortar, about 8 ins wide, and from 2 to 4 ft apart all around the room. These are made to ject from the first coat, out to the intended face of the second one; and while soft are carefully ade perfectly straight, and out of wind with each other, by means of the plumb-line, straight-edge. When they become dry, the second coat is put on, filling up the broad horizontal spaces between em; and is readily brought to a perfectly flat surface, corresponding with that of the screeds, by ans of *long straight-edges* extending over two or more of the latter.

A day's work at plastering.

plasterer, aided by one or two laborers to mix his mortar, and to keep his hawk supplied, ca age from 100 to 200 square yards a day, of first coat; about ⅓ as much of second; and half

much of third, which requires more care. The amount will depend upon the number of angles, size of rooms, whether on ceilings or on walls, &c, &c.

Gen Gillmore's estimate of cost of plastering* 100 square yards with 2 or with 3 coats. Common labor $1 per day.

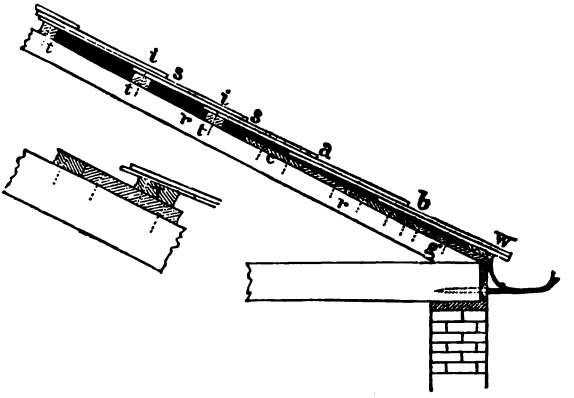
Materials.	Three Coats. Hard finished work.		Two Coats. Slipped coat finish.	
Quicklime	4 casks.	$4.00	3½ casks.	$3.33
" for fine stuff	⅝ "	.85		
Plaster of Paris	⅛ "	.70		
Laths	2000	4.00	2000.	4.00
Hair	4 bushels.	.80	3 bushels.	.60
Common Sand	7 loads.	2.00	6 loads.	1.50
White Sand	2½ bushels.	.25		
Nails	13 lbs.	.90	13 lbs.	.90
Mason's labor	4 days.	7.00	3½ days.	6.11
Laborer	3 days.	3.00	2 days.	2.00
Cartage	.....	2.00	.....	1.20
Cost of 100 square yards	.....	$25.50	.....	$19.65

This amounts to 25½ cts per sq yd for 3 coats; and say 20 cts for 2 coats.

Plastering laths are usually of split white or yellow pine, in lengths of about 3 to 4 ft long; and hence called 3, or 4 ft laths. They are about 1½ ins wide, by ¼ inch thick. They are nailed up horizontally, about ½ inch apart. The upright studs of partitions are spaced at such distances apart, (generally about 15 ins from center to center,) that the ends of the laths may be nailed to them. Laths are sold by the bundle of 1000 each. A square foot of surface requires 1½ four feet laths; or 1000 such laths will cover 666 sq ft. Sawed laths may be had to order, of any required length. A carpenter can nail up the laths for from 40 to 60 sq yds of plastering in a day of 10 hours; depending on the number of angles in the rooms, &c.

SLATING.

ROOFING slates are usually from ⅛ to ¼ inch thick; about $\frac{3}{16}$ being a common average. They may be nailed either to a sheeting of rough boards (*c*, *g*, in the fig) from ¾ to 1¼ inch thick, (which should be, but rarely are, tongued and grooved,)



***Average prices of plastering** in Philada, 1873, in cts per sq yard. *Three coats, including* laths, scaffold &c, 50 to 55 cts. Two coats 35 to 40. Three coats on brick or *stone (no laths reqd,)* 50 to 55. Outside plastering, 60; or if to imitate marble, 75. Simple plaster *cornices, 1 to 2 cts* per inch of girth, per ft run. Plaster center flowers for parlors, $5 to $10 each. *put up. The plastering* of a 20 ft front, 3 story dwelling, with large 3 story back buildings, $900 to *$1300. Stipulate expressly* to pay only for surfaces actually plastered; and thus avoid extras, you *f you have* to pay a few cts more per yard.

ıorizontally from rafter to rafter; or sloping, from purlin to purlin, as the case be; or to stout laths *t t t* about 2 to 3 ins wide, and from 1 to 1¼ thick, nailed e rafters at distances apart to suit the gauge of the slates. Two nails are used to slate; one near each upper corner. They may be either of copper, (which is the durable, but most expensive,) of zinc, or of either galvanized or tinned iron. last two are generally used; or in inferior work, merely plain iron ones, pre- ily boiled in linseed oil, as a partial preservative from rust. Rust, however, times weakens them so much that they break; and the slates are blown off in winds, to the danger of passers by. Since good slate endures for a long series ars, it is true economy to use nails that are equally durable. In iron roofs, the s, instead of being nailed to boards, are sometimes tied directly to the iron ns, by wire.

aboratories, chemical factories, &c, subject to fumes of corrosive acids, it is difficult to provide ıl fastening that will not be eaten away. In such cases it is best to depend chiefly upon a of mortar between the slates. This will harden before the metal fastenings give way; and old the slates in place, while new fastenings are being inserted.

least pitch usually considered advisable for a roof, to prevent rain or snow from being driven gh the interstices between the slates, is about 26½°; or 1 vert to 2 hor; which corresponds to a ¼ the span in a common double pitched roof. But even at steeper pitches, rain, and more ularly snow, will be forced through the roof by violent winds; especially if laths alone be used, n boarding alone. To avoid this, a layer of mortar about ¼ inch thick, may be spread over ıching surfaces of the slates if on laths. If on boards, the same process may be adopted; or ore common one of first covering the boards with a layer of what is called *slating felt;* but in reality is merely thick brown paper, soaked in tar. This is sold in long continuous rolls, wide, and weighing from 40 to 50 lbs. A 50 lb roll will cover about 300 sq ft of roof. With precautions against the admission of rain and snow, a pitch as flat as 1 in 2½, or even 1 in r be adopted.

thickness of slate on a roof is double; except at the *laps is, is, &c,* where it is triple. The measured from the nail hole (under i) of the lower slate, to the lower edge or *tail,* s, of the one; and is usually about 3 ins. In order that the shewing lower edges of the slates shall, laid, form regular straight lines along the roof, the nail holes are made at equal distances from *ower* edges; so that any irregularity of length is concealed from view at the hidden *heads* of ıtes. The slater estimates the length of his slate from the nail hole to the tail, discarding the v strip between the nail hole and the head. If from this reduced length the lap be deducted, ne-half of the remainder will be the *gauge, weathering,* or *margin,* of the slating; or, in other , the *showing* or *exposed* width of the courses of slates. The gauge in ins multiplied by the of a slate in ins, gives the area in sq ins of finished roof covered by a single slate; and if 144) ins in a sq foot) be divided by this area, the quotient will be the number of slates required per f roof. The upper side of a slate is called its *back;* the lower one, its *bed.*

ing, like shingling, must evidently be commenced at the eaves, and extended upward. Since ds of the slates are not exactly parallel to the boarding, and consequently do not rest flat upon se at the lower edge *w* would easily be broken. To prevent this, a *tilting strip* (a stout wide with its upper side planed a little bevelling, to suit the slope of the slates) is first nailed around he eaves, for the tails of the lowest course of slates to rest on. This is shown on a larger scale

e of the best quality has a glistening semi-metallic appearance, somewhat like that of a surface er rubbed with black-lead pencil. That of a dull earthy aspect, is softer, more absorbent, and quently more liable to yield to atmospheric influences, rain, frost, &c. Iron pyrites frequently in slate; and since it always decomposes and leaves holes, should never be admitted on a roof. o qualities of slate, that which absorbs the least weight of water, when pieces of equal size are l for an hour or two, is generally the best; being least liable to split by frost, and become er-worn. This test is easily applied.

England the different sizes of slates are distinguished by absurd names of no meaning. In the l States they are called 6 by 12's; 16 by 24's, &c, according to their measures in inches. They e cut to order, of almost any prescribed dimensions, or shape. Those in common use vary from 7 by 14, to 12 by 18. The first forms about 5 to 6 inch courses; and the last about 7 to 8 inch; ling upon how far from the head the nail holes are pierced. The farther this is, the firmer e slating be.

e roofs, like iron ones, heat the rooms immediately below them very much. This is somewhat shed when the slates are on boards, instead of laths; and still more by a coat of plaster be- They are also liable to break when walked on; less so when bedded in mortar.

eight of slate roofs. Slate weighs about 175 lbs per cub foot; therefore, t, ¼ inch thick, weighs about 1.8 lbs; $\frac{3}{16}$, 2.7 lbs; and ¼ thick, 3.6 lbs. But owing to the pping, a square foot of roof requires about 2½ sq ft of slate of ordinary sizes; and if the slate on boards an inch thick, the weight per sq ft of roof will be increased about 2¼ lbs; or with ıh boards, 2.8 lbs. Laths will weigh about ¼ lb per sq ft of roof.

ce,

	Approx Weight of one sq ft of Slating, in lbs.
Slate ¼ inch thick on laths	4.75
" " on 1 inch boards	6.75
" " on 1¼ " "	7.30
" 3-16 " on laths	7.00
" " " on 1 inch boards	9.00
" " " on 1¼ " "	9.55
" ¼ " on laths	9.25
" " " on 1 inch boards	11.25
" " " on 1¼ " "	11.80

ing felt is used, add ½ lb; or if the slates are bedded in ¼ inch of mortar, add 3 lbs.

[illegible] depend upon the number of angles, the [illegible]

[illegible] of [illegible] of plastering* 100 square yards [illegible] about $1 per day.

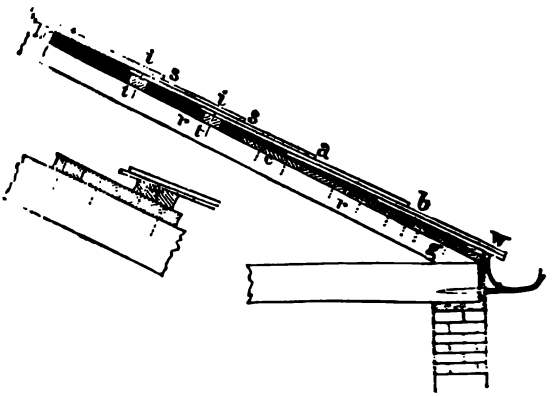
[illegible]	Three Coats. [illegible]		Two Coats. Stripped and finished.	
[illegible]	[illegible]	[illegible]	3½ cwts.	[illegible]
[illegible]	¼ [illegible]	[illegible]		
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	3 bushels.	[illegible]
[illegible]	[illegible]	[illegible]	6 loads.	[illegible]
[illegible]	[illegible]	[illegible]		
[illegible]	1 lbs.	[illegible]	12 lbs.	[illegible]
[illegible]	[illegible]	[illegible]	3½ days.	[illegible]
[illegible]	[illegible]	[illegible]	2 days.	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

Plastering laths are usually of split white or yellow pine, in lengths of [illegible] feet, [illegible] inches wide, by ¼ inch thick. They are about 1½ lbs wide, by ¼ inch thick, [illegible] usually [illegible] feet long. The straight stands of partitions are spaced [illegible] to [illegible] inches apart, generally about [illegible] from center to center.) that the ends of the laths may [illegible] laths are used in the remainder of that work. A square foot of surface requires [illegible] feet of lath which will cover 100 sq ft. Sawed laths may be had to order, of any [illegible] length [illegible] and in the latter for from 40 to 60 sq yds of plastering in a day of [illegible] hours, depending on the number of angles in the rooms, &c.

SLATING.

Roofing slates are usually from ¼ to ¼ inch thick; about $\frac{3}{16}$ being a common average. They may be nailed either to a sheeting of rough boards (*c*, *g*, in the fig. [illegible]) from ⅝ to ¾ inch thick, which should be, but rarely are, tongued and grooved



* **Average prices of plastering** in Philada. 1873 [illegible] per sq yd [illegible] Two coats [illegible] Three coats [illegible] Average plastering 60 [illegible] Plaster mortar [illegible] per sq yd. Plaster mortar [illegible] for [illegible] lbs [illegible] The plastering of a [illegible] room, 3 story [illegible] with large [illegible] and [illegible] [illegible] for surfaces actually plastered, and this [illegible] [illegible] per yard.

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qualities of slate, that which absorbs the least weight of water, when pieces of equal size are
for an hour or two, is generally the best; being least liable to split by frost, and become
worn. This test is easily applied.
gland the different sizes of slates are distinguished by absurd names of no meaning. In the
States they are called 6 by 12's; 16 by 24's, &c, according to their measures in inches. They
cut to order, of almost any prescribed dimensions, or shape. These in common use vary from
by 14, to 12 by 18. The first forms about 5 to 6 inch courses; and the last about 7 to 8 inch;
ng upon how far from the head the nail holes are pierced. The farther this is, the firmer
slating be.
roofs, like iron ones, heat the rooms immediately below them very much. This is somewhat
hed when the slates are on boards, instead of laths; and still more by a coat of plaster be-
They are also liable to break when walked on; less so when bedded in mortar.

ight of slate roofs. Slate weighs about 175 lbs per cub foot; therefore,
¼ inch thick, weighs about 1.8 lbs; $\frac{3}{16}$, 2.7 lbs; and ¼ thick, 3.6 lbs. But owing to the
ping, a square foot of roof requires about 2¼ sq ft of slate of ordinary sizes; and if the slate
on boards an inch thick, the weight per sq ft of roof will be increased about 2¼ lbs; or with
boards, 2.8 lbs. Laths will weigh about ¼ lb per sq ft of roof.

	Approx Weight of one sq ft of Slating, in lbs.
Slate ¼ inch thick on laths	4.75
" " on 1 inch boards	6.75
" " on 1¼ " "	7.30
" 3-16 " on laths	7.00
" " " on 1 inch boards	9.00
" " " on 1¼ " "	9.55
" ¼ " on laths	9.25
" " " on 1 inch boards	11.25
" " " on 1¼ " "	11.80

felt is used, add ¼ lb; or if the slates are bedded in ¼ inch of mortar, add 3 lbs.

should first be thoroughly dissolved in alcohol. The sulphate of zinc causes the wash to become hard in a few weeks.

For brick, masonry, or rough-cast. Slack ½ a bushel of lime as before; then fill the barrel ¾ full of water, and add a bushel of hydraulic cement. Add 2 lbs of sulphate of zinc, previously dissolved in water. The whole should be of the thickness of paint; and may be put on with a whitewash brush. The wash is improved by stirring in a peck of white sand, just before using it. It may be colored, if desired, like the preceding.

He also gives the following cheap oil-paint for outside work on wood, brick, stone, &c; and says it becomes far harder and more durable than common paint: One measure of *ground* fresh quicklime; add the same quantity of fine white sand, or fine coal ashes; and twice as much fresh wood ashes; all the foregoing to be passed through a fine sieve. Mix well together dry. Mix with as much raw linseed oil as will make the mixture as thin as paint. Apply with a painter's brush. It may be colored like the foregoing, taking care to mix the colors well with oil before adding them. It is best to put on two coats; the first thin, and the second thick.

Also, another, said to stand 15 to 20 years: 50 lbs best white lead; 10 quarts raw linseed oil; ½ lb dryer; 50 lbs finely sifted sharp clean sand; 2 lbs raw umber. Add very little, say ½ pint of turpentine. Apply with a *large* brush.

Cement for stopping joints, such as around chimneys, &c, &c. White lead ground in oil, as sold by the keg; mixed with enough pure sand to make a stiff paste that will not run. It grows hard by exposure, and resists heat, cold, and water. Pieces of stone may be strongly cemented together by it, allowing a few months for proper hardening.

Whitewash for inside work, according to Mr. Downing, "is made more fixed and permanent, by adding 2 quarts of thin *size* to a pailful of the wash, just before using. The best size for this purpose is made of shreds of glove leather; but any clean size of good quality will answer," as thin glue-water. We will add, that the common practice of mixing salt with whitewash, should not be permitted. Paper pasted on a wall which has previously been covered with salt whitewash, is very apt to become wet, and loose, and to fall off during damp weather. The whitewash should be scraped off, and the wall or partition covered with a coat or two of thin size, to protect the paper from the effect of the salt that may still adhere to the plaster.

GLASS, AND GLAZING.

WINDOW glass is sold by the box; and whatever may be the size of the panes, a box contains as nearly 50 sq feet of glass as the dimensions of the panes will admit of. In the following table, those numbers which have no + after them, denote that they amount to *precisely* 50 sq ft; while in the others a part of a pane would have to be added to make up the 50 ft.

Panes of any size may be made to order by the manufacturers. The sizes given in the following table, as well as many others, are generally to be had ready made. Ordinary window glass of all the sizes in the table, is about one-sixteenth of an inch thick; and this is the thickness supposed to be intended when a greater one is not specified. *Double-thick* glass is nearly ⅛ inch; and its price is 50 per ct more than the *single* thick. It is of course much stronger than the single.

The panes are confined to the sash by glaziers' putty, made of whiting (powdered chalk) and raw linseed oil; and by small triangular pieces of thin tin, about ¼ inch on a side, which uphold the glass while the putty is being put on; and are allowed to remain afterward, as a protection while the putty continues soft.

TABLE OF NUMBERS OF PANES IN A BOX.

Size in ins.	Panes to a box.	Size in ins.	Panes to a box.	Size in ins.	Panes to a box.	Size in ins.	Panes to a box.
6 × 8	150	11 × 12	54 +	14 × 22	23 +	18 × 28	14 +
6 × 10	120	11 × 14	46 +	14 × 24	21 +	18 × 30	13 +
6 × 12	100	11 × 16	40 +	14 × 26	19 +	18 × 33	12 +
7 × 8	128 +	11 × 18	36 +	14 × 28	18 +	18 × 36	11 +
7 × 10	102 +	11 × 20	32 +	15 × 16	30	20 × 24	15
7 × 12	85 +	11 × 22	29 +	15 × 18	26 +	20 × 26	13 +
7 × 14	73 +	12 × 14	42 +	15 × 20	24	20 × 28	12 +
8 × 10	90	12 × 16	37 +	15 × 22	21 +	20 × 30	12
8 × 12	75	12 × 18	33 +	15 × 24	20	20 × 33	10 +
8 × 14	64 +	12 × 20	30	15 × 26	18 +	20 × 36	10
8 × 16	56 +	12 × 22	27 +	15 × 28	17 +	20 × 40	9
9 × 10	80	12 × 24	25 +	15 × 30	16	22 × 26	12 +
9 × 12	66 +	13 × 14	39 +	16 × 18	25	22 × 30	10 +
9 × 14	57 +	13 × 16	34 +	16 × 20	22 +	22 × 36	9 +
9 × 16	50	13 × 18	30 +	16 × 22	20 +	22 × 40	8 +
9 × 18	44 +	13 × 20	27 +	16 × 24	18 +	22 × 44	7 +
10 × 12	60	13 × 22	25 +	16 × 28	16 +	24 × 27	11 +
10 × 14	51 +	13 × 24	23 +	16 × 32	14 +	24 × 30	10
10 × 15	48	13 × 26	21 +	18 × 20	20	24 × 34	8 +
10 × 16	45	14 × 16	32 +	18 × 22	18 +	24 × 38	7 +
10 × 18	40	14 × 18	28 +	18 × 24	16 +	24 × 42	6 +
10 × 20	36	14 × 20	25 +	18 × 26	15 +	24 × 48	6 +

it qualities of American glass made in the vicinity of Philadelphia, Boston, Pittsburg, &c, ost purely *useful* purposes, as good as those from foreign countries: but when the highest *beauty* is required, as in the lower front windows of first-class dwellings, fancy stores, &c, plate-glass of England, France, or Germany, must be used; although the price for moderate es is from 5 to 8 times as great as that of the best quality single-thick American, as given lowing table.* Its perfectly *smooth* surface, free from distorted reflections, also makes it the covering pictures; still, if carefully selected American panes be used for this purpose, few itics in glass will detect the difference. The polishing of glass plates is not done in the tates as yet.

ick glass is made expressly for flooring, up to 1¼ ins thick, 40 ins by 5 ft dimensions. Also, for skylights, from ¼ to ½ inch thick. This can be fur- order of any size up to 40 ins by 8 or 10 ft. Any of these can also be had *ground*. Grind- nts the entrance of the full glare of the sun; and, moreover, diffuses the light over a much width of space below.

ength of glass. The tensile strength of common glass varies from 2000 s per sq inch. Crushing strength, 6000 to 10000 lbs per sq inch. Transversely, (by the trials,) Millville, N. J., flooring glass, 1 inch square, and 1 foot between the end supports, nder a center load of about 110 lbs; consequently, it is considerably stronger than granite, s regards crushing; in which the two are about equal.

ar. Window and other glass which contains an excess of potash or of soda is very liable to lail in time, owing to the decomposition of those ingredients by atmospheric influences.

WATER.

: water, as boiled and distilled, is composed of the two gases, hydrogen and ; in the proportions of 2 measures hyd, to 1 of ox; or 1 weight of hyd, to 8 Ordinarily, however, it contains several foreign ingredients, as carbonic, and acids; and soluble mineral, or organic substances. When it contains much t is said to be *hard*; and will not make a good lather with soap. The air in inary state contains about 4 grains of water per cub ft.

ary average pressure of the air at sea level, will balance a column of water of 34 ft in vertical At its boiling point of 212° Fah, its bulk is about $\frac{1}{23}$ greater than at 40°.

weight is usually assumed to be 62½ lbs, or 1000 ounces avoir; but 62.3 ld be nearer the truth, as per table below. It is about 815 times heavier than air, when at the temp of 62°; and the barom at 30 ins. According to Mr. James B. Francis, of Lowell, e wt, when pure, is as follows, the barom being as just stated:

p in Deg Fah.	Lbs per Cub F	Temp in Deg of Fah.	Lbs per Cub Ft.
32°	62.375	62°	62.319
40°	62.382	70°	62.272
50°	62.368	80°	62.190
60°	62.331	86°	62.132

he prices for American single-thick glass, per box, in ders are (in Philadelphia, in 1873,) approximately as below. Double-thick 50 per ct more. discounts are made on heavy orders. For ground glass add about $2.50 per box.

Size in Inches.		1st Quality.	2d Quality.	3d Quality.	4th Quality.
From	to				
6 by 8	8 by 10	$ 5.00	$ 4.50	$ 4.25	$ 4.00
8 by 11	10 by 15	5.25	4.75	4.40	4.20
11 by 14	12 by 18	6.00	5.50	5.00	4.65
14 by 16	16 by 24	6.25	5.75	5.25	4.90
18 by 22	18 by 30	7.50	6.75	5.75	5.25
20 by 30	24 by 30	9.25	8.25	6.50	6.00
24 by 31	24 by 36	10.00	8.75	7.00	6.25
28 by 46	30 by 48	11.50	10.50	8.50	
30 by 50	32 by 52	12.25	11.25	9.00	
34 by 58	34 by 60	15.00	14.00	11.00	

harge by glaziers for putting the glass into new windows, including putty, tins, and two coats t to the sash, (one of which is a priming coat,) is (1873) equal to the cost of the glass *at the rices*. See footnote, p 513.

eglazing old sash, removing the broken panes, the charge is about twice as great. In small ies, the following are also approximate prices for American glass: Large plates of ½ inch 5 cts to $1 per sq ft. One inch thick, $1.40 to $1.80; if either is ground, 10 to 15 cts addi- er sq ft. Ribbed glass, ¼ inch thick, 50 cts; ¼ inch, 60 cts per sq ft. *Stained* glass, single thick- $\frac{1}{8}$ inch,) or *figured* white enamelled glass, (single thickness,) 60 to 75 cts per sq ft. Superior *strong figured glass, first ground*, and the transparent figures then formed by polishing away s *of the ground surface*, $1.25 to $2.00 per sq ft.

d glass is an inferior article of fanciful colored patterns, attached by some imperfect process lows them to peel off after a year or two of exposure to the weather.

The weight of water affords an easy way to find the cubic contents of a vessel. First weigh the vessel by itself; and then full of water. The diff will be the weight of the water; and this divided by 62.3 or by the number in the table opp the temp of water, will be the contents in cub ft.

To obtain the size of commercial measures by means of the weight of water.

At the common temperature of from 70° to 75° Fah, a cub foot of fresh water weighs very approximately 62½ lbs avoir. A cubic half foot, (6 ins on each edge,) 7.78125 lbs. A cub quarter foot, (3 ins on each edge.) .97266 lb. A cub yard, 1680.75 lbs; or .75034 ton. A cub half yd, (18 ins on each edge.) 210.094 lbs; or .0938 ton. A cub inch, .036024 lb; or .576384 ounce; or 9.2222 drams; or 252.170 grains. An inch square, and one foot long, .432292 lb; all avoir. Also 1 lb of water contains 27.73008 cub ins; an ounce, 1.735 cub ins; a ton, 35.984 cub ft, all near enough for common use.

Original.

Liquid Measures.

	Lbs Avoir. of Water.
U. S. Gill	.26005*
U. S. Pint	1.0402
U. S. Quart	2.0804
U. S. Gallon	8.3216
U. S. Wine Barrel, 31½ Gall	262.1310

Dry Measures.

U. S. Pint	1.2104
U. S. Quart	2.4208
U. S. Gallon	9.6834
U. S. Peck	19.3668
U. S. Bushel, struck	77.4670

* Or 4 ounces; 2 drams; 15.6625 grs.

Liquid and Dry.

	Lbs Avoir. of Water.
British Imp Gill	.31246
" " Pint	1.2488
" " Quart	2.4975
" " Gallon	9.9900
" " Peck	19.9772
" " Bushel	79.9088

* 4.9942; or very nearly 5 ounces.

French Measures.

Centilitre	.01246
Decilitre	.3246
Litre	2.1246
Decalitre, or Centistere	21.2463
Metre, or Stere	2126.0768

† Or 5.6271 drams; or 153.866 grs.
‡ 3.5169 ounces.

Its max density is when its temp is a little more than 39° Fah; or about 7° warmer than the freezing point. By best authorities 39.2°. From about 39° it expands either by cold, or by heat. When the temp of 32° reduces it to solid ice, its wt is but about 57.2 lbs per cub ft; and its sp gr about .9175, according to the latest determination by L. Dufour. Hence, as ice, it has expanded $\frac{1}{12}$ part of its original bulk as water; and the sudden expansive force exerted at the moment of freezing, is sufficiently great to split iron water-pipes; being probably not less than 30000 lbs per sq inch. Instances have occurred of its splitting cast tubular posts of iron bridges, and of ordinary buildings, when full of rain water from exposure. It also loosens and throws down masses of rock, through the joints of which rain or spring water has found its way. Retaining-walls also are sometimes overthrown, or at least bulged, by the freezing of water which has settled between their backs and the earth filling which they sustain; and walls which are not founded at a sufficient depth, are often lifted upward by the same process.

It is said that in a glass tube ¼ inch in diam, water will not freeze until the temp is reduced to 23°; and in tubes of less than $\frac{1}{200}$ inch, to 3° or 4°. Neither will it freeze until considerably colder than 32°, in rapid running streams. *Anchor ice*, so called, sometimes found at depths as great as 25 ft, consists of an aggregation of small crystals or needles of ice frozen at the surface of rapid open water; and probably carried below by the force of the stream. It does not form under frozen water.

Since ice floats in water; and a floating body displaces a wt of the liquid equal to its own wt, it follows that a cub ft of floating ice weighing 57.2 lbs, must displace 57.2 lbs of water. But 57.2 lbs of water, one ft square, is 11 ins deep; therefore, floating ice of a cubical or parallelopipedal shape, will have $\frac{11}{12}$ of its volume under water; and only $\frac{1}{12}$ above; and a sq ft of ice of any thickness, will require a wt equal to $\frac{1}{11}$ of its own wt to sink it to the surf of the water. In practice, however, this must be regarded merely as a close approximation, since the wt of ice is somewhat affected by enclosed air-bubbles.

Pure water is usually assumed to boil at 212° Fah, in the open air, at the level of the sea; the barom being at 30 ins; and at about 1° less for every 520 ft above sea level, for heights within 1 mile. See levelling by boiling water. In fact, its boiling point varies, like its freezing point, with its purity, the density of the air, the material of the vessel, &c. In a metallic vessel, it may boil at 210°; and in a glass one, at from 212°, to 220°; and it is stated that if all air be previously extracted, it requires 275°.

It evaporates at all temperatures; dissolves more substances than any other agent; and has a greater capacity for heat than any other known substance.

It may be compressed at the rate of about the $\frac{1}{21740}$ part, (or about $\frac{1}{100}$ of an *inch in* $18\frac{1}{10}$ *ft.*) by each atmosphere or pressure of 15 lbs per sq inch. When *the pres is removed*, its elasticity restores its original bulk.

Effect on metals. The lime contained in many waters, forms deposits in *metallic water-pipes*; and in channels of earthenware, or of masonry; especially if the current be

low. Some other substances do the same; obstructing the flow of the water to such an extent, that it is always expedient to use pipes of diameters larger than would otherwise be necessary. See Hydraulics, Art 27. The lime also forms very hard incrustations at the bottoms of boilers; very much impairing their efficiency; and rendering them more liable to burst. It is, therefore, unfit for locomotives. We have seen it stated that the Southwestern R R Co, England, prevent this lime deposit, along their limestone sections, by dissolving 1 ounce of sal-ammoniac to 80 gallons of water.

The salt of sea water forms similar deposits in boilers; as also does mud, and other impurities.

Water, either when very pure, as rain water; or when it contains carbonic acid, (as most water does,) produces carbonate of lead in lead pipes; and as it is is an active poison, such pipes should not be used for such waters. Tinned lead pipes may be substituted for them. If, however, sulphate of lime also be present, as is very frequently the case, this effect is not always produced; and several other substances usually found in spring and river water, also diminish it to a greater or less degree. Fresh water corrodes wrought iron more rapidly than cast; but the reverse appears to be the case with sea water; although it also affects wrought iron very quickly; so that thick flakes may be detached from it with ease. The corrosion of iron or steel by sea water increases with the carbon. Cast-iron cannons from a vessel which had been sunk in the fresh water of the Delaware River for more than 40 years, were perfectly free from rust. Gen Pasley, who had examined the metals found in the ships Royal George, and Edgar, the first of which had remained sunk in the sea for 62 years, and the last for 133 years, "stated that the cast iron had generally become quite soft; and in some cases resembled plumbago. Some of the shot when exposed to the air became hot; and burst into many pieces. The wrought iron was not so much injured, *except when in contact with copper, or brass gun-metal.* Neither of these last were much affected, except when in contact with iron. Some of the wrought iron was reworked by a blacksmith, and pronounced superior to modern iron." "Mr. Cottam stated that some of the guns had been carefully removed in their soft state, to the Tower of London; and in time (within 4 years) *resumed their original hardness.* Brass cannons from the Mary Rose, which had been sunk in the sea for 292 years, were considerably honeycombed in spots only; (perhaps where iron had been in contact with them.) The old cannons, of wrought-iron bars hooped together, were corroded about ¼ inch deep; but had probably been protected by mud. The cast-iron shot became redhot on exposure to the air; and fell to pieces like dry clay!"

"Unprotected parts of cast-iron sluice-valves, on the sea gates of the Caledonian canal, were converted into a soft plumbaginous substance, to a depth of ⅜ of an inch, within 4 years; but where they had been coated with common Swedish tar, they were entirely untinjured. This softening effect on cast iron appears to be as rapid even when the water is but slightly brackish; and that only at intervals. It also takes place on cast iron imbedded in salt earth. Some water pipes thus laid near the Liverpool docks, at the expiration of 20 years were soft enough to be cut by a knife; while the same kind, on higher ground beyond the influence of the sea water, were as good as new at the end of 50 years."

Observation has, however, shown that the rapidity of this action depends much on the quality of the iron; that which is dark-colored, and contains much carbon mechanically combined with it, corrodes most rapidly; while hard white, or light-gray castings remain secure for a long time. Some cast-iron sea-piles of this character, showed no deterioration in 16 years.

Contact with brass or copper is said to induce a galvanic action which greatly hastens decay in either fresh or salt water. Some muskets were recovered from a wreck which had been submerged in sea water for 70 years near New York. The brass parts were in perfect condition; but the iron parts had entirely disappeared. Galvanizing (coating with zinc) acts as a preservative to the iron, but at the expense of the zinc, which soon disappears. The iron then corrodes. If iron be well heated, and then coated with hot coal-tar, it will resist the action of either salt or fresh water for many years. It is very important that the tar be perfectly *purified.* Such a coating, or one of paint, will not prevent barnacles and other shells from attaching themselves to the iron. Asphaltum, if pure, answers as well as coal tar.

No galvanic action has been detected where brass ferules are inserted into the water pipes in Philada.

The most prejudicial exposure for iron, as well as for wood, is that to alternate wet and dry. At some dangerous spots in Long Island Sound, it has been the practice to drive round bars of rolled iron about 4 inches diam, for supporting signals. These wear away most rapidly between high and low water; at the rate of about an inch in depth in 20 years: in which time the 4-inch bar becomes reduced to a 2-inch one, along that portion of it. Under fresh water especially, or under ground, a thin coating of such tar, so applied, will protect iron, such as water-pipes, &c, for a long time. The sulphuric acid contained in the water from coal mines corrodes iron pipes rapidly.

In the fresh water of canals, iron boats have continued in service from 20 to 40 years.

Wood remains sound for centuries under either fresh or salt water, if not exposed to be worn away by the action of currents; or to be destroyed by marine insects.

Sea water differs a little in weight, at diff places; but at the same place it is appreciably the same at all depths; and may be generally assumed at about 64 lbs; or 1¾ per cub ft more than fresh. The additional 1¾ lbs, or $\frac{1}{36}$ part of its entire weight, is chiefly common salt. Sea water freezes at 27° Fah: the ice is fresh.

A teaspoonful of powdered alum, well stirred into a bucket of dirty water, will generally purify it sufficiently within a few hours to be drinkable. If a hole 3 or 4 ft deep be dug in the sand of the seashore, the infiltrating water will usually be sufficiently fresh for washing with soap; or even for drinking. It is also stated that water may be preserved sweet for many years by placing in the containing vessel 1 ounce of black oxide of manganese for each gallon of water.

It is said that water kept in zinc tanks; or flowing through iron tubes galvanized inside, rapidly becomes poisoned by soluble salts of zinc formed thereby; and it is recommended to coat zinc surfaces with asphalt varnish to prevent this. Yet, in the city of Hartford, Conn, service pipes of iron galvanized inside and out, were adopted in 1855, at the recommendation of the water commissioners; and have been in use ever since. They are likewise used in Philadelphia and other cities to a considerable extent. In many hotels and other buildings in Boston, the "*Seamless Drawn Brass Tube*" of the American Tubes Works at Boston, has for many years been in use for service pipe; and has given great satisfaction. It is stated that the softest water may be kept in brass vessels for years without any deleterious result.

The action of lead upon some waters (even pure ones) is highly poisonous. The subject, however

[illegible] of a [illegible]. Then weigh the [illegible] and then divided by [illegible] water, the [illegible] as cubic ft.

To obtain the [illegible] of [illegible] measures of the weights and measures.

[illegible] very approxi- [illegible] each side.) [illegible] grains. [illegible] cub ins.

Liquid Measures.

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

Dry Measures.

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible] 1.[illegible] gr.

Liquids and Dry. Lbs Avdp. of Water.

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible] 1 ounce.

Troy Measures.

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible] from: or 153.[illegible] grs. [illegible] ounces.

The max. density [illegible] is a little more than 39° Fah; or about [illegible] from [illegible] it expands either by [illegible] about 57.2 lbs per cub ft; [illegible] Hence, as ice, it is [illegible] and as water and the sudden expansive force exerted at the [illegible] being probably not less than [illegible] of iron bridges, and of [illegible] and throws down masses of [illegible] Retaining-walls also are [illegible] water which has settled between their back [illegible] and walls which are not founded at a sufficient depth, [illegible]

[illegible] in diam. water will not freeze until the temp is reduced to [illegible] 32° or 4°. Neither will it freeze until considerably other [illegible] *Anchor ice*, so called, sometimes found at depths as great as [illegible] crystals or needles of ice frozen at the surface of rapid open [illegible] carried below by the force of the stream. It does not form under frozen water.

Since ice floats in water; and a floating body displaces a wt of the liquid [illegible] It follows that a cub ft of floating ice weighing 57.2 lbs, must displace 57.2 lbs of [illegible] lbs of water, one ft square, is 11 ins deep; therefore, floating ice of a cubical or parallelopipedal shape, will have $\frac{1}{11}$ of its volume under water; and only $\frac{1}{11}$ above; and a sq ft of its [illegible] thickness, will require a wt equal to $\frac{1}{11}$ of its own wt to sink it to the surf of the water. In practice however this must be regarded merely as a close approximation, since the wt of ice is [illegible] by enclosed air bubbles.

Pure water is usually assumed to boil at 212° Fah, in the open air, at the level of the sea; the barom being at 30 ins; and at about 1° less for every [illegible] ft above sea level [illegible] heights within 1 mile. See levelling by boiling water. In fact, its boiling point varies like its freezing point, with its purity, the density of the air, the material of the vessel, &c. In a metallic vessel, it may boil at 210°; and in a glass ves[illegible] to 200°, and it is stated that if all air be previously extracted, it [illegible]

[illegible] at all temperatures, dissolves more substances than any other agent; [illegible] heat than any other known substance.

[illegible] passed at the rate of about the [illegible] part, or about [illegible] of an [illegible] pressure of [illegible] lbs per sq inch. When [illegible] water [illegible] solid.

[illegible] metals. [illegible] of many various forms depending [illegible] or of masonry, especially if the curves

re are probably but few places in the United States, annual fall of 2 feet may not be safely relied on; and since, as an ordinary average, one- may be collected into reservoirs, we should have for a square mile of drainage, one foot deep, cub ft; equal to 76379 cub ft per day. Allowing 4 cub ft, or 30 gallons per day for each per- making no deduction for evaporation and filtration, this would supply a population of 19095 or a square of 35¼ ft on a side, would in like manner suffice for one person. From $\frac{2}{10}$ to the water annually resulting from rain and snow, passes off into the neighboring rivulets; e into the larger streams and rivers; or may be collected into reservoirs. Under ordinary nces of locality, about ½ may usually be thus secured. The difference is owing chiefly to ce which the water may have to run; the rates of absorption of various soils; the rate of the sides of the valleys leading to the streams; the season of the year, &c, &c.

nch of rain amounts to 3630 cub ft; or 27155 U. S. gals; or 100 acre; or to 2323200 cub ft; or 17379200 U. S. gals; or 64000 tons per sq mile. st destructive rains are usually those which fall upon snow, under which the ground is frozen to absorb water.

SNOW.

ls at diff times by the writer, showed the wt of freshly fallen ary from about 5 to 12 lbs per cub ft; apparently depending chiefly upon the degree of hu- the air through which it had passed. On one occasion when mingled snow and hail had the depth of 6 ins, he found its wt to be 31 lbs per cub ft. It was very dry and incoherent. of heavy snow may, by a gentle sprinkling of water, be converted into about half a cub foot weighing 20 lbs; which will not slide or run off from a shingled roof sloping 30°, if the cold. A cub block of snow saturated with water until it weighed 45 lbs per cub ft, just slid h board inclined at 45°; on a smoothly planed one at 30°; and on slate at 18°; all approxi- prism of snow, saturated to 52 lbs per cub ft; one inch square, and 4 ins high, bore a wt of 7 h at first compressed it about ¼ part of its length. European engineers consider 6 lbs per oof, to be sufficient allowance for the wt of snow; and 8 lbs for the pressure of wind; total, The writer thinks that in the U. S. the allowance for snow should not be taken at *less* than the total for snow and wind, at 20 lbs. There is no danger that snow on a roof will become to the extent just alluded to; because a rain that would supply the necessary quantity of uld also by its violence wash away the snow; but we entertain no doubt whatever that the essures from snow and wind, in our Northern States, do actually at times reach, and even 20 lbs per sq ft of roof. See Table 4, p 301, of Trusses. The limit of perpetual snow at the s at the height of about 16000 ft, or say 3 miles above sea-level; in lat 45° north or south, it alf that height; while near the poles it is about at sea level.

AIR.—ATMOSPHERE.

atmosphere is known to extend to at least 45 miles the earth. Its composition is about .79 measures of nitrogen gas, and ygen gas or about .77 nit, .23 ox, by weight. It generally contains, however, a trace of rbonic acid, and carburetted hydrogen gases; and still less ammonia.

the barom is at 30 ins, and the temperature 60° Fah, air weighs about $\frac{1}{815}$ part as much as 535 grains = 1.224 commercial ounces = .0765 commercial lb, per cub ft. Or 13.072 cub ft); and a cub yd, 2.066 lbs. Or a cube of 30.82 ft on each edge, 1 ton. When colder it weighs cub ft, and vice versa, at the rate of about a grain per deg of Fah. The average weight of atmospheric column, (at least 45 miles high,) at sea level, is 14¾ lbs avoir per sq inch; or nearly a ton) per sq ft; or equal to the weight of a column of water 34.1 ft high. This is usually called the "pressure of the air." At ¼ mile above sea level it is but 14.62 lbs per at ½ mile 13.33; at ¾ mile, 12.66; at 1 mile, 12.02; at 1¼ mile, 11.42; at 1½ mile, 10.88; miles, 9.80 lbs. Therefore, a pump in a high region, will not lift water to as great a height ow one. The pres of air, like that of water, is, at any given point, equal in all directions. en stated that the temperature of the atmosphere lowers or becomes colder, at the rate of 1° ch 300 ft of ascent above the earth's surface; but this is liable to many exceptions, and varies it local causes. Actual observation in balloons seems to show that up to the first 1000 ft, about °, is nearer the truth; at 2000 ft, 250; at 4000 ft, 300; and at a mile, 350.

reathing, a grown person at rest requires from .25 to .35 of a cub ft of inute; which when breathed vitiates from 3½ to 5 cub ft. When walking or hard at work, es and vitiates two to three times as much.

Rose Bridge colliery, England, at the depth of 2424 ft, the temperature of the coal is 93½°

air is a very slow conductor of heat; hence hollow walls serve the heat in dwellings; besides keeping them dry. It rushes into a vacuum near sea level d of about 1157 ft per sec; or 13¼ miles per minute; or about as fast as sound ordinarily rough quiet air. See Sound, p 173.

all other elastic fluids, it expands equally with equal ses of temperature. Every increase of 5° Fah, expands the bulk them slightly more than 1 per cent of that which it has at 0° Fah; or 50° about doubles its ro. *The bulk of any of them diminishes inversely in proportion to the total pressure*

which it is subjected. Thus, if we have a cylinder open at top, and 1 ft deep, full of air: pres of about 15 lbs per sq inch; if by means of a piston we apply an additional pres of inch, making 30 lbs in all, or twice as much as the nat pres, then the air will be comp ins of depth of the cylinder, or one-half of what it occupied before. Or if we apply 45 lb making 60 lbs in all, or 4 times the natural pres, then the air will be compressed into ¼ of the cylinder. Experiment shows that this holds good with air at least up to pressures lbs per sq inch, or 50 times its nat pres; the air in this case occupying the $\frac{1}{50}$ part o bulk. In like manner the bulk will increase as the total pres is diminished; so that if w additional 45 lbs per sq inch, the air in the cylinder will regain its original bulk, and will the cylinder. Substances which follow these laws, are said to be *perfectly elastic*. Un about 5½ tons per sq inch, air would become as dense, or would weigh as much per cub Since the air at the surface of the earth is pressed 14¾ lbs per sq inch by the atmosph and since this is equal to the wt of a column of water 1 inch sq, and 34 ft high, it fo depths of 34, 68, 102 ft, &c, below water, air will be compressed into ½, ⅓, ¼, &c, of it surf; because at those depths it is exposed to pressures equal to 2, 3, 4, &c, times 14¾ lb inasmuch as the pres of the atmosphere on the surf, is in each case to be added to that The pres of the water *alone* at those depths, would be but 1, 2, 3, &c, times 14¾ lbs per

The greatest heat of the air in the sun probably never ex Fah; nor the greatest cold, 65° below zero. About 130° above, and 35° below zero, are in the U. S.; and very unusual.

In a diving-bell, men, after some experience, can readily work hours at a depth of 51 feet; or under a pressure of 2½ atmospheres; or 37½ lbs per sq i 90 ft deep; or under 3.64 atmospheres: or nearly 55 lbs per sq inch, they can work for h hour, without serious suffering from paralysis; or even danger of death. Still at the St. some work was done at a depth of 110½ ft; pres 62.7 lbs per sq inch.

WIND.

The relation between the velocity of wind, and it ure against an obstacle placed either at right angles to its course, or inc has not been well determined; and still less so its pres against curved surfaces. The p large surface is probably proportionally greater than against a small one. It is generally vary nearly as the squares of the vels; and when the obstacle is at right angles to its pres in lbs per sq ft of exposed surface is considered to be equal to the square of the vel hour, divided by 200. On this basis, which is probably quite defective, the following ta by Smeaton, is prepared.

Vel. in Miles per Hour.	Vel. in Ft. per Sec.	Pres. in Lbs. per Sq. Ft.	Remarks.
1	1.467	.005	Hardly perceptible.
2	2.933	.020	Pleasant.
3	4.400	.045	
4	5.867	.080	
5	7.33	.125	
10	14.67	.5	
12½	18.33	.781	Fresh breeze.
15	22.	1.125	
20	29.33	2.	
25	36.67	3.125	Brisk wind.
30	44.	4.5	Strong wind.
40	58.67	8.	High wind.
50	73.33	12.5	Storm.
60	88.	18.	Violent storm.
80	117.3	32.	Hurricane.
100	146.7	50.	Violent hurricane, uprooting la

Tredgold recommends to allow 40 lbs per sq ft of ro pres of wind against it; but as roofs are constructed with a slope, and consequently d the full force of the wind, this is plainly too much.* Moreover, only one-half of a roof i posed, even thus partially, to the wind. Probably the force in such cases varies approxi sines of the angles of slopes. According to observations in Liverpool, in 1860, a wind of hour, produced a pres of 14 lbs per sq ft against an object perp to it; and one of 70 mi (the severest gale on record at that city,) 42 lbs per sq foot. These would make the p more nearly equal to the square of the vel in miles per hour, div by 100; or nearly twic given in Smeaton's table. We should ourselves give the preference to the Liverpool obse very violent gale in Scotland, registered by an excellent anemometer, or wind-gauge, 4 ft. It is stated that as high as 55 lbs has been observed at Glasgow.

The gauge at Girard College, Philada, broke under a st *lbs per sq ft*; a tornado passing at the moment, within ¼ mile.

By inversion of the foregoing rule, if the force in lbs per sq ft, be mult by 100, the sq r will give the vel in miles per hour.

* *The writer thinks* 8 lbs per sq foot of *ordinary double-sloping roofs*, or 18 lbs for a *dent allowance for pres of wind.* See Table 4, p 301; also Snow, p 519.

EVAPORATION, FILTRATION, AND LEAKAGE.

The amount of evaporation from surfaces of water exposed to the natural effects of the open air, is of course greater in summer than in winter; although it is quite perceptible in even the coldest weather. It is greater in shallow water than in deep, inasmuch as the bottom also becomes heated by the sun. It is greater in running, than in standing water; on much the same principle that it is greater during winds than calms. It is probable that the average daily loss from a reservoir of moderate depth, from evaporation alone, throughout the 3 warmer months of the year, (June, July, August,) rarely exceeds about $\frac{3}{10}$ inch, in any part of the United States. Or $\frac{1}{10}$ inch during the 9 colder months; except in the Southern States. These two averages would give a daily one of .15 inch; or a total annual loss of 55 ins, or 4 ft 7 ins.

By some trials by the writer, in the tropics, ponds of pure water 8 ft deep, in a stiff retentive clay, and fully exposed to a very hot sun all day, lost during the dry season, precisely 2 ins in 16 days; or ⅛ inch per day; while the evaporation from a glass tumbler was ¼ inch per day. The air in that region is highly charged with moisture; and the dews are heavy. Every day during the trial the thermometer reached from 115° to 125° in the sun.

The total annual evaporation in several parts of England and Scotland is stated to average from 22 to 38 ins; at Paris, 34; Boston, Mass, 32; many places in the U. S., 30 to 36 ins. This last would give a daily average of $\frac{1}{10}$ inch for the whole year. Such statements, however, are of very little value, unless accompanied by memoranda of the circumstances of the case; such as the depth, exposure, size and nature of the vessel, pond, &c. which contains the water, &c. Sometimes the total annual evaporation from a district of country exceeds the rain fall; and vice versa.

On canals, reservoirs, &c, it is usual to combine the loss by evaporation, with that by filtration. The last is that which soaks into the earth; and of which some portion passes entirely through the banks, (when in embankt;) and if in very small quantity, may be dried up by the sun and air as fast as it reaches the outside; so as not to exhibit itself as water; but if in greater quantity, it becomes apparent, as *leakage*.

E. H. Gill, C E, states the average evaporation and filtration on the Sandy and Beaver canal, Ohio, (38 ft wide at water surface; 26 ft at bottom; and 4 ft deep,) to be but 13 cub ft per mile per minute, *in a dry season*. Here the exposed water surf in one mile is 200640 sq ft; and in order, with this surf, to lose 13 cub ft per min, or 18720 cub ft per day of 24 hours, the quantity lost must be $\frac{18720}{200640} = .0933$ ft, = 1⅛ inch in depth per day. Moreover, one mile of the canal contains 675840 cub ft; therefore, the number of days reqd for the combined evaporation and filtration to amount to as much as all the water in the canal, is $\frac{675840}{18720} = 36$ days. Observations in warm weather on a 22-mile reach of the Chenango canal, N York, (40; 28; and 4 ft,) gave 65½ cub ft per mile per min; or 5 times as much as in the preceding case. This rate would empty the canal in about 8 days. Besides this there was an excessive leakage at the gates of a lock, (of only 5½ ft lift,) of 479 cub ft per min, 22 cub ft per mile per min; and at aqueducts, and waste-weirs, others amounting to 19 cub ft per mile per min. The leakage at other locks with lifts of 8 ft, or less, did not *exceed* about 350 cub ft per min. at each. On other canals, it has been found to be from 50, to 500 ft per min. On the Chesapeake and Ohio canal, (where 50, 32, and 6 ft,) Mr. Fisk, C E, estimated the loss by evap and filtration in 2 weeks of warm weather, to be equal to all the water in the canal. **Professor Rankine assumes 2 ins per day, for leakage of canal bed, and evaporation, on English canals.** J. B. Jervis, C E, estimated the loss from evap, filtration, and leakage through lock-gates, on the original Erie canal, (40, 28, and 4 ft.; at 100 cub ft per mile per min: or 144000 cub ft per day. The water surf in a mile is 211200 sq ft; therefore, the daily loss would be equal to a depth of $\frac{144000}{211200} = .682$ ft, = say 8⅛ ins. See end of Rain. p 519.

On the Delaware division of the Pennsylvania canals, when the supply is temporarily shut off from any long reach, the water falls from 4 to 8 ins per day. The filtration will of course be much greater on embankts, than in cuts. In some of our canals, the depth at high embankts becomes quite considerable; the earth, from motives of economy, not being filled in level under the bottom of the canal; but merely left to form its own natural slopes. At one spot at least, on the Ches and Ohio canal, where one side is a natural face of vertical rock, this depth is 40 ft. Such depths increase the leakage very greatly; especially when, as is frequently the case, the embankts are not puddled; and the practice is not to be commended, for other reasons also.

The total average loss from reservoirs of moderate depths, in case the earthen dams be constructed with proper care, and well settled by time, will not exceed about from ½ to 1 inch per day; but in new ones, it will usually be considerably greater.

The loss from ditches, or channels of small area, is much greater than that from navigable canals; so that long canal feeders usually deliver but a small proportion of the water which enters them at their heads.

HYDROSTATICS.

Art. I. Hydrostatics treats of the pressure of quiet water and other liquids. The pres of liquids against any point of any surf upon which they act, whether said surf be curved or plain, is always at right angles to t

point. At any given depth, the pres of water is equal in every direction; and is in direct proportion to the *vert* depth below the surf. In all cases whatever, the total pres of quiet water against, and perp to any surf, is equal to the wt of a uniform column of water, the area of whose cross-section parallel to its base, is everywhere equal to the area of the surf pressed; and whose height is equal to the *vert* depth of the *cen of grav* of the surf pressed, below the hor surf of the water. This fact is one of those important ones of frequent application, which the young student should impress firmly upon his memory. The wt of a cub ft of fresh water is usually assumed to be 62½ lbs avoir; which is sufficiently correct for ordinary engineering purposes; although 62¼ is nearer the truth for ordinary temperatures of about 70° Fah. Hence,

To find the total pres of quiet water against, and perp to any surf whatever, *as a dam, embkt, lock-gate, &c; or the bottom, side, or top of any containing vessel, water-pipe, &c, whether said surf be vert, hor, or inclined at any angle whatever; or whether it be flat, or curved; or whether it reach to the surf of the water, or be entirely below it:*

RULE. Mult together the area, in sq ft, of the surf pressed; the *vert* depth in ft of its cen of grav below the surf of the water; and the constant number 62.5. The prod will be the reqd pres in pounds.*

EX. 1. The wall A, Fig 1, is 50 ft long; and the depth, *n o*, of water pressing against its *vert* back is uniformly 10 ft. What pres does the wall sustain?

The area of surf pressed is 50 × 10 = 500 sq ft. And the vert depth of its cen of grav below the surf of the water is 5 ft; hence,

500 × 5 × 62.5 = 156250 pounds, or about 70 tons, the pres reqd.

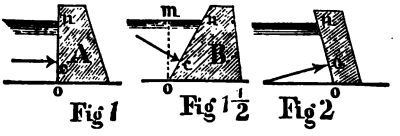

Fig 1 Fig 1½ Fig 2

The pres in this case being perp to a *vert* surf, is *horizontal*; tending either to overturn the wall; or to make it slide on its base. The center of pressure is at *c*; or ⅓ of the vert depth, from the bottom.

EX. 2. As in the foregoing case, the wall B, Fig 1½, is 50 ft long; and the vert depth of water is 10 ft; but it presses against the *sloping* side of the wall; *n o* being 15 ft. What is the total pres, or the pres perp to *n o*; or in the direction of the arrow?

Here the area of surf pressed is 50 × 15 = 750 sq ft. And the vert depth of its cen of grav below the surf of the water is 5 ft, as before; hence,

750 × 5 × 62.5 = 234375 pounds, or about 105 tons, the total pres reqd.

The cen of pres as before, is at *c*, ⅓ the depth, from the bottom.

In such cases, the total pres perp to *n o*, may be considered as resolved into two pressures; one of them acting *hor*, either to overthrow the wall, or to make it slide; and the other acting *vert* to hold it in its place. And if the sloped line *n o* be taken at any scale to represent the total pres, then will the vert line *m o*, measured by the same scale, represent the hor pres; and the hor line *m n*, the vert one. See Art 34, Force in Rigid Bodies. Therefore, so long as the vert depth of water remains the same, the hor pres remains the same, no matter what may be the slope of *n o*; but the vert, as well as the total pres, will increase with *n o*. See Art 4. In Fig 2, the pres tends to *lift* the wall.

REM. 1. This total pres of the water is of course distributed over the entire depth of the wetted part of the back of the wall; being least at top, and gradually increasing toward the bottom; but so far as regards the united action of every portion of it, in tending to overthrow the wall, considered as a single mass of masonry, incapable of being bent or broken, it may all be *assumed* to be applied at *c*; dist from the bottom of the water, ⅓ of its vert depth; or, which amounts to the same thing, at ⅓ of the sloping dist *o n*. Figs 1½ and 2. See Art 57, of Force in Rigid Bodies, p 482.

REM. 2. **It follows, from the foregoing rule, that the amount of pres of water against any surf is entirely independent of the quantity of the water,** so long as the area pressed, and the vert depth of its cen of grav below the level surf remain unchanged. The wall A or B would sustain as great a pres from a layer of water only an inch thick behind it, as if the water had extended back for miles. From this cause, retaining-walls of mortar masonry carelessly backed, have been bulged, and cracked, by the infiltration of rain behind them; while walls of dry masonry would have permitted the water to escape through the open joints; and would therefore have stood safely.

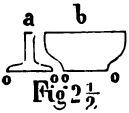

Fig 2½

Also in vessels *a*, *b*, Fig 2½, of any size or shape whatever, if they contain the same vert depth of water; and have equal bases *o o*, pressed by said depths of water, the pressures on the bases will all be equal, without any regard to the quantity of water. Or, if we have two water-pipes of the same diam, both full of water, one standing vertically, 10 feet long; and the other 20 miles long, and laid at an inclination of ½ ft per mile, so as to make its *vert* depth of water also 10 ft, then the pressures at the bases of the two pipes will be the same. This pres of water, independently of its quantity, is called the *hydrostatic paradox*. In the vessel *a*, the pres on the base is much greater than the wt of the water; but in *b*, it is less.

REM. 3. Since the pres of water against any point of a surf, is at right angles to that point, it fol-

* *This is strictly true as regards the pres of the water alone; and this is usually all that is required. But it must be borne in mind that the surf of the water is itself pressed by the air; to the uniform extent (near the level of the sea) of about 14.7 lbs per sq inch; or 2117 lbs, or nearly 1 ton per sq foot. Therefore, to find the true total pres, we should mult the area in sq ft of the surf pressed by the pres of 2117 lbs; and add the prod to the water-pres given by the rule. But in ordinary engineering, or*

RATION, FILTRATION, AND LEAKAGE.

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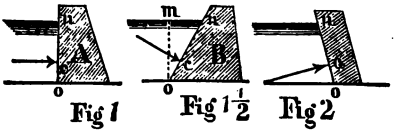

Fig 1 Fig 1½ Fig 2

The pres in this case being perp to a *vert* surf, is *horizontal;* tending either to overturn the wall; or to make it slide on its base. The center of pressure is at *c*; or ⅓ of the vert depth, from the bottom.

EX. 2. As in the foregoing case, the wall B, Fig 1½, is 50 ft long; and the vert depth of water is 10 ft; but it presses against the *sloping* side of the wall; *n o* being 15 ft. What is the total pres, or the pres perp to *n o*; or in the direction of the arrow?

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The cen of pres as before, is at *c*, ⅓ the depth, from the bottom.

In such cases, the total pres perp to *n o*, may be considered as resolved into two pressures; one of them acting *hor*, either to overthrow the wall, or to make it slide; and the other acting *vert* to hold it in its place. And if the sloped line *n o* be taken at any scale to represent the total pres, then will the vert line *m o*, measured by the same scale, represent the hor pres; and the hor line *m n*, the vert one. See Art 34, Force in Rigid Bodies. Therefore, so long as the vert depth of water remains the same, the hor pres remains the same, no matter what may be the slope of *n o*; but the vert, as well as the total pres, will increase with *n o*. See Art 4. In Fig 2, the pres tends to *lift* the wall.

REM. 1. This total pres of the water is of course distributed over the entire depth of the wetted part of the back of the wall; being least at top, and gradually increasing toward the bottom; but so far as regards the united action of every portion of it, in tending to overthrow the wall, considered as a single mass of masonry, incapable of being bent or broken, it may all be *assumed* to be applied at *c*; dist from the bottom of the water, ⅓ of its vert depth; or, which amounts to the same thing, at ⅔ of the sloping dist *o n*, Figs 1½ and 2. See Art 57, of Force in Rigid Bodies, p 482.

REM. 2. **It follows, from the foregoing rule, that the amount of pres of water against any surf is entirely independent of the quantity of the water,** so long as the area pressed, and the vert depth of its cen of grav below the level surf remain unchanged. The wall A or B would sustain as great a pres from a layer of water only an inch thick behind it, as if the water had extended back for miles. From this cause, retaining-walls of mortar masonry carelessly backed, have been bulged, and cracked, by the infiltration of rain behind them; while walls of dry masonry would have permitted the water to escape through the open joints; and would therefore have stood safely.

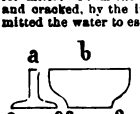

Fig 2½

Also in vessels *a*, *b*, Fig 2½, of any size or shape whatever, if they contain the same vert depth of water; and have equal bases *o o*, pressed by said depths of water, the pressures on the bases will all be equal, without any regard to the quantity of water. Or, if we have two water-pipes of the same diam, both full of water, one standing vertically, 10 feet long; and the other 20 miles long, and laid at an inclination of ½ ft per mile, so as to make its *vert* depth of water also 10 ft, then the pressures at the bases of the two pipes will be the same. This pres of water, independently of its quantity, is called the *hydrostatic paradox.* In the vessel *a*, the pres on the base is much greater than the wt of the water; but in *b*, it is less.

REM. 3. Since the pres of water against any point of a surf, is at right angles to that point, it fol-

* This is strictly true as regards the pres of the water alone; and this is usually all that is required. But it must be borne in mind that the surf of the water is itself pressed by the air; to the average extent (near the level of the sea) of about 14.7 lbs per sq inch; or 2117 lbs, or nearly 1 ton, per sq foot. Therefore, to find the true total pres, we should mult the area, in sq ft, of the surf pressed by the water, 2117 lbs; and add the prod to the water-pres given by the rule. But in ordinary engineering cases

lows that props *p p*, for strengthening such structures as the sloping dam D, Fig 3, should be placed at right angles to them in order to oppose the greatest possible resistance to the pres. Other considerations may at times prevent our doing so; thus the outer prop, *p*, if so placed, would be in danger of being broken by ice, or logs tumbling over the dam; and therefore, had better be more nearly vertical.

REM. 4. It follows, from the foregoing rule, that in a cubical vessel, filled with water, the pres on the base is equal to the weight of the water; that on each of the four sides, to half the weight of the water; and that on the bottom and the 4 sides together, to 3 times the wt of the water. In a conical vessel, forming an entire cone, the pres on its hor base is equal to 3 times the wt of the water; and so likewise in a pyramidal vessel; for in both cases the wt of the water is but ⅓ that of a uniform column of water of the same height. In a full spherical vessel, the total pres against its entire interior surf, is also equal to 3 times the wt of the water, as in a cubical one.

Since the pres against any given area, at any given depth below the surf, is equal to the wt of a uniform column of the fluid having the same transverse area, and depth, as those given; and since a uniform column of water of one sq ft of transverse area, and 1 ft high, weighs very nearly 62½ lbs, the following table gives the pres on each sq ft of area of surf at diff depths; and will show the great pres to which the lower portions of high coffer-dams, &c, are exposed; and the necessity for giving them great strength. Also that in the dam, Fig 3, the props should be nearer together toward the bottom; as should also the hoops of a tank.

Fig 3

Vert. Depth in Feet.	Pres. per Sq. Ft. in Lbs. at that Depth.	Vert. Depth in Feet.	Pres. per Sq. Ft. in Lbs. at that Depth.	Vert. Depth in Feet.	Pres. per Sq. Ft. in Lbs. at that Depth.	Vert. Depth in Feet.	Pres. per Sq. Ft. in Lbs. at that Depth.	Vert. Depth in Feet.	Pres. per Sq. Ft. in Lbs. at that Depth.
1	62.5	11	687.5	21	1312.5	31	1937.5	41	2562.5
2	125.	12	750.	22	1375.	32	2000.	42	2625.
3	187.5	13	812.5	23	1437.5	33	2062.5	43	2687.5
4	250.	14	875.	24	1500.	34	2125.	44	2750.
5	312.5	15	937.5	25	1562.5	35	2187.5	45	2812.5
6	375.	16	1000.	26	1625.	36	2250.	46	2875.
7	437.5	17	1062.5	27	1687.5	37	2312.5	47	2937.5
8	500.	18	1125.	28	1750.	38	2375.	48	3000.
9	562.5	19	1187.5	29	1812.5	39	2437.5	49	3062.5
10	625.	20	1250.	30	1875.	40	2500.	50	3125.

Thus we see that at the depth of 36 ft, the pres of water against a single sq ft of surf, whether hor, vert, or oblique, is fully 1 ton; requiring great precaution to prevent leakage, or breaking. At 72 ft, it would be 2 tons, &c. A pres of 62½ lbs per sq ft gives a pres of .434 lbs per sq inch.

Further; let *a b*, Fig 3½, be a tube of 36 ft vert height; full of water; with a bore so small that the tube would contain say only one pound of water; and let this tube open at its lower end into a vessel also full of water; the top and bottom of which are 8 ft apart. Then the 1 lb of water in the tube, will cause each sq ft of the top of the vessel, (which is 36 ft below the surf of the water in the tube) to be pressed upward with a force of 2250 lbs, as per table. Each sq ft of the bottom of the vessel (which is 44 ft below the surf of the water in the tube) will be pressed downward with a force of 2750 lbs; and any particular sq ft of the sides of the vessel, will be pressed hor outward, with the force given in the table, opposite to the depth of the cen of grav of said sq ft below the same water surf of the top of the tube, whatever said depth may happen to be. Or, suppose, first only the lower vessel to be filled with water, and its inner surf to be sustaining the pres arising therefrom; if we then fill the 36 ft tube with its 1 lb of water, this 1 lb will create on *additional* pres of 2250 lbs against every sq ft of said inner surf; so that if each of the 6 sides of the vessel be 8 ft square; or contain in *all* 384 sq ft of inner surf, this 1 lb of water will produce *additional* pres of 864000 lbs, or full 385 tons, against them.

Fig 3½

Art. 2. Surfaces pressed on both sides; and immersed.

When two bodies of water of diff depths, press against two opposite sides of a plane which is completely immersed, whether vert or sloping; as, for instance, against the two sides *a b*, *n o*, Fig 4; or the two sides *d e*, *c r*, then, the total pres against *i b*, *i e*, *a b*, *n o*, or *c r*, &c, may still be found by the foregoing rule, in Art 1; but the EXCESS *of pres against the part* a b, or d e, *of the immersed plane, beyond the counter-pres against the opposite part* n o, or c r, will be equal to the wt of a column of water whose section is equal to the area of the part *a b*, or *d e*, (as the case may be;) and whose vert height is equal to *m n*, or *x p*, the vert diff of level of the two bodies of water. Consequently, this *excess* of outward pres is found by mult together, the area of *a b* or *d e*, in sq ft; the vert height *m n* or *x p*, in ft; and the constant 62.5 lbs wt of a cub ft of water. Thus, if *a b* is 10 ft high, and 20 ft long; and the vert height *m n*, 12 ft; then the *excess* of pres against *a b*, over that against *n o*, will be 10 × 20 × 12 × 62.5=150000 lbs. The excess will be greater on *d e*, than on *a b*, although both are exposed to the same vert depths *m n*, *x p*; because the area of *d e* is greater than that of *a b*. Moreover, this *excess* of outward pres is *equally* distributed over the entire area of *a b* or *d e*; being no greater at *b* and *e*, than at *a* or *d*; in other words, every sq ft of area of *a b* or *d e* is pressed outward at right angles to its surf, by an *excess* of force equal to the wt of a column of water 1 ft sq; and of a height equal to *m n*, or *x p*.

Fig 4

this pres of the air may, and should be omitted; because it is counterbalanced by an equal pres *against the opposite side, face, or* surf of the pressed body. It becomes necessary, therefore, *to take into consideration only when the opposite face of the body is not exposed to a counterba atmospheric pressure;* as when there is a vacuum on that side.

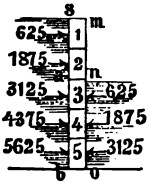

Fig 5

This will be understood by means of Fig 5. which may represent five planks, 1, 2, 3, 4, and 5, forming a dam, and seen endwise; each one 1 ft in depth, and say 20 ft long hor; making the area of each surf pressed, equal to 20 sq ft. The pres in lbs against each separate 20 sq ft of area, calculated by the rule in Art 1, is shown in the fig. Now, the outward pres against the upper *immersed* 20 ft area, or that of plank 3, is 3125 lbs; while the counter-pres against it from the other side is 625 lbs; making the *excess* of outward pres equal to 3125 — 625 = 2500 lbs. Again, at the lowest plank, number 5, the outward pres exceeds the inward one by 5625 — 3125 = 2500 lbs, the same as in the upper one. And so of any other equal area of surf, at any depth whatever; the excess depending upon the vert height of *m n*, will be equally distributed over *a b*. It only remains to show that the total excess of outward pres against *a b*, is equal in amount to the wt of a uniform column of water with a base equal in area to *a b*, and with a height equal to *m n*. Thus, we have seen that in the instance before us, the excess amounts to 3 times 2500 lbs, or to 7500 lbs. Now, the wt of the column of water will be 60 (or area of *a b*) × *m n* (or 2 ft) × 62.5 lbs = 7500 lbs; or the same as the *excess* pres on *a b*.

The excess of pres against the entire side *s b*, over that against *n o*, is evidently the diff between those two pressures calculated respectively by the rule in Art 1.

Art. 3. Surfaces of equal widths, commencing at the level of the water, but extending to diff depths, measured vert; and having the same inclination to the surf of the water; sustain total pressures proportional to the squares of those depths.

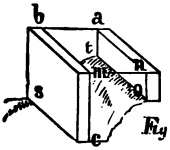

Fig 6

In Fig 6, let the two vert sides, *a n o t*, and *b m c s*, of a vessel, have the same width *a n*, and *b m*; then if the depth *m c*, be 2, 3, 4, 5, &c, times greater than the depth *n o*, the pres against the surf *b m c s*, will be 4, 9, 16, 25, &c, times greater than that against *a n o t*. This will be seen by referring to the pressures figured on the left side of Fig 5, where, as stated in Art 2, the surf of plank 1, exposed to the pres on the left side, is 20 sq ft; that of planks 1 and 2, 40 sq ft; that of planks 1, 2, and 3, 60 sq ft, &c. All these surfs commence at the level of the water; and all of them being vert, are of course at the same inclination with the water surf; but their depths are respectively 1, 2, and 3 ft. The pres against the surf of 1, is 625 lbs; that against the surf of 1, 2, is 625 + 1875 = 2500; and that against the surf of 1, 2, 3, is 625 + 1875 + 3125 = 5625. But 2500 is *four* times 625; and 5625 is *nine* times 625. And the pres against the entire surf *s b*, (which is 5 times as deep as plank 1,) is 25 times as great as that against plank 1; or 625 × 25 = 15625 lbs = the sum of all the pressures marked on the left side of Fig 5.

This follows, from the Rule in Art 1; for twice the area of surf, mult by twice the vert depth of the cen of grav below the surf, must give 4 times the pres; three times the area, by three times the depth, must give 9 times the pres, &c.

It follows, also, that at any particular *point*, or against any *given* area placed at various depths, the pres will increase simply as the vert depth: thus, if there be three areas, each one sq ft, placed in the same positions, but with their centers of grav respectively 8, 16, and 24 ft below the surf, the pres against them will be respectively *as* 8, 16, and 24; or *as* 1, 2, and 3.

Art. 4. The pressure of quiet water, in any one given direction, against any given surf, whether vert, hor, inclined, flat, or curved, is equal to the wt of a uniform column of water, the area of whose section, parallel to its base, is everywhere equal to the area of the *projection* of the pressed surf taken perp to the given direction*; and the height of the column equal to the vert depth of the cen of grav of the pressed surf below the upper surf of the water. Hence the

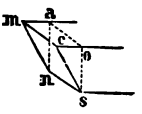

Fig 7

RULE. **To find the pres in lbs,** mult together the area in sq ft of the projection taken at right angles to the given direction; the vert depth in ft of the cen of grav of the pressed surf below the upper surf of the water; and the constant 62.5 lbs wt of a cub ft of water.

Ex. Let *m c s n*, Fig 7, be an inclined surf, sustaining the pres of water which is level with its top *m c*. Then the *total* pres against *m c s n*, and at right angles to it, as found by the rule in Art 1, is an illustration of the present rule; because the projection of *m c s n*, taken at right angles to the given direction, or parallel to *m c s n*, is in fact *m c s n* itself, or equal to it. Hence the rule in Art 1 is merely a simple modification of the present one, applicable to the case of *total* pres against any surf.

But if it be reqd to find only the vert or downward pres against *m c s n*, in pounds, mult together the area of the hor projection *a o c m* in sq ft; the vert depth in ft of the cen of grav of *m c s n* below the surf; and 62.5. Or if only the *hor* pres against *m c s n* be sought, mult together the area of the vert projection *a o s n*; the vert depth of the cen of grav of *m c s n*; and 62.5.

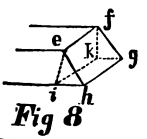

Fig 8

In Fig 8 also, the *total* pres against *e f g h* is found by rule in Art 1; while the *hor* and vert pressures against it are found as in Fig 7, by using the projections *e f k i*, and *k i g h*. In Fig 7 the vert pres is downward; while in Fig 8 it is upward; but this circumstance in no respect affects the rule.

REM. 1. It will be observed in both figs, that the vert projections, *a o s n*, *e f k i*, will remain the same, no matter what may be the inclination of the pressed surfs *m c s n*, and *e f g h*; the degree of inclination therefore does not influence the *hor* pres, but only the total, and the vert ones.

* See Projection, in our glossary.

Again, let Fig 9 represent a conical vessel full of water; its base $b\,c$, 2 ft diam; its vert height $a\,n$, 3 ft; then the circumf of the base will be 6.2832 ft; the area of the base 3.1416 sq ft; the length of its slant side $a\,b$ or $a\,c$, 3.16 ft; the area of its curved slanting sides will be $\frac{6.2832 \times 3.16}{2} = 9.93$ sq ft; and the vert depth of the cen of grav of the slanting sides will be at two-thirds of the vert height $a\,n$ from the apex a, or 2 ft.

Fig 9

Here, to find the total pres against the base, we have by rule in Art 1, $3.1416 \times 3 \times 62.5 = 589.05$ lbs. For the total pres against the slant sides, by the same rule, $9.93 \times 2 \times 62.5 = 1241.25$ lbs. For the vert pres upward against the entire area of the slant sides, we have given the area of the base (which is here the hor projection of the slant sides) $= 3.1416$; and the vert depth of the cen of grav of the slant sides, 2 ft. Therefore $3.1416 \times 2 \times 62.5 = 392.7$ lbs, the upward vert pres.

Finally, for the hor pres in any given direction against the slant sides of *one half* of the cone, we have the vert projection of that half, represented by the triangle $a\,b\,c$, with its base 2 ft, and its perp height 3 ft; and consequently, with an area of 3 sq ft. The depth of its cen of grav is 2 ft; therefore $3 \times 2 \times 62.5 = 375$ lbs, the reqd hor pres.

In Fig 10, which represents a vessel full of water, the *total* pres against the semi-cylindrical surf $a\,v\,e\,m\,d\,k$, and perp to it, must be also hor, because the surf is vert; but inasmuch as the surf is *curved*, this total pres, as found by rule in Art 1, acts against it in many directions, which might be represented by an infinite number of radii drawn from o as a center. But let it be reqd to find the hor pres in lbs, in one direction only, say parallel to $o\,c$, or perp to $a\,d$; which would be the force tending to tear the curved surf away from the flat sides $a\,b\,n\,v$, and $d\,c\,s\,k$, by producing fractures along the lines $a\,v$ and $d\,k$; or which would tend to burst a pipe or other cylinder. In this case, mult together the area of the vert projection $a\,d\,k\,v$ in sq ft; the depth of the cen of grav of the curved surf in ft; (which, in the semi-cylinder would be half of $e\,m$, or of $o\,i$;) and 62.5. Since the resulting pres is resisted equally by the strength of the vessel along the two lines $a\,v$ and $d\,k$, it is plain that each single thickness along those lines need only be sufficient to resist safely *one half* of it; and so in the case of pipes, or other cylinders, such as hooped cisterns or tanks. See Art 16, p 531.

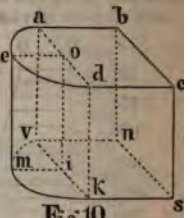

Fig 10

Should the pres against only one half of the curved surf, as $e\,d\,m\,k$ be sought, and in a direction parallel to $o\,d$, tending to produce fractures along the lines $e\,m$, and $d\,k$, then use the vert projection $o\,e\,m\,i$; with the same depth; and 62.5 as before.

It follows, that if the face of a metallic piston be made concave or convex, no more pres will be reqd to force the piston through any dist, than if it were flat; for the pres against the face of the piston in the direction in which it moves, must be measured by the area of a projection of that face, taken at right angles to said direction; and the area of said projection will be the same in all three cases.

Rem. 2. **If a bridge pier, or other construction, Fig 10½, be founded on sand or gravel,** or on any kind of foundation through which water may find its way underneath, even in a very thin sheet, then the upward pres of the water will take effect upon the pier; and will tend to lift it, with a force equal to the wt of the water displaced by the pier; (see Arts 17 and 18;) or in other words, the effective wt of the *submerged* portion of the pier, will be reduced 62½ lbs per cub ft; or nearly the half of the ordinary wt of masonry.


Fig 10½

But if the foundation be on rock, covered with a layer of cement to prevent the infiltration of water beneath the masonry, no such effect will be produced; but on the contrary, the vert pres downward, afforded by the battering sides of the pier, and by its offsets, will tend to hold it down, and thus increase its stability which, in *quiet* water, will then actually be greater than on land.

Art. 5. To divide a rectangular surf, whether vert as $a\,b\,c\,d$, or inclined as $m\,n\,o\,p$, Fig 11, whose top $a\,b$ or $m\,n$ is level with the surf of the water, by a hor line $1\,s$, such that the total pres against the part above said hor line, shall equal that against the part below it.

Rule. Mult one half of the length of $b\,c$, or $m\,p$, as the case may be, by the constant number 1.4142; the prod will be $b\,1$, or $m\,s$.

Ex. Let $b\,c = 12$ ft. Then $6 \times 1.4142 = 8.4852$ ft; or $b\,1$. Let $m\,p = 16$ ft. Then $8 \times 1.4142 = 11.3136$ ft, or $m\,s$.

Rem. The line $s\,1$, thus found, must not be confounded with the *cen of pres*, which is entirely diff. See Art 8.

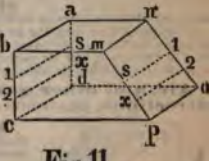

Fig 11

Art. 6. In a rectangular surf, whether vert as $a\,b\,c\,d$, or inclined as $m\,n\,o\,p$, Fig 11, whose top $a\,b$ or $m\,n$ coincides with the surf of the water, to find any number of points, as 1, 2, &c, through which if hor lines, as $1\,s$, $2\,x$, &c, be drawn, they will divide the given surf into smaller rectangles, all of which shall *sustain* equal pressures.

Rule. *First fix on the number of small rectangles reqd. Then for point 1 from the top, mult number 1, by this number of rectangles.* Take the sq rt of the prod. Mult this sq rt by the entire

b c or *m p*, as the case may be. Div the prod by the number of rectangles. The quot will be the dist *b* 1, or *n* 1, as the case may be.

For the dist *b* 2, or *n* 2, proceed in precisely the same way; only instead of the number 1, use the number 2 to be mult by the number of rectangles: and so use successively the numbers 3, 4, 5, &c, if it be reqd to find that number of points.

Ex. Let *b c* = 10 ft; and let it be reqd to find 2 points, 1 and 2, for dividing the rectangular surf *a b c d* into 3 rectangular parts, which shall sustain equal pressures. Here we have for point 1,

$1 \times 3 = 3$. The sq rt of $3 = 1.732$. And 1.732×10 (or *b c*) $= 17.32$. And $\frac{17.32}{3 \text{ rectangles}} = 5.773$ ft $= b\,1$.

For point 2, we have

$2 \times 3 = 6$. The sq rt of $6 = 2.449$. And 2.449×10 (or *b c*) $= 24.49$. And $\frac{24.49}{3 \text{ rectangles}} = 8.163$ ft $= b\,2$.

And so for any number of points.

Rem. 1. **This rule will be found useful in spacing the cross-bars of lock-gates; the hoops around cylindrical cisterns; and the props to a structure, like Fig 3,** Art 1.

Rem. 2. **For dividing any surf, as *o b c d*, Fig 12, which is not rectangular, in the same manner,** with an accuracy sufficient for most practical purposes, perhaps the following method is as convenient as any.

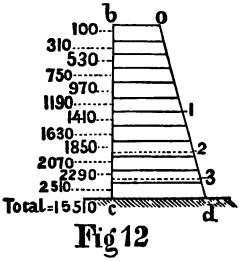

Fig 12

Rule. First div the surf, as in Fig 12, into several small hor parts, equal or not, at pleasure. Then by Rule in Art 1, find the pres on each part separately, as is supposed to be done in the numbers on the left hand of the fig. The sum of these (in this case 15510) is the total pres against the entire surf *o b c d*. Now suppose we wish to div this surf in 4 parts bearing equal pres; first div 15510 by 4 = 3878. Then beginning at the top, add together a number of the separate pressures sufficient to amount to 3878; by this means find point 1. Then proceed with the addition until the sum amounts to twice 3878, or 7756, which will indicate point 2; and in the same manner find point 3, by adding up to three times 3878, or 11634. Then the hor dotted lines ruled through points 1, 2, and 3, will give the reqd divisions approximately. In this manner the hoops of conical, and other shaped vessels, may be spaced nearly enough for practical purposes.

Art. 7. The transmission of pressure through water. Water, in common with other fluids, possesses the important property of transmitting pres equally in all directions. Thus, suppose the vessel, Fig 13, to be entirely closed, and filled with water; and suppose the transverse area of T, C, D, and E, to be each equal to one sq inch. Then, if by means of a piston, or otherwise, a pres of 1 lb, 1 ton, or any other amount, be applied to the one sq inch of area of T, C, D, or E, every sq inch of the inner surf of the vessel, and of the pipe *a*, will instantly receive, at right angles to itself, an equal pres of 1 lb, or 1 ton, &c: in addition to the pres which it before sustained from the water itself; and this will occur if the vessel consist of parts even miles asunder; as, for instance, if T were miles distant from E: and united to it by a long series of tubes. If the vessel were a strong steam boiler full of water, a single pres of a few hundred pounds at T, C, &c, would burst it.

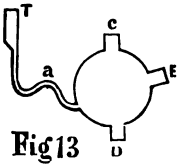

Fig 13

The hydrostatic press acts on this principle. Any body, within the vessel, would also receive an equal additional pres on each sq inch of its surf.

If the top of T be open, the air will press upon the sq inch of the exposed surf of water to the extent of nearly 15 lbs; and the same degree of pres will also be transmitted to every sq inch of the interior surf of the vessel, and its connecting tubes; but no danger of bursting will result from this atmospheric pres, because the air also presses every sq inch of the outside of the vessel to the same extent.

Air, and other gaseous fluids, transmit pres equally in all directions, like liquids; but not as rapidly.

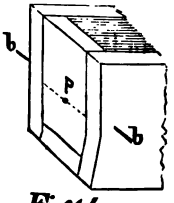

Fig 14

Art. 8. The center of pressure. Let Fig 14 represent a vessel full of water, and suppose the side P to be perfectly loose, so as to be thrown outward by the slightest pres of the water from within. Now, there is but one single point, P, in every surf so pressed, no matter what its shape may be, to which if we apply a force equal to the pres of the water, and in a direction opposite to said pres, the side P will be thereby prevented from yielding. Such point is called the *center of pressure*. It must not be understood by this that the actual amount of pres of the water against that part of the surface which is above the hor dotted line passing through P, is equal to that of the water below said line; but that the products of the several pressures above it, mult by their several leverages, or dists from P, are equal to the products of the pressures below, mult by their leverages; or, in other words, that the *moments* around the point P, of the pressures above the line, are equal to the *moments* of those below it; so that if a hor iron rod *b b* were passed entirely through the side P, at the same level as the dotted line, so as to serve as a hinge, or pivot for the side P to turn on, as shown in fig, the equal moments above and below the bar would prevent the side from turning.

9. To find the cen of pres of a quiet against a plane surface. Fig 15.

cen of pres of a quiet fluid, against one side of any plane *rec*-surface, (that is, a plane surf of uniform width throughout its hether vert as *c o*, or inclined as *c a*, (or inclined in the opposite) and whose top *c*, or *s*, coincides with the hor water surf; is *ert* below the water surf, two-thirds of the vert depth, *s x*, from *r* surf to the bottom of the plane; as at *n*, and *i*. Inasmuch as at ⅔ of the depth of *s x*, intersects both *c a* and *c o* at ⅔ of their spectively, we might say at once that the center of pres against ular plane is at ⅔ of its length below the water surface.

Fig 15

if the hor top *a*, or *o*, Fig 16, of the rectangular plane *a g*, or vered to some depth with water, then the *vert* depth *s m*, of the s *d*, or *e*, below the surf of the water, will be equal to

$$\frac{2}{3} \text{ of } \frac{\text{cube of } s\,c - \text{cube of } s\,w}{\text{square of } s\,c - \text{square of } s\,w}$$

is the vert depth of the bottom, and *s w* the vert depth of the e pressed surf, below the water surf. Or, in words: From the c, take the cube of *s w*; and call the rem *a*. Then, from the *s c*, take the square of *s w*; and call the rem *b*. Div *a* by *b*, two-thirds of the quot for *s m*.

Fig 16

en a plane surf of any shape whatever, whether ar, triangular, or circular, &c; whether vert as , or inclined as *m n*, is *entirely immersed*, so as to d over the entire area of *both* sides; but by *diff* water on its two sides; then the cen of pres coin- the cen *of grav* of the pressed surf.

foregoing figures the supposed surfaces are shown so that their widths do not appear.

Fig 17

any triangular plane surf, whether right-angled, or , as *a b c*, Fig 18; whether vert, or inclined; the *base* ich coincides with the hor surf of the water; the cen will be in the center of the line *c r*, which bisects the

Fig 18

if the triangle, as *a s c*, vert, or inclined, have its the surf of the water; and its base *s c*, *hor*; then the *x*, will also be in the line *a m* which bisects the base; ll be ⅔ of *a m*.

Fig 19

ny plane triangle *a b c*, Fig 19, base up, and hor; have its base d to some depth *n d*, with water; then the cen of pres *o*, will line *c s* which bisects the base; and *n o* will be equal to

$$\frac{m\,x^2 + (2\,m\,x \times m\,a) + 3\,m\,a^2}{(m\,x + 2\,m\,a) \times 2.}$$

center of pres against any *angular* surface, Fig 20, ert as *m n*, or inclined as ; having its top coinciding surf of the water; and y diff depths of water on te sides, as shown in the be *vert* below the *upper* , a dist equal to

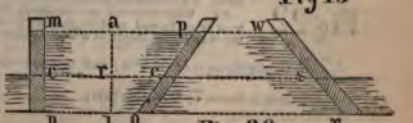

Fig 20

$$\left(\text{surf } m\,n, \text{ or } w\,x \times \frac{\text{sq of vert depth } a\,b}{3}\right) - \left(\text{area of surf } o\,n, \text{ or } e\,o, \text{ or } s\,x \times \frac{\text{sq of vert depth } r\,b}{3}\right) - \left(\text{vert depth } s\,r \times \text{area of surf } o\,n, \text{ or } e\,o, \text{ or } s\,x \times \frac{\text{vert depth } r\,b}{2}\right)$$

$$\left(\text{area of surf } m\,n, \text{ or } p\,o, \text{ or } w\,x \times \text{half of } a\,b\right) - \left(\text{area of surf } o\,n, \text{ or } e\,o, \text{ or } s\,x \times \text{half of } r\,b\right).$$

8. To find the cen of pres against either a circular, or an elliptic surf, pressed on one side only, whether vert, or inclined; and having its top either coinciding with the surf of the water, or below it.

Call the vert depth of the cen of pres below the water surf, h.
The vert (or inclined, as the case may be) *semi*-diam of the surf, r.
The *vert* dist of the *cen* of the pressed surf, below the water surf, d.

Then, $h = \frac{r^2}{4d} + d$.

Art. 10. Walls for resisting the pres of quiet water. A study of what we have said on resisting-walls for earth, will be of service in this connection. It is of course assumed that the water does not find its way under the wall; and that the wall cannot slide. In making calculations for walls to resist the pres of either earth, or water, it is convenient to assume the wall to be but one foot in *length*; (not height, or thickness;) for then the number of cub ft contained in it, is equal to that of the sq ft of area of its cross-section, or profile; so that these sq ft, when mult by the wt of a cub ft of the masonry, give the wt of the wall. In ordinary cases, it is well for safety to assume that the water extends down to the very bottom line of the wall.

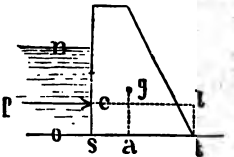

Fig 20 1/2

Now, by Art 1, the total pres of quiet water, against the *rectangular* back of a wall, whether vert or sloping, is found in lbs, by mult together the area in sq ft of the part actually pressed, (or in contact with the water;) half the *vert* depth of the water, in ft, (being the *vert* depth of the cen of grav of a rectangular back, below the surf;) and the constant 62.5 lbs; and this total pres is always *perp to the pressed area*.

When the back of the wall is vert, as in Fig 20½, this pres p is of course less than when it is battered; and is also *hor*; and it tends to overthrow the wall, by making it revolve around its outer toe, or edge t. The cen of pres is at e; $e s$ being ⅓ the *vert* depth $o n$; in other words, the entire pres of the water, so far as regards overthrowing the wall as one mass, (see Art 1, of Force in Rigid Bodies,) may be considered as concentrated at the point e: where it acts with an overthrowing leverage $t l$, (see Arts 41, 49, 50, Force in Rigid Bodies.) The pres in lbs, mult by this leverage in feet, gives the *moment* in ft-lbs of the overturning force; (see Art 49, Force in Rigid Bodies.) The wall, on the other hand, resists in a vert direction $g a$, with a moment equal to its wt, (supposed to be concentrated at its cen of grav g,) mult by the hor dist $a t$, which constitutes the leverage of the wt with respect to the point t as a fulcrum. If the moment of the water is greater than that of the wall, the latter will be overthrown; but if less, it will stand.

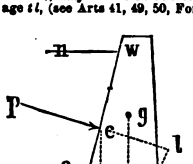

Fig 21

Rem. 1. Art 49 of Force in Rigid Bodies, will sufficiently explain the subjects of moments and leverage; and make it evident that the same principle applies also to sloping backs, as in Fig 21. Here the overturning moment of the water is equal to its calculated pres $p \times$ its leverage $t l$; while the moment of stability of the wall is equal to its wt $\times$ its leverage $a t$. By aid of a drawing to a scale, we may on this principle ascertain whether any proposed wall will stand. For we have only to calculate the pres p; then apply it at e, and at right angles to the back; prolong it to l; measure $t l$ by the same scale. Then calculate the wt of wall; find its cen of grav g: draw $g a$ vert, and measure the leverage $a t$. We then have the data for calculating the two moments. For finding the cen of grav, see Cen of Grav, Trapezoid, p 242.

Rem. 2. If the water, instead of being quiet, is liable to waves, the wall should be made thicker.

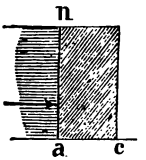

Fig 22

Art. 11. To find the thickness, $a c$, of a vert wall, Fig 22, sustaining quiet water level with its top, and as deep as the wall is high; so as to resist being overturned.

Rule. Div the number 1, or 1½, 2, 3, or &c, (according as the resistance of the wall is reqd to be either just equal to, or 1½, 2, 3, or &c, times as great as the overturning action of the water,) by 3 *times* the sp grav of the material of which the wall is built. Take the sq rt of the quot. Mult this sq rt by the vert depth of the water in ft. The prod will be the reqd thickness, in ft.

The sp gr of a dressed granite wall may be taken at 2.5; of dressed sandstone, 2.2; common mortar rubble, 2; brickwork, 1.8.

Ex. What must be the thickness of a vert wall, built of mortar rubble of a sp gr of 2; and reqd to present a resistance equal to 1.5 times the pres of the water; the depth of water, or height of wall $a n$, each being 20 ft? Here,

$\frac{1.5}{6} = .25$; and the sq rt of $.25 = .5$; and $.5 \times 20 = 10$ ft; the thickness, $a b$, reqd.

The following table is drawn up from this rule:

BLE. To find the thickness of a vert wall to sustain quiet r as deep as the wall is high; mult the height, in feet, e corresponding number in the table. (Original.)

	Sp. Gr. of Wall.	Weight of a Cub. Ft. of Wall.	The Resist of the Wall to be equal to 1½ times the pres of the Water.	The Resist of the Wall to be equal to 2 times the pres of the Water.	The Resist of the Wall to be equal to 3 times the pres of the Water.
		Lbs.			
Granite....	2.5	156	.447	.516	.633
Sandstone.	2.2	137	.477	.550	.674
tubble.....	2.	125	.500	.578	.707
rk.....	1.8	112	.527	.609	.746

the head of Retaining-Walls, will be found an original method for converting a vert wall, in which the back only is vert; and the face battered to any reqd degree; p. 339.

. 12. To find the thickness, *m n*, at the base of a right-d triangular wall, *w m n*, Fig 23, sustaining at its vert , the pres of quiet water level with its top, and as deep wall is high; so as to resist being overturned.

Div the number 1, 1½, or, &c, (according as the resistance of the wall be just equal to, or 1½, 2, or, &c, times as great as the overturning the water.) by *twice* the sp gr of the material of which the wall is ake the sq rt of the quot. Mult this sq rt by the depth of water, or wall, in ft. The prod will be the reqd thickness, *m n*, in feet. *rginal.*) mult the thickness, *m o*, of a vert wall by 1.225. s before; wall of rubble, of sp gr of 2; resistance to be 1.5 times the he water, depth 20 ft. Reqd the thickness, *m n*. Here,

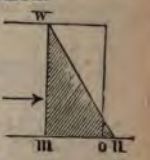

Fig 23

375; the sq rt of .375 = .6124; and .6124 × 20 = 12.25 ft = *m n*. reqd.

se of vert wall in preceding example 10 ft; and 10 × 1.225 = 12.25 fore.

lowing table is drawn up from this rule:

le. Mult the vert height of the wall, or of the water, by the corresponding in the table; the prod will be the base of the triangular wall. (Original.)

	Sp. Gr. of Wall.	Weight of a Cub. Ft. of Wall.	The Resist of the Wall to be equal to 1½ times the pres of the Water.	The Resist of the Wall to be equal to 2 times the pres of the Water.	The Resist of the Wall to be equal to 3 times the pres of the Water.
		Lbs.			
Granite...	2.5	156	.548	.633	.775
Sandstone.	2.2	137	.584	.675	.826
ubble.....	2.	125	.613	.707	.866
k.....	1.8	112	.646	.746	.913

hstanding their greater thickness at base, such triangular walls contain, as seen by the fig, more than half the quantity of masonry reqd for vert ones of equal strength. This is owing ct that their cen of grav is thrown farther back; thus increasing the leverage by which the wall resists overthrow.

13. To find the thickness, *a b*, at ase of a wall. Fig 24, with a vert *s a*, and with a face, *n b*, having iven batter; to sustain the pres iet water level with its top, and ep as the wall is high; so as to being overturned.

Square the vert depth of the water, or wall, in feet. square by 1, or by 1½, 2, 3, or, &c, (according as the e of the wall is reqd to be just equal to, or 1½, 2, 3, or, as great as the overturning action of the water.) Call *a*. Square the entire amount of batter, *h n*, in feet. square by the sp gr of the masonry of which the wall *call the prod. b. Add* together the prods, *a* and *b*. Div *y 3 times the sp gr of the* masonry. Take the sq rt of *This sq rt will be the reqd base, a b.*

Fig 24

ble wall 20 ft high, sp gr 2; batter of face 4 inches to a ft; or total batter *h n*, = 6.666 ft

What must be the thickness $a\,b$, at base, that the resistance of the wall shall be 1.5 ti of the water?

Here, the square of $20 = 400$; and $400 \times 1.5 = 600$, or prod a. Again, the square of 6. and $44.44 \times 2 = 88.88$, or prod b. Now, $600 + 88.88 = 688.88$; and $\frac{688.88}{6} = 114.81$; and 10.715. $= a\,b$, the base reqd.

The following table is drawn up from this rule:

TABLE. Mult height of wall, or of the water, by the sponding number in the table; the prod will be the

Original.

	Sp. gr. of Wall.	Wt. of a cub. ft. of Wall. Lbs.	The Resist of the Wall to be equal to 1½ times the pres of the Water. Batter 1 in to a foot.	Batter 2 ins. to a foot.	Batter 4 ins. to a foot.	Batter 6 ins. to a foot.	The Resist of the Wal to twice the pres of Batter 1 in. to a foot.	Batter 2 ins. to a foot.	Bat 4 ins a fo
Dressed Granite..	2.5	156	.449	.458	.487	.532	.519	.526	.55
Dressed Sandstone	2.2	137	.480	.488	.515	.558	.552	.560	.58
Mortar Rubble....	2.	125	.502	.510	.536	.578	.571	.586	.60
Brickwork	1.8	112	.530	.539	.562	.602	.610	.618	.64

The greater the batter, and consequently the greater the base, the less masonry is re the same strength.

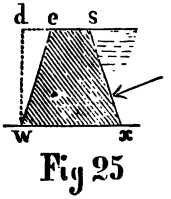

Fig 25

Art. 14. To find the thickness $w\,x$ base of a wall, Fig 25, battering equ the face and on the back; and sust quiet water level with its top; deep as the wall is high; neglecti vert pres of the water upon the b back $x\,s$. From Jamieson's Hydraulics.

RULE. Square the vert depth of water, or wall, in ft. square by 1, 1½, 2, 3, or, &c. (according as the resistance o reqd to be either just equal to, or 1½, 2, 3, or, &c. times as overturning action of the water.) Div the prod by 3 tim of the material of which the wall is built. Call the quot, *one half* of the entire batter in feet, on *one* side; that is, *half* of $d\,e$, in feet. Add this square to the prod a. Tak of the sum. To this sq rt add *half* the batter on *one* side, (or half of $d\,e$.) The sum reqd base, $w\,x$, in ft.

Ex. Rubble wall 20 ft high; sp gr 2; batter on the back, as well as on the face, 3 ins t $= 5$ ft; what must be the thickness, $w\,x$, at its base, that the resistance of the wall shall the pres of the water?

Here, the sq of $20 = 400$; and $400 \times 1.5 = 600$; and $\frac{600}{6} = 100$, or prod a. Again, batter of *one* side, is $\frac{5}{2} = 2.5$; and the sq of $2.5 = 6.25$; and $6.25 + 100 = 106.25$; and 10.308; and $10.308 + 2.5 = 12.808$, or $w\,x$, the reqd base. The thickness at top will of c base minus the batter on both sides, or $12.808 - 10 = 2.808$.

REM. In this case, as in all others, the pres of the water is actually at right angles of the wall; but in omitting the vert portion of the pres, (in order to avoid excessive c in the calculations,) the pres is assumed as that due to a vert back, of the same depth as and consequently as being hor. Hence the assumed *amount* of pres is less than the act assumed *leverage* being consequently increased in a much greater proportion, the result ness greater than necessary; and so much so, that except for very small batters, say no an inch to a ft, we can hardly consider the rule as an approximation. Thus, the base in c presents a resist equal to fully 2½, instead of 1½ times, the actual overturning tendency o The rule is from Jamieson's "Hyd for Practical Men." All such cases may be corr by the mode of *trial* from a drawing made to a scale; as already hinted in Remark 1. A method is very simple, and by no means so tedious as it might at first sight appear to be.

The following table is drawn up from Jamieson's rule; it is of course safe; or gives bas of the truth, especially for great batters.

TABLE. Mult vert height of wall, or of the water, by t responding number in the table. The prod will be the

	Sp. gr. of Wall.	Weight of a cub. foot of Wall. Lbs.	The Resist of the Wall to be equal to 1½ times the pres of the Water. Batter on each side, 1 in. to a ft.	Batter on each side, 2 ins. to a ft.	Batter on each side, 4 ins. to a ft.	The Resist of the equal to twice the Water. Batter on each side, 1 inch to a ft.	Batter each s 2 ins a
ressed Granite...	2.5	156	.492	.539	.646	.560	
essed Sandstone.	2.2	137	.521	.568	.673	.594	
rtar Rubble.....	2.	125	.544	.591	.695	.621	
ckwork.....	1.8	112	.571	.618	.721	.652	

Art. 15. The following table (although not scrupulously orrect) shows how greatly the safety of a wall sustaining ater, is affected by its form; the quantity of masonry re- aining the same. It will be observed that the vert wall is the least safe all in the table. The overturning tendency of the water is here taken as 1.

In France are dam walls from 40 to 70 ft high, with bases of $\frac{7}{10}$ height; the face battering $\frac{4}{10}$ of the height; the back, against which the water presses, being rt. These correspond to No. 12 of the following table, for which the resist is 7.6. But if the water essed against the battered side, the resist would be 4.9; as at No. 6. of the table; and if both sides re battered to the same extent as in No. 18, the resist would be about 36, but with a wider base.

	(Original.) **All these walls contain precisely the same quantity of masonry.** The masonry is supposed to be mortar rubble, weighing 125 lbs per cubic foot; or twice as much as water; or about the same as ordinary rough mortar rubble. If the sp gr of the masonry is actually greater or less than this, the safety also will be greater or less, in precisely the same proportion.	Base in parts of height.	Approx resist of wall.
1	Vertical wall	.5	1.5
2	Face vertical; back batters one-tenth height	.55	1.8
3	" " " " one-fifth "	.6	2.2
4	" " " " one-fourth "	.625	2.6
5	" " " " one-third "	.667	3.5
6	" " " " four-tenths "	.7	4.9
7	" " " " one-half "	.75	14.0
8	Back vertical; face batters one-tenth height	.55	1.8
9	" " " " one-fifth "	.6	2.1
10	" " " " one-fourth "	.625	2.2
11	" " " " one-third "	.667	2.4
12	" " " " four-tenths "	.7	2.6
13	" " " " one-half "	.75	2.9
14	Back and face, each batter one-tenth height	.6	2.2
15	" " " " one-fifth "	.7	3.4
16	" " " " one-fourth "	.75	4.6
17	" " " " one-third "	.833	9.0
18	" " " " four-tenths "	.9	36.0†

It will be observed that in the cases in the table, walls with battered backs are *safer* than these with attered faces; and that the diff increases much more rapidly than the batter; and that these which re battered on both sides, are safer than either.

When the wall is triangular, as in Fig 23, the angle at *m* being 90°, and e sp grav of the masonry 2; if the base *m n* is equal to ½ the height *m w*, the depth of the water ing equal to *m w*, the wall will just *balance* the water, or will be equally safe, whether the pres be s the vert or on the battered side. And in this single case only, the moments of the wall and the ater, when the pres is against the vert side, are each respectively twice as great as when the pres on the battered side. The thickness of a vert wall of the same sp grav, to barely balance water, deep as the wall is high, must be .408 of its height. When the base of the triangular wall, of sp rav 2, is less than ½ the height, the wall is safest when the water presses the vert side; but if the ase is greater than ½ the height, it is safest with the water on the battered side. Thus, with a ase of .6 height, it is safer in the proportion of 1⅓ to 1.

We have used the expression "safe," instead of "strong;" ecause the strength of the wall does not increase in proportion to its safety. The pres of the water greater against the safer walls, than against the less safe ones; and the walls become safer chiefly ecause their form diminishes the length of *leverage* through which the water acts to overturn them. hus, in Fig 24, when the water presses on the vert side *a s*, its leverage about the toe *b* as a fulcrum equal to *o b*; but when it presses on the battered side *n b*, although the pres is actually greater an before, in the proportion of *n b* to *a s*, yet its *leverage a i* is so much smaller than the leverage *b*, that the overturning *moment* of the water is less than before. Thus the wall becomes *safer* when e pres is on its battered side; although it is then actually *weaker* than with the pres on the vert de; that is to say, that if the wall would precisely *balance* a pres *in any given direction* against s vert side; it would overturn under an equal pres. *in that same direction*, against its battered side. ut the direction of the pres of *liquids* against the battered side, is not the same as that against the rt side; in both cases, it is perp to the side pressed.

Art. 16. To find the thickness of a cylinder, to resist safely he pres of water, steam, &c, against its interior.

RULE. Mult the pres in lbs per sq inch, by *half* the bore or inner diam of the cylinder in ins; illing the prod *a*. From the safe cohesive strength in lbs per sq inch, of the material of which the linder is made, subtract the pres in lbs per sq inch. Div the prod *a* by the rem; the quot will be e reqd thickness in ins. *Or, as a formula,*

$$\frac{\textit{Pres in lbs per sq in} \times \text{half bore in ins}}{\textit{safe cohesive strength in lbs per sq inch} - \textit{pres in lbs per sq inch}} = \text{thickness in ins.}$$

. *A wrought-iron cylinder of 3 ft diam, is subjected to an internal pres of 200 lbs per sq i thick must it be, assuming the safe cohesive strength of the iron at 10000 lbs per sq inch.*

e, 200 (lbs) × 18 (ins) = 3600, = prod *a*. And 10000 — 200 = 9800; and $\frac{3600}{9800}$ = .367 of d thickness.

If the cylinder is *single-riveted* wrought iron, it must be made nearly twice as thick as it would otherwise be, to allow for the weakening of the iron by cutting out the rivet holes. See Art 44, of Strength of Materials, p 217.

REM. **Theoretically this rule applies also to cylinders of cast iron,** such as water pipes for sustaining the pres arising from diff heads of water; but with that material, more thickness must be allowed in practice, to compensate for defective casting, whether arising from air-bubbles, or from inequality of thickness; as well as to enable the pipes to bear the necessary handling; and the undue strains to which they are frequently exposed from heavy loads crossing them in the streets, or from irregular settlements of the soil in which they are laid. Observation leads the writer to suggest the following additions to the theoretical thicknesses calculated by the foregoing rule, and exhibited in the following table. Add ¼ inch to the thicknesses in the table. But if the thickness thus found is less than ⅜ inch, make it ⅜ or .375 inch. Finally, if this thickness is less than ⅙ part of the sq rt of the diam in ins, make it equal to said ⅙ part. The strongest pipes, if laid under high embankments, require a very firm foundation to prevent their being broken; so that at times it is expedient to protect them by masonry.

Table of theoretical thicknesses of cast-iron pipes, for a safety of 3 to 1 against internal pres; assuming the ultimate or breakg strength of ordinary cast iron to be 15000 lbs per sq inch; and its safe strength of 3 to 1 against quiet pres, 5000 lbs. If a safety of more than 3 to 1 is reqd, as, for instance, one of 4 to 1, or 5 to 1; then first div 15000 by 4, or by 5, &c, as the case may be; and use the quot 3750, or 3000, &c, instead of 5000, as the safe cohesive strength per sq inch.

(Original.)

Inner Diam. or bore of Pipe, in inches.	Head of Water, in Feet.													
	100	150	200	250	300	400	500	600	800	1000	1200	1400	1600	2000
	Resulting Pressure against sides of Pipe, in pounds per sq inch.													
	43.4	65.1	87	109	130	174	217	260	347	434	521	608	694	868
	Theoretical thickness of cast-iron water-pipe, or cylinder, in inches; for a safety of 3 to 1. In practice, add ¼, or .25 inch. But if the thickness thus found is less than ⅜ of an inch, make it ⅜, or .375 inch. Finally, if this thickness is less than ⅙ part of the square root of the diam in inches, make it equal to said sixth part.*													
2	.009	.013	.018	.022	.027	.036	.045	.055	.075	.095	.116	.139	.161	.210
3	.013	.020	.026	.033	.040	.054	.068	.082	.112	.143	.173	.207	.242	.315
4	.017	.026	.035	.045	.053	.072	.090	.110	.149	.191	.232	.278	.322	.420
5	.022	.033	.044	.056	.067	.090	.113	.137	.186	.237	.290	.347	.402	.525
6	.026	.040	.053	.067	.080	.108	.136	.165	.224	.287	.347	.415	.485	.630
7	.030	.046	.062	.078	.093	.126	.159	.193	.261	.333	.406	.485	.565	.735
8	.034	.053	.071	.089	.107	.144	.181	.220	.298	.382	.465	.556	.644	.840
9	.039	.059	.079	.101	.120	.163	.205	.247	.335	.427	.520	.622	.724	.945
10	.044	.066	.089	.112	.134	.181	.227	.275	.373	.475	.580	.695	.805	1.05
12	.053	.080	.106	.134	.161	.217	.273	.330	.448	.575	.695	.830	.970	1.26
14	.061	.093	.124	.156	.187	.253	.318	.387	.523	.666	.813	.970	1.13	1.47
16	.069	.106	.142	.178	.214	.288	.363	.440	.596	.763	.930	1.11	1.29	1.68
18	.078	.120	.159	.201	.242	.326	.409	.495	.670	.850	1.04	1.24	1.45	1.89
20	.088	.132	.177	.223	.267	.361	.454	.549	.746	.950	1.16	1.39	1.61	2.10
24	.105	.159	.213	.268	.321	.433	.545	.660	.895	1.15	1.39	1.66	1.94	2.52
30	.132	.198	.267	.336	.402	.543	.681	.825	1.12	1.42	1.74	2.08	2.41	3.15
36	.156	.238	.318	.402	.483	.651	.819	.990	1.34	1.71	2.08	2.49	2.91	3.78
42	.184	.279	.372	.469	.562	.759	.955	1.16	1.57	2.00	2.44	2.91	3.39	4.41
48	.210	.317	.425	.535	.641	.866	1.09	1.32	1.79	2.29	2.79	3.32	3.87	5.04

The writer published this table many years ago; but since that time experiments in testing the strength of water pipes by the hydraulic press, have shown that the additional thicknesses then proposed by him, are not as correct as those now suggested.

REM. 1. Under a pressure of 250 lbs per sq inch, equal to a head of 576 feet, (in testing the iron pipes of the Philadelphia water system,) **the water oozes through the pores of pipes ⅝ to ¾ ins thick**; or at times even forms a spray from 2 to 6 ins high. The pipes are of a gray iron, so as to admit of being cut by a chisel. In more porous irons the spray may extend even to some feet; and that under a considerably less pressure. Small particles in the water soon clog the pores; and this oozing then ceases.

* EX. *What must be the thickness of a cast-iron water-pipe, 2 ft. or 24 ins inner diam, to sustain with a safety of 3 to 1, the pres of a head of 100 feet, or 43.4 lbs per sq inch. Here by the table, the theoretical thickness is but .105 of an inch. Adding .25 inch we get .355 inch; but as this is less than .375 inch, we increase it to ⅜ or .375 inch. Finally, the sq rt of 24 is 4.9; the sixth part of which is .82 inch; and since this is greater than ⅜ inch, we make the thickness .82 inch. The addition of ¼ inch is chiefly to guard against inequality of thickness arising from imperfect moulding and casting, which it is almost impossible to avoid to that extent in ordinary water pipes. If the work cannot be insured to be very carefully performed, the addition should be increased to at least ½ inch; and in large pipes, say exceeding 3 ft diam, to ¾ inch. Differences of even 8 or 10 per cent are not at all uncommon in the wts of pipes made at the best foundries, and intended to be precisely the same. They should be cast vertically; and although good ones can be made directly from the pig, they will be stronger and more even, if made from pigs.*

REM. 2. **When pipes have to bear the percussive effect produced by suddenly arresting the flow of the water,** by closing the valves *quickly*, they must be made thicker. The valves or gates of large pipes must be closed very slowly, by means of a screw. See Hydraulics, Art 28, for such valves, p 572.

REM. 3. **The theoretical thickness would be too small for very thin lead pipes,** as well as for iron ones, inasmuch as they would bend and crush under ordinary handling; nor could they well be manufactured so thin; moreover, they would burst if the flow were suddenly arrested. Thus, if we assume the safe cohesive force of milled lead to be 800 lbs per sq inch, or ¼ of its ultimate strength, then the theoretical thickness for a bore of ½ an inch, to sustain safely a head of 100 feet, would be but .014, or $\frac{1}{70}$ of an inch; yet service pipes of that bore, for even less heads, are rarely made *less* than ⅛ or .125 inch thick; or 9 times the theoretical thickness, in order to sustain the percussive force caused by closing the ordinary house stopcock quickly. Thinner ones frequently burst.

The following table shows the theoretical thickness for lead pipes, assuming 800 lbs to be the safe cohesion.

Head in Feet.	Bore ½ inch.	Bore 1 inch.	Bore 2 inches.
	Thickness in Ins.	Thickness in Ins.	Thickness in Ins.
100	.014	.029	.058
200	.030	.061	.122
300	.048	.097	.194
500	.093	.187	.374

Art. 17. The buoyancy of liquids. When a body is placed in a liquid, whether it float or sink, it evidently displaces a bulk of the liquid equal to the bulk of the immersed portion of the body; and the body in both cases, and at any depth, and in any position whatever, is buoyed up by the liquid with a force equal to the wt of the liquid so displaced. Thus, if we immerse entirely in water a piece of cork *c. c*, Fig 26, or any body of less sp gr than water, the cork will by its wt, or force of gravity, tend to descend still deeper; but the upward buoyant force of the water, being greater than the downward force of gravity of the cork, will compel the latter to rise with a force equal to the diff between the two. In this case, the cork receives a total downward pres equal to the wt of the vert column of water above it, shown by the vert lines in vessel 1; and a total upward pres equal to the wt of the column shown in vessel 2. The diff between these two columns is evidently (from the figs) equal to the bulk of the cork itself; therefore the diff between their wts or pressures, (or, in other words, the buoyancy of the water,) is equal to the wt or pres of the water which would have occupied the place of the cork: or, in other words, of the water which is displaced by the cork. This *diff*, or buoyancy, will plainly be the same at any depth whatever of *entire* immersion. Now the cork, if left to itself, will continue to rise until a portion of it reaches above the surf, as in vessel 3; so that the downward pressing column ceases to exist; and the cork is then pressed downward only by its own wt. But as it now remains stationary, we know (from the fact that when two opposite forces keep a body at rest, they must be equal to one another) that the upward pres of the water must be equal to the wt of the cork. But the upward pres of the water arises only from the shaded column shown in vessel 3; and this column is (as in the case of total immersion) equal to the bulk of water displaced. Therefore, *in all cases*, the buoyancy is equal to the wt of water displaced; and when the body *floats* on the surf, the buoyancy, or the wt of water displaced, is also equal to the wt of the body itself.

Fig 26

If the immersed body *c. c.* be of iron, or any other substance specifically heavier than water, the diff between the upward and downward pres will of course remain the same; or equal to the wt of water displaced. But the wt of the body is now greater than that of the water which it displaces; or, in other words, the downward force of gravity of the body is greater than the upward buoyant force of the displaced water; and therefore the body descends, or sinks, with a force equal to the diff between the two. Thus, if the body be a cub ft of cast iron, weighing 450 lbs, while a cub ft of fresh water weighs 62½ lbs, the iron will descend with an effective force of only 450 − 62½ = 387.5 lbs.

If the immersed body has the same sp gr as the fluid, it will neither rise nor sink; but will remain wherever it is placed; because then the wt of the body, and the buoyancy of the water, are equal.

The air also buoys bodies upward to an extent equal to the wt of air displaced; therefore, although a pound of iron, and a pound of feathers, weighed in the air, will balance each other yet in the exhausted bell-glass of an air-pump the feathers will outweigh the iron, by as much as the bulk of air which they displaced outweighs the bulk of air displaced by the iron.

A balloon rises in the air on the same principle that cork rises in water. Its ascending force is equal to the diff between its wt with full of gas, and the wt of the bulk of air which it displaces. The balloon does not actually seek to rise, but to descend; but the air being, bulk for bulk, heavier than the balloon, pushes the balloon upward with more force than the gravity, or the wt of the balloon, exerts to bring it down. So warm smoke has no tendency in itself to rise. It is pushed up by the heavier cold air. No bodies tends to rise; but all tend downward toward the center of the earth.

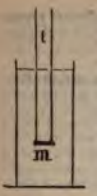

Fig 27

Art. 18. A body lighter than water, if placed at the bottom of a vessel containing water, will not rise unless the water can get under it, to buoy it, or press it upward, as the air presses a balloon or smoke upward. Thus, if one side of a block of light wood, *perfectly* flat and smooth, be placed upon the similarly flat and smooth bottom of a vessel, and held there until the vessel is filled with water, the downward pres will keep it in its place, until water insinuates itself beneath through the pores of the wood. But if the wood be smoothly varnished, to exclude water from its pores, it will remain at the bottom.

On the other hand, a piece of metal may be prevented from sinking in water, by subjecting it to a sufficient *upward* pres only, while the downward pres is excluded. Thus, if the bottom of an open glass tube, *t*, Fig 27, and a plate of iron *m*, be made smooth enough to be water-tight when placed as in the fig; and if in this position they be placed in a vessel of water to a depth greater than about 8 times the thickness of the iron, the upward pres of the water will hold the iron in its place, and prevent its sinking; because it is pressed upward by a column of water heavier than both the column of air, and its own weight, which press it downward. On this principle iron ships float.

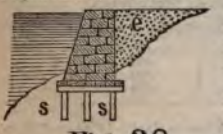

Fig 28

REM. 1. **A retaining-wall, as in Fig 28, founded on piles,** may be strong enough to resist the pres of the earth *e* behind it, in case water does not find its way underneath; and yet may be overthrown if it does; or even if the earth *s s* around the heads of the piles becomes saturated with water so as to form a fluid mud. In either case, the upward pres of the water against the bottom of the wall will virtually reduce the wt of all such parts as are below the water surf, to the extent of 62½ lbs per cub ft; or nearly one-half of the ordinary wt of rubble masonry in mortar.

REM. 2. Although the piles under a wall, as in Fig 28, may be abundantly sufficient to sustain the wt of the wall; and the wall equally strong *in itself* to resist the pres of the backing *e*; yet if the soil *s s* around the piles be soft, both they and the wall may be pushed outward, and the latter overthrown by the pres of the backing *e*. From this cause the wing-walls of bridges, when built on piles in very soft soil, are frequently bulged outward and disfigured. In such cases, the piling, and the wooden platform on top of it, should extend over the whole space between the walls; or else some other remedy be applied.

Art. 19. Draught of vessels. Since a *floating* body displaces a wt of liquid equal to the wt of the body, we may determine the wt of a vessel and its cargo, by ascertaining how many cub ft of water they displace. The cub ft, mult by 62½, will give the reqd wt in lbs. Suppose, for instance, a flat-boat, with vert sides, 60 ft long, 15 ft wide, and drawing unloaded 6 ins, or .5 of a ft. In this case it displaces $60 \times 15 \times .5 = 450$ cub ft of water; which weighs $450 \times 62\frac{1}{2} = 28125$ lbs; which consequently is the wt of the boat also. If the cargo then be put in, and found to sink the boat 2 ft *more*, we have for the wt of water displaced by the cargo alone, $60 \times 15 \times 2 \times 62\frac{1}{2} = 112500$ lbs; which is also the wt of the cargo. So also, knowing beforehand the wt of the boat and cargo, and the dimensions of the boat, we can find what the draught will be. Thus, if the wt as before be 112500 lbs, and the boat 60×15, we have $60 \times 15 \times 62\frac{1}{2} = 56250$; and $\frac{112500}{56250} = 2$ ft, the required draught. In vessels of more complex shapes, as in ordinary sailing vessels, the calculation of the amount of displacement becomes more tedious; but the principle remains the same.

Art. 20. Compressibility of liquids. Liquids are not entirely incompressible; but for most engineering purposes they may be so considered. The bulk of water is diminished but about one-thousandth part by a pres of 300 lbs per sq inch, or 20 atmospheres; varying very slightly with its temperature. It is perfectly elastic; regaining its original bulk when the pres is removed.

HYDRAULICS.

Art. 1. Hydraulics treats of the flow or motion of water through pipes, aqueducts, rivers, and other channels; also through orifices or openings of various kinds; of machinery for raising water; as well as that in which water furnishes the moving power. The science of hydraulics, in many of its departments, is but imperfectly understood; therefore, some of the rules given on the subject are to be regarded merely as furnishing close approximations to the truth.

On the flow of water through pipes.

Inasmuch as the experiments on which the following rules are based, were made with pipes carefully laid in straight lines; and perfectly free from all obstructions to the flow of the water, some allowance must in practice be made for this circumstance. Workmen do not lay long lines of pipes *ds perfectly straight* lines; it is almost impossible to avoid very numerous, although slight devia-*tions, both vert and hor*; the soil itself, in which the pipes are imbedded, especially when in embk, *will settle unequally*; especially in streets liable to heavy traffic, which not only frequently derange, *but occasionally* breaks water pipes whose tops are 3 or 4 ft below the surf. The material used for *alking the joints*, may be carelessly left projecting into the interior, and thus cause obstructions; the *ater is frequently* muddy, or is impregnated with certain salts, or gases, which form deposits, *crustations*, wh ch materially impede the flow. See Art 27. Moreover, the pipes themselves are *t perfectly straight*, or smooth, or of uniform diam; and irregular swellings, by producing eddies

retard the flow as well as *contractions;* and accumulations of air do the same. Under the most favorable circumstances, therefore, it is expedient to make the diams of pipes, even for temporary purposes, sufficiently large to discharge *at least* 5 per cent more than the quantity actually needed; and if there is occasion to anticipate deposit, or incrustation, a still larger allowance should be made in permanent pipes, especially in those of small diam; because in them the same thickness of incrustation occupies a greater *comparative* portion of the area. Perhaps it would be best to allow an equal increase, of say from ½ to 1½ inch, to each diam, whether great or small; inasmuch as the thickness of incrustation will be the same for all diams, or nearly so. The cost of pipes does not increase as rapidly as their discharging capacities; thus, if the diam be increased only $\frac{1}{10}$ part, the disch will be increased about 25 per cent; if ⅕ part, nearly 50 per cent; if ⅓ part, the disch will be doubled. See Table 2. Within these limits, the increase of thickness for the larger diams, and the increased expense of laying, will add but little to the cost; which will therefore augment only a little more rapidly than the diams.

The increased diam involves no waste of water; since the disch may be regulated by stopcocks.

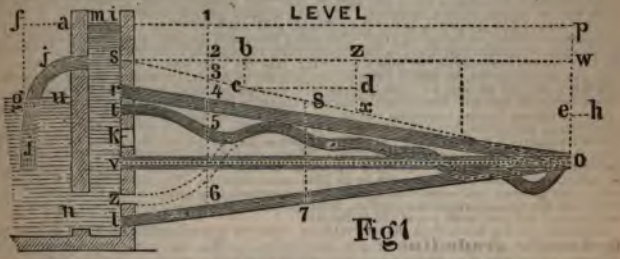

Fig 1

The term **HEAD** or **TOTAL HEAD** of water, as applied to the flowage of water through canals, pipes, or openings in reservoirs, &c. means the *vert* dist *i v* or *p o*, Fig 1, from the level surf, *m i*, of the water in the reservoir, or source of supply, to the center (or more properly to the cen of grav) *o*, of the orifice (whether the end of a pipe, *r o*, *t o*, *v o*, *z o*, *l o*; or any other kind of opening) through which the disch takes place freely, *into the air;* or the vert dist *a u*, or *f p*, from the same surf, *m i*, to the level *surf*, *g u*, of the water in the lower reservoir; when the disch takes place *under water*. Thus, in the case of disch *into the* air, the vert dist *i v* or *p o*, is the total head for either of the pipes *r o*, *t o*, *v o*, *z o*, or *l o*; and *i k* is the head for the orifice, *k*, in the side of the reservoir. And for disch *under water*, *a u*, or *f g*, is the head for either the pipe *f*, or the opening *n*; without any regard whatever to their depths below the surf of the lower water; which, according to the older authorities, do not at all affect their disch. Weisbach, however, a more recent experimenter, says the disch will be about $\frac{1}{75}$ part less under water, than into the air.

In the case of circular pipes or openings; or of rectangular, or many other shaped openings, the center, and the cen of grav coincide; but in triangular or trapezoidal ones, they do not; hence in *all cases of disch into air* the head must be measured to the cen of grav of the disch opening; as it is this head alone that causes the flow. The pres which it imparts to the water, overcomes the friction opposed by the sides and bottom of the channel, and thus enables the water to advance. After water has descended along any channel, to any given dist, that head by which it was driven through that dist, is sometimes called *lost head;* or the water is said to have *lost so much head* in that dist.

A portion of a pipe may have a head greater than the total head of the entire pipe. Thus the point 6 in the pipe *l o*, has a head 6 1; while the entire pipe has only the head *p o*.

Both in theory and in practice it is immaterial as regards the vel, and the quantity of water discharged, whether the pipe is inclined downward, as *r o*, Fig 1; or hor, as *v o*; or inclined upward, as *l o*; provided the total head *p o*, and also the length of the pipe, remain unchanged. If one pipe is longer than another, its sides will evidently present more friction against the water, and thus diminish the vel and the quantity of disch. The inclined pipes, *r o*, *l o*, being of course a little longer than the hor one *v o*, will therefore each disch a trifle less water; but if the hor one were extended slightly beyond *o*, so as to give it the same length as the others, then each of the three would disch the same quantity in the same time.

Rem. 1. It is evidently necessary, however, in every case, that the upper, entry, or reservoir end (*r*, *v*, or *l*) of the pipe, should be placed so far below the surf *m i* of the water, as shall allow the water to flow from the reservoir *into* the pipe, as fast as it afterward flows *along* or *through* the length of the pipe to the lower end. For the total head, *p o*, must be considered as having two distinct duties to perform by means of its pres; and it may be imagined to be divided into two parts respectively appropriated to those duties.

Thus, suppose the total head *i v*, or *p o*, to be so divided into the two parts *p w* or *i s*, and *w o* or *s v*. Then *p w* or *i s* *must be at least* such as would allow the water in the reservoir to flow out of an *opening at v, freely into the* air, with that vel which previous calculation shows that the water in *the pipe will have. The part w o* or *s v* balances or counteracts the resistance which the inner surf *of the pipe opposes to the flow of* water through it. This resistance arises from friction; and *pipes of the same diam*, its amount varies in the same proportion as their lengths. Other resistances *may arise from bends in the pipe;* or from accumulations of air or sediment; but these will be *dered separately. After the part w o* or *s v*, therefore, has perfectly balanced the friction in the *ere is nothing else to prevent the water from flowing along it;* and accordingly it will do

with the vel due to the total head *i v or p o*, but evidently with that due to what remains of it after deducting the part *s v* or *w o*, that was counteracted, or, as it were, destroyed, by friction; that is, to *i s* or *p w*. Hence we see that the lower part performs the duty of counteracting friction; and may therefore be called the **friction head**; while the upper part produces vel, (that is, gives motion,) and is hence called the **velocity head.**

Since the friction, in pipes of the same diam, increases in amount in the same proportion as their lengths, the water when it first *enters* the pipe encounters but little friction, and has great vel; but this gradually decreases as the advancing water encounters the friction along increased lengths of the pipe; and finally becomes slowest when the water fills the whole length, and begins to flow from the disch end *o*. The vel then becomes uniform along the pipe; and will continue to be so, if the vel head *i s* is sufficient to allow the water of the reservoir to enter the pipe with that same vel. If even much more than the actual vel head *i s* is left above the entrance into the pipe, the effect of the surplus is not at all diminished thereby as regards overcoming the friction along the inside of the pipe; and consequently the vel of the disch will undergo no change. Thus, if *i s* be sufficient vel head for a pipe laid direct from *s* to *o*, and if the pipe be afterward changed to the position *i o*, the vel and disch will be the same in both positions. Therefore nothing more is necessary than to be certain that the depth below the surface *m i* of the reservoir shall not be *less* than *i s*. Theoretically, *s* is at the center (or rather, at the cen of grav) of the entry opening of the pipe, or aperture; and the head *i s*, above it, is equal to that found in any case in Table 10, opposite to the vel in the pipe. Thus, if we have found by calculation (by Rule 1, following) that the total head *p o* will produce along the pipe a vel of 6 ft per sec, we find in Table 10, opposite the vel of 6 ft, the head of .56 foot, or 6.72 ins; and this is the true vel head *i s*; or *is* the *least* dist that the center of the end of the pipe must be placed below the surf *m i*. But the end of the pipe should in practice always be entirely below water; otherwise air and floating impurities will be drawn into it, and cause obstructions; therefore, if the pipe is large, say in this case 3 feet in diam, it is evident that the center of its upper end cannot be placed less than 1½ ft under water; or nearly 3 times as deep as the true theoretical vel head. Moreover, the water surface of reservoirs is always liable to considerable changes of height; so that the end of the pipe must be placed at such a depth that the water can flow into it at the lowest stages. As before stated, this will be attended by no diminution of disch. The above vel of 6 ft per sec, is full 4 miles an hour, and is one seldom reached in water pipes in practice; hence we see that in ordinary cases the vel head required by theory, rarely need exceed 6.72 ins; which is usually but a very small part of the total head, *p o*, of the entire pipe.

A straight line *s o*, drawn from the true vel head to the cen of grav *o* of the disch end, is called the **hydraulic grade-line** of any pipe *s o*, *r o*, *t o*, *v o*, *z o*, or *l o*, commencing at the reservoir, and ending at *o*; and any such pipe may be bent into *easy* curves, (having radii not less than 5 diams of the pipe,) as *t o*, *z o*; and still deliver as much water as a straight one *of the same actual length*, provided the tops of its highest bends are kept below this hydraulic grade-line; and provided that arrangements be made for the escape of any air that may accumulate at the tops of the bends. See Art 5, and Fig 42.

If the hydraulic grade-line *s o* be divided into any number of equal parts, as *s c*, *c x*, &c; and if the actual length of any pipe terminating at *o*, be also divided into the same number of equal parts, then will *b c* be the friction head of the first division of the pipe; *x x* that for the first two divisions, &c. If the pipe commences vertically under *s*, like all those in Fig 1; and is straight, like *r o*, *v o*, or *l o*, the equal divisions of the pipe will be vert under those of the line *s o*. But if the pipe be curved either hor, or vert, like *t o* and *z o*, this of course will not be the case.

Rem. 2. **A great diff exists between the condition of a hor pipe, and one inclining downward from the reservoir, in case it should become necessary to prolong them in their original directions.** Thus, if we extend the length of the hor pipe *r o* beyond *o*, it is plain that we do not thereby increase the total head of water; for the vert dist from the surf of the reservoir, to the disch end of the pipe, will still remain equal to *p o*. Consequently the additional friction along the sides of the extended pipe, will cause the water to flow more slowly than before; or, in other words, the disch will be less. Now suppose an inclined pipe *s o*, *starting from the exact vel head s*. It is plain that every equal dist, *s c*, *c x*, &c, along this pipe, has its equal friction head *b c*, *d x*, &c; and however far the pipe may be extended beyond *o*, with the same degree of inclination, its friction head will be extended in the same proportion; and consequently the friction along the additional length will be thereby counteracted, so that no change will take place in the vel of the flow; nor, consequently, in the quantity discharged. The vel head will remain the same as before, for it still has merely to supply the same vel of water as at first. Such a pipe is said to be **in train.**

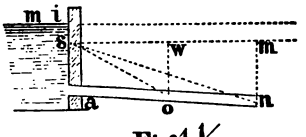

Fig 1¼

This will not be the case *to the same extent* if the entry end of the pipe, as *a*, Fig 1¼, be placed *below s*; for it is evident that if *a o* be the original length of the pipe, and *w o* its friction head; then if we double this length, by extending it to *n*, we do not thereby double the friction head; for the new friction head *m n* is not equal to twice *w o*, and the disch will of course be slower. Still, if *s a* is (as will often be the case in practice) very small in proportion to the friction head *w o* of the original pipe, then *m n* will be so nearly twice *w o*, that the diminution of disch from extending the pipe to any reqd dist, will be but slight.

Rem. 3. **Of the outward, or bursting pressure, of water in pipes.** When any pipe, as, for instance, any of those in Fig 1, is full of water at *rest, this pres is greater than when the water is flowing through it; and is that due to the total head above the point pressed.* Thus, at the point 4 on the pipe *r o*, it is that due to the head 4, 1; at the point 6 in the pipe *l o*, that due to the head 6, 1; at the point o in any of the pipes, that due to *c. Therefore* it may always be readily calculated in lbs, by the Rule in Art 1 of Hydrauli *mely, mult together the area in sq ft, of the portion pressed; the total head, in ft, above its av; and the constant number 62.5.* If the discharge end of the pipe be partially opened, th *l move slowly,* and the pres will become less; and when the water is flowing freely there

full pipe of uniform diam, the ends of which are entirely open, it becomes still more reduced; and if the pipe has its entry end precisely at the vel head *s*, Fig 1, and itself lying upon the hydraulic grade-line *s o*, (a case which never occurs in practice,) there will be no *bursting* pres; and the pipe will experience no pres of any kind, except that of the weight of the running water upon its lower side. But if any part of the running pipe, or all of it, be below the hyd grade-line, as in all the pipes in Fig 1, then every point of such pipe will be subject to a bursting pres due to a head equal to the vert dist of said point, *below the grade-line;* and which consequently may be calculated in lbs, in precisely the same way as in the preceding case of water at rest. Thus the point 4, in the pipe *r o*, Fig 1, will be pressed outward by the head 4, 3; the point 5, in the pipe *t o*, by the head 5, 3; the point 6, in *l o*, by 6, 3; and the point 7 will be pressed outward by the head 7, 8; &c. If at any point in any pipe thus below the hydraulic grade-line, and discharging freely, an open vert communicating pipe be inserted, the flowing water will rise in it to the level of said grade-line. Thus, it will rise in a pipe from 6 to 3, from 5 to 3, from 4 to 3, from 7 to 8, &c. Advantage may be taken of this, to supply water, or establish fountains, at intervals along a great line of pipes. Small vert pipes of this kind, called **piezometers, (pressure measurers,)** either made of glass, or else furnished with a floating index, are sometimes used for detecting the position of accidental obstructions in a line of pipes. If the water in these is found at any time to *fall* below its proper level, it shows that the pres upon it has become diminished by some obstruction in the interval between it and the reservoir; but if the water *rises*, it indicates that the pres upon it has been increased by some obstruction, or partial closing of the pipe, between the piezometer and the outlet, or discharge end. If the outlet be entirely closed, the pres on the piezometer becomes a maximum; and the water will rise in it to a level with the surf *m i* of the water in the reservoir. By having several piezometers, the point at which an obstruction has taken place can be approximately ascertained; and thus much labor saved in searching for it.

If the lower end *o*, Fig 1, of any of the pipes, instead of discharging freely into the air, discharges under water of which *e h* represents the surf, then the hydraulic grade-line must be drawn from *s* to *e*, instead of to *o*; and *p e* becomes the head, instead of *p o*.

REM. 4. **When the pipe rises above the hydraulic grade-line** in any part, an entire change of condition takes place throughout. In Fig 1½, the pipe *a g n o* rises above the grade-line *s o*; which, however, can no longer be properly so called; for the pipe must now be considered as divided into two sections, *a g n*, and *n y o*; each having its own grade-line, as *s n*, *n o*. In a long undulating line, it may thus become necessary to consider many separate sections, with their respective grade-lines. In our fig, the section *a g n* has the friction head *n v* only; and the shorter section *n y o* has the greater friction head *t o*; consequently the water would move more slowly in *a g n* than in *n y o*; and would require a greater diam than it, in order to carry the same quantity of water. Or, if the diam is the same throughout the entire pipe, then the quantity of water delivered at *o*, will be that due to the small head *n h* only. It will then flow from *a* to *n*, *filling* that portion of the pipe; but from *n to o* the pipe will not be full, but will carry off the water as in an iron gutter.

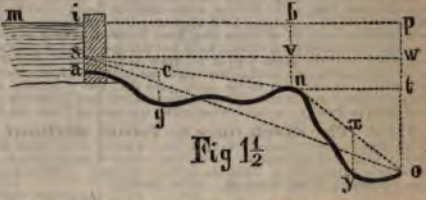

Fig 1½

The bursting pres at any point, as *g*, of the section *a g n*, will be measured by its corresponding vert line, as *g c*; and if the sections have different diams, (proportioned to their vels,) the pres at any point, as *y*, in section *n y o*, will be measured by its corresponding vert, as *y x*. But if both sections be of the same diam, then, since section *n y o* becomes virtually an open gutter, it can experience no bursting pres.

If a pipe be closed suddenly, the arrested momentum of the flowing water will exert a great pres, sufficient in most cases to burst with ease any ordinary street pipe. These are therefore closed very slowly by valves moved by screws; see Figs 35 to 36½. Leaden service-pipes in dwellings are frequently burst by closing the stopcock too quickly.

REM. 5. **Resistance to flow in pipes of diff materials.** It was formerly supposed that the material of which the pipe was made, exerted no influence on the flow, provided the insides were equally smooth; but later observations show that this is not the case. Weisbach states that in wooden pipes of from 2½ to 4½ ins diam, he found the frictional resistance to be as much as 1¾ times as great as in equally smooth cast-iron ones. If this be correct, we infer that under the same friction head, both the vel and the disch through a wooden pipe will be but $\frac{1}{1.75} = \frac{4}{7} = .571$ of that through a cast-iron one; and that the friction head for a wooden one must be 1¾ times that for an iron one; or that with the same friction head, the diam of the wooden one must be a trifle more than one and a quarter (1¼) times that of the iron one, to insure the same discharge. Darcy found that the usual formulas (those here given) agreed sufficiently closely with the actual results with perfectly clean smooth cast-iron pipes; except that they made the disch rather too large in small pipes; and rather too small in large ones; but that when the insides of the pipes were smoothly covered with pitch, the disch was increased about ¼ part; and became about equal to that through tubes made of glass. The vel head *for any given vel*, of course, remains the same for all materials and diams of pipe; only the *friction* head will vary. See Remark 2, Art 4.

Instead of the actual total head, the head in feet per mile is sometimes used, to facilitate over *aluations. The student must be* careful not to mistake one for the other.

Art. 2. **To find the velocity, and the quantity discharg through a straight, smooth, cylindrical cast-iron pipe** *r o, or n o, Fig 1.* **whose length is not shorter than 4 tim diam; knowing its total head *p o*; its length; and its or bore.** *The reservoir end, r, v, or l,* Fig 1, of the pipe or aqueduct, is,

calculations, supposed to be placed sufficiently far below the water surf ce, to allow the water to enter fast enough to keep the pipe *full*. The rule just given in preceding Remark 1, has reference to this fact. The present rule applies equally well to pipes with easy curves, provided the tops of the bends are kept below the hydraulic grade-line. See Rem 2, Art 4, p 542.

RULE 1. Mult the diam in feet, by the total head in feet. Call the prod *a*. Add together the total length of the pipe in ft; and 54 times its diam in ft.* Div the prod *a* by the sum. Take the sq rt of the quot. Mult this sq rt by the constant number 48. The prod will be the reqd vel in ft per sec. Or by formula,

$$\text{Vel in feet per sec. approx.} = 48 \times \sqrt{\frac{\text{Diam in ft.} \times \text{total head in ft.}}{\text{Total length in ft} + 54 \text{ diams in ft.}}}$$

Then to find the discharge in cub ft per sec. mult the vel thus found, by the area of circular transverse section of the pipe in sq ft. Said area may be taken from Table 3.† See foot-note * page 543.

If the disch end of the pipe (its lower end) is under water, the total head is the vert dist between the surfs of the water of the two reservoirs.

Ex. A straight pipe a mile, or 5280 ft long; with a diam of 1 foot, has a total fall of 12 ft, measured from the water surf in the reservoir, to the cen of grav of its lower end or opening. With what vel will the water flow through it; and how much will be dischd per sec?

Here the diam in ft. mult by the total head in ft $= 1 \times 12 = 12 = a$. Again, the length in ft is 5280; and 54 times the diam in ft is 54; and these two added together $= 5334$. And the prod *a*, div by 5334, $= \frac{12}{5334} = .00225$. The sq rt of .00225 is .04743. And $.04743 \times$ constant $48 = 2.27$ ft per sec; the reqd velocity.

Now, for the disch. The area of cross-section of a pipe 1 ft diam, (Table 3,) is .7854 of a sq foot. Hence,

Ft vel per sec		Sq ft of area		
2.27	×	.7854	=	1.782 cub ft dischd per sec‡ approximately.‡

REM. 1. The following Table, No. 1, calculated by this rule, shows the vels and dischgs through a pipe, one mile long, and 1 ft diam, under different heads. But they will be *very* nearly the same for any greater lengths; and also quite approximate for shorter ones not less than 1000 or even 500 diams long, *provided that in all cases they have the same* RATE *of* HEAD; that is, if the given pipe of 1 ft diam, is 2 or 3 miles long, it must have 2 or 3 times as much head as the pipe in the table which is 1 mile long; or if the given pipe of 1 ft diam, is ¼, ⅓, or ½, &c, of a mile long, it must have but ¼, ⅓, or ½ as much total head as the 1 mile one in the table, in order to have very nearly the same vel and disch.‡ If the diam of the given pipe (not less than 1000 or even 500 diams long,) is either greater or less than 1 ft, then the *disch* (but not the *vels*) under the same total head, may be found by mult those in Table 1, by the corresponding numbers in Table 2, p 540.

RULE 2. **The disch may be found without first finding the vel** as per Rule 1; thus: Find, by Table 5, the 5th power of the diam in feet. Mult this 5th power by the head in feet. Call the prod *p*. To the total length of the pipe in ft, add 54 times its diam in ft. Call the sum, *s*. Div *p* by *s*. Take the sq rt of the quot. Mult this sq rt by 37.6. The prod will be the quantity dischd, in cub ft per sec. Or by formula,

$$\text{Discharge in cub ft per sec} = 37.6 \sqrt{\frac{\text{5th power of diam in ft} \times \text{head in ft}}{\text{Length in ft} + 54 \text{ Diams in ft.}}}$$

Ex. How many cub ft per sec will be dischd from a pipe 6 ins diam; 20 ft long; under a total head of 10 feet?

Here the diam 6 ins, is .5 of a foot. And by Table 5, the 5th power of .5 is .031250. And .031250 $\times$ head $10 = .31250$; or *p*. Again, 20 ft + length 27 ft = 47; or *s*. And $\frac{p}{s} = \frac{.31250}{47} = .0066489$; the sq rt of which is .08154. And $.08154 \times 37.6 = 3.066$ cub ft per sec. The foregoing rule gives 3.077 cub ft.

REM. 2. **When we speak of the vel of water** in a pipe, river, &c, we allude to the *mean* vel of the entire cross section of the water. As in a river the vel is usually greater half way across it, and at the surface, than it is at the bottom and sides, so in a pipe the vel is actually greater at the center of its cross section than at its circumf. The mean vel referred to in our rules is an assumed uniform one which would give the same discharge that the actual ununiform one does.

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* When the length of any pipe is so great that this 54 times the diam may be omitted without affecting the result to a serious degree in practice, the pipe is technically called a "long" one by writers. If we take this length to be as short as 500 diams, the omission will give results only about 5 per cent, or $\frac{1}{20}$ part, in excess of the truth; and at 2000 diams the excess will be but about 1 per ct; either of which is near enough for most ordinary cases.

† *To reduce cub ft to U. S. standard galls of 231 cub ins, mult them by 7.48; or to reduce cub ft to British imperial galls of 277.274 cub in, mult them by 6.232.*

‡ *In order to insure the full disch, water-pipes should always have a somewhat greater diam than the calculation requires. Also, it is a good precaution to enlarge the upper end, somewhat as at Fig 7. These rules are by no means strictly correct; but frequently deviate 5, or 6 per ct, or more, from the truth. See Remark 2, p 543. If the pipe be larger than is absolutely necessary, as waste allow, because the disch may be regulated by stopcocks, or valves.*

TABLE 1. Of the actual velocities and discharges through a pipe, 1 foot in diam; 1 mile, or 5280 diams in length; and of cast iron; smooth, and straight. See foregoing Rem 1; also Rem 2, Art 4, p 543.

(Original.)

The head is the vert dist from the surf of the water in the reservoir, to the cen of grav of the lower end of the pipe, when the disch is into the air, as here supposed.

Head in Ft. per 100 Ft.	Head in Ft. per Mile.	Velocity in Ft. per Second.	Discharge in Cub. Ft. per Second.	Discharge in Cub. Ft. per 24 Hours.	Head in Ft. per 100 Ft.	Head in Ft. per Mile.	Velocity in Ft. per Second.	Discharge in Cub. Ft. per Second.	Discharge in Cub. Ft. per 24 Hours.
.0019	.1	.208	.1633	14114	1.515	80.	5.85	4.602	397613
.0038	.2	.293	.2301	19880	1.704	90.	6.23	4.900	423435
.0057	.3	.359	.2819	24360	1.894	100.	6.56	5.144	444312
.0076	.4	.415	.3267	28229	2.083	110.	6.87	5.395	466128
.0095	.5	.464	.3638	31455	2.272	120.	7.18	5.639	487209
.0114	.6	.508	.3989	34464	2.462	130.	7.47	5.866	506822
.0132	.7	.549	.4311	37247	2.652	140.	7.76	6.094	526521
.0151	.8	.585	.4602	39760	2.841	150.	8.05	6.322	546048
.0170	.9	.623	.4901	42343	3.030	160.	8.30	6.534	564576
.0189	1.	.656	.5144	44431	3.219	170.	8.55	6.715	580176
.0237	.25	.735	.5753	49701	3.408	180.	8.80	6.903	596438
.0284	.5	.805	.6322	54604	3.596	190.	9.04	7.100	613440
.0331	.75	.871	.6832	59011	3.788	200.	9.28	7.276	628704
.0379	2.	.928	.7276	62870	4.261	225.	9.84	7.696	664848
.0423	.25	.984	.7696	66484	4.735	250.	10.4	8.168	705728
.0473	.5	1.04	.8168	70572	5.208	275.	10.8	8.482	732844
.0521	.75	1.08	.8482	73284	5.682	300.	11.3	8.914	769824
.0568	3.	1.13	.8914	76982	6.629	350.	12.3	9.621	831168
.0758	4.	1.31	1.028	88862	7.576	400.	13.1	10.28	888624
.0947	5.	1.47	1.150	99403	8.532	450.	13.9	10.91	943056
.1136	6.	1.61	1.264	109209	9.47	500.	14.7	11.50	994032
.1325	7.	1.74	1.366	118022	10.41	550.	15.4	12.09	1044576
.1514	8.	1.86	1.455	125740	11.36	600.	16.1	12.64	1092096
.1703	9.	1.96	1.539	132969	12.30	650.	16.7	13.11	1132704
.1894	10.	2.08	1.633	141145	13.25	700.	17.4	13.66	1180224
.2273	12.	2.27	1.782	153964	14.20	750.	18.0	14.13	1220832
.2652	14.	2.45	1.924	166233	15.15	800.	18.6	14.55	1257408
.3030	16.	2.62	2.057	177724	16.09	850.	19.1	15.00	1296000
.3409	18.	2.78	2.183	188611	17.04	900.	19.6	15.39	1329696
.3788	20.	2.93	2.301	198806	17.99	950.	20.3	15.94	1377216
.4735	25.	3.28	2.572	222156	18.94	1000.	20.8	16.33	1411456
.5682	30.	3.59	2.819	243604	22.73	1200.	22.7	17.82	1539648
.6629	35.	3.88	3.047	263260	26.52	1400.	24.5	19.24	1662336
.7576	40.	4.15	3.267	282288	30.30	1600.	26.2	20.57	1777248
.8523	45.	4.40	3.451	298209	34.08	1800.	27.8	21.83	1886112
.9470	50.	4.64	3.638	314352	37.87	2000.	29.3	23.01	1988064
1.136	60.	5.08	3.989	344649	47.35	2500.	32.8	25.72	2221560
1.326	70.	5.49	4.311	372470	56.81	3000.	35.9	28.19	2436040

To reduce cub ft to U. S. gallons, mult by 7.48. Since, therefore, 8 cub ft are equal to 60 gals, (very nearly,) if we divide the cub ft per 24 hours, by 8, we get the number of persons that may be daily supplied with 60 gals each, by a pipe *constantly running full*, and at the vel given in the third col. This condition does not exist in city water-pipes; the water in them being comparatively stagnant. Therefore, the results of the rule and table do not at all apply to them. See Art 33, p 580.

REM. **If the pipe, instead of being straight, has easy curves,** (say with radii not less than 5 diams of the pipe,) either hor or vert, the disch will not be materially diminished, so long as the total heads, and total actual lengths of pipes remain the same; provided the tops of all the curves are kept below the hydraulic grade-line; (see Rem 1, Art 1,) and provision be made for the escape of air accumulating at the tops of the curves. See Fig 42, p 579.

Notwithstanding what is said about bends on pages 549, 550, we advise to make the radius as much *more* than 5 diams as can conveniently be done.

To find either the area of pipe, opening, or channel-way: or the mean vel; or the quantity discharged, when the other two are given. This applies to openings in the sides of vessels, to rivers, and to all other channels as well as to pipes.

$$\textit{Area in sq feet.} = \frac{\textit{Disch in cub ft per second}}{\textit{mean vel in feet per sec.}} \qquad \textit{Mean vel in ft per sec} = \frac{\textit{Disch in cub ft per second}}{\textit{area in sq feet.}}$$

$$\textit{Disch in cub ft per second} = \textit{area in sq feet} \times \textit{mean vel in ft per second.}$$

Or all the terms may be in inches instead of feet; and minutes or hours instead of second

TABLE 2. (Original.)

Of the diam of long pipe reqd to deliver either more or less water than that of 1 ft diam in Table 1, under the same rate of inclination, or of head in ft per mile. To find the disch (but not the vel) through another pipe, not less than about 1000, or at least 500, of its own diams in length; first take out the disch through the 1 ft one, from Table 1. Div the reqd disch by this tabular one. Look for the quot in the third column of Table 2, of proportions of disch; and opposite to it, in columns 1 and 2, will be found the reqd diam. See Rem 2, p 543. From this table we see that a 13-inch pipe will deliver nearly 1¼ times as much as a 1 ft one; a 14-inch one, nearly 1½ times; a 15-inch one, nearly 1¾ times; and a 16-inch one, fully twice as much as the 1 ft one; &c, of the same length and head.

This use of this table is not sufficiently correct for pipes less than about 1000 (or at furthest 500) diams long; therefore, for shorter ones, use Rule 1 or 2 preceding. For more on finding diams required for a given discharge, see Art 4.

Diam. of long Pipe, in Inches.	Diam. of long Pipe, in Feet.	Proportion of Disch. to that through a 1 ft. *long Pipe*, with the same head per mile.	Diam. of long Pipe, in Inches.	Diam. of long Pipe, in Feet.	Proportion of Disch. to that through a 1 ft. *long Pipe*, with the same head per mile.	Diam. of long Pipe, in Inches.	Diam. of long Pipe, in Feet.	Proportion of Disch. to that through a 1 ft. *long Pipe*, with the same head per mile.
1	.0833	.0020	10½	.8750	.7157	28	2.333	8.319
1½	.1250	.0055	11	.9167	.8044	30	2.5	9.822
2	.1667	.0113	11½	.9583	.8987	30¼	2.521	10.0
2½	.2083	.0198	12	1.	1.	32	2.667	11.6
3	.2500	.0310	12½	1.042	1.106	34	2.833	13.5
3½	.2917	.0458	13	1.083	1.221	36	3.	15.5
4	.3333	.0643	14	1.167	1.470	38	3.167	17.8
4½	.3750	.0857	15	1.250	1.746	40	3.333	20.2
5	.4167	.1119	16	1.333	2.053	42	3.5	22.9
5½	.4583	.1422	17	1.417	2.388	44	3.667	25.7
6	.5	.1767	18	1.5	2.754	48	4.	32.0
6½	.5417	.2159	19	1.583	3.153	54	4.5	42.0
7	.5833	.2600	20	1.667	3.585	60	5.	54.9
7½	.6250	.3090	21	1.75	4.051	66	5.5	69.8
8	.6667	.3631	22	1.833	4.551	72	6.	85.8
8½	.7083	.4220	23	1.917	5.084	78	6.5	104.8
9	.75	.4871	24	2.	5.649	84	7.	126.1
9½	.7917	.5575	24½	2.052	6.000	96	8.	176.
10	.8333	.6337	26	2.167	6.912	120	10.	302.

Examples of the use of Tables 1, and 2. Having a head from a reservoir to a certain point of delivery, of 20 ft, in a dist of 1860 ft, and wishing to receive 6 cub ft of water per sec; what must be the diam of a pipe to accomplish this?

In the first place, we find that a fall of 20 ft in 1860, is equal to a fall of 1.075 ft in 100 ft. Then we see by Table 1, that with a fall of 1.075 ft in 100, a long pipe of 1 ft diam yields about 3.8 cub ft per sec. But we want $\frac{6}{3.8} = 1.58$ times as much as the 1 ft pipe can deliver; and by Table 2, we see that the pipe, to do this, under the same rate of head, must be about 14½ ins in diam. In practice, we should adopt *at least* 15 ins. Near enough, we may say that double the diam gives 5½ times the disch.

TABLE 2½. Weight of water (at 62¼ lbs per cub foot) contained in one foot length of pipes of different bores. (Original.)

Bore. Ins.	Water. Lbs.	Bore. Ins.	Water. Lbs.	Bore. Ins.	Water. Lbs.	Bore. Ins.	Water. Lbs.	Bore. Ins.	Water. Lbs.	Bore. Ins.	Water. Lbs.
⅛	.00531	2.	1.3581	3⅞	5.0980	7½	19.098	13½	61.877	22	164.33
¼	.02122	⅛	1.5331	4.	5.4323	¾	20.392	14.	66.545	23	179.60
⅜	.04775	¼	1.7188	⅛	6.1325	8.	21.729	½	71.384	24	195.56
½	.08488	⅜	1.9150	¼	6.8750	¼	23.109	15.	76.392	25	212.20
⅝	.13263	½	2.1220	⅜	7.6601	½	24.530	½	81.568	26	229.51
¾	.19098	⅝	2.3395	5.	8.4880	¾	25.993	16.	86.916	27	247.51
⅞	.25994	¾	2.5676	⅛	9.3580	9.	27.501	½	92.434	28	266.18
1.	.33952	⅞	2.8063	¼	10.270	½	30.641	17.	98.121	29	285.53
⅛	.42969	3.	3.0557	⅜	11.225	10.	33.952	½	103.97	30	305.57
¼	.53050	⅛	3.3156	6.	12.223	½	37.432	18.	110.00	31	326.27
⅜	.64190	¼	3.5862	⅛	13.262	11.	41.082	½	116.20	32	347.66
½	.76392	⅜	3.8673	¼	14.345	½	44.901	19.	122.56	33	369.74
⅝	.89654	½	4.1591	⅜	15.469	12.	48.891	½	129.09	34	392.48
¾	1.0398	⅝	4.4615	7.	16.636	½	53.049	20.	135.81	35	415.90
⅞	1.1936	¾	4.7745	½	17.846	13.	57.379	21.	149.73	36	440.00

The weights contained in larger pipes, are as the squares of their bores. Thus, a pipe of 40 or 18 bore, will contain 4 times as much as one of 20 or 30 ins bore; and one of $\frac{3}{16}$, ¼ as much as $\frac{3}{8}$ inch. At 62¼ lbs per cub ft, a sq inch of water 1 ft high weighs .432292 of a lb.

TABLE 3. Areas and Contents of Pipes; and square roots of Diams. (Original) Correct.

Diam. in Ins.	Diam. in Feet.	Area in sq ft, also cub ft, in 1 foot length of Pipe.	Sq. rt. of Diam. in Ft.	Diam. in Ins.	Diam. in Feet.	Area in sq ft, also cub ft, in 1 foot length of Pipe.	Sq. rt. of Diam. in Ft.	Diam. in Ins.	Diam. in Feet.	Area in sq ft, also cub ft, in 1 foot length of Pipe.	Sq. rt. of Diam. in Ft.
¼	.0208	.0003	.145	4.	.3333	.0873	.579	15.	1.250	1.227	1.118
5-16	.0260	.0005	.161	¼	.3438	.0928	.588	¼	1.271	1.268	1.127
⅜	.0313	.0008	.177	½	.3542	.0985	.596	½	1.292	1.310	1.136
7-16	.0365	.0010	.191	¾	.3646	.1040	.604	¾	1.313	1.353	1.146
½	.0417	.0014	.204	½	.3750	.1104	.612	16.	1.333	1.396	1.155
9-16	.0469	.0017	.217	¼	.3854	.1167	.621	¼	1.354	1.440	1.163
⅝	.0521	.0021	.228	¾	.3958	.1231	.629	½	1.375	1.485	1.172
11-16	.0573	.0026	.239	⅞	.4063	.1296	.637	¾	1.396	1.530	1.181
¾	.0625	.0031	.250	5.	.4167	.1363	.645	17.	1.417	1.576	1.190
13-16	.0677	.0036	.260	⅛	.4271	.1433	.653	¼	1.437	1.623	1.199
⅞	.0729	.0042	.270	¼	.4375	.1503	.660	½	1.458	1.670	1.207
15-16	.0781	.0048	.280	⅜	.4479	.1576	.669	¾	1.479	1.718	1.216
1.	.0833	.0055	.289	½	.4583	.1650	.677	18.	1.5	1.767	1.224
1-16	.0885	.0062	.297	⅝	.4688	.1725	.685	½	1.542	1.867	1.241
⅛	.0938	.0069	.305	¾	.4792	.1803	.693	19.	1.583	1.969	1.258
3-16	.0990	.0077	.314	⅞	.4896	.1878	.700	½	1.625	2.074	1.274
¼	.1042	.0085	.322	6.	.5	.1964	.707	20.	1.667	2.182	1.291
5-16	.1094	.0094	.330	⅛	.5208	.2131	.722	½	1.708	2.292	1.307
⅜	.1146	.0103	.338	¼	.5417	.2304	.736	21.	1.750	2.405	1.323
7-16	.1198	.0113	.346	⅜	.5625	.2485	.750	½	1.791	2.521	1.339
½	.1250	.0123	.354	7.	.5833	.2673	.764	22.	1.833	2.640	1.354
9-16	.1302	.0133	.361	⅛	.6042	.2867	.777	½	1.875	2.761	1.369
⅝	.1354	.0144	.368	¼	.6250	.3068	.791	23.	1.917	2.885	1.384
11-16	.1406	.0155	.375	⅜	.6458	.3276	.803	½	1.958	3.012	1.399
¾	.1458	.0167	.382	8.	.6667	.3491	.817	24.	2.000	3.142	1.414
13-16	.1510	.0179	.389	⅛	.6875	.3712	.829	25.	2.083	3.409	1.443
⅞	.1563	.0192	.395	¼	.7083	.3941	.841	26.	2.166	3.687	1.472
15-16	.1615	.0205	.402	⅜	.7292	.4176	.854	27.	2.250	3.976	1.500
1.	.1667	.0218	.408	9.	.75	.4418	.866	28.	2.333	4.276	1.528
1-16	.1719	.0232	.414	⅛	.7708	.4667	.879	29.	2.416	4.587	1.555
⅛	.1771	.0246	.420	¼	.7917	.4922	.890	30.	2.500	4.909	1.581
3-16	.1822	.0260	.427	⅜	.8125	.5185	.902	31.	2.584	5.241	1.607
¼	.1875	.0276	.433	10.	.8333	.5454	.913	32.	2.666	5.585	1.633
5-16	.1927	.0291	.440	⅛	.8542	.5730	.924	33.	2.750	5.940	1.658
⅜	.1979	.0308	.445	½	.8750	.6013	.935	34.	2.834	6.305	1.683
7-16	.2031	.0324	.451	⅝	.8958	.6303	.946	35.	2.916	6.681	1.708
½	.2083	.0341	.457	11.	.9167	.6600	.957	36.	3.000	7.069	1.732
9-16	.2135	.0358	.462	⅛	.9375	.6903	.968	38.	3.166	7.876	1.779
⅝	.2188	.0375	.467	½	.9583	.7213	.979	40.	3.333	8.727	1.825
11-16	.2240	.0394	.473	⅞	.9792	.7530	.990	42.	3.500	9.621	1.871
¾	.2292	.0412	.478	12.	1.	.7854	1.000	44.	3.666	10.56	1.914
13-16	.2344	.0432	.484	⅛	1.021	.8184	1.010	48.	4.000	12.57	2.000
⅞	.2396	.0451	.489	½	1.042	.8522	1.020	54.	4.500	15.90	2.121
15-16	.2448	.0471	.495	¾	1.063	.8866	1.031	60.	5.000	19.63	2.236
1.	.2500	.0491	.500	13.	1.083	.9218	1.041	66.	5.500	23.76	2.345
⅛	.2604	.0532	.510	⅛	1.104	.9576	1.051	72.	6.000	28.27	2.449
¼	.2708	.0576	.520	½	1.125	.9940	1.060	78.	6.500	33.18	2.550
⅜	.2813	.0621	.530	⅝	1.146	1.031	1.070	84.	7.000	38.48	2.646
½	.2917	.0668	.540	14.	1.167	1.069	1.080	90.	7.500	44.18	2.739
⅝	.3021	.0716	.550	⅛	1.187	1.108	1.090	96.	8.000	50.27	2.828
¾	.3125	.0767	.560	½	1.208	1.147	1.099				
⅞	.3229	.0819	.570	⅝	1.229	1.187	1.110				

For contents in gallons, see p 46.

Art. 3. To find the total head in feet, that must be given to a straight, smooth, cylindrical cast-iron pipe, not less than 4 diams long, to enable it to disch a given reqd quantity per sec; knowing its diam in feet. See Rem 2, p 543. Also, Rem p 539.

RULE. Square the given disch in cub ft per sec. Add together the total length of the pipe in feet, and 54 times its diam in ft. Mult the sum by the square of the disch just found. Call the prod p. Next div the diam in ft by the dec .235. Take, from Table 5, the fifth power of the quot. Div p by this 5th power. The quot will be the reqd head in ft.

$$\text{Or, as a formula,} \quad \begin{matrix}\textit{Total head}\\ \textit{in feet}\end{matrix} = \frac{\left(\begin{matrix}\textit{Disch in cub}\\ \textit{ft per sec}\end{matrix}\right)^2 \times \left(\begin{matrix}\textit{Length in feet}\\ +\,54\ \textit{diams in ft}\end{matrix}\right)}{\left(\dfrac{\textit{Diam in ft}}{.235}\right)^5}.$$

EX. The same as that in Art 2, Rule 2. A pipe 6 ins, or .5 ft diam, and 20 ft long, is reqd to disch 3.066 cub ft per sec. What total head must it have? Here the square of the disch, or 3.066², is 9.4. The length in ft, added 54 diams in ft = 20 + 27 = 47; and 9.4 × 47 = 441.8, or p. Next, the diam in ft, div by .235, or $\frac{.5}{.235} = 2.128$. The 5th power of 2.128, by Table 5, is 43.64.* Hence the total head is $= \frac{441.8\ (\text{or } p)}{43.64} = 10.12$ ft; instead of 10 ft, as in Art 2; near enough for practice.

REM. The following rule is more simple; but is applicable only when the pipe is so long that the 54 diams may be omitted from the calculation, without affecting the result to a practical degree; in which case writers call it a **long** pipe. If, however, we take it as low as 1000 diams, the resulting head will be but about 6 per ct, or $\frac{1}{16}$ part too small; at 2000 diams, $\frac{1}{32}$ too small, &c.

RULE 2. Having the diam of the pipe in ft, find its area in sq ft, by Table 3. Div the reqd disch in cub ft per sec, by this area. The quot will be the vel in ft per sec. Then

$$\begin{matrix}\text{Head in ft per mile,}\\ \text{in long pipes}\end{matrix} = \frac{\begin{matrix}\text{Square of vel}\\ \text{in ft per sec}\end{matrix} \times 2.3}{\text{Diam in feet.}}$$

EX. How much head in feet per mile, must be given to a long pipe 1 ft diam, to enable it to disch 1.782 cub ft per sec? Here the area of the pipe, by Table 3, is .7854 sq ft. And $\frac{1.782}{.7854} = 2.269$ ft per sec vel. And the square of 2.269 = 5.1484; hence,

$$\begin{matrix}\text{Head in}\\ \text{ft per mile}\end{matrix} = \frac{5.1484 \times 2.3}{1} = 11.84 \text{ ft.}$$

Then say, as

1 mile.	:	Head.	: :	Length of pipe in feet	:	Its head.
5280 ft	:	11.84	: :			

Table 1, which is for a pipe 5280 diams long, gives the head 12 ft per mile; instead of 11.84, which is near enough.

Art. 4. To find the diam of a long, smooth, straight, cylindrical cast-iron

* This 5th power 43.64 was calculated, instead of being taken from the table; in order to illustrate the example more accurately. For practical purposes, however, they can be taken from the table near enough by eye.

d to deliver a given quantity per sec; knowing its length, and total head. No simple direct be given for the diam of *short* pipes. For those longer than about 2000 diams, (also quite ap- te even for 1000.) we may use the following. See Rem 1, after Table 1, p 539.
e the disch in cub ft per sec. Mult this square by the length in ft. Div the prod by the head rom Table 5, p 546, take the 5th root of the quot. Mult this 5th root by the dec .235. The l be the diam in feet. Or by formula,

$$\text{Diam in ft of a long pipe} = \sqrt[5]{\frac{\text{Square of discharge in cub ft per sec} \times \text{length in ft}}{\text{Head in ft}}} \times .235.$$

That diam must be given to a pipe half a mile, or 2640 ft long; with a total head of 6 ft; in furnish 1.782 cub ft per sec?
the sq of the disch, or $1.782^2 = 3.1755$. Hence we have

$$= \sqrt[5]{\frac{3.1755 \times 2640}{6}} \times .235 = \sqrt[5]{1397.2} \times .235 = 4.25 \times .235 = .9988 \text{ ft; or say 1 foot.}$$

the diam of a long pipe, see Tables 1 and 2, pp 539, 540.

1. For the diam of a short pipe, we may first assume any diam ure; and by Rule 1, or Rule 2, p 538, find what disch it would give. If this disch is greater han the reqd one, try again, with a smaller or larger diam; and repeat this operation until the e is found; which can be done in a few minutes, near enough for practice; in which a diam at larger than the exactly calculated one, should always be used.

2. Recent experimenters state that the old formulas in use, h generally sufficiently exact for ordinary practice, are to some extent defective. Thus, Darcy hat for small diams, the disch are somewhat too great; and for large diams, too small; and ch asserts that for vels less than 1½ ft per sec, (full 1 mile per hour,) the heads given by our a are too small; and for higher vels, too great. In other words, these writers agree that the uthorities hitherto followed, have not correctly valued the *friction heads*. See Rem 1, p 535. ch's experiments included much greater vels than those of his predecessors; and his extraor- attainments in science entitle his results to great confidence. The following is his rule for find- friction head in any case. For the total head we have only to add to this the vel head taken able 10, opposite the given vel; usually a very small quantity, rarely *exceeding* 4 or 5 ins. His plies to any lengths exceeding 4 diams; in other words, to both long and short pipes.*

isbach's rule. Square the reqd vel in feet per sec. Div this square by iling the quot a. Next, take the sq rt of the vel. Div the dec .01716 by this sq rt. To the quot dec .0144; calling the sum, b. Div the total length of the pipe in feet, by its bore or inner a feet. Call the quot c. Then,

$$\text{The friction head in feet} = a \times b \times c.$$

ormula,

$$\text{Friction head in feet} = \left(.0144 + \frac{.01716}{\sqrt{\text{vel in ft per sec}}}\right) \times \frac{\text{Length in feet}}{\text{Diam in feet}} \times \frac{\text{Vel}^2 \text{ in ft per sec}}{64.4}.$$

is formula, or by the next table, a pipe 1 ft diam. and a mile long, with a total head of 56 ft, ch 265 cub ft per min, with a vel of 5.6 ft per sec; by the rule in Art 2, or by Table 1, the disch 230 cub ft per min; and the vel 4.9 ft per sec; and the total head reqd for a vel and disch equal bach's, would be 73 ft, instead of his 56 ft.

e following useful table is taken from Weale's well- wn Engineer's Pocket Book.

BLE 4½. Of the vel, and discharge of water through straight, smooth, ical cast-iron pipes; with the friction head required for each 100 feet in length; and also the head. (For these terms, see Remark 1, p 535.) Calculated by means of Weisbach's formula, es Thomson, A M; and George Fuller, C E, Belfast, Ireland. The vel head remains the same *length* of pipe; being dependent only on the *velocity* of the water in the pipe. The vel head, friction head for the entire pipe, together, make the total head. See Rem, p 539.

THE OTHER HAND, many measurements by competent engineers seem to show that the old s give all the accuracy required in common practice. Mr Simpson, Prest of the Soc Civ ondon, states that he has gauged the flow from many pipes from 1 to 10 miles long, and from ins diam, and found them to agree *very exactly* with the formula

$$\text{Vel in ft per sec} = 50\sqrt{\frac{\text{diam} \times \text{head}}{\text{length} + 50 \text{ diams}}},$$

eet; which gives about 4 per ct more than Poncelet's, at top of page 538. Poncelet's is the one Fairbairn in his Treatise on Mills. Perfect accuracy is not attainable in such matters with *sent imperfect knowledge.*

Am edition of Weisbach, of 1848, has .01796, instead of .01716; and .01482, instead of .0144.

TABLE 4½.

Vel. in Feet per Sec.	Vel-head in Feet.	Diam. in Inches. 3		3½		4		4½		5	
		Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min
2.0	.062	.659	5.89	.565	8.02	.494	10.4	.439	13.2	.395	16.3
2.2	.075	.780	6.48	.669	8.82	.585	11.5	.520	14.6	.468	18.0
2.4	.090	.911	7.07	.781	9.62	.683	12.5	.607	15.9	.547	19.6
2.6	.105	1.05	7.65	.901	10.4	.788	13.6	.701	17.2	.631	21.3
2.8	.122	1.20	8.24	1.03	11.2	.900	14.6	.800	18.5	.720	22.9
3.0	.140	1.35	8.83	1.16	12.0	1.02	15.7	.905	19.8	.815	24.5
3.2	.160	1.52	9.42	1.31	12.8	1.14	16.7	1.02	21.2	.915	26.2
3.4	.180	1.70	10.0	1.46	13.6	1.27	17.8	1.13	22.5	1.02	27.8
3.6	.202	1.89	10.6	1.62	14.4	1.41	18.8	1.26	23.8	1.13	29.4
3.8	.225	2.08	11.2	1.78	15.2	1.56	19.9	1.39	25.2	1.25	31.0
4.0	.250	2.28	11.8	1.96	16.0	1.71	20.9	1.52	26.5	1.37	32.7
4.2	.275	2.49	12.3	2.14	16.8	1.87	22.0	1.66	27.8	1.50	34.3
4.4	.302	2.71	12.9	2.33	17.6	2.03	23.0	1.81	29.1	1.63	36.0
4.6	.330	2.94	13.5	2.52	18.4	2.21	24.0	1.96	30.4	1.76	37.6
4.8	.360	3.18	14.1	2.72	19.2	2.38	25.1	2.12	31.8	1.91	39.2
5.0	.390	3.43	14.7	2.94	20.0	2.57	26.2	2.28	33.1	2.05	40.9
5.2	.422	3.68	15.3	3.15	20.8	2.76	27.2	2.45	34.4	2.21	42.5
5.4	.455	3.94	15.9	3.38	21.6	2.96	28.2	2.63	35.8	2.37	44.2
5.6	.490	4.22	16.5	3.61	22.4	3.16	29.3	2.81	37.1	2.53	45.8
5.8	.525	4.50	17.1	3.85	23.2	3.37	30.3	3.00	38.4	2.70	47.4
6.0	.562	4.78	17.7	4.10	24.0	3.59	31.4	3.19	39.7	2.87	49.1
6.2	.600	5.08	18.2	4.36	24.8	3.81	32.4	3.39	41.0	3.05	50.7
6.4	.640	5.39	18.8	4.62	25.6	4.04	33.5	3.59	42.4	3.23	52.3
6.6	.680	5.70	19.4	4.89	26.4	4.28	34.5	3.80	43.7	3.42	54.0
6.8	.722	6.02	20.0	5.16	27.3	4.52	35.6	4.01	45.0	3.61	55.6
7.0	.765	6.35	20.6	5.45	28.0	4.77	36.6	4.24	46.4	3.81	57.2

Vel. in Feet per Sec.	Vel-head in Feet.	Diam. in Inches. 6		7		8		9		10	
		Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min
2.0	.062	.329	23.5	.282	32.0	.247	41.9	.220	53.0	.198	65.4
2.2	.075	.390	25.9	.334	35.3	.293	46.1	.260	58.3	.234	72.0
2.4	.090	.456	28.2	.390	38.5	.342	50.2	.304	63.6	.273	78.5
2.6	.105	.526	30.6	.450	41.7	.394	54.4	.350	68.9	.315	85.1
2.8	.122	.600	32.9	.514	44.9	.450	58.6	.400	74.2	.360	91.6
3.0	.140	.679	35.3	.582	48.1	.509	62.8	.453	79.5	.407	98.2
3.2	.160	.763	37.7	.654	51.3	.572	67.0	.508	84.8	.458	105
3.4	.180	.851	40.0	.729	54.5	.638	71.2	.567	90.1	.510	111
3.6	.202	.943	42.4	.808	57.7	.707	75.4	.629	95.4	.566	118
3.8	.225	1.04	44.7	.892	60.9	.780	79.6	.693	101	.624	124
4.0	.250	1.14	47.1	.979	64.1	.856	83.7	.761	106	.685	131
4.2	.275	1.25	49.5	1.07	67.3	.935	87.9	.832	111	.748	137
4.4	.302	1.35	51.8	1.16	70.5	1.02	92.1	.905	116	.814	144
4.6	.330	1.47	54.1	1.26	73.7	1.10	96.3	.981	122	.883	150
4.8	.360	1.59	56.5	1.36	76.9	1.19	100	1.06	127	.954	157
5.0	.390	1.71	58.9	1.47	80.2	1.28	105	1.14	132	1.03	163
5.2	.422	1.84	61.2	1.58	83.3	1.38	109	1.23	138	1.10	170
5.4	.455	1.97	63.6	1.69	86.6	1.48	113	1.31	143	1.18	177
5.6	.490	2.11	65.9	1.81	89.8	1.58	117	1.40	148	1.26	183
5.8	.525	2.25	68.3	1.93	93.0	1.68	121	1.50	154	1.35	190
6.0	.562	2.39	70.7	2.05	96.2	1.79	125	1.59	159	1.43	196
6.2	.600	2.54	73.0	2.18	99.4	1.90	130	1.69	164	1.52	203
6.4	.640	2.69	75.4	2.31	102	2.02	134	1.79	169	1.61	209
6.6	.680	2.85	77.7	2.44	106	2.14	138	1.90	175	1.71	216
6.8	.722	3.01	80.1	2.58	109	2.26	142	2.01	180	1.81	222
7.0	.765	3.18	82.4	2.72	112	2.38	146	2.12	185	1.90	229

TABLE 4½—(Continued.)

Vel. in Feet per Sec.	Vel. head in Feet.	Diam. in Inches. 11		12		13		14		15	
		Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min
2.0	.062	.180	79.2	.165	94.2	.152	110	.141	128	.132	147
2.2	.075	.213	87.1	.195	103	.180	121	.167	141	.156	162
2.4	.090	.248	95.0	.228	113	.210	133	.195	154	.182	176
2.6	.105	.287	103	.263	122	.242	144	.225	167	.210	191
2.8	.122	.327	111	.300	132	.277	156	.257	179	.240	206
3.0	.140	.370	119	.339	141	.313	166	.291	192	.271	221
3.2	.160	.416	127	.381	151	.352	177	.327	205	.305	235
3.4	.180	.464	134	.425	160	.393	188	.365	218	.340	250
3.6	.202	.514	142	.472	169	.435	199	.404	231	.377	265
3.8	.225	.567	150	.520	179	.480	210	.446	243	.416	280
4.0	.250	.623	158	.571	188	.527	221	.489	256	.457	294
4.2	.275	.680	166	.624	198	.576	232	.534	269	.499	309
4.4	.302	.740	174	.679	207	.626	243	.582	282	.543	324
4.6	.330	.803	182	.736	217	.679	254	.631	295	.589	339
4.8	.360	.867	190	.795	226	.734	265	.682	308	.636	353
5.0	.390	.935	198	.857	235	.791	276	.734	321	.685	368
5.2	.422	1.00	206	.920	245	.850	287	.789	333	.736	383
5.4	.455	1.07	214	.986	254	.910	298	.845	346	.789	397
5.6	.490	1.15	222	1.05	264	.973	309	.903	359	.843	412
5.8	.525	1.22	229	1.12	273	1.04	321	.964	372	.899	427
6.0	.562	1.30	237	1.19	283	1.10	332	1.02	385	.957	442
6.2	.600	1.38	245	1.27	292	1.17	343	1.09	397	1.01	456
6.4	.640	1.47	253	1.35	301	1.24	354	1.15	410	1.08	471
6.6	.680	1.55	261	1.42	311	1.31	365	1.22	423	1.14	486
6.8	.722	1.64	269	1.50	320	1.39	376	1.29	436	1.20	500
7.0	.765	1.73	277	1.59	330	1.46	387	1.36	449	1.27	515

Vel. in Feet per Sec.	Vel. head in Feet.	Diam. in Inches. 16		17		18		19		20	
		Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min
2.0	.062	.123	167	.116	189	.110	212	.104	236	.099	262
2.2	.075	.146	184	.138	208	.130	233	.123	260	.117	288
2.4	.090	.171	201	.161	227	.152	254	.144	283	.137	314
2.6	.105	.197	218	.185	246	.175	275	.166	307	.158	340
2.8	.122	.225	234	.212	265	.200	297	.189	331	.180	366
3.0	.140	.255	251	.240	284	.226	318	.214	354	.204	393
3.2	.160	.286	268	.269	302	.254	339	.241	378	.229	419
3.4	.180	.319	284	.300	321	.283	360	.269	401	.255	445
3.6	.202	.354	301	.333	340	.314	382	.298	425	.283	471
3.8	.225	.390	318	.367	359	.347	403	.328	449	.312	497
4.0	.250	.428	335	.403	378	.380	424	.360	472	.342	523
4.2	.275	.468	352	.440	397	.416	445	.394	496	.374	550
4.4	.302	.509	368	.479	416	.452	466	.429	519	.407	576
4.6	.330	.552	385	.519	435	.490	488	.465	543	.441	602
4.8	.360	.596	402	.561	454	.530	509	.502	567	.477	628
5.0	.390	.642	419	.605	473	.571	530	.541	590	.514	654
5.2	.422	.690	435	.650	492	.614	551	.581	614	.552	680
5.4	.455	.740	452	.696	511	.657	572	.623	638	.592	707
5.6	.490	.791	469	.744	529	.703	594	.666	661	.632	734
5.8	.525	.843	486	.793	548	.749	615	.710	685	.674	759
6.0	.562	.897	502	.844	567	.798	636	.755	709	.718	785
6.2	.600	.953	519	.897	586	.847	657	.802	732	.762	811
6.4	.640	1.01	536	.951	605	.898	678	.851	756	.808	838
6.6	.680	1.07	553	1.01	624	.950	700	.900	780	.855	864
6.8	.722	1.13	569	1.06	643	1.00	721	.951	803	.904	890
7.0	.765	1.19	586	1.12	662	1.06	742	1.00	827	.953	917

TABLE 4½. — (Continued.)

Vel. in Feet per Sec.	Vel-head in Feet.	Diam. in Inches. 22		24		26		28		30	
		Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min	Fr head in Feet.	Cub ft per Min
2.0	.062	.090	316	.082	377	.076	442	.070	513	.066	589
2.2	.075	.106	348	.097	414	.090	486	.083	564	.078	648
2.4	.090	.124	380	.114	452	.105	531	.097	616	.091	707
2.6	.105	.143	412	.131	490	.121	575	.112	667	.105	766
2.8	.122	.164	443	.150	523	.138	619	.128	718	.120	824
3.0	.140	.185	475	.170	565	.157	663	.145	770	.136	883
3.2	.160	.208	507	.191	603	.176	708	.163	821	.152	942
3.4	.180	.232	538	.213	641	.196	752	.182	872	.170	1001
3.6	.202	.257	570	.236	678	.218	796	.202	923	.189	1060
3.8	.225	.284	601	.260	716	.240	840	.223	974	.208	1119
4.0	.250	.311	633	.285	754	.263	885	.244	1026	.228	1178
4.2	.275	.340	665	.312	791	.288	929	.267	1077	.249	1237
4.4	.302	.370	697	.339	829	.313	973	.290	1129	.271	1296
4.6	.330	.401	728	.368	867	.339	1017	.315	1180	.294	1355
4.8	.360	.434	760	.397	905	.367	1062	.341	1231	.318	1414
5.0	.390	.467	792	.428	942	.395	1106	.367	1283	.343	1472
5.2	.422	.502	823	.460	980	.425	1150	.394	1334	.368	1531
5.4	.455	.538	855	.493	1018	.455	1194	.423	1385	.394	1590
5.6	.490	.575	887	.527	1055	.486	1239	.452	1437	.422	1649
5.8	.525	.613	918	.562	1093	.519	1283	.482	1488	.450	1708
6.0	.562	.652	950	.598	1131	.552	1327	.513	1539	.478	1767
6.2	.600	.693	982	.635	1168	.586	1371	.544	1590	.508	1826
6.4	.640	.735	1013	.673	1206	.622	1416	.577	1642	.539	1885
6.6	.680	.778	1045	.713	1244	.658	1460	.611	1693	.570	1943
6.8	.722	.821	1077	.753	1282	.695	1504	.645	1744	.602	2002
7.0	.765	.867	1109	.794	1319	.733	1548	.681	1796	.635	2061

TABLE 5. Of fifth roots and fifth powers.

For most practical purposes of the preceding rules, we may take from this table, the fifth root of the power nearest to that whose root we seek; when the exact one is not in the table.

Power.	No. of Root.	Power.	No. of Root.	Power.	No. of Root.	Power.	No. of Root.	Power.	No. of Root.	Power.	No. of Root.
.0000100	.1	.000142	.170	.004219	.335	.077760	.60	.695688	.93	8.11368	1.33
		.000164	.175	.004544	.340	.084460	.61	.733904	.94	8.66171	1.34
.0000110	.102	.000189	.180	.004888	.345	.091613	.62	.773781	.95	9.23886	1.36
		.000217	.185	.005252	.350	.099244	.63	.815373	.96	9.84658	1.38
.0000122	.104	.000248	.190	.005638	.355	.107374	.64	.858734	.97	10.4858	1.40
		.000282	.195	.006047	.360	.116029	.65	.903921	.98	11.1517	1.42
.0000134	.106	.000320	.200	.006478	.365	.125233	.66	.950990	.99	11.8637	1.44
		.000362	.205	.006934	.370	.135012	.67	1.	1.	12.6049	1.46
.0000147	.108	.000408	.210	.007416	.375	.145393	.68	1.10408	1.02	13.3828	1.48
.0000161	.110	.000459	.215	.007924	.380	.156403	.69	1.21665	1.04	14.1986	1.50
.0000176	.112	.000515	.220	.008459	.385	.168070	.70	1.33823	1.06	15.0537	1.52
.0000193	.114	.000577	.225	.009022	.390	.180423	.71	1.46933	1.08	15.9495	1.54
.0000210	.116	.000644	.230	.009616	.395	.193492	.72	1.61051	1.10	16.8874	1.56
.0000229	.118	.000717	.235	.010240	.400	.207307	.73	1.76234	1.12	17.8690	1.58
.0000249	.120	.000796	.240	.011896	.41	.221901	.74	1.92541	1.14	18.8957	1.60
.0000270	.122	.000883	.245	.013069	.42	.237305	.75	2.10034	1.16	19.9690	1.62
.0000293	.124	.000977	.250	.014701	.43	.253553	.76	2.28775	1.18	21.0906	1.64
.0000318	.126	.001078	.255	.016492	.44	.270678	.77	2.48832	1.20	22.2620	1.66
.0000344	.128	.001188	.260	.018453	.45	.288717	.78	2.70271	1.22	23.4849	1.68
.0000371	.130	.001307	.265	.020596	.46	.307706	.79	2.93163	1.24	24.7610	1.70
.0000401	.132	.001435	.270	.022935	.47	.327680	.80	3.17580	1.26	26.0919	1.72
.0000432	.134	.001573	.275	.025480	.48	.348678	.81	3.43597	1.28	27.4795	1.74
.0000465	.136	.001721	.280	.028248	.49	.370740	.82	3.71293	1.30	28.9255	1.76
.0000500	.138	.001880	.285	.031250	.50	.393904	.83	4.00746	1.32	30.4317	1.78
.0000538	.140	.002051	.290	.034503	.51	.418212	.84	4.32040	1.34	32.0000	1.80
.0000577	.142	.002234	.295	.038020	.52	.443705	.85	4.65259	1.36	33.6351	1.82
.0000619	.144	.002430	.300	.041820	.53	.470427	.86	5.00490	1.38	35.3410	1.84
.0000663	.146	.002639	.305	.045917	.54	.498421	.87	5.37824	1.40	37.1201	1.86
.0000710	.148	.002863	.310	.050328	.55	.527732	.88	5.77353	1.42	38.9763	1.88
.0000754	.150	.003101	.315	.055073	.56	.558406	.89	6.19174	1.44	40.9100	1.90
.0000805	.155	.003355	.320	.060169	.57	.590490	.90	6.63383	1.46	42.9254	1.92
.0000105	.160	.003626	.325	.065636	.58	.624032	.91	7.10082	1.48	45.0253	1.94
.0000122	.165	.003914	.330	.071492	.59	.659082	.92	7.59375	1.50	47.2133	1.96

TABLE 5. Of fifth roots and fifth powers — (Continued.)

Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.	Power.	No. or Root.
88.2735	2.45	2824.75	4.90	85873	9.70	2609193	19.2	20511149	29.0	459165024	54.
97.6562	2.50	2971.84	4.95	96392	9.80	2747949	19.4	21228253	29.2	503284375	55.
107.820	2.55	3125.00	5.00	95099	9.90	2892547	19.6	21965275	29.4	550731776	56.
118 814	2.60	3450.25	5.10	100000	10.0	3043168	19.8	22722628	29.6	601692057	57.
130.686	2.65	3802.04	5.20	110408	10.2	3200000	20.0	23500728	29.8	656356768	58.
143.489	2.70	4181.95	5.30	121665	10.4	3363232	20.2	24300000	30.0	714924299	59.
157.276	2.75	4591.65	5.40	133823	10.6	3535059	20.4	25393654	30.5	777600000	60.
172.104	2.80	5032.84	5.50	146933	10.8	3709677	20.6	28629151	31.0	844596301	61.
188.029	2.85	5507.32	5.60	161051	11.0	3893289	20.8	31013642	31.5	916132832	62.
205.111	2.90	6016.92	5.70	176234	11.2	4084101	21.0	33554432	32.0	992436543	63.
223.414	2.95	6563.57	5.80	192541	11.4	4282322	21.2	36259082	32.5	1073741824	64.
243.000	3.00	7149.24	5.90	210034	11.6	4488166	21.4	39135393	33.0	1160290625	65.
263.936	3.05	7776.00	6.00	228776	11.8	4701850	21.6	42191410	33.5	1252332576	66.
286.292	3.10	8445.96	6.10	248832	12.0	4923597	21.8	45435424	34.0	1350125107	67.
310.136	3.15	9161.33	6.20	270271	12.2	5153632	22.0	48875980	34.5	1453933568	68.
335.544	3.20	9924.37	6.30	293163	12.4	5392186	22.2	52521875	35.0	1564031349	69.
362.591	3.25	10737	6.40	317580	12.6	5639493	22.4	56382167	35.5	1680700000	70.
391.354	3.30	11603	6.50	343597	12.8	5895793	22.6	60466176	36 0	1804229351	71.
421.419	3.35	12523	6.60	371293	13.0	6161327	22.8	64783487	36 5	1934917632	72.
454.354	3.40	13501	6.70	400746	13.2	6436343	23.0	69343957	37.0	2073071593	73.
488.760	3.45	14539	6.80	432040	13.4	6721093	23.2	74157715	37.5	2219606624	74.
525.219	3.50	15640	6.90	465259	13.6	7015834	23.4	79235168	38.0	2373046875	75.
563.822	3.55	16807	7.00	500490	13.8	7320825	23.6	84587005	38.5	2535525376	76.
604.662	3.60	18042	7.10	537824	14.0	7636332	23.8	90224199	39.0	2706784157	77.
647.835	3.65	19349	7.20	577353	14.2	7962624	24.0	96158012	39.5	2887174368	78.
693.440	3.70	20731	7.30	619174	14.4	8299976	24.2	102400000	40.0	3077056399	79.
741.577	3.75	22190	7.40	663383	14.6	8648666	24.4	108962013	40.5	3276800000	80.
792.352	3.80	23730	7.50	710082	14.8	9008978	24.6	115856201	41.0	3486784401	81.
845.870	3.85	25355	7.60	759375	15.0	9381200	24.8	123095020	41.5	3707398432	82.
902.242	3.90	27068	7.70	811368	15.2	9765625	25.0	130691232	42.0	3939040643	83.
961.580	3.95	28872	7.80	866171	15.4	10162550	25.2	138657910	42.5	4182119424	84.
1024.00	4.00	30771	7.90	923896	15.6	10572278	25.4	147008443	43.0	4437053125	85.
1089.62	4.05	32768	8.00	984658	15.8	10995116	25.6	155756538	43.5	4704270176	86.
1158.56	4.10	34868	8.10	1048576	16.0	11431377	25.8	164916224	44 0	4984209207	87.
1230.95	4.15	37074	8.20	1115771	16.2	11881376	26.0	174501858	44.5	5277319168	88.
1306.91	4.20	39390	8.30	1186367	16.4	12345437	26.2	184528125	45.0	5584059449	89.
1386.58	4.25	41821	8.40	1260493	16.6	12823886	26.4	195010046	45.5	5904900000	90.
1470.08	4.30	44371	8.50	1338278	16.8	13317055	26.6	205962976	46.0	6240321451	91.
1557.57	4.35	47043	8.60	1419857	17.0	13825281	26.8	217402615	46.5	6590815232	92.
1649.16	4.40	49842	8.70	1505366	17.2	14348907	27.0	229345007	47 0	6956883693	93.
1745.02	4.45	52773	8.80	1594947	17.4	14888250	27.2	241806543	47.5	7339040224	94.
1845.28	4.50	55841	8.90	1688742	17.6	15443752	27.4	254803968	48.0	7737809375	95.
1950.10	4.55	59049	9.00	1786899	17.8	16015681	27.6	268354383	48.5	8153726976	96.
2059.63	4.60	62403	9.10	1889568	18.0	16604430	27.8	282475249	49.0	8587340257	97.
2174.03	4.65	65908	9.20	1996903	18.2	17210368	28.0	297184391	49.5	9039207968	98.
2293.45	4.70	69569	9.30	2109061	18.4	17833868	28.2	312500000	50.0	9509900499	99.
2418.07	4.75	73390	9.40	2226203	18.6	18475309	28.4	445025251	51.		
2548.04	4.80	77378	9.50	2348493	18.8	19135075	28.6	980204032	52.		
2683.54	4.85	81537	9.60	2476099	19.0	19813557	28.8	418195493	53.		

TABLE 6. Of the square roots of the fifth powers of numbers. In this table the numbers and the roots are supposed to be in the same dimensions; that is, both in inches, or both in feet, &c. See the next table. The foregoing formulas, as given in this book, do not require the use of this table, nor of the next.

No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.	No.	Sq. Rt. of 5th Power.
.25	.031	7.	129.64	17.5	1281.1	31.	5351	49	16807	76	50354
.5	.177	7.25	141.53	18.	1374.6	31.5	5569	50	17678	77	52027
.75	.485	7.5	154.05	18.5	1472.1	32.	5793	51	18575	78	53733
1.	1.	7.75	167.21	19.	1575.6	32.5	6022	52	19499	79	55471
1.25	1.747	8.	181.02	19.5	1679.1	33.	6256	53	20450	80	57243
1.5	2.756	8.25	195.50	20.	1788.9	33.5	6496	54	21428	81	59049
1.75	4.051	8.5	210.64	20.5	1902.8	34.	6741	55	22434	82	60888
2.	5.657	8.75	226.48	21.	2020.9	34.5	6991	56	23468	83	62762
2.25	7.594	9.	243.	21.5	2143.4	35.	7247	57	24529	84	64669
2.5	9.882	9.25	260.23	22.	2270.2	35.5	7509	58	25620	85	66611
2.75	12.541	9.5	278.17	22.5	2401.4	36.	7776	59	26738	86	68588
3.	15.588	9.75	296.83	23.	2537.	36.5	8049	60	27886	87	70601
3.25	19.042	10.	316.23	23.5	2677.1	37.	8327	61	29062	88	72649
3.5	22.918	10.5	357.2	24.	2821.8	37.5	8611	62	30268	89	74737
3.75	27.232	11.	401.3	24.5	2971.1	38.	8901	63	31503	90	76863
4.	32.	11.5	448.5	25.	3125.	38.5	9197	64	32768	91	78996
4.25	37.24	12.	498.8	25.5	3283.6	39.	9498	65	34063	92	81184
4.5	42.96	12.5	552.4	26.	3446.9	39.5	9806	66	35388	93	83408
4.75	49.17	13.	609.3	26.5	3615.1	40.	10119	67	36744	94	85668
5.	55.90	13.5	669.6	27.	3788.	41.	10764	68	38131	95	87965
5.25	63.15	14.	733.4	27.5	3965.8	42.	11432	69	39548	96	90298
5.5	70.94	14.5	800.6	28.	4148.5	43.	12125	70	40996	97	92668
5.75	79.28	15.	871.4	28.5	4336.2	44.	12842	71	42476	98	95073
6.	88.18	15.5	945.9	29.	4528.9	45.	13584	72	43988	99	97519
6.25	97.66	16.	1024.	29.5	4726.7	46.	14351	73	45531	100	100000
6.5	107.72	16.5	1105.9	30.	4929.5	47.	15144	74	47106		
6.75	118.38	17.	1191.6	30.5	5138.	48.	15963	75	48714		

TABLE 6½. The following will be found more convenient than the foregoing in computing discharges through pipes; inasmuch as the diams are here given in inches, and the sq rts of the 5th powers in feet; in which form the latter are sometimes required by formulas.

Intermediate sq roots may be had near enough for practice, by simple proportion.

Diam.	Sq. Rt. of 5th Pow.	Diam.	Sq. Rt. of 5th Pow.	Diam.	Sq. Rt. of 5th Pow.	Diam.	Sq. Rt. of 5th Pow.	Diam.	Sq. Rt. of 5th Pow.
Ins.	Feet.	Ins.	Feet.	Ins.	Feet.	Ins.	Feet.	Ins.	Feet.
¼	.00006	3¾	.0547	12.	1.000	22½	4.813	42	22.92
⅜	.00017	4.	.0641	¼	1.108	23	5.086	43	24.31
½	.00035	¼	.0731	13.	1.221	½	5.365	44	25.74
⅝	.00062	½	.0827	½	1.342	24	5.657	45	27.23
¾	.00098	¾	.0971	14.	1.470	25	6.264	46	28.77
⅞	.00144	5.	.1120	½	1.605	26	6.909	47	30.36
1.	.0020	¼	.1271	15.	1.747	27	7.593	48	32.00
⅛	.0027	½	.1428	½	1.896	28	8.316	49	33.69
¼	.0035	¾	.1590	16.	2.053	29	9.079	50	35.44
⅜	.0044	6.	.1768	½	2.217	30	9.882	51	37.25
½	.0055	½	.2160	17.	2.389	31	10.73	52	39.13
⅝	.0067	7.	.2599	½	2.567	32	11.61	53	41.02
¾	.0081	½	.3088	18.	2.756	33	12.54	54	42.96
⅞	.0096	8.	.3628	½	2.950	34	13.51	55	44.97
2.	.0113	½	.4228	19.	3.155	35	14.53	56	47.05
¼	.0152	9.	.4871	½	3.365	36	15.59	57	49.17
½	.0198	½	.5577	20.	3.586	37	16.69	58	51.35
¾	.0252	10.	.6339	½	3.813	38	17.84	59	53.60
3.	.0312	½	.7162	21.	4.051	39	19.04	60	55.90
¼	.0383	11.	.8043	½	4.297	40	20.29	61	58.27
½	.0459	½	.8990	22.	4.551	41	21.58		

***Art. 5.* On the resistance which curved bends oppose to the flow of water through round pipes.** Well-rounded bends of large rad

whether vert or hor, produce but little resistance; except so far as the first may cause accumulations of sediment, or of air. According to Weisbach, the rules of DuBuat, Navier, and other authorities, are erroneous; and he gives the following one for ascertaining the *additional* head reqd to overcome the resistance produced in a circular pipe, by a bend formed by an arc of a circle: Knowing the rad *r s* of the pipe, (or, in other words, half its diam,) in feet; the rad *r s*, of the axis *r n o* of the bend, in feet; the central angle *r s o* in degrees; (which is equal to the angle *d b x*, or *c b a*,) and the reqd vel of the water in the pipe, in ft per sec.

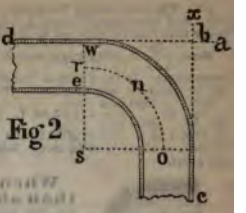

Fig 2

RULE. Div the central angle *r s o* in degrees, by 180. Call the quot *a*. Next, square the reqd vel. Div this square by the constant number 64.4. Call the quot *b*. Div the inner rad *r s* of the pipe, in ft, by the rad *r s* of the axis *r n o* of the bend, in ft. Call the quot *c*. Take from the following Table 7, the number in column *d*, which corresponds to *c*; (unless *c* be *less* than .1, in which case always take .13 as *d*.) Finally, mult together this number *d*, the quot *a*, and the quot *b*. The prod will be the reqd head in feet; which must either be added to the head previously calculated for the straight pipe, if the original vel is required to be maintained; or must be subtracted from it, in case the head does not admit of increase, and a new calculation made to ascertain the diminished vel under the head thus reduced. If there is more than one bend of the same dimensions, an equal alteration of head must be made for each; or, if they are of diff radii, and with diff central angles *r s o*, a separate calculation must be made for each. Rennie's experiments, at the end of this Art, seem to prove that this is by no means the case. So far as the writer is aware, we have no reliable data for calculating the effects of a succession of bends.

In shape of a formula, Weisbach's rule stands thus: R being the rad of the axis of the bend, and r the rad of the pipe:

$$\frac{\text{Head}}{\text{in ft}} = .131 + 1.847 \left(\frac{r \text{ in ft}}{R \text{ in ft}}\right)^{\frac{7}{2}} \times \frac{\text{square of vel in ft per sec}}{64.4} \times \frac{\text{central angle in degrees}}{180.}$$

The expression $\frac{7}{2}$ means the sq root of the 7th power. When the rad of the bend exceeds 5 diams of the pipe, then $1.847 \left(\frac{r}{R}\right)^{\frac{7}{2}}$ becomes inappreciable in practice, and may be omitted from the formula. When the pipe is square, instead of circular, the formula becomes

$$\text{Head} = .124 + 3.104 \left(\frac{r}{R}\right)^{\frac{7}{2}} \times \frac{\text{vel}^2}{64.4} \times \frac{\text{central angle}}{180.}$$

TABLE 7.

c.	d.	c.	d.	c.	d.	c.	d.	c.	d.
.1	.131	.325	.17	.5	.29	.675	.60	.85	1.18
.15	.135	.35	.18	.525	.32	.7	.66	.875	1.29
.2	.138	.375	.195	.55	.35	.725	.73	.9	1.41
.225	.145	.4	.206	.575	.39	.75	.80	.925	1.54
.25	.15	.425	.225	.6	.44	.775	.88	.95	1.68
.275	.155	.45	.24	.625	.49	.8	.98	.975	1.83
.3	.16	.475	.264	.65	.54	.825	1.08	1.	2.

EX. A straight pipe 1 mile long, and 18 ins diam, with a total head of 20 ft, will disch water with a vel of 3.6 ft per sec; but it has been found necessary to introduce a circular bend of 90°, with a rad *s r*, Fig 2, of 5 feet. What addition must be made to the 20 ft head, to compensate for the additional resistance caused by the bend; so that the reqd vel of 3.6 ft per sec may still be maintained?

Here, $\frac{90°}{180} = .5 = a$. Next, the square of the reqd vel in ft per sec, is $3.6 \times 3.6 = 12.96$. And $\frac{12.96}{64.4} = .2012 = b$. The rad *r s* of the pipe (.75 ft), div by the rad *r s* of the bend (5 ft), $= \frac{.75}{5} = .15 = c$; and opposite this .15 in the column *c* of the foregoing table, we find $d = .135$. Finally, $a \times b \times d = .5 \times .2012 \times .135 = .0136$ ft, or about ⅙ inch only, the additional head reqd. See next table, No. 8

Du Buat's rule for the additional head required to overcome the resistance of circular bends in water pipes. Having given the diam of the pipe, in ft; the rad of the bend, in ft; and the vel in ft per sec.† Div *e r*, Fig 2, or *half* the diam of the pipe, by the rad *s w* of the outer side of the bend. The quot will be the nat versed sine of Du Buat's **angle of reflexion.** Take this versed sine from unity, or 1. The rem will be the nat cosine of the same angle. From the Table of Nat Sin and Tang, take both the angle and the nat sine corresponding to this nat cosine. Call the angle R. Also, square the nat sine, and call this square S. Take the angle *w s o* from 180°. Div the rem by *twice* the angle R of reflexion just found. Call the quot T. Finally, mult together the constant dec .00375, the square of the vel in ft per sec, the quot T, and the square S. The prod will be the reqd extra head in feet.

* ***This central angle,*** when the bend is less than a semicircle, will always *be equal to either one of the two exterior angles, a b c, or x b d, Fig 2, formed by the tangents and x c. Either of these angles is called the* **angle of deflection** *of the bend.*

† *When we have given the diam of the pipe in ft, and the quantity of water required to pass, in cubic ft per sec, the vel in ft per sec will be found by dividing the quantity by the area of the pipe. This may be taken from Table 3, Art 2, p 541.*

Ex. The same as the foregoing one for Weisbach's rule; that is, a pipe of 18 ins, or 1.5 ft diam; rad *s w* of *outer* side of bend, 5.75 ft; vel, 3.6 ft per sec. What extra head will the bend require, in order that this vel may not be diminished?

Here, $\frac{.75}{5.75} = .13043$ = nat versed sine of angle of reflexion. And $1 - .13043 = .86957$ = nat cos of same angle. In the Table of Nat Sin and Tang, we find, opposite the nat cos .86957, the angle R = 29° 35′; and its nat sine .4937. The square of .4937 = .2437 = S. Again, 180° − 90° = 90°. And $\frac{90°}{59° \; 10'} = \frac{5400 \text{ min}}{3550 \text{ min}} = 1.52$, or T. Finally, the square of the vel is 12.96; hence, we have .00375 × 12.96 × 1.52 × .2437 = .0172 ft, the reqd extra head; or about $\frac{1}{5}$ of an inch. Weisbach's rule gave .0136 ft, or about ⅙ of an inch. Hence we see that the resistance produced by well-rounded bends is not great.

Fig 3

When the rad *r s*, Fig 2, of the bend, is less than about two diams of the pipe, which will rarely happen, the resistance to the flow of the water increases very rapidly; while, on the other hand, by Weisbach's rule, as we understand it, no advantage appears to be gained by using a rad greater than 5 diams of the pipe.* Employing Weisbach's formula, the writer has drawn up the following table of heads reqd to overcome the resistance of one bend of 90°, for diff vels in ft per sec; and for any diam whatever. This table extends from a rad of 5 diams down to one of ½ diam; which is the smallest possible, inasmuch as it leads to a bend like Fig 3.

A vel of 12 ft per sec is equal to 8.18 miles per hour; one which will rarely occur, inasmuch as it requires a head of about 330 feet per mile.

TABLE 8. Heads required to overcome the resistance in circular bends of 90°. Original.

	Velocity in feet per Second.										
	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.	10 ft.	12 ft.
	HEADS IN FEET.										
Rad = 5 diams of the pipe.	.001	.004	.009	.016	.025	.036	.050	.065	.082	.101	.145
Rad = 3 diams...	.001	.004	.010	.017	.027	.038	.052	.069	.086	.106	.153
Rad = 2 diams...	.001	.005	.011	.019	.029	.042	.057	.074	.094	.116	.167
Rad = 1½ diams...	.001	.005	.012	.021	.033	.048	.066	.086	.108	.134	.192
Rad = 1¼ diam...	.002	.007	.015	.026	.041	.059	.080	.104	.132	.163	.235
Rad = 1 diam...	.002	.009	.020	.036	.056	.081	.110	.144	.182	.225	.324
Rad = ¾ diam...	.005	.018	.041	.072	.113	.162	.221	.288	.365	.450	.649
Rad = ½ diam...	.016	.062	.140	.248	.388	.559	.761	.994	1.26	1.55	2.24

If the central angle *r s o*, Fig 2, should be either greater, or less than 90°, then the heads given in the table, must be increased, or diminished directly in the same proportion.

Experiments by Rennie, with a pipe 15 ft long; and ½ inch bore; with 4 ft head, gave the following disch in cub ft per sec:

Straight	.00699 cub ft.	One bend at right angles near end	.00596
15 semicircular bends	.00617 "	24 bends at right angles	.00253

The mean of many careful experiments tried at Liverpool, England, with a leaden pipe, 75 ft long, ¾ inch bore, under 8 ft head, gave the following number of secs to discharge one gallon of water:

75 ft pipe, straight and horizontal	81.56 sec.	4 vertical bends near discharge end	85.00 sec.
2 hor bends near discharge end	83.33 "	4 vertical bends near supply end	84.00 "
2 hor bends near supply end	81.80 "		

The rad of the bends is not stated. See Minutes Trans Inst Civ Eng, vol 12, page 501.

Fig 4

On knees, or angular bends in water pipes, Fig 4. The bends in lines of water pipes should always form circular arcs; because knees create a much greater resistance. According to Weisbach, the head in ft reqd to overcome the additional resistance caused by a knee in a round pipe is as follows:

$$\text{Head in feet} = \frac{\text{Sq of vel in ft per sec}}{64.4} \times \text{constant in following}$$

table opposite the angle of deflection, a e o, or d e f, Fig 4.

* Notwithstanding this, we advise to use as many more than 5 as can conveniently be done. The constant for any angle of def, is equal to .946 times the square of the nat sine of half the angle of def; × 2.05 times the 4th power of the same sine.

TABLE 9.

Ang. of Def. in Degs.	Constant.	Ang. of Def. in Deg.	Constant.	Ang. of Def. in Degs.	Constant.
140°	2.431	70°	.533	25°	.049
130	2.158	60	.364	20	.030
120	1.861	50	.234	15	.016
110	1.556	40	.139	10	.007
100	1.260	35	.102	5	.002
90	.984	30	.073		
80	.740				

nts intermediate of those in the table may be obtained near enough by simple proportion.
e see by the first table p 539, that under a total head of 20 ft, a straight pipe 1 ft diam.
le long, will disch with a vel of 2.93 ft per sec. What additional head must it have in order
ain the same vel, if a knee bend of 90° be introduced?

$\frac{2.93^2}{64.4} \times .984 = .1333 \times .984 = .131$ ft, the additional head reqd. With a circular arc of 5 ft
ould be but about .01 ft, or but about $\frac{1}{13}$ as much as with the knee.

disch is diminished by swellings, or enlargements in pipes,
s by contractions, bends, and knees; on account of the eddies which they produce, &c.

. 6. Inasmuch as the pres of quiet water against, and perp to, any given
other things being equal) in proportion to the vert height of the water above the cen of grav
essed surf, (see Art 1, Hydrostatics,) it follows that in two pipes of the same diams, as *a b*,
Fig 5, the pres against, and at right
o, the equal bases, *m n* of the vert pipe,
of the inclined one, are equal; because
heights, *a b* and *h g*, of the water above
f grav *a* and *c*, of the equal bases, are
the two pipes. If the base of the inclined
cut so that *c y* becomes the base, then the
no longer a circle, but an ellipse; the
which will always be greater than that
rcular ope; and since the vert height *h g*
unchanged, the pres against the base *f y*,
s to it, will be greater than that against
ne same proportion as the two areas.

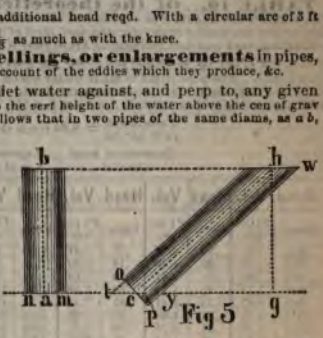

Fig 5

pright pipe may be but 1 ft long; and the
one 1 mile, or 10 miles long, still the pres
ase *m n* will be the same as that at the
so long as the vert height *a b* is equal to
height *g h*. The greater *weight* of the water in *c h*, does not increase the *pres* at its lower end
d weight being sustained by the under part *p w* of the inclined pipe. If, therefore, two steam
were employed, one to force the water up the 1 ft long vert pipe, and another to force it
ime vel up the 10 miles of inclined pipe, both engines would have precisely the same amount
tance to overcome, so far as regards only either the *pres*, or the *weight* of the water in the
ve pipes. The *friction*, however, of the water against the much longer sides of *c h*, would
greater than that along *a b*; and this excess of friction in the inclined pipe would require
tional excess of power in the engine to overcome it.

e amount of this friction is easily ascertained. It is equal
eight of a cylindrical column of water, the diam of which is equal to the diam of the pipe;
vert height of which is equal to the actual head of water (calculated by Art 3; or Table
reqd to impart the given vel to the water flowing through the pipe at its given inclination.
mple: it is required to force water at the rate of 4.4 ft per sec, up a pipe 1 foot diam, 3 miles
d rising 21 ft in the 3 miles. What resistance will the piston of the engine experience from
ct pres upon it of the water in the pipe; and also from the friction of the water along the
the pipe?

ve have first, the direct perp pres against the piston, equal to the weight of a column of water
n, and 21 ft high; or of $1 \times 1 \times .7854 \times 21 = 16.5$ cub ft of water, $= 16.5 \times 62.5 = 1031$ lbs.
umes the diam of the piston to be equal to that of the pipe; if not, the diam of the piston
taken for that also of the pressing column of water against it. As regards the resistance
ction, we see by Table 1, p 539, that to overcome friction, so as to impart a vel of 4.4 ft per
e water in a pipe 1 ft in diam, we must provide a head of 45 ft per mile.* Consequently, in
before us, it would require 45 ft $\times$ 3 miles $= 135$ ft head; so that the resistance from pipe-
will be equal to the weight of a column of water 1 foot diam, and 135 ft high; or to $1 \times 1 \times$
$135 \times 62.5 = 6627$ lbs. And $6627 + 1031 = 7658$ lbs, total resistance arising from the pres of
r; and from its friction against the sides of the pipe, when moving at the rate of 4.4 ft per sec.
case of pumping upward through the vert pipe, there is also a slight friction to overcome, in
to the pres of the water against the piston; and it must be calculated by precisely the same
as in the other case.

. 7. The *flow* of water through openings, or apertures,
e sides or bottom of the containing vessel, or reservoir.
ally, the vel with which water should flow through such an opening, is equal to that which
acquired by a heavy body falling freely through a height equal to the head, or depth of
asured vert from the level surf of the water in the reservoir, to the center of the opening
irectly, to its cen of grav. This theoretical vel is found in ft per sec, by mult the
d, or vert depth in ft, by the constant number 8.03; or, mult the head itself in ft, by 64

bach, or by Table 3½, this head would be considerably less.
is twice 32.2; which last is the "acceleration of gravity" referred to in Note
Hydr Bodies p 455.

and take the sq rt of the prod. In practice, we may use 8 and 64, as near enough. The theoretical, as well as the actual *disch*, or the *quantity* in cub ft, which flows out per sec, is evidently equal in all cases to the prod of the theoretical, or of the actual vel, (as the case may be,) in ft per sec, multd by the area of the opening in sq ft.

These theoretical laws apply equally to all fluids, whatever may be their sp grav; thus, theoretically, mercury, water, air, &c, will all flow with equal vels from openings of equal sizes, under equal heads.

Practically, however, only the mean vel, and the disch through the **vena contracta,** or **contracted vein,** (see Fig 11,) which forms itself just outside of certain kinds of openings, (and which is smaller than the openings themselves,) are actually *very nearly* equal to the theoretical ones; but through the *very opening itself* they are *usually* less. The discrepancy is greater in some cases than in others; depending *chiefly* on the shape of the opening.

On this account, the theoretical vel and disch found by the foregoing rule, must usually be diminished by mult them by certain decimal numbers corresponding to the various kinds of openings; and called *coefficients of discharge*. These coeffs have in many cases been determined by experiment *very approximately;* and will be found in the following articles. It will be seen in Remark 5, p 554, that by the use of a peculiarly formed adjutage, or attachment, to small openings, the *actual* disch may even be increased *beyond* the theoretical one.

The following table will save the trouble of calculating the *theoretical* vel, previously to mult it by the corresponding coeff of disch, for obtaining the actual vel. The coeffs for diff kinds of openings will be found further on.

TABLE 10. Of the theoretical velocities in feet per sec, with which water should flow out into *the air*, under diff heads, through openings in the bottom or sides of the containing reservoir; the surf level of which remains constantly at the same height.

Weisbach says (see first footnote to Art 9) that when water flows out of an opening *under water*, as at *y*, Fig 1, the vel and disch are about $\frac{1}{25}$ part less than when it flows into the open air, under equal heads. When the disch is made under water, the vert dist *b d*, Fig 1, between the surf levels of the two reservoirs, must be taken as the head. These theoretical vels are *very* nearly the actual mean ones at the *contracted vein;* see Art 9, p 554.

$$\text{Theoret vel in ft per sec} = 8.03 \text{ times the sq rt of the head in ft.}$$

$$\text{Theoretical head in feet} = \left(\text{square of vel in ft per sec}\right) \times .0155.$$

Head	Vel.	Head	Vel.	Head	Vel.	Head	Vel.	Head	Vel.	Head	Vel	Head	Vel.
Feet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.	Feet.	Ft per sec.
.005	.57	.29	4.32	.77	7.04	1.50	9.83	7.	21.2	28	42.5	76	69.9
.010	.80	.30	4.39	.78	7.09	1.52	9.90	.2	21.5	29	43.2	77	70.4
.015	.98	.31	4.47	.79	7.13	1.54	9.96	.4	21.8	30	43.9	78	70.9
.020	1.13	.32	4.54	.80	7.18	1.56	10.0	.6	22.1	31	44.7	79	71.3
.025	1.27	.33	4.61	.81	7.22	1.58	10.1	.8	22.4	32	45.4	80	71.8
.030	1.39	.34	4.68	.82	7.26	1.60	10.2	8.	22.7	33	46.1	81	72.2
.035	1.50	.35	4.75	.83	7.31	1.65	10.3	.2	23.0	34	46.7	82	72.6
.040	1.60	.36	4.81	.84	7.35	1.70	10.5	.4	23.3	35	47.4	83	73.1
.045	1.70	.37	4.87	.85	7.40	1.75	10.6	.6	23.5	36	48.1	84	73.5
.050	1.79	.38	4.94	.86	7.44	1.80	10.8	.8	23.8	37	48.8	85	74.0
.055	1.88	.39	5.01	.87	7.48	1.85	10.9	9.	24.1	38	49.5	86	74.4
.060	1.97	.40	5.07	.88	7.53	1.90	11.1	.2	24.3	39	50.1	87	74.8
.065	2.04	.41	5.14	.89	7.57	1.95	11.2	.4	24.6	40	50.7	88	75.3
.070	2.12	.42	5.20	.90	7.61	2.	11.4	.6	24.8	41	51.3	89	75.7
.075	2.20	.43	5.26	.91	7.65	2.1	11.7	.8	25.1	42	52.0	90	76.1
.080	2.27	.44	5.32	.92	7.70	2.2	11.9	10.	25.4	43	52.6	91	76.5
.085	2.34	.45	5.38	.93	7.74	2.3	12.2	.5	26.0	44	53.2	92	76.9
.090	2.41	.46	5.44	.94	7.78	2.4	12.4	11.	26.6	45	53.8	93	77.4
.095	2.47	.47	5.50	.95	7.82	2.5	12.6	.5	27.2	46	54.4	94	77.8
.100	2.54	.48	5.56	.96	7.86	2.6	12.9	12.	27.8	47	55.0	95	78.2
.105	2.60	.49	5.62	.97	7.90	2.7	13.2	.5	28.4	48	55.6	96	78.6
.110	2.66	.50	5.67	.98	7.94	2.8	13.4	13.	28.9	49	56.2	97	79.0
.115	2.72	.51	5.73	.99	7.98	2.9	13.7	.5	29.5	50	56.7	98	79.4
.120	2.78	.52	5.79	1 Ft.	8.03	3.	13.9	14.	30.0	51	57.3	99	79.8
.125	2.84	.53	5.85	1.02	8.10	3.1	14.1	.5	30.5	52	57.8	100	80.3
.130	2.89	.54	5.90	1.04	8.18	3.2	14.3	15.	31.1	53	58.4	125	89.7
.135	2.95	.55	5.95	1.06	8.26	3.3	14.5	.5	31.6	54	59.0	150	98.3
.140	3.00	.56	6.00	1.08	8.34	3.4	14.8	16.	32.1	55	59.5	175	106
.145	3.05	.57	6.06	1.10	8.41	3.5	15.	.5	32.6	56	60.0	200	114
.150	3.11	.58	6.11	1.12	8.49	3.6	15.2	17.	33.1	57	60.6	225	120
.155	3.16	.59	6.17	1.14	8.57	3.7	15.4	.5	33.6	58	61.1	250	126
.160	3.21	.60	6.22	1.16	8.64	3.8	15.6	18.	34.0	59	61.6	275	133
.165	3.26	.61	6.28	1.18	8.72	3.9	15.8	.5	34.5	60	62.1	300	139
.170	3.31	.62	6.32	1.20	8.79	4.	16.0	19.	35.0	61	62.7	350	150
.175	3.36	.63	6.37	1.22	8.87	.2	16.4	.5	35.4	62	63.2	400	160
.180	3.40	.64	6.42	1.24	8.94	.4	16.8	20.	35.9	63	63.7	450	170
.185	3.45	.65	6.47	1.26	9.01	.6	17.2	.5	36.3	64	64.2	500	179
.190	3.50	.66	6.52	1.28	9.08	.8	17.6	21.	36.8	65	64.7	550	188
.195	3.55	.67	6.57	1.30	9.15	5.	17.9	.5	37.2	66	65.2	600	197
.200	3.59	.68	6.61	1.32	9.21	.2	18.3	22.	37.6	67	65.7	700	212
.21	3.68	.69	6.66	1.34	9.29	.4	18.7	.5	38.1	68	66.2	800	227
.22	3.76	.70	6.71	1.36	9.36	.6	19.	23.	38.5	69	66.7	900	241
.23	3.85	.71	6.76	1.38	9.43	.8	19.3	.5	38.9	70	67.1	1000	254
.24	3.93	.72	6.81	1.40	9.49	6.	19.7	24.	39.3	71	67.6		
.25	4.01	.73	6.86	1.42	9.57	.2	20.0	.5	39.7	72	68.1		
.26	4.09	.74	6.91	1.44	9.63	.4	20.3	25.	40.1	73	68.5		
.27	4.17	.75	6.95	1.46	9.70	.6	20.6	.5	40.9	74	69.0		
.28	4.25	.76	6.99	1.48	9.77	.8	20.9	26.	41.7	75	69.5		

t. 8. On the flow of water ugh vertical openings fur- ed with short tubes. When water om a reservoir, Fig 6, through a vert partition , the thickness *a m* of which is about 2½ or 3 times it transverse dimension of the opening, (whether mension be its breadth, or its height;) or when, if ition be very thin, as *n n*, the water flows through as at *t*, the length of which is about 2 or 3 times its ansverse dimension, then the effluent stream will fill the opening, or the tube, as shown in Fig 6; or, ical language, will *run with a full flow; or a full* nd will disch more water in a given time, than if e were either materially longer or shorter. For if han 3 times the least transverse dimension, the l be impeded by the increased friction against the the tube; and if shorter than about twice the least rse dimension, the water will not flow in a full stream, but in a contracted one, as shown by p 554. This will be the case whether the tube be circular, or rectilinear, in its cross-section.

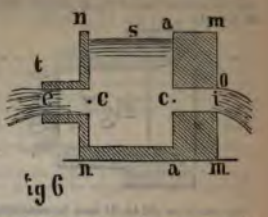

Fig 6

find approximately the actual vel. and disch into the through a tube, or opening, either circular or recti- ar in its outline, or cross-section; and whose length *c i*, e, in the direction of the flow, is about 2½ or 3 times its transverse dimension; when the surface-level, *s*, Fig 6, ains constantly at the same height; and which height t not be below the upper edge of the tube, or opening.

1. Take out the theoretical vel from Table 10, p 552, corresponding to the head measured *vert* e center (or more properly, the cen of grav) *c*, of the opening, to the level water surf *s*. Mult e coeff of disch .81. The prod will be the reqd vel, in ft per sec. Mult this actual vel by the rse area of the opening, in sq ft. If circular, knowing its diam. this area will be found in p 541. The prod will be the quantity of water dischd, in cub ft per sec; within, probably, 3 cent.

2. Find the sq rt of the head in ft. Mult this sq rt by 6.5. The prod will be the actual per sec.

n opening *c o*; or box-shaped tube *c t*, Fig 6, is 3 feet wide, by .25 of a ft high; and its length direction *c i* or *c e* in which the water flows is about .62 of a ft, or about 2½ times its least rse dimension, or its height. The head from the cen of grav *c*, of the opening, to the constant el *s*, is 4 feet. What will be the vel of the water; and how much will be dischd per sec?

ple 1. The theoretical vel (Table 10, p 552,) corresponding to a head of 4 ft is 16 ft per sec. × .81 = 12.96 ft per sec, the actual vel reqd. Again, the transverse area of the opening, or of , is 3 ft × .25 ft = .75 sq ft. And .75 × 12.96 = 9.72 cub ft; the quantity dischd per sec.

ple 2. The sq rt of 4 is 2. And 2 × 6.5 = 13 ft per sec, the reqd vel, as before; the very slight g owing to the omission of small decimals in the coeffs.

1. If the short tube *t* projects partly inside of the vert ition *n n*, the disch will be diminished about ⅛ part. In that case, use .71 tead of the .81 of Rule 1; or 5.7 instead of the 6.5 of Rule 2.

2. When the thickness *a m* of the vert partition *m m a a*; or the length *c e* of the tube *t*, Fig reased to about 4 times the least transverse dimension of the opening; or of the diam, when ; then the additional friction against its sides begins appreciably to lessen the vel and disch. case, or for still greater lengths, up to 100 diams, they may be found approximately, by using of the coeff of disch .81 in Rule 1, the following coeffs, by which to mult the theoretical vels 10, p 552.

TABLE 11.

Length of Pipe in Diams.	Coeff.	Length of Pipe in Diams.	Coeff.
4	.80	40	.62
6...	... .76	50...	... .60
10	.74	60	.57
15...	... .71	70...	... .55
20	.69	80	.52
25...	... .67	90...	... .50
30	.65	100	.48

sch thus obtained agrees quite closely with those given by Rule 1, page 538.

. When the length of the opening or tube, in the direction in which the water flows, becomes about twice its least transverse dimension, the disch is diminished; so that for lengths from s, down to openings in a very thin plate, we may use .61, instead of the .81 of Rule 1. For nings, however, *see Arts* 9 and 10.

. *But on the other hand*, the disch through such short openings and tubes as are shown in *r be increased to nearly the* theoretical ones of Table 10, by merely rounding off nearly the *he entrance end or mouth*, as in Fig 7; which is the shape, and half actual size of one wit *tbsch obtained* .975 of the theoretical vel and discharge, when the head was 10 ft; and .9

with a head of one foot; so that in similar cases, .975, and .968 may be used instead of the coeff in Rule 1.

Fig 8 Fig 9

As much as .92 to .94 may be obtained by widening the opening, *m n*, toward its outer mouth, s Fig. 8, making the divergence, or angle *a*, about 5°; or by widening it toward its inner mouth, as t c, Fig 9; but increasing the angle of divergence, at *b*, to from 11° to 16°. In all cases, we consi the small end as being the opening whose area must be multiplied by the vel to get the discharge.

In some experiments made with large pyramidal woode troughs 9.5 ft long, with an inner mouth of 3.2 × 2.4 ft, and a discharging o of .62 × .44 ft; and under a head of 9½ feet, the discharge was .98 of the theoretical one, due to smaller end. Therefore, .98 may be used in such cases, instead of the .81 of Rule 1.

REM. 5. **The discharge through a short opening of sma transverse section may even be made 50 per cent great than the theoretical one,** by adopting the shape, Fig 10; where *m n* is s posed to be the diameter of the open The best proportions appear to be about follows: $o\,y = 9$ inches; $m\,n = 1$ inch; $= 1.8$ inch; $o\,s = ½$ inch; $a\,d = 2$ ins; curves, *a m*, and *d n*, being quadran the angle, *x*, of divergence, about 5° and the tube of polished metal. In case use 1.55, or more safely, 1.5, inst of the .81 of Rule 1. The only exp ments with this form have been on a small scale. To what extent it may

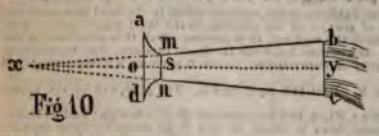

Fig 10

applicable is unknown.

So far as regards the ordinary operations of the engineer, this subject is perhaps more curious th useful; for he will rarely have any difficulty in making his openings large enough, without resortin such aids; except, perhaps, that of a rounding off the inner edges, as in Fig 7; which is usually do

Art. 9. On the disch of water through openings in th vert partitions, with plane or flat faces, *e e*, or *n n*, Fig 11.* If t face *e e*, or *n n*, instead of being plane, and vert, should be curv or inclining in diff directions toward the opening, then the di will be altered. When water flows from a reservoir, Fig 11, throu a vert plane plate or partition *n n*, which is not thicker than ab the least transverse dimension of the opening, whether that dimens be its breadth, or its height *o o*; † or when, if the partition *e e* its is much thicker, we give the opening the shape shown at *b*, (whi evidently amounts to the same thing,) then the effluent stream w not pass out with a *full flow*, as in Fig 6, but will assume the sha shown in Fig 11; forming, just outside of the opening, what called the *vena contracta*, or *contracted vein*. In order that t contraction may take place to its fullest extent, or become comple the inner sharp edges of the opening must not approach either surf of the water, or the bottom or sides of the reservoir, near than about 1½ times the least transverse dimension of the openi The contracted vein occurs at a dist of about half the smallest mension of the orifice, from the orifice itself. In a circular orif at about half the diam dist; and ordinarily its area is about .62 or nearly ⅝ that of the orifice its At this point the actual mean *vel* of the stream is very nearly (about .97) the theoretical vel given Table 10, p 552; and hence the actual *dischs* are but .62, or nearly ⅝ of the theoretical ones.

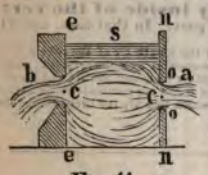

Fig 11

Case 1. **To find the actual disch into air,‡ through either circular or rectilinear§ opening in a thin vert plane part**

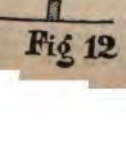

Fig 12

* We believe that these rules for thin plate are also sufficiently approxima for most practical purposes, if the opening be in the *bottom* of the reservo or in an *inclined*, instead of a *vert* side.

† When the side of a reservoir, or the edge of a plank, &c. over which wat flows, has no greater thickness than this, the water is said to flow throug or over, **thin plate,** or **thin partition.**

‡ Should the disch take place under water, as in Fig 12, both surf-levels *maining constant*, then the head to be used is the vert diff *a o*, of the levels. After making the calculation with this head, we should, accord Weisbach, deduct the $\frac{1}{25}$ part; inasmuch as he states that the disch much less when under water, than when it takes place freely into Other experimenters, however, assert that it is precisely the same in

§ If the shape of the opening is oval, triangular, or irregula **must be measured vert from its cen of grav.**

on, when the contraction is complete; and when the surf-vel, *s*, remains constantly at the same height; water being pplied to the reservoir as fast as it runs out at the open-ng.*

ULE 1. When the head, measured vert from the center (or rather from the cen of grav) *c*, of the ening, to the surf-level *s* of the reservoir, is not less than 1 ft, nor more than 10 ft; and when the at transverse dimension of the opening is not less than an inch, mult the theoretical vel in ft per due to the head, (Table 10, p 552,) by the coeff of disch. .62. (See Rem 2.) The prod will be the ual mean vel of the water through the opening. Mult this vel by the area of the opening in sq the prod will be the disch in cub ft per sec, approximately.

When the head is greater than 10 ft, use .6, instead of .62.

ULE 2. Find the sq rt of the head in ft. Mult this sq rt by 5; the prod will be the vel in ft per ; which mult by the area as before for the disch.

EX. What will be the disch through an opening in complete contraction, whose dimensions are 6 , or .5 ft vert; and 4 ft hor; the vert head above the cen of grav of the opening being constantly eet?

By Rule 1. The theoretical vel (Table 10, p 552) corresponding to 6 ft head, is 19.7 ft per sec. And 7 × .62 = 12.214 ft, the reqd vel. Again, the area of the opening = .5 × 4 = 2 sq ft; and 12.214 × = 24.428 cub ft per sec; the disch.

By Rule 2. The sq rt of 6 = 2.45; and 2.45 × 5 = 12.25 ft per sec, the reqd vel; and 12.25 × 2 = 5 cub ft per sec, the disch.

For circular openings in thin plate in full contraction, the ch in cub ft per sec may also be found thus: Square the diam in ft; mult this square by the con-nt 3.91; mult the prod by the sq rt of the head in feet.

EX. What will be the disch through a circular opening 1 ft diam, in thin plate, in full contraction, der a head of 3 ft, measured from the center of the opening?

Here, the square of 1 = 1; and 1 × 3.91 = 3.91; and 3.91 × 1.732 (or sq rt of 3) = 6.77 cub ft per , the reqd disch.

REM. 1. Even when the top of the circular opening coincides with the surf of the water, instead being some dist below it, the rule is quite nearly correct; the calculated disch being then but about ; part too great. The same remark also applies if the orifice be rectangular, or a mere notch, the per edge of which coincides with the water surf.

REM. 2. It will rarely happen that greater accuracy is reqd than that obtained by the foregoing les; but when such does occur, aid may be derived from the following **table deduced om the experiments of Lesbros and Poncelet,** on openings 8 ins de, of diff heights, and with diff heads. Use that coeff in the table which applies to the case, in-ead of the .62 of Rule 1. In some of the cases in this table, the upper edge of the opening is arer the surf-level of the reservoir than 1½ times its least transverse dimension.

ABLE 12. Coefficients for rectangular openings in thin vertical partitions in full contraction.*

Head above cen. of grav. of opening in Feet.	Head above cen. of grav. of opening in Inches.	The breadth in all the openings = 8 inches. HEIGHT OF OPENING. Ins. 8	Ins. 6	Ins. 4	Ins. 3	Ins. 2	Ins. 1	Ins. .4
.033	.4	.....	.....	.....	.....	.....	.....	.70
.0666	.8	.....	.....	.....	.....	.....	.65	.69
.0833	1	.....	.....	.....	.....	.....	.64	.68
.125	1½	.....	.....	.....	.....	.61	.64	.68
.1666	2	.....	.....	.....	.60	.62	.64	.68
.2083	2½	.....	.....	.59	.61	.62	.64	.67
.250	3	.....	.....	.60	.61	.62	.64	.67
.2917	3½	.....	.57	.60	.61	.62	.64	.66
.3333	4	.....	.58	.60	.61	.63	.64	.66
.3750	4½	.56	.59	.60	.61	.63	.64	.66
.4167	5	.57	.59	.61	.62	.63	.64	.66
.6666	8	.59	.60	.61	.62	.63	.64	.65
1	12	.60	.60	.61	.62	.63	.63	.64
3	36	.60	.60	.61	.62	.62	.63	.63
5	60	.60	.60	.61	.61	.62	.62	.62
10	120	.60	.60	.60	.60	.60	.61	.61

t will be observed from this table, that the dischs are proportionally greater through openings ose widths considerably exceed their heights; especially under small heads.

REM. 3. **Careful experiments on openings 4¼ ft wide, and 18 s high,** under heads of from 6 to 15 ft, show that the coeff .62 will give results rect within $\frac{1}{50}$ part, for openings of that size also, under large heads; although the thickness of partition varied on its diff sides, from 12 to 20 ins. It must be recollected, however, that nothing *re than close approximations* are to be attained in such matters.

REM. 4. *It has been asserted* by some writers, that **when two or more *tiguous openings*** are discharging at the same time from the same reser *they disch less in proportion than when only one of them is open.* Other experiments, b *seem to show that this is not the case; it is therefore probable, at least, that the diff. is trifling.*

* See first footnote on preceding page.

Case 2. **The discharge through thin vert partitions in plete contraction, when the surface-level, *m*, Fig 13, des as the water flows out into the air.** In this case, if the res prismatic, that is, if its hor sections are everywhere equal; and if no water is flowing into voir, to supply the place of that which flows out, then, to find the time reqd to disch the r

RULE. Inasmuch as the time in which such a reservoir entirely discharges itself, is twi which the same quantity would flow out under a constant head, as in Case 1, p 554; then culate the disch in cub ft per sec by Rule 1, Art 9; div the number of c tained in the reservoir, above the level *g* of the bottom of the opening, this disch; the quot will be the number of sec in which a volume equal the reservoir, to the depth *g*, would run out in Case 1, of a *constant* he *twice* this number will be the seconds reqd to empty the reservoir in Ca *varying* head.

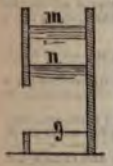

Fig 13

REM. If it should be reqd to find the time in which such a prismatic would *partly* empty itself, as, for instance, from *m* to *n*, Fig 13, first cal the above rule, the secs necessary to empty it if it had only been filled afterward calculate as if it had been filled to *m*. The diff between the will evidently be the time reqd to empty it from *m* to *n*. If the openin complete contraction, see Arts 11, &c.

If the disch is into a lower reservoir, surf-level remains constant, proceed in the same only use the diff of level of the two surfs as the head, and afterward (to Weisbach) increase the time $\frac{1}{75}$ part.

Art. 10. Disch from a reservoir R, Fig 14, the surf-le of which remains constantly at the same height; th an opening, *o*, in thin vert partition; and in complet traction; but entirely under water; and into a pris reservoir, *m*, in which the surf-level rises, as the water into it through *o*.

Fig 14

To find the time reqd to fill the reservoir *m*, from any leve the top of the opening, to any upper level, *d*.

RULE. First find the area in sq ft, of a hor section of the *m*, which is supposed to be of uniform section throughout Mult together this area, the constant number 2, and the sq vert height *a c* in ft. Call the prod *p*. Mult together the ar opening *o*, in sq ft; the coeff of contraction (usually about .62 the disch be into the air, or under water:) and the constant 8 the prod *y*. Div *p* by *y*. The quot will be the reqd time in se

EX. Let the hor dimensions of the reservoir *m* be 10 ft by 2 its area consequently 200 sq ft. Let the dimensions of the ope 2 ft by 3 ft; making its area 6 sq ft. Now, *c* being above the opening; and *c d* being 16 ft; the sq rt of which is 4; how lon reservoir *m* be in filling from *c* to *d*?

Here we have $p = 200 \times 2 \times 4 = 1600$; and $y = 6 \times .62 \times 8.0$

And $\frac{1600}{29.83} = 53.6$ sec, the time reqd.

REM. 1. **If it should be reqd to find the time of filling *m*, its bottom *e*, up to *d*,** we may do so very approximately by calcula the first rule in Art 9, the time reqd from *e* to the center of the opening *o*, as if all that the disch took place into air; and afterward, from the center of the opening to *d*, by the given. This case is similar to that of filling a lock from the canal reach above, in which level may be considered constant.

REM. 2. **If the bottom of the opening *o*, should coincide the bottom of the reservoir,** then the coeff will become greater t See Art 11, for obtaining coeffs for imperfect contraction.

REM. 3. **If the opening, instead of being in complet traction,** is of any of the shapes Figs 6 to 9, then a reference to Art 8 w what coeff must be substituted for .62.

Case 3. **Disch from one prismatic reservoir, Fig 15, W another, X, of any comparative sizes whatever, throu opening *o*, in a plane thin vert partition, and in com contraction; when the water rises in X, while it falls i**

Fig 15

To find the time in which the water, flowing from W into X o, will fall through the dist a s, so as to stand at the same le both reservoirs.

In this case, the water reqd to fill X from *e* to *d*, (*d* being t of the opening *o*,) flows out into the air; and the time necess to do so, must be calculated separately from that reqd above flows into water.

RULE. First from *e* to *d*. Find the hor area of each res sq ft. Mult the hor area of X, by the vert depth *d e* in ft, fo ft contained in that portion. Div these cub ft by the hor ar The quot will be the dist *e m*, in feet, through which the wa must descend, in order to fill X to *d*. Now calculate by Rule the number of secs in which W would empty itself into the a a constant head equal to *a n*; also the time in which it wo itself into air, under a constant head equal to *m n*. Take tween these two times. This diff will be the time in which W would fall from *a* to *m*; and that in X rise from *e* to *d*; stant head equal to *a n*. Mult this time by 2, for the tim actual case before us, under a head varying from *a n* t

ast add that still reqd for the water to fall from m to s; filling from d to c. To find this the square root of the remaining head, $m\,n$, in feet. Mult together, this sq rt; the hor the hor area of X; and the constant number 2. Call the prod p. Next, add together the f W and X. Mult together, the sum of these areas; the constant number 8.02; the area ing o, in sq ft; and the coeff of disch of the opening o (which coeff for opening in com- ction will be about .62). Call the prod y. Div p by y. The quot will be the *additional* n secs, very approximately.

the hor area of W be 100 sq ft; and that of X, 60 sq ft. Let $a\,n$ be 20 ft; and $m\,n$ 16 ft; a of the opening o, 3 sq ft. In what time will the water descend from a to s, and rise ?

h as the method of finding the time for filling from c to d, by the water falling from a to no further exemplification, we will confine ourselves to the *additional* time necessary for d to c, by the water falling from m to s. To find this, we have, the sq rt of the head and the sum of the 2 areas $= 100 + 60 = 160$. Hence, $\frac{4 \times 100 \times 60 \times 2}{160 \times 8.02 \times 3 \times .62} = \frac{p}{y} = \frac{48000}{2386.75} =$

e additional time reqd, very approximately.

. If the opening, as d, Fig 16, reaches very bottom of the reservoirs, we may the water flowing from R into T, as flowing into water, using the head $a\,m$, we at once calculate the time necessary er in the two reservoirs to arrive at the same level $s\,c$, by cess of the preceding rule; or, in other words, by the pro- in the preceding example. But in this case it must be borne at the opening o is no longer in *complete* contraction, inas- e contraction along its lower edge is suppressed. h will consequently be somewhat increased; and a coeff n .62 becomes necessary. The method of finding this, is e following Case 4. A reference to Art 8 will give the coeff opening is shaped as Figs 6 to 9.

Fig 16

II. *Case* 4. **The discharge through openings in plane ert partitions; but in incomplete contraction.**

ing may be such that contraction will take place portion of its perimeter, or at the top of the open- 17; while it is suppressed on another portion; as om and two ends of the opening a; where suppres- sed by the addition of *short* side and bottom pieces it may be caused by the bottom, or ends, or both, with the bottom and sides of the reservoir. In the disch will be greater than in those of complete ; but less than in those of full flow; inasmuch as g now partakes somewhat of the character of the of Art 5; and the coeff will rise from .62, or that *ally* pertains to openings in full contraction; and ich .8, or that of full flow, in proportion to the ex- imeter along which contraction is suppressed; or r .98 by the use of such openings as are shown by

Fig 17

nd approximately a new coeff of disch; and the disch in cases of incomplete contraction.

irst find by the foregoing rules, what would be the disch in the particular case that may onsideration, supposing the contraction to be complete. Then div that portion of the f the opening on which contraction is suppressed, by the entire perimeter. Mult the quot .152 if the opening is rectangular, or by .128 if circular. To the prod add unity, or 1. Call Then say, as unity, or 1, is to g, so is the coeff for complete contraction in ordinary cases t) to the reqd new coeff. Finally, repeat the original calculation, only substituting this new place of .62.

g to this rule, we have the following coeff of discharge for rectangular openings within pro- per cent when contraction is not suppressed on more than ⅙ of the perimeter. The theo- harge multiplied by the corresponding coeff will give the actual discharge. When the con- carried farther, the coeff becomes extremely irregular, and is probably indeterminable.

r complete contraction (ordinarily)	.62
en contraction is suppressed on ¼ the perimeter	.64
" " " " ½ " "	.67
" " " " ¾ " "	.69
" " " *entirely around the orifice*	.80

late ones can be estimated nearly enough, mentally.

. When, instead of a short spout, as in Fig 17, the g is provided with an indefinitely long hor trough, ttached, and open at top, there will be no practically appreciable diminution of disch below gh the simple opening as at a, Fig 11; provided the head measured above the cen of grav ing be at least as great as 2 or 2½ times the height of the opening itself. Therefore, under mstances the disch may be calculated by the rules in Art 9. But with smaller heads the nishes considerably; so when the head above the center becomes but as great as the height ing, it will be but about $\frac{4}{5}$ of the calculated one. With still smaller heads, the flow *s much more rapidly*; but has not been reduced to any rule.

***If, instead* of being hor, the trough is INCLINED**

Case 2. **The discharge through thin vert partitions i plete contraction, when the surface-level, *m*, Fig 13, de as the water flows out into the air.** In this case, if the r prismatic, that is, if its hor sections are everywhere equal; and if no water is flowing in voir, to supply the place of that which flows out, then, to find the time reqd to disch the

RULE. Inasmuch as the time in which such a reservoir entirely discharges itself, is t which the same quantity would flow out under a constant head, as in Case 1, p 554; th culate the disch in cub ft per sec by Rule 1, Art 9; div the number of tained in the reservoir, above the level *g* of the bottom of the opening this disch; the quot will be the number of sec in which a volume equ the reservoir, to the depth *g*, would run out in Case 1, of a *constant twice* this number will be the seconds reqd to empty the reservoir in *varying* head.

Fig 13

REM. If it should be reqd to find the time in which such a prisma would *partly* empty itself, as, for instance, from *m* to *n*, Fig 13, first e the above rule, the secs necessary to empty it if it had only been fille afterward calculate as if it had been filled to *m*. The diff between th will evidently be the time reqd to empty it from *m* to *n*. If the openi complete contraction, see Arts 11, &c.

If the disch is into a lower reservoir, surf-level remains constant, proceed in the sam only use the diff of level of the two surfs as the head, and afterwar to Weisbach) increase the time $\frac{1}{75}$ part.

Art. 10. Disch from a reservoir R, Fig 14, the surf-l of which remains constantly at the same height; t an opening, *o*, in thin vert partition; and in comple traction; but entirely under water; and into a pri reservoir, *m*, in which the surf-level rises, as the wate into it through *o*.

Fig 14

To find the time reqd to fill the reservoir *m*, from any le the top of the opening, to any upper level, *d*.

RULE. First find the area in sq ft, of a hor section of t *m*, which is supposed to be of uniform section throughou Mult together this area, the constant number 2, and the vert height *a c* in ft. Call the prod *p*. Mult together the opening *o*, in sq ft; the coeff of contraction (usually about the disch be into the air, or under water:) and the constan the prod *y*. Div *p* by *y*. The quot will be the reqd time in

EX. Let the hor dimensions of the reservoir *m* be 10 ft b Its area consequently 200 sq ft. Let the dimensions of the o 2 ft by 3 ft; making its area 6 sq ft. Now, *c* being above th opening; and *c d* being 16 ft; the sq rt of which is 4; how l reservoir *m* be in filling from *c* to *d*?

Here we have $p = 200 \times 2 \times 4 = 1600$; and $y = 6 \times .62 \times$

And $\frac{1600}{29.83} = 53.6$ sec, the time reqd.

REM. 1. **If it should be reqd to find the time of filling its bottom *e*, up to *d*,** we may do so very approximately by calcu the first rule in Art 9, the time reqd from *e* to the center of the opening *o*, as if all that the disch took place into air; and afterward, from the center of the opening to *d*, by t given. This case is similar to that of filling a lock from the canal reach above, in whic level may be considered constant.

REM. 2. **If the bottom of the opening *o*, should coinci the bottom of the reservoir,** then the coeff will become greater See Art 11, for obtaining coeffs for imperfect contraction.

REM. 3. **If the opening, instead of being in comple traction,** is of any of the shapes Figs 6 to 9, then a reference to Art 8 what coeff must be substituted for .62.

Case 3. **Disch from one prismatic reservoir, Fig 15, another, X, of any comparative sizes whatever, thro opening *o*, in a plane thin vert partition, and in co contraction; when the water rises in X, while it falls**

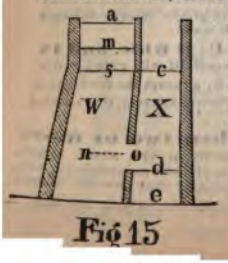

Fig 15

To find the time in which the water, flowing from W into o, will fall through the dist a n, so as to stand at the same both reservoirs.

In this case, the water reqd to fill X from *e* to *d*, (*d* being of the opening *o*,) flows out into the air; and the time nece to do so, must be calculated separately from that reqd abo flows into water.

RULE. First from *e* to *d*. Find the hor area of each r sq ft. Mult the hor area of X, by the vert depth *d e* in ft. ft contained in that portion. Div these cub ft by the ho The quot will be the dist *a m*, in feet, through which must descend, in order to fill X to *d*. Now calculate by the number of secs in which W would empty itself in a *constant* head equal to *a n*; also the time in which itself into air, under a constant head equal to *m n*. tween these two times. This diff will be the time in W would fall from *a* to *m*; and that in X rise from *stant* head equal to *a n*. Mult this time by 2, for actual case before us, under a head varying from

st add that still reqd for the water to fall from m to s; filling from d to c. To find this the square root of the remaining head, $m\,n$, in feet. Mult together, this sq rt; the hor area of X; and the constant number 2. Call the prod p. Next, add together the f W and X. Mult together, the sum of these areas; the constant number 8.02; the area ing o, in sq ft; and the coeff of disch of the opening o (which coeff for opening in com- section will be about .62). Call the prod y. Div p by y. The quot will be the *additional* n secs, very approximately.

the hor area of W be 100 sq ft; and that of X, 60 sq ft. Let $a\,n$ be 20 ft; and $m\,n$ 16 ft; a of the opening o, 3 sq ft. In what time will the water descend from a to s, and rise ?

h as the method of finding the time for filling from c to d, by the water falling from a to no further exemplification, we will confine ourselves to the *additional* time necessary for d to c, by the water falling from m to s. To find this, we have, the sq rt of the head and the sum of the 2 areas $= 100 + 60 = 160$. Hence, $\frac{4 \times 100 \times 60 \times 2}{160 \times 8.02 \times 3 \times .62} = \frac{p}{y} = \frac{48000}{2386.75} =$

e additional time reqd, very approximately.

. If the opening, as d, Fig 16, reaches very bottom of the reservoirs, we may the water flowing from R into T, as flowing into water, using the head $a\,m$, we at once calculate the time necessary er in the two reservoirs to arrive at the same level $s\,c$, by cess of the preceding rule; or, in other words, by the pro- in the preceding example. But in this case it must be borne at the opening o is no longer in *complete* contraction, inas- e contraction along its lower edge is suppressed. h will consequently be somewhat increased; and a coeff u .62 becomes necessary. The method of finding this, is e following Case 4. A reference to Art 8 will give the coeff opening is shaped as Figs 6 to 9.

Fig 16

th. *Case* 4. **The discharge through openings in plane ert partitions; but in incomplete contraction.**

ing may be such that contraction will take place portion of its perimeter, or at the top of the open- 17; while it is suppressed on another portion; as om and two ends of the opening a; where suppres- sed by the addition of *short* side and bottom pieces it may be caused by the bottom, or ends, or both, with the bottom and sides of the reservoir. In the disch will be greater than in those of complete ; but less than in those of full flow; inasmuch as g now partakes somewhat of the character of the of Art 8; and the coeff will rise from .62, or that *ally* pertains to openings in full contraction; and ch .8, or that of full flow, in proportion to the ex- imeter along which contraction is suppressed; or r .98 by the use of such openings as are shown by .

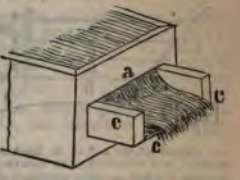

Fig 17

nd approximately a new coeff of disch; and the disch in cases of incomplete contraction.

first find by the foregoing rules, what would be the disch in the particular case that may onsideration, supposing the contraction to be complete. Then div that portion of the f the opening on which contraction is suppressed, by the entire perimeter. Mult the quot .152 if the opening is rectangular, or by .128 if circular. To the prod add unity, or 1. Call Then say, as unity, or 1, is to g, so is the coeff for complete contraction in ordinary cases) to the reqd new coeff. Finally, repeat the original calculation, only substituting this new place of .62.

g to this rule, we have the following coeff of discharge for rectangular openings within pro- t per cent when contraction is not suppressed on more than ⅙ of the perimeter. The theo- harge multiplied by the corresponding coeff will give the actual discharge. When the con- carried farther, the coeff becomes extremely irregular, and is probably indeterminable.

r complete contraction (ordinarily)	.62
en contraction is suppressed on ¼ the perimeter	.64
" " " " ½ " "	.67
" " " " ¾ " "	.69
" " " *entirely around the orifice*	.80

late ones can be estimated nearly enough, mentally.

. When, instead of a short spout, as in Fig 17, the g is provided with an indefinitely long hor trough, ttached, and open at top, there will be no practically appreciable diminution of disch below gh *the simple opening* as at a, Fig 11; provided the head measured above the cen of grav *ng be at least as great as* 2 or 1½ times the height of the opening itself. Therefore, under *stances the disch* may be calculated by the rules in Art 9. But with smaller heads the *shes considerably*; so when the head above the center becomes but as great as the heigh *g, it will be but about* $\frac{4}{5}$ of the calculated one. With still smaller heads, the *much more rapidly*; but has not been reduced to any rule.

If, instead of being hor, the trough is INCLI

as much as 1 in 10, the disch will be *increased* very slightly, (some 3 cent) over that calculated by the rules in Art 9, for the plain opening. These results we by experiments on a very small scale; and should be considered as mere approximations.

Art. 12. In a case like Fig 18, where contraction is su to be suppressed at the bottom, and at both vert sides

Fig 18

opening *o*, in consequence of their c with the bottom and sides of the reservoir; but front of the reservoir, instead of being vert, is slo and when the water, after leaving the opening, over a slightly sloping apron, *g*, then the disch i sec may be approximately found by Rule 1, Ca only substituting .8 in place of .62, when *f* slope or 1 to 1; or .74 when *f* slopes back 63°, or with to a rise of 2. In such cases of inclined fronts, th the opening must be measured *vert*, or rather *at r to the floor of the reservoir*; and not in a lin sloping front.

REM. **When the front, *f*, of the reservoir is vert, and a s apron or trough, *g*, is used,** having its upper edge level with th of the opening, the disch is not appreciably diminished below that which takes place fre air, provided the head above the cen of gray of the opening is not less than from

18 to 24 ins, for an opening 6 to 9 ins high.
12 to 16 " " " " 4 ins high.
9 " " " 2 ins or less, high.

Art. 13. To find, approximately, the time reqd for th tying of a pond, or any other reservoir, as Fig 19, wl not of a prismatic shape; through an opening, *n*, ne bottom.

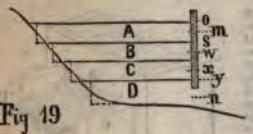

Fig 19

RULE. First ascertain the exact shape and of the reservoir. If large, and irregular, it mu fully surveyed; and soundings taken, and figu correct plan and cross-sections. Next, consider body of water to be divided into a series of thin A, B, C, D; the top line of the lower one bein few ins above the top of the opening *n*. It is no that these strata should be of equal thickness the thinner they are, the more correct will the The depth of the lower one, D, will vary to s with the height of the opening; those next abo not exceed about a foot in thickness, until a depth of 6 or 8 feet is reached; then they niently, and with sufficient accuracy, be increased to about 2 ft, for 6 or 8 ft more; and coming thicker as they approach the surf. By aid of the drawings, calculate the cont stratum in cub ft. Now, since the strata are thin, we may, without serious error, assu them to be prismatic, as shown by the dotted lines; and may assume that the head under stratum (except the lowest) empties itself through *n*, is equal to the vert height from the the opening to the center of the stratum. Thus, *m n* will be the head of A; *w n*, the hea the head of C. Then, for the stratum A, by Rule 1, Art 9, (only using *m n* as the head first and instead of the coeff .62 of that rule (which can only be used if *n* is in complete contrac .64, or whatever other coeff near the end of Art 11 applies to the case, calculate the disc per sec. Div the content of the stratum A by this disch, and the quot will be the number for discharging A. Using the head *w n*, proceed in precisely the same way with the strat using the head *x n*, do the same with C. Finally, for the lower stratum D, find by Rule 1, the same caution as before respecting the proper coeff,) in what time it would empty its *constant* head equal to *y n*, measured from its *surf* to the center of the opening. *Double* th be that reqd to empty itself in the case before us, under its *varying* head. Finally, add these separate times; and their sum will be the entire time reqd to empty the pond, or re proximately enough for practical purposes.

Art. 14. On the discharge of water over weirs, or ove

Experiments and observations on a grand scale, were made on this subject at Lowel **Mr James B. Francis,** C E, one of the most accomplished hydraulicians of th his "Lowell Hydraulic Experiments.") To apply the rule arrived at by Mr Francis, th conditions must exist.

The crest *a*, Fig 20, or top of the weir over which the water discharges, must be a hor, ner, in thin plate, or thin partition. (See p 554; first footnote.) Its inner side must forn straight line, *a h*, with the inner face of the dam, to a depth *a h*, *not less than* twice th *head a m*, measured vert from *a* to the level *o m* of the *hor* portion of the water surf; and *curved surf* of the falling sheet of water. The head *a m* may vary from 6 to 24 ins in he *ends a h*, *a h*, of the weir, Figs 21 and 22, must be vert; and its length, *a*, *a*, not less than *head a m*.

These conditions being observed, we may distinguish 2 cases; namely, Case 1st, that i *in Fig 21*, the weir extends entirely across the reservoir; so that its ends *a h*, *a h*, colu *form portions of*, the sides *s*, *s*, of the reservoir; in which case contraction takes place *upper edge a*, *a*, of the weir, Fig 22, as shown at *a* in Fig 20; but is suppressed entir *so that the water flows out as shown in plan by Fig 23.* And Case 2d, in which, *vert ends a h*, *a h*, as well as the crest *a*, *a*, are formed with a sharp corner in thin moreover, removed from the sides *v*, *v*, of the reservoir, a dist equal at least to the contraction takes place at the ends of the weir, as well as along its crest; and le as shown in plan at *a*, *a*, Fig 24.

To find the disch over a weir in thin plate, when, as in Figs 21 and 23, there is no contraction at its two ends.

Fig 21 Fig 20 Fig 22

Fig 23 Fig 24

RULE. Find the cube of the head $a\,m$, Fig 20, in ft. Take the sq rt of this cube. Mult together this sq rt: the length a, a, of the weir, in ft; and the constant number 3.33. The prod will be the quantity disch, in cub ft per sec. Or, in shape of a formula,

The disch, in cub ft per sec, = *the sq rt of the cube of the head* a m, *in ft* × *the length* a a, *of the overfall, in ft* × *const number* 3.33.

EX. How many cub ft per sec will flow over such a weir in thin plate, 200 ft long; having a head, $a\,m$, of 1.5 ft, measured to the *level* surf $a\,m$, of the reservoir; and with no contraction at either end?

Here, the cube of 1.5 = 3.375. And the sq rt of 3.375 = 1.837. And 1.837 × 200 × 3.33 = 1223.4 cub ft per sec, the reqd disch.*

This rule will also be very approximate even when there is contraction at both ends, provided the length of the weir is at least 10 times as great as the head $a\,m$; and provided the head is not less than 2 or 3 ins in depth. Indeed, it will be within about 6 per cent of the truth, for weirs with contraction at the ends, and whose lengths are but 4 times the head; and for the many cases in which no closer approximation is reqd, the disch may be taken at once from the following table.

To find the discharge over a weir in thin plate; when contraction takes place at both ends, as shown by Figs 22 and 24.

RULE. Proceed precisely the same as in Rule for Case I; except that when there is contraction at *both* ends, $\frac{1}{5}$ part of the head $a\,m$ is to be taken from the length, before using it as a multiplier; and when there is contraction at *one* end only, $\frac{1}{10}$ part of $a\,m$ must first be taken from the length $a\,a$.

EX. How many cub ft per sec will flow over such a weir in thin plate, 200 ft long; having a head $a\,m$, of 1.5 ft; with contraction at both ends?

Here, the cube of 1.5 = 3.375. And the sq rt of 3.375 = 1.837. Again, $\frac{1}{5}$ of $a\,m$ = .3 of a ft. And 200 − .3 = 199.7. Therefore we have 1.837 × 199.7 × 3.33 = 1221.6 cub ft per sec, the reqd disch; or *practically* the same as when there is no contraction, in this case.

REM. If instead of 3.33, we use 3.41, the two foregoing rules will apply to heads $a\,m$, as small as ½ an inch; and coeffs between 3.33 and 3.41 may be used for heads between about 5 inches, and ½ an inch, where more than common accuracy is aimed at. We may also use 3.3 instead of 3.33, for heads greater than 2 feet.

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* **Eytelwein's rule for weirs, over a thin edge;** and extending across the full width of the reservoir; and, of course, having no contraction at the ends, is thus: Mult together, the sq rt of the head; the head itself; the length of overfall; (all in feet;) and the constant number 3.4. The prod will be the disch in cub ft per sec. It gives 1249, instead of the above 1223 cub ft. The sq rt of the head is 1.225; and 1.225 × 1.5 × 200 × 3.4 = 1249. The rules are in fact identical, except in the coefficients.

TABLE 13. Of actual discharges in cub ft per sec, for each foot in length of weir in thin plate; and without contraction at either end; *a b*, Fig 20, being vert, and not less than twice the head *a m*. Very approximate also, when there is contraction at both ends, provided the length be at least 10 times the head. And but about 6 per cent in excess of the truth, if the length be but 4 times the head. (Original.)

The decimal .01 of a foot, is precisely .12 of an inch; or scant ⅛ inch.

Head, *a m*, in Feet.	Cub. ft. per Second.	Head, *a m*, in Feet.	Cub. ft. per Second.	Head, *a m*, in Feet.	Cub. ft. per Second.	Head, *a m*, in Feet.	Cub. ft. per Second.	Head, *a m*, in Feet.	Cub. ft. per Second.
.03	.017	.22	.351	.58	1.47	.94	3.04	2.4	12.2
.04	.027	.24	.401	.60	1.54	.96	3.14	2.5	13.0
.05	.038	.26	.452	.62	1.62	.98	3.24	2.6	13.8
.06	.050	.28	.505	.64	1.70	1.	3.33	2.7	14.6
.07	.063	.30	.560	.66	1.78	1.1	3.85	2.8	15.4
.08	.077	.32	.603	.68	1.86	1.2	4.38	2.9	16.2
.09	.092	.34	.659	.70	1.95	1.3	4.94	3.	17.1
.10	.108	.36	.719	.72	2.03	1.4	5.51	3.1	18.0
.11	.124	.38	.780	.74	2.12	1.5	6.11	3.2	18.9
.12	.142	.40	.842	.76	2.21	1.6	6.73	3.3	19.8
.13	.160	.42	.907	.78	2.30	1.7	7.37	3.4	20.7
.14	.178	.44	.972	.80	2.38	1.8	8.04	3.5	21.6
.15	.198	.46	1.04	.82	2.47	1.9	8.72	3.6	22.5
.16	.218	.48	1.11	.84	2.56	2.	9.42	3.7	23.5
.17	.239	.50	1.18	.86	2.65	2.1	10.0	3.8	24.4
.18	.260	.52	1.25	.88	2.74	2.2	10.8	3.9	25.4
.19	.282	.54	1.32	.90	2.84	2.3	11.5	4.	26.4
.2	.305	.56	1.40	.92	2.94				

In calculating this table, the coeff 3.41 was used for heads from .03 ft to .3 ft; then 3.33 to 2 ft; then 3.3 to the end.

From the Lowell experiments, by Mr Francis, it appears that when the depth, *m a*, is 1 foot, and the entire sheet of water, after passing over the weir, strikes a hor solid floor placed only about 6 ins below the crest *a*, of the weir, the disch is thereby diminished but about the $\frac{1}{1000}$ part; and that when the head *a m* is about 10 ins, and falls into water of considerable depth, no diff whatever is perceptible in the disch, whether the surf of that water be about 4, or about 13 ins below the crest *a*; and that a fall below the crest *a*, equal to one-half of the head *a m*, is quite sufficient.

If the water in the reservoir, or in the feeding canal, instead of being stagnant, has a slight current toward the weir, the disch will be but very little increased thereby when the head *a m* is several ins. Mr Francis observed that a current of 1 foot per sec, or nearly .7 of a mile per hour, increased the disch but about 2 per cent, when the head was 13 ins; and one of 6 ins per sec, about 1 per ct, when the head was 8 ins. Whenever the effect of the current, however, is so great as to require notice, proceed as follows: Find in Table 10, p 552, the theoretical head *h* which corresponds to the observed vel of the approaching current. Add this head to the head *a m*, Fig 20. Cube the sum. Take the sq rt of this cube, calling it *a*. Also cube the theoretical head *h*. Take the sq rt of this cube. Call it *b*. From *a* take *b*. Find the square of the remainder. Take the cube rt of this square, for a new head H′, to be employed in Cases 1 and 2, instead of H, or *a m*; or in shape of a formula,

$$\left((H+h)^{\frac{3}{2}}-h^{\frac{3}{2}}\right)^{\frac{2}{3}}=H' \text{; the new head.}$$

Art. 15. If the inner face of the weir and dam, instead of being vert, as *a b*, Fig 20, is sloped, as *a* or *b*, Fig 25; the contraction on the crest will be diminished; and consequently the disch will be increased. This will also be the case if the inner corner or edge of the crest be rounded off, instead of being left sharp; or if the sides of the reservoir converge more or less as they approach the weir; so as to form wings for guiding the water more directly to it; or if *a b*, Fig 20, be less than twice *a m*. Indeed, so many modifying circumstances exist to embarrass experiments on this, and similar subjects, that some of those which have been made with great care, are rendered inapplicable as other than tolerable approximations, in consequence of the neglect to take into consideration some local peculiarity, which was not at the time regarded as exerting an appreciable effect. Unless, therefore, circumstances admit of our com-

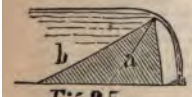


Fig 25

bining all the conditions mentioned in the first part of this article; and thereby securing very ap*proximate results*, we must either resort to an actual measurement of the disch in a vessel of known *capacity*; or else be contented with rules which may lead to errors of 5, 10, or more per cent, in pr*portion* as we deviate from those conditions. Frequently, even 10 per ct of error may be of little *importance*.

The following rule for finding the discharge over an o

fall, or weir, approximately, has been prepared by the author from various data.

Mult together the length, *a*, *a*, of the weir, Figs 21 and 22, in feet; the head, *a m*, Fig 20, measured to the *level* surf of the reservoir, in ft; the theoretical vel in ft per sec, corresponding to the head *a m*, (Table 10, p 552:) and that coeff from the following table, which agrees most nearly with the case. The prod will be the reqd disch in cub ft per sec, near enough for ordinary purposes; and probably quite as close as can be arrived at without actual measurement in each case that presents itself.

Ex. How much water will be dischd over a weir 60 ft long; the crest of which is level, smooth, and 3 ft wide, or thick; and over which the head *a m*, Fig 20, is 8 ins, or .6666 ft thick? Here the theoretical vel for a head of 8 ins, (Table 10, p 552) is 6.55 ft per sec. The coeff for a weir whose crest is level, and 3 ft wide, with a head of 8 ins, is by the following table .31. Consequently, 60×.6666×6.55×.31=81.21 cub ft per sec; the reqd disch approximately.

TABLE 14. Of coefficients of approximate discharge over weirs of different thicknesses, varying from a sharp edge to 3 feet. —(Original.)

Head *a m* in Feet.	Head *a m* in Inches.	Sharp Edge.	2 Inches Thick.	3 Ft Thick; smooth; sloping outward; and downward, from 1 in 12 to 1 in 18.	3 Ft Thick; smooth, and level.
.0833	1	.41	.37	.32	.27
.1666	2	.40	.38	.34	.30
.25	3	.40	.39	.34	.31
.3333	4	.40	.41	.35	.31
.4166	5	.40	.41	.35	.32
.5	6	.39	.41	.35	.33
.5833	7	.39	.41	.35	.32
.6666	8	.39	.41	.34	.31
.8333	10	.38	.40	.34	.31
1.	12	.38	.40	.33	.31
2.	24	.37	.39	.32	.30
3.	36	.37	.39	.32	.30

Fig 26

REM. 1. **When the water, after passing over a weir, Fig 26, instead of falling freely into the air, is carried away by a slightly inclined apron or trough,** T, the floor of which coincides with the crest, *a*, of the weir, then the disch is not appreciably diminished thereby when the head *a m* is 15 ins or more. But if the head *a m* is but 1 ft, then the calculated disch must be reduced about $\frac{1}{10}$ part; if 6 inches, $\frac{2}{10}$; if 2½ inches, $\frac{3}{10}$; and if 1 inch, $\frac{5}{10}$, or one-half, as approximations.

REM. 2. **Professor Thomson, of Dublin, proposed the use of triangular notches, or weirs,** for measuring the disch; inasmuch as then the periphery always bears the same ratio to the area of the stream flowing over it; which is not the case with any other form. Experimenting with a right-angled triangular notch in thin sheet iron, Fig 26¼; with heads of from 2 to 7 ins, measured vert from the bottom of the notch, *to the level surf of the quiet water*, he found the disch in cub ft per sec to be as follows:

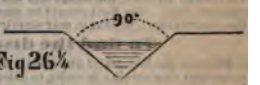


Fig 26¼

Find the fifth power of the head in *inches*, (Table 5, pages 546–7.) Take the sq rt of this 5th power, (Table 6, page 548.) Mult this sq rt by .0051. Or by formula,

$$\text{Disch in cub ft per sec} = \text{Head in inches}^{\frac{5}{2}} \times .0051.$$

REM. 3. **Fig 26½ shows a singular effect observed at Clegg's dam,** across Cape Fear River, N. C. It is from measurements made by Elwood Morris, C E; *by whom they were communicated to the writer.* The dam is of wooden cribwork; *its level crest, 8 ft 5 ins wide, is covered with plank; along which the water glides in a smooth sh 6 ins deep. (at the time of measurement.)* At the upper end of this sheet, and in a dist of show *a head of 9 ins forms itself, as* in the fig. We have no comments to offer; but consider the fac *of sufficient interest to be made more widely known.*

***Art. 16.* On the flow of water through open channel** *following rules, to the end of Art 20,* must be regarded merely as approximations. The ru *volved in much uncertainty.*

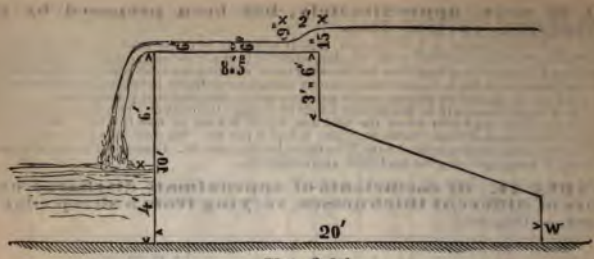


Fig 26½

To ascertain approximately the mean vel of all the water in any given cross-section of a river, canal, or other water-course; having given the greatest surf vel only.

When no great accuracy is reqd, this mean vel may be deduced from the greatest surf vel. Thus, select a place where the stream is for some dist (the longer the better) of tolerably uniform cross-section; and free from counter-currents, slackwater, eddies, rapids, &c. Observe, by a seconds-watch, or pendulum, how long a time a float (such as a small block of wood) placed in the *swiftest* part of the current, occupies in passing through some previously measured dist. From 50 feet for slow streams, to 150 ft for rapid ones, will answer very well. This dist in ft, or ins, div by the entire number of seconds reqd by the float to traverse it, will give the greatest surf vel in ft or ins per sec. Take $\frac{4}{5}$ of this vel; or, in other words, mult it by the dec .8. The prod will be approximately the mean vel of the entire body of water. The result will be somewhat more accurate, if, instead of mult by .8 for every surf vel, we mult by the following decimals in the 4th col.*

Ex. What will be the mean vel of all the water moving through a uniform channel, the greatest surf vel of which is found to be 60 ins per sec? Here, 60 × .83 = 49.8 ins per sec; the reqd vel.

TABLE 15.

SURFACE VELOCITY.			Mult. greatest Surface Vel., by	SURFACE VELOCITY.			Mult. greatest Surface Vel., by
Inches per Sec.	Feet per Sec.	Miles per Hr.		Inches per Sec.	Feet per Sec.	Miles per Hr.	
4	.333	.227	.76	80	6.667	4.545	.85
12	1.	.682	.77	100	8.333	5.682	.86
20	1.667	1.136	.79	120	10.	6.818	.87
40	3.333	2.273	.81	140	11.7	7.955	.88
60	5.	3.409	.83	160	13.3	9.091	.89

The surf vel should be measd in perfectly calm weather, so that the float may not be disturbed by wind; and, for the same reason, the float should not project much above the water. The measurement should be repeated several times to insure accuracy. In very small streams, the banks and bed may be trimmed for a short dist, so as to present a uniform channel-way. The float should be placed in the water a little dist above the point for commencing the observation; so that it may acquire the full vel of the water, before reaching that point.

To find the discharge in the foregoing case.

Measure the breadth of the stream at the surface; and also a sufficient number of depths, taken in a straight line across; in order to calculate the area of its cross-section. This area in sq ft, mult by the mean vel in ft per sec, will evidently give the disch in cub ft per sec.

Rem. If the channel is a small one, and made through common earth, especially if sandy, the loss by soakage into the soil, and by evaporation, will frequently abstract so much water, that the disch will gradually become less and less, the farther down stream it is measured. Long canal feeders thus generally deliver into the canal but a small proportion of the water that enters their upper ends.

Art. 17.* To find approximately the vel at the bottom of a stream; at any part of its cross-section.

Rule. First measure the surf vel over the same part. Mult it by the corresponding decimal in the foregoing table, for a mean vel. Mult this mean vel by 2. From the prod subtract the surf vel. The remainder will be approximately the reqd bottom vel. Ex. Surf vel 100 ins per sec. And 100 × .86 = 86 ins per sec, mean vel; and 86 × 2 = 172; and 172 − 100 = 72 ins per sec, bottom vel.

* As we have already stated, these rules give in many cases very rude results; that in Art. 17 ularly. With the same surface vel, a wide and deep stream will have greater mean and b s than a small shallow one. The writer knows of no reliable rule on the subject.

18. TABLE 16. Of approximate velocities of streams hes per sec; based on the foregoing rules. See Table, p 570.

all the Stream, in Ins. per Sec.	Greatest Bottom Vel., in Ins. per Sec.	Greatest Surface Vel., in Ins. per Sec.	Mean Vel. of all the Stream, in Ins. per Sec.	Greatest Bottom Vel., in Ins. per Sec.	Greatest Surface Vel., in Ins. per Sec.	Mean Vel. of all the Stream, in Ins. per Sec.	Greatest Bottom Vel., in Ins. per Sec.	Greatest Surface Vel., in Ins. per Sec.	Mean Vel. of all the Stream, in Ins. per Sec.	Greatest Bottom Vel., in Ins. per Sec.
2.28	1.54	23	18.2	13.3	48	39.4	30.8	86	73.4	60.9
3.04	2.08	24	19.	14.	50	41.2	32.3	88	75.3	62.7
3.81	2.62	25	19.8	14.6	52	42.9	33.8	90	77.1	64.3**
4.58	3.16*	26	20.6	15.2	54	44.7	35.3	92	78.9	65.9
5.36	3.72	27	21.4	15.9	56	46.4	36.8	94	80.8	67.7
6.14	4.28	28	22.3	16.6	58	48.2	38.4	96	82.7	69.3
6.92	4.84	29	23.1	17.2	60	49.9	39.8¶	98	84.5	71.
7.70	5.40	30	23.9	17.9	62	51.7	41.4	100	86.3	72.6
8.49	5.98†	31	24.8	18.5	64	53.5	43.	105	90.9	76.9
9.29	6.58	32	25.6	19.3	66	55.3	44.6	110	95.6	81.2
0.1	7.16	33	26.5	19.9	68	57.1	46.1	115	100.2	85.3
0.9	7.72‡	34	27.3	20.6	70	58.9	47.7	120	104.9	89.8
1.7	8.34	35	28.1	21.3	72	60.7	49.4	125	109.6	94.3
2.5	8.96§	36	29.	22.	74	62.5	50.9	130	114.3	98.5
3.3	9.58	38	30.7	23.5‖	76	64.3	52.6	135	119.1	103.
4.1	10.2	40	32.5	25.	78	66.2	54.3	140	123.9	108.
4.9	10.8	42	34.2	26.5	80	67.9	55.8	145	128.8	112.
5.7	11.4	44	36.	28.	82	69.8	57.6	150	133.7	117.
6.5	12#	46	37.7	29.3	84	71.7	59.3	160	142.4	125.
7.3	12.7									

a mile per hour; and begins to wear away fine clay. This, and the following, are mere ap-
ions, chiefly deduced by Du Buat from experiments on a trifling scale. More observations
ubject are needed. At the same vel, will not the scour be more rapid at a great depth than
l one?
a mile per hour; and just lifts fine sand.
a mile per hour; lifts sand as coarse as linseed.
a mile per hour; moves fine gravel.
iles per hour; moves pebbles about an inch in diam.
miles per hour; moves pebbles as large as an egg.
miles per hour; begins to wear away soft shistus.

; or 1.4666 ft per sec; or 1056 ins, or 88 ft per min, = 1 mile per hour.
ng to Smeaton, a vel of 8 miles an hour will not derange quarry rubble stones, not exceed-
cub ft, deposited around piers, &c; except by washing the soil from under them.
1 inch per sec, = 5 ft per min, = .056818 of a mile, or 300 ft per hour.
1 foot per sec, = 60 ft per min, = .681816 of a mile, or 3600 ft per hour.

educe inches per sec, to feet per minute, multiply by 5.
" " " " " " " hour, " " 300.
" " " " to miles per hour, divide by 17.6.
ile per hour = 88 ft per min = 1.4667 ft, or 17.6 ins per sec.

19. The simplicity, and easy application of the fore- rules frequently lead to their adoption for the gaug- f streams; but it must be stated that this method is ved in much uncertainty. The experiments of Mr Francis, at ave shown its liability to err at least 15 per cent in *deficiency*; while observations by others m to indicate that it may, under diff circumstances, be as much (if not more) in *excess* of

Mr Francis found that *surface* floats of wax, 2 ins diam, floating down the center of a rec- flume 10 ft wide, and 8 ft deep, actually moved about 6 per cent *slower* than a tin tube 2 ins ching from a few ins above the surf, down to within 1½ ins of the bottom of the flume; and bottom with lead, to insure its maintaining a nearly vert position. While the wax *surf* float the rate of 3.73 ft per sec, the rate of the tube (which was evidently very nearly the same the center *vert* thread of water) was 3.98 ft per sec. Also, that in the same flume, with vels iter tube varying from 1.55, to 4 ft per sec, the vel of the tube was *less* than that of the mean entire cross-section of water in the flume, about as .96 to 1, for the lesser vel; and .93 to 1 reater vel. While, in another rectangular flume 20 ft wide and 8 ft deep, with vels varying to 1.84 ft per sec, that of the tubes was *greater* than that of the entire mass of water, about 1. In a flume 29 ft wide, by 8.1 ft deep, with vels of about 3 ft per sec, it was as 1 to 3; and *56½ ft wide, by 8.4 ft* deep, with vels of about 3½ ft per sec, as 1 to .97.

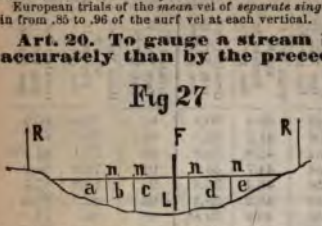
les Ellet, Jr, C E, found in the Mississippi " at diff points o *depths varying* from 54 to 100 ft; and in currents varying from 3 to 7 miles an hour, *f a float supporting* a line 50 ft long, is almost always greater than that of the surf. *same results* were obtained with lines 25 and 75 ft long; the excess of the speed *ng about 2 per cent* over that of the simple floats; and Mr Ellet concludes, therefo *of the entire* cross-section of the Mississippi, instead of being less, is absolutely

by about 2 per cent, than the MEAN *surf* vel. He, however, employed .8 of the *greatest* surf vel as representing approximately, in his opinion, the mean vel of the *entire* cross-section of water. In *shallow* streams, he always found the surf float to travel more rapidly than a line float.

European trials of the *mean* vel of *separate single verticals*, in tolerably deep rivers, have resulted in from .85 to .96 of the surf vel at each vertical. The mean of all may be taken at .9.

Art. 20. To gauge a stream by means of its velocity, more accurately than by the preceding methods.

Fig 27



The following is perhaps the most accurate means that can be adopted for gauging a stream by means of direct measurement of its vel. In case of a large river it will involve some trouble, time, and expense; but where accuracy is necessary, those considerations must yield. Select a place where the cross-section remains for a short dist, tolerably uniform, and free from counter-currents, eddies, still water, or other irregularities. Prepare a careful cross-section, as Fig 27. By means of poles, or buoys, *n, n*, divide the stream into sections, *a, b, c*, &c. Plant two range-poles, R, R, at the upper end; and two others at the lower end of the dist through which the floats are to pass; for observing the time, by a seconds watch, or a pendulum, which they occupy in the passage. Then measure the *mean* vel of each section *a, b, c*, &c, separately, and directly, by means of long floats, as F L, reaching to near the bottom; and projecting a little above the surf. The floats may be tin tubes, or wooden rods; weighted in either case, at the lower end, until they will float nearly vert. They must be of diff lengths, to suit the depths of the diff sections. For this purpose the float may be made in pieces, with screw-joints. The area of each separate section of the stream in sq ft, being mult by the observed mean vel of its water in ft per sec, will give the disch of that section in cub ft per sec. And the discharges of all the separate sections thus obtained, when added together, will give the total disch of the stream. And this total disch, div by the entire area of cross-section of the stream in sq ft, gives the mean vel of *all* the water of the stream, in ft per sec.

Art. 21. To find, approximately, the mean vel, and disch in an open canal, aqueduct, river, &c, of uniform cross-section, and fall throughout; knowing its dimensions, and rate of fall, or descent in feet per foot.

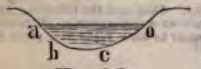


Fig 28

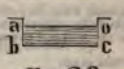


Fig 29

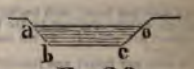


Fig 30

RULE. Find the *area* of the water-way, *a b c o a*, Figs 28, 29, 30, in sq ft. Find the dist *a b c o*, in feet; being that portion of the cross-section of the sides and bottom of the channel bed which is in actual contact with the water. This dist is called the WET PERIMETER, or WET BORDER of the channel. Mult together the area *a b c o a*; the fall in feet per foot; and the constant number 8975; (in practice we may use 9000.) Div the prod by the wet perimeter. Take the sq rt of the quot. From this sq rt subtract the constant dec .1089. The rem will be the mean vel in ft per sec of all the water in the entire cross-section of the stream. Or by formula,

$$\text{Mean vel in ft per sec} = \sqrt{\left(\frac{\text{Area of water-way in sq ft} \times \text{fall, in ft per ft} \times 8975}{\text{wet perimeter in feet.}}\right)} - .1089.$$

EX. A canal of rectangular cross-section, as Fig 29, is to be 10 ft wide; and to have 3.5 ft depth of water. Its bottom is to have a uniform fall, slope, or descent, at the rate .246 of a ft in a length of 492 ft. What will be the mean vel of all the water; and how many cub ft of water will it disch per sec?

Here, the area of water-way $= 10 \times 3.5 = 35$ sq ft. The fall, .246 ft, div by 492 ft (the dist in which it occurs) gives .0005 of a ft fall for every foot of length. The wet perimeter, $= a\,b + b\,c + c\,o = 3.5 + 10 + 3.5 = 17$ ft. Hence we have $\frac{35 \times .0005 \times 8975}{17} = 9.239$. And the sq rt of $9.239 = 3.04$. And $3.04 - .1089 = 2.9311$ ft per sec, the reqd vel. The disch will evidently be, this vel mult by the area of water-way in sq ft; or $2.9311 \times 35 = 102.588$ cub ft per sec. See Remark, p 562; also, Remark 2, of next Art.

REM. **The following more simple rule, is probably as close an approximation as the foregoing.** It is applicable to sewers, &c.

$$\text{Vel in ft per sec} = \sqrt{\frac{\text{Area of water-way, in sq ft}}{\text{wet perimeter in ft}} \times \text{Twice the fall in ft per mile.}}$$

For the foregoing example it gives a vel of 3.3 ft, instead of 2.93 ft; and some experiments with sewers would indicate that 3.3 is nearer the truth; but as before remarked, perfect accuracy in these matters must not be looked for. The degree of smoothness of the channel, and even the muddiness of the water, will affect the result appreciably.

Art. 22. To find, approximately, what fall must be given every foot in length of a uniform canal, or other wa course, the dimensions of which are known; to enabl discharge a reqd quantity, in a given time.

t. Div the number of cub ft of water reqd per sec, by the area in sq ft of the water-way, *a b c o a*, , 29, 30, &c. The quot will be the mean vel which the water must have, in ft per sec, in order to he reqd quantity. Next find the square of this vel. Also find the length of wet perimeter, *a b c o*, Mult together the square of the vel; the wet perim; and the constant dec .0001114. Div the r the area of water-way, *a b c o a*. Call the quot M. Next mult together, the simple vel itself; wet perim; and the constant dec .00002426. Div the prod by the area of water-way, *a b c o a*. Add t to M. The sum will be the reqd fall, in decimals of a foot, which must be given to every foot th of the canal. Or, in shape of a formula,

$$\frac{\textit{Fall in every}}{\textit{foot of length}} = \frac{\textit{Vel}^2 \times \textit{wet perim} \times .0001114}{\textit{area of water-way in sq ft}} + \frac{\textit{Vel} \times \textit{wet perim} \times .00002426}{\textit{area of water-way in sq ft.}}$$

The same as the foregoing. What fall per foot must be given to a rectangular canal, or water- Fig 29, 10 ft wide, and having 3.5 ft depth of water, to enable it to disch 102.55 cub ft per sec?

, the area of water-way, *a b c o a*, = 35 sq ft. And $\frac{102.55}{35} = 2.93$ ft per sec, the vel. And 2.933 9. Also, the wet perim *a b c o* = 3.5 + 10 + 3.5 = 17 feet. Hence,

$$\frac{8.5849 \times 17 \times .0001114}{35} \times \frac{.016258}{35} = .000465 = \text{M}.$$

$$\text{Again;}\ \frac{2.93 \times 17 \times .00002426}{35} = \frac{.0012084}{35} = .000035.$$

lly, .000465 + .000035 = .0005 ft; the reqd fall in every foot of length of the canal; the same as preceding example. See Remark, Art 16, p 562.

M. 1. **Or the following formula, based on that in Remark, 21, may be used.**

$$\left.\begin{matrix}\textit{The square of the vel}\\ \textit{in ft per sec.}\end{matrix}\right\}\textit{divided by}\left\{\frac{\textit{Area of water-way}}{\textit{wet perimeter.}}\right. = \begin{matrix}\textit{Twice the fall}\\ \textit{in ft per mile.}\end{matrix}$$

the foregoing example it gives 2.085 ft per mile; or very nearly .0004 ft per ft, instead of .0005.

M. 2. In designing watercourses, it is important to bear in mind that **when anal, as *a b*, Fig 31, either of uniform cross-section, and cent, or otherwise, is carried directly from a reservoir, hout any enlargement of canal at the point of junc- n, a slight fall forms itself**, somewhat of the shape shown on an erated scale in Fig 31. This fall will not only be an excess of vel at that point, beyond that ich the canal was intended; but will also di- h the calculated depth of water in the canal, *hout its entire length*, to an amount equal to ll. Thus, if *b m* were the depth calculated by les, the actual depth would be *n m*. In large , as in those intended for navigation, this fall ill be approximately equal, in feet, to

Fig 31

$$\frac{\textit{The square of the calculated mean vel, in ft per sec} \times .0155,}{.9}$$

n smaller canals, such as mill courses, to

$$\frac{\textit{The square of the calculated mean vel, in ft per sec.} \times .0155.}{.75}$$

the great mean vel of 3 miles an hour, or 4.4 ft per sec, this t *a* would be as much as .333 of a foot, or 4 ins, in a large ; and .4 of a foot, or 4.8, in an ordinary mill-course. At 1½ per hour, or 2.2 ft per sec, it would be .083 of a ft, or 1 inch, canal; and .1 ft, or 1.2 ins, in the mill-course. Without this e quiescent water of the reservoir would not flow out suffi- y fast to maintain the calculated vel in the canal.

o destroy this fall, or rather, to reduce it to most inappreciable quantity, and thus secure the calculated and disch; at the same time providing a substitute for the t is merely necessary to enlarge the canal at its junction the reservoir; giving it at that point a shape corresponding e degree to that of the *contracted vein*. Thus, if *a b*, Fig 32, e width of the canal, make *c d* equal to .7 of that width; at *eservoir, make e f* = 2 *c d*; and describe two arcs of a circle a e *f with a rad a m* = 1½ *a b*. See Remark, Art 16, p 562.

Fig 32

. 3. ***Mr Poole, C E, England,*** tested the *t, on open canal portions of the Rocquefavour aqueduct, nd found it* to accord closely with fact.

$$\text{Vel in ft per sec} = \sqrt{10000 \frac{\text{total fall in ft} \times \text{area of water-way, in sq ft}}{\text{total length in ft} \times \text{wet perim in ft.}}}$$

The rule in Art 21, gives for the example in that Art, a vel of 3.98 ft per sec; and Mr Poole's rule gives 3.21 feet.

Mr Poole says also, that the fall per mile, in ins, reqd in a canal, to impart any given mean vel, in ins per sec, may be found thus: Div the area of water-way in sq ft, by the wet perim in ft. The quot is what is called **the hydraulic mean depth**, or **mean radius**, of the canal.* Mult this by 12, to reduce it to ins; and call the prod D. Then div the reqd mean vel, by the des .909. Square the quot. Div said square by *twice* D. By formula,

$$\text{The reqd fall, in ins per mile} = \left(\frac{\text{given mean vel in ins per sec}}{\text{const number .909.}}\right)^2 \div \text{twice the mean rad in ins.}$$

For the example at the first part of this Art, Mr Poole's rule gives a fall of 30.3 ins per mile; while the rule at the head of the Art, gives a fall of .0005 of a ft per ft in length; which is equal to 31.68 ins per mile. The two rules, therefore, agree as closely as can be expected in cases of this kind.

Art. 23. The writer has prepared the two following tables for the convenience of forming, at the instant, an approximate idea of the requisite dimensions of channels for fulfilling certain given conditions.

---

* Or of any stream; or of a pipe. In a cylindrical pipe *running full*; or in any *open* channel of *semicircular* cross-section, *running full*, the hyd mean depth thus found is equal to ¼ of the diam of the circle.

TABLE 17. Of mean velocities in feet per second; and of discharges in cubic feet per second, in a rectangular channel 10 ft wide; but with different depths, and falls. (Original)

$$\text{Formula,} \quad \frac{\text{Vel}}{\text{in ft per sec}} = \sqrt{\frac{\text{Area of water-way in sq ft} \times \text{fall per foot in feet}}{\text{wet perimeter in feet}} \times 8975} - .1089.$$

The velocities are *above* the discharges in this table.

Fall per Mile in Feet.	Fall in one Foot of Length, Ft.	DEPTHS IN FEET. .25 Vel. and Dis.	.50 Vel. and Dis.	.75 Vel. and Dis.	1 Vel. and Dis.	1.5 Vel. and Dis.	2 Vel. and Dis.	2.5 Vel. and Dis.	3 Vel. and Dis.	4 Vel. and Dis.	5 Vel. and Dis.	6 Vel. and Dis.	8 Vel. and Dis.
.1	.000019	.091 .227	.168 .840	.223 1.67	.266 2.66	.333 5.00	.383 7.66	.422 10.5	.455 13.6	.505 20.2	.542 27.1	.570 34.2	.612 49.0
.2	.000038	.174 .435	.283 1.42	.361 2.71	.422 4.22	.516 7.74	.587 11.7	.643 16.1	.688 20.6	.760 30.4	.812 40.6	.852 51.1	.911 72.9
.3	.000057	.238 .595	.371 1.85	.467 3.50	.542 5.42	.657 9.86	.743 14.9	.812 20.3	.868 26.0	.954 38.2	1.01 50.5	1.07 64.2	1.14 91.2
.4	.000076	.292 .730	.446 2.23	.556 4.17	.643 6.43	.776 11.6	.875 17.5	.950 23.7	1.02 30.6	1.12 44.8	1.19 59.5	1.25 75.0	1.33 106
.5	.000095	.340 .850	.511 2.55	.635 4.76	.731 7.31	.880 13.2	.992 19.8	1.08 27.0	1.15 34.5	1.26 50.4	1.35 67.5	1.41 84.6	1.51 121
.6	.000114	.383 .957	.571 2.85	.705 5.29	.812 8.12	.975 14.6	1.10 22.0	1.19 29.7	1.27 38.1	1.40 56.0	1.49 74.5	1.56 93.6	1.66 133
.7	.000133	.422 1.05	.626 3.13	.771 5.78	.886 8.86	1.06 15.9	1.19 23.8	1.30 32.5	1.38 41.4	1.52 60.8	1.61 80.5	1.69 101	1.80 144
.8	.000152	.459 1.15	.676 3.38	.832 6.24	.954 9.54	1.14 17.1	1.28 25.6	1.39 34.7	1.48 44.4	1.63 65.2	1.73 86.5	1.81 109	1.94 155
.9	.000170	.494 1.23	.724 3.62	.889 6.67	1.02 10.2	1.22 18.3	1.37 27.4	1.49 37.2	1.57 47.1	1.73 69.2	1.85 92.5	1.93 116	2.06 165
1.	.000189	.526 1.31	.770 3.85	.943 7.07	1.08 10.8	1.29 19.3	1.45 29.0	1.57 39.2	1.68 50.4	1.83 73.2	1.95 97.5	2.04 122	2.17 174
1.5	.000284	.670 1.67	.970 4.85	1.18 8.85	1.35 13.5	1.60 24.0	1.80 36.0	1.95 48.7	2.08 62.4	2.27 90.8	2.41 120	2.52 151	2.70 216
2.	.000379	.790 1.97	1.13 5.65	1.38 10.3	1.58 15.8	1.87 28.0	2.09 41.8	2.27 56.7	2.41 72.3	2.64 106	2.80 140	2.93 175	3.14 251
2.5	.000473	.896 2.24	1.25 6.25	1.56 11.7	1.77 17.7	2.10 31.5	2.35 47.0	2.55 63.7	2.71 81.3	2.96 118	3.15 157	3.29 197	3.53 282
3.	.000568	.992 2.48	1.41 7.05	1.71 12.8	1.95 19.5	2.32 34.8	2.59 51.8	2.80 70.0	2.98 89.4	3.26 130	3.46 173	3.61 217	3.86 309
3.5	.000663	1.08 2.70	1.53 7.65	1.86 14.0	2.12 21.2	2.51 37.6	2.84 56.8	3.04 76.0	3.23 96.9	3.53 141	3.75 187	3.91 235	4.18 334
4.	.000758	1.16 2.90	1.65 8.25	1.99 14.9	2.27 22.7	2.69 40.3	3.01 60.2	3.26 81.5	3.46 104	3.79 148	4.01 200	4.19 251	4.47 357
4.5	.000852	1.24 3.10	1.76 8.80	2.12 15.9	2.41 24.1	2.86 42.9	3.19 63.8	3.46 86.5	3.68 110	4.01 160	4.26 213	4.45 267	4.74 379
5.	.000947	1.31 3.27	1.85 9.25	2.24 16.8	2.55 25.5	3.02 45.3	3.37 67.4	3.65 91.2	3.88 116	4.24 170	4.50 225	4.70 282	5.00 400
7.	.001326	1.57 3.92	2.22 11.1	2.68 20.1	3.04 30.4	3.59 53.8	4.01 80.2	4.34 108	4.61 138	5.03 201	5.34 267	5.58 335	5.93 474
10.	.001894	1.90 4.75	2.67 13.3	3.22 24.1	3.65 36.5	4.32 64.8	4.82 96.4	5.21 130	5.54 166	6.04 242	6.41 320	6.69 401	7.12 570

The mean vel in other uniform rectangular watercourses *with widths diff from those* in the foregoing table, may be found within 2 or 3 per cent at farthe *by merely mult the vels in said table by the corresponding numbers in the following one.*

Ex. What will be the mean vel of all the water in a rectangular channel 15 feet wide, with epth of water, and falling at the rate of 2 feet in a mile? Here, in the foregoing table we fi *channel 10 ft wide, 3 ft deep, and falling at the rate of 2 ft in a mile, has a vel of 2.41 ft* nd in the following table we find that the multiplier for a width of 15 ft, with a depth of 3 *nsequently, 2.41 × 1.07 = 2.58 ft per sec, the reqd vel.* And the disch will be 15 × 3 × ft per sec. See Remark, Art 16, p 562.

TABLE 18. Of multipliers for rectangular canals of widths other than 10 feet. (Original.)

Depths. Feet.	5 ft. Wide.	15 ft. Wide.	20 ft. Wide.	25 ft. Wide.	40 ft. Wide.	60 ft. Wide.	80 ft. Wide.	100 ft. Wide.
8	.77	1.13	1.22	1.28	1.40	1.47	1.51	1.54
6	.79	1.11	1.19	1.25	1.33	1.39	1.42	1.45
5	.80	1.10	1.17	1.22	1.29	1.34	1.37	1.39
4	.81	1.09	1.15	1.18	1.25	1.28	1.30	1.32
3	.83	1.07	1.12	1.15	1.21	1.23	1.25	1.26
2	.86	1.05	1.09	1.11	1.15	1.16	1.17	1.18
1	.91	1.03	1.06	1.07	1.09	1.09	1.10	1.10

Art. 24. Rem. 1. **In calculations of this kind, if the channel is not of masonry, or timber, care must be taken that the bottom vel is not so great as to wear away the soil.** A reference to the table on p 563 Art 18, will show if there is any such danger. If so, artificial means must be applied to protect the channel-way; or it may be advisable to reduce the rate of fall; and increase the cross-section of the channel; so as to secure the same disch, but with less vel.

A liberal increase should also be made in the dimensions of such channels, to compensate for obstructions to the flow, arising from the growth of aquatic plants, or deposits of mud from rain-washes, &c; or even from very strong winds blowing against the current. See also Rem. p 267.

Rem. 2. **The two preceding tables will also serve for giving, approximately, the vel and disch in a channel of trapezoidal cross-section, as *m n o p*, Fig 33.** For, in a trapezoidal channel, of any such form and proportions as would be likely to be employed in practice, the vel will not be more than from 2 to 6 per cent *less* than in a rectangular one, *a b c d*, of the depth *a b*; and having a width *x x*, equal to the average width of the trapezoidal one. If we assume it at 4 per cent less, we will be within 2 or 3 per cent of the truth in any ordinary case. It is rarely that a nearer approximation is required. Now, suppose we wish to know approximately the mean vel of all the water in a channel, *m n o p*, of any ordinary trapezoidal form; the mean width of which, *x x*, is 50 ft; its depth, *a b*, 6 ft; and which falls at the rate of 1.5 ft per mile. Here we see by Table 17, that in a rectangular canal 10 ft wide, 6 ft high, and falling at the rate of 1.5 ft per mile, the vel is 2.52 ft per sec. And by Table 18, we see that in a rectangular channel, *a b c d*, 50 ft wide, and 6 ft deep, the vel is about 1.36 times that of the first; or $2.52 \times 1.36 = 3.43$ ft per sec. But in the trapezoidal canal, *m n o p*, the vel will be a trifle less than in the rectangular one, *a b c d*, of the same area of water-way; because the wet border of the former is greater than that of the latter, and therefore opposes more resistance to the flow of the water. But as we now desire only an approximation, we will assume it to be 4 per cent less than in the rectangular one; or, in other words, that it is only .96 of the rectangular one. This will give us for the vel in the trapezoidal channel, $3.43 \times .96 = 3.29$ ft per sec.

Fig 33

Calculation by the formula, gives 3.32 ft, if the side-slopes are 2 to 1; and 3.40 ft, if they are as steep as 1 to 1.

Rem. 3. Experiments have not yet accurately determined the dist through which either a canal, or a pipe of given uniform dimensions, head, and slope, must be extended before uniform mean vel will be obtained, by the resistance arising from the friction of the bed, and the viscidity of the water itself, becoming equal to the impelling force of gravity. When uniformity of mean vel has commenced, then the water in the pipe, or other channel, is said to be *in train*, or to have *uniformity of motion*. It is said to have *permanency* of motion, when equal quantities flow in equal times through all its cross-sections, whether those sections have equal areas, or not. In the latter case, the mean vel through diff sections must evidently vary *inversely* as the areas of the respective sections.

Rem. 4. **Water running in a channel with a horizontal bed, or bottom, cannot have a uniform vel, or depth, throughout its course;** because the action of gravity due to the inclined plane of a sloping bottom, is wanting in this case; and the water can flow only by forming its *surface* into an inclined plane; which evidently involves a diminution of depth at every successive dist from the reservoir.

Rem. 5. **Mr Wicksteed, an experienced English hydraulician, gives the following table of the least vels and grades or falls, to be given to drain-pipes and sewers in cities,** in order that they may under ordinary circumstances keep themselves clean, or free from deposits. He recommends that no drain pipe, even for a single common dwelling, shall be *less* than 6 ins diam.

Diam. In Inches.	Vel. in ft. per Min.	Grade, 1 in	Grade. Feet per Mile.	Diam. in Inches.	Vel. in ft. per Min.	Grade, 1 in	Grade. Feet per Mile.
4	240	56	146.7	18	180	294	18.0
6	220	65	81.2	21	180	343	15.4
7	220	76	69.5	24	180	392	13.5
8	220	87	60.7	30	180	490	10.8
9	220	98	53.9	36	180	588	9.0
10	210	119	44.4	42	180	686	7.7
11	200	145	36.7	48	180	784	6.8
12	190	175	30.2	54	180	882	6.0
15	180	244	21.6	60	180	980	5.4

e following table was prepared by Mr John Roe, Chief Sur-
o the Metropolitan Commission of Sewers, London. It is based upon 20 years of close practical
ation of sewers themselves; and confirmed by special observations since; including experi-
gaugings of sewers during both short and long storms; and aided by rain-gauges, and every
tion to insure accuracy. The table is approved by the celebrated hydraulic engineer, Mr Hawks-
Mr Bidder, and other practical English engineers.

ble showing the number of acres of covered (as in cities)
ace, from which circular sewers, with junctions properly connected by easy
, will convey away the water fall of a rain of one inch per hour; with house
ge besides. This is a very heavy rain. See Rain, p. 518.

Inclination, Fall, or Slope of Sewer.	Inner Diameter, or Bore of Sewer in Feet.									
	2	2½	3	4	5	6	7	8	9	10
	Ac.	Ac.	Ac.	Ac.	Ar.	Ac.	Ac.	Ac.	Ac.	Ac.
.....	39	67	120	277	570	1020	1725	2850	4125	5825
h in 10 ft; or 1 in 480..	43	75	135	308	630	1117	1925	3025	4425	6250
h in 10 ft; or 1 in 240..	50	87	155	355	735	1318	2225	3500	5100	7175
h in 10 ft; or 1 in 160..	63	113	203	460	950	1692	2875	4500	6575	9250
h in 10 ft; or 1 in 120..	78	143	257	590	1200	2180	3700	5825	7850	11050
h in 10 ft; or 1 in 80...	90	165	295	670	1385	2486	4225	6625		
in 10 ft; or 1 in 60....	115	182	318	730	1500	2675	4550	7125		

culvert will, within the limits of the table, discharge nearly 1¾ times as much as the 10-ft one.
are mile contains 640 acres.
e give also the following

ble of number of acres of town area that may be drained
nall circular drains.* Calculated for a rainfall of *two* ins per hour,
ed on his own actual observations; also, the rates of fall or inclination which
ains must have. Such a rain occurs very rarely.

es.	Bore of Drain in Ins.						Acres.	Bore of Drain in Ins.				
	3	4	5	6	7	8		8	9	12	15	18
	Fall 1 in	Fall 1 in	Fall 1 in	Fall 1 in	Fall 1 in	Fall 1 in		Fall 1 in	Fall 1 in	Fall 1 in	Fall 1 in	Fall 1 in
	120	.....	.....	.....	.....	.....	2.1	60	120	.....	.....	.....
	20	120	.....	.....	.....	.....	2.5	.....	80	.....	.....	.....
16	.....	40	.....	.....	.....	.....	2.75	.....	60	.....	.....	.....
	.....	30	80	.....	.....	.....	4.5	.....	.....	120	.....	.....
	.....	20	60	.....	.....	.....	5.3	.....	.....	80	.....	.....
	.....	.....	20	60	.....	.....	5.8	.....	.....	60	240	.....
	.....	.....	.....	40	20	.....	7.8	.....	.....	.....	120	.....
	.....	.....	.....	20	60	120	9.	.....	.....	.....	80	.....
	.....	.....	.....	.....	.....	80	10.	.....	.....	.....	60	240
	.....	.....	.....	.....	.....	60	17.	.....	.....	.....	.....	120

eight and price per foot run of terra cotta vitrified
s for drains, &c; Philada, 1873.

Wt.	Price.	Bore.	Wt.	Price.	Bore.	Wt.	Price.	Bore.	Wt.	Price.
Lbs.	Cts.	Ins.	Lbs.	Cts.	Ins.	Lbs.	Cts.	Ins.	Lbs.	Cts.
6	11	6	13	28	15	45	125	30	130	450
6	15	8	17	43	18	70	175	36	170	600
8	18	10	21	66	20	80	210	42	200	725
9	23	12	33	83	24	100	300	50	240	900

bores of 12 ins, a bend, or a branch, costs about as much as 5 ft of pipe. These pipes, together
inches, elbows, &c; also, chimney-flues, chimney-tops, roofing-tiles, vases, and other articles
variety and excellent quality, are made at the extensive Moorhead Clay Works, Spring Mill,
, Wilson, proprietor; office 1230 N 9th St, Philada.
ls of the pipes are shaped much as the iron one in Fig 38; and the joints are filled w
rtar; or merely with clay when used for drainage only. Up to 12 ins bore, the pipes
ch thick; 48 inch bore, about 1¼ ins thick.

ar pipes are also made of hydraulic cement.

Art. 25. Obstructions by piers. When the area of section of a channel of running water, is diminished at any point by narrowing the stream; or by placing in it piers, such as those of a bridge, &c, then the same quantity of water which formerly flowed through the wider channel, has to force its way through the narrower one. This it can do only by, as it were, heaping itself up, at a short dist above the obstruction, so as there to form a head, as c o, Fig 34, sufficiently great to overcome the increased resistance; and thereby force along the same quantity of water as before; but of course at an increased vel. If the straight line a b represent the original water surf, then in ordinary cases, the curved line a s c b will approximately represent the new one, (but in an exaggerated manner,) as produced when the ends of the pier are properly rounded.

Fig 34

REM. 1. **It is important to bear in mind,** that before the erection of an obstruction, the vel of the original stream is greatest at the surf; and diminishes gradually toward the bottom; but when a pier, &c, is built, the increased vel produced by the head thereby created, is actually least at the surf; but is *nearly* uniform throughout the entire depth; thus causing a much greater wearing action upon the bottom than before existed, or than is generally supposed. This fact is rarely adverted to by authors. The erection of one or two piers in a quite large stream, will frequently produce an almost incredible amount of scour, if the bottom is at all of a yielding nature. The greatest scour of course takes place during freshets; and near the obstruction.

The two following tables are (with many corrections) from Nicholson's Architecture; and must be looked upon merely as probable approximations. They suppose the piers, &c, to be properly rounded or pointed at their upstream ends, so as to give as free a passage as possible to the water. He says that if they are square-ended, the head will be increased about 50 per ct. The subject is an extremely intricate one, and admits of no precise solution. If the increased vel scours away the bottom until the area of water-way becomes as great as it originally was, the head disappears; and the vel also becomes reduced to its original rate. This is common in soft bottoms.

TABLE 19. Of heads produced by obstructions to streams.

Original Vel. of Stream.*			Kind of Bottom which begins to wear away under Bottom Vel. equal to those in the first three cols.	Proportion of Area of original Water-way, occupied by the Obstructions.								
				$\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$
Per Sec.		Per Hour.		Head of Water produced at the Obstructions; in Feet.								
Ins.	Ft.	Miles.										
3	¼	.170	Ooze, and Mud...	.0003	.0004	.0004	.0006	.001	.0014	.0033	.0067	.0163
6	½	.341	Clay.....	.0011	.0014	.0017	.0023	.004	.0058	.0133	.0267	.0646
12	1	.681	Sand.....	.0045	.0056	.0069	.0091	.015	.0231	.0532	.1069	.2584
24	2	1.36	Gravel.....	.0182	.0225	.0276	.0364	.060	.0924	.2128	.4276	1.036
36	3	2.04	Small Shingle....	.0409	.0507	.0621	.0819	.135	.2079	.4788	.9621	2.328
48	4	2.72	Large "	.0728	.0902	.1104	.1456	.240	.3696	.8412	1.710	4.144
60	5	3.41	Soft Shistus.....	.1137	.1410	.1725	.2275	.375	.5775	1.320	2.672	6.475
72	6	4.09	Stratified Rocks..	.1638	.2030	.2484	.3276	.540	.8316	1.915	3.848	9.304
120	10	6.81	Hard Rocks.....	.4550	.5640	.6901	.9100	1.50	2.310	5.280	10.69	25.9

See Table, Art 18.

TABLE 20. Increased velocities produced at and by rounded, or pointed obstructions. If square, these vels must, according to Nicholson, be increased ⅛ part.

Original Vel. of Stream.*			Proportion of Area of Water-way, occupied by the Obstructions.								
			$\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$
Per Sec.		Per Hour.	Velocity produced at the Obstruction in Feet per Second.								
Ins.	Ft.	Miles.									
3	¼	.170	.28	.29	.30	.32	.35	.394	.52	.7	1.05
6	½	.341	.56	.58	.60	.64	.70	.788	1.05	1.4	2.1
12	1	.681	1.13	1.16	1.20	1.28	1.40	1.58	2.1	2.8	4.2
24	2	1.36	2.27	2.33	2.40	2.52	2.80	3.16	4.2	5.6	8.4
36	3	2.04	3.39	3.48	3.60	3.78	4.20	4.74	6.3	8.4	12.6
48	4	2.72	4.54	4.66	4.80	5.04	5.60	6.32	8.4	11.2	16.8
60	5	3.41	5.60	5.80	6.00	6.40	7.00	7.88	10.5	14.0	21.0
72	6	4.09	6.78	6.96	7.20	7.56	8.40	9.46	12.6	16.8	25.2
120	10	6.81	11.3	11.6	12.0	12.6	14.0	15.8	21.0	28.0	[illegible]

* A very vague expression. Does it refer to the greatest surface vel at mid-channel, or to the mean vel in the section?

Art. 26. The resistance of water against a flat surface moving through it at right angles, is nearly as the squares of the vel; and according to Hutton, its amount is approximately as follows:

$$\text{Resistance in lbs per sq ft} = \text{the square of the vel in ft per sec.}$$

When the moving surf, instead of being at right angles to the direction in which it moves, forms another angle with it, the resistance becomes less in about the following proportions. Therefore when the surf is inclined, first calculate the resistance as if at right angles; and then mult by the following decimals opposite the angle of inclination:

90°....1.00	60°.... .88	40°.... .58	20°.... .16
80 .... .98	55 .... .83	35 .... .46	15 .... .10
70 .... .85	50 .... .76	30 .... .34	10 .... .06
65 .... .92	45 .... .68	25 .... .24	5 .... .02

Art. 27. To calculate the horse-power of falling water, on the ordinary assumption that a horse-power is equal to 33000 lbs lifted 1 foot vert per min. That of average horses is really but about ⅔ as much, or 22000 lbs, 1 foot high per min. Mult together the number of cub ft of water which fall per min; the vert height or head in feet, through which it falls and the number 62.3, (the wt of a cub ft of water in lbs;) and div the prod by 33000. Or, by formula

$$\textit{The number of horse-powers} = \frac{\textit{cub ft per min} \times \textit{vert height in ft} \times \textit{lbs } 62.3}{33000.}$$

Ex. Over a fall 16 ft in vert height, 800 cub ft of water are dischd per min. How many horse powers does the fall afford?

$$\text{Here, } \frac{\overset{\text{cub ft}}{800} \times \overset{\text{ft}}{16} \times \overset{\text{lbs}}{62.3}}{33000} = \frac{797440}{33000} = 24.17 \text{ h-pow.}$$

Water-wheels, however, do not realize all the power inherent in the water, as found by our rule. Thus, undershots realize but from ¼ to ⅓; breast-wheels, ½; overshots, from ⅔ to ¾; turbines, ⅔ to .85 of it; according to the skill of design, and the perfection of workmanship. Even when the wheel revolves in a close-fitting casing, or breast, elbow buckets give considerably more power than plain radial or center-buckets. Of the power actually received by a wheel, part is expended in friction, &c; while the remainder does the *useful* or *paying* net work of raising water, grinding grain, sawing, &c.

Observations by Genl Haupt, in 1866, gave the following results for a small **hydraulic ram.** Head of water to ram = 8.812 ft; diam of drive-pipe = 1½ ins; length 15 ft. Diam of delivery-pipe = ¾ inch; length 200 ft. Vert height to which the water was raised by the delivery-pipe, 63.4 feet. Strokes of ram per min, 170. Quantity of water which worked the ram = 768 cub ins, = 3.31 galls, = 27.73 lbs per min. Quantity raised 63.4 ft high per min, = 48 cub ins, = 1.736 lbs. Hence the power expended per min, was $\overset{\text{lbs water}}{27.73} \times \overset{\text{ft}}{8.812} = \overset{\text{ft-lbs}}{244.35}$. And the useful effect, was $\overset{\text{lbs water}}{1.736} \times \overset{\text{ft}}{63.4} = \overset{\text{ft-lbs}}{110.06}$. Hence the ratio which the *useful effect* bears to the power in this instance, is $\frac{110.06}{244.35}$, or .45. The *actual power* of the ram is, however, greater than this, inasmuch as it has to overcome the friction of the water along the delivery-pipe.*

To find the horse-power of a running stream. Water-wheels with simple float-boards,† instead of buckets, are sometimes driven by the mere force of the ordinary natural current of a stream, without any appreciable fall like that in the foregoing case. In such cases, we must substitute the *virtual* or *theoretic* head; which is that which would impart to it the same vel which it actually has. This virtual head may be taken at once from Table 10. Thus, a stream has a vel of 2.386 miles per hour; or 210 ft per min; or 3½ ft per sec; and in the column of heads in Table 10, opposite to 3.5 vel per sec, we find the reqd head .190 of a ft. Having thus found the head, we must now find the quantity of water which passes any given area of the stream in a min. Thus, suppose that the *immersed* part of a float when vert is 5 ft long, and 1 ft wide or deep; then the area of this part which receives the force of the current, is $5 \times 1 = 5$ square feet. Hence $\overset{\text{area}}{5 \text{ sq ft}} \times \overset{\text{vel}}{210} = 1050$ cub ft per min. Having now the cub ft per min, and the vert height or head, the number of horse-powers *of the stream* of the given area, is found by the foregoing rule, or formula

---

* **A committee of the Franklin Institute,** in 1850, gave .71 as the coefficient for a ram at the Girard College, in which the diam of drive-pipe was 2½ ins: its length 160 ft; fall, 14 ft. Delivery-pipe, 1 inch diam; 2260 ft long; vert rise, or height to which the water was raised, 93 ft. No details of the experiment are given. Some large rams in France give a *useful effect* of from .6 to .65 of the whole power expended. It is an excellent machine for many purposes, and is sometimes used for filling railway tanks at water stations.

† **Such wheels, for floating mills, in Europe,** rarely exceed 15 ft *diam. Whatever the diam, they* may have about 18 to 20 floats. The floats are from 8 to 16 ft long *and about ⅓ to $\frac{1}{10}$ as deep as* the diam of the wheel. They should not dip their entire depth *the water, but nearly so. They* should not be in the same straight line with the radii; but *incline from them 30° up stream,* to produce their full effect. All these remarks apply to *moving freely in a wide or indefinite channel;* as in the case of a floating mill, built on a *anchored out in a stream:* but not to wheels for which the water is dammed up, and acts *tical fall. No great exactness* is to be expected in rules on this subject. The best vel to *be about .4 that of the stream.*

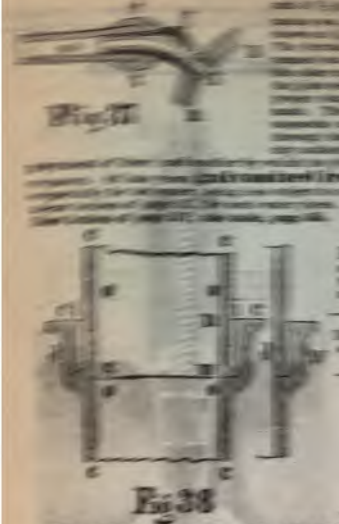


Fig. 38

The weight of lead used per joint, averaged allowed as follows: 20 in. [illegible]

Cost of water pipes 9 ft long, per mile, laid: assuming the pipes to cost 1 cts per lb. or $67.20 per ton, delivered along the streets; lead for the joints, of the weight formerly used in Philadelphia, at 10 cents per lb; and the laying, including digging and refilling the trenches, in earth; making the joints, including the gasket yarn for that purpose; according to the [illegible] which experience has shown to be fair when common labor costs $1 per day. A recent address [illegible] to make it unpaving and repaving are to be done. Owing to the constant fluctuations in prices such tables answer but an imperfect purpose. In 1873 the laying would be nearly twice as great.

Diam. Ins.	Thick's. Ins.	Tons per Mile.	Cost per Mile. $	Lead. Lbs.	Lead. $	Laying per ft. Cts.	Laying. per Mile.	Total. $
30	1	860	57792	66500	6650	100	[illegible]	[illegible]
24	15/16	570	38304	45000	4500	65	[illegible]	[illegible]
20	7/8	400	26880	37300	3730	45	[illegible]	[illegible]
18	3/4	358	24058	31100	3110	40	[illegible]	[illegible]
16	3/4	325	21840	21800	2180	36	[illegible]	[illegible]
14	3/4	267	17943	18000	1800	32	[illegible]	[illegible]
12	3/4	205	13776	15000	1500	28	[illegible]	[illegible]
10	3/4	171	11491	10000	1000	25	[illegible]	[illegible]
8	3/4	111	7459	8100	810	21	[illegible]	[illegible]
7	3/4	84	5645	6300	630	20	[illegible]	[illegible]
6	3/4	58	3898	5000	500	18	[illegible]	[illegible]
5	3/4	33	2218	3200	320	16	[illegible]	[illegible]
4	3/4	24	1613	1500	150	15	[illegible]	[illegible]

sing tightly. The small oblong recess seen below the gate, admits small particles of foreign which might otherwise prevent the gate from closing perfectly. The grooves *a a a a*, in which

36¼

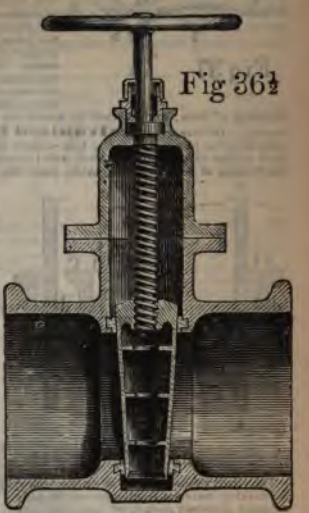


Fig 36½

slides, as well as those parts of the gate which come into contact with them, are carefully ith brass, to diminish friction. At the top of the cover, the screw-stem passes through a -box, which prevents leaking at that point. Very careful workmanship is required throughout. ates, especially of large mains, must be closed very slowly; otherwise, the too sudden arrest- he momentum of the flowing water would be apt to break either them or the covers; or burst es. As a precaution against this, the covers for very large valves are cast with outside ening ribs.*

lf-acting air-valves (see Fig 42) are now placed at street summits, to allow confined air to The fireplugs answer instead. The rad for bor bends in mains is if possible not less than t times their diams; they are made as large as the widths of the streets will admit; usually ft. **Fire-plugs,** Figs 40, are placed as much as possible at summits, so as to serve washing the streets; and for the escape of accumulated air. They average about 8 in num- ach mile of pipe; or 1 to each block of buildings. **No galvanic action** has been d where lead pipes or brass unite with cast-iron ones. **Rust has not very ma- lly affected the pipes** in 30 to 35 years; even without any preservative meas- See Art 34. A coat of whitewash on the outside, impedes rust quite appreciably. Some 22- e which have laid in Philada for 50 years, and 4 inch ones for 65 years, are still in use. In dition.* **No pipe as small as 3 ins diam is carried farther 400 feet,** without being connected with larger ones at both ends. Even this dist is at o great for so small a pipe. None less than 4 inches diam should be laid in cities; and even y for lengths of a few hundred feet. Their insufficiency is chiefly felt in case of fire. Six ins e a better minimum. No more leakage occurs in winter than in summer; except from the t of *private service-pipes* by freezing.

e service-pipes for supplying single dwellings, are of lead;

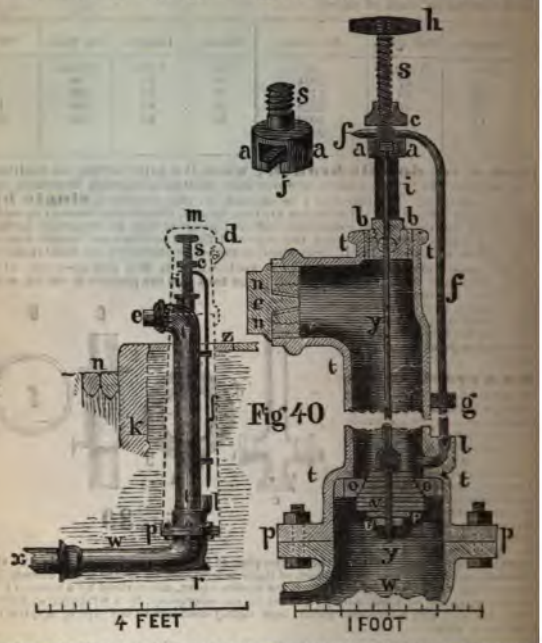
following table gives a tolerable average of the weights of valves, for pipes of various bores, rent makers; with an approximation to average retail prices in 1873.

Bore. Ins.	Wt. Lbs.	Price. $	Bore. Ins.	Wt. Lbs.	Price. $	Bore. Ins.	Wt. Lbs.	Price. $	Bore. Ins.	Wt. Lbs.	Price. $
3	72	21	8	310	56	14	1200	175	22	2300	400
4	105	28	10	575	70	16	1600	220	24	2500	450
6	173	42	12	800	96	20	2100	350	30	5200	950

ment the earth thoroughly against the pipes excludes air, and greatly impedes rust. oded by the leakage of gas through the body as well as the joints of adjacent gas

no provision has been made for such attachment. In this case a piece may be cut out of *f*, as
and a casting *e*, furnished with flanges *m m*, bolted over the opening, by screw-bolts passing th
female screws tapped in the thickness of the pipe. If the new pipe is so large that the openi
if circular, would be inconveniently wide, it may be made *oval*, with the longest diameter in the
tion of the *length* of the pipe *f*, in that case the casting *e* will be oval at its flanges; and circular

In Figs 40, *t t* represent a common street fireplug, or drant. It is bolted by flanges *p*, to the **goose-neck** *w*; and is connecte



the street main by a pipe *x*. The valve *v* is (in Philada) of layers of well-hammered sole-leather;
when closed, shuts against a brass ring-seat *o*; which is confined to its place by a lead joint as
in the fig. The layers of leather are compressed by the nut *u*, and the washer *p*. This valve is
by working down the screw *s*; which, by means of the swivel joint at *a a*, can revolve without
ing the valve-rod *y*. This rod works through a brass stuffing-box at *b b*; the small circle in which
notes the space occupied by the stuffing. The box is lead-jointed to the inside of the fireplug
Two standards *t t* sustain the cross-piece *c*, through which the screw *s* works. When the valve
closed after the plug has been in use, the interior of the plug is full of water; and to prevent
from freezing, and bursting the plug, a small outlet, or *frost-pipe*, *l*, is provided, through which it
escape and soak into the adjacent soil. The closing of the valve *v* by working up the screw *s* in
position in the figs, opens the frost-pipe by means of the rod *f*; the lower end of which is pointed
as to close the frost-pipe when the valve is open; and the upper end of which is bent, and formed
two prongs, which rest upon *a a*; by which means the lower end is drawn out from the frost-pipe
when *a a* rises by the working-up of the screw *s* in closing the valve *v*. The detached Fig. shows
how the cylindrical head of the rod *y* is inserted into *a a*, by means of an opening in one side.
iron ring is then shrunk around *a a*. At *e* is the stopper of the plug. This is of iron; but its
portion is surrounded by a screw-ring of brass, leaded to it; and which works into another, leaded
the fireplug. This brass screw obviates friction from rust; it reduces the diam of the opening, to
The whole plug is protected from injury and frost by a cast-iron box resting on the flanges *p*.
shown by the dotted lines. Its lid *m* is hinged to the body at *d*. The box is in three pieces;
them (each extending only half-way around the plug) reach from the flanges *p*, up to *s*; the
piece, from s to m, reaches entirely around the plug, and clasps the tops of the two lower
keeping them in place. The fig shows the brick paving *x* of the sidewalk, the pebble paving
carriage way, and the curbstone *k*. In Philada there is, on an average, one fireplug to about
150 yards of street pipe.

A great variety of street hydrants, with from 1 to 6 nozzles, with various styles of
are made by the Boston Machine Co.

the thickness of cast-iron pipes of good quality to resist safely the pressures of various heads, 532 of Hydrostatics. Those in the last table have been tested to about 700 feet head, or 300 lbs inch.

he following are the contract prices for laying pipes in city of Brooklyn, N. York, in 1868; they include hauling, unpaving repaving, digging and refilling the trenches, laying the pipe, lead, gasket yarn, but exclusive of cost of pipes.

m.	Length.	Per Foot.	Per Mile.	Diam.	Length.	Per Foot.	Per Mile.
s.	Feet.	Cents.	$	Ins.	Feet.	Cents.	$
5	9	37	1954	16	12	90	4752
5	12	33	1743	20	12	120	6336
5	9	46	2429	30	12	320	16896
5	12	39	2059	48	12	520	27456
2	12	60	3168				

Figs 39, A, is a **double branch;** which is a pipe having, in addition to c at one end, two others, *s* and *i*, to which pipes leading in opposite directions (as at cross-s) may be attached. If either *s* or *i* be omitted, the pipe becomes a **single branch.** pipe is stronger when these extra faucets are near its end, than if they were at its middle. In a line of pipes, for the sake of expedition, different gangs of men are frequently laying detached ns some distance apart; and when two ends of different portions are brought near enough to-r to be united, as *h* and *r*, Fig C, their junction cannot be effected by the usual spigot-and-faucet

In this case a cast-iron **sleeve** *t t* is used, which is first slid upon one of the pieces e; and (after the other piece also is laid) is slid back into the position in the fig, so as to cover int. Sleeves are usually about long; as thick as the pipe; and diam is sufficient to allow the joint of gasket and lead. There ourse such a joint at each end sleeve.

Figs 39

hen a crack takes ce across a pipe, *a a*, Fig eady in use, it is repaired by s of a cast-iron sleeve *g g*, made o parts, bolted together by s of flanges *m* at *n n*. In other ts it is like the preceding . The intermediate white ring lead joint. If the crack is too or otherwise too bad to be rem-by a sleeve, the pipe is broken ces; and the lead joints at its melted out, so as to allow of its al. Then, since an entire new annot now be inserted, owing to the overlapping of the spigot-and-faucet ends, two short pieces be substituted for it. One end of each of these is lead-jointed to the pipes already laid; while her two ends, which will probably be a few inches apart, are covered by a sleeve *t t*, Fig C. cks may at times be temporarily repaired in an emergency, by a wrapping of folds of canvas ughly saturated with white-lead paint; and tightly confined to the pipe by a spiral banding of hoop-iron or wire.

ohn F. Ward's flexible joint for pipes laid across the irregular bottoms eams, is shown at Fig 39½.* A portion, *a o*, of the inside of the socket or faucet *F*, is bored out to form the middle zone of a sphere; and the spigot end the other pipe is cast with two raised collars *o* and *e*. nner collar, *o*, is of such a height as barely to allow it to into the faucet. The outer one, *e*, is a little lower, so as ow melted lead (shown black) to be poured in at *a*. The edge or diam of the spigot end at *o* is carefully turned to fit the turned spherical zone; so that the joint will of considerable play without danger of leaking. In g the pipes under water, the joints are filled with melted as usual, on board of suitable vessels or floats. As fast ey are thus filled, the floats are moved forward, and the , if small, and the water shallow, are passed into the r without further care. But for large pipes in deep wa-uitable apparatus is used for lowering them without un-strain on the joints. Mr Ward has been perfectly suc-il in laying this pipe under water, in one case 40 feet One of the mains of the Philada water-works is thus across the Schuylkill River.

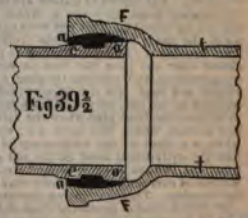


Fig 39½

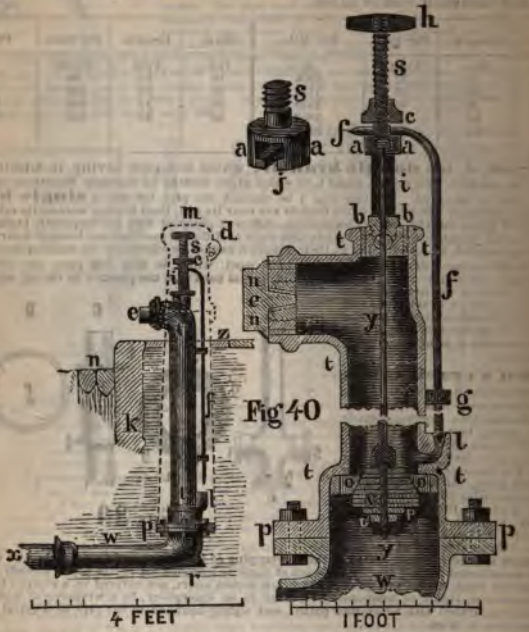
ome cases preliminary dredging may be expedient, to diminish abrupt irregularities of the

attach a pipe *c*, Fig 39, to one *f*, already in use, but in wh

patent is held by Messrs A. W. Craven and John F. Ward, Civ Engs, 38 Broadway, N. Y. er was for many years chief engineer of the New York water-works.

no provision has been made for such attachment. In this case a piece may be cut out of *f*, as at *n n*, and a casting *e*, furnished with flanges *m m*, bolted over the opening, by screw-bolts passing through female screws tapped in the thickness of the pipe. If the new pipe is so large that the opening *n n*, if circular, would be inconveniently wide, it may be made *oval*, with the longest diameter in the direction of the *length* of the pipe *f*. In that case the casting *e* will be oval at its flanges; and circular at *e c*.

In Figs 40, *t t* represent a common street fireplug, or hydrant. It is bolted by flanges *p*, to the **goose-neck** *w*; and is connected with



the street main by a pipe *x*. The valve *v* is (in Philada) of layers of well-hammered sole-leather; and when closed, shuts against a brass ring-seat *o*; which is confined to its place by a lead joint as shown in the fig. The layers of leather are compressed by the nut *u*, and the washer *p*. This valve is opened by working down the screw *s*; which, by means of the swivel joint at *a a*, can revolve without turning the valve-rod *y*. This rod works through a brass stuffing-box at *b b*; the small circle in which denotes the space occupied by the stuffing. The box is lead-jointed to the inside of the fireplug at *t t*. Two standards *i i* sustain the cross-piece *c*, through which the screw *s* works. When the valve *v* is closed after the plug has been in use, the interior of the plug is full of water; and to prevent this from freezing, and bursting the plug, a small outlet, or *frost-pipe*, *l*, is provided, through which it may escape and soak into the adjacent soil. The closing of the valve *v* by working up the screw *s* to the position in the figs, opens the frost-pipe by means of the rod *f*; the lower end of which is pointed, so as to close the frost-pipe when the valve is open; and the upper end of which is bent, and formed into two prongs, which rest upon *a a*; by which means the lower end is drawn out from the frost-pipe when *a a* rises by the working-up of the screw *s* in closing the valve *v*. The detached Fig *f*, shows how the cylindrical head of the rod *y* is inserted into *a a*, by means of an opening in one side. An *iron ring* is then shrunk around *a a*. At *e* is the stopper of the plug. This is of iron; but its inner *portion is surrounded* by a screw-ring of brass, leaded to it; and which works into another, leaded to *the fireplug*. This brass screw obviates friction from rust; it reduces the diam of the opening, to *n n*.

The whole plug is protected from injury and frost by a cast-iron box resting on the flanges *p*. It is *shown by the* dotted lines. Its lid *m* is hinged to the body at *d*. The box is in three pieces: two of *them (each extending* only half-way around the plug) reach from the flanges *p*, up to *n*; the upper *piece, from e to m*, reaches entirely around the plug, and clasps the tops of the two lower pieces, *keeping them in place*. The fig shows the brick paving *g* of the sidewalk, the pebble paving *n* of the *carriage way*, and the curbstone *k*. In Philada there is, on an average, one fireplug to about each *yards of street pipe*.

great variety of street hydrants, with from 1 to 6 nozzles, with various styles of ornamentation *made by the Boston Machine Co.*

Wrought-iron pipes have been found to corrode much more
pidly than cast. Cast-iron pipes laid in Philadelphia in 1804, are still (now 1871) in perfectly good
ndition, and in constant use.

A gutta-percha pipe, ⅛ inch thick, and ¾ inch bore, has sustained safely
n internal pres of more than 250 lbs per sq inch; equal to nearly 600 feet head. It merely swelled
ightly at 337 lbs. The Croton water is carried to Blackwell's Island by a tube of that material,
ank in the East River, N York. It is kept down by weights. It is 2½ ins bore, and about ½ inch
ick. Was laid in 1851, and is still in perfect condition in 1871. Experience has there shown that
a rocky bottom the pipe must be prevented from shifting with the tide, &c; otherwise it will soon
ear through. It is frequently displaced by the anchors of dragging vessels. A wrapping of can-
as, confined by spun yarn, has been used as a protection against abrasion.

Ball's patent iron and cement pipe, made by a company at Jersey
lty, New Jersey, is formed of riveted sheet iron, and lined with about ½ inch thickness of hydraulic
ement. It is laid in a bed of cement mortar, and completely covered with the same. Suitable
eans are provided for making all the attachments, &c, required in city pipes for water and gas.
fore than 400 miles of it are in use in various towns, some of it for 25 years; and it appears to give
eneral satisfaction. Tubercles do not form in these pipes, as they are apt to do in cast-iron ones.
here is every reason to suppose that they are durable. The trenches being dug, the Jersey City Co
rnish pipes and lay them (including the cement) at ⅔, or less, of the cost of cast-iron ones laid.
he invention certainly possesses much merit.

The Wyckoff patent pipe for water or gas, is of wood. Several concentric
ipes of different diams are bored out from the same log. They are coated outside with either hy-
raulic or asphaltic cement. When exposed to pressure from great heads, they are spirally wrapped
r banded with sheet or plate iron. A 10-inch pipe thus banded with iron of 1 × ½ inch; spirals 4
ns apart; sustained a test pressure of 406 lbs per sq inch; equal to a head of water of 935 ft. Their
ost is but about half that of cast-iron pipes. They are made by J. A. Woodward, Williamsport,
enna; and have been quite extensively and successfully used for both water and gas.

Water pipes of bored yellow pine logs, laid in Philada 50 to 60
ears ago, are frequently quite sound, and still fit for use, except where outer sap wood is decayed.
hen this is removed, many of these old pipes have been relaid in factories, &c. Clay *well packed*
round wooden pipes, excludes the contact of air, and thus contributes greatly to their durability.
oose porous soils, such as gravel, &c, on the contrary, are unfavorable.

Pipes made of bituminized paper, prepared under great pressure,
ave been used for both water and gas. They are much less liable to break than cast iron, and do
ot weigh or cost more than about half as much. Pipes of 5 ins bore, and ⅛ inch thick, have resisted
st strains of 220 lbs per sq inch; equal to a water head of 507 ft.

**In Philada, in 1873, there are about 546 miles of street
ipes;** or about 1 mile to every 1300 inhabitants. The population is nearly
00,000; residing in more than 100,000 dwellings.

Art. 29. Reservoirs. In important reservoirs of earth, for storing water
o moderate depths for cities, experience appears not to sanction dimensions *bolder* than 10 ft thick
t top; inner slope 2 to 1; outer slope 1½ to 1.* A top width of 15 ft to 20 ft, and inside slopes of
to 1, are adopted in some important cases; with outer slopes of 2 to 1. Both slopes, however, are
t times made only 1½ to 1. The level water surf should be kept at least 3 or 4 feet below the top of
se embkt; or more, if liable to waves. In a large reservoir, a quite moderate breeze will raise waves
hat will run 3 ft (measured vert) up the inner slope. A low wall, or close fence, *w*, Fig 41, is some-
imes used as a defence against them. The top and the outer slopes should be protected at least by
d or by grass. To assist in keeping the top dry, it should be either a little rounding, or else sloped
ward the outside. The soft soil and vegetable matter should be carefully removed from under the
ntire base of the embkts; which should be carried down to soil itself impervious to water, in order
hat leakage may not take place *under* them. To aid in this, a double row of sheet piles, or a sunk
all of cement masonry, carried to a suitable depth below the bottom, may be placed along the inner
e in bad cases. If there are springs beneath the base, they must either be stopped, or led away by
ipes. The embkt should be carried up in layers, slightly hollowing toward the center, and not ex-
eeding a foot in thickness; and all stones, stumps, and other foreign material, such as *clean* gravel,
and, and decomposed mica shists, &c, that may produce leakage, carefully excluded. These layers
hould be well consolidated by the carts; and the easier the slopes are, the more effectively can this
e done. The layers, however, should not be distinct, and separated by actual plane surfaces; but
ach succeeding one should be well incorporated with the one below. This has sometimes been done
y driving a drove of oxen, or even sheep, repeatedly over each layer: in addition to the carting.
ollers are not to be recommended, as they tend to produce seams between the layers. This might
ossibly be obviated by projections on the circumf of the roller.

Gravelly earth is an excellent material, perhaps the best. The choicest material should be placed
a the slope next to the reservoir; and should be deposited and compacted with special care in that
ortion, so as to prevent the water from leaking into the main body of the dam, and thus weakening
. It is not amiss to introduce a bench *b*, Fig 41, in the outer slope, to diminish danger from rain-
ash by breaking the rapidity of its descent.

If the bottom of the reservoir itself is on a leaky soil, or on fissured rock, through the seams of
hich water may escape, it must be carefully covered with from 1½ to 3 feet of good puddle; which,
a turn, should be protected from abrasion and disturbance, by a layer of gravel; or of concrete,
ither paved or not, according to circumstances.

Reservoirs constructed with the foregoing dimensions, and with care, may remain safe for an in-
efinite period; but where serious damage would result from failure, the following additional pre-
autions should be taken. **The inner slopes should be carefully faced
p to the very top, with at least a close dry rubble-stone
ffching,** not less than about 15 to 18 ins thick; as a protection against wash, and against
akrats. These animals, we believe, always commence to burrow under water. If the slopes

*The writer suggests that a top width equal to 2 ft + twice the sq rt of the height in feet, w
or any height whatever of reservoir properly constructed in other respects.*

much steeper than 2 to 1, this dry pitching will be apt to be overthrown by the sliding down of the softened earth behind it, if the water in the reservoir should for any cause be drawn down rather suddenly. It will be much more effective, but of course more costly, if laid in hydraulic cement; and still more so if laid upon a layer a few ins thick of cement-and-gravel concrete; especially if this last be underlaid by a layer about 1½ to 3 ft thick of good puddle, spread over the face of the slope; the great object being to protect the inner slope from actual contact with the water. If this can be effectually accomplished, slopes as steep as 1½ to 1 will be perfectly secure; for the danger does not arise from any want of weight of the earth for resisting overthrow. Special care should be bestowed upon the inner toe of the slope, to prevent water from finding its way beneath it, and softening the earth so as to undermine the stone pitching. Near the top, reference should be had to danger of derangement by ice, frost, rain, and waves. Flat inner slopes tend not only to prevent the displacement of the pitching; but increase the stability of the embankment, by causing the pressure of the water (which is always at right angles to the slope) to become more nearly vertical; and thus to hold the embankment more firmly to its base than if there were no water behind it. Sometimes the toes of both the inner and outer slopes abut against low retaining-walls in cement. This gives a neat finish, and tends to preservation from injury.

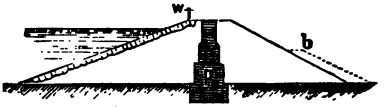


Fig 41

Many engineers, in order to prevent leaking, either through or beneath the embankment, construct a **puddle-wall**, *p*, Fig 41, of well-rammed impervious soil, (gravelly clay is the best,) reaching from the top to several feet below the base. This wall should not be less than 6 or 8 ft thick on top, for a deep reservoir; and should increase downward by *offsets* (and not by slopes, or batters) at the rate of about 1 in total thickness, to 3 or 4 in depth. Other engineers object to these puddle-walls: and contend that leakage should be prevented by making both the inner slopes, and the bottom of the reservoir, water-tight, by means of puddle, concrete, and stone facing in cement, as just alluded to. They argue that if the embankment is well constructed, it is itself a puddle-wall throughout.

It is difficult to prevent water under high pressure from finding its way through considerable distances along seams where earth is in contact with smooth rock, wood, or metal; as, for instance, along the surfaces of iron pipes laid under reservoir embankments; or along the tie-rods sometimes used through the puddle of coffer-dams; and the same is apt to occur under the bases of embankments which rest on smooth rock. Special care should be taken that the earth used in such positions is not of a porous nature; and that it is thoroughly compacted all along the seam; and the straight continuity of the seam should be interrupted or broken as frequently as possible by projections. Faucets or flanges do this to a limited extent in the case of iron pipes; and something similar, but on a larger scale, should at short intervals be constructed in the shape of collars or yokes of cement stonework, in the case of rock or masonry. For more on dams, see Dams, p 583, also 528 &c.

It is usually advisable to divide reservoirs into two parts, so that while the water in one part is being drawn off for use, that in the other may purify itself by settling its sediment. Also, one part may remain in use, while the other is being cleaned or repaired. Many days, or even two or three weeks, sometimes, are required for the complete settlement of the very fine clayey particles in muddy water: depending on the depth of the reservoir.

One or more flights of steps to the bottom of the reservoir, should be provided.

The reservoirs of the London New River Water Co, remained uncleaned for 100 years; at the end of which time the accumulation of mud was about 8 ft deep: or about 1 inch per annum. In the reservoirs at Philada, it is about ¼ inch annually from the Schuylkill; and about 1 inch from the Delaware water. In shallow reservoirs vegetation takes place, and injures the water. A depth of about 20 ft appears to prevent this.

Water flowing through marsh lands is sometimes unfit for drinking purposes. That, for instance, in some sections of the Concord River, Massachusetts, was reported by the eminent hydraulic engineer, Loammi Baldwin, of Boston, to be absolutely *poisonous* from this cause.

The construction of a large deep reservoir is not only a very costly, but a very hazardous undertaking. With every watchfulness and care, it is almost impossible entirely to prevent leaking; although this may not manifest itself for months, or even years. Should a break occur, especially near a city, it would probably be attended by great loss of life and property. If the water once finds its way in a stream, either across the unpaved top, or through the body of the embankment, the rapid destruction of the whole becomes almost certain. In a few cases in Europe, walls of solid masonry have been substituted for earthen embankments; some of them from 40 to 70 feet high. Their face toward the water is vert; the outer one batters $\frac{4}{10}$ of the height; thickness at base $\frac{7}{10}$, and at top $\frac{3}{10}$ of the height. The same proportions answer equally for all depths. The quantity of masonry is equal to that in a wall with a uniform thickness equal to half its height. They cost more than earthen embankments. This is not a strong form of wall for water. See page 531.

Art. 30. STORING RESERVOIRS. The entire annual yield of a stream may be much more than sufficient for supplying a certain population with water; and yet in its natural condition the stream may not be available for this purpose, because it becomes nearly dry in summer, when water is most needed; while, at other seasons, the rains and melted snows produce floods which supply vastly more than is reqd; and which must be allowed to run to waste. A storing reservoir is intended to collect and store up this excess of water, so that it may be drawn off as reqd during the droughts of summer, and thus equalize the supply throughout the entire year. This, when the locality permits, is effected by building a dam across the stream, to form one side of the reservoir; while the hill-slopes of the valley of the stream form the other sides. The stream itself flows into this reservoir at its up-stream end. When the stream is liable to become nearly dry during long summer droughts experience shows that the capacity of the reservoir should be equal to from 4 to 6 months' supply, according to circumstances. During the construction of the dam, a free channel must be provided ass the stream without allowing it to do injury to the work. If the dam were built precisely like , entirely of earth, it would plainly be liable to destruction by being washed away in case the r should become so full that the water would begin to flow over its top. To provide against iv, by means of masonry, or of cribs filled with broken stone, or otherwise, construct a

It is well to allow in addition from ½ inch to 1 inch, or more, (depending on the character of th water,) to each diam; for deposits and concretions.

The water, after reaching the city through one or more large main pipes from the reservoir, mus be distributed through the streets by means of smaller mains branching from the large ones. Th diams of these smaller ones also may be found by Table A. Thus, if a street, with its alleys, &c contains about 6000 persons, (the rate of head being, as before, not less than 50 ft to a mile at an point of the system,) then we see by the table that a 10-inch pipe will answer. It would be well t lay no city street pipes of less than 6 ins diam.

Mains which cross each other should be connected at some of their intersections, to allow the water a more free circulation through out the entire system; so that if the supply at any point is temporarily cut off from one direction b closing the valves for repairs, or is diminished by excessive demand, it may be maintained by th flow from other directions.

RULE 2. *With the same diams, different rates of head will supply the proportionate populations f col 3 of Table B. Or, to find the diams which at different rates of head will supply the same popula tions given in the last col of Table A,* mult the diam given in Table A, by the corresponding numbe in col 4 of Table B; or (approximately) do as directed in col 5.

TABLE B. (Original.)

COL. 1.	COL. 2.	COL. 3.	COL. 4.	COL. 5.
Rate of Head, in Feet per Mile.	Rate of Head, compared with that in Table A.	Proportionate Populations.	Proportionate Diam. to supply the Populations in Table A.	Remarks.
5	.1	.32	1.58	
10	.2	.45	1.37	
12½	.25	.50	1.32	Add one-third.
15	.3	.55	1.27	Add full one-fourth.
20	.4	.64	1.20	Add one-fifth.
25	.5	.71	1.14	Add one-seventh.
30	.6	.78	1.11	Add one-ninth.
35	.7	.84	1.07	Add one-fourteenth.
37½	.75	.87	1.06	Add one-sixteenth.
40	.8	.90	1.05	Add one-twentieth.
45	.9	.95	1.02	Add one-fiftieth.
50	1.0	1.00	1.00	
75	1.5	1.23	.92	Deduct one-thirteenth.
100	2.0	1.41	.88	Deduct one eighth.
125	2.5	1.59	.83	Deduct full one-sixth.
150	3.0	1.73	.80	Deduct one-fifth.
200	4.0	2.00	.76	Deduct nearly one-fourth.
250	5.0	2.25	.73	Deduct nearly two-sevenths.
300	6.0	2.46	.69	Deduct three-tenths.
400	8.0	2.83	.66	Deduct full one-third.
500	10.0	3.18	.63	

EXAMPLE. By Table A we see that with the rate of head of 50 ft per mile, a 30-inch pipe will sup ply a population of 91580; but with three times that rate of head, or 150 ft per mile, we see by col 3 Table B, that the same pipe will supply 1.73 times as many persons, or $91580 \times 1.73 = 158433$ per sons. But if, at this greater rate of head, we still wish to supply only 91580 persons, then we find i col 4, Table B, that we may diminish the diam of the pipe from 30, down to $30 \times .80 = 24$ ins; or, b col 5, we have $30 - 6 = 24$ ins.

Again, after the water has reached the city by the 30-inch pipe of Table A, if we wish to distribute i through the city by say eight branches or smaller mains, we see by col 6, Table A, that each of then must have at least 13½ ins diam. From these eight, other smaller ones may branch off into th cross streets, alleys, &c: and in estimating the supply required for any particular street main, w must evidently add what is reqd also for such cross streets, &c, &c, as are to be fed from said main.

If certain limited parts of a city pipe system have considerably less rates of head than most of th remainder, it may become expedient to supply the former by a special separate main of larger diam which may start either directly from the reservoir; or as a branch from the grand leading main whic feeds the lower parts, according to circumstances.

It must be remembered, that although by increasing the diams, an abundant supply may be ob tained under a small rate of head, as well as under a great one, yet the water will not rise to as grea a height in the service pipes for supplying the different stories of dwellings, &c. Even with th diams in Table A, the water, under ordinary use, will not rise in these pipes to the full height of th surface of the reservoir; and if an unusual drawing-off is going on at the same time at many part of the system, as in case of an extensive fire, or frequently during the hot summer months, it ma not rise to even one-half of that height.

Art. 34. The following has been found very effective for preventing concretions in water pipes. Formerly in Boston, cas iron city pipes, 4 ins diam, became closed up in 7 years: and those of larger diam became sectio reduced in the same time. But during the last 8 years, in which this varnish has been used, no c cretions have *formed.*

Coal-pitch varnish to be applied to pipes and castl

egress to the outer air, by closing the opening at *v*. Now, air carried along by the water, will, on account of its lightness, ascend to the highest points it meets with.

Hence, when such air arrives under the opening at *a a*, it will rise through it, and ascend to *e*; the closed valve preventing it from going farther. Thus successive portions of air ascend, and in time accumulate to such an extent as gradually to force much of the water downward out of the box. When this takes place, the float, which is held up only by the water, of course descends also; and in doing so, pulls down with it the valve *v*. The accumulated air then instantly escapes through the openings at *v* and *n*, into the atmosphere; and the water in the pipe *m m*, immediately ascends again into the box, carrying with it the float; and thus again closing the valve *v*. The valve, and the valve-seat *e*, are faced with brass, to avoid rust, and consequent bad fit. The whole is protected by an iron or wooden cover, reaching to the level of the street, somewhat as in Fig 35, p 572.

Air valves are no longer used in city pipes; their place being supplied by the fireplugs at average distances of about 150 yds apart. These, being placed as much as possible at the summits of undulations in the lines of pipes, for convenience of washing the streets, and being frequently opened for that purpose, permit also the escape of accumulated air.

The escape of compressed air through an air valve, or other opening, has been known to produce bursting of the main pipes; for the escape is instantaneous, and permits the columns of water in the pipes on both sides of the valve, to rush together with great forces, which arrest each other, and react against the pipes.

Water for city use should not be drawn from the very bottom of the reservoir, because it will then be apt to carry along the sediment; which not only injures the water, but creates deposits within the pipes; thus obstructing the flow. In fixing upon the necessary capacity of a reservoir, this must be taken into consideration; inasmuch as all the water below the level for drawing off, must be regarded as lost. When circumstances justify the expense, it is well to curve up the reservoir end of the service main, so as to provide it with valves at diff heights; for drawing off only the purest stratum that may be in the reservoir. With this view, the valve-tower before spoken of generally has such valves communicating with the water in the reservoir; and by this means only the purest is admitted into the tower; and from it, into the city pipes. This refinement, however, is rarely practicable. Such valves must of course be worked by watchmen.

The quantity of water reqd in cities, has been found by experience to increase faster than the population. About 60 gallons, or 8 cub ft, per day, to each inhabitant, is usually considered a fair ample allowance. Many European cities have not half as much; while New York, and some others, use and waste half as much more. With efficient means for preventing waste, 60 gallons would probably suffice for any commercial city; but inasmuch as cleanliness and health are promoted by its free use, as few restrictions as possible should be introduced.

In fixing upon the diams of pipes for supplying cities, it is necessary to bear in mind, that by far the greater portion of the 24 hours' yield is actually drawn from them during only 8 to 12 hours of daylight; and therefore the capacity of the pipes must be sufficient to furnish the daily supply in much less than 24 hours. Again, during the hot summer months, much more water is used than during the winter ones; and this consideration necessitates a still larger diam.

Art. 33. Systems of street pipes for supplying cities. The writer knows of no practical rules for proportioning the diams for such systems. The various complications involved, render a purely scientific investigation of little or no service. With much hesitation, he ventures the following purely empirical rules of his own; based on such limited observations as have casually fallen under his notice.

Rule 1. *When, at no point in a system of city pipes, is the head, or vert dist below the surface of the reservoir, compared with the hor dist from the reservoir, less than at the rate of 50 ft per mile, then the population in the last column of the following Table A, may be abundantly supplied, for all city purposes, by either one pipe of the inner diam or bore in the 1st col; or by 2, 3, &c, pipes of the diams in the other cols.* These diams are given to the nearest safe ¼ inch. The supply is assumed to be about 60 gallons per day to each inhabitant.

TABLE A. (Original.)

NUMBER OF PIPES.								Population.
1	2	3	4	6	8	12	24	
Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	Diam. Ins.	
6	4⅝	3⅞	3½	3	2⅝	2⅜	1⅝	1647
8	6⅛	5¼	4⅝	4	3½	3⅛	2⅝	3465
10	7⅝	6½	5⅞	5	4⅜	3⅞	3	5908
12	9⅛	7⅞	7	5⅞	5¼	4½	3½	9324
14	10⅝	9⅛	8¼	7	6¼	5⅝	4¼	13706
16	12½	10⅝	9¼	7⅞	6⅞	6	4⅝	19141
18	13¾	11⅞	10½	8¾	7¼	6¾	5¼	25677
20	15¼	13	11⅝	9⅜	8¾	7½	5¾	33426
22	16¾	14¼	12¾	10⅞	9½	8¼	6¼	42433
24	18¼	15½	13⅞	11¾	10½	9	6⅞	52671
26	19¾	16¾	15	12¾	11¼	9¾	7½	64447
28	21¼	18⅛	16¼	13¾	12¼	10½	8	77565
30	22⅞	19⅜	17⅜	14¾	13⅛	11¼	8⅝	91580
32	24⅜	20⅝	18½	15⅝	14	11⅞	9	108160
34	25⅞	22	19⅝	16¾	15	12¾	9¾	125840
36	27⅜	23¼	20⅞	17¾	15¾	13½	10¼	144480
40	30½	25⅞	23⅛	19⅝	17¼	15	11½	198870
44	33½	28¾	25⅝	21⅝	19⅛	16⅛	12⅞	258990
48	36½	31	27¾	23¼	21⅛	18	13¾	321790
54	41	34⅞	31¼	26½	23⅞	20¼	15⅝	501790
60	45⅛	38¾	34⅝	29½	26⅜	22½	17⅛	618
66	50⅛	42⅝	38⅞	32⅝	29⅛	24⅝	18¾	5
72	54¾	46¼	41⅝	35⅝	31⅝	26⅞	20⅛	1
	[illegible]	51⅝	46¼	39¼	35⅛	29⅞	22⅛	

It is well to allow in addition from ½ inch to 1 inch, or more, (depending on the character of the water,) to each diam; for deposits and concretions.

The water, after reaching the city through one or more large main pipes from the reservoir, must be distributed through the streets by means of smaller mains branching from the large ones. The diams of these smaller ones also may be found by Table A. Thus, if a street, with its alleys, &c, contains about 6000 persons, (the rate of head being, as before, not less than 50 ft to a mile at any point of the system,) then we see by the table that a 10-inch pipe will answer. It would be well to lay no city street pipes of less than 6 ins diam.

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EXAMPLE. By Table A we see that with the rate of head of 50 ft per mile, a 30-inch pipe will supply a population of 91580; but with three times that rate of head, or 150 ft per mile, we see by col 3, Table B, that the same pipe will supply 1.73 times as many persons, or 91580 × 1.73 = 158433 persons. But if, at this greater rate of head, we still wish to supply only 91580 persons, then we find in col 4, Table B, that we may diminish the diam of the pipe from 30, down to 30 × .80 = 24 ins; or, by col 5, we have 30 − 6 = 24 ins.

Again, after the water has reached the city by the 30-inch pipe of Table A, if we wish to distribute it through the city by say eight branches or smaller mains, we see by col 6, Table A, that each of them must have at least 13½ ins diam. From these eight, other smaller ones may branch off into the cross streets, alleys, &c; and in estimating the supply required for any particular street main, we must evidently add what is reqd also for such cross streets, &c, &c, as are to be fed from said main.

If certain limited parts of a city pipe system have considerably less rates of head than most of the remainder, it may become expedient to supply the former by a special separate main of larger diam; which may start either directly from the reservoir; or as a branch from the grand leading main which feeds the lower parts, according to circumstances.

It must be remembered, that although by increasing the diams, an abundant supply may be obtained under a small rate of head, as well as under a great one, yet the water will not rise to as great a height in the service pipes for supplying the different stories of dwellings, &c. Even with the diams in Table A, the water, under ordinary use, will not rise in these pipes to the full height of the surface of the reservoir; and if an unusual drawing-off is going on at the same time at many parts of the system, as in case of an extensive fire, or frequently during the hot summer months, it may not rise to even one-half of that height.

Art. 34. The following has been found very effective f preventing concretions in water pipes. Formerly in Boston, *from city pipes, 4 ins diam, became closed up in 7 years: and those of larger diam became so reduced in the same time. But during the last 8 years, in which this varnish has been used erviceas have formed.*

Coal-pitch varnish to be applied to pipes and ca

made for the Water Department of Philadelphia, under the following conditions:

First. Every pipe must be thoroughly dressed and made clean, free from the earth or sand which clings to the iron in the moulds; hard brushes to be used in finishing the process to remove the loose dust.

Second. Every pipe must be entirely free from rust when the varnish is applied. If the pipe cannot be dipped immediately after being cleansed, the surface must be oiled with linseed oil to preserve it until it is ready to be dipped: no pipe to be dipped after rust has set in.

Third. The coal-tar pitch is made from coal tar, distilled until the naphtha is entirely removed, and the material deodorized. It should be distilled until it is about the consistency of wax. The mixture of five or six per cent of linseed oil is recommended. Pitch which becomes hard and brittle when cold, will not answer for this use.

Fourth. Pitch of the proper quality having been obtained, it must be carefully heated in a suitable vessel to a temperature of 300 degrees Fahrenheit, and must be maintained at not less than this temperature during the time of dipping. The material will thicken and deteriorate after a number of pipes have been dipped; fresh pitch must therefore be frequently added; and occasionally the vessel must be entirely emptied of its old contents, and refilled with fresh pitch; the refuse will be hard and brittle like common pitch.

Fifth. Every pipe must attain a temperature of 300 degrees Fahrenheit, before it is removed from the vessel of hot pitch. It may then be slowly removed and laid upon skids to drip.

All pipes of 20 inches diameter and upward, will require to remain at least thirty minutes in the hot fluid, to attain this temperature; probably more in cold weather.

Sixth. The application must be made to the satisfaction of the Chief Engineer of the Water Department; and the material be subject at all times to his examination, inspection, and rejection.

Seventh. Payment for coating the pipes will only be made on such pipes as are sound and sufficient according to the specifications, and are acceptable independent of the coating.

Eighth. No pipe to be dipped until the authorized inspector has examined it as to cleaning and rust; and subjected it thoroughly to the hammer proof. It may then be dipped, after which, it will be passed to the hydraulic press to meet the required water proof.

Ninth. The proper coating will be tough and tenacious when cold on the pipes, and not brittle or with any tendency to scale off. When the coating of any pipe has not been properly applied, and does not give satisfaction, whether from defect in material, tools, or manipulations, it shall not be paid for; if it scales off or shows a tendency that way, the pipe shall be cleansed inside before it can be recoated or be receivable as an ordinary pipe.*

FREDERIC GRAFF, Chief Eng Water Department.

Art. 35. The syphon, or siphon † If one leg *a b* of a bent tube or pipe *a b c*, Fig 43, of any diam, filled with water, and with both its ends stopped, be placed in a reservoir of water, as in the fig; and if the stoppers be then removed, the water in the reservoir will begin to flow out at *c*, and will continue to do so until its level is reduced to *t*, which is the same as that of the *highest* end *c* of the pipe or syphon. The flow will then stop. The parts *a b* and *b c* are called the legs of the syphon, *b* being its highest point; and this is correct so far as relates to it merely as a piece of tube; but considering it purely with regard to its character as a hydraulic machine, the part *t a* below the level of the highest end *c*, may be entirely neglected; for the water in the reservoir will not be drawn down below the level of the highest end, whether that be the inner or the outer one. Therefore, if the disch end be above the water in the reservoir, as, for instance, at *w*, no flow will take place. The *vert* height *b o*, from the highest part of the syphon, to the lowest level *t*, to which the reservoir is to be drawn down, must not, theoretically, exceed about 33 or 34 ft; or that at which the pres of the air will sustain a column of water. Practically it must be less, to allow for the friction of the flowing water, and for air which forces its way in. And still less at places far above sea level; for at such the reduced weight of the atmospheric column will not balance so great a height of water. In order readily to understand, or at any time to recall the principle on which the syphon acts, bear in mind that we may theoretically consider the end of the inner leg to be not actually immersed *below* the water surf, but only to be kept precisely *at* it, as the surf descends while the water is flowing out; but may regard the vert dist *b o* as the length of the outer leg; and a varying dist, which at first is *b s*, and finally *b o* (as the surf of the reservoir descends) as the length of the inner leg; and that the flow continues only while *this* outer leg is longer than *this* inner one. The books are wrong in saying that the outer leg *b c must* be longer than the inner one *b a*, in order that the water may run at all. The principle then is simply this: that both these legs *b c*, and *b t*, being first filled with water, (the part *t a* being considered at first as a portion of the *reservoir*, and not of the syphon,) it follows that when the stoppers are removed from the ends *c* and *a*, the air presses equally against these ends; but the great vert head of water *b o* in the outer leg *b c*, presses against the air at *c*, with more force than the small head of water *b s* in the inner leg *b t*, does against the air at *a* or *t*.‡ Consequently, the water in *b c* will tend to fall out more rapidly than that in *b t*; and as it commences to fall, would produce a vacuum at *b*, were it not that the pres of the air against the other end *a* or *t*, forces the water up *t b*, to supply the place of that which flows out at *c*. In this manner the flow continues until the surf of the water *in the reservoir* descends to *t*, on the same level as *c*. The pressures of the vert heads *b o*, *b e*, in the *two legs b c*, *b t*, being then equal, it ceases.

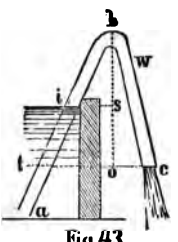


Fig 43

The syphon principle may be employed for draining ponds into lower ground at a considerable dist, *even though* an elevation of several feet (in practice perhaps not exceeding about 25 ft above the level *to which the* pond is to be reduced) may intervene. In such a case an air-valve must be provided *at the summit* (or summits, if there are more than one) of the bends, for the disch of free air, which *will inevitably* enter, and soon stop the flow, unless this precaution be taken. The air valve should

* Such coating is said by Darcy to increase the discharge materially.

† This subject belongs more properly to Pneumatics.

‡ Said pres of the air at *a* or *t*, is of course not direct; but is transmitted through the water d thence upward through the syphon to *t*.

e self-acting, as that at Fig 42. The orifice at this valve may also be used for filling the syphon at ie start; and to prevent the water thus introduced, from running out, stopcocks must be provided t the ends.

The greatest pains must be taken to make all the joints perfectly air-tight. A syphon 3 ins in iam; 1792 ft long; with an intervening rise of 9 ft above the surf of the water to be drained, was ild by Col C Crozet at the tunnel through the Blue Ridge in Virginia; and is perhaps the longest ver executed. Its disch end was 20 ft below the surf of the water in the tunnel which was drained. When in full operation it dischd 43½ galls per min. Owing to the accumulation of air in the air-cassel, which was not a self-acting air valve, the flow would nearly cease in about 2 hours; and the yphon have to be filled by letting in water at its summit, after closing the cocks at the ends.

The motive power or head which causes the flow in a syphon, is the vert dist *s o*, from the surf of he reservoir, to the disch end *c*; or in other words, it is the diff, *s o*, between the theoretical lengths *s* and *b o*, of the two legs. Consequently, the farther *c* is below *s* the more rapid will be the flow; and t is plain that as the surf gradually sinks below *s*, the less rapid will the flow become. Having this ead, the entire length *a b c* of the syphon, and its diam, all in ft, the disch may be found approxi-nately by either of the rules given in Art 2 for straight pipes. These rules give 55½ galls per min, nstead of the 43½ galls actually dischd by Col Crozet's syphon, with a head of 20 ft.

DAMS.

We can devote but little space to this subject, in addition to what is said on earthen dams for re-ervoirs p 577; and on stone ones, p 528 &c. Those we shall now describe will also answer for uch reservoirs, when the perishable nature of timber is not an objection.

Primary objects in the erection of dams, are, a foundation suffi-iently firm to prevent them from settling, and thus leaking; the prevention of leaks through their acks, or under their bases; and the prevention of wear of the bottom of the stream in front of the lam, by the action of the falling water. For the first purpose, hard level rock bottom is of course he best; and should be chosen, if possible. In that case, thick planks, *t t*, Fig 6, (single or double, s the case may be,) closely jointed, and reaching from the crest *c*, to the back lower edge *w*, (where hey should be scribed down to the rock;) with a good backing, *b*, of gravel, will suffice to prevent eaks. Gravel, or rather very gravelly soil, is far better than earth for this purpose; for if the water hould chance to form a void in it, the gravel falls and stops it. To prevent this backing from being isturbed near the crest of the dam, by floating bodies swept along by freshets, a rough pavement of tones, about 15 to 18 ins deep, as shown in Fig 7, should be added for a width of about 10 to 20 ft; r until its top becomes 3 to 5 ft below the crest *c* of the dam, according to circumstances.

In Fig 1, (a dam on the Schuylkill navigation,) the upper timbers, *e*, are all close jointed, and laid ouching, so as not to require planking in addition.

But if the bottom of the stream is gravel or earth, there must in addition to these be used two hicknesses of sheet piles, *p*, Fig 2, &c, close driven, breaking joint, to a depth of several ft, to pre-ent leaking through the soil beneath the base of the dam. Frequently but one thickness is used. If the bottom is soft or open for a depth of only a few ft, it is at times better to remove it, and base the dam on the firmer stratum below; still, however, using the sheet piles. Old decayed timber and ther rubbish should be removed from the base. In very bad soils of greater depth, it may be neces-ary to support the dam entirely upon a platform resting on bearing piles. Here great precautions are necessary against leaks; but the case occurs so rarely, that we shall not stop to consider it.

As to the wearing away of the bottom of the stream by the water falling over the front of the dam, precautions should be used in all cases except that of very hard rock, or of medium rock protected

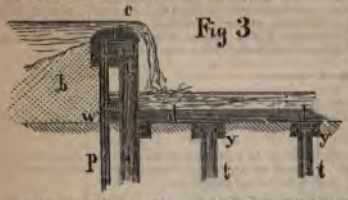
considerable depth of water. The dam, Fig 1, was built upon a tolerably firm mineceous gr nearly vert strata, covered by about 2 feet of water in ordinary stages. In 30 years the roc

worn away in front of the dam, as shown in the fig, to the average depth of 3 feet; or very nearly 1 inch per year. The depth of water on the crest c, was usually from 6 to 18 ins; rarely 5 or 6 ft during freshets; and but a few times during the whole period, 8 or 9 ft.

At Jones's dam, on Cape Fear River; height of dam, 16 ft; front vert; fall, usually 10 ft, into 6 ft depth of water; the soft shale rock, in vert strata, was, in the course of a few years, worn away 16 ft; and the dam was undermined to such an extent as to fall into the cavity.

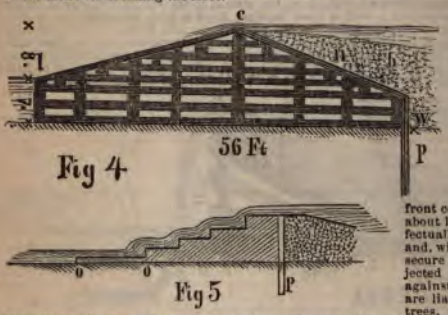
In another case, dam 36 ft high; front vert; the water falling upon nearly vert strata of hard shale rock, usually covered by but about 2 ft of water; in about 20 years wore it to an irregular depth of from 10 to 20 ft; and extending from the very face of the dam, to 70 or 80 ft in front of it.

In Fig 2, upon a stream subject to very violent freshets, the gravel was washed away for a considerable width and depth beyond the apron, as at A. To prevent a repetition, the cavity was filled with cribwork full of stone, clear across the river.

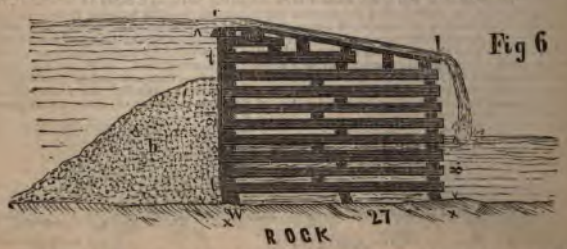


A deposit of blocks of loose stone, of even a ton weight or more, will not serve as a protection in front of a dam exposed to high freshets; but will soon be swept away. A common precaution against this wear, in low dams, is an *apron*, *a a*, Fig 2; or *d d*, Fig 3; of either rough round tree trunks, or of hewn timber, laid close together; extending under the entire base of the dam, and from 15 to 50 ft in front of its face. These are sometimes bolted to pieces, *s s*, Fig 2; or *y y*, Fig 3; laid under them across the stream. In Fig 3, with very soft bottom, these pieces *y y* are supposed to be bolted to short piles *t t*, driven for that purpose.

At times a distinct wide low timber crib, filled with stone, and covered on top with stout plank has been placed in front of the dam, to receive the fall of the water; and is effective in protecting the bottom. Also, in some cases, a dam of less height, and of cheap character, has been built at a short distance down stream from the main one, in order to secure at all times a deep pool in front of the latter for breaking the force.



Another precaution is to substitute a sloping front like *c l*, Fig 4, or such as Figs 1 and 2 would form if reversed, for the nearly vert one of the other figs; thus to some extent reducing the force of the water. This, however, is but a partial remedy, especially for soft bottoms in shallow water; for the sliding sheet still descends with great force. The best form of dam, perhaps, in such cases, is that shown in Fig 5, in which the front consists of a series of steps of about 1 vert, to 3 or 4 hor. These effectually break the force of the water; and, with the addition of an apron co, secure a satisfactory result. It is objected against this form, as also against Figs 4 and 6, that their fronts are liable to be torn by descending trees, ice, and other bodies swept along during freshets; but experience shows that this objection has but little weight; for when such bodies pass, the sheet of water is thicker than usual; and protects the front timbers. On the Sch Nav, the timbers *c l*, Fig 6, scarcely wear thin at the rate of an inch in 10 to 15 years.



The forms of wooden dams are many; (see the figs, which show those most used;) varying with the circumstances of the case, and with the fancy of the designer. In the United States they are usually of cribwork, of either rough round logs with the bark on, or hewn timber; in either case about a foot through. These timbers are merely laid on top of

other, forming in plan a series of rectangles with sides of about 7 to 12 ft. They are not notched together, but simply bolted by 1 inch square bolts (often ragged or jagged) about 2 to 2½ feet long, through every timber at every intersection. These are not found to rust or wear seriously, even when exposed to a current. Square bolts hold best. Round logs are flattened where they lie upon each other. Experience shows that finer but more expensive connections are entirely unnecessary. The cribs are usually, but not always, filled with rough stone. In triangular dams, disposed as in Figs 1, 2, and 7, this stone filling is not so essential as in other forms, because the weight of the water, and of the gravel backing, tends to hold the dam down on its base. Still, even in these, when the lower timbers are not bolted to a rock bottom, or otherwise secured in place, some stone may be necessary to prevent the timbers from floating away while the work is unfinished, and the gravel not yet deposited behind it. On rock, the lowest timbers are often bolted to it, to prevent them from floating away *during construction;* and when the water is some feet deep, this requires coffer-dams.

Fig 7

Or, the cribs may be built at first only a few feet high; then floated into place, and sunk by loading them with stone; for the reception of which a rough platform or flooring will be reqd in the cribs, a little above their lowest timbers. The bolting to the rock may then be dispensed with. The water may flow through the open cribwork as the building higher goes on; attention being paid to adding stone enough to prevent it floating away if a freshet should happen. Or, cribs shown in plan at *c c*, Fig 8, loaded with stone, may be sunk, leaving one or more intervals, like that at *o o o o*, between them, for the free escape of the water. These openings to be finally closed by floating into them closing-cribs shaped like *n*.

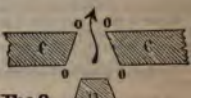


Fig 8

The workmanship of a dam in deep water can of course be much better executed in coffer-dams, than by merely sinking cribs. The joints can be made tighter; the stone filling better packed; the sheet piling more closely fitted, &c.

When a very uneven rock bottom in deep water, or the introduction of sluices in the dam, or any other considerations, make it expedient to build dams within coffer-dams, both should be carried on *in sections;* so as to leave part of the channel-way open for the escape of the water. Commencing at one or both shores, the first section of the coffer-dam may reach say quarter way or more across the stream. In the section of the dam itself built within this enclosing coffer-dam, ample sluices should be left for the water to flow through when we come to build the *closing* section of the coffer-dam. When the dam has been finished clear across, these sluices may be closed by drop-timbers.* Before removing one section of coffer-dam, the outer end of the enclosed section of dam itself must be firmly finished in such a manner as to constitute a part of the inner end of the next section of coffer-dam. It is impossible to give details for every contingency; the engineer must rely upon his own ingenuity to meet the peculiarities of the case before him. In some cases of shallow water, mere mounds of earth may answer for coffer-dams; or rough stone mounds backed with earth or gravel.

After the water has passed beyond the crest, *c* in the figs, there is no necessity for preventing it leaking down among the crib timbers; on the contrary, the thick sheeting planks, (or squared timbers, as occasion may require,) *c l*, Figs 4 and 6, which form the slopes along which the water then flows in some dams, are usually not laid close together, but with open joints of about ½ inch wide between them, for the express purpose of allowing part of the water to fall through them, so as to keep the timbers beneath them partially wet; which, to some extent, renders them more durable. In Figs 1, 4, 6, and 7, the water of the lower pool flows freely back among the crib timbers, and rough quarry stones with which the cribs are filled either partly or entirely. In Figs 4 and 6, these stones are not shown. In the dam, Fig 1, none were used. In Fig 2, they were as shown.

A substantial, and not very expensive dam of the form of Fig 7, may be built of rough stone in cement. Some hewn timbers should be firmly built horizontally into the masonry of the sloping back *c n w*, at a few feet apart, with their tops level with the surf of the masonry. To these must be well spiked close-jointed sheeting-plank *c s w*, for protecting the masonry from the action of the water, and of floating bodies. The gravel backing *b*, may be omitted; but the sheet piles *p*, and an apron in front of the dam, will be as indispensable in yielding soils, as if the dam were of timber.

Figs 1, 2, 4, 6, and 7, are sections drawn to a scale, of existing dams in Pennsylvania, that have stood successfully the force of heavy freshets for a long series of years.† These freshets at times carry along large bodies of ice, trees, houses, bridges, &c.; and have risen to 11 ft above the crests. Fig 1 on the Sch Nav, was built in 1819, and served perfectly for 39 years, until in 1858 the decay of much of its timber, especially of the close-laid top ones, *c*, rendered it necessary to build a new one just in front of it. It was of extremely simple construction; and was never filled with stone. The bottom timbers, *o o*, 10 ft apart, were bolted to the rock; and immediately over each of them, was such a series of inclined timbers as is shown in the fig. The top ones, *c*, however, were close-jointed, and laid touching so as to form the top sheeting, instead of thinner planks. The short pieces at *t* were laid in the same way. No coffer-dam was used; but the bottom pieces were first bolted to the rock; 10 ft apart; then the stringers and the sloping pieces were added. The close covering (*c*) was carried forward from each end of the dam, until at last a space of only about 60 ft was left in the center, for the water to pass. The close covering for this space being then all get ready, a strong force of men was set to work, and the space was covered so rapidly that the river had not time to rise sufficiently high to impede the operation.

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* *Timbers ready prepared for closing an opening through which water is flowing; and such dropped into place by means of grooves or guides of some kind for retaining them in position. Several such timbers may at times be firmly framed together, and then be all dropped at once; the opening or sluice at one operation; especially when it is of small size. In some cases may be sunk on the up-stream side of such an opening, for closing it.*

† *Those on the Schuylkill Navigation were obligingly furnished by James F. Smith, engineer and superintendent of that work. Other valuable information from the same be found in different parts of this volume.*

Fig 2 is a canal feeder dam on the Juniata. Here *ss* are timbers stretching clear across the stream (about 300 ft.) and sustaining the apron *a a*, of stout hewn timbers laid touching. This dam was filled with stone, for the retention of which the front sheeting planks were added.

Fig 6 is on the Sch Nav; was built in 1855. It is a form much approved of on that work, for such situations; namely, firm rock foundation, with a considerable depth of water in front. The highest dam (32 ft) on the Sch Nav, is very similar to it; built in 1851. All the dams on this work are of hewn timber, chiefly white and yellow pine. The water occasionally runs from 8 to 12 feet deep over their crests; and then overflows and surrounds many of the abuts. The vertical back allows the overflowing water to leak down among *all* the lower timbers of the dam, and thus tend to their preservation.

Fig 4 shows the dams on the Monongahela slackwater navigation; W. Milnor Roberts, eng. They are of round logs, with the bark on; flattened at crossings. The longest ones in the fig are 15 feet apart along the length of the dam. Experience shows that such dams possess all the strength necessary for violent streams. On rock, the lowest timbers are bolted to it.

Fig 7 has been successfully used to heights of 40 ft.*

Fig 3 is intended merely as a hint for a very low dam on yielding bottom. Its main supports are piles *i i*, from 4 to 8 ft apart, according to the height of the dam; and other circumstances; and if are short piles for sustaining the apron *d d*. It may be extended to greater heights by adding levees in front; which may be covered by stout planks, to form an inclined slide for the overfalling water. Many effective arrangements of piles, and sloping timbers for dams on soft ground, will suggest themselves to the engineer. Thus, at intervals of several feet, rows of 3 or more piles may be driven transversely of the dam; the top of the outer pile of each row being left at the intended height of the crest, while those behind are successively driven lower and lower; so that when all are afterward connected by transverse and longitudinal timbers, and covered by stout planking, and gravel, they will form a dam somewhat of the triangular form of Fig 7. It would be well to drive the piles with an inclination of their tops up stream.

There is much scope for ingenuity both in designing, and in constructing dams under various circumstances; and in turning the course of the water from one channel to another, by means of flumes, pipes, or troughs, &c., at differing times; aided at times by low temporary dams or mounds of earth, or of sheet piles, &c., or by coffer dams; so as to keep it away from the part being built. Each locality will have its peculiar features; and the engineer must depend on his judgment to make the most of them.

Abutments of dams as a general rule should not contract the natural width of the stream, and if they must do so, as little as possible; for contractions increase the height and violence of the overflowing water in time of freshets; during which a great length of overfall is especially desirable. They should be very firmly connected with the ends of the dams; and should, if the section of the valley admits of it, be so high, and carried so far inland, that the high water of freshets will not sweep either over them, or around their extremities; and thus endanger undermining, and destruction. In wide, flat valleys they cannot be so extended without too much expense, and the dam should be so built as to withstand such action; making their height such that they will, at least, be overflowed but seldom. Their ends adjacent to the dam should be rounded off, so as to facilitate the flow of the water over the crest.

They are best built of large stone in cement; for although sufficient strength may be secured by timber that material is very rapid in such exposures. If of earth only, they are very apt to be carried away if a freshet should overtop them.

Sluices should be placed in every important dam, in order that all the water may be drawn off if necessary, for the purpose of repairs, or of removing mud deposited, or finding lost articles of value, &c. They may be merely strong boxings, with floor, sides, and top of squared timbers, and passing through the breadth of the dam, just above the bottom. To prevent trees, &c. from entering and sticking fast in them, some kind of strong screen is expedient. In common cases a sluice 3 ft and not exceed about 4 ft by 4 ft in cross-section; otherwise it becomes hard to work. Two or more such sluices may be used when much water is to be voided. They should be wear the abutments. The gates or valves for opening and shutting them, should be at the up stream end, so as to keep the sluices free from accumulations of mud, &c. will fill the sluices, and prevent them from working. They are usually of timber, and slide vertically in rebates; being raised and lowered by rack and pinion, or by screws. In very important dams they may be of cast iron. Two sets of sluices are desirable, that one may be always ready for use if the other is stopped for repairs.

The part of the apron in front of the sluice should be particularly firm, so as not to be deranged by the water rushing out under a high head.

Dams are sometimes, but rarely, built in the form of an arch; convex up stream. This form is strong; and when the shores are of rock it may be expected to last. But if the banks are soft, they will be exposed to wear by the current thrown against them at the shore of the arch.

At times dams are built obliquely across the stream, with the object of increasing the straight and consequently shortening the dam of water over the crest is times of freshets. The advantage however, depends on the water in the pool being weight, inasmuch as the [illegible] be exposed to a [illegible] distance up stream from the dam; and would therefore [illegible] the [illegible] effect. [illegible] the [illegible] wed district above. Moreover the [illegible] probably a much more [illegible] commensurate with any advantage gained.

Some dams are subject to "trembling," which have not been [illegible] produced by the [illegible] of water [illegible] of the air, [illegible] or more. [illegible] not [illegible] well secured.

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* **Cost of crib dams.** [illegible] at $1 per day; plank [illegible] $1 per cub yd. [illegible] dams in shallow water, usually [illegible] per cub yd. of crib.

wide crib apron, a few feet high, against the front of the dam, for preventing the abrasion of the bottom. In other cases a series of oblique timbers placed against the front of the dam, and part way up it, at a slope of about 1½ to 1, and covered with plank, has been perfectly effective in stopping it.

The proper time for building dams is of course at the longest period of low stage of water.

To ascertain in advance approximately, the height to which the water will rise above the crest of a dam; or rather, a little back from it; the crest being above the level of the original water. This will vary with the shape of the crest, as may be seen by reference to Fig 26½ of Hydraulics, which, however, is a very peculiar case. Still, until we have more experiments, appreciable deviations from the results of such rules must be expected in practice. Square the disch of the stream in cub ft per sec. Call this square, *s*. Square the length of the overfall in ft. Mult this square by 7. Call the prod *p*. Divide *s* by *p*. Take the cube rt of the quot.* This cube rt will be the reqd approximate height *of rise* in ft.

When, in times of freshets, the water rises above the crest to a height equal to that of the dam itself, there is no perceptible fall at the dam; and boats may pass in safety over the crest.

For measuring the disch over dams, see arts 14 and 15 of Hydraulics. See also Art 1 of Hydrostatics. In shape of a formula, the foregoing rule will be,

$$\text{Rise in ft} = \text{cube rt of} \left(\frac{\text{discharge in cub ft per sec}^2}{7\ (\text{length of overfall in ft}^2)} \right)$$

When the dam is originally a submerged, or drowned one, as D, Fig 9; the following is a rough approximation, *probably somewhat in excess*; *o g* being the natural level of the water previous to building the dam; and *o c* the natural depth in ft, of the water above the intended crest. Then the required depth *a c*, of the up-stream water, above the crest when built, will be, approximately,

Fig 9

$$a\,c = o\,c + \text{cube root of} \left(\frac{\text{discharge}^2}{7 \times \text{length}^2} \right)$$

Having *a c*, deduct *o c*; and the rem will be *a o*, or the required rise produced by the dam.*

The rules given for the varying rise of surface for considerable distances up stream from dams; as well as for some allied subjects in hydraulics; are extremely complicated; and require much greater knowledge of mathematics than is usually found among civil engineers; and so far as regards their application to the actualities of common occurrence, they are probably no less useless than complicated.

The short table on page 317 will at times be of use in finding the dimensions of timbers for sustaining the pressure of different heads of water.

GRAVITY, FALLING BODIES.

Owing to the resistance of the air the following rules will not give perfectly accurate results in practice, especially at great vels. The greater the sp gr of the body the better will be the result. The latitude, and the height above sea level also cause a slight difference; see footnote p 455.

The dist through which a body falls freely in the first second is very nearly 16.1 ft; and it increases as the squares of the times. Therefore to find the number of feet fallen in any case, find the square of the number of sec; and mult it by 16.1.

The dists fallen in equal consecutive times are as 1, 3, 5, 7 &c.

The vel which a falling body acquires at the end of the first sec is very nearly 32.2 ft per sec; and it increases as the times. Therefore to find it in any case, mult the number of seconds by 32.2.

To find the vel in ft per sec acquired in falling through a given dist, mult the dist in ft by 64.4; and take the sq rt of the prod. Or refer to table on p 552.

To find the time in sec, required to fall through a given dist, divide the dist in ft by 16.1, and take sq rt of quot.

To find the fall in ft required to impart a given vel in ft per sec; find the square of the vel, and mult it by the dec .0155.

If a body be thrown vertically upwards with a given vel, it will rise to the same height from which it must have fallen in order to acquire said vel; and its vel will be retarded at the rate of 32.2 ft per sec.

These two rules are from Rankine, who says they apply to "crests either flat or slightly rounded." But that, in itself, is very vague.

SUSPENSION BRIDGES.

Art. 1. Table of data required for calculating the main chains or cables of suspension bridges.

Deflection in parts of the Chord.	Deflection in Decimals of the Chord.	Length of Main Chains between Suspension Piers, in parts of the Chord.	Tension on all the Main Chains at either Suspension Pier, in parts of the entire Suspended Wt. of the Bridge, and its Load.	Tension at the Center of all the Main Chains; in parts of the entire Suspended Wt. of the Bridge, and its Load.	Angle of Direction of the Chains at the Piers.		Natural Sine of the Angle of Direction of the Chains, at the Piers.	Natural Cosine of the Angle of Direction of the Chains at the Piers.
					Deg.	Min.		
1-40	.025	1.002	5.03	5.00	5	43	.0995	.9950
1-35	.0286	1.002	4.40	4.37	6	31	.1135	.9935
1-30	.0333	1.003	3.78	3.75	7	36	.1322	.9912
1-25	.04	1.004	3.16	3.12	9	6	.1580	.9874
1-20	.05	1.006	2.55	2.51	11	19	.1961	.9806
1-19	.0526	1.007	2.43	2.38	11	53	.2060	.9786
1-18	.0555	1.008	2.30	2.25	12	32	.2169	.9762
1-17	.0588	1.009	2.18	2.12	13	14	.2290	.9734
1-16	.0625	1.010	2.06	2.00	14	2	.2425	.9701
1-15	.0667	1.012	1.94	1.87	14	55	.2573	.9663
1-14	.0714	1.013	1.82	1.74	15	57	.2747	.9615
1-13	.0769	1.016	1.70	1.62	17	6	.2941	.9558
1-12	.0833	1.018	1.57	1.49	18	33	.3180	.9480
1-11	.0919	1.022	1.46	1.37	19	59	.3418	.9398
1-10	.1	1.026	1.35	1.25	21	48	.3714	.9285
1-9	.1111	1.033	1.23	1.12	23	58	.4062	.9138
⅛	.125	1.041	1.12	1.00	26	33	.4471	.8945
1-7	.1429	1.053	1.01	.881	29	45	.4961	.8726
3-20	.15	1.058	.972	.833	30	58	.5145	.8574
⅙	.1667	1.070	.901	.750	33	41	.5547	.8320
1-5	.2	1.098	.800	.625	38	40	.6247	.7808
	.225	1.122	.747	.555	42	0	.6690	.7433
¼	.25	1.149	.707	.500	45	00	.7071	.7071
.3	.3	1.205	.651	.417	50	12	.7682	.6401
⅓	.3333	1.247	.625	.375	53	8	.8000	.6000
.4	.4	1.332	.589	.312	58	2	.8483	.5294
9-20	.45	1.403	.572	.278	60	57	.8742	.4855
½	.5	1.480	.559	.250	63	26	.8944	.4472

These calculations are based on the assumption that the curve formed by the main chains is a parabola; which is not strictly correct. In a finished bridge, the curve is between a parabola and a catenary; and is not susceptible of a rigorous determination. **It may save some trouble in making the drawings** of a suspension bridge, to remember that when the deflection does not exceed about $\frac{1}{10}$ of the span, a segment of a circle may be used instead of the true curve; inasmuch as the two then coincide very closely; and the more so as the deflection becomes less than $\frac{1}{10}$. The dimensions taken from the drawing of a segment will answer all the purposes of estimating the quantities of materials.

For some particulars respecting wire for cables, see pages 369 and 380.

The rule commonly given for the length of a parabola, is entirely erroneous. See parabola, in Mensuration. Thus, it makes the last length in the third column 1.666, instead of 1.480: or about 12½ per cent too long. The error diminishes as the curve becomes flatter.

The deflection usually adopted by engineers for great spans is about $\frac{1}{12}$ to $\frac{1}{15}$ the span. As much as $\frac{1}{10}$ is generally confined to small spans. The bridge will be stronger, or will require less area of cable, if the deflection is greater; but it then undulates more readily; and as undulations tend to destroy the bridge by loosening the joints, and by increasing the momentum, they must be specially guarded against as much as possible. The usual mode of doing this is by trussing the hand-railing; which with this view may be made higher, and of stouter timbers than would otherwise be necessary. In large spans, indeed, it may be supplanted by regular *bridge-trusses*, sufficiently high to be braced together overhead, as in the Niagara Railroad bridge, *where the trusses* are 18 ft high; supporting a single-track railroad on top; and a common roadway *of 19 ft clear width*, below.*

* *The writer* believes himself to have been the first person to suggest the addition of very deep *trusses braced* together transversely, for large suspension bridges. Early in 1851, he designed such a *bridge*, *with* four spans of 1000 ft each; and two of 500; with wire cables; and trusses 20 ft high. It was intended for crossing the Delaware at Market Street, Philada. It was publicly exhibited for several months at the Franklin Institute, and at the Merchants' Exchange; and was finally sold from the hall of the latter. Mr Roebling's Niagara bridge, of 800 ft span, with trusses 18 ft high, not commenced until the latter part of 1852; or about 18 months after mine had been publicly exhibited.

Another very important aid is found in deep longitudinal floor timbers, firmly united where their ends meet each other. These assist by distributing among several suspender-rods, and by that means along a considerable length of main cable, the weight of heavy passing loads; and thus prevent the undue undulation that would take place if the load were concentrated upon only two opposite suspenders. With this view, the wooden stringers under the rails on the Niagara bridge are made virtually 4 ft deep. The same principle is evidently good for ordinary trussed bridges.

Another mode of relieving the main cables is by means of *cable-stays*; which are bars of iron, or wire ropes, extending like *c y*, Fig 1, from the saddles at the points of suspension *c*, *d*, obliquely down to the floor, or to some part of the truss. In the Niagara bridge are 64 such stays, of wire ropes of 1¾ inch diam; the longest of which reach more than quarter way across the span from each tower. They transfer much of the strain of the wt of the bridge and its load directly to the saddles at the top of the towers; thereby relieving every part of the cable; as well as diminishing undulation. They end at *c* and *d*, where they are attached, not to the cables, but to the saddles.

The greatest danger arises from the action of strong winds striking below the floor, and either lifting the whole platform, and letting it fall suddenly; or imparting to it violent wavelike undulations. The bridge of 1010 ft span across the Ohio at Wheeling, by Charles Ellet, Jr, was destroyed in this manner. It is said to have undulated 20 ft vertically before giving way. It had no effective guards against undulation; for although its hand-railing was trussed, it was too low and slight to be of much service in so great a span. Many other bridges have been either destroyed or injured in the same way. When the height of the roadway above the water admits of it, the precaution may be adopted of tie-rods, or anchor rods, under the floor at different points along the span, and carried from thence, inclining downward, to the abutments, to which they should be very strongly confined. In the Niagara Railroad bridge 56 such ties, made of wire ropes 1¼ inch diam, extend diagonally from the bottom of the bridge, to the rocks below. They, however, detract greatly from the dignity of a structure.

Mr Brunel, in some cases, for checking undulations from violent winds striking *beneath* the platform, used also *inverted* or *up-curving* cables under the floor. Their ends were strongly confined to the abuts several ft below the platform; and the cables were connected at intervals, with the platform, so as to hold it down.

Art. 2. The angle *a d g*, or *a c i*, Fig 1, which a tang *d g* or *c i* to the curve at either point of suspension *c* or *d*, forms with the *hor* line *c d* or chord, is called **the angle of direction of the main chains,** or cables, at those points. Frequently the ends *c h*, and *d r*, of the chains, called the **backstays,** are carried away from the suspension piers in straight lines; in which case the angles *l d r*, *e c h*, formed between the hor line *e l* and the chain itself, become the angles of direction of the backstays.

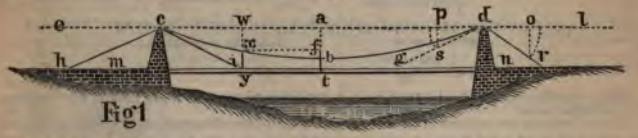


Fig 1

Art. 3. To find the nat sine of the angle of direction, *a d g*; having the span or chord *c d*; and the deff *a b*, at the center of the main chains. See preceding table.

RULE. Mult the deflection by 2. Square the prod; and call the square *w*. Div the chord by 2. Square the quot. Add this square to *w*. Take the sq rt of the sum, and call it *y*. Mult the deflection by 2. Div the prod by *y*. Or, by formula,

$$\textit{Nat sine} = \frac{2\ \textit{deflection}}{\sqrt{(2\ \textit{deflection})^2 + (\tfrac{1}{2}\ \textit{chord})^2}}.$$

Having the nat sine, the angle itself may be taken from the Table of Sines, page 102.

EX. Let the half chord *a d* be 500 feet; and the deff *a b* 50 feet; then,

$$\frac{100}{\sqrt{100^2 + 50^2}} = \frac{100}{\sqrt{10000 + 250000}} = \frac{100}{\sqrt{260000}} = \frac{100}{509.9} = .19611, \text{ nat sine required; and the angle}$$

itself, as found in the Table of Nat Sines, is 11° 19′.

NOTE 1. The direction of the tang *d g* or *c i*, can be laid down on a drawing, thus: Continue the line *a b*, making it twice as long as *a b*; then lines drawn from *d* and *c* to its lower end, will be tangs to the parabolic curve at the points of suspension.

NOTE 2. **If the chord *c d* be not hor,** as sometimes is the case, the angle must be measured from a hor line drawn for the purpose at each point of suspension; as the two angles will in that case be unequal, the piers being of unequal heights.

Art. 4. To find the tension or pull on all the main chains, or cables, together, at either one of the piers, *c* or *d*, Fig 1. See preceding table.

$$\textit{Rule 1.} \quad \textit{Tension of all the main chains, at either pier} = \frac{\textit{one-half of the entire suspended weight of the clear span and load}}{\textit{nat sine of angle of direction, a d g.}}$$

EX. Span, 1000 ft; deff, 50 ft; angle *a d g* of direction, 11° 19′; its nat sine, .1961; half w[illegible] of clear span and load, 900 tons. Then,

$$\textit{Total tension on all cables, at either pier} = \frac{900}{.1961} = 4589\tfrac{1}{2} \text{ tons.}$$

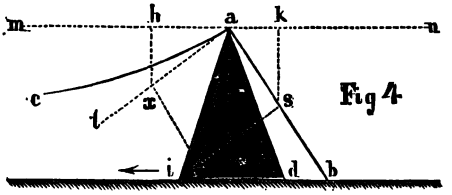


Fig 4

the entire weight of the loaded span.

In Fig 4 also, the angles are unequal; and, consequently, the pres $a\,o$ is not vert; but, in this instance, its amount is *greater* than in Fig 2. In all these cases, if the rollers upon which the chains slide, were supported by single thin posts or pillars, instead of piers, then $a\,o$ would show the proper positions for those pillars, so that the pres should pass precisely along their axes. If we suppose symmetrical piers, $a\,i\,d$, to be used in each case, the base $i\,d$ of that in Fig 2, may be much narrower than in the two other figs; because the direction of $a\,o$ being vert, the pres has no tendency to overturn the pier. In Fig 2, the masonry of the pier should be laid in the usual hor courses, in order that its bed-joints may be at right angles to the pres upon them.

But, in Figs 3 and 4, if the bases were made as narrow as in Fig 2, the lines of direction $a\,o$, of the pres, would fall outside of them; and the piers would consequently be in danger of overturning. Also, the stones of the masonry, if laid in hor courses, would have a considerable tendency to slide on each other. To prevent this, the beds should be at right angles to $a\,o$. Such points, however, will be found to be more fully explained under Force in Rigid Bodies.

In Fig 3, the obliquity of the pres $a\,o$ would tend to slide the base of the pier outward, as shown by the arrow; but in Fig 4, inward. This tendency is produced by the *hor* component of the force $a\,o$. The amount of this may be found thus, in either fig: From a downward draw a vert line; and from o, a hor one, meeting it in e. Then $o\,e$, measured by the same scale of tons as before, will give this hor force; and $a\,e$ will give the vert component of the pres $a\,o$. The effect upon the pier of the one pres $a\,o$, is precisely the same as would be produced upon it by one vert force equal to $a\,e$, and a hor one equal to $o\,e$, acting at the same time; as explained under Comp and Res of Forces.

If, in either fig, we draw the vert lines $x\,h$ and $s\,k$, then $a\,k$, measured by the foregoing scale, will give the tons of *hor* pull; and $k\,s$, the *vert* pres produced on the pier by the backstay; and $h\,s$ and $h\,x$ will in like manner give the corresponding forces produced by the main chain. If we add together $k\,s$ and $h\,x$, they will be found to be equal to $a\,e$; and if we subtract $a\,k$ from $a\,h$, their diff will equal $o\,e$. It is this diff only that tends to slide, or to upset the pier; the other portions of $a\,k$ and $a\,h$, neutralizing each other in that respect.

The foregoing strains may all be *calculated* thus:

The hor pull inward by the main chain = *Tension* × *Nat Cosine of* m a t.
" " " *outward by the backstay* . = *Tension* × *Nat Cosine of* n a s.
Vert pres by main chain..... = *Tension* × *Nat Sine of* m a t.
" " " *backstay*..... = *Tension* × *Nat Sine of* n a s.

The towers, piers, or pillars, which uphold the chains or cables, admit of an endless variety in design. According to circumstances, they may consist each of a single vertical piece of timber, or a pillar of cast or wrought iron; or of two or more such, placed obliquely, either with or without connecting pieces; like the bents of a trestle, as in any of the figs on page 307. Or they may be made (with any degree of ornamentation) of cast-iron plates; as in iron house-fronts. Or they may be of masonry, brick, or concrete; or of any of these combined.

Each of the suspending-rods, through which the floor of the bridge is upheld by the main chains, requires merely strength sufficient to support safely the greatest load that can come upon the interval between it and half-way to the nearest rod on each side of it; including the wt of the platform, &c, along the same interval.

In anchoring the backstays into the ground, it is necessary to secure for them a sufficiently safe resistance against a pull equal to the strain, $a\,s$, upon the backstay. This safety should be as great as that of the main chains against that same amount of tension.

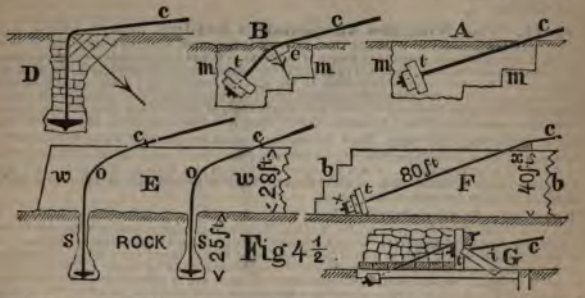
As to the anchorage of the cables below the surface of the ground, natural rock of firm character is the most favorable material that can present itself. When it is not present, serious expense in masonry must be incurred in large spans, in order to secure the necessary weight to resist the pull of the cables. Our Figs 4½ give ideas of the modes most frequently adopted. For a very small bridge, such as a short foot-bridge, for instance, the backstays may simply be anchored to large stones, t, Fig A, buried to a sufficient depth. Or, if the pull is too great for so simple a precaution, the block of masonry, $m\,m$, may be added, enclosing the backstay. A close covering of the mortar or cement of the masonry has a protecting effect upon the iron.

To avoid the necessity for extending the backstays to so great a dist under ground, they are usually curved near where they descend below the surface, as shown at B, D, and E; so as sooner to reach the reqd depth. This curving, however, gives rise to a new strain, in the direction shown by the arrows in Figs B and D. The nature of this strain, and the mode of finding its amount, (knowing *the pull on the* backstay,) are very simple; and fully explained under the head of Funicular Machine, *page 463*. The masonry must be disposed with reference to resisting this strain, as well as *that of the direct* pull of the backstay. With this view, the blocks of stone on which the bend rests *should be laid* in the position shown in Fig D; or by the single block in Fig B. Sometimes the bend *is made over* a cast-iron chair or standard, as at x, Fig F, firmly bolted to the masonry.

Fig E shows the arrangement at the Niagara railway bridge of 821½ ft span. The wire backstays *end at c c;* and from there down to their anchors, they consist of heavy chains; each link of which *is composed of* (alternately) 7 or 8 parallel bars of flat iron, with eye ends, through which pass bolts *as in Figs 41*, page 293.* Each of the 7 bars of each link is 1.4 ins thick, by 7 ins wide, near

* When chains of iron bars are used instead of wire cables, they are usually made as ... nce bar iron has but about half the tensile strength of wire, the chains must have a section ... ice as great as that of a cable.

lowest part of the chain; but they gradually increase from thence upward, until at *c*, *c*, where they unite with the wire cable, the sectional area of each *link* is 93 sq ins. These chain backstays pass in a curve through the massive approach walls, (28 ft high,) and descend vertically down shafts *s*, *s*, 25 ft deep in the solid rock. Here they pass through the cast-iron anchor-plates, to which they are confined below by a bolt 3½ ins diam. The anchor-plates are 6½ feet square, and 2½ ins thick; except for a space of about 20 ins by 26 ins, at the center where the chains pass through, where they are 1



foot thick. Through this thick part is a separate opening for each bar composing the lowest link. From this part also radiate to the outer edges of the lower face of the plate, eight ribs, 2½ ins thick. The shafts *s*, *s*, have rough sides, as they were blasted; and average 3 ft by 7 ft across; except at the bottom, where they are 8 ft square. They are completely filled with cement masonry, with dressed beds, well in contact with the sides of the shafts; and thoroughly grouted, thus tightly enveloping the chains at every point; as does also the masonry of the approach wall *w w*; which extends 28 ft above ground; and is 6 ft thick at top, and 10½ ft thick at its base on the natural rock.

D, Figs 4½, shows a mode that may be used in most cases, for bridges of any span. The depth and the area of transverse section of the shaft, and consequently the quantity of masonry in it, will depend chiefly upon whether it is sunk through rock, or through earth. If through firm rock, then if its sides be made irregular, and the masonry made to fit securely into the irregularities, much reliance may be placed upon it to assist the weight of the masonry in resisting the pull on the backstays. Earth also assists materially in this respect.

F is the arrangement in the Chelsea bridge of 333 feet span, across the Thames, at London; Thos. Page, eng. The space from one wall *b b*, to the opposite one, is 45 feet; and is built up solid with brickwork and concrete; except a passage-way 4 ft wide, and 5 ft high, along the backstay; and a small chamber behind the anchor-plates. It rests chiefly on piles.

The arrangement by Mr Brunel, in the Charing Cross bridge, London, is very similar. In it also the entire abutment rests on piles; and is 40 ft high, 30 ft thick, and solid, except a narrow passage-way along the chains. The backstays extend into it 60 ft. Span 676 feet. Deff 50 feet.

G is intended merely as a general hint, which, variously modified, may find its application in the case of a small temporary, or even permanent bridge; for the number of pieces, *t*, *t*, &c, may be increased to any necessary extent; and they may be made of iron or stone, instead of wood.

In estimating the action of the backstays upon the masonry, &c, to which they are anchored, it is safest to consider the tension along them to continue undiminished to their very ends; although, when they are embedded in masonry, friction causes it to diminish; especially when they are curved, as in E, Figs 4½, in which case the friction is greatly increased, and the tension thereby materially reduced as the ends are approached. Frequently, however, they are not so embedded; for, although embedding preserves the iron, many engineers prefer to leave an open space around the entire length of the anchor-chains; as well as around the anchor-plates; in order that they may be examined from time to time. To this end, the masses *w m* of masonry in Figs 4½, may be made not solid, but to consist of two parallel walls, between which the backstay may pass; and the anchor-stones, or anchor-plates, will extend across the space between the walls, and have their bearing against the ends of the walls. In F, the cable may be supposed either to be tightly surrounded by the masonry, and grouted to it; or else to be surrounded by a cylindrical passage-way like a culvert, so as to be at all times accessible. And the same with regard to the cable in the vertical shafts at D or E.

Art 35, p 465; Art 41, p 470; &c, of Force in Rigid Bodies, will assist in calculating the resistance which the masses *m m* of anchorage masonry oppose to the pull of the backstays. Soft friable stone must be carefully excluded from such parts of these masses as are most directly opposed to this pull.

If blocks of stone large enough for securing good bond are not procurable, heavy bars of iron, or I beams, may be advantageously introduced for that purpose.

The masses must be founded at such a depth as not to slide by the yielding of the earth in front of them.

Experience shows that with due attention to periodical painting, and renewal of woodwork, a properly designed *suspension* bridge will be very durable. The transverse floor joists should be of *wrought iron; to prevent interruption* to travel while putting in new wooden ones.

***Particular* care should be bestowed upon the strength of the joints of the side parapets;** for the undulations and lateral mo*of the bridge expose them* to violent deranging forces in every direction. The parapets sh*high and stout; and not restricted* to mere service as hand-rails, or guards.

As a rule of thumb, one-half the sq rt of the span will be about a good height for them in *cases, provided it is not less* than a hand-rail requires.

We do not think that diagonal horizontal bracing should, as is usual, be omitted under the floor. It may readily be effected by iron rods.

All the cables need not be at the *sides* of the bridge. One or more of them may be over its axis; especially in a wide bridge. One wide footpath in the center may be used, instead of two narrow ones at the sides.

The platform or roadway should be slightly cambered, or curved upward, to the extent say of about $\frac{1}{200}$ of the span.

Art. 11. The Niagara suspension bridge, built in 1852–3, John A Roebling, engineer, consists of a single span of 821½ ft measured straight from center to center of towers; and 800 ft of clear suspended length of roadway. It has two floors or roadways: the upper one, for a single-track railway, is 25½ ft; and the lower one, for common travel, 24½ ft wide, out to out of everything. The lower one is 19 ft wide in the clear of everything. They are 17 ft apart vertically. The trusses are 18 ft total height, throughout. They are on the Pratt arrangement; see p 284; with verticals 5 ft apart from cen to cen; and single oblique iron rods, 1 inch square, running in each direction across four of the 5 ft panels. Where these rods pass each other, they are tied together by 10 or 12 turns of $\frac{1}{16}$ inch wire. Each vertical consists of two pieces of 4¼ by 6¼ timber, placed 4½ ins apart, to allow the oblique rods to pass between them. Both upper and lower floor girders are in two pieces, of 4 by 16 ins each. Pairs 5 ft apart. The tops and bottoms of the verticals pass between the two pieces which form each floor girder. There is no *continuous* upper or lower chord. No tenons or mortises are used in the framing.

There are four cables of iron wire; two on each side of the bridge. Each cable is 10 ins diam. The wire is scant No. 9 of the Birmingham wire gauge, or scant .148 inch diam. Sixty wires have a united transverse section equal to one square inch of *solid* iron. Each of the four cables contains 3640 wires, with a united cross-section of 60.4 sq ins of solid metal. Therefore, the area of solid metal in a section of all the four cables together is 241.6 sq ins, or 1.678 sq ft; weighing 814 lbs per ft of span. The wires of each cable are first made up, in place, into 7 small strands; and these are firmly bound together throughout by a continuous close wrapping of wire. The strength of each individual wire is 1640 lbs, or .73214 of a ton. This is equal to 98400 lbs, or 43.93 tons per sq inch of solid metal; or to 5943360 lbs, or 2653.3 tons per cable; or to 10613.2 tons ultimate strength of the four cables together. One cable on each side of the bridge deflects 54 ft; and the other 64 ft; average deflection 59 ft, or about $\frac{1}{14}$ of the span. With this av defl the tension on the cables at the tops of the towers averages 1.82 times the total suspended wt of the span and its load. See table, Art 1. The wt of the suspended span itself is about 900 tons; and if the greatest extraneous load on the two floors together be taken at 1¼ tons per ft run, we have the total suspended wt 900 + (800 × 1¼) = 1900 tons. And 1900 × 1.82 = 3458 tons tension at towers; or very nearly ⅓ of the ultimate strength of the cables, without any allowance for momentum, or wind. But such loads, although possible, are not permitted to come upon the bridge; and moreover a part of the strain is borne by the upper stays.

The wires were perfectly oiled before being made into strands; and when the strands were being bound together to form a cable, the whole was again thoroughly saturated with oil and paint.

The cables do not hang vertically; but the two upper ones are about 37 ft apart from center to center, where they rest upon the towers, (where all four are on the same level;) and are drawn to within 13 ft of each other at the center of the span; and at the level of the railway track on top of the bridge; while the two lower ones are about 39 ft apart at the towers, and 25 ft at the center of the span, and at the level of about halfway between the two floors.

This drawing-in of the cables contributes much to lateral stability; as do also the upper and lower floor of stout plank. There is no horizontal diagonal floor bracing.

There are 624 suspenders of wire rope, 1½ ins diam, and 5 ft apart, or corresponding with the floor girders, which they uphold; and with the wooden verticals of the trusses. They do not hang vertically; but incline inward.

The masonry towers are all founded on rock. They are 78½ ft high above the bottom of the bridge; and 60½ ft above the upper floor. The two at each end of the span are 39 ft apart from center to center. At the level of the lower floor they are 19 × 20 ft; and 21 ft apart in the clear. At the level of the upper floor they are 15 ft square; and 24 ft apart in the clear. From there they taper regularly to the top, where they are 8 ft square. They are built of limestone, in heavy dressed hor courses; laid in cement; vertical joints grouted. The upper courses are dowelled. On top of each tower is a cast-iron plate, 8 ft sq, and 2½ ins thick, bedded in cement. Part of the top of this plate is planed, as upon it move the rollers which support the cast-iron saddles on which the cables rest. At each tower, each cable has its separate saddle and rollers. Each saddle rests on 10 cast-iron rollers 25½ ins long, and 5 ins diam, carefully planed. They lie loosely, and close together; and are kept in place by side flanges on the bed-plate.

The cast saddles are each 5 ft long, by 25½ ins wide. Their bottoms, which rest on the rollers, are flat, and planed. Their tops are curved to a rad of 6¼ ft; to suit the bend of the cables over the piers; and each saddle has a longitudinal groove, in which the cable lies. The passage of the heaviest trains produces less than ½ an inch of movement in a saddle.

The floors have a camber of 5 feet.

A change of 100° Fah of temperature causes an average variation of about 2½ ft in the deflection of the cables, or in the camber of the roadways; and one of 150°, (about the extreme to which the bridge is exposed,) about 3¾ ft. The passage of a train weighing 291 tons, and covering the entire length of the span, caused a deflection of 10 ins; and an ordinary train deflects it only from 3 to 5 *inches.*

This bridge has, since the year 1853, demonstrated the applicability of the suspension principle to *large span* railway bridges. Its entire cost was not quite $400,000. For more, see top of p 589.

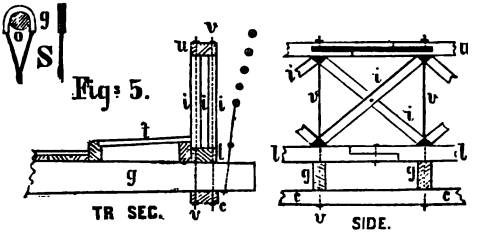
***Art. 12.* The wire suspension bridge near Freyburg, Switzerland,** finished in 1834, Mr. Chaley, engineer, and still in full service, is of *very simple construction*, and has served as the prototype for several in this country. It is for *ommon travel only*; and is narrow: its entire width of platform being but 21½ ft; and its clear *vailable width* but 19 ft. The dist from cen to cen of its towers is 889 feet; and its clear span *ween abutments* 800 ft; or the same as the Niagara. There are 4 cables, each 5 ins diam. Each *m consists of* 1056 wires of No. 10, or full ⅛ inch diam, (or 71 wires to the sq inch of solid m *anged in* 20 strands of about 53 wires each. The four cables, therefore, have a united area *q ins of solid metal*; weighing 202 lbs or .09 of a ton, per ft run of span. All its suspen

vertical; about 5 ft apart; and each upholds one end of a transverse floor girder. It has no sid trussing except the slight one of the wooden hand-railing, which is about 6 feet high; and conse quently, with its great span it is quite flexible. The deflection of the cables is $\frac{1}{11}$ of the span; hene the strain upon them at the top of the towers at either end, is 1.82 times (see table, Art 1,) the wt o the suspended span itself, and its extraneous load; and supposing the wire to be as good as that o the Niagara, the *breaking* strain of the four cables would be $60 \times 44 = 2640$ tons; and their saf strain cannot be taken at *more* than ⅓ as much, or 880 tons. The suspended weight reqd to produc this safe strain would of course be $\frac{880}{1.82} = 484$ tons. The suspended weight of the span itself cann well be less than .3 of a ton per ft run; or 240 tons in all;* thus, leaving $484 - 240 = 244$ tons fo the maximum safe extraneous load. This amounts to .305 of a ton per ft run of span; or 36 lbs pe sq ft of its platform, 19 ft wide *in the clear*. The French allowance is 41 lbs per sq ft; † and since n allowance is here made for momentum or wind, it is plain that this celebrated bridge, on account o its slight cables, and its flexibility, is by no means a strong one. In that respect, as well as steadi ness, it is much inferior to the one next spoken of. It is said, however, to have withstood very sever tempests; and also to have been occasionally completely covered by crowds of people. If so, thei lives were not very secure.

Art. 13. The wire suspension bridge across the Schuylkill at Philada, finished in 1842, Chas Ellet, Jr, engineer, is somewhat similar in character, and in th dimensions of its details, to the preceding; but being of much less span, is much stronger. Its spa from cen to cen of towers is 358 ft; suspended platform between abuts 342 ft. It has ten cables of ins diam; five on each side. Their united sections present 55 sq ins of solid iron; or nearly as muc as the preceding bridge of 800 ft clear span. The five cables on either side have different deflections ranging between the $\frac{1}{12}$ and the $\frac{1}{14}$ of the span from tower to tower. The dist from cen to cen o towers at either end of the span is 35½ ft; and on top of each tower the cables (considerably flattene at that point) lie side by side on a single roller about 30 ins long, and 6 ins smallest diam, which has grooves, for their reception. Each cable is drawn in about 3½ ft at the center of the span. At in tervals of 20 ins the parallel wires of the cable have a close wrapping of finer wire for a distance o 3 ins.

The suspenders are of wire; and are ¾ inch diam; and 4 ft apart. On any one cable they are 2 ft apart. They all incline slightly inward.

The width of the platform from out to out is 27 ft; and in clear of hand-rails 25 ft. Inside of th hand-rail is a footway, 4 ft 4 ins wide, on each side of the bridge. The remaining 16 ft 4 ins serve for a double carriage way, or double-track street railway. Figs 5 show the arrangement of the wood work, on a scale of ½ inch to a ft. The trussing of the parapets is on the Howe system. (see p 283.) which does not appear to be as well adapted as the Pratt, to suspension bridges. The diagonals in the Fairmount bridge work themselves out of place laterally, by the vibrations of the bridge; and we have occasionally seen several of them almost on the point of falling out entirely. Being under municipal charge, it is of course neglected. The upper chords *u*, are 12 ins wide by 6 ins deep; the lower ones *l*, and the stringer *c*, below them, are each 12 wide, by 7 deep The diagonals *i* are all 4 ins wide, by 5 deep. The angle-blocks at their ends are of cast iron, hollow and about ½ inch thick. The vert iron rods *v*, (in pairs,) are ⅝ inch diam near the center of th span; and 1½ at its ends. The top chords are spliced on each vert face by an iron bar, of 5 ft by ins, by ½ inch; with 4 bolts passing through them. The splice of the bottom chord has merely bolts, side by side; (see Figs 5;) which (except S) are to a scale of ½ inch to a ft. The floor girders g 4 ft apart from cen to cen, are 6 by 14 ins at their ends; and 6 by 16 at center.



The floor is of two thicknesses of 2-inch plank; except the footpaths, which are single thickness.

The wires were well oiled when the cables were made; and afterward painted.

At S is shown the mode of uniting a suspender with a cable, *o*, by means of a small cast-iron yok *g*, which straddles the cable; and on the back of which is a groove ¾ of an inch wide, in which th suspender rests. The metal of the yoke is about ¼ inch thick. Since the lower ends of the wire which compose a suspender cannot themselves be formed into a screw-bolt, for upholding the floo girders, they are passed through the eye of a screw-bolt of bar iron; then doubled on themselves, an held by a wrapping of wire. It is well to introduce a yoke here also, to prevent the wear of the wire by friction. The small fig on the right of S is an edge view of the yoke *g*.

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* This is probably nearly its actual weight, as obtained by comparing it with the Fairmount brid which, by a careful *estimate* by the writer, weighs .375 of a ton per ft run; but is considerably w *than the Freyburg; and carries* four lines of light street-rails. But if the Freyburg, has longitu *joists, it will weigh about* .03 ton more per ft run.

† *The greatest load that can come upon an ordinary bridge, is a dense crowd of people;* *the French engineers estimate at* 41 lbs per sq ft of platform. This is certainly as great as *occur under ordinary circumstances*; but it may be considerably exceeded. The French *moreover, includes no allowance for wind, or for the crowd being in motion.* Inclination *writer thinks that no suspension bridge should have a less safety than* 3, against 100 T *added to the weight of the bridge itself.* A less coeff of safety is admissible in a wire *is an iron trussed one, on account of the greater reliability of the material.* See foot-

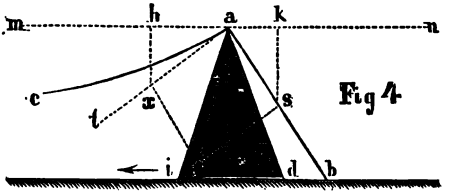


Fig 4

the entire weight of the loaded span.

In Fig 4 also, the angles are unequal; and, consequently, the pres $a\,o$ is not vert; but, in this instance, its amount is *greater* than in Fig 2. In all these cases, if the rollers upon which the chains slide, were supported by single thin posts or pillars, instead of piers, then $a\,o$ would show the proper positions for those pillars, so that the pres should pass precisely along their axes. If we suppose symmetrical piers, $a\,i\,d$, to be used in each case, the base $i\,d$ of that in Fig 2, may be much narrower than in the two other figs; because the direction of $a\,o$ being vert, the pres has no tendency to overturn the pier. In Fig 2, the masonry of the pier should be laid in the usual hor courses, in order that its bed-joints may be at right angles to the pres upon them.

But, in Figs 3 and 4, if the bases were made as narrow as in Fig 2, the lines of direction $a\,e$, of the pres, would fall outside of them; and the piers would consequently be in danger of overturning. Also, the stones of the masonry, if laid in hor courses, would have a considerable tendency to slide on each other. To prevent this, the beds should be at right angles to $a\,o$. Such points, however, will be found to be more fully explained under Force in Rigid Bodies.

In Fig 3, the obliquity of the pres $a\,o$ would tend to slide the base of the pier outward, as shewn by the arrow; but in Fig 4, inward. This tendency is produced by the *hor* component of the force $a\,o$. The amount of this may be found thus, in either fig: From a downward draw a vert line; and from o, a hor one, meeting it in e. Then $o\,e$, measured by the same scale of tons as before, will give this hor force: and $a\,e$ will give the vert component of the pres $a\,o$. The effect upon the pier of the one pres $a\,o$, is precisely the same as would be produced upon it by one vert force equal to $a\,e$, and a hor one equal to $o\,e$, acting at the same time; as explained under Comp and Res of Forces.

If, in either fig, we draw the vert lines $x\,h$ and $x\,k$, then $a\,k$, measured by the foregoing scale, will give the tons of *hor* pull; and $k\,s$, the *vert* pres produced on the pier by the backstay; and $h\,s$ and $h\,x$ will in like manner give the corresponding forces produced by the main chain. If we add together $k\,s$ and $h\,x$, they will be found to be equal to $a\,e$; and if we subtract $a\,k$ from $a\,h$, their diff will equal $o\,e$. It is this diff only that tends to slide, or to upset the pier; the other portions of $a\,k$ and $s\,h$, neutralizing each other in that respect.

The foregoing strains may all be *calculated* thus:

The hor pull inward by the main chain = *Tension* × *Nat Cosine of* m a t.
" " " *outward by the backstay* . = *Tension* × *Nat Cosine of* n a s.
Vert pres by main chain..... = *Tension* × *Nat Sine of* m a t.
" " " *backstay*..... = *Tension* × *Nat Sine of* n a s.

The towers, piers, or pillars, which uphold the chains or cables, admit of an endless variety in design. According to circumstances, they may consist each of a single vertical piece of timber, or a pillar of cast or wrought iron; or of two or more such, placed obliquely, either with or without connecting pieces; like the bents of a trestle, as in any of the figs on page 307. Or they may be made (with any degree of ornamentation) of cast-iron plates; as in iron house-fronts. Or they may be of masonry, brick, or concrete; or of any of these combined.

Each of the suspending-rods, through which the floor of the bridge is upheld by the main chains, requires merely strength sufficient to support safely the greatest load that can come upon the interval between it and half-way to the nearest rod on each side of it; including the wt of the platform, &c, along the same interval.

In anchoring the backstays into the ground, it is necessary to secure for them a sufficiently safe resistance against a pull equal to the strain, $a\,s$, upon the backstay. This safety should be as great as that of the main chains against that same amount of tension.

As to the anchorage of the cables below the surface of the ground, natural rock of firm character is the most favorable material that can present itself. When it is not present, serious expense in masonry must be incurred in large spans, in order to secure the necessary weight to resist the pull of the cables. Our Figs 4½ give ideas of the modes most frequently adopted. For a very small bridge, such as a short foot-bridge, for instance, the backstays may simply be anchored to large stones, t, Fig A, buried to a sufficient depth. Or, if the pull is too great for so simple a precaution, the block of masonry, $m\,m$, may be added, enclosing the backstay. A close covering of the mortar or cement of the masonry has a protecting effect upon the iron.

To avoid the necessity for extending the backstays to so great a dist under ground, they are usually curved near where they descend below the surface, as shown at B, D, and E; so as sooner to reach the reqd depth. This curving, however, gives rise to a new strain, in the direction shown by the arrows in Figs B and D. The nature of this strain, and the mode of finding its amount, (knowing *the pull on the* backstay,) are very simple; and fully explained under the head of Funicular Machine, page 463. The masonry must be disposed with reference to resisting this strain, as well as *that of* the direct pull of the backstay. With this view, the blocks of stone on which the bend rests *should be* laid in the position shown in Fig D; or by the single block in Fig B. Sometimes the bend *is* made over a cast-iron chair or standard, as at x, Fig F, firmly bolted to the masonry.

Fig E shows the arrangement at the Niagara railway bridge of 821½ ft span. The wire backstays *end at c c*; and from there down to their anchors, they consist of heavy chains; each link of which *is* composed of (alternately) 7 or 8 parallel bars of flat iron, with eye ends, through which pass bolts as in Figs 41, page 293.* Each of the 7 bars of each link is 1.4 ins thick, by 7 ins wide, near

* When chains of iron bars are used instead of wire cables, they are usually made as follows. Since bar iron has but about half the tensile strength of wire, the chains must have a section as great as that of a cable.

bridge would therefore safely sustain a greater load per foot run of span, than the Freyburg.

These old bridges frequently failed by the rotting of the end posts; or were carried away by freshets; but we have never heard of a failure from the breaking of the chains. Many of them were built in a much more perfect style than the one just described; and on the most used roads in the Union.

Art. 17. As it is sometimes convenient to form a rough idea at the moment, of the size of cables required for a bridge, we suggest the following rule for finding approximately the area in sq ins of *solid* iron in the wire required to sustain, with a safety of 3,* the weight of the bridge itself, together with an extraneous load of 1.205 tons per foot run of span; which corresponds to 100 lbs per sq ft of platform of 27 ft *clear available* width. This suffices for a double carriage-way, and two footways. The deflection is assumed at $\frac{1}{12}$ of the span; and the wire to have an ultimate strength of 36 tons per solid square inch, as per table, page 369; and which can be procured without difficulty. **For spans of 100 ft or more,**

RULE. Mult the span in feet, by the square root of the span. Divide the prod by 100. To the quot add the sq rt of the span. Or, as a formula,

$$\textit{Area of solid metal of all the cables; in square ins; for spans over 100 feet} = \frac{\textit{span} \times \textit{sq rt of span}}{100} + \textit{sq rt of span.}$$

For spans less than 100 feet, proportion the area to that at 100 ft.

If a defl of $\frac{1}{10}$ is adopted instead of $\frac{1}{12}$, the area of the cables may be reduced very nearly $\frac{1}{7}$ part.

The following table is drawn up from this rule. The 3d col gives the united areas of all the actual wire cables, when made up, including voids. (Original.)

Span in Feet.	Solid Iron in all the Cables.	Areas of all the Finished Cables.	Span in Feet.	Solid Iron in all the Cables.	Areas of all the Finished Cables.	Span in Feet.	Solid Iron in all the Cables.	Areas of all the Finished Cables.
	Sq. Ins.	Sq. Ins.		Sq. Ins.	Sq. Ins.		Sq. Ins.	Sq. Ins.
1000	348	446	400	100	128	150	30.6	39.2
900	300	385	350	84	108	125	25.2	32.3
800	254	326	300	69	89	100	20	25.6
700	212	272	250	55	71	75	15	19.2
600	171	219	200	42	54	50	10	12.8
500	134	172	175	36.4	46.7	25	5	6.4

Having the areas of all the actual cables, we can readily find their diam. Thus, suppose with a span of 500 ft, we intend to use four cables. Then the area of each of them will be $\frac{172}{4} = 43$ sq ins; and from the table of circles, page 18, we see that the corresponding diam is full 7⅝ ins.

The above areas are supposed to allow for the increased wt of a depth of truss, and other additions necessary to secure the bridge from violent winds, and from undue vibrations from passing loads.

When these considerations are neglected, and a less maximum load assumed, the foregoing descriptions of the Wheeling and Freyburg bridges show what reductions are practicable. Weight, sufficiently provided for, is of great service in reducing undulation.

FRICTION.

Art. 1. (See Arts 60, 61, 62, 63, &c, of Force in Rigid Bodies.) Friction is that resistance (produced by an interlocking of the roughnesses of surfaces in contact with each other) which opposes the sliding of one body along another. It occurs in all machinery; in water, gas, or air, moving along pipes, or other channels; and in all cases whatever where one body moves along another in contact with it. **It is the principal cause of what is called "loss of power,"** in all these cases; which means simply that the whole of any moving force (such as steam, gravity, moving water, wind, muscular force, &c) cannot be directly applied through machinery, to give motion to other bodies, because a portion of it is *lost* in overcoming friction in the machinery. Strictly speaking, it is not *lost;* but is quite as useful as the remaining portion; for if friction were not first overcome, there could be no motion at all. In hydraulics, this loss of power is usually termed "loss of head," meaning that portion of the head of water, which by its pres counteracts the fric of the water in the pipes, &c, and thus permits the other portion to produce motion or flow. In nearly all the cases which come under the notice of the engineer, the surfaces in contact are rendered more or less smooth by art; and the smoother they are made, the less is the friction, because the roughnesses are diminished; and of course take less hold on each other. But in even the most highly polished surfaces, there is sufficient roughness to produce some friction. Although th

* *The writer must not be understood to advocate a safety of 3 against 100 lbs per sq ft, in addi to the weight of the bridge,* in all cases. He believes that limit to be about a sufficient one for *perly designed wire suspension* bridge for ordinary travel; but for an important railroad bri *would (according to position, exposure, &c)* adopt a safety of at least from 4 to 6 against the *possible load, added to the wt of the bridge.* A train of cars opposes a great surface to the *side winds; and trains must run during violent storms,* as well as during calms; but a *bridge for common travel* is not likely to be densely crowded with people during a severe

roughness may not be visible to the naked eye, it becomes very apparent under a powerful microscope. Could it be entirely removed, there would be no friction; and then the smallest conceivable force would slide hor the greatest conceivable weight; nor could the most powerful vise prevent the lightest body from falling out of its jaws.

So long as the force which presses any given sliding surfaces together, is considerably less than would be necessary to abrade, or wear them away appreciably, the degree of friction (and of the force reqd to overcome it) *bears a certain proportion, or ratio, to the* **pressing force;** *and is entirely independent of the* **size of the surfaces themselves.** A brick will slide just as readily when lying on its broad side, as when on a narrow one. This proportion, however, is diff for diff materials; and for diff degrees of smoothness; and moreover, is less when the surfs are clean and well lubricated, than when dirty and dry. Whatever this proportion may be in any particular case, it is called the *coefficient of friction* for that case. Thus, if we wish to slide hor a block of dry cut stone weighing 1 ton, upon a surf of the same kind, we shall have to exert a force of about ⅙ of a ton; or if we first place a weight of 2 tons upon the moving stone, making the total pres upon its base 3 tons, we must exert a sliding force of ⅙ of 3 tons. Because the fric of dry out stone upon dry out stone averages about ⅙ of the force which presses the two surfs together; or in other words, the coeff of fric of dry cut stone on dry cut stone, is about ⅙, or .66.

But if the pres upon the sliding surfs is sufficient to produce abrasion (indeed, while it is much less,) the fric becomes greater: but no precise law has yet been discovered for estimating it. The fric between surfs actually in motion, is less than when they have been for some time quiescent or stationary. Hence it is usual to consider it under the heads of *moving*, and *quiescent* fric. Experience, however, has shown that slight jarring suffices to remove this diff; and since all structures, even the heaviest, are subject to occasional jarring, (as a bridge, or a neighboring building, or even a hill, during the passage of a train; or a large factory, by the motion of its machinery; or in numberless cases, by the action of the wind.) it is considered expedient, in construction, not to rely on fric for stability, any further than the coeff for *moving* friction may justify.

Beside the foregoing subdivisions of *sliding* fric, we have *rolling*, and *journal* fric. At present we shall confine ourselves to sliding.

The coeff of moving fric is the same at all vels; and this, in connection with the foregoing statements that it is in proportion to the pres, so long as this is considerably below the abrading point; and that it is independent of the amount of surf; are considered the three grand laws of fric. Unfortunately, the experiments on this subject by Morin, which are those by which practical men are chiefly guided; were made with slight pressures; not exceeding about 30 lbs per sq inch of contact-area of the bodies, even with the strongest bodies, such as iron, &c; which, in machinery very often have to move under far greater pres. It is well known that fric, under much heavier pres, increases very considerably beyond the extent assigned by Morin; as will be seen by our table of results found by Rennie; Art 3. Locomotives would frequently be unable to draw the loads they do, if their fric (miscalled *adhesion*) on the rails did not greatly exceed the limits of iron on iron assigned by Morin. The fact is, that although it is customary to consider our knowledge of the laws of fric to be very complete, it is extremely defective. The common assertion that "*fric* is the same at all vels," may lead to mistakes, unless we clearly distinguish between the *coeff*, and the *amount*. If one man drags a sled one mile in one hour; and another man drags it one mile in ¼ of an hour; then, inasmuch as both the coeff, and the dist, are the same in both cases; each man has overcome the same *amount* of fric. But if one drags it 4 miles, in the same time that the other drags it 1 mile, then, although the *coeff*, and the *time*, remain unchanged, the *amount of fric* is 4 times as great in the first case as in the last; and has reqd the man to expend 4 times as much force to overcome it alone; without any regard to the force expended in moving himself so rapidly. In other words, the quantity, or amount of fric; and the total amount of power reqd to overcome it, is in proportion to the *space* passed over; without any reference to the time reqd. So a train moving at 60 miles per hour, overcomes just as great an amount of fric in 1 mile, as does a train moving at 10 miles per hour; but the fast one overcomes 6 times as much per hour, min, or sec, as the slow one. So with pivots, journals, &c, in machinery; the more rapidly they revolve, the greater is the *amount* of friction they must overcome in a given time; although the *degree*, or *coeff*, or *intensity* of the fric remains the same at all vels.

Art. 2. To find the proportion, or ratio, of fric, to the pres which produces it, (or the coeff of fric,) for diff materials. This may be ascertained by making of one of them an inclined plane: and finding by trial, what slope it must have, to enable the other just to begin to slide down it. Div the vert height of this slope, by its hor base; and the quot will be the reqd coeff. It will also be the nat tang of the angle which the slope makes with the hor; so that by looking for the coeff in a table of nat tang, the angle itself will be found opposite to it. If the weight of the sliding body be mult by the coeff, the quot will be the force reqd to drag it *horizontally* along such a surf as that of the inclined plane; or more correctly, the force required to barely balance the fric; still more force must be added to produce motion. The angle of slope is usually called **the angle of fric**; sometimes **the angle of repose**; and **the limiting angle of resistance.** See Force in Rigid Bodies, Art 63, p 486.

Experiments on fric with unguents cannot well be made on a small scale by means of an inclined plane: on account of the cohesion, or the stickiness of the unguent; which when the sliding body is very light, requires a proportionally greater slope than when it is heavy.

In trying experiments on fric for himself, the student must not expect by any means to arrive at *the same* results as the following; because the slightest deviation in point of hardness, smoothness, *dryness, dust, &c,* will make a very appreciable diff in the coeff. Even dampness in the air will effect *this. Where fric* is to be depended upon for stability, it is best to assume it at less than in the table: *and when it is to* be regarded as a resistance, which we must provide force for overcoming, it should *be taken* at considerably more than the tabular statement. When not otherwise stated, the fibres are *parallel to the motion.*

Table of moving friction, of perfectly smooth, clean, and dry, plane surfaces, chiefly from Morin.

Materials Experimented with.	Coeff of Fric: or Proportion of Fric to the Pres.	Angle of Fric. Deg.	Angle of Fric. Min.
Oak on oak; all the fibers parallel to the motion	.48	25	38
" " moving fibres at right angles to the others; and to the motion	.32	17	45
" " all the fibres at right angles to the motion	.34	18	47
" " moving fibres on end: resting fibres parallel to the motion	.19	10	46
" cast iron, fibres at right angles to motion	.37	20	19
Elm on oak, fibres all parallel to motion	.43	23	17
Oak on elm, " " "	.25	14	3
Elm on oak, moving fibres at right angles to the others, and to motion	.45	24	16
Ash on oak, fibres all parallel to motion	.40	21	49
Fir on oak, " " " "	.36	19	48
Beech on oak " " " "	.36	19	48
Wrought iron on oak, fibres parallel to motion	.62	31	47
Wrought iron on elm, " " " "	.25	14	3
Wrought iron on cast iron, fibres parallel to motion	.19	10	46
" " on wrought iron, fibres all parallel to motion	.14	7	58
Wrought iron on brass	.17	9	39
Wrought iron on soft limestone, well dressed	.49	26	6
" " " hard " " "	.24	13	30
" " " " " " wet	.30	16	42
" " or steel on hard marble, sawed. By the writer ... about	.17	9	39
" " " " " smoothly planed, and rubbed mahogany, fibres parallel to motion	.18	10	12
" " " " " smoothly planed wh pine	.16	9	6
Cast iron on oak, fibres parallel to motion	.49	26	6
" " " elm, " " " "	.20	11	19
" " " cast iron	.15	8	32
" " " brass	.15	8	32
Steel on cast iron	.20	11	19
Steel on steel. By the writer	.14	7	59
Steel on brass	.15	8	32
Steel on polished glass. By the writer ... about	.11	6	17
" quite smooth, but not polished; on perfectly dry planed wh pine, fibres parallel to motion ... about	.16	9	6
" quite smooth, but not polished; on perfectly dry planed and smoothed mahogany, fibres parallel to motion ... about	.18	10	12
Yellow copper on cast iron	.19	10	46
" " on oak	.62	31	48
Brass on cast iron	.22	12	25
" on wrought iron, fibres parallel to motion	.16	9	6
" on brass	.20	11	19
" on perfectly dry planed wh pine, fibres parallel to motion ... about	.19	10	46
" " " " " and smoothed mahogany, fibres parallel to motion ... about	.24	13	30
Polished marble on polished marble. By the writer	.16	9	6
" " on common brick	.44	23	45
Common brick on common brick	.64	32	38
Soft limestone well dressed, on the same	.64	32	38
Common brick, on well-dressed soft limestone	.65	33	2
" " " " " hard "	.60	31	00
Oak across the grain, on soft limestone, well dressed	.38	20	48
" " " " " hard " " "	.38	20	48
Hard limestone on hard limestone, both " "	.38	20	48
" " " soft " " " "	.67	33	50
Soft " " hard " " " "	.65	33	2
Wood on metal, generally, .2 to .62 ... mean	.41	22	18
Wood, *very smooth*, on the same, generally, .25 to .5 ... "	.38	20	48
Wood, " " on metal, " .2 to .62 ... "	.41	22	18
Metal on metal, very smooth, dry " .15 to .22 ... "	.18	10	12
Masonry and brickwork, dry " .6 to .7 ... "	.65	33	2
" " " with wet mortar ... about	.47	25	30
" " " " slightly damp mortar ... "	.74	36	30
" on dry clay ... "	.51	27	00
" " moist " ... "	.33	18	15
Marble, sawed; on the same; both dry. By the writer ... "	.4	21	49
" " " " " both damp ... "	.55	28	49
" " on *perfectly dry* planed wh pine. " ... "	.45	24	11
" " *on damp* planed wh pine ... " ... "	.6	31	00
" *polished, on perfectly dry* planed wh pine " ... "	.28	14	30
White pine, perfectly dry; planed; on the same; all the fibres parallel to motion ... about	.4	21	49
" " *damp, planed*; on the same ... "	.6	31	00

Art. 3. Table of coefficients of moving friction of smooth plane surfaces, when kept perfectly lubricated. (Morin.)

Substances.	Dry Surfs.	Olive Oil.	Tallow.	Lard.	Lard & Pow. Sup.
Oak on oak. Fibres parallel to motion. [illegible]	.100	.....	.071	.067	
" " " Fibres perpendicular to motion.	.....	.....	.065	.072	
" on elm. Fibres parallel to motion.	.100	.....	.071	.060	
" on cast iron. Fibres parallel to motion.	.....	.....	.080		
" on wrought iron. " " "	.....	.....	.069		
Beech on oak. Fibres " " "	.....	.....	.065		
Elm on oak. " " "	.100	.....	.070	.060	
" on elm. " " "	.100				
" cast iron. " " "	.....	.....	.066		
Wrought iron on oak. Fibres parallel, greased and wet, .326.					
" " " " Fibres parallel to motion.	.210	.....	.065		
" " on elm. " " "	.....	.066	.070	.070	
" " on cast iron. " " "	.....	.066	.061	.074	
" " on wrought iron. " " "	.....	.070	.062	.061	
" " on brass. Fibres " " "	.....	.070	.061	.075	
Cast iron on oak, fibres parallel to motion.	.100				
" " " " " " " greased and wet, .319	.....	.075	.070	.075	
" " on elm. " " "	.....	.062	.077	....	.06
" " on cast iron, with water, .316.	.097	.064	.060	.070	.04
" " on brass.	.....	.075	.061	.075	
Copper on oak. Fibres parallel to motion.	.....	.....	.069		
Yellow copper on cast iron.	.....	.066	.077	.060	
Brass on cast iron.	.....	.077	.064		
" on wrought iron.	.....	.072	.061	.065	.08
" on brass.	.....	.065			
Steel on cast iron.	.....	.079	.165	.061	
" on wrought iron.	.....	.....	.063	.076	
" on brass.	.....	.064	.056	.065	.07
Tanned outside on cast iron, greased and very wet, .365	.....	.152	.159		
" " on brass.	.....	.181	.241		
" " on oak, with water, .29.					

It will be seen from the foregoing table, that when kept *perfectly* well lubricated with oil, lard or tallow, the fric of motion of woods and metals, however combined, generally ranges between .07 and .1, or from $\frac{1}{14}$ to $\frac{1}{10}$, of the pres. The angles of repose, or of fric, corresponding to these coeffs, are respectively 4° and 5° 43'. Oil and lard will also be seen to be superior to tallow as a lubricant.

Coeffs of friction of dry surfaces, under pressures gradually increased up to the limits of abrasion. (By G. Rennie, C.E.)

Pres. in Lbs. per Square Inch.	Wrought Iron on Wrought Iron.	Wrought Iron on Cast Iron.	Steel on Cast Iron.	Brass on Cast Iron.
32.5	.149	.174	.166	.157
196	.250	.275	.300	.295
224	.271	.292	.333	.219
336	.312	.333	.347	.215
448	.376	.365	.354	.208
560	.409	.367	.358	.233
672	.....	.376	.403	.233
709	.....	.434	.....	.234
784	.....	.....	.....	.232
821	.....	.....	.....	.273

The irregularities in the last column are remarkable.

Art. 4. Pivot friction. To find the amount of power consumed by the fric of pivots. The base of the pivot is supposed to be flat; that being the best form. The moment of pivot fric, or, in other words, its tendency to prevent the pivot from revolving: is found by mult the amount of the fric itself, by its leverage; which last is equal to ⅔ (two thirds) of the *rad of the pivot*. Hence, first find the fric, by mult the entire pres on the pivot in *lbs*, by the corresponding coeff taken from one of the foregoing tables. Mult the fric so obtained, by ⅔ of the *circumf* of the pivot, in *feet* * The prod will be the amount of power consumed by fric at each turn of the pivot, expressed in foot pounds. And these ft-lbs, mult by the number of turns made per min, will give the power expended per min in overcoming the fric.

Ex. A pres of 22100 lbs is supported by a steel pivot 6 ins diam, revolving on a cast-iron step; and *kept well lubricated* with oil. What power must the prime mover expend in overcoming the fric?

Here, the diam of the pivot being 6 inches. Its circumf is 18.84 ins; and two-thirds of 18.84 is 12.56 *ins*, or 1.05 *ft*. Also, the corresponding number for steel on cast iron in the table, is about .08. Consequently,

$$22100 \times .08 \times 1.05 = 1881.6 \text{ ft-lbs of fric per revolution.}$$

* Equal to the whole circumf described by ⅓ of the rad.

Now, if the pivot makes say 60 turns per min, fric will consume 1881.6 × 60 = 112896 ft-lbs of powe per min. A horse-power is 33000 ft-lbs per min; consequently fric consumes $\frac{112896}{33000}$ = 3.42 horse powers of the prime-mover.

NOTE. The diam of the pivot should be as small as considerations of strength will admit; fo although the amount of fric remains the same whether the diam be large or small, yet its resistanc to motion increases with the greater leverage of the larger one.

When the pres on a pivot does not exceed 150 lbs per sc inch for cast iron, or 300 lbs for steel, with good lubrication, it will revolv rapidly in machinery for a long time, without sensible wear. A flat base or foot is the best for pivot. It should be kept well lubricated, and free from dust or grit.

Art. 5. The friction of the journals of axles, gudgeons, o trunnions, in their boxes or bearings, is a species of sliding fric; yet somewha distinct from the foregoing. It bears a less proportion to the pres than in the case of flat surfaces Neither the fric itself, nor the power reqd to overcome it, is affected by the *length* of the journal i its bearings; but its resistance to motion increases with its diam: and in the same proportion; be cause the leverage with which the fric resists motion, is as the diam. Therefore, it becomes impo tant to employ strong materials for journals, as well as for pivots, in order to reduce the diam as muc as possible.

To find the amount of power consumed by the fric of jour nals: Mult together, the weight or the pres sustained by the journals, in pounds the corresponding number taken from the *following* table; and the circumf of the journal *in fee* The prod will be the loss of power at each rev, expressed in ft-lbs.

Ex. A *pres* of 22400 pounds is sustained by two journals of cast iron, 6 ins diam, and runnin in cast-iron bearings well lubricated with lard. How much power of the prime mover is consume by fric, at each rev?

Here, we have the pres on the journals, 22400 lbs; the corresponding number from the followin table, .054; and the circumf of the journals 18.84 ins, or 1.57 ft; consequently,

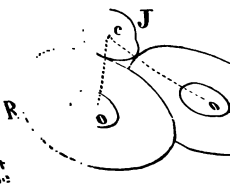
Power consumed = 22400 × .054 × 1.57 = 1899 ft-lbs per rev.

A horse-power is 33000 ft-lbs per min; hence, if the journals are meant to revolve say 60 times pe min, their fric will every min consume 1899 × 60 = 113940 ft-lbs of the power of the prime mover; o $\frac{113940}{33000}$ = 3.45 horse-powers; one-half of it by each journal.

Table of journal, or axle friction.

For the boxes of journals, cast iron perfectly smoothed, is as good a material for light pressure as any, *if kept constantly well oiled;* but is bad if not, as it then cuts the journal rapidly. Brass or bronze is much used. Being softer than cast iron it does not cut so much when badly oiled; but is less durable. Babbitt or other soft metals are useful under great pressures.	Dry; or a little greasy.	Greasy and wet.	Greased and wet.	Oil, Tallow, or Lard. In the usual way; or at intervals.	Oil, Tallow, or Lard. Continuously.	Very soft, purified carriage-grease.	Hog's lard and Plumbago.
Bell-metal on bell metal	.....	.....	.....	.097			
Cast-iron on bell-metal	.194	.161	.....	.075	.054	[illegible]	
Wrought-iron on bell-metal	.251	.189	.....	.075	.054	[illegible]	.111
Wrought-iron on cast iron	.....	.....	.....	.075	.054		
Cast-iron on cast iron	.....	.137	.079	.075	.054		
Wrought iron on lignumvitæ	.188	.....	.....	.125			
Cast iron on lignumvitæ	.185	.....	.....	.100	[illegible]	.....	.109
Lignumvitæ on cast iron	.....	.....	.....	.116			
Lignumvitæ on lignumvitæ	.....	.....	.....	.....	[illegible]		
Cast iron on brass	.190	.160	.....	.075	[illegible]		
Wrought iron on brass	.250	.190	.....	.07[illegible]			
Brass on brass	.....	.....	.....	.1[illegible]			
Brass on cast iron	.....	.....	.....	...			

Art. 6. Friction rollers. If a journal J, instead of revolving on ordinary bearings, be supported on friction rollers R, R, its fric will be reduced in nearly the same *proportion that the diam of the axle o or o of the rollers, is less than the diam of the rollers themselves.*



Thus, if the fric of J when in ordinary bearings be 1000 pounds, it will, if placed on rollers 12 ins diam, revolving on axles 3 ins diam, be reduced almost to 250 pounds, or to one-fourth, or as 12 is to 3.

When more exactness is reqd, find the angle o c o in degrees and minutes; and div it by 2. From the Table of Nat Sines, &c, take the nat cosine of this half-angle. Mult this cos by the diam of a roller. Div the diam of the axle

of a roller, by the prod just obtained. Finally mult the fric of the journal J in a common bearing by the quot. The prod will be the fric of J when placed on the rollers.

Ex. Let the fric of J in ordinary bearings be 1000 lbs; the diam of *o* or *o*, 3 ins; the diam of R, 12 ins; and the angle *o c o*, 20°.

Then, the half-angle will be 10°, and its nat cos .9848. And .9848 × 12 = 11.8176. And $\frac{3}{11.8176}$ = .254. And, finally, 1000 × .254 = 254 pounds, the fric of J on the rollers.

Art. 7. Rolling friction is that which takes place where the circumf of a wheel, or of any rolling body, comes in contact with the surf on which it rolls. It appears to follow the same law that applies to sliding friction, viz, that so long as abrasion does not take place, it increases as the pres. It is also *inversely* as the diams. In practice there are usually two ways of applying the force reqd to overcome rolling fric. The first is at the *axis* of the rolling body; as the force of a horse is applied at the axis of a wheel in a wagon; or of a man, at that of a wheelbarrow. The second is at the *circumf* of the roller; as when workmen push along a heavy timber laid on top of two or more rollers; or as the ends of an iron bridge play backward and forward by contraction and expansion, on top of rollers, or balls of metal. The fric is much less in the second case, than in the first; for although in the second there are two rolling frics to be overcome, (one at the top, and one at the bottom of the roller,) yet these two are much less than the one rolling, and the one axle fric of the first case. The few experiments that have been made on the coeffs of rolling fric, disconnected from axle fric, have been too incomplete to serve as a basis for practical rules.

Art. 8. Rolling, and axle friction combined; as in railroad cars. A body will not slide, or roll down an inclined plane, unless the plane is so steep that the sliding force of grav is sufficient to overcome the sliding or rolling fric (as the case may be) of the body. See Art 62, of Force in Rigid Bodies. The fric is in proportion to the weight, or rather to the *pres*, of the body; and the sliding force of gravity is in proportion to the height of the plane as compared with its hor length or base. Therefore, if the height of the plane has to be $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{100}$, &c, of its base, before the body begins to slide, or to roll down it, we know that the fric of the body is $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{100}$, &c, of its weight; or, more correctly speaking, of its *pres* on the plane. This pres is equal to the weight only when the plane is level, or hor; and becomes less than the weight, as the plane becomes steeper; but the diff is so slight on moderate railroad grades, that it may be neglected in such cases as that now before us. See Table, p 486, Force in Rigid Bodies.

It is found that railroad cars with wheels and journals of the ordinary diams, (about 28 to 32 ins for wheels, and about 3 ins for journals,) begin to roll down a grade when it is as steep as from 16 to 24 ft per mile. This diff is owing to a varying condition of the rails as to smoothness, dust, and irregularities; and to diff proportions between the diams of the wheels and journals; the kind of springs; the kind and quantity of oil used in lubricating; and to other minor considerations. There are 5280 ft in a mile; therefore, 16 and 24 ft per mile give for the height of the plane in proportion to its base, $\frac{5280}{16}$ = 1 to 330; and $\frac{5280}{24}$ = 1 to 220. Therefore, the combined rolling and axle fric of the car vary from $\frac{1}{330}$ to $\frac{1}{220}$ part of its weight; or since there are 2240 lbs in a ton, it varies from $\frac{2240}{330}$ to $\frac{2240}{220}$ = 6.8 to 10.2 lbs per ton weight of car; and the coeff of its fric varies from $\frac{1}{330}$ or .00303, to $\frac{1}{220}$ or .00454. Perhaps 8½ lbs per ton, or $\frac{1}{264}$ of the weight, (or .00379,) which corresponds to $\frac{5280}{264}$ = 20 ft grade per mile, (or to an inclination of 13 mins,) may in practice be assumed as the average on American railroads; and that about 3½ lbs of this may be ascribed to rolling; and 5 lbs to axle or journal friction.

Under the same circumstances, this fric, and its coeff, remain unchanged at all vels: that is, if the fric is 8½ lbs per ton at a vel of 1 mile per hour, it will be 8½ lbs per ton at 60 miles per hour; and precisely the same amount of it will be generated at *each rev* of the wheel in either case; and the same amount of motive power will in either case be required to overcome it *during one rev*. This being the case, it may at first sight appear paradoxical that the motive power or traction reqd to overcome the fric must increase with the vel. But it is evident that this follows from the fact that the fric at *each* rev requires the same amount of power to overcome it, at whatever vel; consequently, since at 60 miles an hour there are 60 revs in the same time that there is 1 at 1 mile an hour, there must be 60 times more power continuously applied during the high vel than during the low one. Still the *total amount* of fric, and also the *total amount* of power called into action in travelling any given *dist*, will be the same at all vels; for whether the car travels 60 miles in 1 hour, or in 60 hours, its wheels make the same number of revs, and generate the same total amount of friction. At 1 mile per hour, we apply our continuous power in a small stream, but for a long time; and at 60 miles per hour, we apply it in a stream 60 times larger, but for only one 60th as long a time. The engine will expend the same total amount of power against fric during a 60-mile trip at 1 mile an hour, as during the same 60 miles at 60 miles an hour; but still, an engine to perform the 60 miles in 1 hour, must be 60 times as powerful as one that can barely do 1 mile per hour; in other words, the fast engine must be able to apply 60 times as much force *at any one instant*. The same principle applies to the fric of pivots, bands, &c, at diff vels. The effect of *grades* is not here considered.

In them all, the *coeff* of fric remains unchanged at all vels; but the **quantity** *of fric to be overcome* **in a given time**, of course varies as the vels; and, therefore, as in the case of the cars, greater vel requires greater power during the *time* of motion.*

When a train moves rapidly, other resistances are generated; such as that of the air, (which is almost inappreciable at low speeds;) jolts against irregularities of the rails; fric against the rails caused by the cars swaying from side to side of the track, &c. These, however, do not affect the fric *just spoken of*.

A double purchase crane, with a weight of 7000 lbs suspended from it, showed a fric of $\frac{1}{9}$ the weight.

* *The young* student should reflect well on this subject, and familiarize himself with the broad distinction between the **unvarying coeff** of fric, and the **varying amount** of fric at diff vels.

or nearly 800 lbs. One ton suspended at each end of a chain passing over 2 cast-iron sheaves of 2 feet diam; with wrought-iron journals, working in brass bearings, well oiled, gave $\frac{1}{14}$ of the weight; or $\frac{2 \text{ tons}}{14} = 320$ lbs; or 160 lbs per ton. The fric of an unloaded locomotive, is about 12 lbs per ton of its weight; with a train attached, this is increased about 1 lb per ton of train.

Morin says the fric of a sled on dry ground is ¾ of the pres. Babbage states that a block of stone of 1090 lbs was drawn along a rock surface by 755 lbs; or fric 70 per ct; on a wooden sled on a wooden floor, 60 per cent; with both wooden surfaces greased, only 6 per cent; and with the block on top of wooden rollers 3 ins diam, only 2.6 per cent.

TRACTION.

Traction on common roads, and canals; or the power reqd to draw vehicles and boats along them. In connection with this subject read Art 9 of the preceding one; also Hauling by Horses.

The following table shows tolerable approximations to the force in lbs per ton, reqd to draw a stage coach and passengers, up ascents on the Holyhead turnpike road in England, (a fine road,) by horses; as ascertained by means of a dynamometer. The entire weight was 1½ tons; but in the table, the results are given per single ton. From the nature of such cases, no great accuracy is attainable.

Proportional Ascent.	Ascent in Ft. per Mile.	At 4 Miles per Hour.	At 6 Miles per Hour.	At 8 Miles per Hour.	At 10 Miles per Hour.
		Lbs.	Lbs.	Lbs.	Lbs.
1 in 15½	340.7	210	216	225	240
1 " 20	264.	196	202	212	229
1 " 26	203.1	155	160	166	175
1 " 30	176.	137	142	147	154
1 " 40	132.	114	120	124	130
1 " 64	82.5	109	115	120	126
1 " 118	44.7	102	107	113	120
1 " 138	38.3	99	103	109	117
1 " 156	33.9	98	101	106	112
1 " 245	21.6	93	96	101	107
1 " 600	8.8	81	85	91	96
Level.	0.	76	80	85	91

The following results, most of them with the same instrument, are also in lbs per ton; with a four-wheeled wagon, at a slow pace, on a level; and the roads in fair condition.

On a cubical block pavement..... 32 lbs per ton.
" McAdam road, of small broken stone..... 62 " " " probably to 75.
" gravel road..... 140 " " "
" Telford road, of small stone on a paving of spawls 46 " " " " " 75.
" broken stone, on a bed of cement concrete..... 46 " " " " " 75.
" common earth roads..... 200 to 300. On a plank road 30, to 50 lbs.

The tractive power of a horse diminishes as his speed increases; and perhaps, within certain limits, say from ¾ to four miles per hour, nearly in inverse proportion to it. Thus, the average traction of a horse, *on a level*, and actually pulling for 10 hours in the day, may be assumed approximately as follows:

Miles per hour.	Lbs. Traction.	Miles per hour.	Lbs. Traction.
¾.....	333.33	2¼.....	111.11
1	250.	2½.....	100.
1¼.....	200.	2¾.....	90.91
1½.....	166.66	3	83.33
1¾.....	142.86	3½.....	71.43
2	125.	4	62.50

If he works for a smaller number of hours, his traction may increase as the hours diminish; down to about 5 hours per day and for speeds of about from 1¼ to 3 miles per hour. Thus, for 5 hours per day his traction at 2½ miles per hour will be 200 lbs, &c. When ascending a hill, his power diminishes so rapidly, from having partially to raise his own weight, (which averages about 1000 to 1100 lbs,) that up a slope of 5 to 1, he can barely struggle along without any load. On such an ascent, (see table, p 486, of Force in Rigid Bodies,) he must exert a force equal to 439 lbs per ton, or of 196 lbs for the 1000 lbs of his own weight. Assuming that on a level piece of good turnpike, he would when hauling a cart and load, together weighing 1 ton, have to exert a traction of 60 lbs; then on ascending a hill of 4° inclination, (or 1 in 14.3; or 369¼ ft per mile,) he would have to exert 156 lbs, against the gravity of the 1 ton: and 67 lbs, against that of his own weight; or 223 lbs in all. He may, for a few mins, exert without injury, about twice his regular traction. This calculation shows that up a hill of 4°, an *average horse is fully tasked in drawing a total load of one ton*; and should, therefore, be *allowed, in such a case, to choose his own gait*; and to rest at short intervals. A fair load for a strong *horse with a cart, at a variable walking pace, working 10 hours per day, on a common unfinished road in good order, is about half a ton*, in addition to the cart, which will be about half a ton. *With two horses to this same cart, the load alone may be about 1½ tons.*

Rem. *Since the action of gravity is the same on good roads and bad ones, it follows that ascents become more objectionable the better the roads.* Thus, on an ascent of 2°, or 184.4 ft per mile, gravity alone requires a traction of 78.7

which is about 10 times that on a level railroad at 6 miles an hour; but only about equal to that on a level common turnpike road, at the same speed. Therefore, (to speak somewhat at random,) it would require 10 locomotives instead of 1; but only 2 horses instead of 1. A grade of 1 in 35; or 150 ft to a mile; or 1° 38', is about the steepest that permits horses to be driven down a hard smooth road, in a fast trot, without danger. It should, therefore, not be exceeded except when absolutely necessary, especially on turnpikes.

On canals and other waters, the liquid is the resisting medium that takes the place of friction on level roads. But unlike friction, its resistance varies as the squares of the vels; (see Art 26 of page 571,) at least from the vel of 2 ft per sec, or 1.364 miles per hour; to that of 11½ ft per sec, or 7.84 m per h. As the speed falls below 1½ m per h, the resistance varies less and less rapidly; and this is the case whether the moved body floats partly above the surface; or is entirely immersed. In towing along stagnant canals, &c, the vel is usually from 1 to 2½ m per h; for freight most frequently from 1½ to 2. Less force is required to tow a boat at say 2 m per h, where there is no current, than at say 1¼ m per h, against a current of ¾ m per h, because in the last case the boat has to be *lifted up* the very gradual inclined plane or slope which produces the current. Therefore, a steamboat which has its speed increased say 3 or 4 m per h by a descending current, will have it retarded more than 3 or 4 m per h when returning against the current.

The force required to tow a boat along a canal depends *greatly* upon the comparative transverse sectional areas of the channel, and of the immersed portion of the boat. When the width of a canal at water-line is at least 4 times that of the boat; and the area of its transverse section as great as at least 6½ times that of the *immersed* transverse section of the boat, the towing at usual canal vels will be about as easy as in wider and deeper water. With less dimensions, it becomes more difficult. (D'Aubuisson.) Much also depends on the shape of the bow and other parts of the boat; and on the proportion of its length to its breadth and depth. Hence it is seen that the mere weight of the load is by no means so controlling an element as it is on land. The whole subject, however, is too intricate to be treated of here. Morin states that naval constructors estimate the resistance to sailing and steam vessels at sea, at but from about .5 to .7 of a ℔ for every sq ft of immersed transverse section, when the vel is 3 ft per sec, or 2.046 miles per hour. It is far greater on canals.

On the Schuylkill Navigation of Pennsylvania, of mixed canal and slackwater, for 108 miles, the regular load for 3 horses or mules, is a boat of very full build; and to keel: 100 ft long, 17½ ft beam; and 8 ft depth of hold; drawing 5½ ft when loaded.* Weight of boat about 65 tons; load 175 tons of coal, (2240 ℔s;) total weight 240 tons, or 80 tons per horse or mule. On the down trip with the loaded boats, for 4 days, the animals are at work, *actually towing,* (except at the locks,) for 18 hours out of the 24; thus exceeding by far the limits of time usually allowed for continuous effort.

On the canal sections, (which have 60 ft water-line; and 6 ft depth,) the speed is 1¾ miles per hour; and on the deep wide pools, 2 miles.

On the up trip with the empty 65-ton boats, the average speed is about 2½ miles per hour. The empty boats draw 16 to 18 ins water; and frequently keep on without stopping to rest day or night through the entire distance of 108 miles. The animals generally have 2 or 3 days' rest at each end of the trip; but are materially deteriorated at the end of the boating season.

If our preceding assumption of 143 ℔s traction of a horse at 1¾ miles per hour, is correct, the traction of the loaded boats on the canal sections is $\frac{143 \text{ ℔s}}{80 \text{ tons}} = 1.83$ ℔s per ton.

The intelligent engineer and superintendent of the Sch Nav, James F Smith, gives as the results of his own extensive observation, that one of these large boats loaded (240 tons in all) may, without distressing the animals, be drawn along the canal sections, for 10 hours per day, as follows: By one average horse or mule, at the rate of 1 mile; by two animals, at 1½ miles; and by three, at 1¾ miles per hour. When four animals are used the gain of time is very trifling. At a time of rivalry among the boatmen, one of them used 8 horses; but with these could not exceed 2½ miles per hour in the canal portions. Two or more horses together cannot for hours pull as much as when working separately.

If our preceding short table of the traction of a horse at diff vels for 10 hours is correct, then the traction of the above loaded coal boats (240 tons) on the canal sections of the navigation, is as follows: The last column shows the traction in ℔s per sq ft of area of immersed transverse section where largest; viz, about 95 sq ft.

No. of Horses.	Miles per Hour.		Traction in Lbs. per Ton.	Traction in ℔s per Sq Ft of immersed sec.
1	1	$\frac{250}{240}$	1.04	2.63
2	1½	$\frac{333}{240}$	1.39	3.50
3	1¾	$\frac{428}{240}$	1.78	4.50
3 on pools	2	$\frac{375}{240}$	1.56	3.95
8	2½	$\frac{800}{240}$	3.33	8.42
3 up-trip	2½	$\frac{300}{65}$	4.61	12.50

On the Lachine canal in Canada, 120 ft wide at water-line; 80 ft at bottom; depth on mitre sills 9 ft; 6 horses tow loaded schooners with ease.

Before the enlargement of the **Erie canal,**† its dimensions were 40 ft water-line; 28 ft bottom; 4 ft depth of water. The average weight of the boats was about 30 tons. With 75 tons of load, or 105 tons total, they were towed by 2 horses, at the rate of about 2 miles per hour; which by our table gives a traction of nearly 2.4 ℔s per ton. The boats were about 80 ft long; 14 ft beam; full 3½ ft draught *loaded; hence the* traction by our table would be about 5.7 ℔s per sq ft of immersed transverse section.

* *The cost of these* boats in 1872 averages about $2500; and their repairs about $150 per annum. *They last from* 9 *to* 12 *years.* Before the canal was enlarged, boats weighing about 22 tons, and carry*ing 60 tons of* coal, were used. Length 70 ft; beam 13; draft empty, about 1 foot; when loaded, 4 ft; *cost $700. These* are approximately the dimensions of the freight boats, on our ordinary canals. *With 2 horses,* speed 1¾ miles.

† *Length* 363 miles; cost $19680 per mile. The enlarged canal has 70 ft water-line; and 7 ft of w r; *and cost* $90800 per mile for the enlargement only. The cost of the several canals in Pennsylv a *has ranged* between $23000 and $50000 per mile.

for the 82-ton loaded boats in the footnote, on the smaller canal, (the boats nearly touch-
ttom,) the traction at 1¾ miles, would be 3½ lbs per ton; or about twice as great as the above
s. It also would be 5.7 lbs per sq ft of immersed section.

action on a level straight railroad, at speeds not exceeding about
s per hour, is from 7 to 10 lbs per ton, depending on diam of wheels and journals; lubrication
t 25 miles an hour, 12 to 15 lbs per ton.

ANIMAL POWER.

t. 1. So far as regards horses, this subject has been partially considered
the preceding head, Traction. All estimates on this subject must to a certain extent be vague,
to the diff strengths and speeds of animals of the same kind; as well as to the extent of their
g to any particular kind of work. Authorities on the subject differ widely; and sometimes
s themselves in a loose manner that throws doubt on their meaning. We believe, however,
e following will be found to be as close approximation to practical averages as the nature of
se admits of with our present imperfect knowledge. We suppose a good average trained horse,
ng not less than about ½ a ton, well fed and treated. Such a one, when actually walking for
rs a day, at the rate of 2½ miles per hour, on a good *level* road, such as the tow-path of a canal,
rcular horse-path,* **can exert a continuous pull, draught, power,
raction, of 100 lbs.**
, 2½ miles per hour, is 220 ft per min, or 3⅔ ft per sec; and since 10 hours contain 600 min,
y's work of actual hauling on a level, at that speed, amounts to

$$600 \text{ min} \times 220 \text{ ft} \times 100 \text{ lbs} = 13\,200\,000 \text{ ft-lbs per day.}$$

000 ft-lbs per min, or 366⅔ ft-lbs per sec.† Which means that he exerts force enough during the
lift 13 200 000 lbs 1 foot high; or 1 320 000 lbs 10 feet high; or 132 000 lbs 100 ft high, &c. He may
this force either in *traction* (hauling) or in *lifting* loads. If he has to raise a small load to a
height, the machinery through which he does it must be so geared as to gain speed, at the loss
(only but improperly so expressed) of power. Whether he lifts the great weight through a
height, or the small weight through a great height, he exerts precisely the same amount of
r power. In connection with this subject, the student should read Arts 5, 9, 11, &c, of Force
id Bodies. Also, see Hauling by Horses and Carts.
erience shows that within the limits of 5 and 10 hours per day, (the speed remaining the same,)
**draft of a horse may be increased in about the same pro-
tion as the time is diminished;** so that when working from 5 to 10 hours
y, it will be about as shown in the following table. Hence, the total amount of 13 200 000 ft-lbs
y may be accomplished, whether the horse is at work 5, 6, or 8, &c, hours per day.‡ This, of
, supposes him to be actually lifting or hauling *all the time;* and makes no allowance for stop-
for any purpose.

ble of draft of a horse, at 2½ miles per hour, on a level.

Hours per day.	Lbs.	Hours per day.	Lbs.
10	100	7	142⅞
9	111 1/9	6	166⅔
8	125	5	200

**xperience also shows that at speeds between ¾ and 4
es an hour, his force or draught will be inversely in pro-
tion to his speed.** Thus, at 2 miles an hour, for 10 hours of the day, his
t will be

miles		miles		lbs		lbs
2	:	2½	: :	100	:	125 draught.

¼ miles, it would be 166⅔ lbs; at 3 miles, 83⅓ lbs; and at 4 miles, 62½ lbs; as per table in
on.
refore, in this case also, the entire amount of his day's work remains the same; ‡ and within

o enable a horse to work with ease **in a circular horse-walk,** its diam
not be less than 25 ft; 30 or 35 would be still better.
nominal horse-power is 33000 ft-lbs per minute; this being the rate
ed by Boulton and Watt in selling their engines; so that purchasers wishing to substitute
for horses, should not be disappointed. Their assumption can be carried out by a very strong
day after day for 8 or 10 hours; but as the engine can work day and night for months without
ng, which a horse cannot, it is plain that a one-horse engine can do much more work than any
ch horse. Hence many object to the term horse-power as applied to engines; but since every-
nderstands its plain meaning, and such a term is convenient, it is not in fact objectionable.
n and Watt meant that a one-horse engine would at any moment perform the work of a very
horse. An *average* horse will do but 22000 ft-lbs per min.
is plain that although the *day's* labor will be the same, that of an hour, or of a min, will vary
he number of hours taken as a day's work. It must be remembered that a *working day* of a
number of hours, by no means implies, in every case, that number of hours of *actual work;*
cludes intermissions and rests.
**his remark about speed will not apply to loads towed
ough the water.** Thus, if his draught at 2 miles an hour be 125 lbs; and
iles, 62½ lbs; he will on land draw loads in these proportions; but in hauling a boat through
ter at the greater speed, he has to encounter the increased resistance of the water used, which
ice at 4 miles is much more than twice as great as at 2 miles; probably 4 times as great
re, at 4 miles on a canal, his draught of 62½ lbs would not suffice for a load half as great
tow with his draft of 125 lbs at 2 miles.

all the foregoing limits of hours and speed, may be practically taken to be about 13 200 000 ft-lbs per day; or 22000 ft-lbs per min of a day of 10 hours. But it does not follow that the horse can always in practice actually *lift loads* at that rate; because generally a part of his power is expended in overcoming the *friction* of the machinery which he puts in motion; and moreover, the nature of the work may require him to stop frequently; so that in a *working day* of 8 or 10 hours, the horse may not actually be *at work* more than 5, 6, or 7 hours.

As a rough approximation, to allow for the waste of force in overcoming the friction of hoisting machinery, and the weight of the hoisting chains, buckets, &c, we may say that **the useful or paying daily net work of a horse, in hoisting by a common gin,** is about 10 000 000 ft-lbs. That is, he will raise equivalent to 10 000 000 lbs net of water, or ore, &c, 1 foot. The load which he can raise at once, including chains, bucket, and an allowance for friction, will be as much greater than his own direct force, as the diam of the horse walk is greater than that of the winding drum; and it will move that much slower than he does. His own direct force will vary according to the number of hours per day that he may be required to work, as in the foregoing table. With these data, the size of the buckets can be decided on; and of these there should be at least two, so that the empty one at the bottom may be filled while the full one at top is being emptied; so as to save time. The same when the work is done by men.

Art. 2. A practised laborer hauling along a level road, by a rope over his shoulders; or in a circular path, pushing before him a hoe lever, at a speed of from 1½ to 3 miles per hour, exerts about ⅙ lb part as much force as a horse; or 2 200 000 ft-lbs per day; or 3666⅔ ft-lbs per min of a day of 10 hours of actual hauling or pushing.

But laborers frequently have to work under circumstances less advantageous for the exertion of their force than when hauling or pushing in the manner just alluded to; and in such cases they cannot do as much per day. Thus in turning a winch or crank like that of a grindstone, or of a crane, the continual bending of the body, and motion of the arms, is more fatiguing. **The size of a winch should not exceed 18 ins,** or the rad of a circle of 3 ft diam; and against it a laborer can exert a force of about 16 lbs, at a vel of 2½ ft per sec, or 150 ft per min, making very nearly 16 turns per min; for 8 hours per day. To these 8 hours an addition must be made of about ⅓ part, for short rests. Or if a *working day* is taken at 8, or 10, &c, hours, ⅓ part must generally be taken from it for such rests. On the foregoing data an hour's *work* of 60 min of *actual hoisting* would be

$$\overset{\text{lbs}}{16} \times \overset{\text{ft}}{150} \times \overset{\text{min}}{60} = 144000 \text{ ft-lbs};$$

or, deducting ⅓ part for rests, 115200 ft-lbs per hour of *time, including rests.* In practice, however, a further deduction must be made for the fric of the machine, and for the wt of the hoisting chains; and in case of raising water, stone, ore, &c, from pits, for the wt of the buckets also. As a rough average we may assume that these will leave but 100 000 ft-lbs of paying, or useful work per hour; that is, that **a man at a winch will actually lift equivalent to 100000 lbs of water, ore, &c, 1 foot high per hour's time, including rests.** This is equal to 1666⅔ ft-lbs per min of a day of 10 hours, including rests. Therefore, in a day of 10 working hours he would raise 1 000 000 lbs net, 1 foot high; **or just $\frac{1}{10}$ part of what a horse would do with a gin** in the same time. We have before seen that in hauling along a level road, he can at a slow pace perform about ⅙ of the daily duty of a horse. He may also work the winch with greater force, say up to 30 or even 40 lbs; but he will do it at a proportionately slower rate; thus, accomplishing only the same daily duty. **With a gin, like those for horses,** but lighter, with 2 or more buckets, a practised laborer will in a working day of 10 hours, raise from 1 200 000 to 1 400 000 ft-lbs net of water, ore, &c. With a shallow well or pit, more time is lost in emptying buckets than in a deep one; but the deep one will require a greater wt of rope. To save time in all such operations on a large scale, there should be at least two buckets; the empty one to be filled while the full one is being emptied. It is also best to employ 2 or more men to hoist at the same time, by winches, at both ends of the axis; and the men will work with more ease if the winches are at right angles to each other. Each winch handle may be long enough for 2 or 3 men. An extra man should be employed to empty the buckets. He may take turns with the hoisters. The same remarks apply in some of the following cases.

On a treadwheel a practised laborer will do about 40 per cent more daily duty than at a winch; or in a working day* of 10 hours, including rests, he will do about 1 400 000 ft-lbs. And he can do this whether he works at the outer circumf of the wheel, stepping upon foot-boards, or tread-boards, on a level with its axis; or walks inside of it near its bottom. In both cases he acts by his wt, usually about 130 to 140 lbs, and not by the muscular strength of his arms. When at the level of the axis, his wt acts more directly than when he walks on the bottom of the wheel; but in the first case he has to perform a slow and fatiguing duty resembling that of walking up a continuous flight of steps; while in the second he has as it were merely to ascend a very slightly inclined plane; which he can do much more rapidly for hours, with comparatively little fatigue; and this rapidity compensates for the less direct action of his wt. Therefore, in either case, as experience has shown, he accomplishes about the same amount of daily duty. Treadwheels may be from 5 to 8 ft in diam, according to the nature of the work. They are generally worked by several men at once; and may at times be advantageously used in pile-driving, as well as in hoisting water, stone, &c.

By a good common pump, properly proportioned, a practised laborer will in a day of 10 working hours, raise about 1 000 000 ft-lbs of water, net.†

Bailing with a light bucket or scoop, he can accomplish about 200 000 ft lbs net of water. **By a bucket and swape,** (a long lever rocking vertically; *and weighted at one end so as to balance the full bucket hung from the other*; often seen at country

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* *The working day must be understood to include necessary rests, and such intermissions as the nature of the work demands*; but does not include time lost at meals. A working day of 10 hours *may, therefore,* have but 8, 7, or 6, &c hours of actual labor. This will be understood when we here *after speak of a working day, or simply a day.*

† *Dessagulier's estimates of daily work of men and horses exceed the above, but are entirely too great*

wells,) 600 000 to 800 000. In the last he has only to pull down the empty bucket, and thereby raise the counterweight. **By 2 buckets at the ends of a rope suspended over a pulley,** 500 000 to 600 000. Here he works the buckets by pulling the rope by hand.

By a tympan, or tympanum,* worked by a treadwheel, about 1 200 000 to 1 400 000.

By a Persian wheel,† a chain-pump, a chain of buckets,‡ or an Archimedes screw, all worked by a treadwheel, from 800 000 to 1 000 000 ft-lbs. Of these four, the first three lose useful effect by either spilling, leaking, or the necessity for raising the water to a level somewhat higher than that at which it is discharged.

When any of the five foregoing machines are worked by men at winches, the result will be about ⅓ less than by treadwheels. They are all frequently worked also by either steam, water, or horse-power.

By walking backward and forward, on a lever which rocks on its center, a man may, according to Robison's Mech Philosophy, perform a much greater duty than by any of the preceding modes. He states that a young man weighing 135 lbs, and loaded with 30 lbs in addition, worked in this manner for 10 hours a day without fatigue; and raised 9¼ cubic feet of water, 11½ ft high per min. This is equal to 3 984 000 ft-lbs per day of 10 hours; or 6640 ft-lbs per min; or nearly $\frac{4}{10}$ of the net daily work of a horse in a gin.

A laborer standing still, can barely sustain for a few min, a load of 100 lbs, by a rope over his shoulder, and thence passing off hor over a pulley. And scarcely as much, when (facing the load and pulley) he holds the end of a hor rope with his hands before him. He cannot push hor with his hands at the height of his shoulders, with more than about 30 lbs force.

Weisbach states from his own observation, that 4 practised men raised a **dolly** (a wooden beetle or rammer, of wood; with 4 hor projecting round bars for handles; weighing 120 lbs, 4 ft high, at the rate of 34 times per min, for 4½ min; and then rested for 4½ min; and so on alternately through the 10 hours of their working day. Therefore, 5 of these hours were lost in rests; and the duty performed by each man during the other 5 hours, or 300 mins, was

$$\frac{120 \times 4 \times 34 \times 300}{4} = 1\,224\,000 \text{ ft-lbs.}$$

In the old mode of driving piles, where the ram of 400 to 1200 lbs suspended from a pulley, was raised by 10 to 40 men pulling at separate cords, from 35 to 40 lbs of the ram were allotted to each man, to be lifted from 12 to 18 times per min, to a height of 3¾ to 4¾ feet each time, for about 3 min at a spell, and then 3 min rest. It was very laborious; and the gangs had to be changed about hourly, after performing but ½ an hour's actual labor.

Hauling by horses. See Traction. When working all day, say 10 working hours, the average rate at which a horse walks while hauling a full load, and while returning with the empty vehicle, is about 2 to 2¼ miles per hour; but to allow for stoppages to rest, &c, it is safest to take it at but about 1.8 miles per hour, or 160 ft per min. The time lost on each trip, in loading and unloading, may usually be taken at about 15 min. Therefore, to find the number of loads that can be hauled to any given dist in a day, first find the time in min reqd in hauling one load, and returning empty. Thus: div twice the dist in ft to which the load is to be hauled; or in other words, div the length in ft, of the round trip, by 160 ft. The quot is the number of min that the horse is in motion during each round trip. To this quot add 15 min lost each trip while loading and unloading; the sum is the total time in min occupied by each round trip. Div the number of min in a working day (600 min in a day of 10 working hours) by this number of min reqd for each trip; the quot will be the number of trips, or of loads hauled per day.

Ex. How many loads will a horse haul to a dist of 960 ft, in a day of 10 working hours, or 600 min? Here, $960 \times 2 = 1920$ ft of round trip at each load. And $\frac{1920}{160} = 12$ min, occupied in walking. And $12 + 15$ in loading, &c) $= 27$ min reqd for each load. Finally, $\frac{600}{27} = \frac{\text{min in 10 hours}}{\text{min per trip}} = 22.2$, or say 22 trips; or loads hauled per day.

Table of number of loads hauled per day of 10 working hours. The first col is the distance to which the load is actually hauled; or half the length of the round trip. The cost of hauling per load, is supposed to be for one-horse carts; the driver doing the loading and unloading; rating the expense of horse, cart, and driver at $2 per day. See Cost of Earthwork, page 437.

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* The tympan revolves on a hor shaft; and is a kind of large wheel, the spokes, arms, or radii of which are gutters, troughs, or pipes, which at their outer ends terminate in scoops, which dip into the water. As the water is gradually raised, it flows along the arms of the wheel to its axis, where it is dischd. The scoop wheel is a modification of it. It is an admirable machine for raising large quantities of water to moderate heights. We cannot go into any detail respecting this and other hydraulic machines.

† A kind of large wheel with buckets or pots at the ends of its radiating arms; revolves on a hor axis; discharges at top. The buckets are attached loosely, so as to hang vert, and thus avoid spilling until they arrive at the proper point, where they come into contact with a contrivance for tilting and emptying them. The noria is similar, except that the buckets are firmly held in place, and thus spill much water. It is therefore inferior to the Persian wheel.

‡ An endless revolving vert chain of buckets. D'Aubuisson and some others erroneously call this the noria. It is an effective machine.

Dist. Feet.	No. of Loads.	Cost per Load.	Dist. Feet.	No. of Loads.	Cost per Load.	Dist. Miles.	No. of Loads.	Cost per Load.
		Cts.			Cts.			Cts.
50	38	5.26	1500	18	11.11	1	7	28.57
100	37	5.41	2000	15	13.33	1¼	6	33.33
200	34	5.88	2500	13	15.39	1½	5	40.00
300	32	6.25	3000	11	18.18	2	4	50.00
400	30	6.67	3500	10	20.00	3	3	66.67
600	27	7.41	4000	9	22.22	4	2	100.00
1000	22	9.09	5000	7	28.57	9	1	200.00

If the loading and unloading is such as cannot be done by the driver alone; but requires the help of cranes, or other machinery, an addition of from 10 to 50 cts per load may become necessary. Hauling can generally be more cheaply done by using 2 or 3 horses, and one driver, to a vehicle. The net load per horse, in addition to the vehicle, will usually be from ½ to 1 ton, depending on the condition, and grades of the road. From 13 to 15 cub ft of solid stone; or from 23 to 27 cub feet of broken stone, make 1 ton. **In estimating for hauling rough quarry stone for drains, culverts, &c,** bear in mind that each cub yard of common scabbled rubble masonry, requires the hauling of about 1.2 cub yds of the stone as usually piled up for sale in the quarry; or about ⅔ of a cub yd of the original rock in place. **A cub yd of solid stone, when broken into pieces, usually occupies about 1.9 cub yds perfectly loose;** or about 1⅓ when piled up. A strong cart for stone hauling, will weigh about ⅔ ton; or 1500 lbs; and will hold stone enough for a perch of rubble masonry; or say 1.2 pers of the rough stone in piles. The average weight of a good working horse is about ½ a ton.

Morin gives the following results from careful experiments made by him for the French Government. The draft of the same wheeled vehicle on a road, may in practice be considered to be,

1st. On hard turnpikes, and pavements; in proportion to the loads; inversely as the diams of the wheels; and nearly independent of the width of tire. It increases to uncertain extents with the inequalities of the road; the stiffness (want of spring) of the vehicle; and the speed; (considerably less than as the square roots of the last.)

2d. On soft roads, the draft is less with wide tires than with narrower ones; and for farming purposes he recommends a width of 4 ins. With speeds from a walk to a fast trot, the draft does not vary sensibly.

CHORDS TO A RADIUS 1. (See p 147.)

M.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	M.
0'	.0000	.0175	.0349	.0524	.0698	.0872	.1047	.1221	.1395	.1569	.1743	0'
2	.0006	.0180	.0355	.0529	.0704	.0878	.1053	.1227	.1401	.1575	.1749	2
4	.0012	.0186	.0361	.0535	.0710	.0884	.1058	.1233	.1407	.1581	.1755	4
6	.0017	.0192	.0366	.0541	.0715	.0890	.1064	.1238	.1413	.1587	.1761	6
8	.0023	.0198	.0372	.0547	.0721	.0896	.1070	.1244	.1418	.1592	.1766	8
10	.0029	.0204	.0378	.0553	.0727	.0901	.1076	.1250	.1424	.1598	.1772	10
12	.0035	.0209	.0384	.0558	.0733	.0907	.1082	.1256	.1430	.1604	.1778	12
14	.0041	.0215	.0390	.0564	.0739	.0913	.1087	.1262	.1436	.1610	.1784	14
16	.0047	.0221	.0396	.0570	.0745	.0919	.1093	.1267	.1442	.1616	.1789	16
18	.0052	.0227	.0401	.0576	.0750	.0925	.1099	.1273	.1447	.1621	.1795	18
20	.0058	.0233	.0407	.0582	.0756	.0931	.1105	.1279	.1453	.1627	.1801	20
22	.0064	.0239	.0413	.0588	.0762	.0936	.1111	.1285	.1459	.1633	.1807	22
24	.0070	.0244	.0419	.0593	.0768	.0942	.1116	.1291	.1465	.1639	.1813	24
26	.0076	.0250	.0425	.0599	.0774	.0948	.1122	.1296	.1471	.1645	.1818	26
28	.0081	.0256	.0430	.0605	.0779	.0954	.1128	.1302	.1476	.1650	.1824	28
30	.0087	.0262	.0436	.0611	.0785	.0960	.1134	.1308	.1482	.1656	.1830	30
32	.0093	.0268	.0442	.0617	.0791	.0965	.1140	.1314	.1488	.1662	.1836	32
34	.0099	.0273	.0448	.0622	.0797	.0971	.1145	.1320	.1494	.1668	.1842	34
36	.0105	.0279	.0454	.0628	.0803	.0977	.1151	.1325	.1500	.1674	.1847	36
38	.0111	.0285	.0460	.0634	.0808	.0983	.1157	.1331	.1505	.1679	.1853	38
40	.0116	.0291	.0465	.0640	.0814	.0989	.1163	.1337	.1511	.1685	.1859	40
42	.0122	.0297	.0471	.0646	.0820	.0994	.1169	.1343	.1517	.1691	.1865	42
44	.0128	.0303	.0477	.0651	.0826	.1000	.1175	.1349	.1523	.1697	.1871	44
46	.0134	.0308	.0483	.0657	.0832	.1006	.1180	.1355	.1529	.1703	.1876	46
48	.0140	.0314	.0489	.0663	.0838	.1012	.1186	.1360	.1534	.1708	.1882	48
50	.0145	.0320	.0494	.0669	.0843	.1018	.1192	.1366	.1540	.1714	.1888	50
52	.0151	.0326	.0500	.0675	.0849	.1023	.1198	.1372	.1546	.1720	.1894	52
54	.0157	.0332	.0506	.0681	.0855	.1029	.1204	.1378	.1552	.1726	.1900	54
56	.0163	.0337	.0512	.0686	.0861	.1035	.1209	.1384	.1558	.1732	.1905	56
58	.0169	.0343	.0518	.0692	.0867	.1041	.1215	.1389	.1564	.1737	.1911	58
60	.0175	.0349	.0524	.0698	.0872	.1047	.1221	.1395	.1569	.1743	.1917	60

'able of Chords, in parts of a rad 1; for protracting—Continued.

r.	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	M.
0'	.1917	.2091	.2264	.2437	.2611	.2783	.2956	.3129	.3301	.3473	0'
2	.1923	.2096	.2270	.2443	.2616	.2789	.2962	.3134	.3307	.3479	2
4	.1928	.2102	.2276	.2449	.2622	.2795	.2968	.3140	.3312	.3484	4
6	.1934	.2108	.2281	.2455	.2628	.2801	.2973	.3146	.3318	.3490	6
8	.1940	.2114	.2287	.2460	.2634	.2807	.2979	.3152	.3324	.3496	8
0	.1946	.2119	.2293	.2466	.2639	.2812	.2985	.3157	.3330	.3502	10
2	.1952	.2125	.2299	.2472	.2645	.2818	.2991	.3163	.3335	.3507	12
4	.1957	.2131	.2305	.2478	.2651	.2824	.2996	.3169	.3341	.3513	14
6	.1963	.2137	.2310	.2484	.2657	.2830	.3002	.3175	.3347	.3519	16
8	.1969	.2143	.2316	.2489	.2662	.2835	.3008	.3180	.3353	.3525	18
0	.1975	.2148	.2322	.2495	.2668	.2841	.3014	.3186	.3358	.3530	20
2	.1981	.2154	.2328	.2501	.2674	.2847	.3019	.3192	.3364	.3536	22
4	.1986	.2160	.2333	.2507	.2680	.2853	.3025	.3198	.3370	.3542	24
6	.1992	.2166	.2339	.2512	.2686	.2858	.3031	.3203	.3376	.3547	26
8	.1998	.2172	.2345	.2518	.2691	.2864	.3037	.3209	.3381	.3553	28
0	.2004	.2177	.2351	.2524	.2697	.2870	.3042	.3215	.3387	.3559	30
2	.2010	.2183	.2357	.2530	.2703	.2876	.3048	.3221	.3393	.3565	32
4	.2015	.2189	.2362	.2536	.2709	.2881	.3054	.3226	.3398	.3570	34
6	.2021	.2195	.2368	.2541	.2714	.2887	.3060	.3232	.3404	.3576	36
8	.2027	.2200	.2374	.2547	.2720	.2893	.3065	.3238	.3410	.3582	38
0	.2033	.2206	.2380	.2553	.2726	.2899	.3071	.3244	.3416	.3587	40
2	.2038	.2212	.2385	.2559	.2732	.2904	.3077	.3249	.3421	.3593	42
4	.2044	.2218	.2391	.2564	.2737	.2910	.3083	.3255	.3427	.3599	44
6	.2050	.2224	.2397	.2570	.2743	.2916	.3088	.3261	.3433	.3605	46
8	.2056	.2229	.2403	.2576	.2749	.2922	.3094	.3267	.3439	.3610	48
0	.2062	.2235	.2409	.2582	.2755	.2927	.3100	.3272	.3444	.3616	50
2	.2067	.2241	.2414	.2587	.2760	.2933	.3106	.3278	.3450	.3622	52
4	.2073	.2247	.2420	.2593	.2766	.2939	.3111	.3284	.3456	.3628	54
6	.2079	.2253	.2426	.2599	.2772	.2945	.3117	.3289	.3462	.3633	56
8	.2085	.2258	.2432	.2605	.2778	.2950	.3123	.3295	.3467	.3639	58
0	.2091	.2264	.2437	.2611	.2783	.2956	.3129	.3301	.3473	.3645	60

M.	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	M.
0'	.3645	.3816	.3987	.4158	.4329	.4499	.4669	.4838	.5008	.5176	0'
2	.3650	.3822	.3993	.4164	.4334	.4505	.4675	.4844	.5013	.5182	2
4	.3656	.3828	.3999	.4170	.4340	.4510	.4680	.4850	.5019	.5188	4
6	.3662	.3833	.4004	.4175	.4346	.4516	.4686	.4855	.5024	.5193	6
8	.3668	.3839	.4010	.4181	.4352	.4522	.4692	.4861	.5030	.5199	8
0	.3673	.3845	.4016	.4187	.4357	.4527	.4697	.4867	.5036	.5204	10
2	.3679	.3850	.4022	.4192	.4363	.4533	.4703	.4872	.5041	.5210	12
4	.3685	.3856	.4027	.4198	.4369	.4539	.4708	.4878	.5047	.5216	14
6	.3690	.3862	.4033	.4204	.4374	.4544	.4714	.4884	.5053	.5221	16
8	.3696	.3868	.4039	.4209	.4380	.4550	.4720	.4889	.5058	.5227	18
0	.3702	.3873	.4044	.4215	.4386	.4556	.4725	.4895	.5064	.5233	20
2	.3708	.3879	.4050	.4221	.4391	.4561	.4731	.4901	.5070	.5238	22
4	.3713	.3885	.4056	.4226	.4397	.4567	.4737	.4906	.5075	.5244	24
6	.3719	.3890	.4061	.4232	.4403	.4573	.4742	.4912	.5081	.5249	26
8	.3725	.3896	.4067	.4238	.4408	.4578	.4748	.4917	.5086	.5255	28
0	.3730	.3902	.4073	.4244	.4414	.4584	.4754	.4923	.5092	.5261	30
2	.3736	.3908	.4079	.4249	.4420	.4590	.4759	.4929	.5098	.5266	32
4	.3742	.3913	.4084	.4255	.4425	.4595	.4765	.4934	.5103	.5272	34
6	.3748	.3919	.4090	.4261	.4431	.4601	.4771	.4940	.5109	.5277	36
8	.3753	.3925	.4096	.4266	.4437	.4607	.4776	.4946	.5115	.5283	38
0	.3759	.3930	.4101	.4272	.4442	.4612	.4782	.4951	.5120	.5289	40
2	.3765	.3936	.4107	.4278	.4448	.4618	.4788	.4957	.5126	.5294	42
4	.3770	.3942	.4113	.4283	.4454	.4624	.4793	.4963	.5131	.5300	44
6	.3776	.3947	.4118	.4289	.4459	.4629	.4799	.4968	.5137	.5306	46
8	.3782	.3953	.4124	.4295	.4465	.4635	.4805	.4974	.5143	.5311	48
0	.3788	.3959	.4130	.4300	.4471	.4641	.4810	.4979	.5148	.5317	50
2	.3793	.3965	.4135	.4306	.4476	.4646	.4816	.4985	.5154	.5322	[illegible]
4	.3799	.3970	.4141	.4312	.4482	.4652	.4822	.4991	.5160	.5328	[illegible]
6	.3805	.3976	.4147	.4317	.4488	.4658	.4827	.4996	.5165	.5334	[illegible]
8	.3810	.3982	.4153	.4323	.4493	.4663	.4833	.5002	.5171	.5339	[illegible]
0	.3816	.3987	.4158	.4329	.4499	.4669	.4838	.5008	.5176	.5345	[illegible]

Table of chords, in parts of a rad 1; for protracting—Continued.

M.	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°	M.
0'	.5345	.5513	.5680	.5847	.6014	.6180	.6346	.6511	.6676	.6840	0
2	.5350	.5518	.5686	.5853	.6020	.6186	.6352	.6517	.6682	.6846	2
4	.5356	.5524	.5691	.5859	.6025	.6191	.6357	.6522	.6687	.6851	4
6	.5362	.5530	.5697	.5864	.6031	.6197	.6363	.6528	.6693	.6857	6
8	.5367	.5535	.5703	.5870	.6036	.6202	.6368	.6533	.6698	.6862	8
10	.5373	.5541	.5708	.5875	.6042	.6208	.6374	.6539	.6704	.6868	10
12	.5378	.5546	.5714	.5881	.6047	.6214	.6379	.6544	.6709	.6873	12
14	.5384	.5552	.5719	.5886	.6053	.6219	.6385	.6550	.6715	.6879	14
16	.5390	.5557	.5725	.5892	.6058	.6225	.6390	.6555	.6720	.6884	16
18	.5395	.5563	.5730	.5897	.6064	.6230	.6396	.6561	.6725	.6890	18
20	.5401	.5569	.5736	.5903	.6070	.6236	.6401	.6566	.6731	.6895	20
22	.5406	.5574	.5742	.5909	.6075	.6241	.6407	.6572	.6736	.6901	22
24	.5412	.5580	.5747	.5914	.6081	.6247	.6412	.6577	.6742	.6906	24
26	.5418	.5585	.5753	.5920	.6086	.6252	.6418	.6583	.6747	.6911	26
28	.5423	.5591	.5758	.5925	.6092	.6258	.6423	.6588	.6753	.6917	28
30	.5429	.5597	.5764	.5931	.6097	.6263	.6429	.6594	.6758	.6922	30
32	.5434	.5602	.5769	.5936	.6103	.6269	.6434	.6599	.6764	.6928	32
34	.5440	.5608	.5775	.5942	.6108	.6274	.6440	.6605	.6769	.6933	34
36	.5446	.5613	.5781	.5947	.6114	.6280	.6445	6610	.6775	.6939	36
38	.5451	.5619	.5786	.5953	.6119	.6285	.6451	.6616	.6780	.6944	38
40	.5457	.5625	.5792	.5959	.6125	.6291	.6456	.6621	.6786	.6950	40
42	.5462	.5630	.5797	.5964	.6130	.6296	.6462	.6627	.6791	.6955	42
44	.5468	.5636	.5803	.5970	.6136	.6302	.6467	.6632	.6797	.6961	44
46	.5474	.5641	.5808	.5975	.6142	.6307	.6473	.6638	.6802	.6966	46
48	.5479	.5647	.5814	.5981	.6147	.6313	.6478	.6643	.6808	.6971	48
50	.5485	.5652	.5820	.5986	.6153	.6318	.6484	.6649	.6813	.6977	50
52	.5490	.5658	.5825	.5992	.6158	.6324	.6489	.6654	.6819	.6982	52
54	.5496	.5664	.5831	.5997	.6164	.6330	.6495	.6660	.6824	.6988	54
56	.5502	.5669	.5836	.6003	.6169	.6335	.6500	.6665	.6829	.6993	56
58	.5507	.5675	.5842	.6009	.6175	.6341	.6506	.6671	.6835	.6999	58
60	.5513	.5680	.5847	.6014	.6180	.6346	.6511	.6676	.6840	.7004	60

M.	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	M.
0'	.7004	.7167	.7330	.7492	.7654	.7815	.7975	.8135	.8294	.8452	0'
2	.7010	.7173	.7335	.7498	.7659	.7820	.7980	.8140	.8299	.8458	2
4	.7015	.7178	.7341	.7503	.7664	.7825	.7986	.8145	.8304	.8463	4
6	.7020	.7184	.7346	.7508	.7670	.7831	.7991	.8151	.8310	.8468	6
8	.7026	.7189	.7352	.7514	.7675	.7836	.7996	.8156	.8315	.8473	8
10	.7031	.7195	.7357	.7519	.7681	.7841	.8002	.8161	.8320	.8479	10
12	.7037	.7200	.7362	.7524	.7686	.7847	.8007	.8167	.8326	.8484	12
14	.7042	.7205	.7368	.7530	.7691	.7852	.8012	.8172	.8331	.8489	14
16	.7048	.7211	.7373	.7535	.7697	.7857	.8018	.8177	.8336	.8495	16
18	.7053	.7216	.7379	.7541	.7702	.7863	.8023	.8183	.8341	.8500	18
20	.7059	.7222	.7384	.7546	.7707	.7868	.8028	.8188	.8347	.8505	20
22	.7064	.7227	.7390	.7551	.7713	.7873	.8034	.8193	.8352	.8510	22
24	.7069	.7232	.7395	.7557	.7718	.7879	.8039	.8198	.8357	.8516	24
26	.7075	.7238	.7400	.7562	.7723	.7884	.8044	.8204	.8363	.8521	26
28	.7080	.7243	.7406	.7568	.7729	.7890	.8050	.8209	.8368	.8526	28
30	.7086	.7249	.7411	.7573	.7734	.7895	.8055	.8214	.8373	.8531	30
32	.7091	.7254	.7417	.7578	.7740	.7900	.8060	.8220	.8378	.8537	32
34	.7097	.7260	.7422	.7584	.7745	.7906	.8066	.8225	.8384	.8542	34
36	.7102	.7265	.7427	.7589	.7750	.7911	.8071	.8230	.8389	.8547	36
38	.7108	.7270	.7433	.7595	.7756	.7916	.8076	.8236	.8394	.8552	38
40	.7113	.7276	.7438	.7600	.7761	.7922	.8082	.8241	.8400	.8558	40
42	.7118	.7281	.7443	.7605	.7766	.7927	.8087	.8246	.8405	.8563	42
44	.7124	.7287	.7449	.7611	.7772	.7932	.8092	.8251	.8410	.8568	44
46	.7129	.7292	.7454	.7616	.7777	.7938	.8098	.8257	.8415	.8573	46
48	.7135	.7298	.7460	.7621	.7782	.7943	.8103	.8262	.8421	.8579	48
50	.7140	.7303	.7465	.7627	.7788	.7948	.8108	.8267	.8426	.8584	50
52	.7146	.7308	.7471	.7632	.7793	.7954	.8113	.8273	.8431	.8589	52
54	.7151	.7314	.7476	.7638	.7799	.7959	.8119	.8278	.8437	.8594	54
56	.7156	.7319	.7481	.7643	.7804	.7964	.8124	.8283	.8442	.8600	56
58	.7162	.7325	.7487	.7648	.7809	.7970	.8129	.8289	.8447	.8605	58
60	.7167	.7330	.7492	.7654	.7815	.7975	.8135	.8294	.8452	.8610	60

of chords, in parts of a rad 1; for protracting—Continued.

1°	52°	53°	54°	55°	56°	57°	58°	59°	60°	M.
10	.8767	.8924	.9080	.9235	.9389	.9543	.9696	.9848	1.0000	0'
15	.8773	.8929	.9085	.9240	.9395	.9548	.9701	.9854	1.0005	2
21	.8778	.8934	.9090	.9245	.9400	.9553	.9706	.9859	1.0010	4
26	.8783	.8940	.9095	.9250	.9405	.9559	.9711	.9864	1.0015	6
31	.8788	.8945	.9101	.9256	.9410	.9564	.9717	.9869	1.0020	8
36	.8794	.8950	.9106	.9261	.9415	.9569	.9722	.9874	1.0025	10
42	.8799	.8955	.9111	.9266	.9420	.9574	.9727	.9879	1.0030	12
47	.8804	.8960	.9116	.9271	.9425	.9579	.9732	.9884	1.0035	14
52	.8809	.8966	.9121	.9276	.9430	.9584	.9737	.9889	1.0040	16
57	.8814	.8971	.9126	.9281	.9436	.9589	.9742	.9894	1.0045	18
63	.8820	.8976	.9132	.9287	.9441	.9594	.9747	.9899	1.0050	20
68	.8825	.8981	.9137	.9292	.9446	.9599	.9752	.9904	1.0055	22
73	.8830	.8986	.9142	.9297	.9451	.9604	.9757	.9909	1.0060	24
78	.8835	.8992	.9147	.9302	.9456	.9610	.9762	.9914	1.0065	26
84	.8841	.8997	.9152	.9307	.9461	.9615	.9767	.9919	1.0070	28
89	.8846	.9002	.9157	.9312	.9466	.9620	.9772	.9924	1.0075	30
94	.8851	.9007	.9163	.9317	.9472	.9625	.9778	.9929	1.0080	32
99	.8856	.9012	.9168	.9323	.9477	.9630	.9783	.9934	1.0086	34
05	.8861	.9018	.9173	.9328	.9482	.9635	.9788	.9939	1.0091	36
10	.8867	.9023	.9178	.9333	.9487	.9640	.9793	.9945	1.0096	38
15	.8872	.9028	.9183	.9338	.9492	.9645	.9798	.9950	1.0101	40
20	.8877	.9033	.9188	.9343	.9497	.9650	.9803	.9955	1.0106	42
26	.8882	.9038	.9194	.9348	.9502	.9655	.9808	.9960	1.0111	44
31	.8887	.9044	.9199	.9353	.9507	.9661	.9813	.9965	1.0116	46
36	.8893	.9049	.9204	.9359	.9512	.9666	.9818	.9970	1.0121	48
41	.8898	.9054	.9209	.9364	.9518	.9671	.9823	.9975	1.0126	50
47	.8903	.9059	.9214	.9369	.9523	.9676	.9828	.9980	1.0131	52
52	.8908	.9064	.9219	.9374	.9528	.9681	.9833	.9985	1.0136	54
57	.8914	.9069	.9225	.9379	.9533	.9686	.9838	.9990	1.0141	56
62	.8919	.9075	.9230	.9384	.9538	.9691	.9843	.9995	1.0146	58
67	.8924	.9080	.9235	.9389	.9543	.9696	.9848	1.0000	1.0151	60

1°	62°	63°	64°	65°	66°	67°	68°	69°	70°	M.
51	1.0301	1.0450	1.0598	1.0746	1.0893	1.1039	1.1184	1.1328	1.1472	0'
56	1.0306	1.0455	1.0603	1.0751	1.0898	1.1044	1.1189	1.1333	1.1476	2
61	1.0311	1.0460	1.0608	1.0756	1.0903	1.1048	1.1194	1.1338	1.1481	4
66	1.0316	1.0465	1.0613	1.0761	1.0907	1.1053	1.1198	1.1342	1.1486	6
71	1.0321	1.0470	1.0618	1.0766	1.0912	1.1058	1.1203	1.1347	1.1491	8
76	1.0326	1.0475	1.0623	1.0771	1.0917	1.1063	1.1208	1.1352	1.1495	10
81	1.0331	1.0480	1.0628	1.0775	1.0922	1.1068	1.1213	1.1357	1.1500	12
86	1.0336	1.0485	1.0633	1.0780	1.0927	1.1073	1.1218	1.1362	1.1505	14
91	1.0341	1.0490	1.0638	1.0785	1.0932	1.1078	1.1222	1.1366	1.1510	16
96	1.0346	1.0495	1.0643	1.0790	1.0937	1.1082	1.1227	1.1371	1.1514	18
01	1.0351	1.0500	1.0648	1.0795	1.0942	1.1087	1.1232	1.1376	1.1519	20
06	1.0356	1.0504	1.0653	1.0800	1.0946	1.1092	1.1237	1.1381	1.1524	22
11	1.0361	1.0509	1.0658	1.0805	1.0951	1.1097	1.1242	1.1386	1.1529	24
16	1.0366	1.0514	1.0662	1.0810	1.0956	1.1102	1.1246	1.1390	1.1533	26
21	1.0370	1.0519	1.0667	1.0815	1.0961	1.1107	1.1251	1.1395	1.1538	28
26	1.0375	1.0524	1.0672	1.0820	1.0966	1.1111	1.1256	1.1400	1.1543	30
31	1.0380	1.0529	1.0677	1.0824	1.0971	1.1116	1.1261	1.1405	1.1548	32
36	1.0385	1.0534	1.0682	1.0829	1.0976	1.1121	1.1266	1.1409	1.1552	34
41	1.0390	1.0539	1.0687	1.0834	1.0980	1.1126	1.1271	1.1414	1.1557	36
46	1.0395	1.0544	1.0692	1.0839	1.0985	1.1131	1.1275	1.1419	1.1562	38
51	1.0400	1.0549	1.0697	1.0844	1.0990	1.1136	1.1280	1.1424	1.1567	40
56	1.0405	1.0554	1.0702	1.0849	1.0995	1.1140	1.1285	1.1429	1.1571	42
61	1.0410	1.0559	1.0707	1.0854	1.1000	1.1145	1.1290	1.1433	1.1576	44
66	1.0415	1.0564	1.0712	1.0859	1.1005	1.1150	1.1295	1.1438	1.1581	46
71	1.0420	1.0569	1.0717	1.0863	1.1010	1.1155	1.1299	1.1443	1.1586	48
76	1.0425	1.0574	1.0721	1.0868	1.1014	1.1160	1.1304	1.1448	1.1590	50
81	1.0430	1.0579	1.0726	1.0873	1.1019	1.1165	1.1309	1.1452	1.1595	52
86	1.0435	1.0584	1.0731	1.0878	1.1024	1.1169	1.1314	1.1457	1.1600	54
91	1.0440	1.0589	1.0736	1.0883	1.1029	1.1174	1.1319	1.1462	1.1605	56
96	1.0445	1.0593	1.0741	1.0888	1.1034	1.1179	1.1323	1.1467	1.1609	58
01	1.0450	1.0598	1.0746	1.0893	1.1039	1.1184	1.1328	1.1472	1.1614	60

Table of Chords, in parts of a rad 1; for protracting—Continued.

M.	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	M.
0	1.1614	1.1756	1.1896	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	0
2	1.1619	1.1760	1.1901	1.2041	1.2180	1.2318	1.2455	1.2591	1.2726	1.2860	2
4	1.1624	1.1765	1.1906	1.2046	1.2184	1.2322	1.2459	1.2595	1.2731	1.2865	4
6	1.1628	1.1770	1.1910	1.2050	1.2189	1.2327	1.2464	1.2600	1.2735	1.2869	6
8	1.1633	1.1775	1.1915	1.2055	1.2194	1.2332	1.2468	1.2604	1.2740	1.2874	8
10	1.1638	1.1779	1.1920	1.2060	1.2198	1.2336	1.2473	1.2609	1.2744	1.2878	10
12	1.1642	1.1784	1.1924	1.2064	1.2203	1.2341	1.2478	1.2614	1.2748	1.2882	12
14	1.1647	1.1789	1.1929	1.2069	1.2208	1.2345	1.2482	1.2618	1.2753	1.2887	14
16	1.1652	1.1793	1.1934	1.2073	1.2212	1.2350	1.2487	1.2623	1.2757	1.2891	16
18	1.1657	1.1798	1.1938	1.2078	1.2217	1.2354	1.2491	1.2627	1.2762	1.2896	18
20	1.1661	1.1803	1.1943	1.2083	1.2221	1.2359	1.2496	1.2632	1.2766	1.2900	20
22	1.1666	1.1807	1.1948	1.2087	1.2226	1.2364	1.2500	1.2636	1.2771	1.2905	22
24	1.1671	1.1812	1.1952	1.2092	1.2231	1.2368	1.2505	1.2641	1.2775	1.2909	24
26	1.1676	1.1817	1.1957	1.2097	1.2235	1.2373	1.2509	1.2645	1.2780	1.2914	26
28	1.1680	1.1821	1.1962	1.2101	1.2240	1.2377	1.2514	1.2650	1.2784	1.2918	28
30	1.1685	1.1826	1.1966	1.2106	1.2244	1.2382	1.2518	1.2654	1.2789	1.2922	30
32	1.1690	1.1831	1.1971	1.2111	1.2249	1.2386	1.2523	1.2659	1.2793	1.2927	32
34	1.1694	1.1836	1.1976	1.2115	1.2254	1.2391	1.2528	1.2663	1.2798	1.2931	34
36	1.1699	1.1840	1.1980	1.2120	1.2258	1.2396	1.2532	1.2668	1.2802	1.2936	36
38	1.1704	1.1845	1.1985	1.2124	1.2263	1.2400	1.2537	1.2672	1.2807	1.2940	38
40	1.1709	1.1850	1.1990	1.2129	1.2267	1.2405	1.2541	1.2677	1.2811	1.2945	40
42	1.1713	1.1854	1.1994	1.2134	1.2272	1.2409	1.2546	1.2681	1.2816	1.2949	42
44	1.1718	1.1859	1.1999	1.2138	1.2277	1.2414	1.2550	1.2686	1.2820	1.2954	44
46	1.1723	1.1864	1.2004	1.2143	1.2281	1.2418	1.2555	1.2690	1.2825	1.2958	46
48	1.1727	1.1868	1.2008	1.2148	1.2286	1.2423	1.2559	1.2695	1.2829	1.2962	48
50	1.1732	1.1873	1.2013	1.2152	1.2290	1.2428	1.2564	1.2699	1.2833	1.2967	50
52	1.1737	1.1878	1.2018	1.2157	1.2295	1.2432	1.2568	1.2704	1.2838	1.2971	52
54	1.1742	1.1882	1.2022	1.2161	1.2299	1.2437	1.2573	1.2708	1.2842	1.2976	54
56	1.1746	1.1887	1.2027	1.2166	1.2304	1.2441	1.2577	1.2713	1.2847	1.2980	56
58	1.1751	1.1892	1.2032	1.2171	1.2309	1.2446	1.2582	1.2717	1.2851	1.2985	58
60	1.1756	1.1896	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	1.2989	60

M.	81°	82°	83°	84°	85°	86°	87°	88°	89°	M.
0	1.2989	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018	0
2	1.2993	1.3126	1.3257	1.3387	1.3516	1.3644	1.3771	1.3897	1.4022	2
4	1.2998	1.3130	1.3261	1.3391	1.3520	1.3648	1.3776	1.3902	1.4026	4
6	1.3002	1.3134	1.3265	1.3396	1.3525	1.3653	1.3780	1.3906	1.4031	6
8	1.3007	1.3139	1.3270	1.3400	1.3529	1.3657	1.3784	1.3910	1.4035	8
10	1.3011	1.3143	1.3274	1.3404	1.3533	1.3661	1.3788	1.3914	1.4039	10
12	1.3015	1.3147	1.3279	1.3409	1.3538	1.3665	1.3792	1.3918	1.4043	12
14	1.3020	1.3152	1.3283	1.3413	1.3542	1.3670	1.3797	1.3922	1.4047	14
16	1.3024	1.3156	1.3287	1.3417	1.3546	1.3674	1.3801	1.3927	1.4051	16
18	1.3029	1.3161	1.3292	1.3421	1.3550	1.3678	1.3805	1.3931	1.4055	18
20	1.3033	1.3165	1.3296	1.3426	1.3555	1.3682	1.3809	1.3935	1.4060	20
22	1.3038	1.3169	1.3300	1.3430	1.3559	1.3687	1.3813	1.3939	1.4064	22
24	1.3042	1.3174	1.3305	1.3434	1.3563	1.3691	1.3818	1.3943	1.4068	24
26	1.3046	1.3178	1.3309	1.3439	1.3567	1.3695	1.3822	1.3947	1.4072	26
28	1.3051	1.3183	1.3313	1.3443	1.3572	1.3699	1.3826	1.3952	1.4076	28
30	1.3055	1.3187	1.3318	1.3447	1.3576	1.3704	1.3830	1.3956	1.4080	30
32	1.3060	1.3191	1.3322	1.3452	1.3580	1.3708	1.3834	1.3960	1.4084	32
34	1.3064	1.3196	1.3326	1.3456	1.3585	1.3712	1.3839	1.3964	1.4089	34
36	1.3068	1.3200	1.3331	1.3460	1.3589	1.3716	1.3843	1.3968	1.4093	36
38	1.3073	1.3204	1.3335	1.3465	1.3593	1.3721	1.3847	1.3972	1.4097	38
40	1.3077	1.3209	1.3339	1.3469	1.3597	1.3725	1.3851	1.3977	1.4101	40
42	1.3082	1.3213	1.3344	1.3473	1.3602	1.3729	1.3855	1.3981	1.4105	42
44	1.3086	1.3218	1.3348	1.3477	1.3606	1.3733	1.3860	1.3985	1.4109	44
46	1.3090	1.3222	1.3352	1.3482	1.3610	1.3738	1.3864	1.3989	1.4113	46
48	1.3095	1.3226	1.3357	1.3486	1.3614	1.3742	1.3868	1.3993	1.4117	48
50	1.3099	1.3231	1.3361	1.3490	1.3619	1.3746	1.3872	1.3997	1.4122	50
52	1.3104	1.3235	1.3365	1.3495	1.3623	1.3750	1.3876	1.4002	1.4126	52
54	1.3108	1.3239	1.3370	1.3499	1.3627	1.3754	1.3881	1.4006	1.4130	54
56	1.3112	1.3244	1.3374	1.3503	1.3631	1.3759	1.3885	1.4010	1.4134	56
58	1.3117	1.3248	1.3378	1.3508	1.3636	1.3763	1.3889	1.4014	1.4138	58
60	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018	1.4142	60

Logarithms of Numbers, from 0 to 1000.*

No.	0	1	2	3	4	5	6	7	8	9	Prop.
0	0	00000	30103	47712	60206	69897	77815	84510	90309	95424	
10	00000	00432	00860	01283	01703	02118	02530	02938	03342	03742	415
11	04139	04532	04921	05307	05690	06069	06445	06818	07188	07554	379
12	07918	08278	08636	08990	09342	09691	10037	10380	10721	11059	349
13	11394	11727	12057	12385	12710	13033	13353	13672	13987	14301	323
14	14613	14921	15228	15533	15836	16136	16435	16731	17026	17318	300
15	17609	17897	18184	18469	18752	19033	19312	19590	19865	20139	281
16	20412	20682	20951	21218	21484	21748	22010	22271	22530	22788	264
17	23045	23299	23552	23804	24054	24303	24551	24797	25042	25285	248
18	25527	25767	26007	26245	26481	26717	26951	27184	27415	27646	236
19	27875	28103	28330	28555	28780	29003	29225	29446	29666	29885	223
20	30103	30319	30535	30749	30963	31175	31386	31597	31806	32014	212
21	32222	32428	32633	32838	33041	33243	33445	33646	33845	34044	202
22	34242	34439	34635	34830	35024	35218	35410	35602	35793	35983	194
23	36173	36361	36548	36735	36921	37106	37291	37474	37657	37839	185
24	38021	38201	38381	38560	38739	38916	39093	39269	39445	39619	177
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330	171
26	41497	41664	41830	41995	42160	42324	42488	42651	42813	42975	164
27	43136	43296	43456	43616	43775	43933	44090	44248	44404	44560	158
28	44716	44870	45024	45178	45331	45484	45636	45788	45939	46089	153
29	46240	46389	46538	46686	46834	46982	47129	47275	47421	47567	148
30	47712	47856	48000	48144	48287	48430	48572	48713	48855	48995	143
31	49136	49276	49415	49554	49693	49831	49968	50105	50242	50379	138
32	50515	50650	50785	50920	51054	51188	51321	51454	51587	51719	134
33	51851	51982	52113	52244	52374	52504	52633	52763	52891	53020	130
34	53148	53275	53402	53529	53655	53781	53907	54033	54157	54282	126
35	54407	54530	54654	54777	54900	55022	55145	55266	55388	55509	122
36	55630	55750	55870	55990	56110	56229	56348	56466	56584	56702	119
37	56820	56937	57054	57170	57287	57403	57518	57634	57749	57863	116
38	57978	58092	58206	58319	58433	58546	58658	58771	58883	58995	113
39	59106	59217	59328	59439	59549	59659	59769	59879	59988	60097	110
40	60206	60314	60422	60530	60638	60745	60852	60959	61066	61172	107
41	61278	61384	61489	61595	61700	61804	61909	62013	62117	62221	104
42	62325	62428	62531	62634	62736	62838	62941	63042	63144	63245	102
43	63347	63447	63548	63648	63749	63848	63948	64048	64147	64246	99
44	64345	64443	64542	64640	64738	64836	64933	65030	65127	65224	98
45	65321	65417	65513	65609	65705	65801	65896	65991	66086	66181	96
46	66276	66370	66464	66558	66651	66745	66838	66931	67024	67117	94
47	67210	67302	67394	67486	67577	67669	67760	67851	67942	68033	92
48	68124	68214	68304	68394	68484	68574	68663	68752	68842	68930	90
49	69020	69108	69196	69284	69372	69460	69548	69635	69722	69810	88
50	69897	69983	70070	70156	70243	70329	70415	70500	70586	70671	86
51	70757	70842	70927	71011	71096	71180	71265	71349	71433	71516	84
52	71600	71683	71767	71850	71933	72015	72098	72181	72263	72345	82
53	72428	72509	72591	72672	72754	72835	72916	72997	73078	73158	81
54	73239	73319	73399	73480	73559	73639	73719	73798	73878	73957	80
55	74036	74115	74193	74272	74351	74429	74507	74585	74663	74741	78
56	74818	74896	74973	75050	75127	75204	75281	75358	75434	75511	77
57	75587	75663	75739	75815	75891	75966	76042	76117	76192	76267	75
58	76342	76417	76492	76566	76641	76715	76789	76863	76937	77011	74
59	77085	77158	77232	77305	77378	77451	77524	77597	77670	77742	73
60	77815	77887	77959	78031	78103	78175	78247	78318	78390	78461	72
61	78533	78604	78675	78746	78816	78887	78958	79028	79098	79169	71
62	79239	79309	79379	79448	79518	79588	79657	79726	79796	79865	70
63	79934	80002	80071	80140	80208	80277	80345	80413	80482	80550	69
64	80618	80685	80753	80821	80888	80956	81023	81090	81157	81224	68
65	81291	81358	81424	81491	81557	81624	81690	81756	81822	81888	67

*Each log is supposed to have the decimal sign . before it.

Table of Chords, in parts of a rad 1; for protracting—Continued.

M.	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	M.
0	1.1614	1.1756	1.1896	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	0
2	1.1619	1.1760	1.1901	1.2041	1.2180	1.2318	1.2455	1.2591	1.2726	1.2860	2
4	1.1624	1.1765	1.1906	1.2046	1.2184	1.2322	1.2459	1.2595	1.2731	1.2865	4
6	1.1628	1.1770	1.1910	1.2050	1.2189	1.2327	1.2464	1.2600	1.2735	1.2869	6
8	1.1633	1.1775	1.1915	1.2055	1.2194	1.2332	1.2468	1.2604	1.2740	1.2874	8
10	1.1638	1.1779	1.1920	1.2060	1.2198	1.2336	1.2473	1.2609	1.2744	1.2878	10
12	1.1642	1.1784	1.1924	1.2064	1.2203	1.2341	1.2478	1.2614	1.2748	1.2882	12
14	1.1647	1.1789	1.1929	1.2069	1.2208	1.2345	1.2482	1.2618	1.2753	1.2887	14
16	1.1652	1.1793	1.1934	1.2073	1.2212	1.2350	1.2487	1.2623	1.2757	1.2891	16
18	1.1657	1.1798	1.1938	1.2078	1.2217	1.2354	1.2491	1.2627	1.2762	1.2896	18
20	1.1661	1.1803	1.1943	1.2083	1.2221	1.2359	1.2496	1.2632	1.2766	1.2900	20
22	1.1666	1.1807	1.1948	1.2087	1.2226	1.2364	1.2500	1.2636	1.2771	1.2905	22
24	1.1671	1.1812	1.1952	1.2092	1.2231	1.2368	1.2505	1.2641	1.2775	1.2909	24
26	1.1676	1.1817	1.1957	1.2097	1.2235	1.2373	1.2509	1.2645	1.2780	1.2914	26
28	1.1680	1.1821	1.1962	1.2101	1.2240	1.2377	1.2514	1.2650	1.2784	1.2918	28
30	1.1685	1.1826	1.1966	1.2106	1.2244	1.2382	1.2518	1.2654	1.2789	1.2922	30
32	1.1690	1.1831	1.1971	1.2111	1.2249	1.2386	1.2523	1.2659	1.2793	1.2927	32
34	1.1694	1.1836	1.1976	1.2115	1.2254	1.2391	1.2528	1.2663	1.2798	1.2931	34
36	1.1699	1.1840	1.1980	1.2120	1.2258	1.2396	1.2532	1.2668	1.2802	1.2936	36
38	1.1704	1.1845	1.1985	1.2124	1.2263	1.2400	1.2537	1.2672	1.2807	1.2940	38
40	1.1709	1.1850	1.1990	1.2129	1.2267	1.2405	1.2541	1.2677	1.2811	1.2945	40
42	1.1713	1.1854	1.1994	1.2134	1.2272	1.2409	1.2546	1.2681	1.2816	1.2949	42
44	1.1718	1.1859	1.1999	1.2138	1.2277	1.2414	1.2550	1.2686	1.2820	1.2954	44
46	1.1723	1.1864	1.2004	1.2143	1.2281	1.2418	1.2555	1.2690	1.2825	1.2958	46
48	1.1727	1.1868	1.2008	1.2148	1.2286	1.2423	1.2559	1.2695	1.2829	1.2962	48
50	1.1732	1.1873	1.2013	1.2152	1.2290	1.2428	1.2564	1.2699	1.2833	1.2967	50
52	1.1737	1.1878	1.2018	1.2157	1.2295	1.2432	1.2568	1.2704	1.2838	1.2971	52
54	1.1742	1.1882	1.2022	1.2161	1.2299	1.2437	1.2573	1.2708	1.2842	1.2976	54
56	1.1746	1.1887	1.2027	1.2166	1.2304	1.2441	1.2577	1.2713	1.2847	1.2980	56
58	1.1751	1.1892	1.2032	1.2171	1.2309	1.2446	1.2582	1.2717	1.2851	1.2985	58
60	1.1756	1.1896	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	1.2989	60

M.	81°	82°	83°	84°	85°	86°	87°	88°	89°	M.
0	1.2989	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018	0
2	1.2993	1.3126	1.3257	1.3387	1.3516	1.3644	1.3771	1.3897	1.4022	2
4	1.2998	1.3130	1.3261	1.3391	1.3520	1.3648	1.3776	1.3902	1.4026	4
6	1.3002	1.3134	1.3265	1.3396	1.3525	1.3653	1.3780	1.3906	1.4031	6
8	1.3007	1.3139	1.3270	1.3400	1.3529	1.3657	1.3784	1.3910	1.4035	8
10	1.3011	1.3143	1.3274	1.3404	1.3533	1.3661	1.3788	1.3914	1.4039	10
12	1.3015	1.3147	1.3279	1.3409	1.3538	1.3665	1.3792	1.3918	1.4043	12
14	1.3020	1.3152	1.3283	1.3413	1.3542	1.3670	1.3797	1.3922	1.4047	14
16	1.3024	1.3156	1.3287	1.3417	1.3546	1.3674	1.3801	1.3927	1.4051	16
18	1.3029	1.3161	1.3292	1.3421	1.3550	1.3678	1.3805	1.3931	1.4055	18
20	1.3033	1.3165	1.3296	1.3426	1.3555	1.3682	1.3809	1.3935	1.4060	20
22	1.3038	1.3169	1.3300	1.3430	1.3559	1.3687	1.3813	1.3939	1.4064	22
24	1.3042	1.3174	1.3305	1.3434	1.3563	1.3691	1.3818	1.3943	1.4068	24
26	1.3046	1.3178	1.3309	1.3439	1.3567	1.3695	1.3822	1.3947	1.4072	26
28	1.3051	1.3183	1.3313	1.3443	1.3572	1.3699	1.3826	1.3952	1.4076	28
30	1.3055	1.3187	1.3318	1.3447	1.3576	1.3704	1.3830	1.3956	1.4080	30
32	1.3060	1.3191	1.3322	1.3452	1.3580	1.3708	1.3834	1.3960	1.4084	32
34	1.3064	1.3196	1.3326	1.3456	1.3585	1.3712	1.3839	1.3964	1.4089	34
36	1.3068	1.3200	1.3331	1.3460	1.3589	1.3716	1.3843	1.3968	1.4093	36
38	1.3073	1.3204	1.3335	1.3465	1.3593	1.3721	1.3847	1.3972	1.4097	38
40	1.3077	1.3209	1.3339	1.3469	1.3597	1.3725	1.3851	1.3977	1.4101	40
42	1.3082	1.3213	1.3344	1.3473	1.3602	1.3729	1.3855	1.3981	1.4105	42
44	1.3086	1.3218	1.3348	1.3477	1.3606	1.3733	1.3860	1.3985	1.4109	44
46	1.3090	1.3222	1.3352	1.3482	1.3610	1.3738	1.3864	1.3989	1.4113	46
48	1.3095	1.3226	1.3357	1.3486	1.3614	1.3742	1.3868	1.3993	1.4117	48
50	1.3099	1.3231	1.3361	1.3490	1.3619	1.3746	1.3872	1.3997	1.4122	50
52	1.3104	1.3235	1.3365	1.3495	1.3623	1.3750	1.3876	1.4002	1.4126	52
54	1.3108	1.3239	1.3370	1.3499	1.3627	1.3754	1.3881	1.4006	1.4130	54
56	1.3112	1.3244	1.3374	1.3503	1.3631	1.3759	1.3885	1.4010	1.4134	56
58	1.3117	1.3248	1.3378	1.3508	1.3636	1.3763	1.3889	1.4014	1.4138	58
60	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018	1.4142	60

Logarithms of Numbers, from 0 to 1000.*

No.	0	1	2	3	4	5	6	7	8	9	Prop.
0	0	00000	30103	47712	60206	69897	77815	84510	90309	95424	
10	00000	00432	00860	01283	01703	02118	02530	02938	03342	03742	415
11	04139	04532	04921	05307	05690	06069	06445	06818	07188	07554	379
12	07918	08278	08636	08990	09342	09691	10037	10380	10721	11059	349
13	11394	11727	12057	12385	12710	13033	13353	13672	13987	14301	323
14	14613	14921	15228	15533	15836	16136	16435	16731	17026	17318	300
15	17609	17897	18184	18469	18752	19033	19312	19590	19865	20139	281
16	20412	20682	20951	21218	21484	21748	22010	22271	22530	22788	264
17	23045	23299	23552	23804	24054	24303	24551	24797	25042	25285	249
18	25527	25767	26007	26245	26481	26717	26951	27184	27415	27646	236
19	27875	28103	28330	28555	28780	29003	29225	29446	29666	29885	223
20	30103	30319	30535	30749	30963	31175	31386	31597	31806	32014	212
21	32222	32428	32633	32838	33041	33243	33445	33646	33845	34044	202
22	34242	34439	34635	34830	35024	35218	35410	35602	35793	35983	194
23	36173	36361	36548	36735	36921	37106	37291	37474	37657	37839	185
24	38021	38201	38381	38560	38739	38916	39093	39269	39445	39619	177
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330	171
26	41497	41664	41830	41995	42160	42324	42488	42651	42813	42975	164
27	43136	43296	43456	43616	43775	43933	44090	44248	44404	44560	158
28	44716	44870	45024	45178	45331	45484	45636	45788	45939	46089	153
29	46240	46389	46538	46686	46834	46982	47129	47275	47421	47567	148
30	47712	47856	48000	48144	48287	48430	48572	48713	48855	48995	143
31	49136	49276	49415	49554	49693	49831	49968	50105	50242	50379	138
32	50515	50650	50785	50920	51054	51188	51321	51454	51587	51719	134
33	51851	51982	52113	52244	52374	52504	52633	52763	52891	53020	130
34	53148	53275	53402	53529	53655	53781	53907	54033	54157	54282	126
35	54407	54530	54654	54777	54900	55022	55145	55266	55388	55509	122
36	55630	55750	55870	55990	56110	56229	56348	56466	56584	56702	119
37	56820	56937	57054	57170	57287	57403	57518	57634	57749	57863	116
38	57978	58092	58206	58319	58433	58546	58658	58771	58883	58995	113
39	59106	59217	59328	59439	59549	59659	59769	59879	59988	60097	110
40	60206	60314	60422	60530	60638	60745	60852	60959	61066	61172	107
41	61278	61384	61489	61595	61700	61804	61909	62013	62117	62221	104
42	62325	62428	62531	62634	62736	62838	62941	63042	63144	63245	102
43	63347	63447	63548	63648	63749	63848	63948	64048	64147	64246	99
44	64345	64443	64542	64640	64738	64836	64933	65030	65127	65224	98
45	65321	65417	65513	65609	65705	65801	65896	65991	66086	66181	96
46	66276	66370	66464	66558	66651	66745	66838	66931	67024	67117	94
47	67210	67302	67394	67486	67577	67669	67760	67851	67942	68033	92
48	68124	68214	68304	68394	68484	68574	68663	68752	68842	68930	90
49	69020	69108	69196	69284	69372	69460	69548	69635	69722	69810	88
50	69897	69983	70070	70156	70243	70329	70415	70500	70586	70671	86
51	70757	70842	70927	71011	71096	71180	71265	71349	71433	71516	84
52	71600	71683	71767	71850	71933	72015	72098	72181	72263	72345	82
53	72428	72509	72591	72672	72754	72835	72916	72997	73078	73158	81
54	73239	73319	73399	73480	73559	73639	73719	73798	73878	73957	80
55	74036	74115	74193	74272	74351	74429	74507	74585	74663	74741	78
56	74818	74896	74973	75050	75127	75204	75281	75358	75434	75511	77
57	75587	75663	75739	75815	75891	75966	76042	76117	76192	76267	75
58	76342	76417	76492	76566	76641	76715	76789	76863	76937	77011	74
59	77085	77158	77232	77305	77378	77451	77524	77597	77670	77742	73
60	77815	77887	77959	78031	78103	78175	78247	78318	78390	78461	72
61	78533	78604	78675	78746	78816	78887	78958	79028	79098	79169	71
62	79239	79309	79379	79448	79518	79588	79657	79726	79796	79865	70
63	79934	80002	80071	80140	80208	80277	80345	80413	80482	80550	69
64	80618	80685	80753	80821	80888	80956	81023	81090	81157	81224	68
65	81291	81358	81424	81491	81557	81624	81690	81756	81822	81888	67

*Each log is supposed to have the decimal sign . before it.

Logarithms of Numbers, from 0 to 1000*—(Continued.)

No.	0	1	2	3	4	5	6	7	8	9	Prop.
66	81954	82020	82085	82151	82216	82282	82347	82412	82477	82542	66
67	82607	82672	82736	82801	82866	82930	82994	83058	83123	83187	65
68	83250	83314	83378	83442	83505	83569	83632	83695	83758	83821	64
69	83884	83947	84010	84073	84136	84198	84260	84323	84385	84447	63
70	84509	84571	84633	84695	84757	84818	84880	84941	85003	85064	62
71	85125	85187	85248	85309	85369	85430	85491	85551	85612	85672	61
72	85733	85793	85853	85913	85973	86033	86093	86153	86213	86272	60
73	86332	86391	86451	86510	86569	86628	86687	86746	86805	86864	59
74	86923	86981	87040	87098	87157	87215	87273	87332	87390	87448	58
75	87506	87564	87621	87679	87737	87794	87852	87909	87966	88024	57
76	88081	88138	88195	88252	88309	88366	88422	88479	88536	88592	56
77	88649	88705	88761	88818	88874	88930	88986	89042	89098	89153	56
78	89209	89265	89320	89376	89431	89487	89542	89597	89652	89707	55
79	89762	89817	89872	89927	89982	90036	90091	90145	90200	90254	54
80	90309	90363	90417	90471	90525	90579	90633	90687	90741	90794	54
81	90848	90902	90955	91009	91062	91115	91169	91222	91275	91328	53
82	91381	91434	91487	91540	91592	91645	91698	91750	91803	91855	53
83	91907	91960	92012	92064	92116	92168	92220	92272	92324	92376	52
84	92427	92479	92531	92582	92634	92685	92737	92788	92839	92890	51
85	92941	92993	93044	93095	93146	93196	93247	93298	93348	93399	51
86	93449	93500	93550	93601	93651	93701	93751	93802	93852	93902	50
87	93951	94001	94051	94101	94151	94200	94250	94300	94349	94398	49
88	94448	94497	94546	94596	94645	94694	94743	94792	94841	94890	49
89	94939	94987	95036	95085	95133	95182	95230	95279	95327	95376	48
90	95424	95472	95520	95568	95616	95664	95712	95760	95808	95856	48
91	95904	95951	95999	96047	96094	96142	96189	96236	96284	96331	48
92	96378	96425	96473	96520	96567	96614	96661	96708	96754	96801	47
93	96848	96895	96941	96988	97034	97081	97127	97174	97220	97266	47
94	97312	97359	97405	97451	97497	97543	97589	97635	97680	97726	46
95	97772	97818	97863	97909	97954	98000	98045	98091	98136	98181	46
96	98227	98272	98317	98362	98407	98452	98497	98542	98587	98632	45
97	98677	98721	98766	98811	98855	98900	98945	98989	99033	99078	45
98	99122	99166	99211	99255	99299	99343	99387	99431	99475	99519	44
99	99563	99607	99651	99694	99738	99782	99825	99869	99913	99956	44

* Each log is supposed to have the decimal sign before it.

The log of 2870 is 3.45788	The log of .287 is — 1.45788
" " " 287 is 2.45788	" " " .028 is — 2.44716
" " " 28.7 is 1.45788	" " " .002 is — 3.30103
" " " 2.87 is 0.45788	" " " .0002 is — 4.30103

What is the log of 2873?

Here, log of 2870 = 3.45788
And prop 153 × 3 = 459

3.458339

To find roots divide the log (with its index) of the given number, by that number which expresses the kind of root. The quotient will be the log of the required root.

Example. What is the cube root of 2870?

Here, the log of 2870, with its index, is 3.45788. And $\frac{3.45788}{3} = 1.15263$. Hence the cube root is 14.2

The Hyperbolic, or Napierian logarithm is the common log of *the table multiplied by* 2.3025851.

GLOSSARY OF TERMS.

the flat square member on top of a column.
r *abscissa;* any portion of the axis of a curve, from the vertex to any point from which
s the axis at right angles, and extends to meet the curve itself; said line being called an

; an upward slope, or ascent of ground, &c.
horizontal passage into a mine, &c.
well-known curved cutting instrument, for dressing or chipping horizontal surfaces.
l. Motion is imparted to the water in a line of pipes, by the *forward* stroke of the piston
acting pump; but during the *backward* stroke, this motion is stopped; and the water in
omes to rest. Therefore, at the next forward stroke, all the water has to be again set in
d the force that must be exerted by the pump to do this is much greater than would be
the motion previously imparted had been maintained during the time of the backstroke.
n of an air-vessel secures this maintenance of motion, and thus effects a great saving of
ides diminishing the danger of bursting the pipes at each forward stroke. It is merely a
ong air-tight iron box, usually cylindrical, strongly bolted on top of the pipes just beyond
and communicating freely with them through an opening in its base. It is full of air.
d stroke of the piston then forces water not only along the pipes, but also into the lower
air-vessel, through the opening in its base; thus compressing its contained air. But
backstroke, this compressed air, being relieved from the pressure of the pump, expands;
ding presses upon the water in the pipes, and thus keeps it in motion until the next for-
e; and so on. An air-vessel also acts as an *air-cushion;* permitting the piston to apply
the water in the pipes gradually; thus preserving both the pipes and the pump from vio-
. The air in the vessel, however, becomes by degrees absorbed and taken away by the
l its action as a regulator then ceases. To prevent this, fresh air must be forced into the
time to time by a condenser, or forcing air-pump. A *double-acting* pump does not so
an air-vessel. There is no particular rule for the size or capacity of air-vessels. In prac-
ars to vary from about 5 to 50 times that of the pump; with a height equal to two or more
lam. A stand-pipe, which see, is sometimes used instead of an air-vessel.
ng motion; up and down, or backward and forward, instead of revolving, &c.
ad, or plaster bead; a bead nailed to projecting angles in rooms, to protect the plaster on
from injury.
ck; a triangular block against which the ends of the braces and counters abut in a Howe

velocity. See footnote to Art 6 of Force in Rigid Bodies. Page 447.
to toughen some of the metals, glass, &c, by first heating them, and then causing them to
owly. This process however lessens the tensile strength.
l axis; in geology; a line from which the strata of rocks slope away downward in oppo-
ns, like the slates on the roof of a house; the ridge of the roof representing the axis.
a covering of timber, stone, or metal, to protect a surface against the action of water flow-
Has many other meanings.
See Journal.
ce; that part of an entablature which is next above the columns. Applies also when there
mns. Also, the mouldings around the sides and tops of doors and windows, attached to
nner or outer face of the wall.
sharp edge formed by any two surfaces which meet at an angle. The edges of a brick are

a facing of cut stone, applied to a backing of rubble or rough masonry, or brickwork.
; a small moulding, about semi-circular or semi-elliptic, and either plain or ornamented by

imaginary line passing through a body, which may be supposed to revolve around it; as
f a sphere. Any piece that passes through and supports a body which revolves; in which
alled an axle, or shaft.
. See Journal-box.
; an axle which remains fixed while the wheel revolves around it, as in wagons, &c.
. The azimuth of a body is that arc of the horizon that is included between the meridian
e given place, and another great circle passing through the body.
; the rough masonry of a wall faced with finer work. Earth deposited behind a retaining-

beams; the long top beams of lock-gates, by which they are pushed open or shut.
large beam of timber.
broken stone, sand or gravel, &c. on which railroad cross-ties are laid.
t; a cistern valve at one end of a lever, at the other end of which is a floating ball. The
nd falls with the water in the cistern; and thus opens or shuts the valve.
se. See Valve.
rds; boards nailed against the outer face of a wall, along the slopes of a gable end of a
ide the rafters, &c; and to make a neat finish.
sometimes affectedly *batir.*) or *talus;* the sloping backward of a face of masonry.
bridges, &c, sometimes a panel; sometimes a span.
a ornament either composed of a straight cylindrical rod; or carved or cast in that shape
face.
f the course by a compass. The span or length in the clear between the points of support
&c. The points of support themselves of a beam, shaft, axle, pivot, &c.
blings; ornamental mouldings on the lower face of a projecting cornice, &c.
s; a large plate of iron laid as a foundation for something to rest on.
t heavy wooden rammer, such as pavers use.
k. See Crank.
rk; a level mark cut at the foot of a tree for future reference, as being more permanent
.
erme; a horizontal surface, as if for a pathway, and forming a kind of step along the t

of sloping ground. In canals, the level top of the embankment opposite and corresponding to the towpath is called the berm.

Bessemer steel is formed by forcing air into a mass of melted cast iron; by which means the excess of carbon in the iron is separated from it, until only enough remains to constitute cast steel. The carbon is *chemically* united with the steel, but *mechanically* with the iron.

Beton; concrete of hydraulic cement, with broken stone and bricks, gravel, &c.

Bevel; the slope formed by trimming away a sharp edge, as of a board, &c. Edges of common drawing rulers and scales are usually bevelled. See 13, Figs 42, of Trusses, p 294.

Bevel gear; cog-wheels with teeth so formed that the wheels can work into each other at an angle.

Bilge; the nearly flat part of the bottom of a ship on each side of the keel. Also, the swelled part of a barrel, &c. To bilge is to spring a leak in the bilge, or to be broken there.

Bitts; the small boring points used with a brace.

Blast-pipes; in a locomotive; those through which the waste steam passes from the cylinder into the smoke-pipe, and thus creates an artificial draft in the chimney, or smoke-pipe.

Boasting; dressing stone with a broad chisel called a beaster, and mallet. The beaster gives a smoother surface after the use of the point, or the narrow chisel called a tool.

Bond; the disposing of the blocks of stone or brickwork so as to form the whole into a firm structure, by a judicious overlapping of each other, so as to break joint. Applies also to timber, &c, in various ways.

Bonnet; a cap over the end of a pipe, &c. A cast-iron plate bolted down as a covering over an aperture.

Bore; inner diameter of a hollow cylinder.

Boss; an increase of the diameter at any part of a shaft for any purpose. A projection in shape of a segment of a sphere, or somewhat so, whether for use or for ornament; often carved, or cast.

Box-drain; a square or rectangular drain of masonry or timber, under a railroad, &c.

Brace; a kind of curved handle used for boring holes with bitts. The head of the brace remains stationary, being pressed against by the body of the person using it, while the other part with the bitt is turned round by his hand. Also, an inclined beam, bar, or strut, for sustaining compression.

Bracket; a projecting piece of board, &c, frequently triangular, the vertical leg attached to the face of a wall, and the horizontal one supporting a shelf, &c. Often made in ornamental shape for supporting busts, clocks, &c. Also, the supports for shafting; as pendent, wall, and pedestal brackets.

Brake; an arrangement for preventing or diminishing motion by means of friction. The friction is usually applied at the circumference of a revolving wheel, by means of levers. On railroads, the car-brakes should be worked by steam, as those of Loughridge, Westinghouse, and Creamer. Also, such a handle as that of a common pump.

Brass is composed of copper and zinc.

Brasses; fittings of brass in many plummer-blocks, and in other positions, for diminishing the friction of revolving journals which rest upon them.

Braze; to unite pieces of iron, copper, or brass, by means of a hard solder, called spelter solder, and composed, like brass, of copper and zinc, but in other proportions.

Break joint; to so overlap pieces that the joints shall not occur at the same place, and thus produce a bad bond.

Breast-summer; a beam of wood, iron, or stone, supporting a wall over a door or other opening; a kind of lintel.

Breast-wall; one built to prevent the falling of a vertical face cut into the *natural* soil; in distinction to a retaining-wall or revetment, which is built to sustain earth *deposited* behind it.

Breech; the hind part of a cannon, &c.

Bridge, or *bridge-piece*, or *bridge bar;* a narrow strip placed across an opening, for supporting something without closing too much of the opening.

Bronze is composed of copper and tin.

Bulkhead; on ships, &c, the timber partitions across them. Also, a long face of wharf parallel to the stream.

Buoy; a floating body, fastened by a chain or rope to some sunk body, as a guide for finding the latter. Sometimes also used to indicate channels, shoals, rocks, &c.

Burnish; to polish by rubbing; chiefly applies to metals.

Bush; to line a circular hole by a ring of metal, to prevent the hole from wearing larger. Also, when a piece is cut out, and another piece neatly inserted into the cavity, the last piece is sometimes said to be bushed in; sometimes it is called a plug.

Buttress; a vertical projecting piece of brickwork or masonry, built in front of a wall to strengthen it.

Caisson; a large wooden box with sides that may be detached and floated away.

Caliber; the inner diameter, or bore.

Calipers; compasses or dividers with curved legs, for measuring outside diameters.

Calk, or *caulk;* to fill seams or joints with something to prevent leaking.

Calking iron; a tool for forcing calking into a joint.

Camb, or *cam*, or *wiper;* a piece fixed upon a revolving shaft in such a manner as to produce an alternating or reciprocation motion in something in contact with the cam. An eccentric.

Camber; a slight upward curve given to a beam or truss, to allow for settling.

Camel; a kind of barges or hollow floating vessels, which, when filled with water, are fastened to the sides of a ship; and the water being then pumped out, they rise by their buoyancy; and lift the ship so that she can float in shallower water.

Cantilevers; projecting pieces for supporting an upper balcony, &c.

Cants, rims, or *shroudings;* the pieces forming the ends of the buckets of water-wheels, to prevent the water from spilling endwise.

Capstan; a long hollow rope-drum surrounding a strong vertical pivot, upon the head of which it rests, and around which it turns. Its top is a thick projecting circular piece, having holes around its outer edge or circumference, for the insertion of the ends of levers; or capstan-bars. It is a kind of *vertical windlass*.

Case-harden; to convert the outer surface of wrought iron into steel, by heating it while in contact with charcoal.

Casemate; in fortification; the small apartment in which a cannon stands.

Castors; rollers usually combined with swivels; as those used under heavy furniture, &c.

Causeway; a raised footway or roadway.

Cavetto; a moulding consisting of a receding quadrant of a circle.

Cementation; the process of converting wrought iron into steel, by heating it in contact with char

process produces blisters on the steel bars; hence *blister* steel. These are removed, and ompacted, by reheating it, and then subjecting it to a tilt-hammer. It is then *tilted* steel, teel. Or if the blister steel is broken up; remelted; and then run into ingots or blocks; it *et*, or *ingot steel;* which is harder and closer-grained than tilted steel. It may be softened, ecome less brittle, by annealing. The ingots may be converted into bars by either rolling ing, the same as shear and blister.
the supports of an arch while being built.
' *gravity*. See Art 56, of Force, p 681. Also see Cen of Grav, p 442.
' *gyration*. Suppose a body free to revolve around an axis which passes through it in any or to oscillate like a pendulum hung from a point of suspension. Then suppose in either tain given amount of force to be applied to the body, at a certain given dist from the axis, e point of suspension, so as to impart to the body an angular vel; or in other words, to describe a number of *degrees* per sec. Now, there will be a certain point in the body, such entire wt of the body were there concentrated, then the same force as before, applied at the rom the axis, or from the point of suspension as before, would impart to the body the same tion as before. This point is the center of gyration; and its dist from the axis, or from the pension, is the *Radius of gyration*, of the body. To find the position of this center, or the his rad, see Moment of Inertia, in this glossary. One use of the center of gyration is to calculate the momentum or moving force in a revolving or oscillating body. This force in is found by mult the wt of the body in lbs, by the vert height in ft through which it to fall, in order to acquire the vel which its center of gyration has. This height may be ble 10, p 552, of Hydraulics.
' *oscillation*, or *of vibration*. See Rem 2, of Pendulums, p 173.
' *percussion*, in a moving body, is that point which would strike an opposing body with e than any other point would. If the opposing body is immovable, it will receive *all* the igid moving body which strikes with its center of percussion. See Pendulum, page 173.
a shallow well for receiving waste water, filth, &c.
: means much the same as bevel; but applies more especially when two edges are cut away n either a chamfer-groove, (see 14, p 294, of Trusses,) or a projecting sharp edge.
two flat parallel pieces confining something between them. See *w*, at 15, of Figs 21½, of 165.
chill-hardening, or *chill-casting;* giving great hardness to the outside of cast-iron, by ato a mould made of iron instead of wood. The iron mould causes the outside or skin of to cool very rapidly; and this for some unknown reason increases its hardness. This pro- ently confounded with case-hardening.
ny piece used for filling up a chance hole, or vacancy.
he arrangement attached to the revolving shaft, arbor, or mandril of a lathe, for holding be turned.
ill; a long iron bar, with a cutting end of steel; much used in quarrying, and worked by ad letting it fall. When worked by blows of a hammer or sledge it is called a jumper.
cyma: a moulding nearly in shape of an S. When the upper part is concave, it is called a; when convex, a cima reversa. See page 67.
ve. See Valve.
a piece fastened by tongue and groove, transversely along the end of others, to keep them ag. A kind of open collar, which, being closed by a clam-screw, holds tight what it sur- se Cramp.
rds; short thin boards, shingle-shaped, and used instead of shingles.
split provided at the end of an iron bar, or of a hammer, &c, to take hold of the heads of kes for drawing them out; as in a common claw-hammer.
piece merely bolted to another to serve as a support for something else; as at 7, 8, 9, 10, f Trusses. Often used on shipboard for fastening ropes to, as at 11. Also a piece of 1 across two or more other boards, for holding them together, as is often done in temporary

ee Shackle.
ee Ratchet.
stening like that on the tops of the Y's of a spirit level; being a kind of half collar opening

pplied to various arrangements at the ends of separate shafts, and which by clutching or to each other cause both shafts to revolve together. A kind of coupling.
kind of valve for the discharge of liquids, air, steam, &c.
t; a coefficient of friction, safety, tensile strength, &c, may usually be taken to be a num- shows the proportion (or rather the ratio) which friction, safety, tensile strength, &c. bear something else which is not generally expressed at the time, but is well understood. Thus, ay that the coeff of friction of one body upon another is $\frac{1}{10}$, &c, it is understood that the n the proportion of $\frac{1}{10}$th of the *pressure* which produces it. A coeff of safety of 3, means fety has a proportion or ratio of 3 to 1 to the *theoretical breaking load*. A coeff of 500 lbs; s, &c, of tensile strength of any material, denotes that said strength is in the proportion r of 20 tons, &c, to each *square inch of transverse section*, &c.
m; an enclosure built in the water, and then pumped dry, so as to permit masonry or to be carried on inside of it.
tooth of a cog-wheel.
flat ring surrounding anything closely.
sm; a horizontal timber stretching from one to another of two rafters which meet at top; he main tie-beam. See 21, Figs 42, of Trusses. Page 294.
: artificial stone formed by mixing broken stone, gravel, &c, with common lime. When ement is used instead of lime, the mixture is called beton. The terms "lime concrete" nt concrete" would be convenient.
g-rod; a piece which connects a crank with something which moves it, or to which it n.
a kind of ornamental bracket, somewhat in shape of an S; much used in cornices, &c, *ng ornamental* mouldings above it.
lat plates of stone, iron, &c, placed on the tops of walls exposed to the weather.
horizontal projecting piece which assists in supporting one resting upon it, which prop

Core; anything serving as a mould for anything else to be formed around. A term much used in foundries.

Cornice; the ornamental projection at the eaves of a building, or at the top of a pier, or of any other structure.

Cotter-bolt, or *key-bolt;* a bolt which, instead of a screw and nut at one end, has a slot cut through it near that end, for the insertion of a wedge-shaped key or cotter, for keeping it in its place. Sometimes the ends of these keys are split, so as to spread open after being inserted, so as not to be jolted out of place.

Counterfort; vertical projections of masonry or brickwork built at intervals along the *back* of a wall to strengthen it; and generally of very little use.

Counter-shaft; a secondary shaft or axle which receives motion from the principal one.

Countersunk. See Reaming.

Counter-weight; or counter-balance; any weight used to balance another.

Couplings; a term of very general application to arrangements for connecting two shafts so that they shall revolve together.

Crab; a short shaft or axle, which serves as a rope-drum in raising weights; and is revolved either by cog-wheels, a winch, or by levers or handspikes, inserted in holes around its circumference like a windlass, or capstan, of which it is a variety. It may be either vertical or horizontal. It is often set in a frame, to be carried from place to place.

Cradle; applied to various kinds of timber supports, which partly enclose the mass sustained.

Cramp; a short bar of metal, having its two ends bent downward at right angles for insertion into two adjoining pieces of stone, wood, &c, to hold them together. Much used at the ends of coping-stones. Also a similar bent piece, with a set-screw passing through one of the bent ends, for holding things tight between it and the other end. This last is also called a clamp.

Crane; a hoisting machine consisting of a revolving vertical *post* or *stalk;* a projecting *jib;* and a *stay* for sustaining the outer end of the jib. The stay may be either a strut or a tie. There are also cog-wheels, a rope drum or barrel, with a winch, ropes, pulleys, &c. In a crane the post, jib, and stay do not change their relative positions, as they do in a derrick.

Crank; a double bend at right angles, somewhat like a Z, at the end of a shaft or axle, and forming a kind of handle by which the axle may be made to revolve. Sometimes, as in common grindstones, this crank is formed of a separate piece removable at pleasure. That part of this piece which has the square opening in it for fitting it to the square end of the axle, is called the *crank-arm;* and the other part the *crank-handle.* A *bell-crank* consists of 4 bends at right angles at the center of an axle, forming in it a kind of U. A *double crank* consists of two bell cranks arranged thus, $\sqcup\!\sqcap$. The bend in the U forms the *crank-wrist.* The term bell crank is applied also to those used in fixing common dwelling house bells: and to larger ones on the same principle. A *crank-pin* is a pin projecting from a revolving *wheel,* disk, or other body, and serving as a crank-handle. A *crank-shaft* is a shaft which has a crank in it, or at its end. A *cranked* shaft has it in it only. A ship or other vessel is said to be crank when its breadth is so small in proportion to its depth as to make it liable to upset easily; or when the same liability is caused by want of sufficient ballast.

Crest; that top part of a dam over which the water pours.

Cross-cut saw; a large horizontal saw worked by two men, one at each end.

Cross-head; a piece attached across the end (or near it) of another piece, and at right angles to it, so as to form a kind of T or cross. Often seen on piston rods, which they serve to keep in place by resting on the slides, or guides.

Crowbar; a bar of iron used as a lever for various purposes; often pointed at one end.

Crown, or *contrate wheel;* a cog-wheel in which the teeth stand not upon its outer circumference as usual, but upon the plane of its circle.

Curb; a broad flat circular ring of wood, iron, or stone, placed under the bottoms of circular walls, as in a well, or shaft, to prevent unequal settlement; or built into the walls at intervals, for the same purpose. Has many other meanings.

Cut-off; an arrangement for cutting off the steam from a cylinder before the piston has made its full stroke. Also a channel cut through a narrow neck of land, to straighten the course of a river.

Cutwater, or starling; the projecting ends of a bridge pier, &c, usually so shaped as to allow water, ice, &c, to strike them with but little injury.

Damper; a door or valve to regulate the admission of air to a furnace, stove, &c.

Dead load; the cars, engine, &c, in a train; non-paying load.

Dead points; those two points in the revolution of a crank, when the crank arm is parallel with the rod which connects it with the moving power; and at which said rod neither pulls nor pushes the crank-arm.

Declination, of the sun, or of a star, is its latitude, or angle north or south of the earth's equator at the time of observation.

Declivity; a downward slope or descent of ground, &c.

Dentils; blocks constituting ornaments in a cornice; placed at short intervals apart, they resemble teeth. When, instead of mere blocks, they are handsomely carved in various shapes, they are called modillions.

Derrick; a kind of crane, differing from common ones, chiefly in the fact that the rope or chain which forms the stay may be let out or hauled in at pleasure, thus raising or lowering the inclination of a jib; thereby enabling the raised load to be placed vertically at the required spot. This cannot be done with a crane, which, therefore, is not as well adapted for laying heavy masonry, especially at great heights.

Diaphram; a thin plate or partition placed across a tube or other hollow body.

Die; that part of a stamp that gives the impression. Dies are also two flat plates of hardened steel, on an edge of each of which is hollowed out a semicircular half of a short female screw. When these plates are put in contact they form a complete female screw, like that in a nut; and being strongly held together by an iron boxing called the die-stocks, which have long handles for revolving them, they constitute a mould or cutter for forming threads on a male screw. Also the main body of a pedestal.

Dip; in geology, either the angle which the slope of a stratum forms with a horizontal; or the direction by compass, toward which it slopes. In surveying, the inclination at which an unbalanced compass-needle rests on its pivot after being magnetized.

Disk; a flat circular piece.

Dock; an artificial enclosure, either partial or total, in which ships and other vessels are placed for being loaded or unloaded, or repaired. The first is a wet dock; the last a dry one.

Dog-iron; a short bar of iron, forming a kind of cramp, with its ends bent down at right angles and pointed, so as to hold together two pieces into which they are driven. Often used for temporary purposes. It is also called a dog-iron when only one end is bent down and pointed for driving.

other end being formed into an eye or a handle by which the piece into which the other end is driven may be hauled or towed away.

Donkey-engine; a small steam engine attached to a large one, and fed from the same boiler. It is used for pumping water into the boiler.

Double crank. See Crank.

Double keys. See K, Figs 42, of Trusses; page 234.

Dovetail; a joint like 20, Figs 42 of Trusses; it is a poor one for timber when there is much strain, being then apt to draw out more or less.

Dowel; a straight pin of wood or metal, inserted part way into each of two faces which it unites.

Draft; the depth to which a floating vessel sinks in the water; in other words the water it draws.

Draught; a drawing. A narrow level stripe which a stonecutter first cuts around the edges of a rough stone, to guide him in dressing off the face thus enclosed by the draught.

Draw-plate; a plate of very hard steel, pierced with small circular holes of different diameters, through which in succession rods of iron are drawn, and thus lengthened out into wire. Sometimes the holes are drilled through diamond or ruby, &c, instead of steel.

Drift; a horizontal or inclined passage-way, or small tunnel, in mines, &c. To float away with a current. Trees, &c, carried along by freshets.

Drip; a small channel cut under the lower projecting edge of coping, &c, so that rain when it reaches that point will drip or fall off, instead of finding its way horizontally beneath to the wall, which it would make damp.

Drop; short pieces of nearly complete cylinders, placed at small distances apart, in a row like teeth, as an ornament to cornices, &c.

Drum; a revolving cylinder around which ropes or belts either travel or are wound. When narrow and used with belts they are called pulleys.

Dry-rot; decay in such portions of the timber of houses, bridges, &c, as are exposed to dampness, especially in confined warm situations. The timber in cellars and basement stories is more liable to it than in other parts, owing to the greater dampness absorbed by the brickwork from the ground. Contact with lime or mortar hastens dry rot. The ends of girders, joists, &c, resting on damp walls, may be partially protected by placing pieces of slate or sheet iron under them. The painting or tarring of *unseasoned* timber expedites internal dry rot. A thorough soaking of timber in a solution of 28 grains of quicklime to 1 gallon of water is said to be a preventive of dry rot; but the best process for that purpose is saturation with creosote or carbolic acid by the Robbins mode.

Dyke; mounds of earth, &c, built to prevent overflow from rivers or the sea. A kind of geological irregularity or disturbance, consisting of a stratum of rock injected as it were by volcanic action, between or across strata of rocks of another kind. A levee.

Eccentric; a circular plate or pulley, surrounded by a loose ring, and attached to a revolving shaft, and moving around with it, but not having the same center; for producing an alternate motion. Often used instead of a crank, as they do not weaken the axle by requiring it to be bent. There are many modifications.

Escarpment; a nearly vertical natural face of rock or soil.

Escutcheon; the little outside movable plate that protects the keyhole of a lock from dust.

Eye; a circular hole in a flat bar, &c, for receiving a pin, or for other purposes.

Eye and strap; a hinge common for outside shutters, &c, one part consisting of an iron strap one end of which is forged into a pin at right angles to it; and the other part, of a spike with an eye, through which the pin passes. When the eye is on the strap, and the pin on the spike, it is called a hook and strap. Such hinges are sometimes called "backflaps."

Eye-bolt; a bolt which has an eye at one end.

Face-wall; one built to sustain a face cut into natural earth, in distinction to a retaining-wall, which supports earth deposited behind it.

Fall; the rope used with pulleys in hoisting.

False-works; the scaffold, center, or other temporary supports for a structure while it is being built. In very swift streams it is sometimes necessary to sink cribs filled with stone, as a base for false-works to foot upon.

Fascines; bundles of twigs and small branches, for forming foundations on soft ground.

Fatigue; of materials; the increase of weakness produced by frequent bending; or by sustaining heavy loads for a long time.

Faucet; a short tube for emptying liquids from a cask, &c; the flow is stopped by a spigot. The wider end of a common cast-iron water or gas pipe.

Feather; a slightly projecting narrow rib lengthwise of a shaft, and which, catching into a corresponding groove in anything that surrounds and slides along the shaft, will hold it fast at any required part of the length of the feather. Has other applications.

Feather-edge; when one edge of a board, &c, is thinner than the other.

Felloe, or *felly;* the circular rim of a wheel, into which the outer ends of the spokes fit; and which is often surrounded by a tire.

Felt; a kind of coarse fabric or cloth made of fibres of hair, wool, coarse paper, &c, by pressure, and not by weaving.

Fender; a piece for protecting one thing from being broken or injured by blows from another; frequently vertical timbers along the outer faces of wharves, to prevent injury from the rubbing of vessels.

Fender-piles; piles driven to ward off accidental floating bodies.

Ferrule; a broad metallic ring or thimble put around anything to keep it from splitting or breaking. A small sleeve.

Fillet; a plain narrow flat moulding in a cornice, &c. See Flatband.

Fish; to join two beams, &c, by fastening other long pieces to their sides.

Flags; broad flat stones for paving.

Flange; a projecting ledge or rim.

Flashings; broad strips of sheet lead, copper, tin, &c, with one edge inserted into the joints of brickwork or masonry an inch or two above a roof, &c; and projecting out several inches, so as to be flattened down close to the roof, to prevent rain from leaking through the joint between the roof and the brick chimney, &c, which projects above it.

Flasks; upper and lower; the two parts of the box which contains the mould into which melted iron is poured for castings.

Flatting; causing painting to have a dead or dull, instead of a glossy finish, by using turpentine instead of oil in the last coat.

Flight; a straight flight of steps in a stairway.

Floodgate; a gate to let off excess of water in floods, or at other times.

Flume; a ditch, trough, or other channel of moderate size for conducting water. The ditches or culverts through which surplus water passes from an upper to a lower reach of a canal.

Flush; forming an even continuous line or surface. To clean out a line of pipes, sewers, gutters, &c, by letting on a sudden rush of water.

Fluxes; various substances used to prevent the instantaneous formation of rust when welding two pieces of hot metal together. Such rust would cause a weak weld. Borax is used for wrought iron; a mixture of borax and sal ammoniac for steel; chloride of zinc for zinc; sal ammoniac for copper or brass; tallow or resin for lead.

Fly-wheel; a heavy revolving wheel for equalizing the motion of machinery.

Foaming; an undue amount of boiling, caused by grease or dirt in a boiler.

Follower; any cog-wheel that is driven by another; that other is the *leader.*

Forceps; any tools for holding things, as by pincers, or pliers.

Forebay, or *penstock;* the reservoir from which the water passes immediately to a water-wheel.

Forge; to work wrought iron into shape by first softening it by heat, and then hammering it into the required form.

Forge-hammer; a heavy hammer for forging large pieces; and worked by machinery.

Fuzzail; a thin wedge inserted into a slit at the lower end of a pin, so that as the pin is driven down, the wedge enters it and causes it to swell, and hold more firmly.

Frame; to put together pieces of timber or metal so as to form a truss, door, or other structure. The thing so framed.

Friction-rollers; hard cylinders placed under a body, that it may be moved more readily than by sliding. See Fig 43; page 295.

Friction-wheels; wheels so placed that the journals of a shaft may rest upon their rims, and thus be enabled to revolve with diminished friction. See page 601.

Frieze; in architecture, the portion between the architrave and cornice. The term is often applied when there is no architrave.

Fulcrum; the point about which a lever turns.

Furrings; pieces placed upon others which are too low, merely to bring their upper surfaces up to a required level; as is often done with joists, when one or more are too low; a kind of chock.

Fuze, or *fuse;* to melt. A slow match, which, by burning for some time before the fire reaches the powder, gives the men engaged in blasting time to get out of the way of flying fragments of stone.

Gasket; rope-yarn or hemp, used for stuffing at the joints of water-pipes, &c.

Gearing; a train of cog-wheels. Now much supplanted by belts.

Gib; the piece of metal somewhat of this shape, ⊔, often used in the same hole with a wedge-shaped key for confining pieces together. In common use for fastening the strap to the stub-end of the connecting-rod of an engine.

Gin; a revolving vertical axis, usually furnished with a rope-drum, and having one or more long arms or levers, by means of which it is worked by horses walking in a circle around it. Used for hoisting. Cotton-gin, a machine for separating cotton from its seeds.

Girder; a beam larger than a common joist, and used for a similar purpose.

Glacis; in fortification, an easy slope of earth.

Gland. See Stuffing-box. Also, a kind of coupling for shafts.

Glue; a cement for wood, prepared chiefly from the gelatine furnished by boiling the parings of hides. Good glue will hold two pieces of wood together with a force of from 400 to 750 lbs per sq in.

Governor; two balls so attached to an upright revolving axis as to fly outward by their centrifugal force, and thus regulate a valve.

Grapnel; a kind of compound hook with several curved points, for finding things in deep water.

Grillage; a kind of network of timbers laid crossing each other at right angles; frequently placed on the heads of piles, for supporting piers of bridges, and other masonry.

Groin; an arch formed by two segmental arches or vaults intersecting each other at right angles. Also, a kind of pier built from the shore outward, to intercept shingle or gravel.

Groove; a small channel, as at 17 and 18 of p 294 of Trusses. A triangular one is called a chamfered groove.

Ground-swell; waves which continue after a storm has ceased; or caused by storms at a distance.

Grout; thin mortar, to be poured into the interstices between stones or bricks.

Gudgeons; the metal journals of a horizontal shaft, such as that of a water-wheel. The diam of a gudgeon should not be less than its length; and this being assumed, the diam in ins of a cast-iron gudgeon may be found thus: Divide the wt of the wheel and water, or whatever wt is sustained, by 2. Find sq rt of rem. Divide this sq rt by 20. For wrought iron, add $\frac{1}{20}$th part.

Gun-metal, or bronze: a compound of copper and tin, sometimes used for cannon. Also, a quality of cast iron fit for the same purpose.

Gussets; plain triangular pieces of plate iron, riveted by their vertical and horizontal legs to the sides, tops, and bottoms of box-girders, tubular bridges, &c, inside, for strengthening their angles.

Guys; ropes or chains used to prevent anything from swinging or moving about.

Gyrate; to revolve around a central axis, or point.

Halving; to notch together two timbers which cross each other, so deeply that the joint thickness shall equal only that of one whole timber.

Hammer dress; to dress the face of a stone by slight blows of a hammer with a cutting edge. The *patent* hammer for such purposes has several such edges placed parallel to each other, each of which may be removed and replaced at pleasure.

Hand-lever; in an engine, a lever to be worked by hand instead of by steam.

Handspike; a wooden lever for working a capstan or windlass; or other purposes.

Hand-wheel; a wheel used instead of a spanner, wrench, winch, or lever of any kind, for screwing nuts, or for raising weights, or for steering with a rudder, &c.

Hangers, or *pendent brackets;* fixtures projecting below a ceiling, to support the journals of long lines of shafting; and for other purpose. Should be "self-adjusting."

Hasp; a piece of metal with an opening for folding it over a staple.

Hatchway; a horizontal opening or doorway in a floor, or in the deck of a vessel.

Haunches; the parts of an arch from the keystone to the skewback.

Head-block; a block on which a pillow-block rests.

Header; a stone or brick laid lengthwise at right angles to the face of the masonry.

Heading; in tunnelling, a small driftway or passage excavated in advance of the main body of the tunnel, but forming part of it; for facilitating the work.

Headway; the clear height overhead. Progress.

Heel-post; that on which a lock gate turns on its pivot.

Helve; the handle of an axe.

hose commonly used on the doors of dwellings are called butts, or butt hinges. See Eye
Rising hinges are such as allow the door to rise a little as it is opened, and these cause
shut itself.
or *hipped roof;* one that slopes four ways; thus forming angles called hips.
; a temporary close fence of boards, placed around a work in progress, to exclude

plates, or *anchors;* strong broad plates of iron sunk into the ground, and generally sur-
masonry; for resisting the pull of the cables of suspension bridges; and for other simi-
s.
l strap. See Eye and strap.
the sloping timbers which carry the steps in a staircase.
; in rolling mills, &c, the vertical supports for the boxes in which the journals revolve.
nave; the central part of a wheel, through which the axletree passes, and from which
radiate.
the upper part of a pier from which an arch springs.
lump of cast metal, generally somewhat wedge shaped. A pig of cast iron is an ingot.
an inverted arch frequently built under openings, in order to distribute the pressure more
the foundation.
aising instrument, consisting of an iron rack, in connection with a short stout timber
orts it, and worked by cog-wheels and a winch. A *screw-jack* is a large screw working
frame, the base of which serves for it to stand on; and which is caused to revolve and
ng the load on top of it, by turning a nut, or otherwise.
ers, or common rafters; small rafters laid on the purlins of a roof, for supporting the
ths, &c.
; a spike whose sides are jagged or notched, with the mistaken idea that its holding power
nuch increased. If a spike or bolt is first put into its place loosely, and then has melted
ound it, the jagging does assist; but not when it is driven into wood.
he sides of an opening through a wall, &c; as door, window, and fireplace jambs.
inge; the facing of woodwork with which jambs are covered and hidden.
opening, often V-shaped, the inner edges of which are for holding something in place.
jetty; a pier, mound, or mole projecting into the water; as a wharf-pier, &c.
upper projecting member or arm of a crane, supported by the stay.
a very narrow thin saw worked vertically by machinery, and used for sawing curved
in boards.
joint like that at 3 or 4, &c, p 294, of Trusses, for receiving the pressure of a strut at
s or nearly so. Also applied to squared blocks of stone sometimes inserted between
masonry to prevent sliding, &c.
nding joists are girders for sustaining common joists. The common ones are then called
ists. *Ceiling* joists are small ones under roof trusses, or under girders, and for sustain-
the plastered ceiling.
box; a fixture upon which a journal rests and revolves, instead of a plummer-block.
; the cylindrical supporting ends of a horizontal revolving shaft. Their length is usually
1½ times their diam. In lines of shafting 4 diams. To find the diam, see Gudgeon.
a drill used for boring holes in stone by aid of blows of a sledge-hammer.
small anchor.
the pieces of metal or wood which keep a sliding bolt in its place, and guide it in sliding.
s opening or narrow slit made in sawing.
See Cotter-bolt.
; the center stone of an arch.
he bucket used for raising earth, stone, &c, from shafts or mines.
t, king-rod; the center post, vertical piece, or rod, in a truss; all those on each side of it
osts, or queen-rods. Frequently called simply kings and queens.
piece of metal or wood bent at an angle; to serve as a bracket, or as a means of uniting
s which form with each other a similar angle.
or *sheeting;* a covering of loose plank; as that placed upon centers, and supporting the
Also, an outer wooden casing to locomotive boilers and others.
; the resting-place at the end of a flight of stairs.
wheel. See Trundle.
place one piece upon another, with the edge of one reaching beyond that of the other.
ing; welding together pieces that have first been lapped; in distinction to butt-welding.
onounced *leed;*) in locomotives, a certain amount of opening of the port-valve before
of the piston begins. The distance to which earth is hauled or wheeled.
a cog-wheel that gives motion to the next one or follower.
beam; leading-pit; one placed as a guide for placing others.
wheels; in a locomotive, those frequently placed in front of the driving-wheels.
the cogs of pinions.
part projecting over like a shelf; a rock so projecting. A narrow strip of board nailed
r boards, to hold them together, as in temporary ledge-doors.
n arrangement composed of 2 or 3 pieces of metal let into a wedge-shaped hole in a block
r which to raise the block.
a scow, raft, or other vessel, used for unloading vessels out from the shore.
; a pin near the end of an axle, to hold the wheel on.
e of the divisions of a chain; or a piece shaped like one.
ion; a device for regulating the movement of the main or port valve in a locomotive.
horizontal beam across an opening in a wall, as seen in windows, doors, &c. When of
and supporting heavy brickwork or masonry, it is called a breast-summer, or bressummer.
ose common door locks which are entirely concealed within the thickness of the door, are
ice locks; those which are screwed against the face of a door, rim locks. It must be remem-
locks are "right and left."
a kind of vertical window, frequently at the tops of roofs of depots, &c, provided with hor-
s, which permit ventilation, and exclude rain.
: the shape of a rhomb; often called diamond-shaped.
asting, small projections from the general surface, and for various purposes, such as for
ody; or for a flange for joining it to another; or for a support for something else.
e *wooden hammer* used by stonecutters.

Mandrel; an iron rod used as a core around which a flat piece may be bent into a cylindrical shape. Also the shaft that carries the chuck of a lathe.

Manhole; an opening by which a man can enter a boiler, culvert, &c, to clean or repair it.

Mattock; a kind of pick with broad edges for digging.

Maul; a heavy wooden hammer.

Mean, arithmetical; half the sum of two numbers.

" , *geometrical;* the sq rt of the product of two numbers.

Mean-proportional; the same as the geometrical mean.

Meridian; a north and south line. Noon.

Mitre-joint; a joint formed along the diagonal line where the ends of two pieces are united at an angle with each other.

Mitre-sill; the sill against which the lock gates of a canal shut.

Modulus; a datum serving as a means of comparison.

Modulus of elasticity; see pp 177 and 632.

Moment; tendency of force acting with leverage.

Moment of inertia, of a body either revolving (or imagined to revolve) around an axis, as a grindstone; or oscillating, like a pendulum. Suppose that the shortest dist from the axis (or from the point of suspension, as the case may be,) to each single individual particle of the body, has been measured; also that each of these dists has been squared; and that each square has been mult by the weight of that particle to which the dist was measured; also that all these last products are added into one sum. That sum is the moment of inertia of the body; and is the I of scientific writers. In practice we may suppose the body to be divided into portions of a cub inch, or some other size; and use these instead of the theoretical infinitely small particles. When the moment of inertia of a mere *surface* is wanted, (instead of that of a *solid*,) the surf must be supposed to be div into an infinite number of small *areas*, which must be used instead of the *weights* of the particles of the solid body. For an example, see Remark to Art 25, of Strength of Materials. If the moment of inertia is div by the wt of the entire solid body; or by the area of the entire surf, as the case may be; the square root of the quot will be the *radius of gyration*, of the body or surf. If the length of the rad of gyration be laid off perp from the axis, or from the point of suspension, it will reach to the *center of gyration*. See Cen of Gyration, in this Glossary.

Moment of rupture, or *of bending;* the tendency which any load or force exerts to break or bend a body by the aid of leverage. Its amount is found in foot-pounds by multiplying the force in lbs, by the length of leverage in feet between it and that part of the body upon which the tendency is exerted.

Moment of stability. See Art 69, of Force in Rigid Bodies, page 489.

Momentum; moving force. See page 448, Force in Rigid Bodies.

Monkey; the hammer or ram of a pile-driver.

Monkey-wrench, or *screw-wrench;* a spanner, the gripping end of which can be adjusted by means of a screw to fit objects of different sizes.

Moorings; fixtures to which ships, &c, can make fast.

Mortise; a hole cut in one piece, for receiving the tenon which projects from another piece.

Muck; soft surface soil containing much vegetable matter.

Muntins, or mullions; the vertical pieces which separate the panes in a window-sash.

Nailing-blocks; blocks of wood inserted in walls of stone or brick, for nailing washboards, &c, to.

Nave; the main body of a building, having connecting wings or aisles on each side of it. The hub of a wheel.

Newel; the open space surrounded by a stairway.

Newel-post; a vertical post sometimes used for sustaining the outer ends of steps. Also the large baluster often placed at the foot of a stairway.

Nippers; pincers. An arrangement of two curved arms for catching hold of anything.

Normal; perpendicular to. According to rule, or to correct principles.

Nosing; the slight projection often given to the front edge of the tread of a step; usually rounded.

Nut, or *burr;* the short piece with a central female screw, used on the end of a screw-bolt, &c, for keeping it in place.

Ogee; a moulding in shape of an S, the same as a cima.

Ordinate; a line drawn at right angles from the axis of a curve, and extending to the curve.

Oscillate; to swing backward and forward like a pendulum.

Out of wind, pronounced *wynd;* perfectly straight or flat.

Ovolo; a projecting convex moulding of quarter of a circle; when it is concave it is a cavetto, or hollow.

Packing; the material placed in a stuffing-box, &c, to prevent leaks.

Packing-pieces; short pieces inserted between two others which are to be riveted or bolted together, to prevent their coming in contact with each other.

Pall, or *pawl.* See Ratchet.

Parapet; a wall or any kind of fence or railing to prevent persons from falling off.

Parcel; to wrap canvas or rags round a rope.

Parge; to make the inside of a flue smooth by plastering it.

Patent hammer; a hammer with several parallel sharp edges for dressing stone.

Pay. To cover a surface with tar, pitch, &c. A ship word.

Pay out. To slacken, or let out rope.

Pediment; the triangular space in the face of a wall that is included between the two sloping sides of the roof and a line joining the eaves.

Penstock. See Forebay.

Pier; the support of two adjacent arches. The wall space between windows, &c. A structure built out into the water.

Pierre perdue; lost stone; *random stone*, or rough stones thrown into the water, and let find their own slope.

Pilaster; a thin flat projection from the face of a wall, as a kind of ornamental substitute for a column.

Pile-planks; planks driven like piles.

Pillow-block, or *plummer-block;* a kind of metal chair or support, upon which the journals of horizontal shafts are generally made to rest, and on which they revolve.

Pinion; a small cog wheel which gives motion to a larger one.

Pintle; a vertical projecting pin like that often placed at the tops of crane-posts, and over which the holding rings at the tops of the wooden guys fit. Also, such as is used for the hinges of rudders, or of window-shutters to turn around.

lope of a roof, &c. The distance from center to center of the teeth of a cog-wheel, or
a screw. Boiled tar.
onnecting-rod for transmitting motion from a prime mover to machinery at a distance,

arge saw worked vertically by two men, one of whom (the pitman) stands in a pit.
wer end of a vertical revolving shaft, whether a part of the shaft itself, or attached to
be flat; and both it and the step or socket upon which it rests should be of hard steel.
has to revolve rapidly and continuously, it is well to proportion its diam. so as not to
more than 250 lbs per sq inch; otherwise it will wear quickly. Dust and grit should
ason be carefully guarded against. Pivots which revolve but seldom, and slowly, as
oad turntable, may be trusted with half a ton, or even a whole ton per sq inch. As a
-iron pivots should not be loaded with more than half as much as steel ones. A steel
ided to the foot of its cast-iron shaft; or may be inserted part way into it; and the
ened by iron bands shrunk on.
polish metals by rubbing with a hard smooth tool.
outfit of machinery, &c, necessary for carrying on any kind of work.
; a small vertical strip of iron or wood nailed along projecting angles in rooms, to
ster at those parts.
plain, flat, wide, slightly projecting strip, generally for ornament. When narrow, it
.
ad of pincers.
square lowest member of the base of a column or pillar.
e inserted to stop a hole. *Screw-plug*, a plug that is screwed into a hole.
ical.
plumb-bob; a weight at the lower end of a string, for testing verticality.
kind of solid piston, or one without a valve.
d of pointed chisel for dressing stone. To put a finish to masonry by touching up the
oints. To dress stone with a point and mallet.
a longitudinal timber resting on the ends of tie-beams of roofs; and for supporting
common or jack rafters, when such are used.
ening or passage controlled by a valve.
ut on the first coat of paint. Priming also is when water passes into a steam cylinder
steam.
If parallel straight lines be imagined to be drawn in any one given direction, from
any surface *s*, whether flat, curved, or irregular, then if all these lines be supposed to
or cut by a plane, either at right angles to their direction, or obliquely, the figure
oss-section thus made would form upon said plane, is called the projection of the sur-
h lines be supposed to be drawn from a person's face, in a direction in front of him,
y a plane at right angles to their direction, their projection on the plane would be the
ice portrait. If the lines be drawn *sideways* from his face, the projection will be his
projection of a globe upon a flat plane, will evidently be a circle if the plane cuts the
angles; and an ellipse if it cuts them obliquely. Shadows cast by the sun are projec-

See Ratio.
th well rammed into a trench, &c, to prevent leaking. A process for converting cast
ght by a puddling furnace.
mill for tempering clay for bricks or pottery, &c.
rcular hoop which carries a belt in machinery.
machinery, a small short pedestal or stand. *Puppet-valve*. See Valves.
horizontal pieces placed on rafters, for supporting the roof covering.
put-locks; horizontal pieces supporting the floor of a scaffold; one end being inserted
les left for that purpose in the masonry.
arf.
hollow into which a quoin-post of a canal lock-gate fits. Stones, usually dressed, placed
cal angles of buildings, chiefly for ornament.
the vertical post on which a lock-gate turns. The heel-post.
bate; a half groove along the edge of a board, &c. See 16, p 294, of Trusses, where
e shown overlapping each other.
annel which conducts water either to or from a water-wheel: the first is a head race;
race. The waves produced by the meeting of strong opposing currents; also, a rapid
st.
nion; the rack is a straight row of cogs on a bar, and called a rack-bar; and the pinion
wheel working into it.
ration. See Center of gyration.
ee Jag-spike.
one with teeth or pins which catch into the links of a chain.
orizontal pieces in a door.
ammer of a pile-driver.
e; rip rap, or rough stones thrown promiscuously into the water, to form a founda-

rse file.
pall: the former is sometimes a straight bar, at others a wheel: in either case it is
teeth between which the pall drops and prevents backward motion. Used for safety
chinery, &c. The pall is sometimes called a click.
ple ratio is a number denoting how often one quantity is contained in another. Thus,
o 10 is $\frac{5}{10}$, or $\frac{1}{2}$; and the ratio of 10 to 5 is $\frac{10}{5}$, or 2. When, of four numbers, two
ther the same ratio that the other two have, the numbers are said to be *in proportion*
Thus, 6 has the same ratio (2) to 3, as 100 has to 50; therefore, 6, 3, 100, and 50, are
roportion; or, as 6 : 3 :: 100 : 50. In other words, an *equality of ratios* is called pro-
o and proportion are often confounded with one another; but the error is one of no

ole, wider at top than at bottom, (see 19, p 294, of Trusses,) through which a screw,
e inserted, so that its head shall not project above the general surface, is said to be
ned out; or to be countersunk.
a number is the quotient found by dividing 1 by that number.
motion. See Alternating motion.

Re-entering angle; an angle or corner projecting inward.

Revetment; steep facing of stone to the sides of a ditch or parapet in fortification. A retaining-wall.

Rib; the curved pieces which form the arches of iron or wooden bridges, &c. Also, those to which the outer planking of a sailing vessel, &c, are fastened.

Ridge of a roof; its peak, or the sharp edge along its very top. Has various similar applications.

Ridge-pole, ridge-piece, or *ridge-plate;* the highest horizontal timber in a roof, extending from top to top of the several pairs of rafters of the trusses; for supporting the heads of the jack-rafters.

Right and left; a lock which in its proper position suits one flap of a pair of folding doors, will not suit if fastened to the other flap; nor even to the same flap if required to open to the right instead of to the left, or vice versa, according to whether it is a right or a left-hand lock. And so with many other things, as, for instance, certain arrangements for working railway switches, &c. Right and left boots and shoes are a familiar illustration; also, right and left screws. Therefore, in ordering several of anything, it is necessary to consider whether they may all be of the same pattern, or whether some must be right-hand, and others left-hand ones.

Right shore of a river; that which is on the right hand when descending the river.

Right-solid body; one which has its axis at right angles to its base; when not so, it is *oblique.*

Ring-bolt; a bolt with an eye and a ring at one end.

Rip-rap. See Random stone.

Roadstead; anchorage at some distance from shore.

Rock-shaft; a shaft which only rocks or makes part of a revolution each way, instead of revolving entirely around.

Rockwork; squared masonry in which the face is left rough to give a rustic appearance.

Rubble; masonry of rough, undressed stones. Scabbled rubble has only the roughest irregularities knocked off by a hammer. Ranged rubble has the stones in each course rudely dressed to nearly a uniform height.

Rundle, or *round;* the step of a ladder.

Rustic; much the same as rockwork.

Saddle; the rollers and fixtures on top of the piers of a suspension bridge, to accommodate expansion and contraction of the cables. The top piece of a stone cornice of a pediment. Has many other applications.

Sag; to bend downward.

Salient; projecting outward.

Sandbag; a bag filled with sand for stopping leaks.

Scabble; to dress off the rougher projections of stones for rubble masonry, with a stone-axe, or scabbling hammer.

Scantling; the depth and breadth of pieces of timber; thus we say, a scantling of 8 by 10 ins, &c.

Scarf; the uniting of two pieces by a long joint, aided by bolts, &c.

Scarp; a steep slope. In fortification, the inner slope of a ditch.

Scotia; a receding moulding consisting of a semi-circle or semi-ellipse, or similar figure.

Screeds; long narrow strips of plaster put on horizontally along a wall, and carefully faced out of wind, to serve as guides for afterward plastering the wide intervals between them.

Screw-bolt; a bolt with a screw cut on one end of it.

Screw-jack. See Jack.

Screw-wrench. See Wrench.

Scribe; to trim off the edge of a board, &c, so as to make it fit closely at all points, to an irregular surface. The lower edges of an open caisson are scribed to fit the irregularities of a rocky river bottom.

Scroll; an ornamental form consisting of volutes or spirels arranged somewhat in the shape of S.

Scupper nails; nails with broad heads for nailing down canvas, &c.

Scuppers; on shipboard, holes for allowing water to flow off from the deck into the sea.

Scuttle; a small hatchway. To make holes in a vessel to cause sinking.

Sea-wall; a wall built to prevent encroachment of the sea.

Secret nailing; so nailing down a floor by nails along the edges of the boards, that the nail-heads do not show.

Serve; to wrap twine or yarn, &c, closely round a rope to keep it from rubbing.

Set-screw, or *tightening-screw;* a screw for merely pressing one thing tightly against another at will; such as that which confines the movable leg of a pair of dividers in its socket.

Shackle, or *clevis;* a link in a chain shaped like a U, and so arranged that by drawing out a bolt or pin, which fits into two holes at the ends of the U, the chain can be separated at that point.

Shaft; a vertical pit like a well. The body of a column. A large axle.

Shank; the body of a bolt exclusive of its head. The long straight part of many things, as of an anchor, a key, &c.

Shears, or *sheers;* two tall timbers or poles, with their feet some distance apart, and their tops fastened together; and supporting hoisting tackle.

Sheave; a wheel or round block with a groove around its circumference for guiding a rope.

Sheeting, or *sheathing;* covering a surface with boards, sheet iron, felt, &c.

Shingle; the pebbles on a seashore.

Shoes; certain fittings at the ends of pieces; as the pointed iron shoes for piles. The wall shoes into which the lower ends of iron rafters generally fit, &c.

Shore; a prop.

Shot; the edge of a board is said to be shot when it is planed perfectly straight.

Shrink. When an iron hoop or band is first heated, and then at once placed upon the body which it is intended to surround, it shrinks or contracts as it cools, and therefore clasps the body more firmly. This is called *shrinking on* the hoop.

Shuttle; a small gate for admitting water to a water-wheel, or out of a canal lock, &c.

Siding; a short piece of railroad track, parallel to the main one, to serve as a passing place.

Silt; soft fine mud deposited by rivers, &c.

Siphon culvert; a culvert built in shape of a U, for carrying a stream under an obstacle, and allowing it afterward to rise again to its natural level. The term is improper, inasmuch as the principle of the siphon is not involved.

Skewback; the inclined stone from which an arch springs.

Skids; vertical fenders, on a ship's sides. Two parallel timbers for rolling things upon.

Skirting; narrow boards nailed along a wall, as the washboards in dwellings.

Sledge; a heavy hammer.

Sleeper; any lower or foundation piece in contact with the ground.

Sleeve; a hollow cylinder slid over two pieces to hold them together.

Slide-bars, or *slides;* bars for anything to slide along; as those for the cross-heads of piston-rods, &c. Often called guides.

Slings; pieces of rope or chain to be put around stones, &c, for raising them by.

Slip; the sliding down of the sides of earth-cuts or banks. A long narrow water space or dock between two wharf-piers.

Slope-wall; a wall, generally thin and of rubble stone, used to preserve slopes from the action of water in the banks of canals, rivers, reservoirs, &c; or from the action of rain.

Slot; a long narrow hole cut through anything.

Sluice; a water-channel of wood, masonry, &c; or a mere trench. The flow is usually regulated by a sluice-gate.

Smoke-box; in locomotives, that space in front of the boiler, through which the smoke passes to the chimney.

Snag; a lug with a hole through it, for a bolt.

Socket; a cavity made in one piece for receiving a projection from, or the end of, another piece; as that into which the movable leg of a pair of dividers fits.

Soffit; the lower or underneath surface of an arch, cornice, window, or door-opening, &c.

Solder; a compound of different metals, which when melted is used for uniting pieces of metal also heated. Soft solder is a compound of lead and tin, and is used for uniting lead or tin. There are various hard solders, such as spelter solder, composed of copper and zinc, for uniting iron, copper, or brass.

Sole; that lining around a water-wheel which forms the bottoms of the buckets.

Spandrel; the space, or the masonry, &c, between the back or extrados of an arch and the roadway.

Spanner; a kind of wrench, consisting of a handle or lever with a square eye at one end of it; much used for tightening up the nuts upon screw-bolts, &c. The eye fits over or surrounds the nut.

Spar; a beam; but generally applied to round pieces like masts, &c.

Spelter; zinc.

Spigot; the pin or stopper of a faucet. The smaller end of a common cast-iron water or gas pipe.

Spindle; a thin delicate shaft or axle.

Splay; to widen or flare, like the jambs of a common fireplace, or those of many windows; or like the wing-walls of most culverts.

Splice; to unite two pieces firmly together.

Spokes; the radii of wheels.

Springer; the lowest stone of an arch.

Spur-wheel; a common cog-wheel, in which the teeth radiate from a common cen, like those of a spur.

Square-head; a square termination like that upon which a watch-key fits for winding; or that upon which the eye of the handle of a common grindstone fits for turning it, &c.

Staging; the temporary flooring of a scaffold, platform, &c.

Stanchion; a vertical prop or strut.

Standing-bolt, or *stud-bolt;* a bolt with a screw cut upon each end; one end to be screwed permanently into something, and the other end to hold by means of a nut something else that may be required to be removed at times.

Stand-pipe; a stand-pipe is sometimes used for the same purpose as an air-vessel; which see. It is a tall pipe, open to the air at top; and communicating freely at its foot with the water-pipe, in the same manner as in an air-vessel. Its top must be somewhat higher than that to which the pump has to force the water through the system of pipes; otherwise the water would be wasted by flowing over its top. The area of its transverse section should be *at least* equal to that of the pipe or pipes which conduct the water *from* it; but it is at times better to have it much larger, as a stand-pipe may then answer, especially in a small town, *as a reservoir*, if the pumping should cease for a few hours. A stand-pipe should be cylindrical, not conical; for if thick ice should form on top of the water in a conical one, a sudden forcing of it upward by the pump might strain the stand-pipe seriously. There are four or five stand-pipes connected with the Philadelphia Water-Works. They are (with one exception) cylindrical, from 125 to 170 ft high; 5 ft diam; and made of riveted boiler-iron about ⅜ inch thick near the base, and about ¼ inch near the top. They have no protection from the weather; nor are they braced in any manner; but retain their positions by their own inherent strength, although exposed at times to violent winds.

Staple; a kind of double pin in shape of a U; its two sharp points are driven into timber, and curved part is left projecting, to receive a hoop, pin, or hasp, &c.

Startings; the projecting up and down-stream ends or cutwaters of a bridge pier.

Stay; variously applied to props, struts, and ties, for staying anything or keeping it in place.

Stay-bolts; long bolts placed across the inside of a boiler, &c, to give it greater strength.

Steam-chest; the iron box in locomotive engines and others, through which the steam is admitted to the cylinders.

Steam-pipe; the one which leads steam from a boiler to the steam-chest.

Step; a cavity in a piece for receiving the pivot of an upright shaft; or the end of any upright piece.

Stiles; the flat vertical pieces between and at the sides of the panels in doors, &c.

Stock; the eye with handles for turning it, in which the dies for the cutting of screws are held.

Stove-up, or *stoved*, or *upset;* when a rod of iron is heated at one end, and then hammered endwise so that that part becomes of greater diameter or stouter than the remainder. The heads of bolts are frequently made in one piece with the shank in this way; and the screw ends of long screw-rods are often upset, so that the cutting of the threads of the screw may not reduce the strength of the bar.

Strap; a long thin narrow piece of metal bolted to two bodies to hold them together. A strap-hinge is a strap fastened to a shutter, &c, and having an eye or a pin at one end for fitting it to the other part of the hinge which is attached to the wall.

Stratum; a layer, or bed; as the natural ones in rocks, &c.

Stretcher; a brick, or a block of masonry laid *lengthwise* of a wall. A frame for stretching anything upon.

Stretcher-course; a course of masonry all of stretchers, without any headers.

Strike; an imaginary horizontal line drawn upon the inclined face of a stratum of rocks. Thus, if the slates or shingles on a roof represent inclined strata of rocks, then either the ridge or the eaves of the roof, or any horizontal line between them, will represent their strike. The inclination is called the *dip* of the strata; and the strike is always at right angles to it by compass.

String; variously applied to longitudinal pieces.

String-board; the boarding (often ornamented) at the outer ends of steps in staircases. It hides the horses, as the inclined timbers which carry the steps are called.

String-course; a long horizontal course of brick or masonry projecting a little beyond the other and often introduced for ornament.

Strut; a prop. A piece that sustains compression, whether vertical or inclined.

Strut-tie, or *tie-strut;* a piece adapted to sustain both tension and compression.

Stub-end; a blunt end.

Stud; a short stout projecting pin. A prop. The vertical pieces in a stud partition.

Stud-bolt. See Standing-bolt.

Stuffing-box; a small boxing on the end of a steam cylinder, and surrounding the piston-rod like a collar; or in other positions where a rod is required to move backward and forward, or to revolve, in an opening through any kind of partition, without allowing the escape of steam, air, or water, &c, as the case may be. The box is filled with greased hemp or other packing, which is kept pressed close around the moving rod by means of a top-piece or kind of cover called the *gland,* which may be screwed down more or less tightly upon it at pleasure. The rod passes through the gland also. See Figs 40 of Hydraulics, where *b b* is the gland of the stuffing-box just below it between *t* and *t*; and the small circle shows the cavity for the packing or stuffing.

Sumpt, or *sump;* a draining well into which rain or other water may be led by little ditches from different parts of a work to which it would do injury.

Surbase; the inside horizontal mouldings just under a window-sill. Also those around the top of a pedestal, or of wainscoting, &c.

Sloage, or *swedge;* a kind of hammer, on the face of which is a semi-cylindrical, or other shaped groove or indentation; and which, being held upon a piece of hot iron and struck by a heavy hammer, leaves the shape of the indentation upon the iron.

Switch; the movable tongue or rail by which a train is directed from one track to another.

Swivels; devices for permitting one piece to turn readily in various directions upon another, without danger of entanglement or separation. At 18, p 265, of Trusses, is a tightening swivel; the casters under the legs of heavy furniture are swivelled rollers.

Synclinal axis; in geology, a valley axis, or one toward which the strata of rocks slope downward from opposite directions. The line of the gutter in a valley roof may represent such an axis.

T's; pieces of metal in that shape, whether to serve as straps, or for other purposes. So also with L's, S's, W's, +'s, &c. See figs to Welded Iron Tubes, p 365.

Tackle; a combination of ropes and pulleys.

Tilus; the same as batter.

Tamp; to fill up with sand or earth, &c, the remainder of the hole in which the powder has been poured for blasting rock.

Tap; a kind of screw made of hard steel, and having a square head which may be grasped by a wrench for turning it around, and thus forcing it through a hole around the inside of which it cuts an interior screw. To strike with moderate force. To make an opening in the side of any vessel.

Tappet; a pin or short arm projecting from a revolving shaft; or from an alternating bar, and intended to come into contact with, or tap, something at each revolution or stroke.

Teeth; or cogs of wheels.

Temper; to change the hardness of metals by first heating, and then plunging them into water, oil, &c. To mix mortar, or to prepare clay for bricks, &c.

Templet: the outline of a moulding or other article, cut out of sheet metal or thin wood, to serve as a pattern for stonecutters, carpenters, &c.

Tenon; a projecting tongue fitting into a corresponding cavity called a mortise.

Terra cotta; baked clay. Brick is a coarse kind.

Thimble; an iron ring with its outer face curved into a continuous groove. A rope being doubled around this and tied, the thimble acts as an eye for it, and prevents that part of the rope from wearing. Also, a short piece of tube slid over another piece, or over a rod, &c, to strengthen a joint, &c.

Thread; the continuous spiral projection or worm of a screw.

Through-stone; a stone that extends entirely through a wall.

Throw; the radius, or distance to which a crank "throws out" its arm. Applies in the same way to lathes. Some use it to express the diameter instead of the radius. To avoid mistakes, the terms "single" and "double" throw might be used.

Tides; those well-known rises and falls of the surface of the sea and of some rivers, caused by the attraction of the sun and moon. There are two rises, floods, on high tides; and two falls, ebbs, or low tides, every 24 hours and 50 minutes; a lunar day; making the average of 6 hours 12½ minutes between high and low water. These intervals are, however, subject to great variations; as are also the heights of the tides; and this not only at different places, but at the same place. These irregularities are owing to the shape of the coast line, the depth of water, winds, and other causes. *Usually* at new and full moon, or rather a day or two after, (or twice in each lunar month, at intervals of two weeks,) the tides rise higher, and fall lower than at other times; and these are called *spring* tides. Also, one or two days after the moon is in her *quarters,* twice in a lunar month, they both rise and fall less than at other times; and are then called *neap* tides. From neap to spring they rise and fall more daily; and vice versa. The *time* of high water at any place, is generally two or three hours after the moon has passed over either the upper or lower meridian; and is called the *establishment* of that place; because, when this time is established, the time of high water on any other day may be found from it in most cases. The total height of spring tides is *generally* from 1½ to 2 times as great as that of neaps. The great tidal wave is merely an *undulation,* unattended by any current, or progressive motion of the particles of water. Each successive high tide occurs about 24 minutes later than the preceding one; and so with the low tides.

Tie; any piece that sustains tension or pull.

Tightning-ring. See 14, of Figs 21½, of Trusses, p 265.

Tightning-screw. See Set-screw.

Tire; the iron ring placed around the outer circumference of the felloe of a wheel.

Toggle-joint. In Fig 16, of Force in page 461, suppose *a m* and *a n* to be two stiff bars, hinged together at *a*. It is plain that if we press downward at *a*, the result will be a great pushing force against any bodies placed at the ends *m* and *n*. Such an arrangement of two bars for producing such pressure is a toggle-joint.

Tongue; a long slightly projecting strip to be inserted into a corresponding groove, as in tongued and grooved floors.

Tooling; dressing stone by means of a tool and mallet; the tool being a chisel with a cutting edge of 1 to 2 inches wide. Tooling is generally done in parallel stripes across the stone.

Torus; a projecting semi-circular, or semi-elliptic moulding; often used in the bases of columns. It is the reverse of a scotia.

Trailing-wheels; in a locomotive, those sometimes placed behind the driving-wheels.

Train; a number of cog-wheels working into each other.

Tramway; any two smooth parallel tracks upon which wheels without flanges may run. In w

ıe rails themselves have flanges; but in wide stone ones for common vehicles, none are

· a beam across the opening for a door, &c. Also, a horizontal piece dividing a high wino stories, &c, &c.
ıe horizontal part of a step.
a kind of foot-lever, for turning a lathe, grindstone, &c, by the foot.
a long wooden pin.
· a short cross-timber framed into two joists so as to sustain the ends of intermediate
event the latter from entering a chimney-flue, or interfering with a window, &c.
mer, or *tilt-hammer;* a large hammer worked by camb machinery, and used for heavy
ıspecially for hammering irregular masses into the shape of bars, &c.
kind of small wagon consisting of a platform on two or more low wheels. Also, those
wheels usually placed under railroad cars and engines, and which, by means of a pintle
:he two, allows them to vibrate or move laterally to some extent independently of each other.
lantern-wheel, or *wallower;* used instead of a cog-wheel, and consisting of two parallel
ces some distance apart, and united by a central axis, and by cylindrical rods placed
parallel to the axis, to serve instead of cogs or teeth.
ı long wooden boxing forming a water channel.
s; cylindrical projections, as at the sides of a cannon, forming as it were an interrupted
t for supporting the cannon on its carriage; and allowing it to revolve vertically through
oe.
· a kind of spring catch, which at the proper moment falls or tumbles into a notch or
ed for it in a piece; thus holding the piece in position until the tumbler is lifted out of the

-*shaft;* in locomotives, a shaft used in the "link motion."
a passage-way excavated beneath the surface of the ground. For railroads, they should,
be straight, especially when there is but a single track; inasmuch as collisions or other
a tunnel would be peculiarly disastrous. A tunnel will rarely be expedient before the
tting expeeds 60 feet. Firm rock of moderate hardness, and of a durable nature, is the
ble material for a tunnel; especially if free from springs, and lying in horizontal strata.
, or in shales, (even if hard and firm at first,) or in earth, a lining of hard brick or manent, is necessary. A tunnel should have a grade or inclination in one direction, for ease
ainage and ventilation. No special arrangement is essential for ventilation either during
ı, or after, if the length does not exceed about 1000 ft; but beyond that, generally during
ı either shafts are resorted to, or means provided for forcing air into the tunnel through
its ends. But after the work is finished, except under peculiar circumstances, nothing of
necessary. The Blue Ridge tunnel, on the Virginia Central, 4240 ft long, and many others
have no artificial ventilation. The Mont Cenis tunnel under the Alps, completed in 1871,
long, and has no shafts, although it grades up from each end, which is the most unfavorconditions for ventilation without shafts. It was made so for facilitating drainage. Its venmaintained by air forced in from the ends. The Hoosick tunnel, Mass, 4½ miles long, has
of them 1030 ft deep; but they are for expediting the work. Shafts generally cost from
es as much per cub yd as the main tunnel, owing to the greater difficulty of excavating
ng the material; and getting rid of the water, all of which must be done by hoisting.
ıgh earth, they must be lined as well as the tunnel; and the lining must usually be an
ing process. Or the lining may first be built *over* the intended shaft, and then sunk by
ıg it gradually; see page 327. Their sectional area commonly varies from about 40 to 100
y have the great advantage of expediting the work by increasing the number of points at
n be carried on; but if placed too close together, their cost more than compensates for
air in some tunnels, while being constructed, is much more foul than in others; so that
ork has been commenced, temporary shafts may be found expedient where they were not
. In excavating the tunnel itself, a *heading* or passage-way, 5 or 8 ft high, and 3 to 12 ft
iven and maintained a short distance (10 to 100 ft, or more, according to the firmness of
ıl) in advance of the main work. In rock, the heading is just below the top of the tunnel,
men can conveniently drill holes in its floor for blasting; but in earth, the heading is
g the bottom of the tunnel, that being the most convenient for enlarging the aperture to
nel size, by undermining the earth, and letting it fall. In earth, the top and sides of the
ı well as of the tunnel, must be carefully prevented from caving in before the lining is
this is done by means of rows of vertical rough timber props, and horizontal caps or over-
, between which and the earth rough boards are placed to form temporary supporting sides
to the excavation. The props and caps are placed first; and the boards are then driven
them and the earthen sides of the excavation. These are gradually removed as the lining
orward. The lining, when of brick, is usually from 2 to 3 bricks thick (17 to 26 inches) at
ı from 1½ to 2½ bricks thick at top; and when of rough rubble in cement, about half
ick. It is important that the bricks or stone should be of excellent hard quality, and laid
ıent. The bricks should be moulded to the shape of the arch. As the lining is finished
ıgths, and before the centers are removed, any cavities or voids between it and the earth
carefully and compactly filled up. Even in rock, if much fissured, or if not of durable
as common shale, lining is necessary. The cross-section of a single-track railroad tunnel,
· of everything, and for cars of 11 ft extreme width, should not be *less* than about 15 feet
feet high; nor a double-track one, less than 27 ft wide, by 24 ft high; unless in the last
ıterial is firm rock, in which a high arch is not necessary for lining. The roof may then
ıtter, so that a height of 20 ft may answer. With cars of 10 ft extreme width, the width
nel may be reduced to 25 ft; or with 9 ft cars, to 23 ft. Many have been made 22 ft.
Cenis is 26 ft wide, by 25 high. The rate of daily progress from *each* face of a tunnel
ı 18 inches to 9 feet of length per 24 hours, with three relays of workmen. On the Mont
several years past, the extremes have been about 4 to 9 ft daily for a whole year, from
On that work greater expedition than usual was secured, by the use of improved drills
compressed air; although they were employed only in the headings, which were 12 ft wide
ı. Ordinarily, from 1½ to 3 ft may be taken as averages. The difference of rate of proen a single and a double track tunnel, is not so great as might be supposed; inasmuch as
ce can be employed on the wider one. If the tunnel is in earth, the construction of its
t makes up for the slower excavation of one in rock. In rock, with labor at $1 per day
usually vary with the character of the rock, from $2 to $5 per cub yd for the main tunnel
m *$3 to $10* for the heading; while shafts will average about 50 per ct more than tunnel
, *however*, is liable to serious contingencies which cannot be foreseen. Since the

and roof are rough as blasted, the width and height should each be estimated to the contractor as about 18 ins or 2 ft greater than the established *clear* ones. At any rate, the mode of measurement should be clearly stated in the specifications for the work. When a tunnel is made with a uniform grade, the work generally progresses in a more satisfactory manner from the lower end, because the descent favors the drainage of the spring water that is usually met with; whereas, at the upper end, it must be removed by pumps or by bailing. The upper end has, however, the advantage of sooner getting rid of the smoke in blasting. Before commencing a tunnel, or even deciding upon one, trial shafts should be sunk to ascertain the nature of the material. In long ones, the greatest care and accuracy are necessary for preserving the line of direction, so that the work from both ends shall meet properly at the center. The cost of a single-track tunnel, when common labor is $1 per day, will generally range between $30 and $75 per foot of length. Our limits prevent our saying more on this important subject, which of itself would require a volume.

Turnbuckle; variously applied, as to the ordinary fastenings at the outer face of a wall, for holding window-shutters back when opened; also, to the tightening swivel at 13, page 265, of Trusses, &c.

Turnout; a kind of siding, or short piece of track, for permitting railroad trains to pass each other on a single-track road.

Turntable; the well-known arrangement for turning locomotives at rest. See page 428.

Undermine; to excavate beneath anything.

Underpin; to add to the height of a wall already constructed, by excavating and building beneath it. Also, to introduce additional support of any kind beneath anything already completed.

Upset. See Stove-up.

Valves; various devices for permitting or stopping at pleasure the flow of water, steam, gas, &c. A SAFETY VALVE is one so balanced as to open of itself when the pressure becomes too great for safety. A SLIDE VALVE is one that slides backward and forward over the opening through which the flow takes place. A BALL VALVE, or spherical valve, is a sphere, which in any position fits the opening. When the pressure below it raises it off from its seat, it is prevented from rolling away by means of a kind of open caging which surrounds it. A CONICAL or PUPPET VALVE is a horizontal slice of a cone, which fits into a corresponding conical seat made in the opening. In rising and falling it is kept in position by a vertical valve-stem or spindle, which passes through its center, and which plays through guide holes in bridge-pieces placed above and below the valve. A TRAP, CLACK, FLAP, or DOOR VALVE, is a plate with hinges like a door. When two such valves are used, with their hinged edges adjacent to each other, so that in opening and shutting they flap like the wings of a butterfly, they constitute a butterfly valve. A THROTTLE VALVE is one which when closed forms a partition across a pipe; and opens by partially revolving upon an axis placed along its diameter. A ROTARY VALVE works like a common stopcock. A SHIFTING VALVE is one which lets out steam under water; and is so called from the shifting noise thereby produced. The PORT VALVE is the sliding one which admits steam from the steam-chest into the cylinders. A DOUBLE SEAT, or DOUBLE SEAT VALVE is a peculiar one with two seats, one above the other; and so arranged that the pressure of steam or water against it when shut, does not oppose its being opened. A CUP VALVE is in shape of an inverted cylindrical cup, with a length somewhat greater than its diameter. Its lower or open edge is ground to fit the seat over which it rests. As this cup rises and falls, it is kept in place by a cylindrical caging closed at top, and having for its sides four or more vertical pieces, against the inner sides of which the sides of the cup play. A CHECK VALVE is any kind so placed as to check or prevent the return of the fluid after its passage through the valve into the pipe or vessel beyond it.

Vault; an arch long in comparison with its span. The space covered by such an arch.

Veneer; a very thin sheet of ornamental wood glued over a more common variety.

Wainscot; a wooden facing to walls in rooms, instead of plaster, or over a facing of plaster; usually not more than 3 or 4 feet high above the floor.

Wales; long longitudinal timbers in the sides of a ship, coffer-dam, caisson, &c.

Wallow; a water-wheel, &c, is said to wallow when it does not revolve evenly on its journals.

Willower. See Trundle.

Wall-plate, or *raising-plate;* a timber laid along the tops of walls for the roof trusses or rafters to rest on, so as to distribute their weight more equally upon the wall.

Warped; twisted, as a board, or the face of a stone, &c, which is not perfectly flat. *To warp;* to haul a vessel ahead by means of an anchor dropped some distance ahead. To flood an extent of ground with water for a short time to increase its fertility.

Washboards; boards nailed around the walls of rooms at the floor, so as to prevent injury to the plaster when washing the floors.

Washers; broad pieces of metal surrounding a bolt, and placed between the faces of the timber through which the bolt passes, and the head and nut of the bolt, so as to distribute the pressure over a larger surface, and prevent the timber from being crushed when the bolt is tightly screwed up.

Waste-weir; an overfall provided along a canal, &c, at which the water may discharge itself in case of becoming too high by rain, &c. Sometimes called a tumbling-bay.

Water-shed; the sloping ground from which rain-water descends into a stream.

Water-table; a slight projection of the lower masonry or brickwork on the outside of a wall, and reaching to a few feet above the ground surface, as a partial protection against rain, or as ornament.

Ways; the inclined timbers along which a vessel glides when being launched.

Weather-boards; boards used instead of bricks or masonry for the outsides of a building, or bridge, &c. They are nailed to vertical and inclined indoor timbers; and may be either vertical or hor. When hor, they are so placed that the lower edge of one overlaps the upper edge of the one below. When vert, their edges should be tongued and grooved; and narrow slips be nailed over the vert joints, to keep out rain, &c.

Weir, or *wier;* a dam, or an overfall.

Weld; to join two pieces of metal together by first softening them by heat, and then hammering them in contact with each other. In this operation fluxes are used.

Wharf; a level space upon which vessels lying along its sides can discharge their cargoes; or from which it can receive them.

Wheel-base; the distance from center to center from the extreme front wheels, to the extreme hind ones in a locomotive, car, &c.

Wicket; a small door or gate made in a larger one; as the shuttle or valve in a lock-gate, for letting out the water.

Winch; a handle bent at right angles, and used for turning an axis; that of a common grindstone.

Wind. See Out of wind.

Winders; those steps (often triangular) in a staircase by which we wind, or turn angles.

Windlass; the wheel and axle, or winch and drum, as often used in common wells. Also, a hor

zontal shaft on shipboard, by which the anchor is raised; the windlass being revolved by means of wooden levers called *handspikes*.

Wing-dam; a projection carried out part way across a shallow stream, so as to force all the water to flow deeper through the channel thus contracted.

Wings; applied in many ways to projections. The flanges which radiate out from a gudgeon; and by which it is fastened to the shaft. Small buildings projecting from a main one. The wings or flaring wing-walls of a culvert or bridge.

Wing-walls; the retaining-walls which flare out from the ends of bridges, culverts, &c.

Wiper. See Camb.

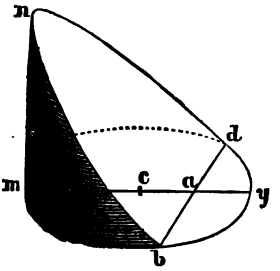
Working-beam, or *walking-beam;* a beam vibrating vertically on a rock-shaft at its center, as seen in some steam-engines; one end of it having a connection with the piston-rod; and the other end with a crank, or with a pump-rod, &c.

Worm; the so-called endless screw, which by revolving without advancing gives motion to a cog-wheel (worm-wheel), the teeth of which catch in the thread of the screw.

Wrench; a long handle having at one end an eye or jaw which may catch hold of anything to be twisted or turned around, as a screw-nut, &c. When it has a jaw which by means of a screw is adaptable to nuts, &c, of different sizes, it is a monkey-wrench, or screw-wrench.

APPENDIX.

P. 22, near bottom; before Rem 1. **To find the area of a circular segment.** If the height, *d b*, exceeds half the diam of the circle, the quotient will exceed .5; and will not be found in the table. In such cases, subtract the height from the diam; and use the remainder as the height. Then find the area of a segment having this new height; and subtract it from the area of the entire circle. The rem will plainly be the reqd area.



P. 32; just above Circular Rings.

When *a m* exceeds *c m*, for the **solidity** of the ungula *d n m b a*, find the cube of *a b*, or of *a d*, and take ⅔ of it, which call *p*. Mult the area of the base *a d m b* by *a c*. Add the prod to *p*. Mult the sum by the height *m n*. Divide by *a m*.

And for the **area of the convex surface,** mult the diam *y m* by *a b* or by *a d*. Call the prod *p*. Take *a d* from *a m*. Mult the rem by the length of the arc *d m b*. Add the prod to *p*. Call the sum *s*. Divide the height *m n* by *a m*. Mult the quot by *s*.

P. 34. For the solidity of such cylindric ungulas as at the 8th fig on page 34, use the prismoidal rule given near the foot of page 33.

P. 312. Cost of ashlar facing masonry. If the stone be **sandstone** with good natural beds, the getting out may be put at $3.00 per cubic yard. Face dressing at 26 cts per sq ft: say $3.64 per cubic yd. Beds and joints 13 cts per sq ft: say $6.76 per cub yd. The neat cost, laid, $17.00, instead of the $21.86 per cub yd for granite; and the total cost, $19.55, instead of the $25.14 for granite. And **the total cost of large well scabbled ranged sandstone masonry in mortar,** may be taken at about $10 per cub yd, at the wages on p 312.

Embankment, shrinkage of.

Although earth, when first dug, and loosely thrown out, *swells* about $\frac{1}{5}$ part, so that a cub yd in *place* averages about $1\frac{1}{5}$ or 1.2 cub yds when dug; or 1 cub yd dug is equal to $\frac{5}{6}$, or to .8333 of a cub yd in place; yet when made into embankment it gradually subsides, settles, or shrinks, into a less bulk than it occupied *before being dug*.

The following are approximate averages of the shrinkage; or, in other words, the earth measured in place in a cut, will, when made into embankment, occupy a bulk less than before by about the following proportions:

Gravel or sand.....	about	8 per ct;	or 1 in 12½ less.
Clay.....	"	10 per ct;	or 1 in 10 less.
Loam.....	"	12 per ct;	or 1 in 8⅓ less.
Loose vegetable surface soil	"	15 per ct;	or 1 in 6⅔ less.
Puddled clay.....	"	25 per ct;	or 1 in 4 less.

The writer thinks, from some trials of his own, that 1 cub yd of any hard rock in place, will make from 1½ to 1¾ cub yds of embankment; say on an average 1.7 cub yds. Or that 1 cub yd of rock *embankment* requires .5882 of a cub yd in place. He found that a solid cub yd when broken into *fragments*, made about 1.9 cub yds of loose heap; * 1¾ yds carelessly piled; † and 1.6 yds carefully *piled*.‡ Or 1½ cub yds of very carelessly scabbled rubble; § or 1¼ yds of somewhat carefully scabbled.‖ See p 504.

*	Solid part	.526;	voids	.474	of the entire bulk.			
†	"	"	.570;	"	.430	"	"	"
‡	"	"	.630;	"	.370	"	"	"
§	"	"	.670;	"	.330	"	"	"
‖	"	"	.800;	"	.200	"	"	"

Approximate cost of buildings per cubic foot, at Philada prices in 1873; including every cub ft of space from roof to cellar floor. Plain brick dwellings, such as most of those in Philada, 12 to 15 cts. Better class, highly finished throughout, 15 to 18 cts. First class, with cut stone fronts, 20 to 30 cts. Plain brick churches, public schools, court-houses, theaters, &c, 13 to 18 cts. Ornate Gothic churches with much cut stone facing, 30 to 45 cts, exclusive of spires. Large plain brick or rubble R R shops, depots, station-houses, &c, 9 to 12 cts; or with ornamental finish and best materials, 15 to 20 cts. First class city stores, marble fronts, high stories, fire-proof, (so called,) throughout to roof; best materials and workmanship, 18 to 25 cts.

Small buildings cost more per cub ft than large ones of the same finish. Also isolated or corner buildings, cost more than those which have two party-walls.

In Philada dwellings of brick, the carpentry and lumber usually cost each about one-fourth of the entire building.

P 327. The plenum process as applied at the South St bridge, Philada, by Mr. John W. Murphy, contracting engineer, differs materially from that described on p 327; and moreover deserves notice on account of the great simplicity and efficacy of his plant.* This consisted partly of two canal boats, decked, each 100 ft long, by 17½ ft wide, and 8 ft depth of hold. They were anchored parallel to each other, 15 ft apart. Supported by the boats, and over the space between them, was a strong four-legged shears about 50 ft high; at the top of which was attached tackle for handling the cast iron cylinders. In the hold of one of the boats was a **Burleigh Compressor** having two pistons of 10 ins diam, and 9 ins stroke; together with its boiler. On the deck of the same boat stood a vertical **air-tank or regulator,** 22 ft long, by 2 ft diam, made of quarter inch boiler iron. This served to maintain a supply of compressed air in the submerged cylinder in case of an accidental stopping of the compressor; which otherwise would probably be fatal to the laborers in the cylinder. The condensed air flowed from this air-tank to the air-lock of the cylinder through a hose 4 ins diam, made of gum elastic and canvas, and so long, and so placed, as to extend itself as the cylinder went down, thus maintaining the communication at all times. Entirely across both boats, and across the interval between them, extended two heavy wooden **clamps,** each 3 ft wide by 18 ins high; each composed of three pieces of 12 × 18 inch timber strongly bolted together. At the centers of these clamps the two inner vertical sides which faced each other were hollowed out to the depth of a foot by concavities corresponding to the curve of the cylinders. The distance apart of the clamps was regulated by two strong iron rods, having screws and nuts at their ends for that purpose. Thus when a section of a cylinder was hoisted by means of the shears into its position over the space between the two boats, the two concavities of the clamps were brought into contact with it, and the nuts being then screwed up, the cylinder was firmly held in place by the clamps. The shears could then be used to raise another section of the cylinder to its place upon the first one, that the two might be bolted together. By repeating this process the height of the cylinder would soon become too great to allow the shears to place another section upon it; in which case the nuts of the screws were slightly loosened, and the cylinder was allowed to slip down slowly into the water until its top was but a little above the surface. The screws were then again tightened, and the cylinder again held fast until other sections were added and bolted to it. When there was danger that the upward pressure of the condensed air might lift a cylinder, the clamps were raised by the shears clear of the boats; then tightened to the cylinder, and a platform of planks laid upon them, and loaded with stone.

The **air-lock** was so arranged as not to require to be removed when a new section was to be bolted on. This was effected as follows. Sections of the cylinder were bolted together in the manner just described, until its foot rested on the bottom, with its top a few feet above high water. A heavy cast iron **diaphragm** 1¼ inches thick, to form the floor of the air-lock, was then placed on top. Then was added another 10 ft high section of the cylinder, to form the chamber of the air-lock. These were bolted together; and then another diaphragm was added at top to form the roof of the air-lock. These diaphragms were furnished with openings, and with doors and valves corresponding with those shown in Fig 17, p 327, and remained permanently in the cylinders when the work was finished. If the depth of soil to be passed through before reaching rock is so great as to require other sections of cylinder to be bolted on above the top of the air-lock, this may be done to any extent, inasmuch as it is immaterial whether the air-lock is under water or not. **To keep the cylinder both air- and water-tight** the faces of the flanges before being bolted together were smeared with a mixture of red and white lead and cotton fiber.

At the South St bridge the cylinders were 4, 6, and 8 ft diam; in lengths or sections 10 ft long. They were all 1¼ inch thick. Inside flanges 2¾ ins wide, 1¼ thick, with bolt-holes 1¼ inch diam, by 5 ins apart from center to center. The bottom edge has no flange. A 10 ft section of an 8 ft cylinder weighs 14600 lbs; of a 6 ft one, 10800; of a 4 ft one, 6800. An 8 ft diaphragm, 2800 lbs; 6 ft, 1600; 4 ft, 783. The rock under the soil was quite uneven in places; but was levelled off as the cylinders went down. These were then bolted to it by cast iron brackets. **The work went on, day and night,** summer and winter: with no interruption from the tides, floods, or floating ice; and the thirteen columns were sunk, filled with concrete, and completed in 11 months; much of which was consumed in levelling off the rock, and bolting the brackets.

Rise and fall of tide about 7 ft. Greatest depth of soil, gravel, &c, passed through, 30 ft; least, 6 ft. Depth of water about 25 ft. The work was under charge of John Anderson, a very skillful and energetic superintendent of such matters. **The entire neat cost** of the cylinders in place, and filled with hydraulic concrete, was probably not far from $120 per ft of total length for the 8 ft ones; $80 for the 6 ft; and $50 for the 4 ft diams. There were three gangs of workmen; and each gang worked 4 hours at a time.

Such cylinders have cracked through, around their entire circumference, in many parts of the U. S. in very cold weather; owing to the diff of contraction of the iron, and of the concrete filling.

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* *See a full and very instructive description with engravings, by D. M. Stauffer, Superintendent Engineer for the city, in the Journal of the Franklin Inst, Nov. 1872. From it the above facts are taken.*

P 177. Moduluses of Elasticity in lbs per sq inch. For the meaning of the term see p 177. Authors differ considerably in their data on this subject. Where it was possible to adopt an average without error of practical importance, we have done so; but where the differences were too great for this, we have omitted the material entirely. Fortunately it is a matter of but little importance to the engineer; for, with the exception of iron and steel, he rarely need care to know the exact degree to which his materials are lengthened or shortened by their loads.

It must be carefully remembered that in practice the principle of mod of elas applies only within the elas limit of materials. This limit may ordinarily be safely taken at about one-third of the breaking tensile strengths given on pages 177 to 180. This is done in our 5th column. It is, however, but a tolerable approximation. There is reason to doubt the accuracy of many of the items in this table.

MATERIAL.	Modulus of Elasticity.	Stretch or Compression in a length of 10 ft, under a load of 1000 lbs per sq in.	Stretch or Compression in a length of 10 ft, under a load of 1 ton per sq in.	Approx elas limit.
	lbs per sq in.	Ins.	Ins.	lbs per sq in.
Ash	1 600 000	.075	.168	4500
Beech	1 300 000	.092	.207	4000
Birch	1 400 000	.086	.192	5000
Brass, cast	9 200 000	.013	.029	6000
" wire	14 200 000	.009	.019	16000
Chestnut	1 000 000	.120	.269	4500
Copper, cast	18 000 000	.007	.015	6500
" wire	18 000 000	.007	.015	10000
Elm	1 000 000	.120	.269	2000
Glass	8 000 000	.015	.034	3200
Iron, cast	12 000 000	.010	.022	4500
	to	to	to	to
	23 000 000	.005	.012	8000
" " average	17 500 000	.007	.015	6250
" wrought, in either bars, sheets or plates	22 000 000	.005	.012	13000
	to	to	to	to
	32 000 000	.004	.008	26000
" " " average	27 000 000	.004	.010	19500
" " " wire, hard	26 000 000	.005	.010	27000
" " wire ropes	15 000 000	.008	.018	13000
Larch	1 100 000	.109	.244	2300
Lead, sheet	720 000	.167	.....	1100
" wire	1 000 000	.120	.....	1100
Mahogany	1 400 000	.086	.192	2700
Oak	1 000 000	.120	.269	.....
	to	to	to	
	2 000 000	.060	.134	.....
" average	1 500 000	.080	.179	3300
Pine, white or yellow	1 600 000	.075	.168	3300
Slate	14 500 000	.008	.018	3700
Spruce	1 600 000	.075	.168	3300
Steel bars	29 000 000	.004	.009	34000
	to	to	to	to
	42 000 000	.003	.006	44000
" " average	35 500 000	.003	.007	39000
Sycamore	1 000 000	.120	.269	4000
Teak	2 000 000	.060	.134	5000
Tin, cast	4 600 000	.026	.....	1500

P 603. Friction of Hydraulic Presses. Careful experiments by Mr. John Hick, C. E., England, show that this is independent both of the width of the leather collar, and of the length of the ram in the cylinder; and that in a press in good order it may be found very approximately thus,

Rule. Mult the diam of the ram in inches by 25. Divide the total pressure against the bottom of the ram by the product.

Example. In a press of 18 ins diam the total pressure is 680000 lbs; how much must be deducted from this for friction? Here,

$$18 \times 25 = 450. \quad \text{And } \frac{680000}{450} = 1511 \text{ lbs, answer.}$$

Hence in an 18 inch press the ratio of the friction to the total pressure is $\frac{1511}{680000} = .00222$; or $\frac{1}{.00222} = \frac{1}{450}$ part of the total pressure. Thus may be found the ratio for any other diam.

In a press 24 ins diam it will be $\frac{1}{600}$; 18 ins, $\frac{1}{450}$; 15 ins, $\frac{1}{375}$; 12 ins, $\frac{1}{300}$; 9 ins, $\frac{1}{225}$; *6 ins*, $\frac{1}{150}$; *3 ins*, $\frac{1}{75}$ part of the entire pressure.

Table of Ordinates 5 ft apart. Chord 100 ft.

For Railroad Curves.

For the radii corresponding to the angles of deflection, see page 418.

Ordinates for angles intermediate of those in the table can at once be found by simple proportion. This table was calculated by rule p 20.

Distances of the Ordinates from the end of the 100 feet Chord.

Ang. of Defl.	Mid. 50 ft.	45 ft.	40 ft.	35 ft.	30 ft.	25 ft.	20 ft.	15 ft.	10 ft.	5 ft.
° ′										
4	.014	.014	.014	.013	.012	.010	.008	.008	.005	.003
8	.029	.029	.028	.026	.024	.022	.018	.015	.010	.005
12	.043	.043	.041	.038	.037	.033	.028	.022	.015	.008
16	.058	.058	.056	.052	.049	.044	.037	.030	.020	.011
20	.073	.072	.070	.066	.061	.055	.047	.037	.026	.014
24	.087	.086	.083	.077	.074	.066	.056	.045	.031	.017
28	.102	.101	.098	.092	.086	.077	.065	.052	.036	.019
32	.116	.115	.112	.106	.098	.088	.075	.058	.042	.022
36	.131	.130	.126	.119	.110	.099	.084	.066	.047	.024
40	.145	.144	.140	.133	.123	.110	.093	.074	.052	.027
44	.160	.158	.153	.145	.135	.121	.103	.081	.057	.030
48	.174	.172	.167	.158	.147	.132	.112	.088	.062	.033
52	.189	.187	.181	.171	.159	.143	.122	.095	.068	.035
56	.204	.202	.195	.185	.171	.154	.131	.103	.073	.038
1	.218	.216	.209	.198	.183	.164	.140	.111	.078	.041
4	.233	.231	.223	.211	.196	.175	.150	.118	.083	.043
8	.247	.245	.237	.224	.208	.186	.159	.125	.088	.046
12	.262	.260	.252	.237	.220	.196	.168	.133	.094	.049
16	.276	.274	.265	.251	.232	.207	.177	.140	.099	.052
20	.291	.288	.279	.264	.244	.218	.187	.148	.104	.055
24	.306	.303	.293	.277	.256	.229	.197	.155	.109	.057
28	.320	.317	.307	.291	.269	.240	.206	.163	.114	.060
32	.334	.331	.321	.304	.281	.251	.215	.171	.120	.063
36	.349	.345	.335	.317	.293	.262	.224	.178	.125	.066
40	.364	.360	.349	.330	.305	.273	.233	.185	.130	.069
44	.378	.374	.363	.343	.318	.284	.242	.192	.135	.072
48	.393	.389	.377	.356	.330	.295	.251	.200	.141	.075
52	.407	.403	.391	.370	.342	.305	.261	.208	.147	.077
56	.422	.418	.405	.383	.354	.316	.270	.215	.152	.080
2	.436	.432	.419	.397	.366	.327	.280	.222	.157	.083
4	.451	.446	.433	.409	.379	.338	.289	.230	.162	.086
8	.465	.461	.447	.425	.391	.349	.298	.237	.167	.088
12	.480	.475	.461	.437	.403	.360	.308	.245	.173	.090
16	.495	.490	.475	.450	.415	.371	.317	.252	.178	.093
20	.509	.504	.489	.463	.428	.382	.326	.260	.183	.096
24	.523	.518	.503	.476	.440	.393	.334	.267	.188	.099
28	.538	.533	.517	.489	.452	.404	.346	.275	.194	.102
32	.552	.547	.531	.503	.465	.415	.355	.282	.199	.104
36	.567	.562	.545	.516	.477	.425	.364	.289	.204	.107
40	.582	.576	.559	.529	.489	.436	.373	.297	.209	.110
44	.596	.590	.573	.542	.501	.447	.382	.304	.214	.113
48	.611	.605	.587	.555	.513	.458	.391	.312	.219	.116
52	.625	.619	.601	.569	.526	.469	.401	.319	.225	.118
56	.640	.634	.615	.582	.538	.480	.410	.326	.230	.121
3	.654	.648	.629	.595	.550	.491	.419	.334	.235	.124
4	.669	.662	.643	.608	.562	.502	.428	.341	.240	.127
8	.683	.677	.657	.621	.574	.512	.438	.349	.246	.130
12	.698	.691	.671	.635	.587	.523	.448	.357	.251	.132
16	.713	.705	.685	.649	.599	.534	.457	.364	.257	.135
20	.727	.720	.699	.662	.611	.545	.466	.371	.262	.138
24	.742	.734	.713	.675	.623	.556	.475	.378	.267	.141
28	.756	.749	.727	.688	.635	.567	.485	.386	.272	.144
32	.771	.763	.741	.702	.648	.578	.494	.394	.278	.146
36	.786	.777	.755	.715	.660	.589	.503	.401	.283	.149
40	.800	.792	.769	.728	.673	.600	.512	.408	.288	.152
44	.814	.806	.783	.741	.685	.611	.521	.415	.293	.155
48	.829	.821	.797	.754	.697	.621	.531	.423	.298	.158
52	.843	.835	.811	.768	.709	.632	.541	.431	.304	.160
56	.858	.850	.825	.781	.721	.643	.550	.438	.309	.163
4	.873	.864	.839	.794	.734	.655	.559	.445	.314	.166
10	.909	.900	.874	.827	.764	.682	.582	.464	.327	.173
20	.945	.936	.909	.860	.795	.709	.606	.482	.340	.179
30	.981	.972	.944	.893	.825	.736	.629	.501	.354	.186
40	1.017	1.008	.979	.926	.855	.764	.652	.519	.367	.193
50	1.054	1.044	1.014	.959	.886	.791	.676	.538	.380	.200
5	1.091	1.080	1.048	.993	.917	.818	.699	.557	.393	.207
10	1.127	1.116	1.083	1.026	.947	.845	.722	.576	.406	.214
20	1.164	1.152	1.118	1.058	.978	.872	.746	.594	.419	.221
30	1.200	1.188	1.153	1.092	1.009	.900	.769	.613	.432	.228

Table of Ordinates 5 ft apart.—(Continued.)

Distances of the Ordinates from the end of the 100 feet Chord.

Ang. of Defl.	Mid. 50 ft.	45 ft.	40 ft.	35 ft.	30 ft.	25 ft.	20 ft.	15 ft.	10 ft.	5 ft.
° ′										
5 40	1.236	1.224	1.188	1.124	1.039	.927	.792	.631	.445	.235
50	1.273	1.260	1.223	1.157	1.070	.954	.816	.649	.458	.241
6	1.309	1.296	1.258	1.191	1.100	.982	.839	.668	.472	.248
10	1.345	1.332	1.293	1.224	1.130	1.009	.862	.686	.485	.255
20	1.382	1.368	1.328	1.256	1.161	1.036	.886	.705	.498	.262
30	1.419	1.404	1.362	1.290	1.192	1.064	.909	.724	.511	.269
40	1.455	1·440	1.397	1.323	1.222	1.091	.932	.742	.524	.276
50	1.491	1.476	1.432	1.355	1.253	1.118	.956	.761	.537	.283
7	1.528	1.512	1.467	1.389	1.284	1.146	.979	.779	.551	.290
10	1.564	1.548	1.502	1.422	1.314	1.173	1.002	.798	.564	.297
20	1.600	1.584	1.537	1.454	1.345	1.200	1.026	.816	.576	.304
30	1.637	1.620	1.572	1.488	1.375	1.228	1.048	.835	.590	.311
40	1.673	1.656	1.607	1.521	1.405	1.255	1.071	.854	.603	.318
50	1.710	1.692	1.641	1.553	1.436	1.282	1.095	.872	.616	.324
8	1.746	1.728	1.677	1.587	1.467	1.310	1.118	.891	.629	.332
30	1.855	1.836	1.782	1.687	1.559	1.392	1.188	.946	.669	.353
9	1.965	1.944	1.886	1.787	1.651	1.474	1.258	1.002	.708	.373
30	2.074	2.052	1.991	1.887	1.742	1.556	1.328	1.057	.748	.394
10	2.183	2.161	2.096	1.987	1.834	1.637	1.398	1.114	.787	.415
30	2.292	2.269	2.201	2.087	1.926	1.719	1.468	1.170	.827	.436
11	2.401	2.377	2.306	2.186	2.018	1.802	1.538	1.226	.866	.457
30	2.511	2.486	2.411	2.286	2.110	1.884	1.609	1.282	.906	.478
12	2.620	2.594	2.516	2.386	2.203	1.967	1.680	1.339	.946	.499
30	2.730	2.703	2.621	2.485	2.295	2.049	1.750	1.395	.985	.520
13	2.839	2.811	2.726	2.585	2.387	2.132	1.820	1.451	1.025	.541
30	2.949	2.920	2.832	2.685	2.479	2.214	1.891	1.507	1.065	.562
14	3.058	3.028	2.937	2.785	2.571	2.297	1.961	1.564	1.105	.583
30	3.168	3.136	3.042	2.884	2.664	2.379	2.031	1.620	1.144	.604
15	3.277	3.245	3.147	2.984	2.756	2.462	2.102	1.676	1.184	.625
30	3.387	3.354	3.252	3.084	2.848	2.544	2.172	1.732	1.224	.646
16	3.496	3.462	3.358	3.184	2.941	2.627	2.243	1.789	1.264	.667
17	3.716	3.680	3.569	3.384	3.125	2.792	2.384	1.902	1.344	.709
18	3.935	3.897	3.779	3.584	3.310	2.958	2.525	2.014	1.424	.751
19	4.155	4.115	3.990	3.784	3.495	3.123	2.666	2.127	1.504	.793
20	4.375	4.332	4.201	3.984	3.680	3.288	2.808	2.240	1.583	.836
22	4.815	4.768	4.624	4.386	4.050	3.620	3.093	2.467	1.744	.922
24	5.255	5.204	5.048	4.789	4.423	3.952	3.379	2.695	1.905	1.008
26	5.697	5.642	5.473	5.192	4.798	4.286	3.665	2.924	2.068	1.094
28	6.139	6.079	5.898	5.595	5.171	4.622	3.952	3.154	2.232	1.181
30	6.582	6.517	6.323	5.999	5.544	4.958	4.239	3.385	2.396	1.268
32	7·027	6.957	6.751	6.406	5.922	5.297	4.530	3.619	2.565	1.356
34	7.472	7.398	7.179	6.813	6.300	5.637	4.822	3.854	2.733	1.445
36	7.918	7.841	7.609	7.222	6.679	5.978	5.115	4.090	2.901	1.535
38	8.367	8.286	8.041	7.633	7.060	6.320	5.410	4.327	3.069	1.626
40	8.816	8.731	8.474	8.044	7.442	6.663	5.705	4.565	3.238	1.718

INDEX.

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Elevation of the outer Rail on Curves

ve	M=15	M=20	M=25	M=30	M=40	M=50
	012	022	034	049	088	137
	025	044	068	099	175	274
	037	066	103	148	263	411
	049	088	137	197	351	548
	062	110	171	247	438	685
	074	131	205	296	526	822
	086	153	240	345	613	958
	099	175	274	394	701	1.095
	111	197	308	443	788	1.232
	123	219	342	493	876	1368

No. of case	Condition of Beam	How loaded	General moment of flexure	Maximum moment of stress	Relative moment	Rel. maximum deflection or coefficient of $\frac{l^3}{EI}$
I	Fixed at one end	Load at one end	$EI\frac{d^2y}{dx^2} = Px$ (53)	Pl	24	(57) $y = \Delta = -\frac{Pl^3}{3EI}$
II	Fixed at one End	Uniform load	$EI\frac{d^2y}{dx^2} = \frac{1}{2}wx^2$ (58)	$\frac{1}{2}Wl$	12	(61) $y = \Delta = -\frac{Wl^3}{8EI}$
III	Fixed at one end	uniform load load at one end	$EI\frac{d^2y}{dx^2} = Px + \frac{1}{2}wx^2$			(62) $y = \Delta = -\frac{l^3}{3EI}\left(P + \frac{3}{8}W\right)$
IV	Supported at both ends	Load at middle	$EI\frac{d^2y}{dx^2} = -\frac{1}{2}Px$ (70)	$\frac{1}{4}Pl$	6	(73) $y = \Delta = -\frac{Pl^3}{48EI}$
V	Supported at both ends	uniform load	$EI\frac{d^2y}{dx^2} = \frac{1}{2}w(-lx + x^2)$ (74)	$\frac{1}{8}Wl$	3	(76) $y = \Delta = \frac{5Wl^3}{384EI}$
VI	Supported at both ends	uniform load load at middle	$EI\frac{d^2y}{dx^2} = \frac{1}{2}Px + \frac{1}{2}wx^2 - \frac{1}{2}wlx$ (77)			(78) $y = \Delta = \frac{l^3}{48EI}\left(P + \frac{5}{8}W\right)$
VII	fixed at one end supported at other	Load at ~~one~~ middle				$y = \Delta = 8 = \frac{7}{48.16} \cdot \frac{Pl^3}{EI}$ *
VIII	fixed at one end supported at other	uniform load	$EI\frac{d^2y}{dx^2} = \frac{1}{8}wx(4x - 3l)$ (112)	$\frac{1}{8}Wl$ (116)	3	(118) $y = \Delta = 0.0054\frac{Wl^3}{EI}$
IX	fixed at both ends	load at middle	$EI\frac{d^2y}{dx^2} = \frac{1}{8}P(l - 4x)$ (117)	$\frac{1}{8}Pl$ (121)		(120) $y = \Delta = \frac{Pl^3}{192EI}$
X	fixed at both ends	uniform load	$EI\frac{d^2y}{dx^2} = \frac{W}{12l}(l^2 - 6x(l - x))$ (122)	$\frac{1}{12}Wl$ (126)		(125) $y = \Delta = \frac{1}{384} \cdot \frac{Wl^3}{EI}$

k. In this case, the max strain will be at a point 0.634 of the length of the beam from the fixed end.

For AD,

$$EI\frac{d^2y}{dx^2} = P\left(nl - x - \tfrac{1}{2}n^2/3-n)(l-x\right.$$

For DB,

$$EI\frac{d^2y}{dx^2} = -\tfrac{1}{2}Pn^2(3-n)(l-x)-$$

In these formulae,

$I = \frac{1}{12}bd^3$ (51) for a rectangle

$I = \frac{1}{4}\pi r^4 = \frac{1}{64}\pi d^4$ (52) for a circle

E = coef. of elasticity

Cross-section Triangles.

ng	Slope 1 : 1	Slope 1/4 : 1	Slope 1 1/2 : 1
	.005	.001	.007
	.02	.005	.03
	.04	.012	.067
	.08	.020	.12
	.12	.03	.19
	.18	.05	.27
	.24	.06	.37
	.32	.08	.48
	.40	.10	.61
	.50	.13	.75
	.60	.15	.91
	.72	.18	1.08
	.84	.21	1.27
	.98	.25	1.47
	1.12	.28	1.69
	1.28	.32	1.92
	1.44	.36	2.17
	1.62	.41	2.43
	1.80	.45	2.71
	2.0	.50	3.00
	2.21	.55	3.31

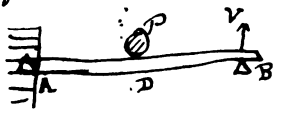
Cotton	1:1	1/4:1	1 1/2:1
2.3	2.64	.67	3.97
2.4	2.88	.72	4.32
2.5	3.12	.78	4.69
2.6	3.38	.84	5.07
2.7	3.64	.91	5.46
2.8	3.92	.98	5.88
2.9	4.20	1.05	6.30
3.0	4.50	1.12	6.75
3.	4.80	1.20	7.20
3.2	5.12	1.28	7.68
3.3	5.44	1.36	8.16
3.4	5.78	1.44	8.67
3.5	6.12	1.53	9.16
3.6	6.48	1.62	9.72
3.7	6.84	1.71	10.26
3.8	7.27	1.80	10.90
3.9	7.61	1.90	11.41
4.0	8.00	2.00	12.00
4.	8.41	2.10	12.60
4.2	8.82	2.20	13.23
4.3	9.24	2.31	13.86
4.4	9.68	2.42	14.52
4.5	10.12	2.53	15.18
[illegible]	10.58	2.64	15.87
[illegible]	11.04	2.76	16.56
[illegible]	11.52	2.88	17.28
[illegible]	[illegible]	[illegible]	[illegible]

12.50	3.12	18.75
13.01	3.25	19.50
13.52	3.38	20.28
14.04	3.51	21.06
14.58	3.64	21.87
15.12	3.78	22.68
15.68	3.92	23.52
16.24	4.06	24.36
16.82	4.20	25.23
17.40	4.35	26.10
18.00	4.50	27.00
18.61	4.65	27.90
19.22	4.80	28.83
19.84	4.96	29.76
20.48	5.12	30.72
21.12	5.28	31.68
21.78	5.44	32.67
22.44	5.61	33.66
23.12	5.78	34.68
23.70	5.92	35.55
24.50	6.12	36.75
25.20	6.30	37.80
25.92	6.48	38.88
26.64	6.66	39.96
27.38	6.84	41.07
28.12	7.03	42.18
28.88	7.22	43.32

[illegible]	1 : 1	$\frac{1}{4}$: 1	$1\frac{1}{2}$: 1
7.7	29.64	7.41	44.46
7.8	30.42	7.60	45.63
7.9	31.20	7.80	46.80
8.0	32.00	8.00	48.00
8.1	32.80	8.20	49.20
8.2	33.62	8.40	50.43
8.3	34.44	8.61	51.66
8.4	35.28	8.82	52.92
8.5	36.12	9.03	54.18
8.6	36.98	9.24	55.47
8.7	37.84	9.46	56.76
8.8	38.72	9.68	58.08
8.9	39.60	9.90	59.40
9.0	[illegible]	10.12	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]

	1:1	1/4 : 1	1 1/2 : 1
	54.08	13.52	81.12
	55.12	13.78	82.68
	56.18	14.04	84.27
	57.24	14.31	85.86
	58.32	14.58	87.48
	59.40	14.85	89.10
	60.50	15.12	90.75
	61.60	15.40	92.40
	62.72	15.68	94.08
	63.84	15.96	95.76
	64.98	16.24	97.47
	66.12	16.53	99.18
	67.28	16.82	100.92
	68.44	17.11	102.66
	69.62	17.41	104.43
	70.80	17.70	106.20
	72.00	18.00	108.00
	73.20	18.30	109.80
	74.42	18.61	111.63
	75.64	18.91	113.46
	76.88	19.22	115.32
	78.12	19.53	117.18
	79.38	19.84	119.07
	80.63	20.16	120.99
	81.92	20.48	122.98
	82.70	20.67	124.05

*. In this case, the max strain will be at a point 0.634 of the length of the beam from the fixed end.



For AD,

$$EI\frac{d^2y}{dx^2} = P(nl - x - \tfrac{1}{2}n^2/3-n)(l-x)$$

For DB,

$$EI\frac{d^2y}{dx^2} = -\tfrac{1}{2}Pn^2(3-n)(l-x)-($$

In these formulae,

$I = \frac{1}{12}bd^3$ (5·1) for a rectangle

$I = \frac{1}{4}\pi r^4 = \frac{1}{64}\pi d^4$ (5·2) for a circle

E = coef. of elasticity

Cross-section Triangles.

ing	Slope 1 : 1	Slope 1/4 : 1	Slope 1½ : 1
	.005	.001	.007
	.02	.005	.03
3	.04	.012	.067
	.08	.020	.12
	.12	.03	.19
	.18	.05	.27
	.24	.06	.37
	.32	.08	.48
	.40	.10	.61
	.50	.13	.75
	.60	.15	.91
	.72	.18	1.08
	.84	.21	1.27
	.98	.25	1.47
	1.12	.28	1.69
	1.28	.32	1.92
7	1.44	.36	2.17
	1.62	.41	2.43
	1.80	.45	2.71
	2.0	.50	3.00
	2.21	.55	3.31

Tin	1:1	1/4:1	1/2:1
2.3	2.64	.67	3.97
2.4	2.88	.72	4.32
2.5	3.12	.78	4.69
2.6	3.38	.84	5.07
2.7	3.64	.91	5.46
2.8	3.92	.98	5.88
2.9	4.20	1.05	6.30
3.0	4.50	1.12	6.75
3.	4.80	1.20	7.20
3.2	5.12	1.28	7.68
3.3	5.44	1.36	8.16
3.4	5.78	1.44	8.67
3.5	6.12	1.53	9.16
3.6	6.48	1.62	9.72
3.7	6.84	1.71	10.26
3.8	7.27	1.80	10.90
3.9	7.61	1.90	11.41
4.0	8.00	2.00	12.00
4.	8.41	2.10	12.60
4.2	8.82	2.20	13.23
4.3	9.24	2.31	13.86
4.4	9.68	2.42	14.52
4.5	10.12	2.53	15.18
[illegible]	10.58	2.64	15.87
[illegible]	11.04	2.76	16.56
[illegible]	11.52	2.88	17.28
[illegible]	[illegible]	[illegible]	[illegible]

12.50	3.12	18.75
13.01	3.25	19.50
13.52	3.38	20.28
14.04	3.51	21.06
14.58	3.64	21.87
15.12	3.78	22.68
15.68	3.92	23.52
16.24	4.06	24.36
16.82	4.20	25.23
17.40	4.35	26.10
18.00	4.50	27.00
18.61	4.65	27.90
19.22	4.80	28.83
19.84	4.96	29.76
20.48	5.12	30.72
21.12	5.28	31.68
21.78	5.44	32.67
22.44	5.61	33.66
23.12	5.78	34.68
23.70	5.92	35.55
24.50	6.12	36.75
25.20	6.30	37.80
25.92	6.48	38.88
26.64	6.66	39.96
27.38	6.84	41.07
28.12	7.03	42.18
28.88	7.22	43.32

[illegible]	1:1	$\frac{1}{4}$:1	$1\frac{1}{2}$:1
7.7	29.64	7.41	44.46
7.8	30.42	7.60	45.63
7.9	31.20	7.80	46.80
8.0	32.00	8.00	48.00
8.1	32.80	8.20	49.20
8.2	33.62	8.40	50.43
8.3	34.44	8.61	51.66
8.4	35.28	8.82	52.92
8.5	36.12	9.03	54.18
8.6	36.98	9.24	55.47
8.7	37.84	9.46	56.76
8.8	38.72	9.68	58.08
8.9	39.60	9.90	59.40
9.0	40.50	10.12	60.75
9.1	41.40	10.35	62.10
9.2	42.32	10.58	63.48
9.3	43.24	10.81	64.86
9.4	44.18	11.04	66.27
9.5	45.12	11.28	67.78
9.6	46.08	11.52	69.12
9.7	47.04	11.76	70.56
9.8	48.02	12.00	72.03
9.9	49.00	12.25	73.50
10.0	50.00	12.50	75.00
10.1	51.00	12.75	76.50
10.2	52.02	13.00	78.03
10.3	53.04	13.26	79.56

ing	1:1	$\frac{1}{4}$:1	$1\frac{1}{2}$:1
4	54.08	13.52	81.12
5	55.12	13.78	82.68
6	56.18	14.04	84.27
7	57.24	14.31	85.86
8	58.32	14.58	87.48
9	59.40	14.85	89.10
.0	60.50	15.12	90.75
1	61.60	15.40	92.40
2	62.72	15.68	94.08
3	63.84	15.96	95.76
.4	64.98	16.24	97.47
.5	66.12	16.53	99.18
6	67.28	16.82	100.92
7	68.44	17.11	102.66
.8	69.62	17.41	104.43
.9	70.80	17.70	106.20
2.0	72.00	18.00	108.00
.1	73.20	18.30	109.80
.2	74.42	18.61	111.63
2.3	75.64	18.91	113.46
2.4	76.88	19.22	115.32
.5	78.12	19.53	117.18
2.6	79.38	19.84	119.07
2.7	80.63	20.16	120.99
.8	81.92	20.48	122.98
[illegible]	[illegible]	[illegible]	[illegible]

Cutting	1:1	1/4:1	1 1/2:1
13.0	84.50	21.12	126.75
13.1	85.80	21.45	128.70
13.2	87.12	21.78	130.68
13.3	88.44	22.11	132.66
13.4	89.78	22.44	134.67
13.5	91.12	22.78	136.68
13.6	92.48	23.12	138.72
13.7	93.84	23.46	140.76
13.8	95.22	23.80	142.83
13.9	96.60	24.15	144.90
14.0	98.00	24.50	147.00
14.1	99.40	24.85	149.10
14.2	100.82	25.20	151.20
14.3	102.24	25.56	153.36
14.4	103.68	25.92	155.52
14.5	105.12	26.28	157.68
14.6	106.58	26.64	159.87
14.7	108.04	27.01	162.06
14.8	109.52	27.38	164.28
14.9	111.00	27.75	166.50
15.0	112.50	28.12	168.75
15.1	114.00	28.50	171.00
15.2	115.52	28.88	173.28
15.3	117.02	29.25	175.56
15.4	118.58	29.64	177.87
15.5	120.12	30.03	180.18
15.6	121.68	30.42	182.52

123.24	30.81	184.86
124.82	31.20	187.23
126.40	31.60	189.60
128.00	32.00	192.00
129.60	32.40	194.40
131.22	32.80	196.83
132.84	33.21	199.26
134.48	33.62	201.72
136.12	34.03	204.18
137.78	34.45	206.67
139.44	34.86	209.16
141.12	35.28	211.68
142.80	35.70	214.20
144.50	36.12	216.75
146.20	36.55	219.30
147.92	36.98	221.88
149.64	37.41	224.46
151.38	37.85	227.07
153.12	38.28	229.68
154.88	38.72	232.32
156.64	39.16	234.96
158.42	39.60	237.63
160.20	40.05	240.30
162.00	40.50	243.00
163.80	40.95	245.70
165.62	41.40	248.43

Cutting	1:1	1/4:1	1 1/2:1
18.3	167.44	41.86	251.16
18.4	169.28	42.32	253.92
18.5	171.12	42.78	256.68
18.6	172.98	43.24	259.47
18.7	174.84	43.71	262.26
18.8	176.72	44.18	265.08
18.9	178.60	44.65	267.90
19.0	180.50	45.12	270.75
19.1	182.40	45.60	273.60
19.2	184.32	46.08	276.48
19.3	186.24	46.56	279.36
19.4	188.18	47.04	282.27
19.5	190.12	47.53	285.18
19.6	192.08	48.02	288.12
19.7	194.04	48.51	291.06
19.8	196.02	49.00	294.03
19.9	198.00	49.50	297.00
20.0	200.00	50.00	300.00
20.1	202.00	50.50	303.00
20.2	204.02	51.00	306.03
20.3	206.04	51.51	309.06
20.4	208.08	52.02	312.12
20.5	210.12	52.53	315.18
20.6	212.18	53.04	318.27
20.7	214.24	53.56	321.36
20.8	216.32	54.08	324.48

1:1	1/4:1	1 1/2:1
218.40	54.60	327.60
220.50	55.12	330.75
222.60	55.65	333.90
224.72	56.18	337.08
226.84	56.71	340.26
228.98	57.24	343.47
231.12	57.78	346.68
233.28	58.32	349.92
235.44	58.86	353.16
237.62	59.40	356.43
239.80	59.95	359.70
242.00	60.50	363.00
244.20	61.05	366.30
246.42	61.60	369.63
248.64	62.13	372.96
250.88	62.72	376.32
253.12	63.28	379.68
255.38	63.84	383.07
257.64	64.41	386.46
259.92	64.98	389.88
262.20	65.55	393.30
264.50	66.12	396.75
266.80	66.70	400.20
269.12	67.28	403.68
271.44	67.86	407.16
273.78	68.44	410.67
276.12	69.03	414.18

Cutting	1:1	1/4:1	1 1/2:1
23.6	278.48	69.62	417.72
23.7	280.84	70.21	421.26
23.8	283.22	70.80	424.83
23.9	285.60	71.40	428.40
24.0	288.00	72.00	432.00
24.1	290.40	72.60	435.60
24.2	292.82	73.20	439.23
24.3	295.24	73.81	442.86
24.4	297.68	74.42	446.52
24.5	300.12	75.03	450.18
24.6	302.58	75.64	453.87
24.7	305.04	76.26	457.56
24.8	307.52	76.88	461.28
24.9	310.00	77.50	465.00
25.0	312.50	78.12	468.75
25.1	315.00	78.75	472.50
25.2	317.52	79.38	476.28
25.3	320.04	80.01	480.60
25.4	322.58	80.64	483.87
25.5	325.12	81.28	487.68
25.6	327.68	81.92	491.52
25.7	330.24	82.56	495.36
25.8	332.82	83.20	499.23
25.9	335.40	83.85	503.10
26.0	338.00	84.50	507.00
26.1	340.60	85.15	510.90
26.2	343.22	85.81	514.83

ing	1:1	1/4:1	1 1/2:1
3	345.84	86.46	518.76
4	348.48	87.12	522.72
5	351.12	87.78	526.68
6	353.78	88.45	530.67
7	356.44	89.11	534.66
8	359.12	89.78	538.68
9	361.80	90.45	542.70
0	364.50	91.15	546.75
1	367.20	91.80	550.80
2	369.92	92.48	554.88
3	372.64	93.16	558.96
4	375.38	93.85	563.07
5	378.12	94.53	567.18
6	380.88	95.22	571.32
7	383.64	95.91	575.46
8	386.42	96.57	579.63
9	389.20	97.30	583.80
0	392.00	98.00	588.00
1	394.80	98.70	592.20
2	397.62	99.41	596.43
3	400.44	100.11	600.66
4	403.28	100.82	604.92
5	406.12	101.53	609.18
6	408.98	102.25	613.47
7	411.84	102.96	617.76
8	[illegible]	[illegible]	[illegible]

Cutting	1 : 1	1/4 : 1	1 1/2 : 1
29.0	420.50	105.12	630.75
29.1	423.40	105.85	635.10
29.2	426.32	106.58	639.48
29.3	429.24	107.31	643.86
29.4	432.18	108.05	648.27
29.5	435.12	108.78	652.68
29.6	438.08	109.52	657.12
29.7	441.04	110.26	661.56
29.8	444.02	111.01	666.03
29.9	447.01	111.75	670.51
30.0	450.00	112.50	675.00
30.1	453.01	113.25	679.51
30.2	456.02	114.01	684.03
30.3	459.04	114.76	688.57
30.4	462.08	115.52	693.12
30.5	465.12	116.28	697.69
30.6	468.18	117.05	702.27
30.7	471.24	117.81	706.87
30.8	474.32	118.58	711.48
30.9	477.40	119.35	716.1[illegible]
31.0	480.50	120.12	720.75
31.1	483.60	120.90	725.41
31.2	486.72	121.68	730.08
31.3	489.84	122.46	734.77
31.4	492.98	123.24	739.47
31.5	496.12	124.03	744.19
[illegible]	[illegible]	[illegible]	[illegible]

1:1	1/4:1	1 1/2:1
502.44	125.61	753.67
505.62	126.40	758.43
508.80	127.20	763.21
512.00	128.00	768.00
515.20	128.80	772.81
518.42	129.61	777.63
521.64	130.41	782.48
524.88	131.22	787.32
528.12	132.03	792.19
531.38	132.84	797.07
534.64	133.66	801.97
537.92	134.48	806.88
541.20	135.30	811.81
544.50	136.12	816.75
547.80	136.95	821.71
551.12	137.78	826.68
554.44	138.61	831.67
557.78	139.44	836.67
561.12	140.28	841.69
564.48	141.12	846.72
567.84	141.96	851.77
571.22	142.81	856.83
574.60	143.65	861.91
578.00	144.50	867.00
581.40	145.35	872.11
584.82	146.21	877.23
588.24	147.06	882.37

Cutting	1:1	1/4:1	1 1/2:1
34.4	591.68	147.92	887.52
34.5	595.12	148.78	892.69
34.6	598.58	149.64	897.87
34.7	602.04	150.51	903.07
34.8	605.52	151.38	908.28
34.9	609.00	152.25	913.51
35.0	612.50	153.12	918.75
35.1	616.00	154.00	924.01
35.2	619.52	154.88	929.28
35.3	623.04	155.76	934.57
35.4	626.58	156.64	939.87
35.5	630.12	157.53	945.19
35.6	633.68	158.44	950.52
35.7	637.24	159.31	955.87
35.8	640.82	160.20	961.23
35.9	644.40	161.10	966.61
36.0	648.00	162.00	972.00
36.1	651.60		977.41
36.2	655.22		982.83
36.3	658.84		988.27
36.4	662.48		993.72
36.5	666.12		999.19
36.6	669.78		1004.67
36.7	673.44		1010.17
36.8	677.12		1015.68
36.9	680.80		1021.20
37.0	684.50		1026.75

...tting	1:1	1/4 : 1	1 1/2 : 1
37.1	688.20		1032.31
37.2	691.92		1037.88
37.3	695.64		1043.47
37.4	699.38		1049.07
37.5	703.12		1054.69
37.6	706.88		1060.32
37.7	710.64		1066.97
37.8	714.42		1071.63
37.9	718.20		1077.31
38.0	722.00		1083.00
38.1	725.80		1088.71
38.2	729.62		1094.43
38.3	733.45		1100.17
38.4	737.28		1105.92
38.5	741.12		1111.69
38.6	744.98		1117.47
38.7	748.84		1123.27
38.8	752.72		1129.08
38.9	756.60		1134.91
39.0	760.50		1140.75
39.1	764.40		1146.61
39.2	768.32		1152.48
39.3	772.24		1158.37
39.4	776.18		1164.27
39.5	780.12		1170.19
39.6	784.08		1176.12
39.7	788.04		1182.07

Cutting	1:1	1/4:1	1 1/2:1
39.8	792.02		1188.03
39.9	796.00		1194.01
40.0	800.00		1200.00
40.1	804.00		1206.01
40.2	808.02		1212.03
40.3	812.04		1218.07
40.4	816.08		1224.12
40.5	820.12		1230.19
40.6	824.18		1236.27
40.7	828.24		1242.37
40.8	832.32		1248.48
40.9	836.40		1254.61
41.0	840.50		1260.75
41.1	844.60		1266.91
41.2	848.72		1273.08
41.3	852.84		1279.27
41.4	856.98		1285.47
41.5	861.12		1291.69
41.6	865.28		1297.92
41.7	869.44		1304.17
41.8	873.62		1310.43
41.9	877.80		1316.71
42.0	882.00		1323.00
42.1	886.20		1329.32
42.2	890.42		1335.63
42.3	894.64		1341.96
42.4	898.88		1348.32

Cutting	1:1	1¼:1	1½:1
45.1			1525.51
45.2			1532.28
45.3			1539.07
45.4			1545.87
45.5			1552.69
45.6			1559.52
45.7			1566.37
45.8			1573.23
45.9			1580.11
46.0			1587.00
46.1			1593.91
46.2	1081.26		1600.83
46.3			1607.77
46.4			1614.72
46.5			1621.69
46.6			1628.67
46.7			1635.67
46.8			1642.68
46.9			1649.75
47.0			1656.85
47.1			1663.88
47.2			1670.88
47.3			1677.97
47.4			1685.07
47.5			1692.19
47.6			1699.32

t	1:1	1¼:1	1½:1
47			1702.49
47.8			1718.83
47.9			1720.81
48.0			1728.00
48.1			1735.21
48.2			1742.43
48.3			1749.67
48.4			1756.92
48.5			1764.19
48.6			1771.47
48.7			1778.77
48.8			1786.08
48.9			1793.41
49.0			1800.75
49.1			1808.11
49.2			1815.48
49.3			1822.87
49.4			1830.27
49.5			1837.69
49.6			1845.12
49.7			1852.57
49.8			1860.03
49.9			1867.51
50.0			1875.01

Cutting	1:1	1/4:1	1 1/2:1
45.1			1525.51
45.2			1532.28
45.3			1539.07
45.4			1545.87
45.5			1552.69
45.6	- - - -	- -	1559.52
45.7			1566.37
45.8			1573.23
45.9			1580.11
46.0			1587.00
46.1			1593.91
46.2	1081.26		1600.83
46.3			1607.77
46.4			1614.72
46.5			1621.69
46.6			1628.67
46.7			1635.67
46.8			1642.68
46.9			1649.75
47.0			1656.85
47.1			1663.88
47.2			1670.88
47.3			1677.97
47.4			1685.07
47.5			1692.39
47.6			1699.32

but	1 : 1	1¼ : 1	1½ : 1
47.7			1702.49
47.8			1718.83
47.9			1720.81
48.0			1728.00
48.1			1735.21
48.2			1742.43
48.3			1749.67
48.4			1756.92
48.5			1764.19
48.6			1771.47
48.7			1778.77
48.8			1786.08
48.9			1793.41
49.0			1800.75
49.1			1808.11
49.2			1815.48
49.3			1822.87
49.4			1830.27
49.5			1837.69
49.6			1845.12
49.7			1852.57
49.8			1860.03
49.9			1867.51

Cutting	1:1	$\frac{1}{4}$:1	$1\frac{1}{2}$:1
2.3	2.64	.67	3.97
2.4	2.88	.72	4.32
2.5	3.12	.78	4.69
2.6	3.38	.84	5.07
2.7	3.64	.91	5.46
2.8	3.92	.98	5.88
2.9	4.20	1.05	6.30
3.0	4.50	1.12	6.75
3.1	4.80	1.20	7.20
3.2	5.12	1.28	7.68
3.3	5.44	1.36	8.16
3.4	5.78	1.44	8.67
3.5	6.12	1.53	9.16
3.6	6.48	1.62	9.72
3.7	6.84	1.71	10.26
3.8	7.27	1.80	10.90
3.9	7.61	1.90	11.41
4.0	8.00	2.00	12.00
4.1	8.41	2.10	12.60
4.2	8.82	2.20	13.23
4.3	9.24	2.31	13.86
4.4	9.68	2.42	14.52
4.5	10.12	2.53	15.18
[illegible]	10.58	2.64	15.87
[illegible]	11.04	2.76	16.56
[illegible]	11.52	2.88	17.28
[illegible]	[illegible]	3.00	18.00

ting	1:1	1/4:1	1 1/2:1
7	502.44	125.61	753.67
8	505.62	126.40	758.43
9	508.80	127.20	763.21
.0	512.00	128.00	768.00
1	515.20	128.80	772.81
2	518.42	129.61	777.63
3	521.64	130.41	782.48
.4	524.88	131.22	787.32
5	528.12	132.03	792.19
.6	531.38	132.84	797.07
7	534.64	133.66	801.97
8	537.92	134.48	806.88
.9	541.20	135.30	811.81
.0	544.50	136.12	816.75
1	547.80	136.95	821.71
2	551.12	137.78	826.68
.3	554.44	138.61	831.67
.4	557.78	139.44	836.67
.5	561.12	140.28	841.69
.6	564.48	141.12	846.72
.7	567.84	141.96	851.77
.8	571.22	142.81	856.83
.9	574.60	143.65	861.91
.0	578.00	144.50	867.00
1	581.40	145.35	872.11
2	584.82	146.21	877.23
3	588.24	147.06	882.37

Cutting	1:1	1/4:1	1 1/2:1
13.0	84.50	21.12	126.75
13.1	85.80	21.45	128.70
13.2	87.12	21.78	130.68
13.3	88.44	22.11	132.66
13.4	89.78	22.44	134.67
13.5	91.12	22.78	136.68
13.6	92.48	23.12	138.72
13.7	93.84	23.46	140.76
13.8	95.22	23.80	142.83
13.9	96.60	24.15	144.90
14.0	98.00	24.50	147.00
14.1	99.40	24.85	149.10
14.2	100.82	25.20	151.20
14.3	102.24	25.56	153.36
14.4	103.68	25.92	155.52
14.5	105.12	26.28	157.68
14.6	106.58	26.64	159.87
14.7	108.04	27.01	162.06
14.8	109.52	27.38	164.28
14.9	111.00	27.75	166.50
15.0	112.50	28.12	168.75
15.1	114.00	28.50	171.00
15.2	115.52	28.88	173.28
15.3	117.02	29.25	175.56
15.4	118.58	29.64	177.87
15.5	120.12	30.03	180.18
15.6	121.68	30.42	182.52

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	123.24	30.81	184.86
[illegible]	124.82	31.20	187.23
[illegible]	126.40	31.60	189.60
0	128.00	32.00	192.00
[illegible]	129.60	32.40	194.40
2	131.22	32.80	196.83
3	132.84	33.21	199.26
4	134.48	33.62	201.72
5	136.12	34.03	204.18
6	137.78	34.45	206.67
7	139.44	34.86	209.16
8	141.12	35.28	211.68
[illegible]	142.80	35.70	214.20
0	144.50	36.12	216.75
[illegible]	146.20	36.55	219.30
2	147.92	36.98	221.88
3	149.64	37.41	224.46
[illegible]	151.38	37.85	227.07
5	153.12	38.28	229.68
[illegible]	154.88	38.72	232.32
7	156.64	39.16	234.96
8	158.42	39.60	237.63
[illegible]	160.20	40.05	240.30
0	162.00	40.50	243.00
[illegible]	163.80	40.95	245.70
[illegible]	165.62	41.40	248.43

Cutting	1:1	1/4:1	1 1/2:1
18.3	167.44	41.86	251.16
18.4	169.28	42.32	253.92
18.5	171.12	42.78	256.68
18.6	172.98	43.24	259.47
18.7	174.84	43.71	262.26
18.8	176.72	44.18	265.08
18.9	178.60	44.65	267.90
19.0	180.50	45.12	270.75
19.1	182.40	45.60	273.60
19.2	184.32	46.08	276.48
19.3	186.24	46.56	279.36
19.4	188.18	47.04	282.27
19.5	190.12	47.53	285.18
19.6	192.08	48.02	288.12
19.7	194.04	48.51	291.06
19.8	196.02	49.00	294.03
19.9	198.00	49.50	297.00
20.0	200.00	50.00	300.00
20.1	202.00	50.50	303.00
20.2	204.02	51.00	306.03
20.3	206.04	51.51	309.06
20.4	208.08	52.02	312.12
20.5	210.12	52.53	315.18
20.6	212.18	53.04	318.27
20.7	214.24	53.56	321.36
20.8	216.32	54.08	324.48

1:1	1/4:1	1 1/2:1
218.40	54.60	327.60
220.50	55.12	330.75
222.60	55.65	333.90
224.72	56.18	337.08
226.84	56.71	340.26
228.98	57.24	343.47
231.12	57.78	346.68
233.28	58.32	349.92
235.44	58.86	353.16
237.62	59.40	356.43
239.80	59.95	359.70
242.00	60.50	363.00
244.20	61.05	366.30
246.42	61.60	369.63
248.64	62.13	372.96
250.88	62.72	376.32
253.12	63.28	379.68
255.38	63.84	383.07
257.64	64.41	386.46
259.92	64.98	389.88
262.20	65.55	393.30
264.50	66.12	396.75
266.80	66.70	400.20
269.12	67.28	403.68
271.44	67.86	407.16
273.78	68.44	410.67
276.12	69.03	414.18

Cutting	1:1	¼:1	1½:1
23.6	278.48	69.62	417.72
23.7	280.84	70.21	421.26
23.8	283.22	70.80	424.83
23.9	285.60	71.40	428.40
24.0	288.00	72.00	432.00
24.1	290.40	72.60	435.60
24.2	292.82	73.20	439.23
24.3	295.24	73.81	442.86
24.4	297.68	74.42	446.52
24.5	300.12	75.03	450.18
24.6	302.58	75.64	453.87
24.7	305.04	76.26	457.56
24.8	307.52	76.88	461.28
24.9	310.00	77.50	465.00
25.0	312.50	78.12	468.75
25.1	315.00	78.75	472.50
25.2	317.52	79.38	476.28
25.3	320.04	80.01	480.60
25.4	322.58	80.64	483.87
25.5	325.12	81.28	487.68
25.6	327.68	81.92	491.52
25.7	330.24	82.56	495.36
25.8	332.82	83.20	499.23
25.9	335.40	83.85	503.10
26.0	338.00	84.50	507.00
26.1	340.60	85.15	510.90
26.2	343.22	85.81	514.83

Cutting	1:1	1/4:1	1 1/2:1
26.3	345.84	86.46	518.76
26.4	348.48	87.12	522.72
26.5	351.12	87.78	526.68
26.6	353.78	88.45	530.67
26.7	356.44	89.11	534.66
26.8	359.12	89.78	538.68
26.9	361.80	90.45	542.70
27.0	364.50	91.15	546.75
27.1	367.20	91.80	550.80
27.2	369.92	92.48	554.88
27.3	372.64	93.16	558.96
27.4	375.38	93.85	563.07
27.5	378.12	94.53	567.18
27.6	380.88	95.22	571.32
27.7	383.64	95.91	575.46
27.8	386.42	96.57	579.63
27.9	389.20	97.30	583.80
28.0	392.00	98.00	588.00
28.1	394.80	98.70	592.20
28.2	397.62	99.41	596.43
28.3	400.44	100.11	600.66
28.4	403.28	100.82	604.92
28.5	406.12	101.53	609.18
28.6	408.98	102.25	613.47
28.7	411.84	102.96	617.76
28.8	414.72	103.68	622.08
28.9	[illegible]	[illegible]	[illegible]

Cutting	1:1	1/4:1	1 1/2:1
29.0	420.50	105.12	630.75
29.1	423.40	105.85	635.10
29.2	426.32	106.58	639.48
29.3	429.24	107.31	643.86
29.4	432.18	108.05	648.27
29.5	435.12	108.78	652.68
29.6	438.08	109.52	657.12
29.7	441.04	110.26	661.56
29.8	444.02	111.01	666.03
29.9	447.01	111.75	670.51
30.0	450.00	112.50	675.00
30.1	453.01	113.25	679.51
30.2	456.02	114.01	684.03
30.3	459.04	114.76	688.57
30.4	462.08	115.52	693.12
30.5	465.12	116.28	697.69
30.6	468.18	117.05	702.27
30.7	471.24	117.81	706.87
30.8	474.32	118.58	711.48
30.9	477.40	119.35	716.11
31.0	480.50	120.12	720.75
31.1	483.60	120.90	725.41
31.2	486.72	121.68	730.08
31.3	489.84	122.46	734.77
31.4	492.98	123.24	739.47
31.5	496.12	124.03	744.19

Ting	1:1	1/4:1	1 1/2:1
7	502.44	125.61	753.67
.8	505.62	126.40	758.43
.9	508.80	127.20	763.21
2.0	512.00	128.00	768.00
.1	515.20	128.80	772.81
.2	518.42	129.61	777.63
.3	521.64	130.41	782.48
.4	524.88	131.22	787.32
.5	528.12	132.03	792.19
2.6	531.38	132.84	797.07
.7	534.64	133.66	801.97
.8	537.92	134.48	806.88
.9	541.20	135.30	811.81
3.0	544.50	136.12	816.75
.1	547.80	136.95	821.71
.2	551.12	137.78	826.68
.3	554.44	138.61	831.67
3.4	557.78	139.44	836.67
.5	561.12	140.28	841.69
.6	564.48	141.12	846.72
.7	567.84	141.96	851.77
.8	571.22	142.81	856.83
3.9	574.60	143.65	861.91
4.0	578.00	144.50	867.00
.1	581.40	145.35	872.11
.2	584.82	146.21	877.23

Cutting	1:1	1/4:1	1 1/2:1
34.4	591.68	147.92	887.52
34.5	595.12	148.78	892.69
34.6	598.58	149.64	897.87
34.7	602.04	150.51	903.07
34.8	605.52	151.38	908.28
34.9	609.00	152.25	913.51
35.0	612.50	153.12	918.75
35.1	616.00	154.00	924.01
35.2	619.52	154.88	929.28
35.3	623.04	155.76	934.57
35.4	626.58	156.64	939.87
35.5	630.12	157.53	945.19
35.6	633.68	158.44	950.52
35.7	637.24	159.31	955.87
35.8	640.82	160.20	961.23
35.9	644.40	161.10	966.61
36.0	648.00	162.00	972.00
36.1	651.60		977.41
36.2	655.22		982.83
36.3	658.84		988.27
36.4	662.48		993.72
36.5	666.12		999.19
36.6	669.78		1004.67
36.7	673.44		1010.17
36.8	677.12		1015.68
36.9	680.80		1021.21

ing	1:1	1¼:1	1½:1
1	688.20		1032.31
2	691.92		1037.88
3	695.64		1043.47
4	699.38		1049.07
2.5	703.12		1054.69
2.6	706.88		1060.32
2.7	710.64		1066.97
2.8	714.42		1071.63
2.9	718.20		1077.31
3.0	722.00		1083.00
3.1	725.80		1088.71
3.2	729.62		1094.43
3.3	733.45		1100.17
3.4	737.28		1105.92
3.5	741.12		1111.69
3.6	744.98		1117.47
3.7	748.84		1123.27
3.8	752.72		1129.08
3.9	756.60		1134.91
4.0	760.50		1140.75
4.1	764.40		1146.61
4.2	768.32		1152.48
4.3	772.24		1158.37
4.4	776.18		1164.27
4.5	780.12		1170.19
4.6	784.08		1176.12
4.7	788.04		1182.07

Cutting	1:1	1/4:1	1 1/2:1
39.8	792.02		1188.03
39.9	796.00		1194.01
40.0	800.00		1200.00
40.1	804.00		1206.01
40.2	808.02		1212.03
40.3	812.04		1218.07
40.4	816.08		1224.12
40.5	820.12		1230.19
40.6	824.18		1236.27
40.7	828.24		1242.37
40.8	832.32		1248.48
40.9	836.40		1254.61
41.0	840.50		1260.75
41.1	844.60		1266.91
41.2	848.72		1273.08
41.3	852.84		1279.27
41.4	856.98		1285.47
41.5	861.12		1291.69
41.6	865.28		1297.92
41.7	869.44		1304.17
41.8	873.62		1310.43
41.9	877.80		1316.71
42.0	882.00		1323.00
42.1	886.20		1329.32
42.2	890.42		1335.63
42.3	894.64		1341.96
42.4	898.88		1348.32

	1:1	1/4:1	1 1/2:1
9	903.12		1354.69
	907.38		1361.07
	911.64		1367.47
	915.92		1373.88
	920.20		1780.31
0	924.50		1386.75
1	928.80		1393.22
2	933.12		1399.68
3	937.44		1406.17
.4	941.78		1412.67
3.5	946.12		1419.17
.6	950.48		1425.72
3.7			1432.27
3.8			1438.83
3.9			1445.41
4.0			1452.00
4.1			1458.61
4.2			1465.23
4.3			1471.87
4.4			1478.52
4.5			1485.19
4.6			1491.87
4.7			1498.57
4.8			1505.28
4.9			1512.01
5.0			1518.75

[illegible]	[illegible]	[illegible]	[illegible]
45.1			1525.61
45.2			1532.28
45.3			1539.07
45.4			1545.87
45.5			1552.69
45.6	- - - -	- -	1559.52
45.7			1566.37
45.8			1573.23
45.9			1580.11
46.0			1587.00
46.1			1593.91
46.2	1081.26		1600.83
46.3			1607.77
46.4			1614.72
46.5			1621.69
46.6			1628.67
46.7			1635.67
46.8			1642.68
46.9			1649.75
47.0			1656.85
47.1			1663.88
47.2			1670.88
47.3			1677.97
47.4			1685.07
.5			1692.39
6			1699.32

t	1 : 1	1¼ : 1	1½ : 1
47.7			1700.47
47.8			1718.83
47.9			1720.81
48.0			1728.00
48.1			1735.21
48.2			1742.43
48.3			1749.67
48.4			1756.92
48.5			1764.19
48.6			1771.47
48.7			1778.77
48.8			1786.08
48.9			1793.41
49.0			1800.75
49.1			1808.11
49.2			1815.48
49.3			1822.87
49.4			1830.27
49.5			1837.69
49.6			1845.12
49.7			1852.57
49.8			1860.03
49.9			1867.51
50.0			1875.00